Sorting algorithms

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Overview

1. Introduction
2. Bubblesort
   - Stable sort
   - In-place
3. Selection sort
4. Other algorithms
5. Quicksort
   - Divide & Conquer
6. Comparing
7. Quiz
8. Recap

Sorting

C

Sorting is one of the classic problems for learning algorithms.

- Requirement for everything.
- Obvious applications like sorting text, statistics (median calculations).

- Less obvious, sorting objects in games for FOV (Field Of View) calculations.
- Route planning.

Different algorithms

C

Lots of different algorithms, different ways to achieve the same thing.

- Going to be looking at several common/well known algorithms.
  - Bubblesort.
  - Selection sort.
  - Quick sort.
- Comparing and contrasting, advantages and disadvantages.

Bubblesort

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Very simple sort.

- Compares each item to the next in the sequence.
  - Swap items if in wrong order.

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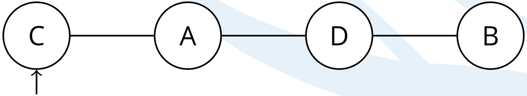

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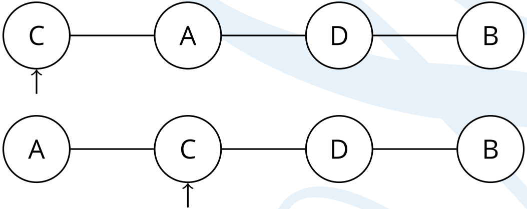

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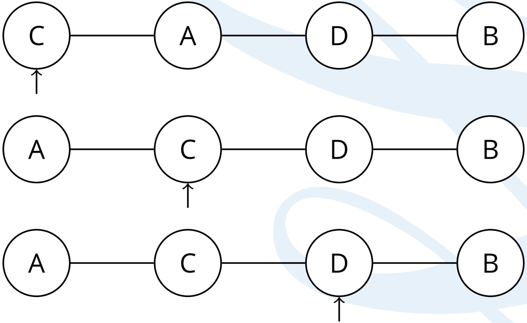

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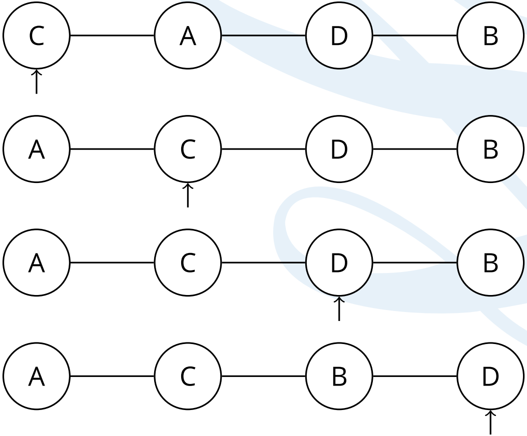

Bubblesort

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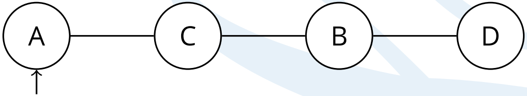
Iterating over the sequence once isn't typically enough.

- Keep iterating over the sequence until elements are sorted.



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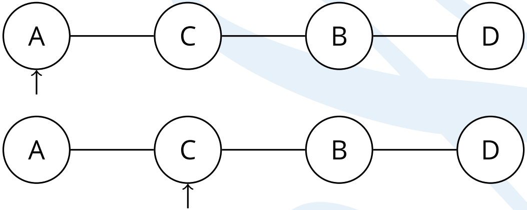

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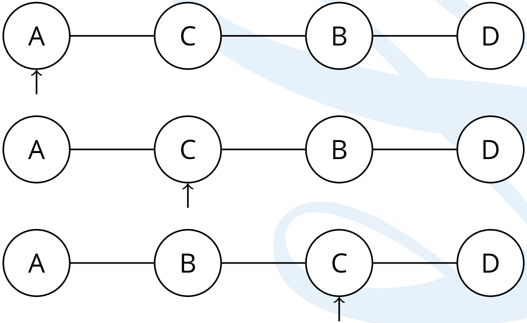

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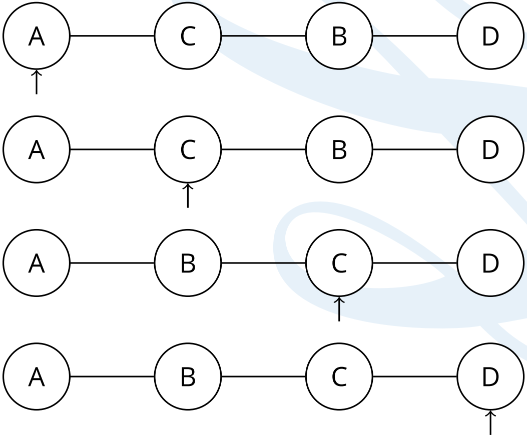

Stable sort

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Stable meaning that equivalent elements do not change their relative orders.

