Attention-Over-Actions Option-Critic

Computer Science Extended Essay Word Count:

Research Question

How are localized options trained in the Option-Critic architecture?

1 Introduction

As human we operate in high level actions. For example when driving a car, we make decisions about turning left or right instead of thinking about which muscle to contract. Human have the ability to group a chain of actions into one single high level action.

In Reinforcement Learning, we have a way to capture this idea of grouping action by using options [1]. When the options are defined, learning how to use them are very simple. However, if the options are not given and need to be learned, things get a lot harder since it requires knowing what makes an option good.

Many people argue that a good options should be diverse and localized, and many recent algorithms have followed this argument. Indeed, these algorithms have been able to discover options that are localized, so in this essay, I will try to derive a framework for localization in order to answer my research question, and I will develop an algorithm out of the framework to see whether it can actually train localized options.

This essay will be structured as follows: First, preliminary and related work will be presented to give a context to what I am trying to do. Second, previous work will be analyzed to figure out how localization is achieved. Third, a framework will be proposed based on the observations made in the analysis. Forth, an algorithm will be derived from the framework. Finally, the algorithm will be tested in the Four Rooms environment.

2 Preliminary

This section only acts as a summary. You are assumed to have basic knowledge about Reinforcement Learning and Options.

Markov Decision Process

Markov decision process (MDP) [4] is a mathematical framework for modeling decision making in a stochastic environment. It is defined as a tuple: $\langle S, A, r, \gamma, P \rangle$ where:

 \mathcal{S} is the set of states.

 \mathcal{A} is the set of actions

 $r: \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is the reward function

 $\gamma \in [0,1)$ is the discount factor that ensure the cumulative reward $\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)$ converges

 $P: \mathcal{S} \times A \times \mathcal{S} \to [0,1]$ is the transition model which gives the probability for a particular transition to occur.

All MDPs must follow the Markov Property, which means that everything is stateless and does not depend on the history. In an MDP, A policy $\pi: \mathcal{S} \times \mathcal{A} \to [0,1]$ is responsible for choosing the action, after an action is chosen, the environment transitions to a new state according to P(s, a, s'), and a reward is given to the policy. This process repeats until the environment enters a terminal state.

Reinforcement Learning

Reinforcement Learning (RL) [4] is a machine learning paradigm which allows an agent to learn from interaction in an MDP. The agent's goal is to maximizes a certain objective function.

Actor-Critic [5] is one of the popular classes of algorithms that harvest the advantage of both Q-Learning and Policy Gradient. An actor network is trained to choose the best action, while a critic network is trained to evaluate the decision made by the actor network.

Option Framework

In the Option Framework[1], instead of only using 1 policy, we use a set of options, each option is defined as a tuple: $\langle \mathcal{I}_{\omega}, \pi_{\omega}, \beta_{\omega} \rangle$, where:

 $\mathcal{I}_{\omega} \subseteq \mathcal{S}$ is the initiation set that define which state the option can be selected

 $\pi_{\omega}: \mathcal{S} \times \mathcal{A} \to [0,1]$ is the internal policy

 $\beta_{\omega}: \mathcal{S} \to [0,1]$ is the termination probability.

These options take turns to choose the action. A policy-over-options $\pi_{\Omega}: \mathcal{S} \times \Omega \to [0,1]$, where Ω is the set of options, decides which option to use. When an option is chosen, actions are chosen by its internal policy π_{ω} from then on, until it terminated according to its β_{ω} , then the policy-over-options π_{Ω} chooses an option again. An option has a chance to terminate every time environment transitions to a new state.

If the options are defined, the policy-over-options π_{Ω} can be learned by using SMDP Q-Learning [6] or Intra-Option Learning [7]. However, options are not always predefined and need to be discovered.

Four Rooms Environment

The Four Rooms environment is commonly used to evaluate algorithms that use options. The agent have 4 actions: up, down, left and right, which will move the agent in that corresponding direction. The choice of agent has 1/3 chance to fail and a random action will be chosen instead.

+50 reward will be given to the agent when it arrives to the goal state, and the episode will terminate after that.

Each cell position in the environment is mapped to a state number and is given to the agent in each step. The agent starts in a random state when an episode starts.

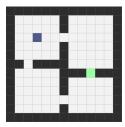


Figure 1: The Four Room environment. The black cells indicate the wall. The white cells indicate the area the agents can be in. The blue cell indicates the current position of the agent. The green cell indicates the goal.

3 Related Work

In this section, some option discovery algorithms will be summarized.

Option-Critic

Option-Critic [8] is an RL algorithm inspired by Action-Critic [5], where options are trained to maximize expected return, while an option-value function Q_{Ω} is trained to evaluate the decision the options.

Deliberation Cost

Since optimal policy can be achieved even without using options, if options are trained only to maximize expected return, they may degenerate and either terminate every steps or never terminate. Deliberation Cost [9] is a way to encourage longer option duration by punishing option switching.

Interest Option-Critic

The original Option-Critic assumes that options can be initiated everywhere, Interest Option-Critic [2] tries to remove this assumption by introducing interest functions $I: \mathcal{S} \times \Omega \to [0,1]$ as a replacement for the initiation set. Experimental result shows that options learned by Interest Option-Critic is localized.

Termination-Critic

Termination-Critic [10] changes the objective of the termination function β from maximizing the expected return to minimize the entropy of the termination state. Since entropy can be interpreted as the information gain, this means minimizing the information gained from knowing the termination state, or in other words, making the termination state more predictable.

Attention Option-Critic

Attention Option-Critic [3] implements attention mechanism into Option-Critic. Different options are trained to attend to different features of the state. The attention units were trained to not only maximize the expected return, but also other things like maximizing difference between attention of different options.

4 Exploration

An analysis on localization will be conducted in this section.

What is Localization?

Localization is about options each responsible for a sub-task, or another way of looking at it is options each representing a skill. However, defining and measuring localization quantitatively is hard, which is why most work evaluate these option discovery algorithms qualitatively, by observing the agent acting for an episode in the environment. For example in the Four Rooms environment, only using one option in each room is considered as localization.

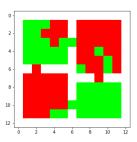


Figure 2: Red and green each represents an option, the options are pretty localized here.

Why Localization?

To understand why we want localization, first we need to answer a fundamental question: Why do we even use options in the first place? Is it to maximize expected return? However, optimal policy can

be achieved using only primitive actions. If options cannot give us a higher return, why do we even need options? Some researchers suggest that options should speed up planning [9] [10] and also options should be transferable [2] [3].

If this is what a good option should be like, then a set of localized options would be beneficial. Localized options are easy to interpret, which made it easily reusable when transferred to a different environment. Also, easy-to-interpret options can speed up planning because each options have its clear purpose and usage.

How Localization is achieved?

Now I will analyze how some of the previous work achieve localization of options.

Attention Option-Critic

In Attention Option-Critic [3], each options are trained to attend to different features of the state. My hypothesis is that the attention mechanism can act as a constraint on what kind of policy each option can have. Each features of the state represents a piece of information about the state. When performing a sub-task, not all the features are necessary. Each sub-task requires different subset of features. Since the attention mechanism limits the subset of features given to an option, the option cannot learn sub-task that requires features outside of the subset of features it was given, or else the option will perform poorly. In the algorithm, each option is trained to have diverse attention, which force each option to learn to complete a different sub-task.

