A Dynamic Model of COVID-19: Contagion and Implications of Isolation Enforcement

Miguel Casares
Universidad Pública de Navarra

Hashmat Khan
Carleton University

Working Paper
D.T. 2001

Departamento de Economía
Universidad Pública de Navarra

Universidad Pública de Navarra
Nafarroako Unibertsitate Publikoa

A Dynamic Model of COVID-19: Contagion and Implications of Isolation Enforcement

Miguel Casares*
Universidad Pública de Navarra

Hashmat Khan†
Carleton University

First Draft: March 26, 2020
Updated: April 1, 2020

Abstract

We present a dynamic model that produces day-to-day changes in key variables due to the COVID-19 contagion: both accumulated and currently infected people, deaths, recovered, and infected people who require hospitalization. The model is carefully calibrated to Spanish data and we conduct simulation exercises to study the effects of isolation enforcement. We find that virus containment from isolation exhibits increasing returns. Our model simulations show that the Spanish government's declaration of the State of Alarm (March 14th, 2020) is estimated to have cut the number of deaths by 95% and the number of hospital beds needed by 96%. The simulations also indicate that both the intensity and, especially, the timing of isolation enforcement are important for the evolution of the virus spread and the smoothing of the hospitalization needs.

*Departamento de Economía and INARBE, Universidad Pública de Navarra, 31006, Pamplona, Spain. E-mail: mcasares@unavarra.es (Miguel Casares).

†Department of Economics and CMFE, Carleton University, Ottawa, ON K1S 5B6, Canada. E-mail: hashmat.khan@carleton.ca (Hashmat Khan).

1 Introduction

On March 11th 2020, the World Health Organization declared the Coronavirus Disease 2019 (COVID-19) outbreak a pandemic—a worldwide spread of the disease. As of March 30th, there are 33,579 confirmed deaths due to COVID-19 worldwide and the six countries with each over 2,500 confirmed deaths, namely, Italy (10,781), Spain (6,528), China (3,311), Iran (2,640), France (2,606) and the US (2,509). The total number of confirmed cases has reached 715,660 (Roser et al. (2020)).

Unfortunately, the pandemic is still in progress unleashing a global health crisis and putting enormous pressure on health care systems. In addition to the travel related source of virus spread there is now a full-blown 'community spread' where the initial source of the infection remains unidentified. Governments and public authorities are implementing mandatory actions to contain the virus spread such as travel restrictions, lockdowns, closures of public spaces, institutions, and businesses, social (and physical) distancing, and self-isolation.

Drawing on the epidemiological Susceptible-Infected-Recovered (SIR) methodology, pioneered by Kermack and McKendrick (1927), we present a discrete-time dynamic model to predict the COVID-19 contagion. Even though the model is simple, it captures the main characteristics of the contagion process and provides insights valuable for policy orientation. The model can help understand the day-to-day effects of changes in medical and social policies. The model may also be suitable for transferring knowledge and communicating the response of health and political authorities to the public.

As one clear-cut applied exercise, we calibrate the model parameters to Spanish data and present simulations to show the dramatic implications of enforcing mobility constraints over the COVID-19 spread in Spain.[1] Our paper is related to three recent contributions. Wang et al. (2019) estimates the evolution of the COVID-19 cases in Wuhan, China, while Atkeson (2020) investigates the impact of social distancing for the virus spread in the US, and Ferguson et al. (2020) analyzes the impact of non-pharmaceutical interventions to contain the virus expansion in the Great Britain. We use the global epidemiological data provided in Anderson et al. (2020) for the calibration of some of the model parameters.

[1]While our focus is on studying the effects of immediate mobility controls in dealing with the ongoing health crisis, their unavoidable drastic effects on economic activity are underway. Eichenbaum et al. (2020) embed an epidemiological model in a macroeconomic general equilibrium model to study the tradeoff between the severity of decline in output and lives saved.

2 Model description

For any given day t, we have the decomposition

$$N = x_t + z_t$$

where N is the total population on the arrival day of the first person infected by COVID-19, x_t is the accumulated number of people infected by COVID-19 on day t and z_t is the accumulated number of people never infected on day t.[2] On day 1, $x_1 = 1$ and $z_1 = N - 1$. For any future day t, the law of motion for x_t is

$$x_t = x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} \tag{1}$$

that adds up to its value on the previous day, x_{t-1}, the number of newly infected people $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1}$. In the latter term, $0 < \alpha < 1$ is the contagion probability on each encounter between one non-infected person and one infected person, $y > 0$ is the number of people each person meets per day, $\widetilde{x}_{t-1}$ is the number of people currently infected as of day $t-1$, and k_{t-1} is the accumulated number of deaths caused by COVID-19 as of day $t-1$.[3]

The ratio $\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$ provides the share of currently infected people with respect to the surviving population at the end of day $t-1$, which determines the probability of meeting someone infected. Thus, the product of the number of encounters by the rate of infected people, $y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$, is the number of infected people every person meets on day t. Once we multiply it by the contagion probability on each encounter, we have $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$ as the effective daily contagion rate per person. The number of people who have *never* been infected at the end of day $t-1$ is z_{t-1}, and they are the potential newly infected people (susceptible people in the SIR methodology). Therefore, the second term on the right side of (1), $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1}$, is the number of newly infected people on day t. It explains how the number of new cases depends

[2] We assume that the initial population N remains constant in this decomposition to consider that all deaths caused by the virus infection will determine the fatality rate of the virus. The same result would be obtained if there where no migration flows and the daily natality rate would be the same as the mortality rate not-related to COVID-19. Given the short time horizon of the analysis and the focus of the paper, we have decided to keep N fixed.

