122COM: Introduction to algorithms

David Croft

Coventry University

david.croft@coventry.ac.uk

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Overview

Introduction

C

Introduction to algorithms module.

- What is an algorithm?

Introduction

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Introduction to algorithms module.

- What is an algorithm?
- Not the same as code.
- Not the same as a program.

Introduction

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Introduction to algorithms module.

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Task/algorithm/code

C

A task is a problem that needs to be solved.

- I.e. bake me a cake.

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An algorithm is a generalised set of instructions to perform a specific task.

- A strategy to solve a given problem.
  - Many different strategies to solve same task.
- Like a recipe.

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An algorithm is a generalised set of instructions to perform a specific task.

- A strategy to solve a given problem.
  - Many different strategies to solve same task.
- Like a recipe.

Code is a specific set of instructions to perform a specific task.

- An implementation of a strategy in a specific language/system.
- Have to adapt the algorithm to the specific features and abilities of the language.

Fibonacci sequence algorithm

C

Task - calculate the fibonacci sequence.

Fibonacci sequence algorithm

C

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Algorithm

1. Starting with 0 and 1.
2. Sum the two numbers to make a third.
3. Discard the lowest number.
4. Repeat from step 2.

Fibonacci sequence algorithm

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Recursive Python

```python
def fibonacci( a, b ):
    c = a + b
    a, b = b, c

    print( a )
    fibonacci( a, b )

fibonacci( 0, 1 )
```

Iterative C++

```cpp
for( int a=0, b=1, c;
     a>=0;
     c=a+b, a=b, b=c )
{
    cout << a << endl;
}
```

Solvable/Unsolvable

C

Some problems we can solve perfectly.

- Easy problems.
  - Fibonacci sequence.
  - Searching algorithms.
  - Polynomial time.

Some problems we can't solve.

- Hard problems.
  - Because they are provably unsolvable.
    - Literally impossible.
    - Investigate the Halting State problem.
  - Because they take too long to solve.

Heuristics

C

Some problems we could solve perfectly if only we had infinite computers/time.

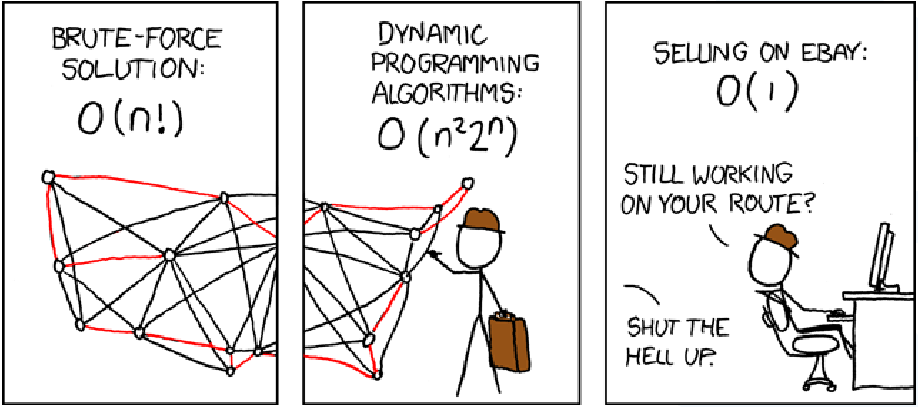
- Travelling salesman.
  - Hard problem, non-polynomial (will discuss later).
  - Can only solve very simple versions of the problem perfectly.
  - 5 cities = 120 possible solutions, 20 cities = 2 432 902 008 176 640 000 possible solutions.

Heuristic algorithms.

- Don’t promise to find the best solution.
- Quickly find a ‘good enough’ solution.

Heuristics

A



https://xkcd.com/399/

Module content

Looking at searching and sorting algorithms in later weeks.

Will be tested on some algorithmic concepts.

- Implement simple algorithms.
- Describe advantages/disadvantages of certain algorithms.
- Big O notation.
  - How algorithms scale.
- Calculate an algorithm's $O()$ notation.

