122COM: Introduction to algorithms

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Overview

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Introduction

C

Introduction to algorithms module.

- What is an algorithm?
- Not the same as code.
- Not the same as a program.

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Task/algorithm/code C

A task is a problem that needs to be solved.
- I.e. bake me a cake.

An algorithm is a generalised set of instructions to perform a specific task.
- A strategy to solve a given problem.
  - Many different strategies to solve same task.
- Like a recipe.

Code is a specific set of instructions to perform a specific task.
- An implementation of a strategy in a specific language/system.
- Have to adapt the algorithm to the specific features and abilities of the language.

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Fibonacci sequence algorithm   C

Task - calculate the fibonacci sequence.
Algorithm

1. Starting with 0 and 1.
2. Sum the two numbers to make a third.
3. Discard the lowest number.
4. Repeat from step 2.

Recursive Python

```python
def fibonacci( a, b ):
    c = a + b
    a, b = b, c

    print( a )
    fibonacci( a, b )

fibonacci( 0, 1 )
```

Iterative C++

```cpp
for( int a=0, b=1, c;
     a>=0;
     c=a+b, a=b, b=c )
{
    cout << a << endl;
}
```

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Solvable/Unsolvable   C

Some problems we can solve perfectly.

- Easy problems.
  - Fibonacci sequence.
  - Searching algorithms.
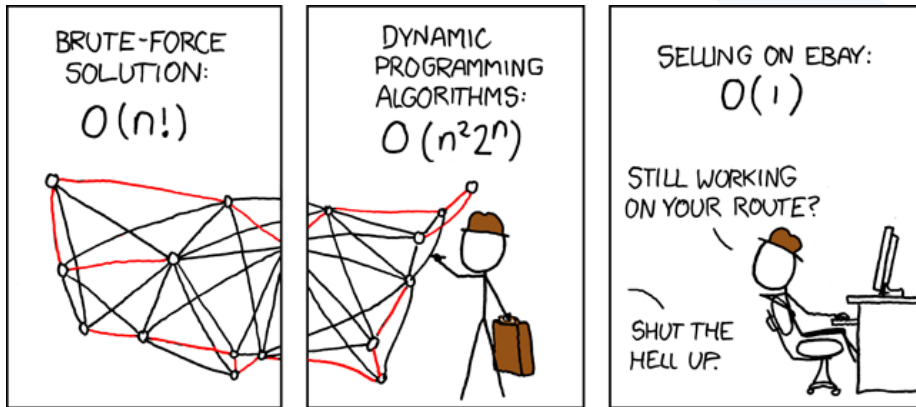 - Polynomial time.

Some problems we can't solve.

- Hard problems.
  - Because they are provably unsolvable.
    - Literally impossible.
    - Investigate the Halting State problem.
  - Because they take too long to solve.

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Fibonacci example

Difficulty

Module content

Profiling
Efficiency
Optimization

O() notation
Simple algorithms
Good algorithms
Bad algorithms

Recap

Heuristics

Some problems we could solve perfectly if only we had infinite computers/time.

- Travelling salesman.
  - Hard problem, non-polynomial (will discuss later).
  - Can only solve very simple versions of the problem perfectly.
  - 5 cities = 120 possible solutions, 20 cities = 2 432 902 008 176 640 000 possible solutions.

Heuristic algorithms.
- Don't promise to find the best solution.
- Quickly find a 'good enough' solution.

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Heuristics

https://xkcd.com/399/

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Looking at searching and sorting algorithms in later weeks.

Will be tested on some algorithmic concepts.

- 🟢 Implement simple algorithms.
- 🟢 Describe advantages/disadvantages of certain algorithms.
- 🟡 Big *O* notation.
  - How algorithms scale.
- 🔴 Calculate an algorithm's *O*() notation.

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Efficiency C

When writing software need to think about its efficiency.

- Time.
- Memory.
- Time vs Memory.
    - Can you trade one for the other
    - I.e. data stored in RAM costs memory but saves time.
    - I.e. data stored on hard drive saves memory but costs time.

But don't be too concerned with performance.

Optimization C

"Premature optimization is the root of all evil"

–Donald Knuth

For any large piece of code you should:

- Write clear, easily understood code. Focus on getting the behaviour right, not on performance.
    - If it doesn't work then no-one cares how efficiently it fails.
- Test the performance.
    - It may be fine.
- Measure your code to get the baseline performance.
    - So that you know if you are making things better or worse.
- Ideally using profiling tools.
    - Investigate in your own time.

