

# Finding Cliques in Networks: Annotated Bibliography

Cassandra Overney and Emma Pan

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## 1 Standard Clique References

These references involve finding the maximum number of cliques in a graph.

- [1] Moon, J.W. & Moser, L. “On cliques in graphs.” Israel J. Math. (1965) 3: 23. Paper. <http://users.monash.edu.au/~davidwo/MoonMoser65.pdf>.

We originally tried to understand the section of the paper with the proof explaining why any  $n$ -vertex graph has at most  $3^{(n/3)}$  maximal cliques, but after struggling to understand the confusing mathematical notation, we transitioned to another paper.

- [2] Wood, D.R. “On the Maximum Number of Cliques in a Graph.” Graphs and Combinatorics (2007) 23: 337. Paper. <https://arxiv.org/pdf/math/0602191.pdf>.

We referenced this paper for the maximum number of cliques proof in our supplementary document. The relevant proof starts on page 13. We basically re-wrote it using the proof by induction techniques we learned this semester.

## 2 Relaxed Clique References

These references involve deriving 8 types of relaxed cliques from standard cliques.

- [1] Balasundaram, Balabhaskar & Butenko, Sergiy & Hicks, I. & Sachdeva, S. (2007). “Clique Relaxations in Social Network Analysis: The Maximum  $k$ -Plex Problem.” Operations Research. Paper. <https://www.caam.rice.edu/~ivhicks/kplex4web.pdf>.

We referenced this paper to get a better understanding of  $s$ -plexes and learn about their applications.

- [2] Butenko Sergiy. “Clique Relaxation Models in Networks: Theory, Algorithms, and Applications.” AACIMP 2011 Summer School. Operational Research stream. Lecture. <https://www.slideshare.net/ssakpi/clique-relaxation-models-in-networks-theory-algorithms-and-applications>.

We used this presentation to get a better understanding of relaxed cliques (more specifically  $k$ -cores,  $s$ -plexes,  $s$ -cliques, and  $s$ -clubs) and to learn about their different applications.

- [3] Pardalos Panos. “Cliques, Quasi-cliques and Clique Partitions in Graphs.” Lecture. [https://www.imus.us.es/DOC-COURSE10/PDF/seminars/May\\_2010\\_clique.pdf](https://www.imus.us.es/DOC-COURSE10/PDF/seminars/May_2010_clique.pdf).

We used this presentation to learn about y-quasi-cliques, s-defective cliques, k-blocks, and s-bundles.

- [4] Timo Gschwind & Stefan Irnich & Fabio Furini & Roberto Wolfler Calvo, 2015. "Social Network Analysis and Community Detection by Decomposing a Graph into Relaxed Cliques." Working Papers 1520, Gutenberg School of Management and Economics, Johannes Gutenberg-Universität Mainz. Paper. [http://wiwi.uni-mainz.de/Papers/DiscussionPaper\\_1520.pdf](http://wiwi.uni-mainz.de/Papers/DiscussionPaper_1520.pdf).

We used this paper as an introduction into relaxed cliques. We based the relaxed cliques section of our supplementary document on the 8 types of relaxed cliques explained in this paper.

### 3 Application-Based References

These references include tutorials and articles we referenced while trying to complete our clique applications involving email and co-purchasing networks.

- [1] J. Yang and J. Leskovec. "Defining and Evaluating Network Communities based on Ground-truth." ICDM, 2012. Paper. <https://arxiv.org/abs/1205.6233>.

We used the Amazon co-purchasing dataset first introduced in this paper. We obtained the actual data from the Stanford Network Analysis Project <https://snap.stanford.edu/data/com-Amazon.html>.

- [2] Kapoor Amita. "Social Network Analysis in Python." Data Camp. October 2nd, 2018. Tutorial. <https://www.datacamp.com/community/tutorials/social-network-analysis-python>.

We went through this tutorial to learn how to use Python's NetworkX library for network visualization.

- [3] K. Ben. "Book Recommendation (Python)." Medium. Feb 18, 2018. Tutorial. <https://medium.com/@baemaek/amazon-book-recommendation-system-analysis-d9d72b9a7173>.

This article was really useful in helping us read in the Amazon co-purchasing data.