Efficiency

C

When writing software need to think about its efficiency.

- Time.
- Memory.

Efficiency

C

When writing software need to think about its efficiency.

- Time.
- Memory.
- Time vs Memory.
  - Can you trade one for the other
  - I.e. data stored in RAM costs memory but saves time.
  - I.e. data stored on hard drive saves memory but costs time.

But don't be too concerned with performance.

Optimization

C

"Premature optimization is the root of all evil"

–Donald Knuth

For any large piece of code you should:

Optimization

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For any large piece of code you should:

- Write clear, easily understood code. Focus on getting the behaviour right, not on performance.
  - If it doesn't work then no-one cares how efficiently it fails.
- Test the performance.
  - It may be fine.
- Measure your code to get the baseline performance.
  - So that you know if you are making things better or worse.
- Ideally using profiling tools.
  - Investigate in your own time.

O() notation

C

Measuring performance identifies how your code is running for **those** inputs on **that** machine.

- Not good at measuring how code will scale.
  - Change in response to different inputs.
  - Change in response to problem size.
- Algorithmic complexity.
  - *O*() notation or Big-O notation.
- Certain algorithms are known to be better than other algorithms.

$O()$ notation

Used to describe complexity in terms of time and/or space.

- Commonly encountered examples...
  - $O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(2^n)$ and $O(n!)$
- n refers to the size of the problem.
  - E.g. n values to be sorted.
  - E.g. n values to be searched.
- $O()$ notation describes the worst case scenario.
  - Usually, unless otherwise stated.
- $O()$ notation is discussed in detail next year.
  - Main idea is to capture the dominant term: the thing that is most important when the size of the input (n) gets big.

$O(n)$

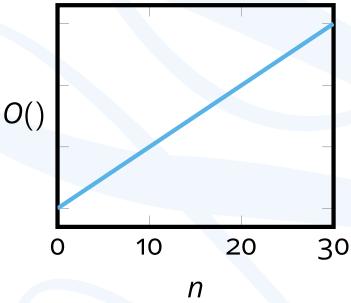
C

Linear complexity.

- n is directly proportional to time/space required
  - E.g. n doubles then time/space doubles.
- E.g. linear/sequential search.

```
a = [ 0, 1, 2, 3, 4, 5, 6, 7, 42 ]

for i in a:
  if i == 42:
    print('Found it')
    break
```



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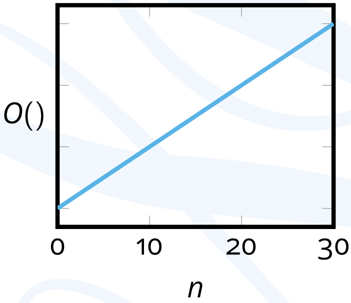
(n)



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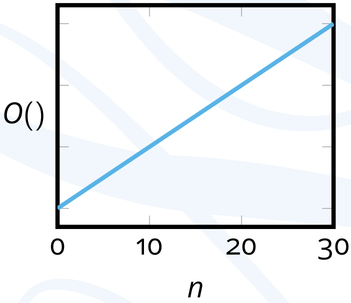

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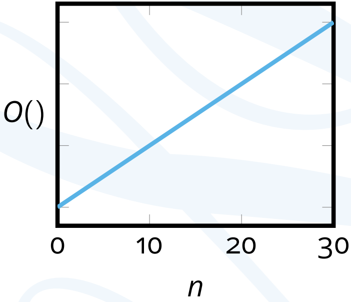
for i in a:                          (n)
  if i == 42:                        (n)
    print('Found it')                (1)
    break
```



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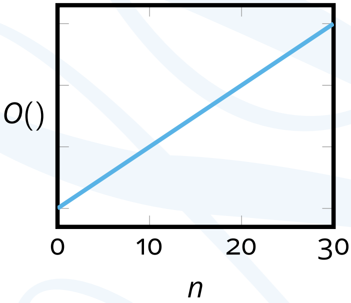


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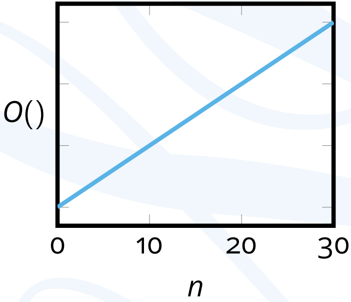


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for i in a:                              (n)
  if i == 42:                            (n)
    print('Found it')                    (1)
    break                                (1)
```

- So the algorithm takes $n + n + 1 + 1 = 2n + 2$ operations.
  - BUT! We would say it has complexity $O(n)$, constant values are irrelevant.