For example, there is 3 options and an RGB 2D image is the features of the state. Suppose the 3 options each attend to one of the RGB channels, and one of the sub-tasks is checking if there is a purple circle on the image. In this case, only the option with attention on the green channel can complete this sub-task.

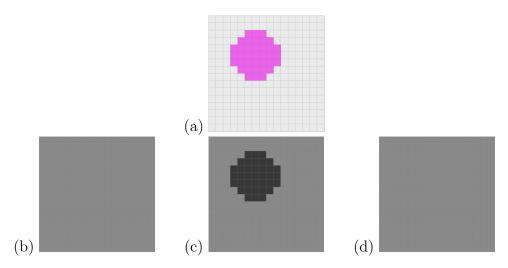


Figure 3: (a) is the RGB input, (b), (c) and (d) are the red, green and blue channel respectively. The circle can only be seen in the green channel.

Deliberation Cost

In Deliberation Cost [9], options are encouraged to be more temporally extended. Since options that terminates every step must be non-localized, Deliberation Cost can increase the chance of achieving localization.

Termination-Critic

In Termination-Critic [10], option termination states' entropy is being minimized, and experimental

results show that option trained by this usually choose to terminate in bottleneck states (frequently visited states). My hypothesis is that bottleneck states are usually the start or end of a sub-task, having the option terminate at these states essentially chains termination with initiation.

I will illustrate this with a simple example: In the Four Rooms environment, assume that the sub-task is walking from one doorway to another. The two doorway are bottleneck states because the agent must go through them. Since the agent can take on many paths, all the other states are not bottleneck states. When the agent get to the next doorway, another option can be immediately initiated.

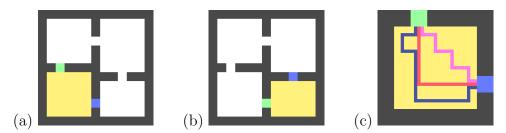


Figure 4: (a) and (b) are two options in the theoretic example. Green is the start of the option, blue is the end of the option, yellow is the intermediate states the option may encounter. (c) is a zoomed-in version of the bottom left room. Dark blue, red and pink lines are some of the paths the option can take.

Interest Option-Critic

In Interest Option-Critic. My hypothesis is that the interest function made the policy-over-options bias to choosing one of the options in a state, and since the paper uses a neural network as the interest function, the policy-over-options will also bias to choosing that option in a neighboring states.

5 The Localization Framework

Naturally, the next question that will be asked is: What do all of these algorithms have in common? Now I will propose a framework that can act as an abstraction for all of these algorithms.

The Localization Framework

1. Grouping

Group states into meaningful sub-tasks based on a certain criterion

2. Assignment

Assign the sub-tasks to different options

3. Optimization

Train options to perform well in the sub-task it is given and also improve the initial grouping of sub-tasks

4. Selection

Form the policy-over-options to select option in different state

This framework is inspired by Adaboost [11], which is an Ensemble Learning algorithm from Supervised Learning. There are a lot of similarities between Ensemble Learning and Option Learning, this has already been pointed out in previous work [12], the individual weak classifiers can be thought of as options.

Each weak classifier is responsible for classifying a small subset of the training data, just like how each option is responsible for a sub-task. In Adaboost, a bunch of weak classifiers are trained sequentially, each of them focuses on training data that is classified poorly by the previous weak classifiers.

Since this training process involves dividing training example into groups, then assign it to different weak classifiers, it inspires the Grouping and Assignment steps in the Localization Framework. Also, the

Optimization step in the framework is reminiscent of the weak classifiers learning to classify the training examples. After a lot of weak classifiers are trained, Adaboost combines them together to form a boosted classifier. The boosted classifier is the weighted sum of all the weak classifiers, the weighting is somewhat like a selection process, so it inspires the Selection step in the framework.

	Grouping	Assignment	Optimization	Selection
Attention Option- Critic	Group states that need the same sets of features	The algorithm assign an attention mecha- nism to each option	Options and attention mechanisms maximize return	Choose the option with maximum expected return
Termination- Critic	Group states between two bottleneck states	Each option terminates in a bottleneck state	Internal policy maximize return, termination function minimize entropy	Choose the option with maximum expected return
Interest Option- Critic	Group states that are close together	The algorithm assign an interest function to each option	Options and interest functions maximize return	Choose the option with high expected return and interest

Table 1: Previously mentioned algorithms can be fitted into the Localization Framework

This framework is the abstraction of algorithms that produce localized options, so it is very useful in deriving a new algorithm in the next section.

6 Attention-Over-Actions Option-Critic

Now that there is a framework, I can just follow the framework and derive a new algorithm. The following algorithm will be called Attention-Over-Actions Option-Critic because it perform abstraction on the action space, this algorithm is largely inspired by Attention Option-Critic.

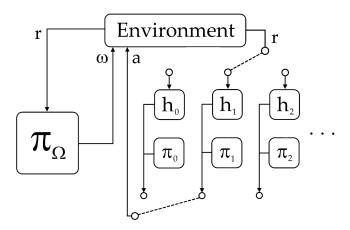


Figure 5: Visualization of the interaction between the environment and the Attention-Over-Actions Option-Critic algorithm.

1. Grouping

In Attention Option-Critic, the Grouping step divide sub-tasks based on features needed, this works because each sub-task requires a different subset of features. The following algorithm borrow the idea of attention over state features and apply it to actions, so each option will attend to a different sets of actions.

The intuition for this is that each sub-task will not need to use all the actions, for example, you would not consider performing a kicking action when you are driving a car. This is essentially performing actions abstraction, since the option can only attend to a subset of the actions.

2. Assignment

The following algorithm assigns the sub-task in a similar way as Attention Option-Critic, an attention mechanism $h_{\omega,\phi}:\mathcal{A}\to[0,1]$ parameterized by ϕ will be given to each option. But instead of masking the state features, it masks the probability for choosing each actions. So the final probability $\pi_{h_{\omega}}$ for option ω to choose action a will be:

$$\pi_{h_{\omega}}(a|s) = \frac{\pi_{\omega,\theta}(a|s)h_{\omega,\phi}(a)}{\sum_{a'} \pi_{\omega,\theta}(a'|s)h_{\omega,\phi}(a')}$$

where $\pi_{\omega,\theta}$ is the internal policy of option ω and is parameterized by θ .

3. Optimization

The optimization of the attention mechanism and that of the option will be described separately.

Option Optimization

Let's first consider the optimization of the option. I will train the internal policy and termination function just like in Option-Critic. They will maximize the expected return by using gradient ascent:

$$\theta \leftarrow \theta + \alpha_{\theta} \nabla_{\theta} Q_{\Omega}(s, \omega)$$

$$\nu \leftarrow \nu + \alpha_{\nu} \nabla_{\nu} U_{\Omega}(s', \omega)$$

where θ and ν are the parameters for the internal policy and termination function respectively, $Q_{\Omega}(s,\omega)$ and $U_{\Omega}(s',\omega)$ are the expected return for choosing the option ω in state s and entering the state s when using option ω respectively.