[3] For simplicity, the contagion probability α is both constant over time and identical for all meetings, which ignores heterogeneity in the meeting duration, the degree of physical contact, the viral load of the transmitter, etc. Hence, α is considered to represent contagion probability under average circumstances. We also assume the number of daily social contacts, y, is constant and exogenous, which must be interpreted as the behavior of the representative individual.

on both the contagion probability α, and on the intensity at which the disease spreads in the matching between infected and non-infected individuals, $y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}z_{t-1}$.

The difference between x_t and the number of people (still) currently infected, $\widetilde{x}_t$, comes from the fact that the COVID-19 disease is neither chronic nor necessarily lethal. Let us assume, for simplicity and taking a realistic average, that the duration of the disease is T days, while the incubation period of the virus is T_i days with $T_i < T$. Thus, T days after catching the virus the individual either recovers (with an associated survival probability $0 < 1-\lambda < 1$) or dies (with an associated fatality probability $0 < \lambda < 1$). Both outcomes together reduce $\widetilde{x}_t$ by one. On day t, the number of infected people who will either get cured or die are $x_{t-T} - x_{t-T-1}$, i.e. those who were infected between day $t-T-1$ and day $t-T$. Therefore, the law of motion for the number of people currently infected by COVID-19 is

$$\widetilde{x}_t = \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1} - \left(x_{t-T} - x_{t-T-1}\right) \tag{2}$$

With the model elements that have been introduced, we can also define the accumulated number of deaths on day t as

$$k_t = k_{t-1} + \lambda \left(x_{t-T} - x_{t-T-1}\right)$$

where λ is the fatality rate. Naturally, the accumulated number of healed people, h_t, is

$$h_t = h_{t-1} + (1-\lambda)\left(x_{t-T} - x_{t-T-1}\right)$$

Since $x_t = h_t + k_t + \sum_{j=0}^{T}\left(x_{t-j} - x_{t-j-1}\right)$, from $N = x_t + z_t$ we also get

$$N = h_t + k_t + \sum_{j=0}^{T}\left(x_{t-j} - x_{t-j-1}\right) + z_t \tag{3}$$

which means that total population, N, comprises four groups of people: those who have already recovered, h_t, those who have already died, k_t, those who are infected with their outcome not yet known, $\sum_{j=0}^{T}\left(x_{t-j} - x_{t-j-1}\right)$, and those who have never been infected, z_t.

COVID-19 is an infectious virus that typically causes mild symptoms similar to the common flu, and only a minor fraction of infected people who test positive need hospitalization.[4]

[4]In fact, some of the people infected with COVID-19 are asymptomatic, which makes the spreading out of the epidemic more difficult to prevent and control by the health authorities. Anderson et al. (2020) say that

Nevertheless, the contagion rate of COVID-19 is very high and the capacity of hospitals to give treatment to sick people is severely constrained. In the model, we assume that a fraction θ of the infected people who have passed the incubation period, T_i, suffer from severe complications (typically, respiratory difficulties and pneumonia) and need hospitalization. Thus, the number of hospital beds, b_t, required to treat COVID-19 positive people on day t is

$$b_t = \theta \sum_{j=T_i}^{T} \left(x_{t-j} - x_{t-j-1}\right)$$

where $\sum_{j=T_i}^{T} \left(x_{t-j} - x_{t-j-1}\right)$ is the total number of infected people who have passed the incubation period, T_i, on day t.

To summarize, we have a dynamic system of 6 equations as follows:

$$\begin{aligned}
x_t &= x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} \\
\widetilde{x}_t &= \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} - \left(x_{t-T} - x_{t-T-1}\right) \\
N &= x_t + z_t \\
k_t &= k_{t-1} + \lambda \left(x_{t-T} - x_{t-T-1}\right) \\
h_t &= h_{t-1} + (1 - \lambda) \left(x_{t-T} - x_{t-T-1}\right) \\
b_t &= \theta \sum_{j=T_i}^{T} \left(x_{t-j} - x_{t-j-1}\right)
\end{aligned}$$

which, given initial values, determine the evolution of the 6 endogenous variables $\{x_t, \widetilde{x}_t, z_t, k_t, h_t, b_t\}$.

3 Model calibration for Spain

The baseline calibration is aimed at representing the outbreak of COVID-19 in a medium-size country. We take the case of Spain as one representative country and assume realistic values for the medical parameters that provide the epidemiological characteristics of COVID-19 based on Anderson et al. (2020). Table 1 provides the calibration values for the seven model parameters.

"Estimates suggest that about 80% of people with COVID-19 have mild or asymptomatic disease...'.