O(1)

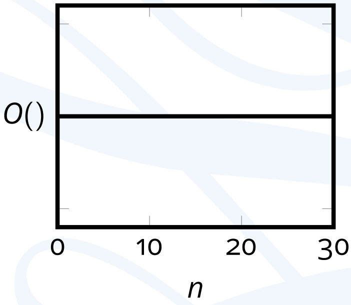
C

Constant complexity.

- *n* doesn't matter.
- Always takes same time/space.
- E.g. getting first item in an array.

```python
a = [ i for i in range(100) ]
b = [ i for i in range(1000000) ]

print(a[0])
print(b[0])
```



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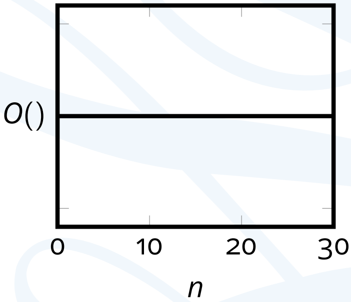

$O(1)$

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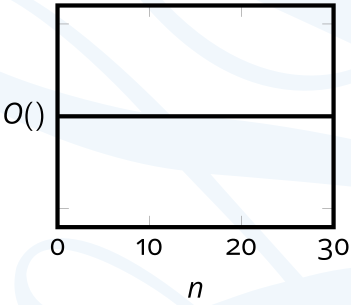

$O(1)$

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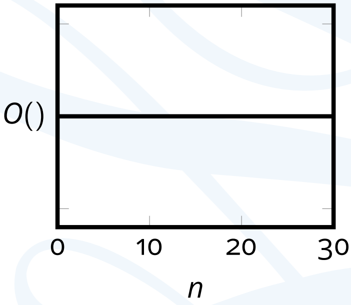


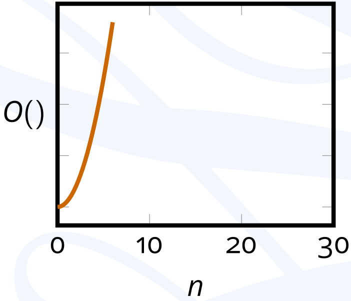
$O(n^2)$

Quadratic complexity.

- A lot of simple sorting algorithms are $O(n^2)$.
- Nested `for` loops are common example.
- $O(n^3)$, $O(n^4)$, $O(n^m)$ etc. are all possible.
- Polynomial time.

```
print('The n times tables')

for i in range(n):
  for j in range(n):
    print(i*j)
```



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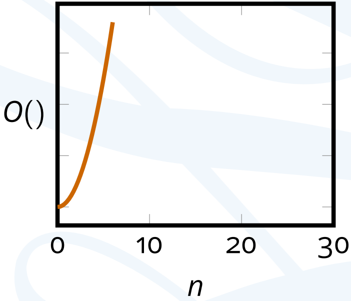

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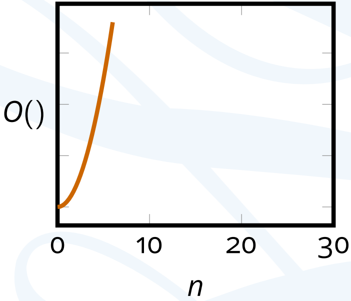