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$O()$ notation   C

Measuring performance identifies how your code is running for **those** inputs on **that** machine.

- Not good at measuring how code will scale.
    - Change in response to different inputs.
    - Change in response to problem size.
- Algorithmic complexity.
    - $O()$ notation or Big-O notation.
- Certain algorithms are known to be better than other algorithms.

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$O()$ notation

Used to describe complexity in terms of time and/or space.

- Commonly encountered examples…
  - $O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$,

    $O(2^n)$ and $O(n!)$
- n refers to the size of the problem.
  - E.g. n values to be sorted.

  - E.g. n values to be searched.
- $O()$ notation describes the worst case scenario.
  - Usually, unless otherwise stated.

- $O()$ notation is discussed in detail next year.
  - Main idea is to capture the dominant term: the thing that is most important when the size of the input (n) gets big.

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$O(n)$ C

Linear complexity.

- n is directly proportional to time/space required
  - E.g. n doubles then time/space doubles.
- E.g. linear/sequential search.

```
a = [ 0, 1, 2, 3, 4, 5, 6, 7, 42 ]

for i in a:                        (n)
  if i == 42:                      (n)
    print('Found it')             (1)
    break                          (1)
```


$O()$ vs n
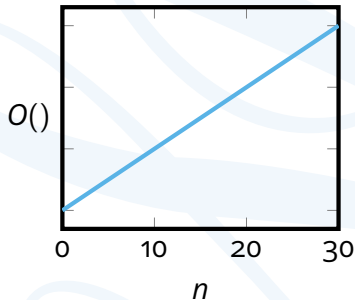
● So the algorithm takes $n + n + 1 + 1 = 2n + 2$ operations.
  - BUT! We would say it has complexity $O(n)$, constant values are irrelevant.

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$O(1)$ C

Constant complexity.

- *n* doesn't matter.
- Always takes same time/space.
- E.g. getting first item in an array.

```python
a = [ i for i in range(100) ]      (n)
b = [ i for i in range(1000000) ]  (m)

print(a[0])                        (1)
print(b[0])                        (1)
```
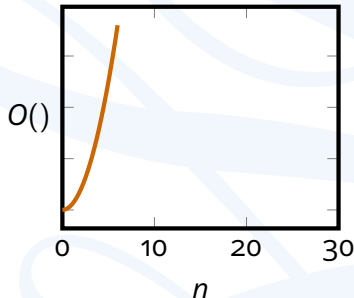

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$O(n^2)$ **I**
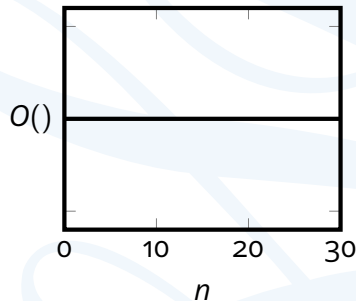
Quadratic complexity.

- A lot of simple sorting algorithms are $O(n^2)$.
- Nested `for` loops are common example.
- $O(n^3)$, $O(n^4)$, $O(n^m)$ etc. are all possible.
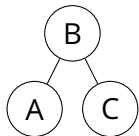- 🔴 Polynomial time.

```python
print('The n times tables')      (1)

for i in range(n):               (n)
  for j in range(n):             (n*n)
    print(i*j)                   (n*n)
```


$O()$ plotted against n (axis marked 0, 10, 20, 30).
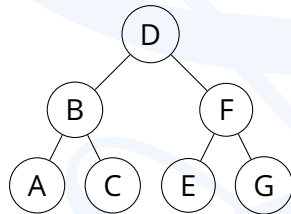
$O(\log n)$

Logarithmic complexity.

- Bit more complicated.
- Imagine you want to find a node in a tree (i.e. binary search).
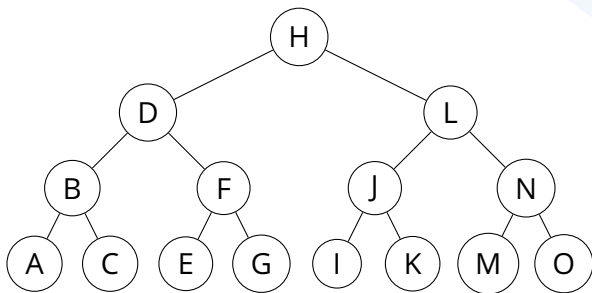 - How many nodes would you have to check?.

$$n = 3$$
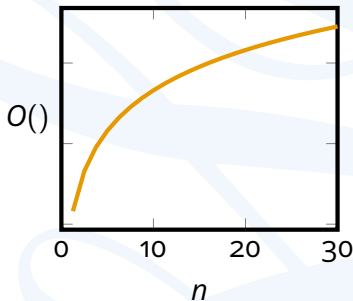$$O(\log n) = 1.58 \Rightarrow 1$$

$$n = 7$$
$$O() = 2.81 \Rightarrow 2$$

$O(\log n)$ cont.

