2

# Regular Expressions, Text Normalization, Edit Distance

User: I am unhappy.

ELIZA: DO YOU THINK COMING HERE WILL HELP YOU NOT TO BE UNHAPPY

User: I need some help, that much seems certain.
ELIZA: WHAT WOULD IT MEAN TO YOU IF YOU GOT SOME HELP
User: Perhaps I could learn to get along with my mother.

ELIZA: TELL ME MORE ABOUT YOUR FAMILY User: My mother takes care of me.

ELIZA: WHO ELSE IN YOU FAMILY TAKES CARE OF YOU

User: My father. ELIZA: YOUR FATHER

User: You are like my father in some ways.

Weizenbaum (1966)

**ELIZA** 

The dialogue above is from **ELIZA**, an early natural language processing system that could carry on a limited conversation with a user by imitating the responses of a Rogerian psychotherapist (Weizenbaum, 1966). ELIZA is a surprisingly simple program that uses pattern matching to recognize phrases like "I need X" and translate them into suitable outputs like "What would it mean to you if you got X?". This simple technique succeeds in this domain because ELIZA doesn't actually need to *know* anything to mimic a Rogerian psychotherapist. As Weizenbaum notes, this is one of the few dialogue genres where listeners can act as if they know nothing of the world. ELIZA's mimicry of human conversation was remarkably successful: many people who interacted with ELIZA came to believe that it really *understood* them and their problems, many continued to believe in ELIZA's abilities even after the program's operation was explained to them (Weizenbaum, 1976), and even today such **chatbots** are a fun diversion.

chatbots

Of course modern conversational agents are much more than a diversion; they can answer questions, book flights, or find restaurants, functions for which they rely on a much more sophisticated understanding of the user's intent, as we will see in Chapter 15. Nonetheless, the simple pattern-based methods that powered ELIZA and other chatbots play a crucial role in natural language processing.

We'll begin with the most important tool for describing text patterns: the **regular expression**. Regular expressions can be used to specify strings we might want to extract from a document, from transforming "I need X" in ELIZA above, to defining strings like \$199 or \$24.99 for extracting tables of prices from a document.

normalization

tokenization

We'll then turn to a set of tasks collectively called **text normalization**, in which regular expressions play an important part. Normalizing text means converting it to a more convenient, standard form. For example, most of what we are going to do with language relies on first separating out or **tokenizing** words from running text, the task of **tokenization**. English words are often separated from each other by whitespace, but whitespace is not always sufficient. *New York* and *rock 'n' roll* are sometimes treated as large words despite the fact that they contain spaces, while sometimes we'll need to separate *I'm* into the two words *I* and *am*. For processing tweets or texts we'll need to tokenize **emotions** like:) or **hashtags** like #nlproc.

lemmatization

Some languages, like Japanese, don't have spaces between words, so word tokenization becomes more difficult.

stemming

sentence segmentation Another part of text normalization is **lemmatization**, the task of determining that two words have the same root, despite their surface differences. For example, the words *sang*, *sung*, and *sings* are forms of the verb *sing*. The word *sing* is the common *lemma* of these words, and a **lemmatizer** maps from all of these to *sing*. Lemmatization is essential for processing morphologically complex languages like Arabic. **Stemming** refers to a simpler version of lemmatization in which we mainly just strip suffixes from the end of the word. Text normalization also includes **sentence segmentation**: breaking up a text into individual sentences, using cues like periods or exclamation points.

Finally, we'll need to compare words and other strings. We'll introduce a metric called **edit distance** that measures how similar two strings are based on the number of edits (insertions, deletions, substitutions) it takes to change one string into the other. Edit distance is an algorithm with applications throughout language processing, from spelling correction to speech recognition to coreference resolution.

# 2.1 Regular Expressions

regular expression

corpus

One of the unsung successes in standardization in computer science has been the **regular expression** (often shortened to **regex**), a language for specifying text search strings. This practical language is used in every computer language, word processor, and text processing tools like the Unix tools grep or Emacs. Formally, a regular expression is an algebraic notation for characterizing a set of strings. Regular expressions are particularly useful for searching in texts, when we have a **pattern** to search for and a **corpus** of texts to search through. A regular expression search function will search through the corpus, returning all texts that match the pattern. The corpus can be a single document or a collection. For example, the Unix command-line tool grep takes a regular expression and returns every line of the input document that matches the expression.

A search can be designed to return every match on a line, if there are more than one, or just the first match. In the following examples we generally underline the exact part of the pattern that matches the regular expression and show only the first match. We'll show regular expressions delimited by slashes but note that slashes are *not* part of the regular expressions.

Regular expressions come in many variants. We'll be describing **extended regular expressions**; different regular expression parsers may only recognize subsets of these, or treat some expressions slightly differently. Using an online regular expression tester is a handy way to test out your expressions and explore these variations.

### 2.1.1 Basic Regular Expression Patterns

concatenation

The simplest kind of regular expression is a sequence of simple characters; putting characters in sequence is called **concatenation**. To search for *woodchuck*, we type /woodchuck/. The expression /Buttercup/ matches any string containing the substring *Buttercup*; grep with that expression would return the line *I'm called little Buttercup*. The search string can consist of a single character (like /!/) or a sequence of characters (like /urgl/) (see Fig. 2.1).

Regular expressions are case sensitive; lower case /s/ is distinct from upper

Regex	Example Patterns Matched	
/woodchucks/	"interesting links to woodchucks and lemurs"	
/a/	"Mary Ann stopped by Mona's"	
/!/	"You've left the burglar behind again!" said Nori	

case /S/ (/s/ matches a lower case s but not an upper case S). This means that the pattern /woodchucks/ will not match the string Woodchucks. We can solve this problem with the use of the square braces [ and ]. The string of characters inside the braces specifies a **disjunction** of characters to match. For example, Fig. 2.2 shows that the pattern /[wW]/ matches patterns containing either w or W.

Regex	Match	Example Patterns
/[wW]oodchuck/	Woodchuck or woodchuck	"Woodchuck"
/[abc]/	'a', 'b', <i>or</i> 'c'	"In uomini, in sold <u>a</u> ti"
/[1234567890]/	any digit	"plenty of <u>7</u> to 5"

Figure 2.2 The use of the brackets [] to specify a disjunction of characters.

The regular expression /[1234567890]/ specifies any single digit. While such classes of characters as digits or letters are important building blocks in expressions, they can get awkward (e.g., it's inconvenient to specify

```
/[ABCDEFGHIJKLMNOPQRSTUVWXYZ]/
```

Some simple regex searches.

to mean "any capital letter"). In cases where there is a well-defined sequence associated with a set of characters, the brackets can be used with the dash (-) to specify any one character in a **range**. The pattern /[2-5]/ specifies any one of the characters 2, 3, 4, or 5. The pattern /[b-g]/ specifies one of the characters b, c, d, e, f, or g. Some other examples are shown in Fig. 2.3.

Regex	Match	Example Patterns Matched
/[A-Z]/	an upper case letter	"we should call it ' <u>D</u> renched Blossoms'
/[a-z]/	a lower case letter	"my beans were impatient to be hoed!"
/[0-9]/	a single digit	"Chapter 1: Down the Rabbit Hole"

**Figure 2.3** The use of the brackets [] plus the dash - to specify a range.

The square braces can also be used to specify what a single character *cannot* be, by use of the caret  $\hat{}$ . If the caret  $\hat{}$  is the first symbol after the open square brace [, the resulting pattern is negated. For example, the pattern  $/[\hat{}a]/$  matches any single character (including special characters) except a. This is only true when the caret is the first symbol after the open square brace. If it occurs anywhere else, it usually stands for a caret; Fig. 2.4 shows some examples.

/ F^ A 77 /	
/[^A-Z]/ not an upper case letter "Oyfn pripetchik"	
/[^Ss]/ neither 'S' nor 's' "I have no exquisite reason for't	,,
/[^.]/ not a period "our resident Djinn"	
/[e^]/ either 'e' or '^' "look up _ now"	
$/a^b/$ the pattern 'a^b' "look up $\underline{a^b}$ now"	

Figure 2.4 The caret ^ for negation or just to mean ^. See below re: the backslash for escaping the period.

How can we talk about optional elements, like an optional s in woodchuck and woodchucks? We can't use the square brackets, because while they allow us to say

range

Figure 2.1

CHAPTER 2 • REGULAR EXPRESSIONS, TEXT NORMALIZATION, EDIT DISTANCE

"s or S", they don't allow us to say "s or nothing". For this we use the question mark /?/, which means "the preceding character or nothing", as shown in Fig. 2.5.

Regex	Match	Example Patterns Matched
/woodchucks?/	woodchuck or woodchucks	"woodchuck"
/colou?r/	color or colour	" <u>color</u> "

Figure 2.5 The question mark? marks optionality of the previous expression.