Table 1: Calibration of model parameters for Spain

1.	Population	$N = 47 \times 10^6$
2.	Fatality rate	$\lambda = 0.0075$
3.	Disease duration (days)	$T = 20$
4.	Incubation period (days)	$T_i = 6$
5.	Hospitalization rate	$\theta = 0.0528$
6.	Daily meetings per person	$y = 25$
7.	Contagion probability	$\alpha = 0.011$

The total population is $N = 47$ million people to coincide approximately with the population of Spain in 2020. For the fatality rate, λ, we follow Anderson et al. (2020) who provide an estimated range between 0.3% and 1% with reference to the data released by the World Health Organization.[5] Typically, the Infection Fatality Rate (IFR) $\left(= \frac{\text{confirmed COVID-19 deaths}}{\text{confirmed+unconfirmed cases}}\right)$ is lower than the Case Fatality Rate $\left(= \frac{\text{confirmed COVID-19 deaths}}{\text{confirmed cases}}\right)$ as some of the cases are not reported because they are either asymptomatic or the tests have not been taken. Our model produces the IFR. Spain may experience a relatively high IFR due to the population aging (in 2019 people over 75 years old represented 9.54% of the total population) and the much stronger severity of COVID-19 on the elderly. By contrast, health coverage is guaranteed by the government with a well-developed public provision of hospitals and treatments. Balancing out both arguments, we set $\lambda = 0.0075$ (0.75%), slightly above the median value of the range suggested by Anderson et al. (2020).

Anderson et al. (2020) report that the incubation period for COVID-19 is about 5 or 6 days and there is an average period of 10 days or more (longer than a common flu) of confrontation between the immune system and the virus.[6] Therefore, we set an average disease duration at $T = 20$ days and the incubation period to last for 6 days, $T_i = 6$.

Ferguson et al. (2020) estimate the COVID-19 hospitalization rate for the population of the Great Britain using a subset of cases obtained from China. Their estimate is set at 4.4%. For Spain, since the population has a higher fraction of elderly people than in either Great Britain or China, we set the hospitalization rate 20% higher at $\theta = 0.0528$ (5.28%).

The number of two-people encounters per day is subject to heterogeneity because it clearly depends on the specific social and economic characteristics of the individuals: type of job,

[5]Recently, Wu et al. (2020) have lowered the estimate of the case fatality risk (measured as the the probability of dying after developing symptoms) of COVID-19 in Wuhan to 1.4%.

[6]Anderson et al. (2020) cast some doubts about the length of the disease after the COVID-19 incubation as they say "*...perhaps lasting for 10 days or more after the incubation period*".

social/leisure activities, age, etc, as well as on the social norms and habits of a country or territory. People gatherings for social and economic activities are quite common in Spain. Thus, we set $y = 25$ meetings to represent an average behavior of a Spanish citizen, though recognizing the uncertainty and variance that affect this model parameter.

The contagion probability α measures the speed at which the virus spreads. A standard way of measuring this speed in the data is the time it takes for the infected people to pass on the infection to the same number of people: the doubling time (also called serial interval). According to Anderson et al. (2020), COVID-19 is spreading more rapidly than the 2009 Influenza A H1N1 pandemic with a doubling time between 4.4 and 7.5 days (similar to SARS). In Spain, the COVID-19 is showing exponential growing patterns with doubling times of confirmed cases around 4 days and of deaths between 2 and 4 days (2 days in the early stages of the outbreak and close to 4 in the latest observations).[7] Hence, we search for a value of α required in model simulations to match the speed that has characterized the daily series of deaths caused by COVID-19 in Spain.[8] This search determined setting $\alpha = 0.011$.

4 Simulations

We have programmed the simulations in MATLAB.[9] For initial values, we consider that on day 1, $t = 1$, there is one imported contagion and one person gets infected while the rest of the population had no virus, i.e. $x_1 = \widetilde{x}_1 = 1$. Then, we run the calibrated six-equation model forward over the next 365 days. In order to test the effects of policies aimed at people isolation and restrictions to mobility, we simulate the model under values for the number of daily encounters, y, different from the one set in the baseline calibration.

Before commenting on the results, we briefly discuss the role of y in the dynamics of the model. If the number of individual contacts, y, is high, there will be an increase in the effective contagion rate per person, $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$, which raises the daily variation in both the accumulated number of infected people (Equation (1)) and the number of currently infected people (Equation (2)). The downward phase turns also faster with more interpersonal contacts, y. As T days pass, there will be more infected people who will recover and a constant

[7] Ferguson et al. (2020) assume a doubling time of the confirmed cases of COVID-19 at 5 days.

[8] A direct observation of the contagion probability α is not possible because the incubation period of COVID19 is typically long (5 or 6 days) and many infected people are not tested. The true number of infected cases at the time of the contagion is impossible to obtain. These difficulties justify the criterion chosen to calibrate α based on the matching between ex post observations of model simulations and the data.