I can directly reuse the result from the Option-Critic paper:

$$\nabla_{\theta} Q_{\Omega}(s, \omega) = E[\nabla_{\theta} \log \pi_{\omega, \theta}(a|s) Q_{U}(s, \omega, a)]$$

$$\nabla_{\nu} U_{\Omega}(s', \omega) = E[\nabla_{\nu} \beta_{\omega, \nu}(s') A(s', \omega)]$$

where $Q_U(s,\omega,a)$ is the expected return for choosing action a when using option ω in state s.

The reason why I ignored the attention mechanism here is that the internal policy should not know the about the attention mechanism, or else it may want to revert the effect of the attention mechanism, i.e. Assigning a high weight to an action with low attention.

Attention Mechanism Optimization

Now let's consider the optimization of $h_{\omega,\phi}$. I want the grouping to be different for all options because or else all options will just aim for the sub-task with highest return. I also want the options to focus on as little actions as possible while still having acceptable performance. Essentially what I need is the algorithm to consider the trade-off between these objectives and achieve a balance between them. A nice way to do this is to add all of these objective up and then perform gradient ascent on the sum:

$$\phi \leftarrow \phi + \alpha_{\phi} \nabla_{\phi} \sum_{o} (w_{o} O_{o})$$

where o is the index of an objective, w_o is the weight of the objective, O_o is the objective function. This method has been used for Attention Option-Critic too. Now I will list out the objectives that I want the option to consider:

- 1. Perform well
- 2. Different from other options
- 3. The components of the attention mechanism is close to 0 or 1
- 4. Focus on small set of actions

For the first objective, I can just use $Q_{\Omega}(s,\omega)$ like in Attention Critic.

$$\max_{b} O_1 = \max_{b} Q_{\Omega}(s, \omega)$$

For the second objective, I will minimize cosine similarity just like in Attention Critic.

$$\min_{h} O_2 = \min_{h} \sum_{h' \neq h} C(h, h') = \min_{h} \sum_{h' \neq h} \frac{\langle h', h \rangle}{||h'|| \times ||h||}$$

For the third objective, I will minimize entropy in the attention mechanism. Entropy measures the uncertainty in a probability distribution, so h should be normalized first.

$$\max_{h} O_3 = \max_{h} H(\frac{h}{||h||}) = \max_{h} < \frac{h}{||h||}, \log \frac{h}{||h||} >$$

For the forth objective, I will minimize the length of the attention mechanism, which discourage focusing on too many actions.

$$\min_{h} O_4 = \min_{h} ||h||$$

4. Selection

Any policy-over-options that favor higher Q-value options will work in this case, because the option will need the right set of actions in order to perform well, or else it will fail horribly. So the Q-value already encoded which option has the right set of actions. This means that policies like ϵ -greedy should work for this algorithm.

Algorithm 1 Pseudocode for Attention-Over-Actions Option-Critic (AOAOC)

```
s \leftarrow s_0
Choose \omega according to the policy-over-options \pi_{\Omega}(s)
repeat
     Choose a according to \pi_{h_{\omega}}(a|s)
     Take action a in s, observe s', r
     1. Options evaluation:
     \delta \leftarrow r - Q_U(s, \omega, a)
     if s' is non-terminal then
      \delta \leftarrow \delta + \gamma (1 - \beta_{\omega,\nu}(s')) Q_{\Omega}(s',\omega) + \gamma \beta_{\omega,\nu}(s') \max_{\omega'} Q_{\Omega}(s',\omega')
     Q_U(s,\omega,a) \leftarrow Q_U(s,\omega,a) + \alpha\delta
     1. Options improvement:
     \theta \leftarrow \theta + \alpha_{\theta} \nabla_{\theta} \log \pi_{\omega,\theta}(a|s) Q_U(s,\omega,a)
     \nu \leftarrow \nu + \alpha_{\nu} \nabla_{\nu} \beta_{\omega,\nu}(s') (Q_{\Omega}(s',\omega) - V_{\Omega}(s'))
     \phi \leftarrow \phi + \alpha_{\phi} \nabla_{\phi} \sum_{o} (w_{o} O_{o})
     if \beta_{\omega,\nu} terminates in s' then
      choose new \omega according to the policy-over-options \pi_{\Omega}(s)
     end if
     s \leftarrow s'
until s' is terminal
```

7 Experiment

In this section the algorithm AOAOC will be tested in the Four Rooms environment.

Initial Setup of the Experiment

 Q_{Ω} and Q_U each is represented as a tensor. I do not need to use a neural network here since the state space is discrete in Four Rooms.

 π_{Ω} is an ϵ -greedy policy for the Q_{Ω} , i.e. the policy-over-options has a probability of $1 - \epsilon + \frac{\epsilon}{n_{\Omega}}$ to select the best option, where n_{Ω} is the number of options.

 ν is a tensor, and $\beta_{\omega,\nu}$ is parameterized by ν . More precisely, $\beta_{\omega,\nu} = \sigma(\nu)$, where σ is the sigmoid function $\frac{1}{1+e^{-x}}$. This can ensure that $\beta_{\omega,\nu}$ is a probability.

 θ is also a tensor, and $\pi_{\omega,\theta}$ is parameterized by θ . More precisely, $\pi_{\omega,\theta}$ is the softmax over the components of different actions in θ . This can ensure that $\pi_{\omega,\theta}$ is a probability distribution over actions.

 ϕ is also a tensor, and $h_{\omega,\phi}$ is parameterized by ϕ . Just like as $\beta_{\omega,\nu}$, $h_{\omega,\phi} = \sigma(\phi)$. This can ensure that $h_{\omega,\phi}$ is between 0 and 1.

Also, I would like the agent to have a larger incentive to go to the goal in a shorter duration, so I added a punishment of -2 for each step taken by the agent.

Problematic Attention Mechanism

When running the experiment, two problem is encountered:

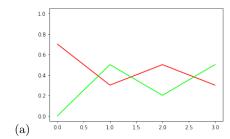
- 1. The attention mechanism h_{ω} often becomes all one.
- 2. The attention objectives conflict with one another.

Problem 1 leads to degenerate attention mechanism because the option is paying the same amount of attention to all actions. Suppose each of the component in the attention mechanism is a constant k, i.e. $h_{\omega} = [k, k, k, \ldots]$. The final policy will be:

$$\pi_{h_{\omega}}(a|s) = \frac{k \times \pi_{\omega}(a|s)}{\sum_{a'} k \times \pi_{\omega}(a'|s)} = \frac{\pi_{\omega}(a|s)}{\sum_{a'} \pi_{\omega}(a'|s)}$$

which is the same as not using attention at all.

This problem is probably caused by the objective 1, which is to maximizing $Q_{\Omega}(s,\omega)$. Having a uniform attention always yield a higher return because there will not be a constraint to which action it can use.



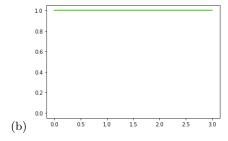


Figure 6: x-axis is the actions. y-axis is the attention on that action. Red and green lines each represents an attention mechanism. (a) is what the attention mechanism should look like. (b) is when it becomes all ones.