We can think of the question mark as meaning "zero or one instances of the previous character". That is, it's a way of specifying how many of something that we want, something that is very important in regular expressions. For example, consider the language of certain sheep, which consists of strings that look like the following:

baa!

baaa!

baaaa!

baaaaa!

. . .

This language consists of strings with a b, followed by at least two a's, followed by an exclamation point. The set of operators that allows us to say things like "some number of as" are based on the asterisk or \*, commonly called the **Kleene** \* (generally pronounced "cleany star"). The Kleene star means "zero or more occurrences of the immediately previous character or regular expression". So /a\*/ means "any string of zero or more as". This will match a or aaaaaaa, but it will also match the empty string at the start of Off Minor since the string Off Minor starts with zero a's. So the regular expression for matching one or more a is /aa\*/, meaning one a followed by zero or more as. More complex patterns can also be repeated. So /[ab]\*/ means "zero or more a's or b's" (not "zero or more right square braces"). This will match strings like aaaa or ababab or bbbb.

For specifying multiple digits (useful for finding prices) we can extend /[0-9]/, the regular expression for a single digit. An integer (a string of digits) is thus  $/[0-9][0-9]^*/$ . (Why isn't it just  $/[0-9]^*/$ ?)

Sometimes it's annoying to have to write the regular expression for digits twice, so there is a shorter way to specify "at least one" of some character. This is the **Kleene +**, which means "one or more occurrences of the immediately preceding character or regular expression". Thus, the expression / [0-9]+/ is the normal way to specify "a sequence of digits". There are thus two ways to specify the sheep language: /baaa\*!/ or /baa+!/.

One very important special character is the period (/./), a **wildcard** expression that matches any single character (*except* a carriage return), as shown in Fig. 2.6.

Regex	Match	Example Matches
/beg.n/	any character between beg and n	begin, beg'n, begun
E'anna 2 ( Thank		

Figure 2.6 The use of the period . to specify any character.

The wildcard is often used together with the Kleene star to mean "any string of characters". For example, suppose we want to find any line in which a particular word, for example, *aardvark*, appears twice. We can specify this with the regular expression /aardvark.\*aardvark/.

Kleene \*

Kleene +

Anchors are special characters that anchor regular expressions to particular places

in a string. The most common anchors are the caret ^ and the dollar sign \$. The caret ^ matches the start of a line. The pattern / The/ matches the word *The* only at the start of a line. Thus, the caret ^ has three uses: to match the start of a line, to indicate a negation inside of square brackets, and just to mean a caret. (What are the contexts that allow grep or Python to know which function a given caret is supposed to have?) The dollar sign \$ matches the end of a line. So the pattern \_\$ is a useful pattern for matching a space at the end of a line, and / The dog\.\$/ matches a line that contains only the phrase *The dog*. (We have to use the backslash here since we want the . to mean "period" and not the wildcard.)

Regex	Match
^	start of line
\$	end of line
\b	word boundary
\B	non-word boundary

Figure 2.7 Anchors in regular expressions.

There are also two other anchors: \b matches a word boundary, and \B matches a non-boundary. Thus, /\bthe\b/ matches the word *the* but not the word *other*. More technically, a "word" for the purposes of a regular expression is defined as any sequence of digits, underscores, or letters; this is based on the definition of "words" in programming languages. For example, /\b99\b/ will match the string 99 in *There are 99 bottles of beer on the wall* (because 99 follows a space) but not 99 in *There are 299 bottles of beer on the wall* (since 99 follows a number). But it will match 99 in \$99 (since 99 follows a dollar sign (\$), which is not a digit, underscore, or letter).

### 2.1.2 Disjunction, Grouping, and Precedence

Suppose we need to search for texts about pets; perhaps we are particularly interested in cats and dogs. In such a case, we might want to search for either the string *cat* or the string *dog*. Since we can't use the square brackets to search for "cat or dog" (why can't we say /[catdog]/?), we need a new operator, the **disjunction** operator, also called the **pipe** symbol |. The pattern /cat|dog/ matches either the string cat or the string dog.

disjunction

Sometimes we need to use this disjunction operator in the midst of a larger sequence. For example, suppose I want to search for information about pet fish for my cousin David. How can I specify both *guppy* and *guppies*? We cannot simply say /guppy|ies/, because that would match only the strings *guppy* and *ies*. This is because sequences like guppy take **precedence** over the disjunction operator |. To make the disjunction operator apply only to a specific pattern, we need to use the parenthesis operators ( and ). Enclosing a pattern in parentheses makes it act like a single character for the purposes of neighboring operators like the pipe | and the Kleene\*. So the pattern /gupp(y|ies)/ would specify that we meant the disjunction only to apply to the suffixes y and ies.

precedence

The parenthesis operator ( is also useful when we are using counters like the Kleene\*. Unlike the | operator, the Kleene\* operator applies by default only to a single character, not to a whole sequence. Suppose we want to match repeated instances of a string. Perhaps we have a line that has column labels of the form Column 1 Column 2 Column 3. The expression /Column\_[0-9]+\_\* will not match any number of columns; instead, it will match a single column followed by

operator precedence

6

any number of spaces! The star here applies only to the space \_ that precedes it, not to the whole sequence. With the parentheses, we could write the expression /(Column\_[0-9]+\_\*)\*/ to match the word *Column*, followed by a number and optional spaces, the whole pattern repeated zero or more times.

This idea that one operator may take precedence over another, requiring us to sometimes use parentheses to specify what we mean, is formalized by the **operator precedence hierarchy** for regular expressions. The following table gives the order of RE operator precedence, from highest precedence to lowest precedence.

> Parenthesis ()\* + ? {} Counters Sequences and anchors the 'my end\$ Disjunction

because counters have a higher precedence than sequences, /the\*/ matches theeeee but not thethe. Because sequences have a higher precedence than disjunction, /the | any/ matches the or any but not thany or theny.

Patterns can be ambiguous in another way. Consider the expression /[a-z]\*/ when matching against the text *once upon a time*. Since /[a-z]\*/ matches zero or more letters, this expression could match nothing, or just the first letter o, on, onc, or *once*. In these cases regular expressions always match the *largest* string they can; we say that patterns are **greedy**, expanding to cover as much of a string as they can.

There are, however, ways to enforce non-greedy matching, using another meaning of the ? qualifier. The operator \*? is a Kleene star that matches as little text as possible. The operator +? is a Kleene plus that matches as little text as possible.

#### A Simple Example 2.1.3

Suppose we wanted to write a RE to find cases of the English article the. A simple (but incorrect) pattern might be:

/the/

One problem is that this pattern will miss the word when it begins a sentence and hence is capitalized (i.e., *The*). This might lead us to the following pattern:

/[tT]he/

But we will still incorrectly return texts with the embedded in other words (e.g., other or theology). So we need to specify that we want instances with a word boundary on both sides:

```
/\b[tT]he\b/
```

Suppose we wanted to do this without the use of /\b/. We might want this since /\b/ won't treat underscores and numbers as word boundaries; but we might want to find *the* in some context where it might also have underlines or numbers nearby (the\_ or the25). We need to specify that we want instances in which there are no alphabetic letters on either side of the *the*:

```
/[^a-zA-Z][tT]he[^a-zA-Z]/
```

But there is still one more problem with this pattern: it won't find the word the when it begins a line. This is because the regular expression [^a-zA-Z], which

greedy non-greedy

+?

we used to avoid embedded instances of *the*, implies that there must be some single (although non-alphabetic) character before the *the*. We can avoid this by specifying that before the *the* we require *either* the beginning-of-line or a non-alphabetic character, and the same at the end of the line:

$$/(^|[^a-zA-Z])[tT]he([^a-zA-Z]|$)/$$

false positives false negatives The process we just went through was based on fixing two kinds of errors: **false positives**, strings that we incorrectly matched like *other* or *there*, and **false negatives**, strings that we incorrectly missed, like *The*. Addressing these two kinds of errors comes up again and again in implementing speech and language processing systems. Reducing the overall error rate for an application thus involves two antagonistic efforts:

- Increasing **precision** (minimizing false positives)
- Increasing **recall** (minimizing false negatives)

We'll come back to precision and recall with more precise definitions in Chapter 4.

#### 2.1.4 More Operators

Figure 2.8 shows some aliases for common ranges, which can be used mainly to save typing. Besides the Kleene \* and Kleene + we can also use explicit numbers as counters, by enclosing them in curly brackets. The regular expression  $/\{3\}$ / means "exactly 3 occurrences of the previous character or expression". So  $/a \ 24\ z$ / will match a followed by 24 dots followed by z (but not a followed by 23 or 25 dots followed by a z).