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Stable meaning that equivalent elements do not change their relative orders.

- Not important if e.g. sorting people by height.
- Important if e.g. you are sorting people by height and then sorting them by surname.
    - People with the same surname would still be in height order.
    - Can have performance benefits.

With unstable sorting algorithm the relative orders of equivalent elements can be changed.

In-place

I

In-place meaning that it only needs a small amount of additional memory in order to work.

- More memory efficient than the alternative.
  - Can be slower though.
- Can be important if...
  - ...dealing with large amounts of data.
  - ...have limited resources (i.e. embedded systems).
- Bubble sort only needs a few extra variables to swap the elements and to step through the sequence.

Bubblesort

C

One of the simplest sorting algorithms.

- Explained here to introduce you to sorting concepts.
  - In-place, stable.

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  - ● Horrible performance, average is $O(n^2)$.

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  - In-place, stable.

- Is rubbish.
  - 🔴 Horrible performance, average is $O(n^2)$.
  - 🟡 But best case is only $O(n)$.

Order

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The time taken to sort a sequence depends on:

- The starting order of the sequence.

For example, Bubblesorting a 100 elements:

So sorting algorithms have 3 $O()$ values.

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For example, Bubblesorting a 100 elements:

- Best case, already sorted.
    - Iterate over sequence once.
    - 100 comparisons.
- Worst case, in reverse order.
    - Iterate over sequence 100 times.
    - 10,000 comparisons.

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For example, Bubblesorting a 100 elements:

- Best case, already sorted.
  - Iterate over sequence once.
  - 100 comparisons.
- Worst case, in reverse order.
  - Iterate over sequence 100 times.
  - 10,000 comparisons.
- Average case, random order.
  - Somewhere in between.

So sorting algorithms have 3 $O()$ values.

Break

Selection sort

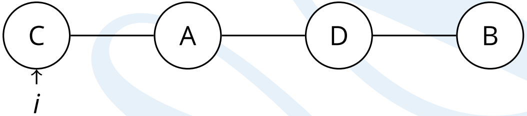
C

- Divides sequence into sorted and unsorted regions.
- Stable/Unstable, depends on implementation.
- In place.

1. Iterate over sequence.
2. For each element search the remaining elements on its right for the smallest value.
3. Swap smallest element with current element.

Selection sort II

C



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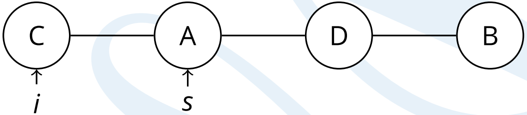

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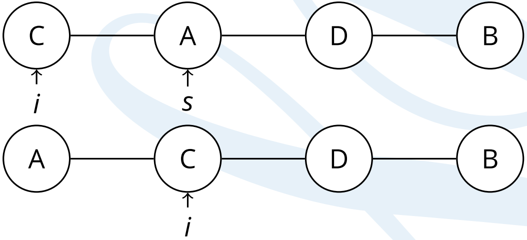

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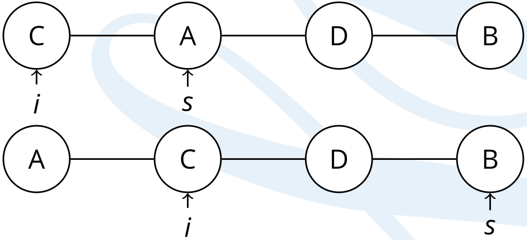

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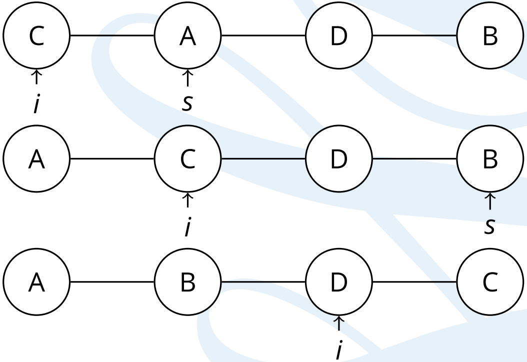

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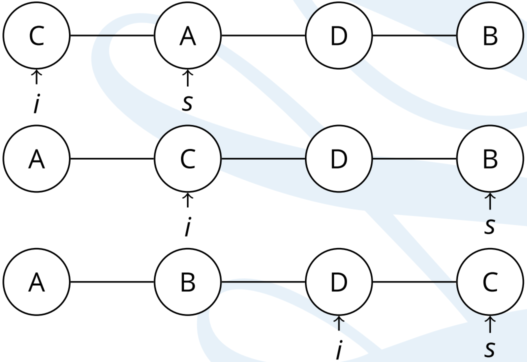