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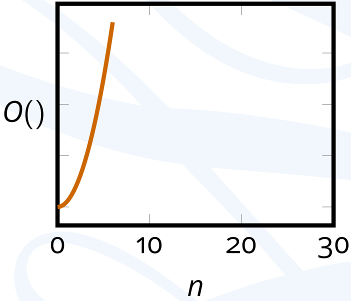

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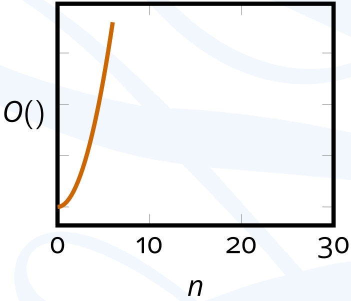
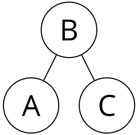


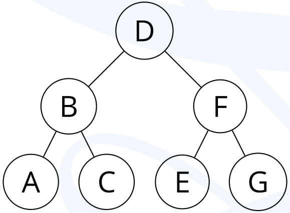
$O(\log n)$

I

Logarithmic complexity.

- Bit more complicated.
- Imagine you want to find a node in a tree (i.e. binary search).
  - How many nodes would you have to check?.





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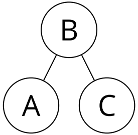
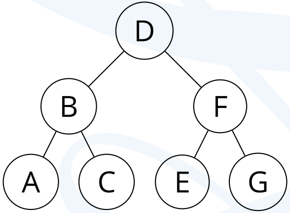


$n = 3$



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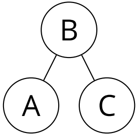
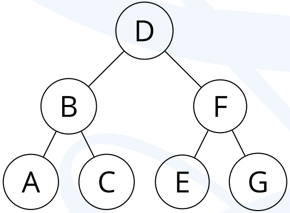


$$n = 3$$
$$O(\log n) = 1.58 \Rightarrow 1$$



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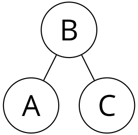

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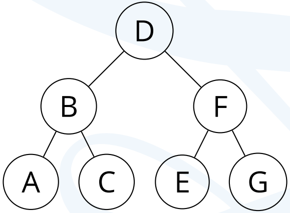

$n = 7$

$O(\log n)$

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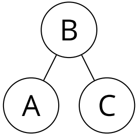

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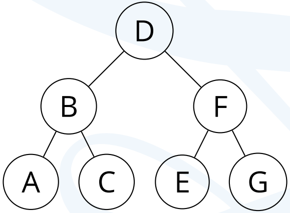
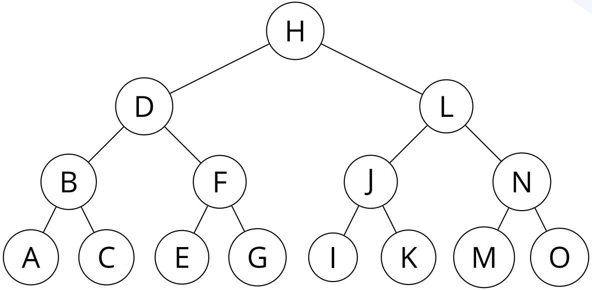


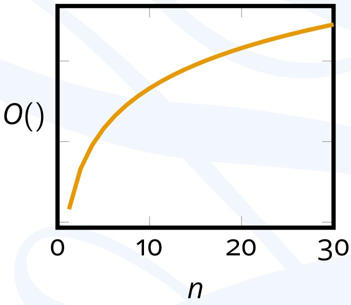
$$n = 7$$
$$O() = 2.81 \Rightarrow 2$$

$O(\log n)$ cont.

$O(\log n)$ complexity.

- Rate of increase gets lower and lower.
- $\log_2(100)$ is only 6.
- $\log_2(1000000000000)$ (trillion) is only 39.





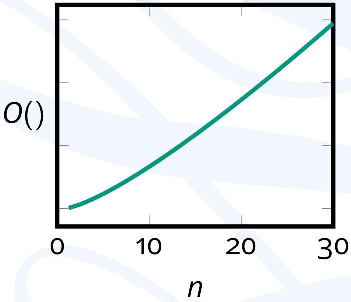
$$n = 15$$
$$O() = 3.91 \Rightarrow 3$$

$O(n \log n)$

A

Loglinear complexity.