Regex	Expansion	Match	First Matches
\d	[0-9]	any digit	Party_of_ <u>5</u>
\D	[^0-9]	any non-digit	<u>B</u> lue∟moon
\w	[a-zA-Z0-9_]	any alphanumeric/underscore	<u>D</u> aiyu
\W	[^\w]	a non-alphanumeric	<u>!</u> !!!!
\s	[	whitespace (space, tab)	in_Concord
\S	[^\s]	Non-whitespace	<u>i</u> n_Concord

Figure 2.8 Aliases for common sets of characters.

A range of numbers can also be specified. So  $/\{n,m\}/$  specifies from n to m occurrences of the previous char or expression, and  $/\{n,\}/$  means at least n occurrences of the previous expression. REs for counting are summarized in Fig. 2.9.

Regex	Match
*	zero or more occurrences of the previous char or expression
+	one or more occurrences of the previous char or expression
?	zero or one occurrence of the previous char or expression
{n}	exactly $n$ occurrences of the previous char or expression
{n,m}	from $n$ to $m$ occurrences of the previous char or expression
{n,}	at least <i>n</i> occurrences of the previous char or expression
{,m}	up to m occurrences of the previous char or expression
Figure 2.0	Dagular averaggion operators for counting

Figure 2.9 Regular expression operators for counting.

Finally, certain special characters are referred to by special notation based on the backslash (\) (see Fig. 2.10). The most common of these are the **newline** character

\n and the tab character \t. To refer to characters that are special themselves (like ., \*, [, and \), precede them with a backslash, (i.e.,  $\langle \cdot \cdot \cdot \rangle$ ,  $\langle \cdot \cdot \rangle$ , and  $\langle \cdot \cdot \rangle$ ).

Regex	Match	First Patterns Matched
/*	an asterisk "*"	"K <u>*</u> A*P*L*A*N"
\.	a period "."	"Dr. Livingston, I presume"
\?	a question mark	"Why don't they come and lend a hand?"
\n	a newline	
\t	a tab	

Figure 2.10 Some characters that need to be backslashed.

#### 2.1.5 **A More Complex Example**

Let's try out a more significant example of the power of REs. Suppose we want to build an application to help a user buy a computer on the Web. The user might want "any machine with at least 6 GHz and 500 GB of disk space for less than \$1000". To do this kind of retrieval, we first need to be able to look for expressions like 6 GHz or 500 GB or Mac or \$999.99. In the rest of this section we'll work out some simple regular expressions for this task.

First, let's complete our regular expression for prices. Here's a regular expression for a dollar sign followed by a string of digits:

Note that the \$ character has a different function here than the end-of-line function we discussed earlier. Most regular expression parsers are smart enough to realize that \$ here doesn't mean end-of-line. (As a thought experiment, think about how regex parsers might figure out the function of \$ from the context.)

Now we just need to deal with fractions of dollars. We'll add a decimal point and two digits afterwards:

This pattern only allows \$199.99 but not \$199. We need to make the cents optional and to make sure we're at a word boundary:

$$/(^|\W)$$
\$[0-9]+(\.[0-9][0-9])?\b/

One last catch! This pattern allows prices like \$199999.99 which would be far too expensive! We need to limit the dollars:

$$/(^{|\mathbb{W})}[0-9]\{0,3\}(\.[0-9][0-9])?\b/$$

How about disk space? We'll need to allow for optional fractions again (5.5 GB); note the use of? for making the final s optional, and the use of /\_\*/ to mean "zero or more spaces" since there might always be extra spaces lying around:

$$\b| (0-9] + (\.[0-9] +)? | *(GB| [Gg] igabytes?) | b/$$

Modifying this regular expression so that it only matches more than 500 GB is left as an exercise for the reader.

#### 2.1.6 Substitution, Capture Groups, and ELIZA

substitution

An important use of regular expressions is in **substitutions**. For example, the substitution operator s/regexp1/pattern/ used in Python and in Unix commands like vim or sed allows a string characterized by a regular expression to be replaced by another string:

s/colour/color/

It is often useful to be able to refer to a particular subpart of the string matching the first pattern. For example, suppose we wanted to put angle brackets around all integers in a text, for example, changing the 35 boxes to the <35> boxes. We'd like a way to refer to the integer we've found so that we can easily add the brackets. To do this, we put parentheses ( and ) around the first pattern and use the **number** operator  $\1$  in the second pattern to refer back. Here's how it looks:

 $s/([0-9]+)/<\1>/$ 

The parenthesis and number operators can also specify that a certain string or expression must occur twice in the text. For example, suppose we are looking for the pattern "the Xer they were, the Xer they will be", where we want to constrain the two X's to be the same string. We do this by surrounding the first X with the parenthesis operator, and replacing the second X with the number operator  $\1$ , as follows:

/the (.\*)er they were, the \ler they will be/

Here the \1 will be replaced by whatever string matched the first item in parentheses. So this will match the bigger they were, the bigger they will be but not the bigger they were, the faster they will be.

capture group

register

This use of parentheses to store a pattern in memory is called a **capture group**. Every time a capture group is used (i.e., parentheses surround a pattern), the resulting match is stored in a numbered **register**. If you match two different sets of parentheses, \2 means whatever matched the *second* capture group. Thus

```
/the (.*)er they (.*), the \ler we \2/
```

will match the faster they ran, the faster we ran but not the faster they ran, the faster we ate. Similarly, the third capture group is stored in \3, the fourth is \4, and so on.

Parentheses thus have a double function in regular expressions; they are used to group terms for specifying the order in which operators should apply, and they are used to capture something in a register. Occasionally we might want to use parentheses for grouping, but don't want to capture the resulting pattern in a register. In that case we use a **non-capturing group**, which is specified by putting the special commands?: after the open parenthesis, in the form (?: pattern).

non-capturing group

```
/(?:some|a few) (people|cats) like some <math>1/
```

will match some cats like some cats but not some cats like some some.

Substitutions and capture groups are very useful in implementing simple chatbots like ELIZA (Weizenbaum, 1966). Recall that ELIZA simulates a Rogerian psychologist by carrying on conversations like the following:

User<sub>1</sub>: Men are all alike. ELIZA<sub>1</sub>: IN WHAT WAY

User<sub>2</sub>: They're always bugging us about something or other. ELIZA<sub>2</sub>: CAN YOU THINK OF A SPECIFIC EXAMPLE

User<sub>3</sub>: Well, my boyfriend made me come here.

ELIZA<sub>3</sub>: YOUR BOYFRIEND MADE YOU COME HERE

User<sub>4</sub>: He says I'm depressed much of the time.

ELIZA<sub>4</sub>: I AM SORRY TO HEAR YOU ARE DEPRESSED

ELIZA works by having a series or cascade of regular expression substitutions each of which matches and changes some part of the input lines. Input lines are first uppercased. The first substitutions then change all instances of *MY* to *YOUR*, and *I'M* to *YOU ARE*, and so on. The next set of substitutions matches and replaces other patterns in the input. Here are some examples:

```
s/.* I'M (depressed|sad) .*/I AM SORRY TO HEAR YOU ARE \1/
s/.* I AM (depressed|sad) .*/WHY DO YOU THINK YOU ARE \1/
s/.* all .*/IN WHAT WAY/
s/.* always .*/CAN YOU THINK OF A SPECIFIC EXAMPLE/
```

Since multiple substitutions can apply to a given input, substitutions are assigned a rank and applied in order. Creating patterns is the topic of Exercise 2.3, and we return to the details of the ELIZA architecture in Chapter 15.

#### 2.1.7 Lookahead Assertions

Finally, there will be times when we need to predict the future: look ahead in the text to see if some pattern matches, but not advance the match cursor, so that we can then deal with the pattern if it occurs.

lookahead

zero-width

These **lookahead** assertions make use of the (? syntax that we saw in the previous section for non-capture groups. The operator (?= pattern) is true if pattern occurs, but is **zero-width**, i.e. the match pointer doesn't advance. The operator (?! pattern) only returns true if a pattern does not match, but again is zero-width and doesn't advance the cursor. Negative lookahead is commonly used when we are parsing some complex pattern but want to rule out a special case. For example suppose we want to match, at the beginning of a line, any single word that doesn't start with "Volcano". We can use negative lookahead to do this:

```
/^(?!Volcano)[A-Za-z]+/
```

### 2.2 Words

corpus corpora Before we talk about processing words, we need to decide what counts as a word. Let's start by looking at one particular **corpus** (plural **corpora**), a computer-readable collection of text or speech. For example the Brown corpus is a million-word collection of samples from 500 written English texts from different genres (newspaper, fiction, non-fiction, academic, etc.), assembled at Brown University in 1963–64 (Kučera and Francis, 1967). How many words are in the following Brown sentence?

He stepped out into the hall, was delighted to encounter a water brother.