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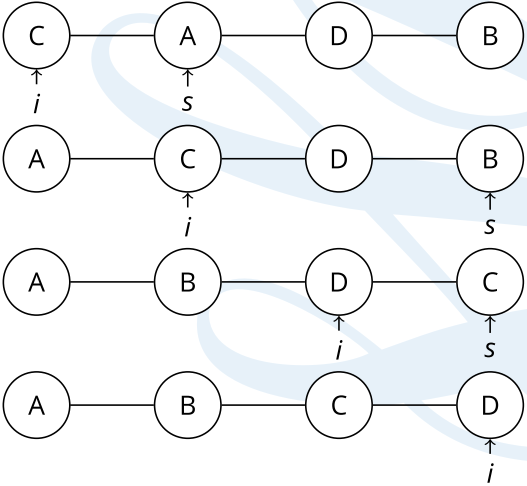

$O()$ I

Bubblesort is $O(n^2)$ worst and average case .
Selection sort is $O(n^2)$ worst and average case.

- Selection sort is generally faster than bubble.
  - But have same $O()$ complexity.
  - What?

$O()$

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Selection sort is $O(n^2)$ worst and average case.

- Selection sort is generally faster than bubble.
  - But have same $O()$ complexity.
  - What?

- $O()$ notation describes how an algorithm will grow.
- Not good at absolute performances.
- Selection sort typically does fewer comparisons and swaps than bubblesort.
  - Therefore typically faster.

- Best case bubblesort is $O(n)$, selection is $O(n^2)$.
  - So is occasionally faster.

Sorting Algorithms

A

Many sorting algorithms

- Different trade-offs, performances.
- Some are just jokes.

1. Bead
2. Bogo
3. Bubble
4. Circle
5. Cocktail
6. Comb
7. Counting
8. Cycle
9. Gnome
10. Heap
11. Insert
12. Merge
13. Pancake
14. Patience
15. Permutation
16. Quick
17. Radix
18. Selection
19. Shell
20. Sleep
21. Stooge
22. Strand
23. Tree

Break

`https://www.youtube.com/watch?v=ZZuD6iUe3Pc`

Quicksort

C

Neither bubble or selection sort are very good.

- Simple algorithms but slow.
- Not (typically) used in real code.

One of the fastest sorting algorithms.

- Used in real life.
- Recursively breaks the sequence in half.
  - Divide & Conquer.

Quicksort II

I

1. Select a value from the sequence, this is the pivot.
2. Put all values $<$ pivot in one group.
3. Put all values $\geq$ pivot in another group.
4. Treat each group as a new sequence and repeat from step 1.

Quicksort III

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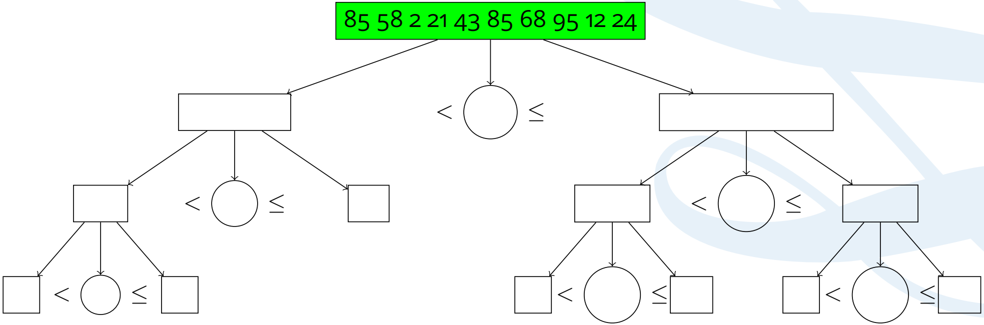

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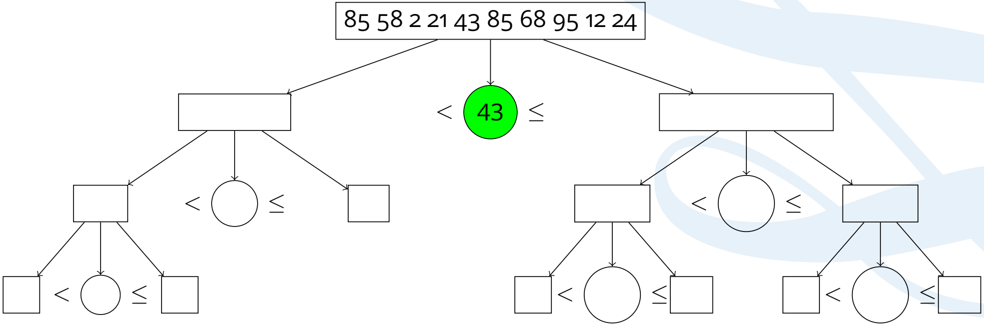