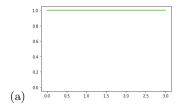
Problem 2 is follows directly from problem 1. Objective 1 will try to increase all the components to 1. Objective 3 will try to make only one of the components to 1, while the others all 0. Objective 4 will try to decrease all the components to 0.

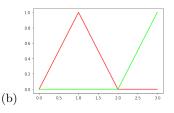
My original intention is that the algorithm will automatically balance each objective and find the optimal attention mechanism. However, it turns out that the attention mechanism almost always result in 1 of 3 situations:

1. All components are 1

- 2. All components are 0
- 3. Only 1 of the component is one, while the others are all zero

For most of the time, 1 of the objectives completely dominates and results in 1 of the 3 situations above.





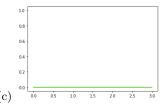


Figure 7: (a), (b) and (c) is the perfect end result for objective 1,3 and 4 respectively. All of which are degenerated attention mechanism.

8 Conclusion

9 Future Direction

This algorithm currently only focuses on discrete action space, a possible future direction might be extending this to continuous space. For vector actions, it might be good to attend to only some of the components of the vector, while ignoring other components by either randomly selecting values for them or keeping them the same as in the last state of the previous option. For scalar action, one direction is to use a one dimensional gaussian distribution as the attention mechanism, and multiply it with the original action distribution. However, some sort of trick may need to be deployed to speed up the process of normalization.

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Appendix

Notations

Markov Decision Process

S — Set of states

 \mathcal{A} — Set of actions

r — Reward function or reward

 γ — Discount factor

P — Transition model

 π — Policy

s — State

a — Action

 s_T — Terminal state

Option Framework

 Ω — Set of options

 $\pi_{\omega,\theta}$ — Internal policy of option ω

 β_{ω} — Termination probability of option ω

 \mathcal{I}_{ω} — Initiation set of option ω

 π_{Ω} — Policy-over-options

 Q_U — Expected return for choosing an action

 Q_{Ω} — Expected return for choosing an option

 A_{Ω} — Expected advantage for choosing an option

 V_{Ω} — Expected return in a state

 U_{Ω} — Expected return for arriving in a state

 θ — Parameter for internal policy

 ν — Parameter for termination probability

 ω — option

Attention-Over-Actions Option-Critic

 $h_{\omega,\phi}$ — Attention Mechanism

 $\pi_{h_{\omega}}$ — Final probability ϕ — Parameter for attention mechanism

 α — Learning rate for Q_U

 α_{θ} — Learning rate for internal policy

 α_{ν} — Learning rate for termination probability

 α_{ϕ} — Learning rate for attention mechanism

 O_o — Objective function

 w_o — Weight of objective function

o — Objective

 δ — One step Q-value error

Operations

 ∇ — Gradient

 $E[\]$ — Expected value

B Proof

B.1 Derivative of Objective 1

Let h_{ω}^{a} be the element in the attention vector h_{ω} that corresponds to the action a, i.e. $h_{\omega}=[h_{\omega}^{a_{0}},h_{\omega}^{a_{1}},...]$

$$\begin{split} \frac{\partial Q_{\Omega}(s,\omega)}{\partial h_{\omega}^{a}} &= \sum_{s,\omega} \mu_{\Omega}(s,\omega|s_{0},\omega_{0}) \sum_{a} \frac{\partial \pi_{h_{\omega}}(a|s)}{\partial h_{\omega}^{a}} Q_{U}(s,\omega,a) \\ &= E_{s,\omega,a \sim \pi_{h_{\omega}}} \left[\frac{\partial \log \pi_{h_{\omega}}(a|s)}{\partial h_{\omega}^{a}} Q_{U}(s,\omega,a) \right] \end{split}$$

The steps above follows directly from the Option-Critic appendix. The next step is to express $\frac{\partial \log \pi_{h_{\omega}}(a|s)}{\partial h_{\omega}^{a}}$ in a simpler form.

$$\begin{split} \frac{\partial \log \pi_{h_{\omega}}(a|s)}{\partial h_{\omega}^{a}} &= \frac{\partial}{\partial h_{\omega}^{a}} [\log \frac{\pi_{\omega}(a|s)h_{\omega}^{a}}{\sum_{a'} \pi_{\omega}(a'|s)h_{\omega}^{a'}}] \\ &= \frac{\partial}{\partial h_{\omega}^{a}} [\log \pi_{\omega}(a|s) + \log h_{\omega}^{a} - \log \sum_{a'} \pi_{\omega}(a'|s)h_{\omega}^{a'}] \\ &= \frac{\partial}{\partial h_{\omega}^{a}} [\log \pi_{\omega}(a|s)] + \frac{\partial}{\partial h_{\omega}^{a}} [\log h_{\omega}^{a}] - \frac{\partial}{\partial h_{\omega}^{a}} [\log \sum_{a'} \pi_{\omega}(a'|s)h_{\omega}^{a'}] \end{split}$$

 $\pi_{\omega}(a|s)$ is independent of h_{ω}^{a} , hence $\frac{\partial}{\partial h_{\omega}^{a}}[\log \pi_{\omega}(a|s)] = 0$

$$\begin{split} &= \frac{\partial}{\partial h_{\omega}^{a}} [\log h_{\omega}^{a}] - \frac{\partial}{\partial h_{\omega}^{a}} [\log \sum_{a'} \pi_{\omega}(a'|s) h_{\omega}^{a'}] \\ &= \frac{1}{h_{\omega}^{a}} - \frac{\pi_{\omega}(a|s)}{\sum_{a'} \pi_{\omega}(a'|s) h_{\omega}^{a'}} \\ &= \frac{1}{h_{\omega}^{a}} - \frac{\pi_{\omega}(a|s)}{\sum_{a'} \pi_{\omega}(a'|s) h_{\omega}^{a'}} \times \frac{h_{\omega}^{a}}{h_{\omega}^{a}} \end{split}$$

By definition $\pi_{h_{\omega}}(a|s) = \frac{\pi_{\omega}(a|s)h_{\omega}^{a}}{\sum_{a'} \pi_{\omega}(a'|s)h_{\omega}^{a'}}$

$$= \frac{1}{h_{\omega}^{a}} - \frac{\pi_{h_{\omega}}(a|s)}{h_{\omega}^{a}}$$

$$= \frac{1 - \pi_{h_{\omega}}(a|s)}{h_{\omega}^{a}}$$

Substitute back to the original calculation,

$$\frac{\partial Q_{\Omega}(s,\omega)}{\partial h_{\omega}^{a}} = E_{s,\omega,a \sim \pi_{h_{\omega}}} \left[\frac{1 - \pi_{h_{\omega}}(a|s)}{h_{\omega}^{a}} Q_{U}(s,\omega,a) \right]$$

B.2 Derivative of Objective 2

Let h_{ω}^{a} be the element in the attention vector h_{ω} that corresponds to the action a, i.e. $h_{\omega} = [h_{\omega}^{a_{0}}, h_{\omega}^{a_{1}}, ...]$ Since we would like to minimize the cosine similarity, the negative cosine similarity will be used instead.