This sentence has 13 words if we don't count punctuation marks as words, 15 if we count punctuation. Whether we treat period ("."), comma (","), and so on as words depends on the task. Punctuation is critical for finding boundaries of things (commas, periods, colons) and for identifying some aspects of meaning (question marks, exclamation marks, quotation marks). For some tasks, like part-of-speech tagging or parsing or speech synthesis, we sometimes treat punctuation marks as if they were separate words.

The Switchboard corpus of American English telephone conversations between strangers was collected in the early 1990s; it contains 2430 conversations averaging 6 minutes each, totaling 240 hours of speech and about 3 million words (Godfrey et al., 1992). Such corpora of spoken language don't have punctuation but do introduce other complications with regard to defining words. Let's look at one utterance from Switchboard; an **utterance** is the spoken correlate of a sentence:

utterance

I do uh main- mainly business data processing

disfluency fragment filled pause This utterance has two kinds of **disfluencies**. The broken-off word *main*- is called a **fragment**. Words like *uh* and *um* are called **fillers** or **filled pauses**. Should we consider these to be words? Again, it depends on the application. If we are building a speech transcription system, we might want to eventually strip out the disfluencies.

But we also sometimes keep disfluencies around. Disfluencies like *uh* or *um* are actually helpful in speech recognition in predicting the upcoming word, because they may signal that the speaker is restarting the clause or idea, and so for speech recognition they are treated as regular words. Because people use different disfluencies they can also be a cue to speaker identification. In fact Clark and Fox Tree (2002) showed that *uh* and *um* have different meanings. What do you think they are?

Are capitalized tokens like *They* and uncapitalized tokens like *they* the same word? These are lumped together in some tasks (speech recognition), while for part-of-speech or named-entity tagging, capitalization is a useful feature and is retained.

How about inflected forms like *cats* versus *cat*? These two words have the same **lemma** *cat* but are different wordforms. A **lemma** is a set of lexical forms having the same stem, the same major part-of-speech, and the same word sense. The **wordform** is the full inflected or derived form of the word. For morphologically complex languages like Arabic, we often need to deal with lemmatization. For many tasks in English, however, wordforms are sufficient.

lemma

wordform

word type

word token

How many words are there in English? To answer this question we need to distinguish two ways of talking about words. **Types** are the number of distinct words in a corpus; if the set of words in the vocabulary is V, the number of types is the vocabulary size |V|. **Tokens** are the total number N of running words. If we ignore punctuation, the following Brown sentence has 16 tokens and 14 types:

They picnicked by the pool, then lay back on the grass and looked at the stars.

When we speak about the number of words in the language, we are generally referring to word types.

Fig. 2.11 shows the rough numbers of types and tokens computed from some popular English corpora. The larger the corpora we look at, the more word types we find, and in fact this relationship between the number of types |V| and number of tokens N is called **Herdan's Law** (Herdan, 1960) or **Heaps' Law** (Heaps, 1978) after its discoverers (in linguistics and information retrieval respectively). It is shown in Eq. 2.1, where k and  $\beta$  are positive constants, and  $0 < \beta < 1$ .

Herdan's Law Heaps' Law

1	^
	,

Corpus	Tokens = N  Types =  V
Shakespeare	884 thousand 31 thousand
Brown corpus	1 million 38 thousand
Switchboard telephone conversations	2.4 million 20 thousand
COCA	440 million 2 million
Google n-grams	1 trillion 13 million

**Figure 2.11** Rough numbers of types and tokens for some English language corpora. The largest, the Google n-grams corpus, contains 13 million types, but this count only includes types appearing 40 or more times, so the true number would be much larger.

The value of  $\beta$  depends on the corpus size and the genre, but at least for the large corpora in Fig. 2.11,  $\beta$  ranges from .67 to .75. Roughly then we can say that the vocabulary size for a text goes up significantly faster than the square root of its length in words.

Another measure of the number of words in the language is the number of lemmas instead of wordform types. Dictionaries can help in giving lemma counts; dictionary **entries** or **boldface forms** are a very rough upper bound on the number of lemmas (since some lemmas have multiple boldface forms). The 1989 edition of the Oxford English Dictionary had 615,000 entries.

### 2.3 Corpora

Words don't appear out of nowhere. Any particular piece of text that we study is produced by one or more specific speakers or writers, in a specific dialect of a specific language, at a specific time, in a specific place, for a specific function.

Perhaps the most important dimension of variation is the language. NLP algo-

rithms are most useful when they apply across many languages. The world has 7097 languages at the time of this writing, according to the online Ethnologue catalog (Simons and Fennig, 2018). It is important to test algorithms on more than one language, and particularly on languages with different properties; by contrast there is an unfortunate current tendency for NLP algorithms to be developed or tested just on English (Bender, 2019). Even when algorithms are developed beyond English, they tend to be developed for the official languages of large industrialized nations (Chinese, Spanish, Japanese, German etc.), but we don't want to limit tools to just these few languages. Furthermore, most languages also have multiple varieties, often spoken in different regions or by different social groups. Thus, for example, if we're processing text that uses features of African American English (AAE) or African American Vernacular English (AAVE)—the variations of English used by millions of people in African American communities (King 2020)—we must use NLP tools that function with features of those varieties. Twitter posts might use features often used by speakers of African American English, such as constructions like iont (I don't in Mainstream American English (MAE)), or talmbout corresponding to MAE talking about, both examples that influence word segmentation (Blodgett et al. 2016, Jones 2015).

code switching

AAE

MAE

It's also quite common for speakers or writers to use multiple languages in a single communicative act, a phenomenon called **code switching**. Code switching is enormously common across the world; here are examples showing Spanish and (transliterated) Hindi code switching with English (Solorio et al. 2014, Jurgens et al. 2017):

- (2.2) Por primera vez veo a @username actually being hateful! it was beautiful:) [For the first time I get to see @username actually being hateful! it was beautiful:)]
- (2.3) dost that or ra- hega ... don't wory ... but dherya rakhe ["he was and will remain a friend ... don't worry ... but have faith"]

Another dimension of variation is the genre. The text that our algorithms must process might come from newswire, fiction or non-fiction books, scientific articles, Wikipedia, or religious texts. It might come from spoken genres like telephone conversations, business meetings, police body-worn cameras, medical interviews, or transcripts of television shows or movies. It might come from work situations like doctors' notes, legal text, or parliamentary or congressional proceedings.

Text also reflects the demographic characteristics of the writer (or speaker): their age, gender, race, socioeconomic class can all influence the linguistic properties of the text we are processing.

And finally, time matters too. Language changes over time, and for some languages we have good corpora of texts from different historical periods.

Because language is so situated, when developing computational models for language processing from a corpus, it's important to consider who produced the language, in what context, for what purpose. How can a user of a dataset know all these details? The best way is for the corpus creator to build a **datasheet** (Gebru et al., 2020) or **data statement** (Bender and Friedman, 2018) for each corpus. A datasheet specifies properties of a dataset like:

**Motivation**: Why was the corpus collected, by whom, and who funded it?

**Situation**: When and in what situation was the text written/spoken? For example, was there a task? Was the language originally spoken conversation, edited text, social media communication, monologue vs. dialogue?

**Language variety**: What language (including dialect/region) was the corpus in?

**Speaker demographics**: What was, e.g., the age or gender of the text's authors?

**Collection process**: How big is the data? If it is a subsample how was it sampled? Was the data collected with consent? How was the data pre-processed, and what metadata is available?

**Annotation process**: What are the annotations, what are the demographics of the annotators, how were they trained, how was the data annotated?

**Distribution**: Are there copyright or other intellectual property restrictions?

### 2.4 Text Normalization

Before almost any natural language processing of a text, the text has to be normalized. At least three tasks are commonly applied as part of any normalization process:

- 1. Tokenizing (segmenting) words
- 2. Normalizing word formats
- 3. Segmenting sentences

In the next sections we walk through each of these tasks.

datasheet

#### 2.4.1 Unix Tools for Crude Tokenization and Normalization

Let's begin with an easy, if somewhat naive version of word tokenization and normalization (and frequency computation) that can be accomplished for English solely in a single UNIX command-line, inspired by Church (1994). We'll make use of some Unix commands: tr, used to systematically change particular characters in the input; sort, which sorts input lines in alphabetical order; and uniq, which collapses and counts adjacent identical lines.

For example let's begin with the 'complete words' of Shakespeare in one file, sh.txt. We can use tr to tokenize the words by changing every sequence of non-alphabetic characters to a newline ('A-Za-z' means alphabetic, the -c option complements to non-alphabet, and the -s option squeezes all sequences into a single character):

```
tr -sc 'A-Za-z' '\n' < sh.txt
```

The output of this command will be:

THE
SONNETS
by
William
Shakespeare
From
fairest
creatures
We
...