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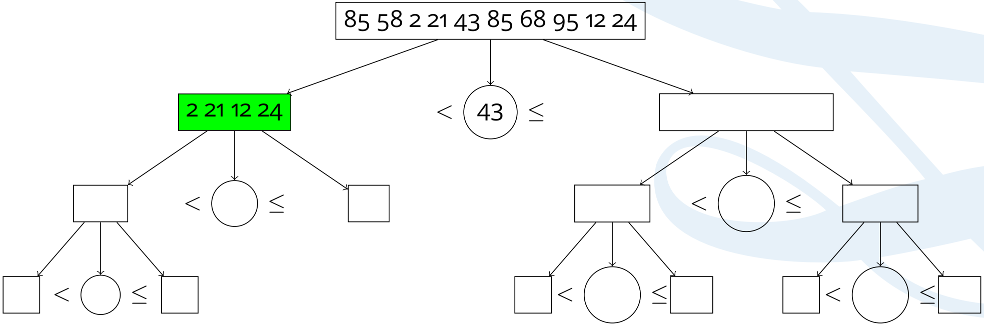

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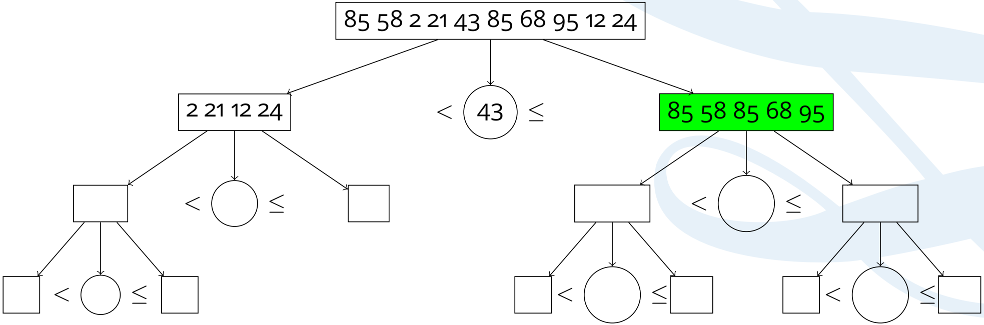

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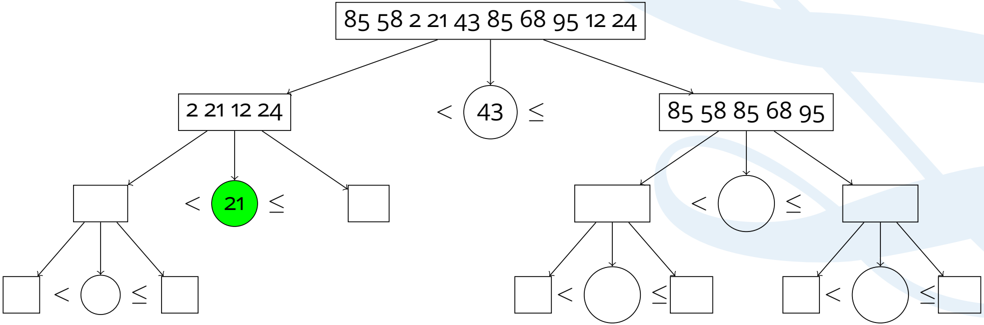

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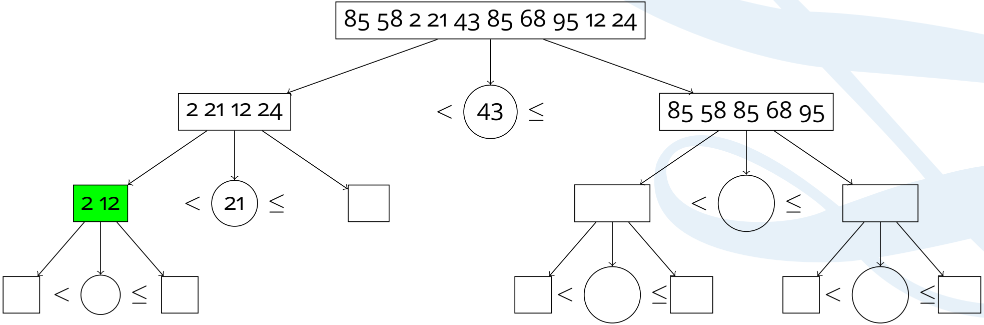