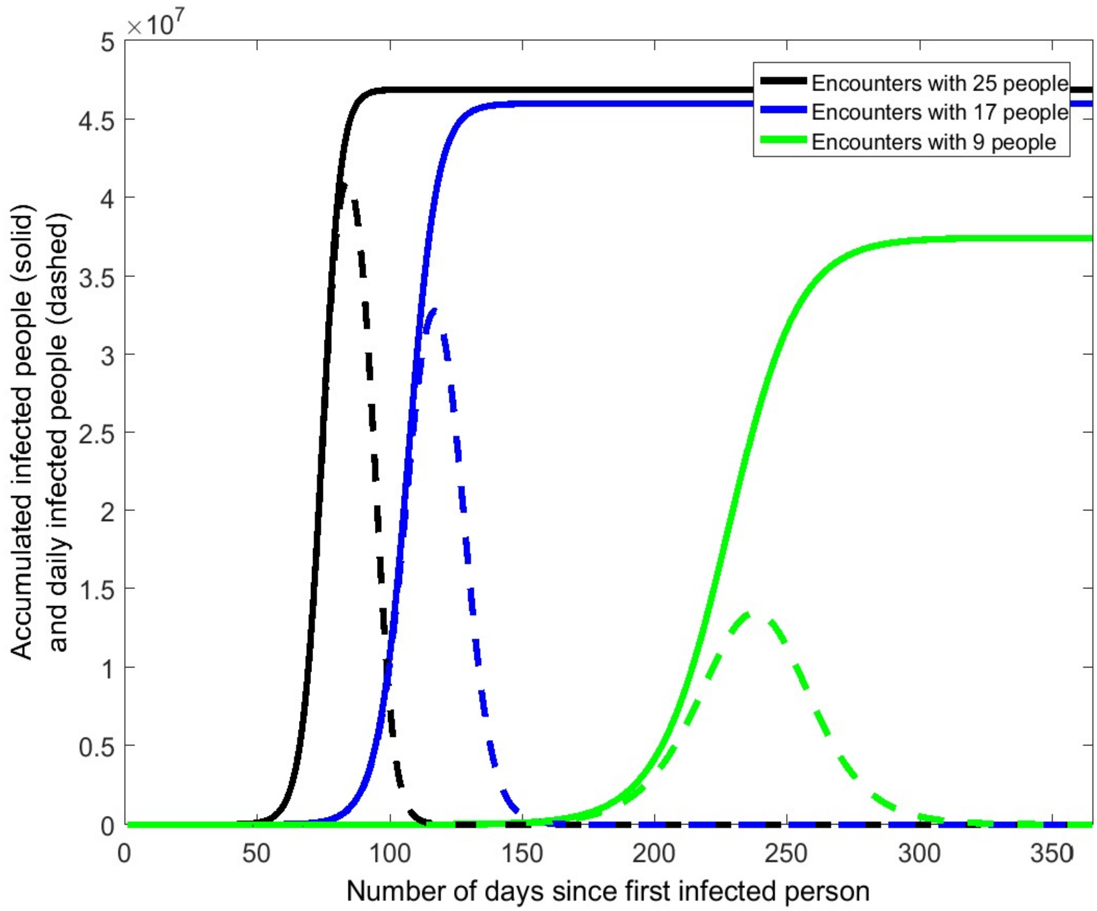
[9] The MATLAB code written to carry out the model simulations is available upon request.

fraction of them will die. Both daily recovered and deaths are cutting down the number of currently infected people (Equation (2)).

4.1 Social interaction

Our first exercise illustrates the impact of social interaction on the extent and length of the virus spread. We have simulated the model under three alternative values for the number of daily physical contacts among individuals: the value assigned in the calibration ($y = 25$) and two lower values that result from initially subtracting 8 daily encounters ($y = 17$), and additionally 8 more encounters subtracted ($y = 9$). Figures 1-3 and Table 2 show the results.

Figure 1: **Simulated series of COVID-19 infected people in Spain depending on social interaction**



As Figure 1 shows, the case with the highest social interactions ($y = 25$ encounters) displays a fast and sharp contagion pattern. There is a rapid increase of infected cases around day 50 and the epidemic is mostly over by day 100 (black line). Almost all the population get infected (46.89 million). Table 2 reports nearly 352 thousand deaths (0.75% of all the accumulated infected population) and the maximum number of people who need hospitalization on a single day is more than 1.9 million (observed on day 85).

Table 2: Model simulation results with alternative levels of social interaction

	$y = 25$	$y = 17$	$y = 9$
• Accumulated infected people, millions	46.89	45.99	37.44
• Accumulated deaths, thousands	351.7	344.9	280.8
• Daily peak of hospitalized people, thousands	1906.5	1420.1	545.5
Peak day	85	118	239

If we would consider that the Spanish people had less social interaction and, therefore, a lower number of daily encounters at $y = 17$ meetings per individual, the contagion pattern is slower and less pronounced (see blue lines in Figure 1). With $y = 17$, the number of COVID-19 cases increases around day 75, reaching a maximum daily value around day 110, and falling to close to zero levels by day 150. Still, the epidemic turns out to have very severe consequences, with most of the population getting infected (45.99 million) and 345 thousand deaths (see Table 2). The peak value of health coverage needs is reported on day 118 when over 1.4 million infected people must be hospitalized.