$O(\log n)$ complexity.

- Rate of increase gets lower and lower.
- $\log_2(100)$ is only 6.
- $\log_2(1000000000000)$ (trillion) is only 39.
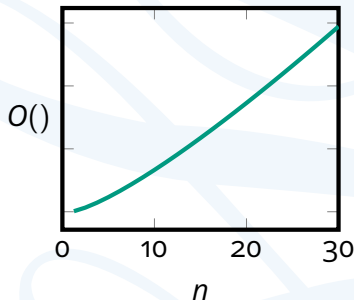
$n = 15$

$O() = 3.91 \Rightarrow 3$

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$O(n \log n)$ A

Loglinear complexity.

- Looks more difficult than it is.
- $O(n \log n)$ means, do $O(\log n)$ n times.
- E.g. binary search for n items.
  - Binary search is $O(\log n)$.
  - Doing n binary searches.
  - So $O(n \log n)$.
- Lots of good sorting algorithms are $O(n \log n)$.

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**Bad algorithms**
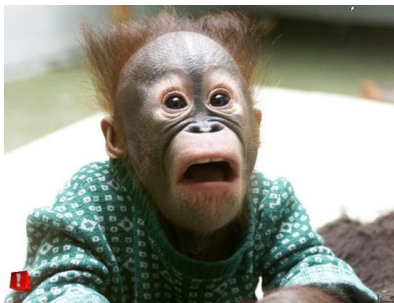
Recap

$O(2^n)$ **A**

Exponential complexity.

- Very, very bad.
- Each additional value doubles the time/space.
- Doesn't scale.
- $O(3^n)$, $O(4^n)$ etc. are all possible.

$O()$

n

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$O(n!)$ A
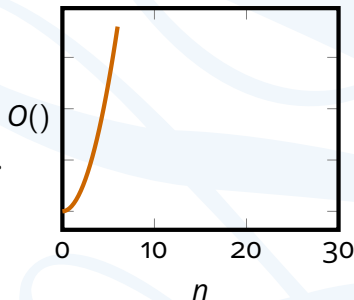
Factorial complexity.

- Just awful.
- Every possible combination of *n* items.
- Brute force travelling salesman is $O(n!)$.
- Totally impractical even for small values of *n*.

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Scaling
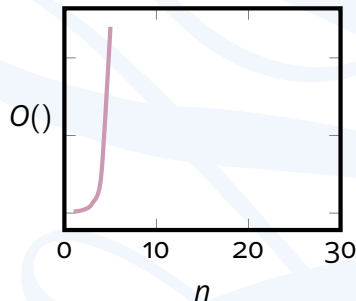
Different $O()$ == wildly different complexity.

		n		
		2	10	100
Best	$O(1)$	1	1	1
	$O(\log n)$	1	3	6
↑	$O(n)$	2	10	100
↓	$O(n \log n)$	2	33	664
	$O(n^2)$	4	100	10000
	$O(2^n)$	4	1024	$1.27 \cdot 10^{30}$
Worst	$O(n!)$	2	3628800	$9.33 \cdot 10^{157}$

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Comparison   A

Legend:
- $O(1)$
- $O(\log n)$
- $O(n)$
- $O(n \log n)$
- $O(n^2)$
- $O(2^n)$
- $O(n!)$