$$\sum_{h_{\omega'} \neq h_{\omega}} \frac{\partial C(h_{\omega}, h_{\omega'})}{\partial h_{\omega}^{a}} = \sum_{h_{\omega'} \neq h_{\omega}} \frac{\partial}{\partial h_{\omega}^{a}} \frac{\langle h_{\omega}, h_{\omega'} \rangle}{||h_{\omega}|| \times ||h_{\omega'}||}$$

$$= \sum_{h_{\omega'} \neq h_{\omega}} \frac{||h_{\omega}|| \times ||h_{\omega'}|| \frac{\partial}{\partial h_{\omega}^{a}} \langle h_{\omega}, h_{\omega'} \rangle - \langle h_{\omega}, h_{\omega'} \rangle \frac{\partial}{\partial h_{\omega}^{a}} ||h_{\omega}|| \times ||h_{\omega'}||}{(||h_{\omega}|| \times ||h_{\omega'}||)^{2}}$$

I will calculate each of the derivative separately:

$$\begin{split} \frac{\partial}{\partial h^a_\omega} < h_\omega, h_{\omega'} > &= \frac{\partial}{\partial h^a_\omega} \sum_{a'} h^{a'}_\omega h^{a'}_{\omega'} = h^a_{\omega'} \\ \\ \frac{\partial}{\partial h^a_\omega} ||h_\omega|| \times ||h_{\omega'}|| &= ||h_{\omega'}|| \frac{\partial}{\partial h^a_\omega} \sqrt{\sum_{a'} (h^{a'}_\omega)^2} = ||h_{\omega'}|| \frac{h^a_\omega}{\sqrt{\sum_{a'} (h^{a'}_\omega)^2}} \end{split}$$

Substitute back to the original calculation.

$$\begin{split} \sum_{h_{\omega'} \neq h_{\omega}} \frac{\partial}{\partial h_{\omega}^{a}} \frac{\langle h_{\omega}, h_{\omega'} \rangle}{||h_{\omega}|| \times ||h_{\omega'}||} &= \sum_{h_{\omega'} \neq h_{\omega}} \frac{||h_{\omega}|| \times ||h_{\omega'}||h_{\omega'}^{a} - \langle h_{\omega}, h_{\omega'} \rangle ||h_{\omega'}|| \frac{h_{\omega}^{a}}{\sqrt{\sum_{a'} (h_{\omega}^{a'})^{2}}}}{(||h_{\omega}|| \times ||h_{\omega'}||)^{2}} \\ &= \sum_{h_{\omega'} \neq h_{\omega}} \frac{h_{\omega'}^{a}}{||h_{\omega}|| \times ||h_{\omega'}||} - \frac{\langle h_{\omega}, h_{\omega'} \rangle h_{\omega}^{a}}{(||h_{\omega}|| \times ||h_{\omega'}||) \times ||h_{\omega}||^{2}} \end{split}$$

B.3 Derivative of Objective 3

Let h_{ω}^{a} be the element in the attention vector h_{ω} that corresponds to the action a, i.e. $h_{\omega} = [h_{\omega}^{a_{0}}, h_{\omega}^{a_{1}}, ...]$ Since entropy usually applies to probabilities, I will normalize the attention unit into $\overline{h_{\omega}} = \frac{h_{\omega}}{\sum_{a'} h_{\omega}^{a'}}$

$$\begin{split} \frac{\partial H(\overline{h_{\omega}})}{\partial h_{\omega}^{a}} &= \frac{\partial \sum_{a'} \overline{h_{\omega}^{a'}} \log \overline{h_{\omega}^{a'}}}{\partial h_{\omega}^{a}} \\ &= \frac{\partial \overline{h_{\omega}^{a}} \log \overline{h_{\omega}^{a}}}{\partial h_{\omega}^{a}} + \sum_{\overline{h_{\omega}^{a'}} \neq \overline{h_{\omega}^{a}}} \frac{\partial \overline{h_{\omega}^{a'}} \log \overline{h_{\omega}^{a'}}}{\partial h_{\omega}^{a'}} \\ &= \frac{\partial \overline{h_{\omega}^{a}} \log \overline{h_{\omega}^{a}}}{\partial \overline{h_{\omega}^{a}}} \times \frac{\partial \overline{h_{\omega}^{a}}}{h_{\omega}^{a}} + \sum_{\overline{h_{\omega}^{a'}} \neq \overline{h_{\omega}^{a}}} \frac{\partial \overline{h_{\omega}^{a'}} \log \overline{h_{\omega}^{a'}}}{\partial \overline{h_{\omega}^{a'}}} \times \frac{\partial \overline{h_{\omega}^{a'}}}{h_{\omega}^{a'}} \end{split}$$

I will calculate each of the derivative separately:

$$\begin{split} \frac{\partial \overline{h_{\omega}^{a}} \log \overline{h_{\omega}^{a}}}{\partial \overline{h_{\omega}^{a}}} &= \log \overline{h_{\omega}^{a}} \frac{\partial \overline{h_{\omega}^{a}}}{\partial \overline{h_{\omega}^{a}}} + \overline{h_{\omega}^{a}} \frac{\partial \log \overline{h_{\omega}^{a}}}{\partial \overline{h_{\omega}^{a}}} = \log \overline{h_{\omega}^{a}} + 1 \\ \frac{\partial \overline{h_{\omega}^{a}}}{\partial h_{\omega}^{a}} &= \frac{\partial}{\partial h_{\omega}^{a}} \frac{h_{\omega}^{a}}{\sum_{b} h_{\omega}^{b}} = \frac{\sum_{b} h_{\omega}^{b} - h_{\omega}^{a}}{(\sum_{b} h_{\omega}^{b})^{2}} \\ \frac{\partial \overline{h_{\omega}^{a'}}}{\partial h_{\omega}^{a}} &= \frac{\partial}{\partial h_{\omega}^{a}} \frac{h_{\omega}^{a'}}{\sum_{b} h_{\omega}^{b}} = \frac{-h_{\omega}^{a'}}{(\sum_{b} h_{\omega}^{b})^{2}} \end{split}$$

Substitute back to the original calculation,

$$\begin{split} \frac{\partial \overline{h_{\omega}^{a}} \log \overline{h_{\omega}^{a}}}{\partial \overline{h_{\omega}^{a}}} &= (\log \overline{h_{\omega}^{a}} + 1) \frac{\sum_{b} h_{\omega}^{b} - h_{\omega}^{a}}{(\sum_{b} h_{\omega}^{b})^{2}} + \sum_{\overline{h_{\omega}^{a'}} \neq \overline{h_{\omega}^{a}}} (\log \overline{h_{\omega}^{a'}} + 1) \frac{-h_{\omega}^{a'}}{(\sum_{b} h_{\omega}^{b})^{2}} \\ &= \frac{(\log \overline{h_{\omega}^{a}} + 1)}{\sum_{b} h_{\omega}^{b}} + \sum_{h_{\omega}^{a'}} (\log \overline{h_{\omega}^{a'}} + 1) \frac{-h_{\omega}^{a'}}{(\sum_{b} h_{\omega}^{b})^{2}} \end{split}$$