If we have a second same-sized reduction in the number of daily meetings to just $y = 9$, the results show even a greater containment of the COVID-19 outbreak. The "*flattening of the curve*" is clearly observed in the green lines of Figure 1, both in terms of the slowing down in the pace of the accumulated cases and also in the number of people who currently suffer the infection.[10] As reported in Table 2, the hospitalization needs turn clearly lower with a maximum of 545.5 thousand hospitalized people (around 3/10th of the number reported when $y = 25$ and 2/5th of the number when $y = 17$). Notably, the slow down of the contagion process is really remarkable. The daily peak on the hospitalization of infected people takes place on day 239, which means more than 7 months after the start of the outbreak. Since COVID-19 is a seasonal virus, such late effects could tentatively be discarded due to

[10] See, for example, https://www.flattenthecurve.com/.

a half-year seasonal change (from Winter to Summer) that would significantly weaken the virus contagion capacity in warmer temperatures. Similarly, a future period of no further contagion could be considered if the COVID-19 vaccine were clinically tested successfully, and vaccinations could be administered to all the individuals.[11]

Figure 2: **Simulated series of COVID-19 hospitalization needs in Spain depending on social interaction**

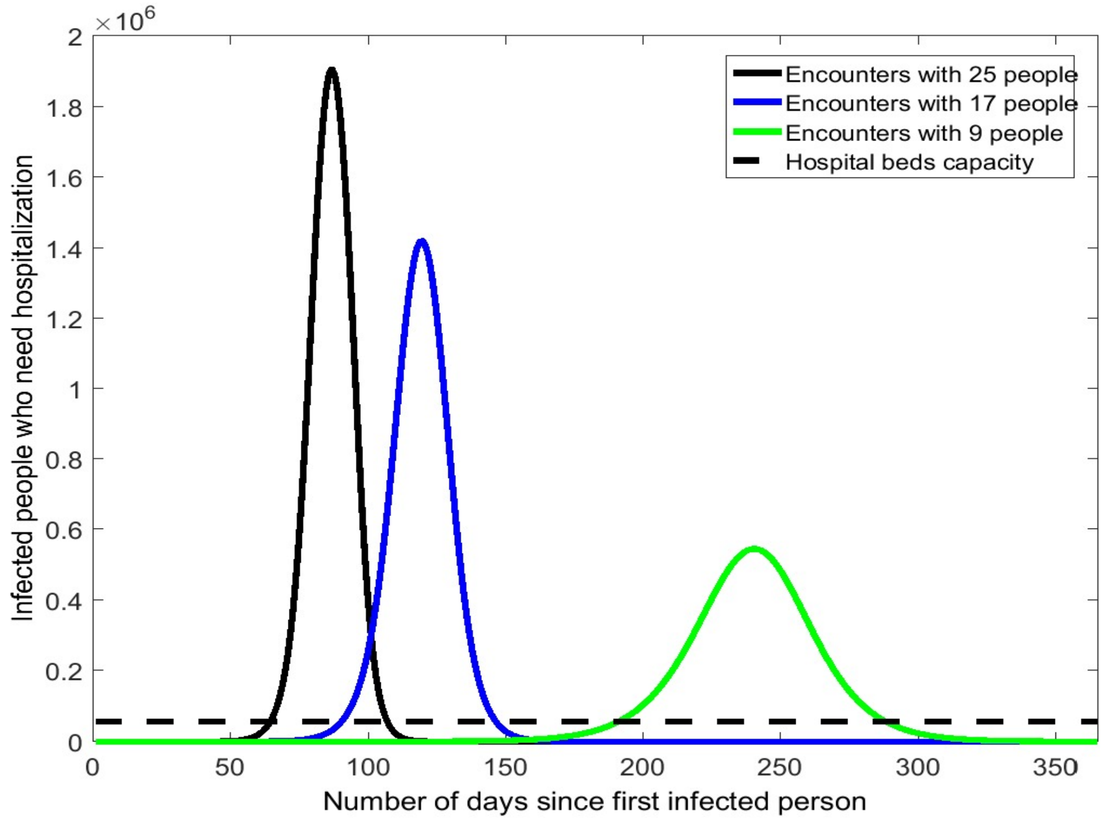


Figure 2 shows the needs for hospitalization and treatment depending on the three scenarios of social interactions. Spain has approximately 300 hospital beds per 100,000 people (below the EU average of about 372 beds), which brings an overall amount of around 141,000

[11] An alternative model setup could have included $\overline{T}$ as the number of days after the first infected person from which the virus cannot spread out any further due to either climate conditions or a universal administration of vaccinations. This would imply $\alpha = 0$ if $t \geqslant \overline{T}$, and we could rewrite the low of motion for x_t and $\widetilde{x}_t$ as follows:

$$x_t = \left\{ \begin{array}{ll} x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1} & \text{if } t < \overline{T} \\ x_{t-1} & \text{if } t \geqslant \overline{T} \end{array} \right\}$$

$$\widetilde{x}_t = \left\{ \begin{array}{ll} \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1} - (x_{t-T} - x_{t-T-1}) & \text{if } t < \overline{T} \\ \widetilde{x}_{t-1} - (x_{t-T} - x_{t-T-1}) & \text{if } t \geqslant \overline{T} \end{array} \right\}$$

units. Let us suppose that in normal times the capacity utilization rate is 60%. Thus, the hospital beds capacity to cope with the COVID-19 spread in Spain is assumed to be 40% of 141,000 which is 56,400 units. We represent this as the horizontal line of hospitals bed capacity in Figure 2. The three scenarios of social interaction clearly give a number of people who need hospitalization that exceed the Spanish capacity (though the deficit is significantly smaller in the least social interaction case). This result indicates the urgent need of a policy action in Spain to reduce the number of daily meetings below $y = 9$, to prevent the health care system from collapsing.