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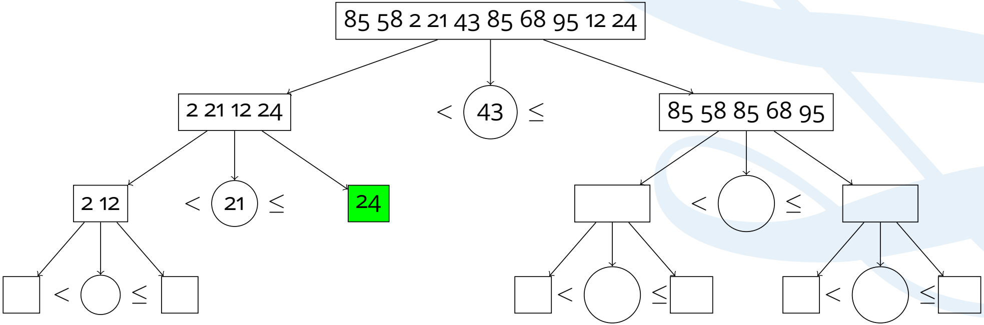

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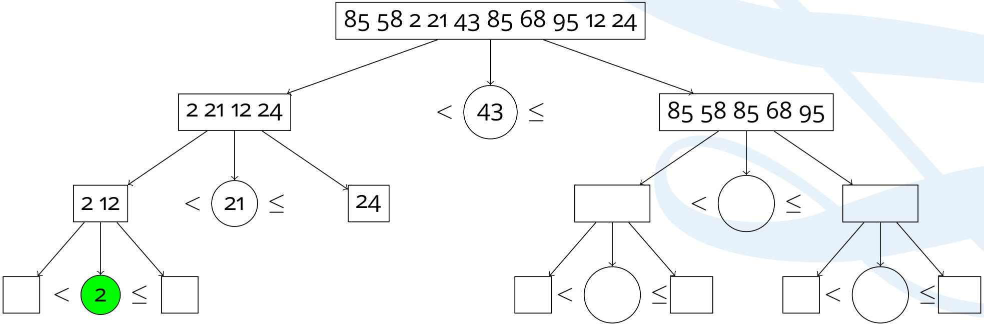

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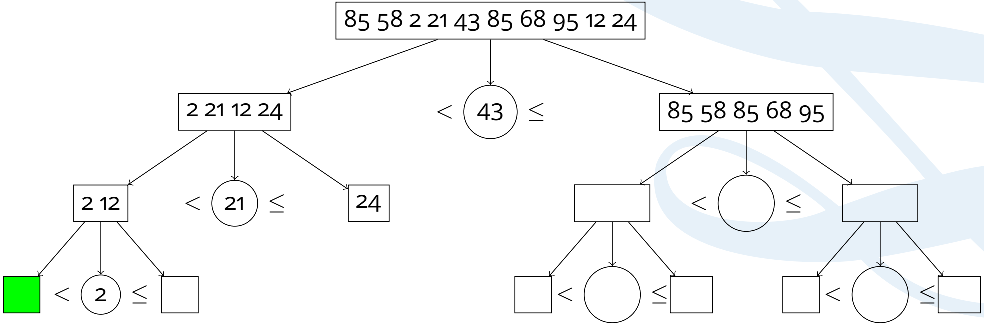

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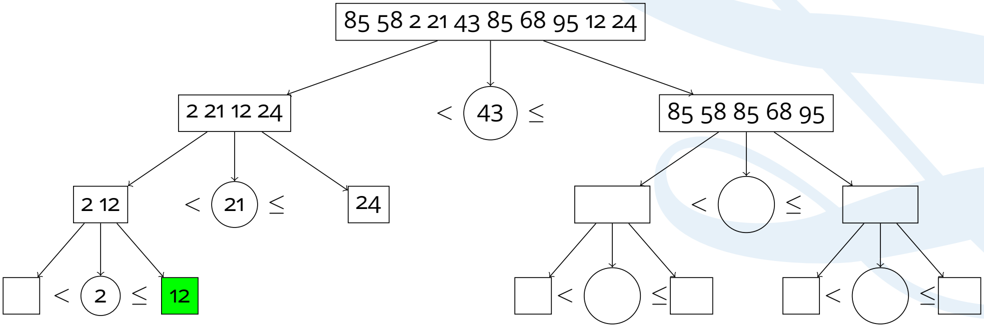

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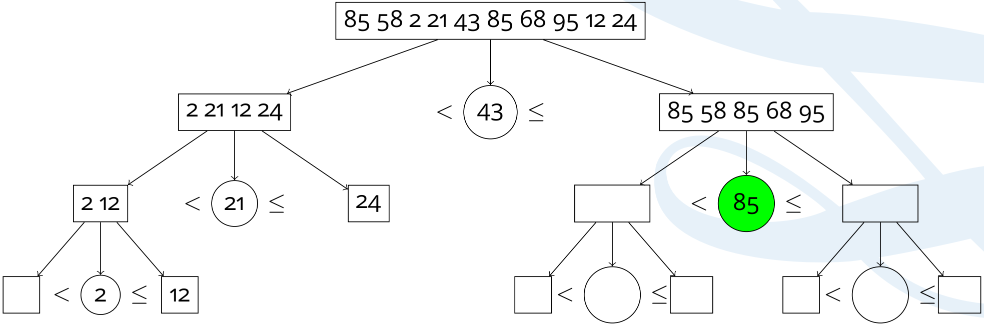