Now that there is one word per line, we can sort the lines, and pass them to uniq -c which will collapse and count them:

```
tr -sc 'A-Za-z' '\n' < sh.txt | sort | uniq -c
with the following output:
1945 A
72 AARON
19 ABBESS
25 Aaron
6 Abate
1 Abates
5 Abbess
6 Abbey
3 Abbot
....</pre>
```

Alternatively, we can collapse all the upper case to lower case:

```
tr -sc 'A-Za-z' '\n' < sh.txt | tr A-Z a-z | sort | uniq -c
whose output is
14725 a
    97 aaron
    1 abaissiez
10 abandon
2 abandoned</pre>
```

```
2 abase
1 abash
14 abate
3 abated
3 abatement
```

Now we can sort again to find the frequent words. The -n option to sort means to sort numerically rather than alphabetically, and the -r option means to sort in reverse order (highest-to-lowest):

```
tr -sc 'A-Za-z' '\n' < sh.txt | tr A-Z a-z | sort | uniq -c | sort -n -r
```

The results show that the most frequent words in Shakespeare, as in any other corpus, are the short **function words** like articles, pronouns, prepositions:

```
27378 the
26084 and
22538 i
19771 to
17481 of
14725 a
13826 you
12489 my
11318 that
11112 in
```

Unix tools of this sort can be very handy in building quick word count statistics for any corpus in English. While in some versions of Unix these command-line tools also correctly handle Unicode characters and so can be used for many languages, in general for handling most languages outside English we use more sophisticated tokenization algorithms.

#### 2.4.2 Word Tokenization

tokenization

The simple UNIX tools above were fine for getting rough word statistics but more sophisticated algorithms are generally necessary for **tokenization**, the task of segmenting running text into words.

While the Unix command sequence just removed all the numbers and punctuation, for most NLP applications we'll need to keep these in our tokenization. We often want to break off punctuation as a separate token; commas are a useful piece of information for parsers, periods help indicate sentence boundaries. But we'll often want to keep the punctuation that occurs word internally, in examples like *m.p.h.*, *Ph.D.*, *AT&T*, and *cap'n*. Special characters and numbers will need to be kept in prices (\$45.55) and dates (\$1/\$\02/\$\06\$); we don't want to segment that price into separate tokens of "45" and "55". And there are URLs (https://www.stanford.edu), Twitter hashtags (#nlproc), or email addresses (someone@cs.colorado.edu).

Number expressions introduce other complications as well; while commas normally appear at word boundaries, commas are used inside numbers in English, every three digits: 555,500.50. Languages, and hence tokenization requirements, differ on this; many continental European languages like Spanish, French, and German, by contrast, use a comma to mark the decimal point, and spaces (or sometimes periods) where English puts commas, for example, 555 500,50.

clitic

A tokenizer can also be used to expand **clitic** contractions that are marked by apostrophes, for example, converting what're to the two tokens what are, and we're to we are. A clitic is a part of a word that can't stand on its own, and can only occur when it is attached to another word. Some such contractions occur in other alphabetic languages, including articles and pronouns in French (j'ai, l'homme).

Depending on the application, tokenization algorithms may also tokenize multiword expressions like New York or rock 'n' roll as a single token, which requires a multiword expression dictionary of some sort. Tokenization is thus intimately tied up with **named entity recognition**, the task of detecting names, dates, and organizations (Chapter 8).

Penn Treebank tokenization One commonly used tokenization standard is known as the **Penn Treebank tokenization** standard, used for the parsed corpora (treebanks) released by the Linguistic Data Consortium (LDC), the source of many useful datasets. This standard separates out clitics (doesn't becomes does plus n't), keeps hyphenated words together, and separates out all punctuation (to save space we're showing visible spaces ' $\_$ ' between tokens, although newlines is a more common output):

"\_does\_n't\_charge\_\$\_10\_"\_.

In practice, since tokenization needs to be run before any other language processing, it needs to be very fast. The standard method for tokenization is therefore to use deterministic algorithms based on regular expressions compiled into very efficient finite state automata. For example, Fig. 2.12 shows an example of a basic regular expression that can be used to tokenize English with the nltk.regexp\_tokenize function of the Python-based Natural Language Toolkit (NLTK) (Bird et al. 2009; https://www.nltk.org).

```
>>> text = 'That U.S.A. poster-print costs $12.40...'
>>> pattern = r'''(?x)  # set flag to allow verbose regexps
...  (?:[A-Z]\.)+  # abbreviations, e.g. U.S.A.
... | \w+?:(-\w+)*  # words with optional internal hyphens
... | \$?\d+(?:\.\d+)?%?  # currency, percentages, e.g. $12.40, 82%
... | \.\.\  # ellipsis
... | [][.,;"'?():_`-]  # these are separate tokens; includes ], [
... '''
>>> nltk.regexp_tokenize(text, pattern)
['That', 'U.S.A.', 'poster-print', 'costs', '$12.40', '...']
```

**Figure 2.12** A Python trace of regular expression tokenization in the NLTK Python-based natural language processing toolkit (Bird et al., 2009), commented for readability; the (?x) verbose flag tells Python to strip comments and whitespace. Figure from Chapter 3 of Bird et al. (2009).

Carefully designed deterministic algorithms can deal with the ambiguities that arise, such as the fact that the apostrophe needs to be tokenized differently when used as a genitive marker (as in *the book's cover*), a quotative as in *'The other class'*, *she said*, or in clitics like *they're*.

Word tokenization is more complex in languages like written Chinese, Japanese, and Thai, which do not use spaces to mark potential word-boundaries. In Chinese, for example, words are composed of characters (called **hanzi** in Chinese). Each

character generally represents a single unit of meaning (called a **morpheme**) and is pronounceable as a single syllable. Words are about 2.4 characters long on average. But deciding what counts as a word in Chinese is complex. For example, consider the following sentence:

(2.4) 姚明进入总决赛
"Yao Ming reaches the finals"

As Chen et al. (2017) point out, this could be treated as 3 words ('Chinese Treebank' segmentation):

(2.5) 姚明 进入 总决赛 YaoMing reaches finals

or as 5 words ('Peking University' segmentation):

(2.6) 姚 明 进入 总 决赛 Yao Ming reaches overall finals

Finally, it is possible in Chinese simply to ignore words altogether and use characters as the basic elements, treating the sentence as a series of 7 characters:

(2.7) 姚 明 进 入 总 决 赛 Yao Ming enter enter overall decision game

In fact, for most Chinese NLP tasks it turns out to work better to take characters rather than words as input, since characters are at a reasonable semantic level for most applications, and since most word standards, by contrast, result in a huge vocabulary with large numbers of very rare words (Li et al., 2019).

However, for Japanese and Thai the character is too small a unit, and so algorithms for **word segmentation** are required. These can also be useful for Chinese in the rare situations where word rather than character boundaries are required. The standard segmentation algorithms for these languages use neural **sequence models** trained via supervised machine learning on hand-segmented training sets; we'll introduce sequence models in Chapter 8 and Chapter 9.

### 2.4.3 Byte-Pair Encoding for Tokenization

There is a third option to tokenizing text. Instead of defining tokens as words (whether delimited by spaces or more complex algorithms), or as characters (as in Chinese), we can use our data to automatically tell us what the tokens should be. This is especially useful in dealing with unknown words, an important problem in language processing. As we will see in the next chapter, NLP algorithms often learn some facts about language from one corpus (a **training** corpus) and then use these facts to make decisions about a separate **test** corpus and its language. Thus if our training corpus contains, say the words *low*, *new*, *newer*, but not *lower*, then if the word *lower* appears in our test corpus, our system will not know what to do with it.

To deal with this unknown word problem, modern tokenizers often automatically induce sets of tokens that include tokens smaller than words, called **subwords**. Subwords can be arbitrary substrings, or they can be meaning-bearing units like the morphemes *-est* or *-er*. (A morpheme is the smallest meaning-bearing unit of a language; for example the word *unlikeliest* has the morphemes *un-*, *likely*, and *-est*.) In modern tokenization schemes, most tokens are words, but some tokens are frequently occurring morphemes or other subwords like *-er*. Every unseen word like *lower* can thus be represented by some sequence of known subword units, such as *low* and *er*, or even as a sequence of individual letters if necessary.

word segmentation

subwords

**BPE** 

Most tokenization schemes have two parts: a **token learner**, and a **token segmenter**. The token learner takes a raw training corpus (sometimes roughly preseparated into words, for example by whitespace) and induces a vocabulary, a set of tokens. The token segmenter takes a raw test sentence and segments it into the tokens in the vocabulary. Three algorithms are widely used: **byte-pair encoding** (Sennrich et al., 2016), **unigram language modeling** (Kudo, 2018), and **WordPiece** (Schuster and Nakajima, 2012); there is also a **SentencePiece** library that includes implementations of the first two of the three (Kudo and Richardson, 2018).