Figure 3: **Distribution of the Spanish population following the COVID-19 outbreak depending on social interaction**

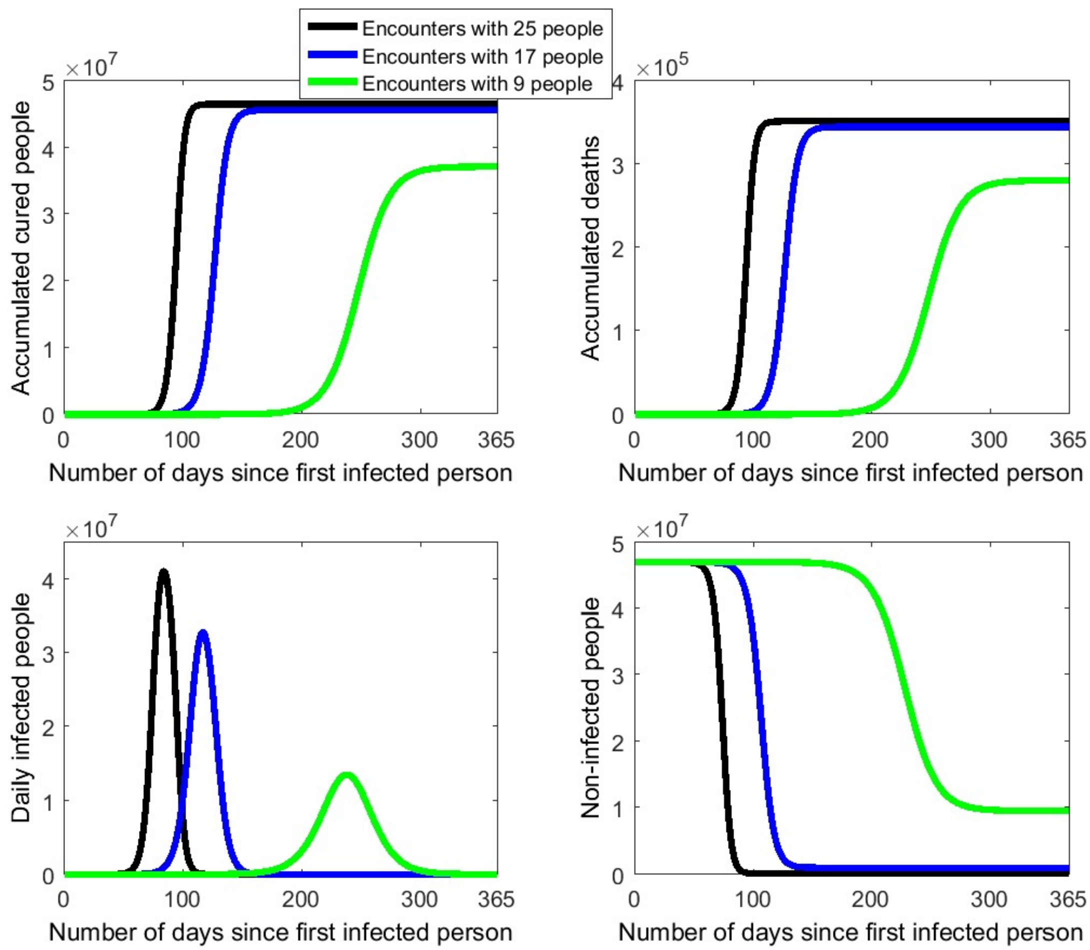


Figure 3 illustrates the distribution of the initial population between the four possible daily states indicated in Equation (3): in the top left-hand side, the people who have already healed, h_t; in the top right-hand side, the people who have already died, k_t; in the bottom left-hand side, the people who are still infected, $\sum_{j=0}^{T}\left(x_{t-j}-x_{t-j-1}\right)$, and in the bottom right-hand side, the people who have never been infected, z_t. The impact of social distancing is very significant in the population allocation. As individuals meet less people, the speed of contagion is lower, people get infected later and, subsequently, people either recover or die later. Although there are less infected people in the case with little social interaction (y=9), the slower contagion pace raises the epidemic duration (see the wider curves with daily infected people in the bottom left-hand side cell of Figure 3).