B.4 Derivative of Objective 4

Let h_{ω}^{a} be the element in the attention vector h_{ω} that corresponds to the action a, i.e. $h_{\omega} = [h_{\omega}^{a_0}, h_{\omega}^{a_1}, ...]$ Since we would like to minimize the length, the negative length will be used instead.

$$\frac{\partial ||h_{\omega}||}{\partial h_{\omega}^{a}} = \frac{\partial}{\partial h_{\omega}^{a}} \sqrt{\sum_{a'} (h_{\omega}^{a'})^{2}}$$
$$= \frac{h_{\omega}^{a}}{\sqrt{\sum_{a'} (h_{\omega}^{a'})^{2}}} = \frac{h_{\omega}^{a}}{||h_{\omega}||}$$

C Experimental Details

D Code

The code below is based on codes in the ioc repository.[13]

fourrooms.py

```
import numpy as np
  from random import uniform
  class Fourrooms:
      def __init__(self, initstate_seed, punishEachStep, deterministic, modified, easier):
           self.punishEachStep = punishEachStep
           self.deterministic = deterministic
           self.modified = modified
           if easier:
               self.layout = """\
11
  WWWWWWWWWW
12
13
14
15
16
17
18
19
20
21
22
  WWWWWWWWWWW
24
            self.layout = """\
26
27
28
     W
29
        W
               W
30
  W
               W
```

```
w w
                W
32
33
  ww wwww
                W
  W
         www www
34
35
  W
         w
36
  W
         W
                W
37
  W
                W
38
  W
         W
39
  WWWWWWWWWWW
40
41
42
           self.occupancy = np.array([list(map(lambda c: 1 if c=='w' else 0, line)) for line in
43
                 self.layout.splitlines()])
45
           self.action_space = 4
46
           self.observation_space = int(np.sum(self.occupancy == 0))
47
48
49
           # 0 - Up
           # 1 - Down
# 2 - Left
50
51
           # 3 - Right
52
53
           self.directions = [np.array((-1,0)), np.array((1,0)), np.array((0,-1)), np.array((0,-1))]
54
                ((0,1))]
55
           self.rng = np.random.RandomState(1234)
56
57
           self.initstate_seed = initstate_seed
58
           self.rng_init_state = np.random.RandomState(self.initstate_seed)
59
60
           self.tostate = {}
61
62
           self.occ_dict = dict(zip(range(self.observation_space),
63
                                       np.argwhere(self.occupancy.flatten() == 0).squeeze()))
64
65
           statenum = 0
66
67
           for i in range(13):
                for j in range(13):
68
                     if self.occupancy[i, j] == 0:
    self.tostate[(i, j)] = statenum
69
70
                         statenum += 1
71
72
           self.tocell = {v:k for k,v in self.tostate.items()}
73
74
           self.goal = 62
75
           self.init_states = list(range(self.observation_space))
76
77
           self.init_states.remove(self.goal)
78
79
       def empty_around(self, cell):
80
           avail = []
81
           for action in range(self.action_space):
82
                nextcell = tuple(cell + np.multiply(self.directions[action], self.inQuad24(self.
83
                    currentcell)))
                if not self.occupancy[nextcell]:
84
                    avail.append(nextcell)
85
86
           return avail
87
       def reset(self, test=None):
88
           if test:
89
                state = test
90
91
                state = self.rng_init_state.choice(self.init_states)
92
93
           self.currentcell = self.tocell[state]
           return state
94
95
       def step(self, action):
96
```

```
reward = -2 * int(self.punishEachStep)
97
           if self.rng.uniform() < 1/3 and not(self.deterministic):</pre>
                empty_cells = self.empty_around(self.currentcell)
99
100
                nextcell = empty_cells[self.rng.randint(len(empty_cells))]
           else:
               nextcell = tuple(self.currentcell + np.multiply(self.directions[action], self.
                    inQuad24(self.currentcell)))
           if not self.occupancy[nextcell]:
                self.currentcell = nextcell
106
           state = self.tostate[self.currentcell]
           if state == self.goal:
               reward = 50
           done = state == self.goal
           return state, reward, float(done), None
116
       def inQuad24(self, cell):
           if not(self.modified):
               return np.array([1,1])
           if cell[1] > 6:
                if cell[0] < 7:</pre>
120
                    return np.array([1, -1])
12
           else:
122
                if cell[0] > 6:
123
                    return np.array([1, -1])
124
           return np.array([1, 1])
```