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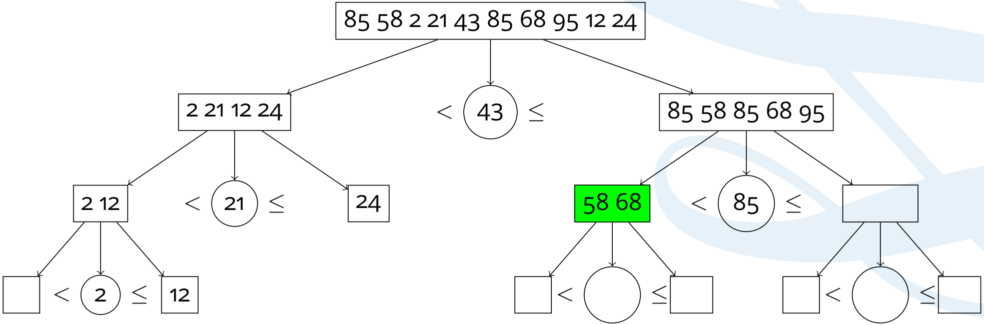

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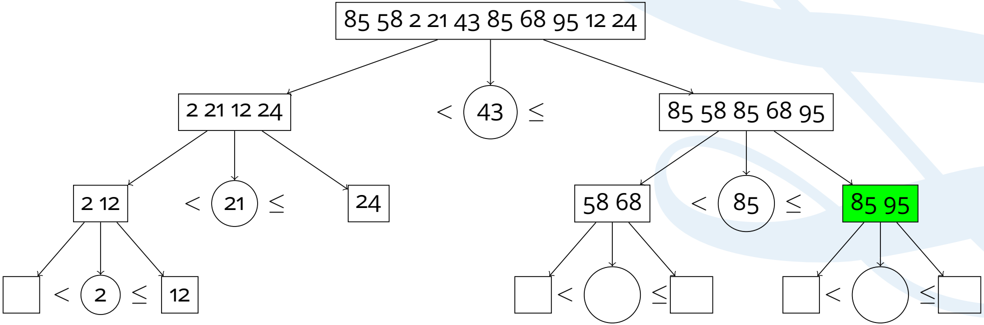

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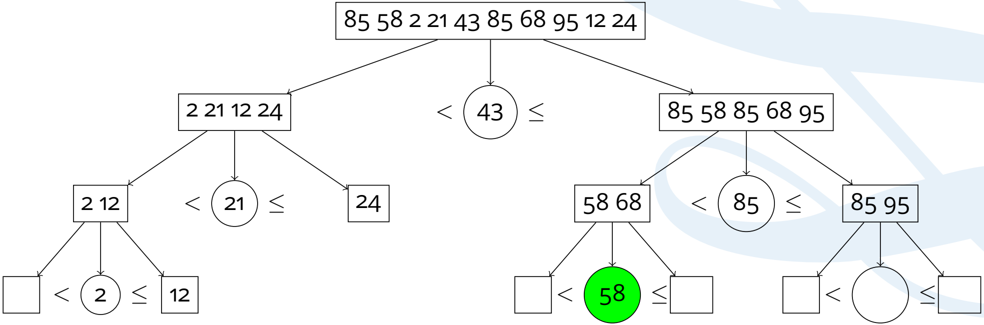

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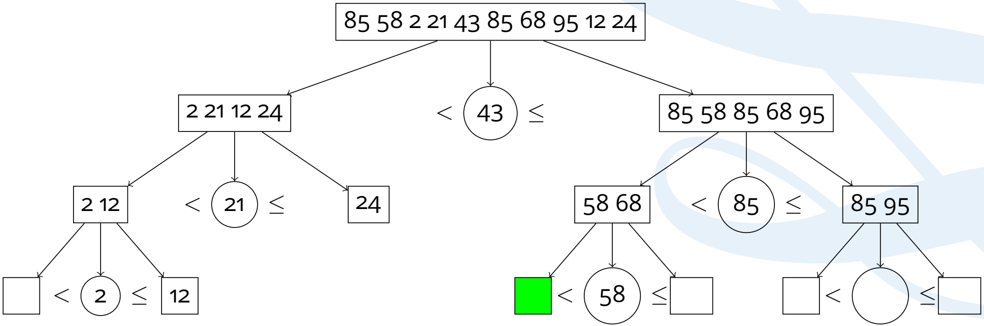