- Looks more difficult than it is.
- $O(n \log n)$ means, do $O(\log n)$ n times.
- E.g. binary search for n items.
    - Binary search is $O(\log n)$.
    - Doing n binary searches.
    - So $O(n \log n)$.
- Lots of good sorting algorithms are $O(n \log n)$.

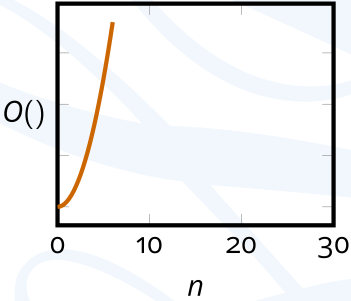


$O(2^n)$

A

Exponential complexity.

- Very, very bad.
- Each additional value doubles the time/space.
- Doesn't scale.
- $O(3^n)$, $O(4^n)$ etc. are all possible.

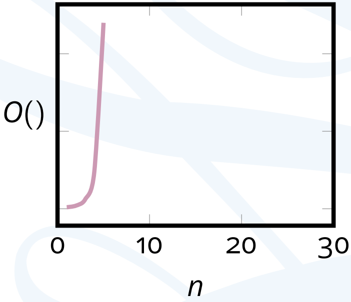


$O(n!)$

A

Factorial complexity.

- Just awful.
- Every possible combination of n items.
- Brute force travelling salesman is $O(n!)$.
- Totally impractical even for small values of n.



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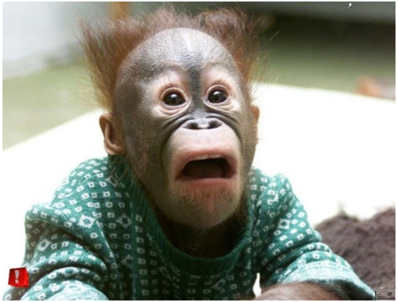
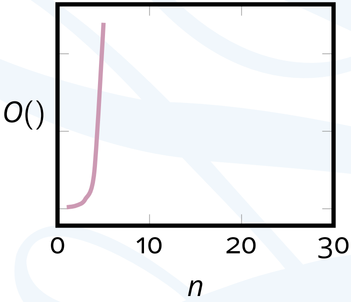




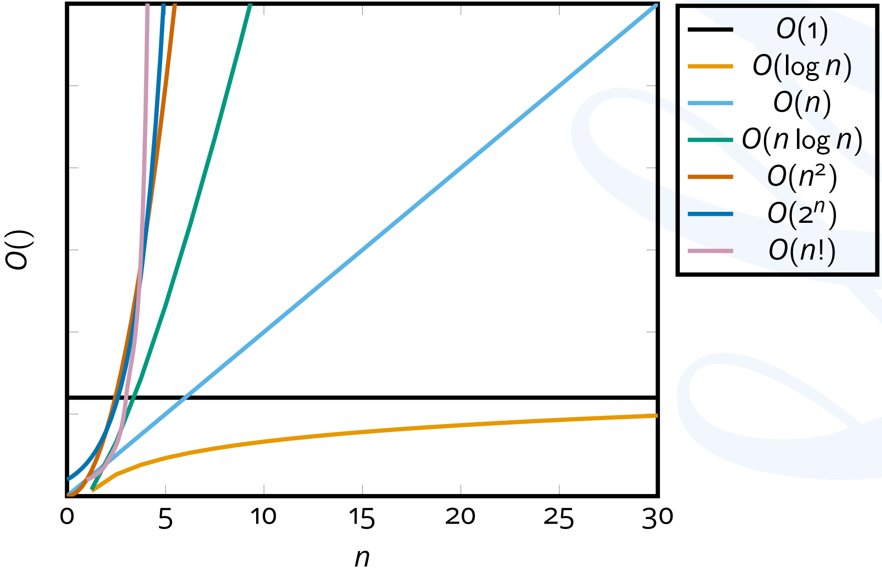
Scaling

Different $O()$ == wildly different complexity.

