Consider that **BST<E>** is a class that represents a **generic binary search tree** and **Node<E>** is a nested class that represents a **node of that tree**, as shown below.

```
public class BST<E extends Comparable<E>> implements BSTInterface<E>
    protected Node<E> root = null;      // root of the tree
...
//----------------------------------------------------------------
protected static class Node<E> {
  private E element;                 // an element stored at this node
  private Node<E> left;            // a reference to the left child (if any)
  private Node<E> right;          // a reference to the right child (if any)
...
}
//----------------------------------------------------------------
}
```

1. Implement the class TREE by inheriting from the BST, and add to it the following methods:

    a) `public boolean contains(E element)` that verifies if an element is in the tree.

    b) `public boolean isLeaf(E element)` that verifies if an element is in a leaf.

    c) `public Iterable<E> ascdes()` - returns an iterable list with the elements of the left subtree in increasing order and the elements of the right subtree in descending order. According to the figure, the result is: 60, 65, 70, 75, 80, 95, 90, 85.
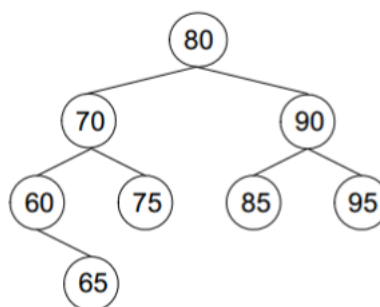

    d) `public BST<E> autumnTree()` – returns a new binary search tree, identical to the original, but without the leaves.

    e) `public int[] numNodesByLevel()` – returns an array with the number of nodes in each level of the tree, position 0 number of nodes at level 0, position 1 number of nodes at level 1, …

    f) Make a function that indicates whether a TREE is a perfectly balanced tree, i.e., a tree in which all nodes have balance factor zero.

    g) Complete the test classe to test the perfectBalanced method.

2. Add to the TREE<E> class a method that calculates the lowest common ancestor between two nodes in a binary search tree. For example in the tree below the lowest common ancestor between nodes 8 and 60 is node 54, the lowest common ancestor between nodes 19 and 80 is node 65.
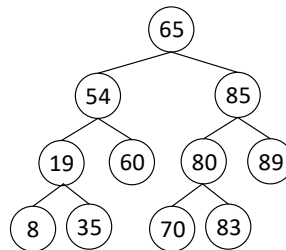
3. Add to the TREE<E> class a method that returns the minimum complete subtree that contains two nodes passed as a parameter. For example in the tree shown below, given nodes 1 and 15, the returned tree must be {13, 11, 3, 1, 17, 15, 20} .
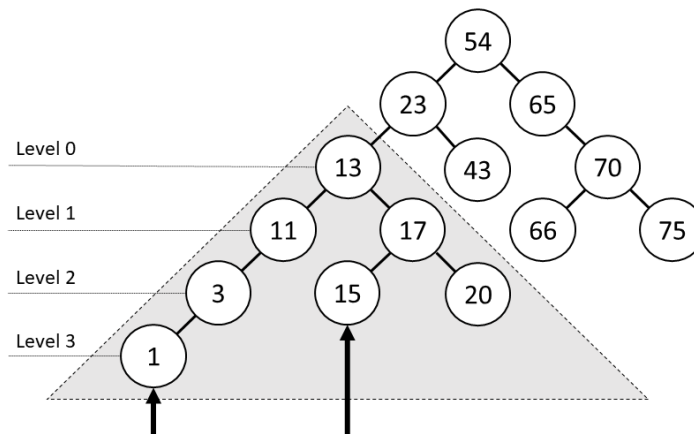
Complementary Exercises

1. Insert the following sequence of nodes: A, Z, B, C, X, D, Q, E, V, M, Q in AVL tree and draw the tree after each insertion, showing the rotations made.

2. Remove B, A, F, P, C elements and draw the state of the tree after each deletion, with the rotations made.