Remarkably, all these effects are substantially more pronounced when social distancing switches from moderate to strict ($y = 17$ to $y = 9$) than from loose to moderate ($y = 25$ to $y = 17$). Therefore, we find increasing returns to isolation, which is confirmed with the numbers reported in Table 2. Going through one example from numbers displayed in Table 2, we first see that if the number of interactions falls from y=25 to y=17, hospitalization needs are reduced by 438 thousand (a 21.5% reduction). Then, the same 8-unit reduction in encounters, from y=17 to y=9, results in a deeper decrease of the hospitalization needs by 865 thousand (a 54.2% reduction).

4.2 Policy intervention

On March 14th, 2020, the Spanish government declared a state of emergency, the "State of Alarm" (SoA) in response to the COVID-19 outbreak in Spain. The decree contemplated mobility restrictions, school and some production activity suspensions and home confinement for the population. Fifteen days after the SoA declaration (March 29th, 2020), the government passed further actions and enforced home confinement to every person whose job is not related to either health care or basic needs. This is a natural response of most countries to the COVID-19 expansion worldwide. The calibrated model can represent the SoA as a policy intervention that significantly reduces the number of physical contacts among citizens.

We have characterized Spain in normal times with a high degree of social interaction, $y = 25$ face-to-face contacts per day. Once the COVID-19 infection arrives in Spain, we can simulate the effects of the SoA intervention by switching, on the day when the isolation is enforced, from $y = 25$ daily encounters per person to a much lower value of $y = 3$ encounters. The tighter lockdown actions, that came into force 15 days past the SoA declaration, are supposed to cut the number of personal contacts 40% further down to $y = 1.8$ encounters per day.[12]

The choice of the day in which the isolation is enforced can be crucial for the posterior extension of the disease (as we will document below). Thus, we paid special attention to selecting the day of our simulated series when the policy intervention took place. The SoA decree was published on March 14th when the coronavirus death toll in Spain was 136 people. We have used this information in the calibrated model with $y = 25$ and searched for the day on which the number of deaths in the simulations is the closest one to 136. This is day 61 (with 121 deaths in the simulation of the calibrated model), which we consider as the moment for the SoA declaration in Spain and the isolation enforcement reduces sharply the number of daily interpersonal encounters to y=3. The tightening of the SoA, which lowers y by 40% ($y = 1.8$), takes place on day 76 in model simulations. We also evaluate the impact of such isolation enforcement in two alternative timing scenarios: SoA declared four days earlier (day 57) and SoA declared four days later (day 65).

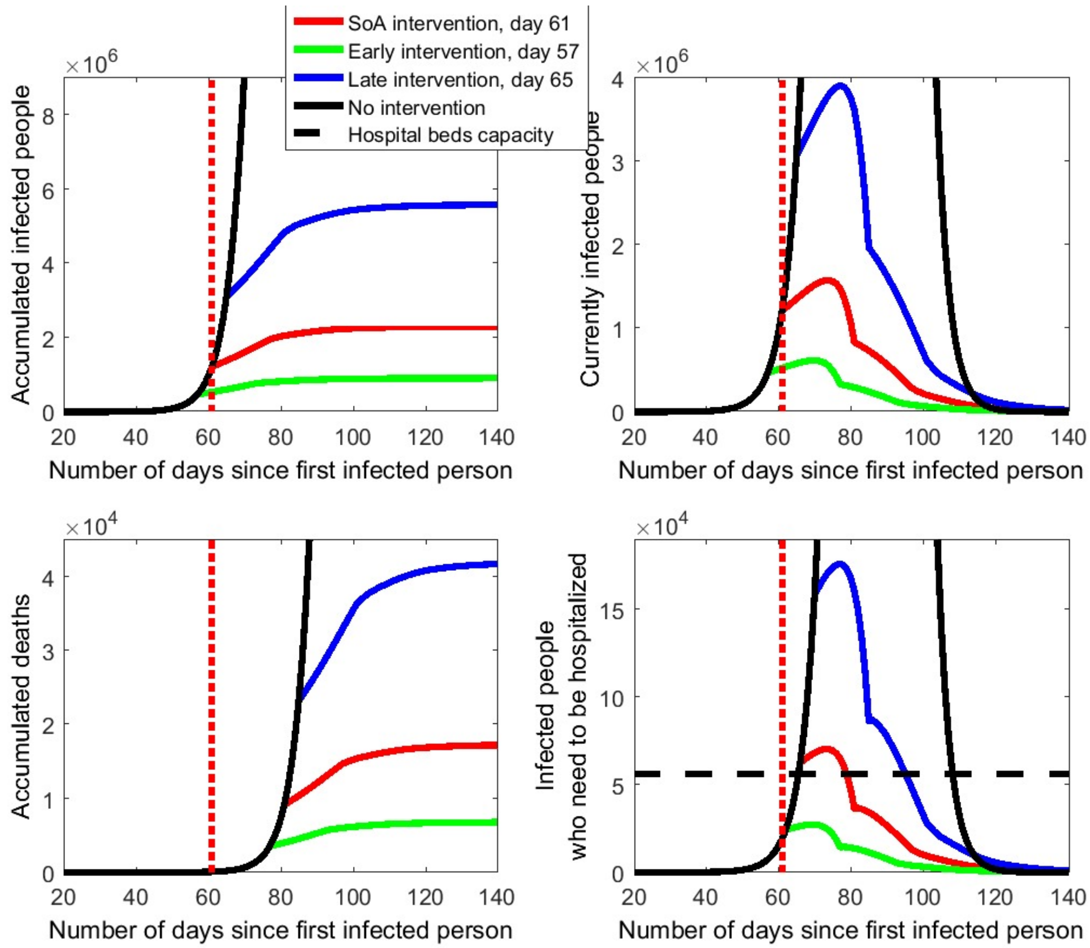
Figure 4 and Table 3 show the results. The benchmark case for comparison is the "no intervention" scenario, keeping $y = 25$ which would lead to having almost all the Spanish people infected and over 350 thousand deaths.