Axes: $O()$ versus n

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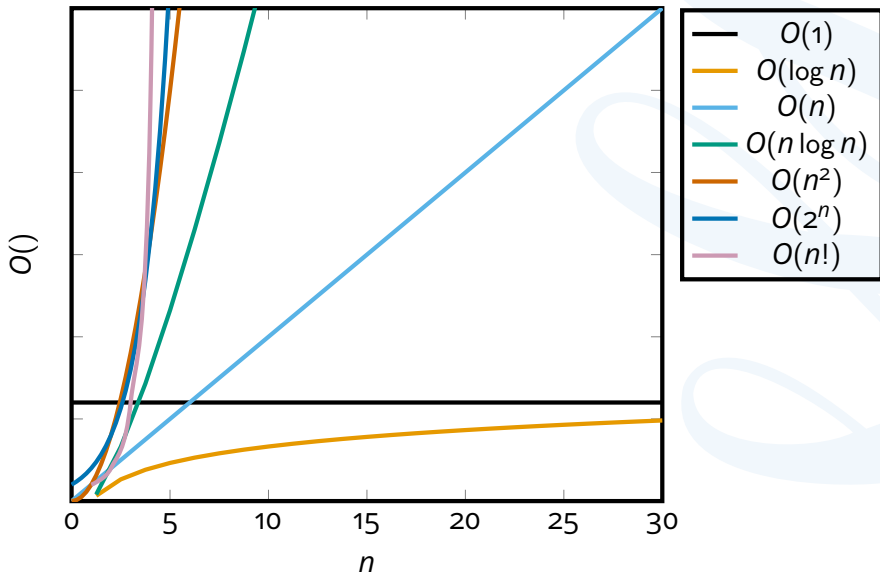
Complexity vs. Time
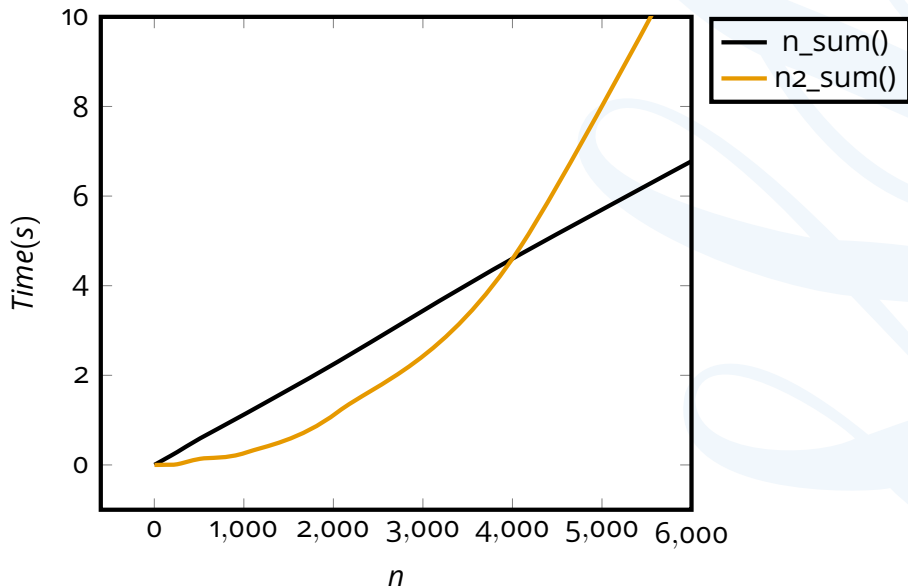
Complexity isn't the same as efficiency.

- A good $O(n^2)$ implementation can be better than a bad $O(n)$.
  - For a while.
- Eventually, as *n* increases, $O(n)$ will always outperform $O(n^2)$ etc.

```python
def n_sum(sequence):
    total = 0
    for i in range(len(sequence)):
        total += sequence[i]
        time.sleep(0.001)
    return total
```
lec_fast_slow_functions.py

```python
def n2_sum(sequence):
    total = 0
    for i in range(len(sequence)):
        counter = 0
        while counter < i:
            counter += 1
        total += sequence[counter]

    return total
```

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Time results

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Recap

Why do I care? C

- Everyone
    - Thinking algorithmically is critical programming skill.
    - Learning how to break down a problem into small steps.
        - Functional decomposition.
    - Evaluate algorithms.
        - Does this algorithm actually work?
    - Interview questions.
    - Without $O()$ notation we can't discuss how algorithms compare.
    - Without $O()$ can't discuss why some tasks are effectively impossible (travelling salesman).
- Ethical Hackers - $O()$ important in discussing password security.
- Games Tech - $O()$ explains the need for path finding and graphics work arounds.

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Recap

- 🟢 What is an algorithm.
- 🟢 Code vs. algorithms.
- 🟡 Heuristics = good enough solutions.
- 🟢 *O*() describes algorithm complexity in time and/or space.
- 🟢 How your code *should* scale.
- 🟡 $O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$
- 🔴 $< O(n^2)$ means polynomial.
- 🔴 $\geq O(2^n)$ means exponential.
- 🔴 Polynomial = easy problems.
- 🔴 Non-polynomial = hard problems.

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The End