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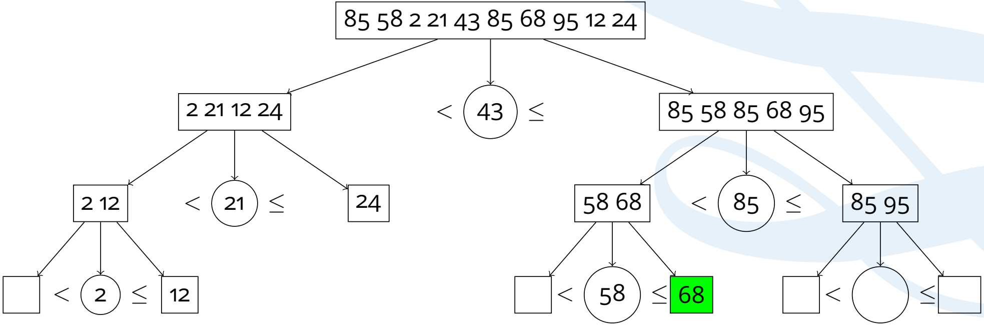

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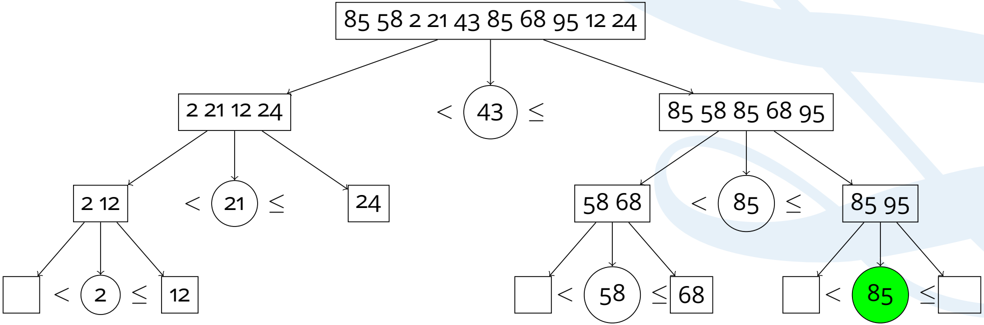

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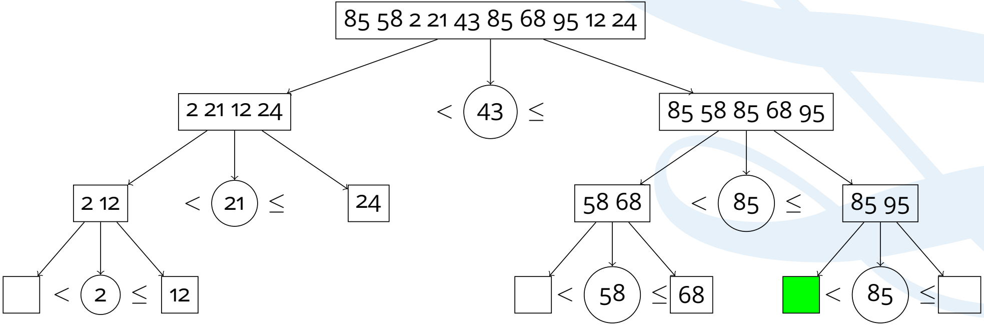

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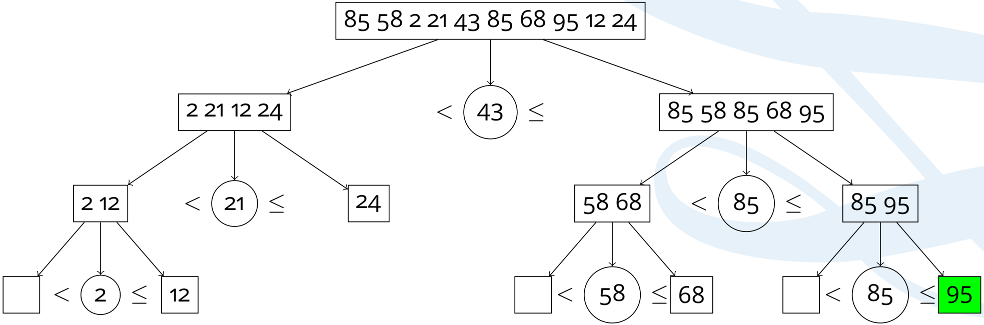