		n = 2	10	100
Best	$O(1)$	1	1	1
	$O(\log n)$	1	3	6
↑	$O(n)$	2	10	100
↓	$O(n \log n)$	2	33	664
	$O(n^2)$	4	100	10000
	$O(2^n)$	4	1024	$1.27 \cdot 10^{30}$
Worst	$O(n!)$	2	3628800	$9.33 \cdot 10^{157}$

Comparison

A



Complexity vs. Time

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Complexity isn't the same as efficiency.

- A good $O(n^2)$ implementation can be better than a bad $O(n)$.
  - For a while.
- Eventually, as n increases, $O(n)$ will always outperform $O(n^2)$ etc.

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  - For a while.
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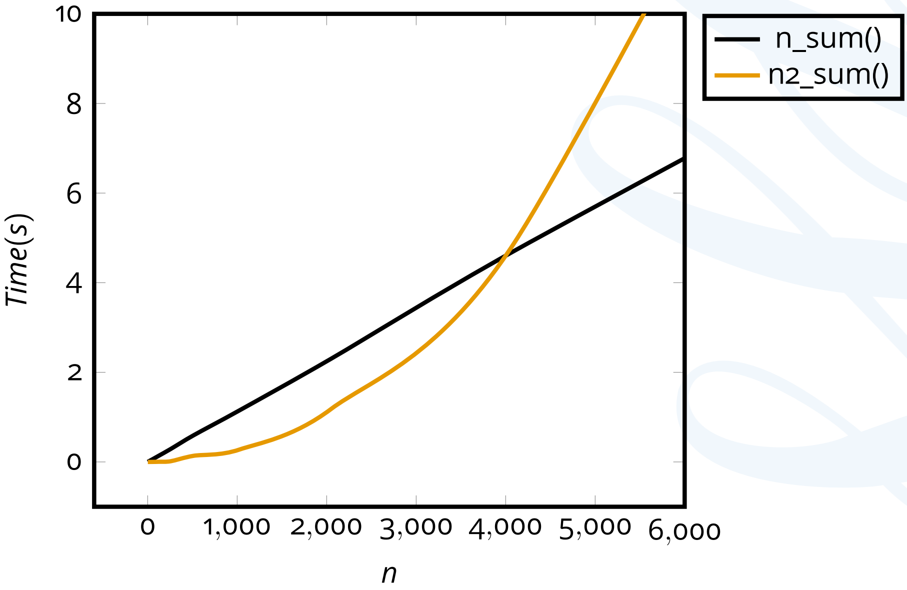
```python
def n_sum(sequence):
    total = 0
    for i in range(len(sequence)):
        total += sequence[i]
        time.sleep(0.001)
    return total
```

lec_fast_slow_functions.py

```python
def n2_sum(sequence):
    total = 0
    for i in range(len(sequence)):
        counter = 0
        while counter < i:
            counter += 1
        total += sequence[counter]

    return total
```

Time results



Why do I care?

C

- Everyone
  - Thinking algorithmically is critical programming skill.
  - Learning how to break down a problem into small steps.
    - Functional decomposition.
  - Evaluate algorithms.
    - Does this algorithm actually work?
  - Interview questions.
  - Without *O*() notation we can't discuss how algorithms compare.
  - Without *O*() can't discuss why some tasks are effectively impossible (travelling salesman).
- Ethical Hackers - *O*() important in discussing password security.
- Games Tech - *O*() explains the need for path finding and graphics work arounds.

Recap

- What is an algorithm.
- Code vs. algorithms.
- Heuristics = good enough solutions.
- $O()$ describes algorithm complexity in time and/or space.
- How your code *should* scale.
- $O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$
- $< O(n^2)$ means polynomial.
- $\geq O(2^n)$ means exponential.
- Polynomial = easy problems.
- Non-polynomial = hard problems.

The End