In this section we introduce the simplest of the three, the **byte-pair encoding** or **BPE** algorithm (Sennrich et al., 2016); see Fig. 2.13. The BPE token learner begins with a vocabulary that is just the set of all individual characters. It then examines the training corpus, chooses the two symbols that are most frequently adjacent (say 'A', 'B'), adds a new merged symbol 'AB' to the vocabulary, and replaces every adjacent 'A' 'B' in the corpus with the new 'AB'. It continues to count and merge, creating new longer and longer character strings, until k merges have been done creating k novel tokens; k is thus a parameter of the algorithm. The resulting vocabulary consists of the original set of characters plus k new symbols.

The algorithm is usually run inside words (not merging across word boundaries), so the input corpus is first white-space-separated to give a set of strings, each corresponding to the characters of a word, plus a special end-of-word symbol \_\_, and its counts. Let's see its operation on the following tiny input corpus of 18 word tokens with counts for each word (the word *low* appears 5 times, the word *newer* 6 times, and so on), which would have a starting vocabulary of 11 letters:

The BPE algorithm first counts all pairs of adjacent symbols: the most frequent is the pair e r because it occurs in *newer* (frequency of 6) and *wider* (frequency of 3) for a total of 9 occurrences. We then merge these symbols, treating er as one symbol, and count again:

```
        corpus
        vocabulary

        5
        low _ _ _ _ , d, e, i, l, n, o, r, s, t, w, er

        2
        lowest _ _ _

        6
        newer _ _ _

        3
        wider _ _ _

        2
        new _ _
```

Now the most frequent pair is er \_\_, which we merge; our system has learned that there should be a token for word-final er, represented as er\_\_:

```
    corpus
    vocabulary

    5
    1 o w _ _ _ _ , d, e, i, l, n, o, r, s, t, w, er, er_

    2
    1 o w e s t _ _

    6
    n e w er_ _ _

    3
    w i d er_ _

    2
    n e w _ _
```

<sup>&</sup>lt;sup>1</sup> Note that there can be ties; we could have instead chosen to merge r \_ first, since that also has a frequency of 9.

Next n e (total count of 8) get merged to ne:

```
    corpus
    vocabulary

    5
    1 o w _ _ _ _ , d, e, i, 1, n, o, r, s, t, w, er, er__, ne

    2
    1 o w e s t _ _

    6
    ne w er__

    3
    w i d er__

    2
    ne w _ _
```

If we continue, the next merges are:

```
        merge
        current vocabulary

        (ne, w)
        __, d, e, i, l, n, o, r, s, t, w, er, er__, ne, new

        (l, o)
        __, d, e, i, l, n, o, r, s, t, w, er, er__, ne, new, lo

        (lo, w)
        __, d, e, i, l, n, o, r, s, t, w, er, er__, ne, new, lo, low

        (new, er__)
        __, d, e, i, l, n, o, r, s, t, w, er, er__, ne, new, lo, low, newer__

        (low, __)
        __, d, e, i, l, n, o, r, s, t, w, er, er__, ne, new, lo, low, newer__, low__
```

```
function BYTE-PAIR ENCODING(strings C, number of merges k) returns vocab V

V \leftarrow all unique characters in C # initial set of tokens is characters

for i = 1 to k do # merge tokens k times

t_L, t_R \leftarrow Most frequent pair of adjacent tokens in C

t_{NEW} \leftarrow t_L + t_R # make new token by concatenating

V \leftarrow V + t_{NEW} # update the vocabulary

Replace each occurrence of t_L, t_R in C with t_{NEW} # and update the corpus

return V
```

**Figure 2.13** The token learner part of the BPE algorithm for taking a corpus broken up into individual characters or bytes, and learning a vocabulary by iteratively merging tokens. Figure adapted from Bostrom and Durrett (2020).

Once we've learned our vocabulary, the token segmenter is used to tokenize a test sentence. The token segmenter just runs on the test data the merges we have learned from the training data, greedily, in the order we learned them. (Thus the frequencies in the test data don't play a role, just the frequencies in the training data). So first we segment each test sentence word into characters. Then we apply the first rule: replace every instance of e r in the test corpus with er, and then the second rule: replace every instance of er in the test corpus with er, and so on. By the end, if the test corpus contained the character sequence n e w e r \_, it would be tokenized as a full word. But the characters of a new (unknown) word like 1 o w e r \_ would be merged into the two tokens low er\_.

Of course in real settings BPE is run with many thousands of merges on a very large input corpus. The result is that most words will be represented as full symbols, and only the very rare words (and unknown words) will have to be represented by their parts.

### 2.4.4 Word Normalization, Lemmatization and Stemming

normalization

Word **normalization** is the task of putting words/tokens in a standard format, choosing a single normal form for words with multiple forms like USA and US or uh-huh and uhhuh. This standardization may be valuable, despite the spelling information that is lost in the normalization process. For information retrieval or information

20

case folding

extraction about the US, we might want to see information from documents whether they mention the US or the USA.

Case folding is another kind of normalization. Mapping everything to lower case means that *Woodchuck* and *woodchuck* are represented identically, which is very helpful for generalization in many tasks, such as information retrieval or speech recognition. For sentiment analysis and other text classification tasks, information extraction, and machine translation, by contrast, case can be quite helpful and case folding is generally not done. This is because maintaining the difference between, for example, US the country and us the pronoun can outweigh the advantage in generalization that case folding would have provided for other words.

For many natural language processing situations we also want two morphologically different forms of a word to behave similarly. For example in web search, someone may type the string *woodchucks* but a useful system might want to also return pages that mention *woodchuck* with no s. This is especially common in morphologically complex languages like Polish, where for example the word *Warsaw* has different endings when it is the subject (*Warszawa*), or after a preposition like "in Warsaw" (w *Warszawie*), or "to Warsaw" (do Warszawy), and so on.

**Lemmatization** is the task of determining that two words have the same root, despite their surface differences. The words *am*, *are*, and *is* have the shared lemma *be*; the words *dinner* and *dinners* both have the lemma *dinner*. Lemmatizing each of these forms to the same lemma will let us find all mentions of words in Polish like *Warsaw*. The lemmatized form of a sentence like *He is reading detective stories* would thus be *He be read detective story*.

How is lemmatization done? The most sophisticated methods for lemmatization involve complete **morphological parsing** of the word. **Morphology** is the study of the way words are built up from smaller meaning-bearing units called **morphemes**. Two broad classes of morphemes can be distinguished: **stems**—the central morpheme of the word, supplying the main meaning—and **affixes**—adding "additional" meanings of various kinds. So, for example, the word *fox* consists of one morpheme (the morpheme *fox*) and the word *cats* consists of two: the morpheme *cat* and the morpheme -s. A morphological parser takes a word like *cats* and parses it into the two morphemes *cat* and s, or parses a Spanish word like *amaren* ('if in the future they would love') into the morpheme *amar* 'to love', and the morphological features *3PL* and *future subjunctive*.

#### **The Porter Stemmer**

Lemmatization algorithms can be complex. For this reason we sometimes make use of a simpler but cruder method, which mainly consists of chopping off word-final affixes. This naive version of morphological analysis is called **stemming**. For example, the **Porter stemmer**, a widely used stemming algorithm (Porter, 1980), when applied to the following paragraph:

This was not the map we found in Billy Bones's chest, but an accurate copy, complete in all things-names and heights and soundings-with the single exception of the red crosses and the written notes.

produces the following stemmed output:

Thi wa not the map we found in Billi Bone s chest but an accur copi complet in all thing name and height and sound with the singl except of the red cross and the written note

morpheme stem affix

stemming Porter stemmer The algorithm is based on series of rewrite rules run in series: the output of each pass is fed as input to the next pass. Here are some sample rules (more details can be found at https://tartarus.org/martin/PorterStemmer/):

```
ATIONAL \rightarrow ATE (e.g., relational \rightarrow relate)

ING \rightarrow \epsilon if the stem contains a vowel (e.g., motoring \rightarrow motor)

SSES \rightarrow SS (e.g., grasses \rightarrow grass)
```

Simple stemmers can be useful in cases where we need to collapse across different variants of the same lemma. Nonetheless, they do tend to commit errors of both over- and under-generalizing, as shown in the table below (Krovetz, 1993):

Errors of Co	mmission	Errors of	<b>Errors of Omission</b>			
organization	organ	European	Europe			
doing	doe	analyzes	analysis			
numerical	numerous	noisy	noise			
policy	police	sparsity	sparse			

#### 2.4.5 Sentence Segmentation

sentence segmentation

**Sentence segmentation** is another important step in text processing. The most useful cues for segmenting a text into sentences are punctuation, like periods, question marks, and exclamation points. Question marks and exclamation points are relatively unambiguous markers of sentence boundaries. Periods, on the other hand, are more ambiguous. The period character "." is ambiguous between a sentence boundary marker and a marker of abbreviations like *Mr.* or *Inc.* The previous sentence that you just read showed an even more complex case of this ambiguity, in which the final period of *Inc.* marked both an abbreviation and the sentence boundary marker. For this reason, sentence tokenization and word tokenization may be addressed jointly.