aoaoc_tabular.py

```
import numpy as np
  from fourrooms import Fourrooms
  from scipy.special import logsumexp, expit, softmax
  ======CLASS MAP======
  Option
      - FinalPolicy pi_h
          - Internal Policy (SoftmaxPolicy) pi_omega
           - Attention Unit (LearnableAttention/PredefinedAttention) h_omega
               - Value Objective (ValueObj) o1
               - Cosine Similarity Objective (CoSimObj) o2
               - Entropy Objective (EntropyObj) o3
12
               - Length Objective (LengthObj) o4
      - Termination Function (SigmoidTermination) beta_omega
14
      - Q_omega (Q_0)
  Policy Over Options (POO) pi_Omega
16

    Policy (EgreedyPolicy)

18
      - Q_Omega (Q_U)
  #=====Option======
20
  class Option:
21
22
      def __init__(self, rng, nfeatures, nactions, args, policy_over_options, index):
          self.weights = np.zeros((nfeatures, nactions))
23
          self.policy = FinalPolicy(rng, nfeatures, nactions, args, self.weights, index)
24
          self.termination = SigmoidTermination(rng, nfeatures, args)
25
26
          self.Qval = Q_U(nfeatures, nactions, args, self.weights, policy_over_options)
27
28
      def sample(self, phi):
          return self.policy.sample(phi)
29
30
      def terminate(self, phi, value=False):
31
32
          if value:
              return self.termination.pmf(phi)
33
```

```
else:
34
               return self.termination.sample(phi)
35
36
37
       def _Q_update(self, traject, reward, done, termination):
           self.Qval.update(traject, reward, done, termination)
38
39
       def _H_update(self, traject):
4(
           qVal = self.Qval.value(traject[0][0], traject[2])
41
           self.policy.H_update(traject, qVal)
42
43
       def _B_update(self, phi, option, advantage):
44
           self.termination.update(phi, option, advantage)
45
46
       def _P_update(self, traject, baseline):
47
           self.policy.P_update(traject, baseline)
48
49
       def update(self, traject, reward, done, phi, option, termination, baseline, advantage):
50
           self._Q_update(traject, reward, done, termination)
51
           self._H_update(traject)
           self._P_update(traject, baseline)
53
54
           self._B_update(phi, option, advantage)
55
56
   #=====Final Policy======
57
   class FinalPolicy:
58
       def __init__(self, rng, nfeatures, nactions, args, qWeight, index):
59
           self.rng = rng
60
           self.nactions = nactions
61
           self.internalPI = SoftmaxPolicy(rng, nfeatures, nactions, args, qWeight)
62
           if (args.h_learn):
63
               self.attention = LearnableAttention(nactions, args, index)
64
6.
               self.attention = PredefinedAttention(args, index)
66
67
       def pmf(self, phi):
68
           pi = self.internalPI.pmf(phi)
69
           h = self.attention.pmf()
70
71
           normalizer = np.dot(pi, h)
           return (pi*h)/normalizer
72
73
       def sample(self, phi):
74
75
           return int(self.rng.choice(self.nactions, p=self.pmf(phi)))
76
       def H_update(self, traject, qVal):
77
           self.attention.update(traject, self.pmf(traject[0][0]), qVal)
78
79
       def P_update(self, traject, baseline):
80
81
           self.internalPI.update(traject, baseline)
82
83
   #======Internal Policy======
84
   class SoftmaxPolicy:
85
       def __init__(self, rng, nfeatures, nactions, args, qWeight):
86
           self.rng = rng
87
           self.nactions = nactions
           self.temp = args.temp
89
           self.weights = np.zeros((nfeatures, nactions))
90
91
           self.qWeight = qWeight
           self.lr = args.lr_intra
92
93
       def _value(self, phi, action=None):
94
           if action is None:
95
               return np.sum(self.weights[phi, :], axis=0)
96
           return np.sum(self.weights[phi, action], axis=0)
97
98
       def pmf(self, phi):
99
           v = self._value(phi)/self.temp
100
           return np.exp(v - logsumexp(v))
```

```
def sample(self, phi):
           return int(self.rng.choice(self.nactions, p=self.pmf(phi)))
104
       def update(self, traject, baseline):
106
           actions_pmf = self.pmf(traject[0][0])
           critic = self.qWeight[traject[0][0], traject[2]]
108
           if baseline:
                critic -= baseline
           self.weights[traject[0][0], :] -= self.lr*critic*actions_pmf
           self.weights[traject[0][0], traject[2]] += self.lr*critic
   #===== Attention ======
   class LearnableAttention():
       def __init__(self, nactions, args, index):
117
           self.weights = np.random.uniform(low=-1, high=1, size=(nactions,))
           self.lr = args.lr_attend
           self.o1 = ValueObj(args)
           self.o2 = CoSimObj(args, index)
121
122
           self.o3 = EntropyObj(args)
           self.o4 = LengthObj(args)
123
           CoSimObj.add2list(self)
124
           self.normalize = args.normalize
125
126
       def pmf(self):
127
           if self.normalize:
               return np.clip(softmax(self.weights), 0.05, None)
129
           return expit(self.weights)
130
131
       def _grad(self):
           attend = self.pmf()
133
           return attend*(1. - attend)
134
135
       def attention(self, a):
136
           return self.pmf()[a]
137
138
139
       def update(self, traject, finalPmf, qVal):
           hPmf = self.pmf()
140
141
           gradList = [self.o1.grad(traject[0][0], traject[2], hPmf, finalPmf, qVal), self.o2.
                grad(hPmf), self.o3.grad(hPmf), self.o4.grad(hPmf)]
           self.weights += self.lr * np.sum(gradList, axis=0) * self._grad()
           if self.normalize:
143
               self.normalizing()
144
145
       def normalizing(self):
146
           self.weights -= np.mean(self.weights)
147
148
149
   class PredefinedAttention():
       def __init__(self, args, index):
           if (index==0):
                self.weights = np.array([1, 1, 1, 1])
           if (index == 1):
                self.weights = np.array([1, 1, 1, 1])
156
       def pmf(self):
157
158
           return self.weights
       def attention(self, a):
160
           return self.pmf()[a]
161
162
       def update(self, traject, finalPmf, qVal):
163
           pass
164
165
   #======Objectives======
168 class Objective:
```

```
def __init__(self, weight):
169
170
            self.weight = weight
171
172
       def grad(self):
            return None
173
       def loss(self):
175
            return None
176
177
179
   class ValueObj(Objective):
       def __init__(self, args):
180
            super().__init__(args.wo1)
181
182
       def grad(self, phi, a, hPmf, finalPmf, qVal):
183
            return self.weight * ((finalPmf + 1)/hPmf[a]) * qVal
184
185
       def loss(self):
186
187
            pass
188
189
   class CoSimObj(Objective):
190
       hList = []
191
192
       def __init__(self, args, index):
193
            super().__init__(args.wo2)
194
            self.index = index
195
196
       def grad(self, hPmf):
197
            gradient = []
198
            for i in range(len(hPmf)):
199
                derivative = 0.
200
                exclude = 0
201
202
                for a in self.hList:
                     if exclude == self.index:
203
                         continue
204
                     exclude +=1
205
206
                     normalizer = np.linalg.norm(hPmf)*np.linalg.norm(a.pmf())
205
                     term1 = a.pmf()[i]/normalizer
208
                     term2 = hPmf[i]*np.dot(hPmf,a.pmf()) / (normalizer*np.power(np.linalg.norm()))
209
                         hPmf),2))
                     derivative += -1*(term1 - term2)
                gradient.append(derivative)
211
            return self.weight * np.array(gradient)
212
213
       def loss(self):
214
            return np.sum([np.dot(hPmf,a.pmf())/(np.linalg.norm(hPmf)*np.linalg.norm(a.pmf()))
215
                for a in self.hList])
216
       @classmethod
217
       def add2list(cls, attention):
218
            cls.hList.append(attention)
219
220
221
       @classmethod
       def reset(cls):
222
            cls.hList = []
223
224
225
   class EntropyObj(Objective):
226
       def __init__(self, args):
            super().__init__(args.wo3)
228
229
       def grad(self, hPmf):
230
231
            gradient = []
            normalizer = np.sum(hPmf)
232
            normh = hPmf/normalizer
233
           for i in range(len(hPmf)):
234
```

```
term1 = (1.+np.log(normh[i]))/normalizer
235
                term2 = np.sum([(1.+np.log(normh[index]))*hPmf[index]/(normalizer**2) for index
                    in range(len(hPmf))])
237
                gradient.append((term1-term2)*(self.loss(hPmf)-0.69))
           return self.weight * np.array(gradient)
239
       def loss(self, hPmf):
240
           normalizer = np.sum(hPmf)
241
           normh = hPmf/normalizer
242
           return -1*np.sum(normh * np.log(normh))
243
244
245
   class LengthObj(Objective):
246
       def __init__(self, args):
247
248
           super().__init__(args.wo4)
249
       def grad(self, hPmf):
           return -1 * hPmf / self.loss(hPmf)
251
252
253
254
       def loss(self, hPmf):
           return np.linalg.norm(hPmf)
256
257
   #=====Termination Function======
258
   class SigmoidTermination:
259
       def __init__(self, rng, nfeatures, args):
260
           self.rng = rng
261
           self.weights = np.zeros((nfeatures,))
262
           self.lr = args.lr_term
263
           self.dc = args.dc
26
265
       def pmf(self, phi):
266
           return expit(np.sum(self.weights[phi]))
267
268
       def sample(self, phi):
269
           return int(self.rng.uniform() < self.pmf(phi))</pre>
270
271
       def _grad(self, phi):
279
273
            terminate = self.pmf(phi)
           return terminate*(1. - terminate), phi
274
275
       def update(self, phi, option, advantage):
276
           magnitude, direction = self._grad(phi)
277
            self.weights[direction] -= self.lr*magnitude*(advantage+self.dc)
278
280
   #=====Q-Value Individual Option======
281
   class Q_U:
282
       def __init__(self, nfeatures, nactions, args, weights, policy_over_options):
283
           self.weights = weights
284
           self.lr = args.lr_criticA
285
286
           self.discount = args.discount
           self.policy_over_options = policy_over_options
287
       def value(self, phi, action):
289
           return np.sum(self.weights[phi, action], axis=0)
290
291
       def update(self, traject, reward, done, termination):
292
           update_target = reward
            if not done:
294
                current_values = self.policy_over_options.value(traject[1][0])
                update_target += self.discount*((1. - termination)*current_values[traject[0][1]]
296
                     + termination*np.max(current_values))
297
           tderror = update_target - self.value(traject[0][0], traject[2])
            self.weights[traject[0][0], traject[2]] += self.lr*tderror
299
300
```

```
301
   #=====Policy Over Option======
302
   class POO:
303
304
       def __init__(self, rng, nfeatures, args):
            self.weights = np.zeros((nfeatures, args.noptions))
305
            self.policy = EgreedyPolicy(rng, args, self.weights)
306
           self.Q_Omega = Q_O(args, self.weights)
307
308
       def update(self, traject, reward, done, termination):
309
           self.Q_Omega.update(traject, reward, done, termination)
310
311
       def sample(self, phi):
312
           return self.policy.sample(phi)
313
314
       def advantage(self, phi, option=None):
315
            values = np.sum(self.weights[phi],axis=0)
316
            advantages = values - np.max(values)
317
            if option is None:
318
                return advantages
319
           return advantages[option]
32
       def value(self, phi, option=None):
322
            if option is None:
323
                return np.sum(self.weights[phi, :], axis=0)
324
           return np.sum(self.weights[phi, option], axis=0)
325
327
   class EgreedyPolicy:
       def __init__(self, rng, args, weights):
329
            self.rng = rng
330
           self.epsilon = args.epsilon
331
           self.noptions = args.noptions
332
           self.weights = weights
333
334
       def _value(self, phi, action=None):
335
            if action is None:
336
                return np.sum(self.weights[phi, :], axis=0)
337
338
           return np.sum(self.weights[phi, action], axis=0)
330
340
       def sample(self, phi):
            if self.rng.uniform() < self.epsilon:</pre>
341
                return int(self.rng.randint(self.weights.shape[1]))
342
           return int(np.argmax(self._value(phi)))
343
344
345
   #=====Q-Value All Option======
346
   class Q_0:
347
       def __init__(self, args, weights):
348
           self.weights = weights
349
           self.lr = args.lr_critic
350
           self.discount = args.discount
351
352
353
       def _value(self, phi, option=None):
            if option is None:
354
355
                return np.sum(self.weights[phi, :], axis=0)
           return np.sum(self.weights[phi, option], axis=0)
356
357
358
       def update(self, traject, reward, done, termination):
            update_target = reward
359
            if not done:
360
                current_values = self._value(traject[1][0])
361
                update_target += self.discount*((1. - termination)*current_values[traject[0][1]]
362
                     + termination*np.max(current_values))
363
           tderror = update_target - self._value(traject[0][0], traject[0][1])
364
           self.weights[traject[0][0],traject[0][1]] += self.lr*tderror
365
366
367
```

```
#=====Standard======
368
   # Follow the code standard of the ioc repository
369
   class Tabular:
370
371
       def __init__(self, nstates):
            self.nstates = nstates
372
373
       def __call__(self, state):
374
            return np.array([state,])
375
376
       def __len__(self):
377
            return self.nstates
```