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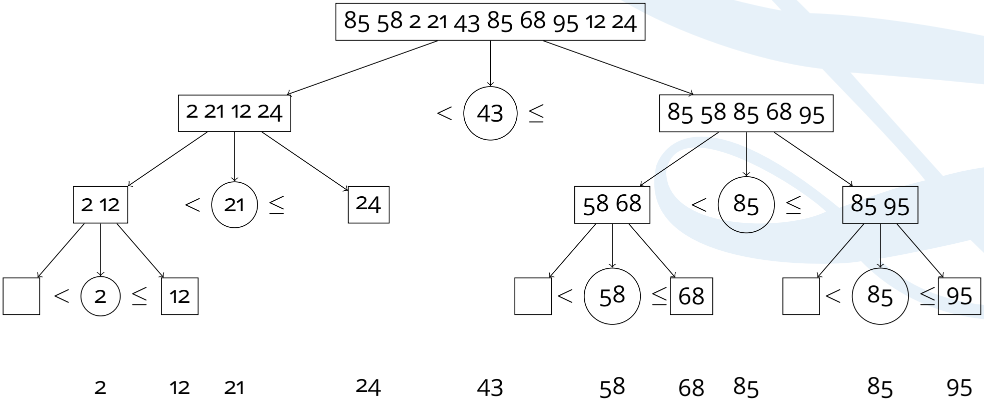

Quicksort

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Quicksort is...

- ...sometimes in-place.
  - Depends on implementation.
- ...sometimes stable.
  - Depends on implementation.

Some issues with the original algorithms (1959).

- Choosing the pivot.
  - First element.
  - Middle element.
  - Average of first, middle and last.
- Repeated elements.
  - Fat partition.

Divide and Conquer

C

Quicksort is a divide and conquer algorithm.

- Too hard to sort the whole sequence?
- Divide the problem.
    - Still too hard?
    - Divide the problem.
        - Still too hard?
        - Divide the problem.
        - Etc, etc, etc.

Naturally suited for parallelism.

- Each sub problem can be processed separately.

Comparing algorithms

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Have seen there are many ways to sort.

- Best sorting algorithm depends on multiple factors.
- Good in one situation is bad in another.

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- Stability? In place?
- What are you sorting?
  - Linked lists?
  - Sequential memory (arrays)?

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- What are you sorting?
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  - Sequential memory (arrays)?
- ● Where are you sorting?
  - RAM?
  - EEPROM? cheap to read, expensive to write.

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Have seen there are many ways to sort.

- Best sorting algorithm depends on multiple factors.
- Good in one situation is bad in another.

- Stability? In place?
- What are you sorting?
  - Linked lists?
  - Sequential memory (arrays)?
- ● Where are you sorting?
  - RAM?
  - EEPROM? cheap to read, expensive to write.
- ● Size of *n*.
  - Insertion sort with small *n*.

Comparing algorithms

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Quiz

Q1

Bubblesort performs best (has $O(n)$ performance) when

____

- The sequence is already in order.
- The sequence is in a random order.
- The sequence is in reverse order.
- The sequence contains a few distinct values that are repeated.

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Q2

Divide & Conquer algorithms work by ______

- Dividing the problem in half.
- Breaking problems down into smaller easier problems.
- Simplifying the code so that they run faster.
- Invading Czechoslovakia.

Q2

Divide & Conquer algorithms work by _____

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Q3

Which of the following algorithms are NOT divide & conquer?

- Bubblesort.
- Bubblesort and selection sort.
- Selection sort.
- Quicksort.

Q3

Which of the following algorithms are NOT divide & conquer?

- Bubblesort.
- Bubblesort and selection sort.
- Selection sort.
- Quicksort.

Q4

Which algorithm uses a pivot value to repeatedly halve the sequence?

- Bubblesort.
- Selection sort.
- Quicksort.
- All of the above.

Q4

Which algorithm uses a pivot value to repeatedly halve the sequence?

- Bubblesort.
- Selection sort.
- Quicksort.
- All of the above.

Q5

The worst sorting algorithm is ____

- Bubblesort.
- Bogo sort.
- Sleep sort.
- Selection sort.

Q5

The worst sorting algorithm is _____

- Bubblesort.
- Bogo sort.
- Sleep sort.
- Selection sort.

Why do I care?

Everyone

- Sorting algorithms are key to understanding many important concepts.
    - I.e. Binary Search Trees.
- Key to writing efficent code.
- Key to understanding memory/processor trade offs.
- Useful in teaching algoritmic thinking.
    - Algorithm design.
    - Comparing and contrasting different algorithms.
    - Divide and Conquer concepts.
- Employability skill, popular questions for programming interviews.

Recap

- 🟢 Many sorting algorithms.
- 🟢 Bubblesort.
- 🟢 Selection sort.
- 🟡 Quicksort
- 🟢 Advantages/disadvantages.
  - 🟢 In place.
  - 🟢 Stable.
  - 🟡 Divide and Conquer.
- 🟢 Performance
  - 🟡 O()
  - 🔴 Sequence type.
  - 🔴 Read/writes.
  - 🔴 Size of *n*.

The End