In general, sentence tokenization methods work by first deciding (based on rules or machine learning) whether a period is part of the word or is a sentence-boundary marker. An abbreviation dictionary can help determine whether the period is part of a commonly used abbreviation; the dictionaries can be hand-built or machine-learned (Kiss and Strunk, 2006), as can the final sentence splitter. In the Stanford CoreNLP toolkit (Manning et al., 2014), for example sentence splitting is rule-based, a deterministic consequence of tokenization; a sentence ends when a sentence-ending punctuation (., !, or ?) is not already grouped with other characters into a token (such as for an abbreviation or number), optionally followed by additional final quotes or brackets.

### 2.5 Minimum Edit Distance

Much of natural language processing is concerned with measuring how similar two strings are. For example in spelling correction, the user typed some erroneous string—let's say graffe—and we want to know what the user meant. The user probably intended a word that is similar to graffe. Among candidate similar words, the word giraffe, which differs by only one letter from graffe, seems intuitively to be more similar than, say grail or graf, which differ in more letters. Another

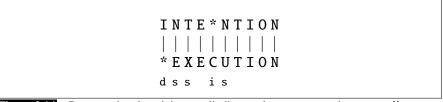
example comes from **coreference**, the task of deciding whether two strings such as the following refer to the same entity:

Stanford President Marc Tessier-Lavigne Stanford University President Marc Tessier-Lavigne

Again, the fact that these two strings are very similar (differing by only one word) seems like useful evidence for deciding that they might be coreferent.

**Edit distance** gives us a way to quantify both of these intuitions about string similarity. More formally, the **minimum edit distance** between two strings is defined as the minimum number of editing operations (operations like insertion, deletion, substitution) needed to transform one string into another.

The gap between *intention* and *execution*, for example, is 5 (delete an i, substitute e for n, substitute x for t, insert c, substitute u for n). It's much easier to see this by looking at the most important visualization for string distances, an **alignment** between the two strings, shown in Fig. 2.14. Given two sequences, an **alignment** is a correspondence between substrings of the two sequences. Thus, we say I **aligns** with the empty string, N with E, and so on. Beneath the aligned strings is another representation; a series of symbols expressing an **operation list** for converting the top string into the bottom string: **d** for deletion, **s** for substitution, **i** for insertion.



**Figure 2.14** Representing the minimum edit distance between two strings as an **alignment**. The final row gives the operation list for converting the top string into the bottom string: d for deletion, s for substitution, i for insertion.

We can also assign a particular cost or weight to each of these operations. The **Levenshtein** distance between two sequences is the simplest weighting factor in which each of the three operations has a cost of 1 (Levenshtein, 1966)—we assume that the substitution of a letter for itself, for example, t for t, has zero cost. The Levenshtein distance between *intention* and *execution* is 5. Levenshtein also proposed an alternative version of his metric in which each insertion or deletion has a cost of 1 and substitutions are not allowed. (This is equivalent to allowing substitution, but giving each substitution a cost of 2 since any substitution can be represented by one insertion and one deletion). Using this version, the Levenshtein distance between *intention* and *execution* is 8.

### 2.5.1 The Minimum Edit Distance Algorithm

How do we find the minimum edit distance? We can think of this as a search task, in which we are searching for the shortest path—a sequence of edits—from one string to another.

The space of all possible edits is enormous, so we can't search naively. However, lots of distinct edit paths will end up in the same state (string), so rather than recomputing all those paths, we could just remember the shortest path to a state each time we saw it. We can do this by using **dynamic programming**. Dynamic programming is the name for a class of algorithms, first introduced by Bellman (1957), that

minimum edit distance

alignment

dynamic programming

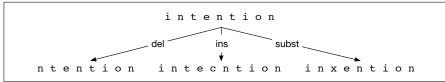


Figure 2.15 Finding the edit distance viewed as a search problem

apply a table-driven method to solve problems by combining solutions to subproblems. Some of the most commonly used algorithms in natural language processing make use of dynamic programming, such as the **Viterbi** algorithm (Chapter 8) and the **CKY** algorithm for parsing (Chapter 17).

The intuition of a dynamic programming problem is that a large problem can be solved by properly combining the solutions to various subproblems. Consider the shortest path of transformed words that represents the minimum edit distance between the strings *intention* and *execution* shown in Fig. 2.16.

**Figure 2.16** Path from *intention* to *execution*.

Imagine some string (perhaps it is *exention*) that is in this optimal path (whatever it is). The intuition of dynamic programming is that if *exention* is in the optimal operation list, then the optimal sequence must also include the optimal path from *intention* to *exention*. Why? If there were a shorter path from *intention* to *exention*, then we could use it instead, resulting in a shorter overall path, and the optimal sequence wouldn't be optimal, thus leading to a contradiction.

The **minimum edit distance algorithm** was named by Wagner and Fischer (1974) but independently discovered by many people (see the Historical Notes section of Chapter 8).

Let's first define the minimum edit distance between two strings. Given two strings, the source string X of length n, and target string Y of length m, we'll define D[i,j] as the edit distance between X[1..i] and Y[1..j], i.e., the first i characters of X and the first j characters of Y. The edit distance between X and Y is thus D[n,m].

We'll use dynamic programming to compute D[n,m] bottom up, combining solutions to subproblems. In the base case, with a source substring of length i but an empty target string, going from i characters to 0 requires i deletes. With a target substring of length j but an empty source going from 0 characters to j characters requires j inserts. Having computed D[i,j] for small i,j we then compute larger D[i,j] based on previously computed smaller values. The value of D[i,j] is computed by taking the minimum of the three possible paths through the matrix which arrive there:

$$D[i,j] = \min \begin{cases} D[i-1,j] + \text{del-cost}(source[i]) \\ D[i,j-1] + \text{ins-cost}(target[j]) \\ D[i-1,j-1] + \text{sub-cost}(source[i],target[j]) \end{cases}$$
 (2.8)

minimum edit distance algorithm If we assume the version of Levenshtein distance in which the insertions and deletions each have a cost of 1 (ins-cost(·) = del-cost(·) = 1), and substitutions have a cost of 2 (except substitution of identical letters have zero cost), the computation for D[i, j] becomes:

$$D[i,j] = \min \begin{cases} D[i-1,j] + 1 \\ D[i,j-1] + 1 \\ D[i-1,j-1] + \begin{cases} 2; & \text{if } source[i] \neq target[j] \\ 0; & \text{if } source[i] = target[j] \end{cases} \end{cases}$$
(2.9)

The algorithm is summarized in Fig. 2.17; Fig. 2.18 shows the results of applying the algorithm to the distance between *intention* and *execution* with the version of Levenshtein in Eq. 2.9.

```
function MIN-EDIT-DISTANCE(source, target) returns min-distance
  n \leftarrow \text{LENGTH}(source)
  m \leftarrow \text{LENGTH}(target)
  Create a distance matrix D[n+1,m+1]
  # Initialization: the zeroth row and column is the distance from the empty string
  D[0,0] = 0
  for each row i from 1 to n do
      D[i,0] \leftarrow D[i-1,0] + del\text{-}cost(source[i])
  for each column j from 1 to m do
      D[0,j] \leftarrow D[0,j-1] + ins-cost(target[j])
  # Recurrence relation:
  for each row i from 1 to n do
        for each column j from 1 to m do
          D[i, j] \leftarrow MIN(D[i-1, j] + del\text{-}cost(source[i]),
                           D[i-1, j-1] + sub\text{-}cost(source[i], target[j]),
                           D[i, j-1] + ins-cost(target[j])
  # Termination
  return D[n,m]
```

**Figure 2.17** The minimum edit distance algorithm, an example of the class of dynamic programming algorithms. The various costs can either be fixed (e.g.,  $\forall x$ , ins-cost(x) = 1) or can be specific to the letter (to model the fact that some letters are more likely to be inserted than others). We assume that there is no cost for substituting a letter for itself (i.e., sub-cost(x, x) = 0).

**Alignment** Knowing the minimum edit distance is useful for algorithms like finding potential spelling error corrections. But the edit distance algorithm is important in another way; with a small change, it can also provide the minimum cost **alignment** between two strings. Aligning two strings is useful throughout speech and language processing. In speech recognition, minimum edit distance alignment is used to compute the word error rate (Chapter 16). Alignment plays a role in machine translation, in which sentences in a parallel corpus (a corpus with a text in two languages) need to be matched to each other.