visualize.py

```
import numpy as np
  import matplotlib.pyplot as plt
  from fourrooms import Fourrooms
  from time import sleep
  # 0 - Red
  # 1 - Green
# 2 - Blue
  # 3 - Black
  class Visualization:
      def __init__(self, fRoom, args, nactions, colorList
           =[[255,0,0],[0,255,0],[0,0,255],[0,0,0]]):
           assert args.noptions <= len(colorList), "Lengthuofucolorulistumustumatchunumberuofu
               options"
           self.colorList = colorList
           self.layout = fRoom.layout
14
           self.occupancy = fRoom.occupancy
self.tostate = fRoom.tostate
16
           self.tocell = fRoom.tocell
1.5
           self.screen = np.array([list(map(lambda c: [0,0,0] if c=='w' else [255,255,255],
18
               line)) for line in self.layout.splitlines()])
           self.lastphi = None
20
           self.noptions = args.noptions
           self.nactions = nactions
21
22
      def showMap(self, phi, option):
           color = self.colorList[option]
24
25
           self._draw(self.lastphi, [255,255,255])
           self._draw(phi, color)
26
27
           self.lastphi = phi
           plt.figure(figsize=(5,5))
28
           plt.subplot(111)
29
30
           plt.imshow(self.screen, vmax=255, vmin=0)
           plt.show()
31
           sleep(0.05)
32
33
34
       def showAttention(self, options):
           x = np.array([i for i in range(self.nactions)])
35
           plt.plot(x, np.array([int(i != 0) for i in range(self.nactions)]), color=[1,1,1])
36
37
           for i in range(self.noptions):
               plt.plot(x, options[i].policy.attention.pmf(), color=np.array(self.colorList[i])
38
                   /255.)
           plt.show()
39
40
41
      def showPref(self, weight): # policy_over_options.weightsP or options[index].weightsP
           for weight
           pref = np.zeros((13,13,3), dtype="int")
           for i in range (13):
43
44
               for j in range(13):
                   if self.occupancy[i,j] == 0:
45
                        choice = np.argmax(weight[self.tostate[(i,j)],:])
46
                        pref[i,j] = np.array(self.colorList[choice])
```

```
else:
48
                       pref[i,j] = np.array([255,255,255])
          plt.figure(figsize=(5,5))
50
51
          plt.subplot(111)
          plt.imshow(pref, vmax=255, vmin=0)
52
          plt.show()
53
54
      def savePref(self, weight, algo=None, wo=None, dc=None, run=None):
55
          pref = np.zeros((13,13,3), dtype="int")
56
          for i in range(13):
57
58
               for j in range(13):
                   if self.occupancy[i,j] == 0:
59
                       choice = np.argmax(weight[self.tostate[(i,j)],:])
60
61
                       pref[i,j] = np.array(self.colorList[choice])
62
                       pref[i,j] = np.array([255,255,255])
63
          plt.figure(figsize=(5,5))
64
          plt.subplot(111)
65
          plt.imshow(pref, vmax=255, vmin=0)
66
          if wo != None:
67
               plt.savefig("../result/{0}/wo_{1}/dc_{2}/run_{3}.png".format(algo, str(wo), str(
68
                   dc), str(run)))
          else:
69
               plt.savefig("../result/{0}/dc_{1}/run_{2}.png".format(algo, str(dc), str(run)))
70
71
72
      def resetMap(self, phi):
          self.screen = np.array([list(map(lambda c: [0,0,0] if c=='w' else [255,255,255],
73
               line)) for line in self.layout.splitlines()])
          self.lastphi = phi
74
          self._draw([62],[200,200,200])
75
76
      def _draw(self, phi, rgb):
77
          self.screen[self.tocell[phi[0]]] = np.array(rgb)
```