To extend the edit distance algorithm to produce an alignment, we can start by visualizing an alignment as a path through the edit distance matrix. Figure 2.19 shows this path with boldfaced cells. Each boldfaced cell represents an alignment

Src\Tar	#	e	X	e	С	u	t	i	0	n
#	0	1	2	3	4	5	6	7	8	9
i	1	2	3	4	5	6	7	6	7	8
n	2	3	4	5	6	7	8	7	8	7
t	3	4	5	6	7	8	7	8	9	8
e	4	3	4	5	6	7	8	9	10	9
n	5	4	5	6	7	8	9	10	11	10
t	6	5	6	7	8	9	8	9	10	11
i	7	6	7	8	9	10	9	8	9	10
0	8	7	8	9	10	11	10	9	8	9
n	9	8	9	10	11	12	11	10	9	8

**Figure 2.18** Computation of minimum edit distance between *intention* and *execution* with the algorithm of Fig. 2.17, using Levenshtein distance with cost of 1 for insertions or deletions, 2 for substitutions.

of a pair of letters in the two strings. If two boldfaced cells occur in the same row, there will be an insertion in going from the source to the target; two boldfaced cells in the same column indicate a deletion.

Figure 2.19 also shows the intuition of how to compute this alignment path. The computation proceeds in two steps. In the first step, we augment the minimum edit distance algorithm to store backpointers in each cell. The backpointer from a cell points to the previous cell (or cells) that we came from in entering the current cell. We've shown a schematic of these backpointers in Fig. 2.19. Some cells have multiple backpointers because the minimum extension could have come from multiple previous cells. In the second step, we perform a **backtrace**. In a backtrace, we start from the last cell (at the final row and column), and follow the pointers back through the dynamic programming matrix. Each complete path between the final cell and the initial cell is a minimum distance alignment. Exercise 2.7 asks you to modify the minimum edit distance algorithm to store the pointers and compute the backtrace to output an alignment.

# i e c t X e u 0 n # 0 ← 3 ← 4 ← 5 ← 6 7 ← 8 ← 1 9 i **↑ 1** <--> 5 ← 7  $\nwarrow \leftarrow \uparrow$ <--↑4 <->↑ 7  $\leftarrow$ 2 **\←↑8 ↑** 7 7 <-↑ **3** <-↑4 <-↑ 5 7  $\wedge \leftarrow \uparrow$ **↑** 3 t <-↑4 <->↑ 5 <-↑ 7 <-↑8 **←**↑8  $\uparrow 8$ **†** 4 3 **←**↑ 8 **\**←↑9 **\←**↑ 10 **†9 †** 5 **†** 4 <-↑ 5 <-↑ 7 **~**←↑8 <-↑ 10 <-↑ 11 10 n <-↑ 7 ← 9 t. **↑6** <-↑8 10 ← 9 i ↑ **7 †** 6 <-↑ 7 <-↑8 <-↑9 <-↑ 10 **↑9** 8 10 **+ 8 \←**↑ 10 ↑ 1<del>0</del> **↑9** ₹ 8 <-↑8 **\**←↑9 9 0 **↑9**  $\uparrow 9$ <-↑ 10 <-↑ 11 ↑ 11 ↑ 10 n

**Figure 2.19** When entering a value in each cell, we mark which of the three neighboring cells we came from with up to three arrows. After the table is full we compute an **alignment** (minimum edit path) by using a **backtrace**, starting at the **8** in the lower-right corner and following the arrows back. The sequence of bold cells represents one possible minimum cost alignment between the two strings. Diagram design after Gusfield (1997).

While we worked our example with simple Levenshtein distance, the algorithm in Fig. 2.17 allows arbitrary weights on the operations. For spelling correction, for example, substitutions are more likely to happen between letters that are next to

backtrace

each other on the keyboard. The **Viterbi** algorithm is a probabilistic extension of minimum edit distance. Instead of computing the "minimum edit distance" between two strings, Viterbi computes the "maximum probability alignment" of one string with another. We'll discuss this more in Chapter 8.

### 2.6 Summary

This chapter introduced a fundamental tool in language processing, the **regular expression**, and showed how to perform basic **text normalization** tasks including **word segmentation** and **normalization**, **sentence segmentation**, and **stemming**. We also introduced the important **minimum edit distance** algorithm for comparing strings. Here's a summary of the main points we covered about these ideas:

- The **regular expression** language is a powerful tool for pattern-matching.
- Basic operations in regular expressions include concatenation of symbols, disjunction of symbols ([], |, and .), counters (\*, +, and {n,m}), anchors (^, \$) and precedence operators ((,)).
- Word tokenization and normalization are generally done by cascades of simple regular expression substitutions or finite automata.
- The **Porter algorithm** is a simple and efficient way to do **stemming**, stripping off affixes. It does not have high accuracy but may be useful for some tasks.
- The minimum edit distance between two strings is the minimum number of
  operations it takes to edit one into the other. Minimum edit distance can be
  computed by dynamic programming, which also results in an alignment of
  the two strings.

# Bibliographical and Historical Notes

Kleene 1951; 1956 first defined regular expressions and the finite automaton, based on the McCulloch-Pitts neuron. Ken Thompson was one of the first to build regular expressions compilers into editors for text searching (Thompson, 1968). His editor *ed* included a command "g/regular expression/p", or Global Regular Expression Print, which later became the Unix grep utility.

Text normalization algorithms have been applied since the beginning of the field. One of the earliest widely used stemmers was Lovins (1968). Stemming was also applied early to the digital humanities, by Packard (1973), who built an affix-stripping morphological parser for Ancient Greek. Currently a wide variety of code for tokenization and normalization is available, such as the Stanford Tokenizer (https://nlp.stanford.edu/software/tokenizer.shtml) or specialized tokenizers for Twitter (O'Connor et al., 2010), or for sentiment (http://sentiment.christopherpotts.net/tokenizing.html). See Palmer (2012) for a survey of text preprocessing. NLTK is an essential tool that offers both useful Python libraries (https://www.nltk.org) and textbook descriptions (Bird et al., 2009) of many algorithms including text normalization and corpus interfaces.

For more on Herdan's law and Heaps' Law, see Herdan (1960, p. 28), Heaps (1978), Egghe (2007) and Baayen (2001); Yasseri et al. (2012) discuss the relationship with other measures of linguistic complexity. For more on edit distance, see the excellent Gusfield (1997). Our example measuring the edit distance from 'intention'

to 'execution' was adapted from Kruskal (1983). There are various publicly available packages to compute edit distance, including Unix diff and the NIST sclite program (NIST, 2005).

In his autobiography Bellman (1984) explains how he originally came up with the term *dynamic programming*:

"...The 1950s were not good years for mathematical research. [the] Secretary of Defense ...had a pathological fear and hatred of the word, research... I decided therefore to use the word, "programming". I wanted to get across the idea that this was dynamic, this was multistage... I thought, let's ... take a word that has an absolutely precise meaning, namely dynamic... it's impossible to use the word, dynamic, in a pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It's impossible. Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to."

### **Exercises**

- **2.1** Write regular expressions for the following languages.
  - 1. the set of all alphabetic strings;
  - 2. the set of all lower case alphabetic strings ending in a b;
  - 3. the set of all strings from the alphabet *a*,*b* such that each *a* is immediately preceded by and immediately followed by a *b*;
- **2.2** Write regular expressions for the following languages. By "word", we mean an alphabetic string separated from other words by whitespace, any relevant punctuation, line breaks, and so forth.
  - 1. the set of all strings with two consecutive repeated words (e.g., "Humbert Humbert" and "the the" but not "the bug" or "the big bug");
  - 2. all strings that start at the beginning of the line with an integer and that end at the end of the line with a word;
  - 3. all strings that have both the word *grotto* and the word *raven* in them (but not, e.g., words like *grottos* that merely *contain* the word *grotto*);
  - 4. write a pattern that places the first word of an English sentence in a register. Deal with punctuation.
- 2.3 Implement an ELIZA-like program, using substitutions such as those described on page 10. You might want to choose a different domain than a Rogerian psychologist, although keep in mind that you would need a domain in which your program can legitimately engage in a lot of simple repetition.
- **2.4** Compute the edit distance (using insertion cost 1, deletion cost 1, substitution cost 1) of "leda" to "deal". Show your work (using the edit distance grid).
- **2.5** Figure out whether *drive* is closer to *brief* or to *divers* and what the edit distance is to each. You may use any version of *distance* that you like.
- 2.6 Now implement a minimum edit distance algorithm and use your hand-computed results to check your code.
- 2.7 Augment the minimum edit distance algorithm to output an alignment; you will need to store pointers and add a stage to compute the backtrace.

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