The estimated effects of the actual SoA intervention are displayed as red lines in Figure 4. In comparison to the no intervention scenario (black lines), the curves of infected people and hospitalized people shift down and widen up as a clear example of the "flattening of the curve" pattern. Compared to the no intervention scenario, the SoA brings a fall in the accumulated number of infected people from 46.89 million to 2.3 million, the maximum number of people who need daily hospitalization is reduced by 96% (from 1.9 million people to 70 thousand people). Accumulated deaths are estimated to decrease from almost 352 thousand to 17.2 thousand (an impressive 95.1% cut).

As shown in the bottom right-hand cell of Figure 4, the number of people who need to be hospitalized show no apparent variation after the SoA declaration in comparison to the no intervention case. The reason for this lack of effects is that the reduction in the newly infected people will not be realized before the end of the 6-day incubation period. Precisely, it is day 67 (6 days after the SoA day) when the slope of the red line flattens as there are fewer infected people who develop symptoms and need to be hospitalized. The model also estimates that between days 64 and 77 the demand for hospital beds exceeds capacity. The

[12]The Spanish Minister of internal affairs commented on a press conference on the first day after the suspension of all non-basic economic activities that traffic in public transportation fell 34% compared to the previous working day.

Figure 4: **Alternative timings for the isolation policy in Spain following the COVID-19 outbreak**



downward phase is fast for some days after the peak day but it turns slower on day 81 onwards (coinciding with the 20-day average duration of the infection we have assumed in the calibration). To illustrate the slowing down on the downwards phase, the simulation indicates that two months after the SoA declaration (day 122, May 14th) still 1630 persons would require medical treatment (2.3% of the number of people found on peak day).

A 4-day earlier intervention (day 57) would have been prevented many infections and reduced the number of deaths and the hospitalization needs (see the flattening and pushing down of the green lines in Figure 4 relative to the red lines). Numbers reported in Table 3 support an important point on undertaking early action. In a scenario with social distancing enforced 4 days earlier, the model estimates a reduction by over 60% in the number of

Table 3: Simulation results of the timing of social distancing in Spain

	No intervention	Day 57	Day 61 (SoA)	Day 50
• Accumulated infected people, millions	46.89	0.91	2.30	5.59
• Accumulated deaths, thousands	351.7	6.8	17.2	41.9
• Daily peak of hospitalized people, thousands	1906.5	37.3	70.4	176.2
Peak day	85	68	72	76

infected people and deaths (more than 10,000 lives are estimated to be saved). Moreover, the number of required hospitalizations drops to nearly half of its value, from 70,400 to 37,300, which can be totally covered by the Spanish health care system (daily numbers always lie down below the dashed horizontal line of hospital beds capacity drawn in the corresponding cell of Figure 4).

The 4-day postponement of the intervention to day 65 would increase infected people, hospitalization needs, and deaths by a factor higher than 2 (see the blue lines in Figure 3 and the numbers reported in Table 3). The situation would have been catastrophic for the health assistance of more than 175 thousand people who need medical treatment on the peak day, when this number is more than 3 times the Spanish hospitalization capacity.

In short, the simulation results indicate that the choice of the day for setting the enforcement of social distancing has critical consequences on the evolution of the virus spread.

Finally, we examine the effects of different degrees of intensity of the social distancing action taken by the Spanish government. Figure 5 and Table 4 document the sensitivity of the results to assuming scenarios with either a stronger or a weaker enforcement of social isolation in the SoA. Thus, we compare the cases of $y = 2$ (more intensity on isolation) and $y = 4$ (less intensity on isolation) to the calibrated setting of $y = 3$ for the SoA procurement. Once again, the quantitative effects are very large (although somehow not as large as they were for the timing of the intervention). Both the numbers reported in Table 4 and the green lines on Figure 5 indicate that only reducing the SoA enforcement in one more meeting would produce an estimated decrease in the number of deaths by 21.5% and in the peak number of people who need hospitalization by 7% (which would place the curve always below the capacity line as shown in Figure 5). By contrast, a looser implementation of the SoA with $y =$ 4 daily encounters per person would have an important cost in human lives (the accumulated number of deaths would increase by almost 6,000) and on the number of people who need hospitalization (on peak day around 7,000 more people). Actually, the health care system would be on the verge of collapsing because for 15 consecutive days (from day 65 to day 79, both included) that would require more hospital beds than the installed capacity.

Figure 5: **Alternative intensities for the isolation policy in Spain following the COVID-19 outbreak**

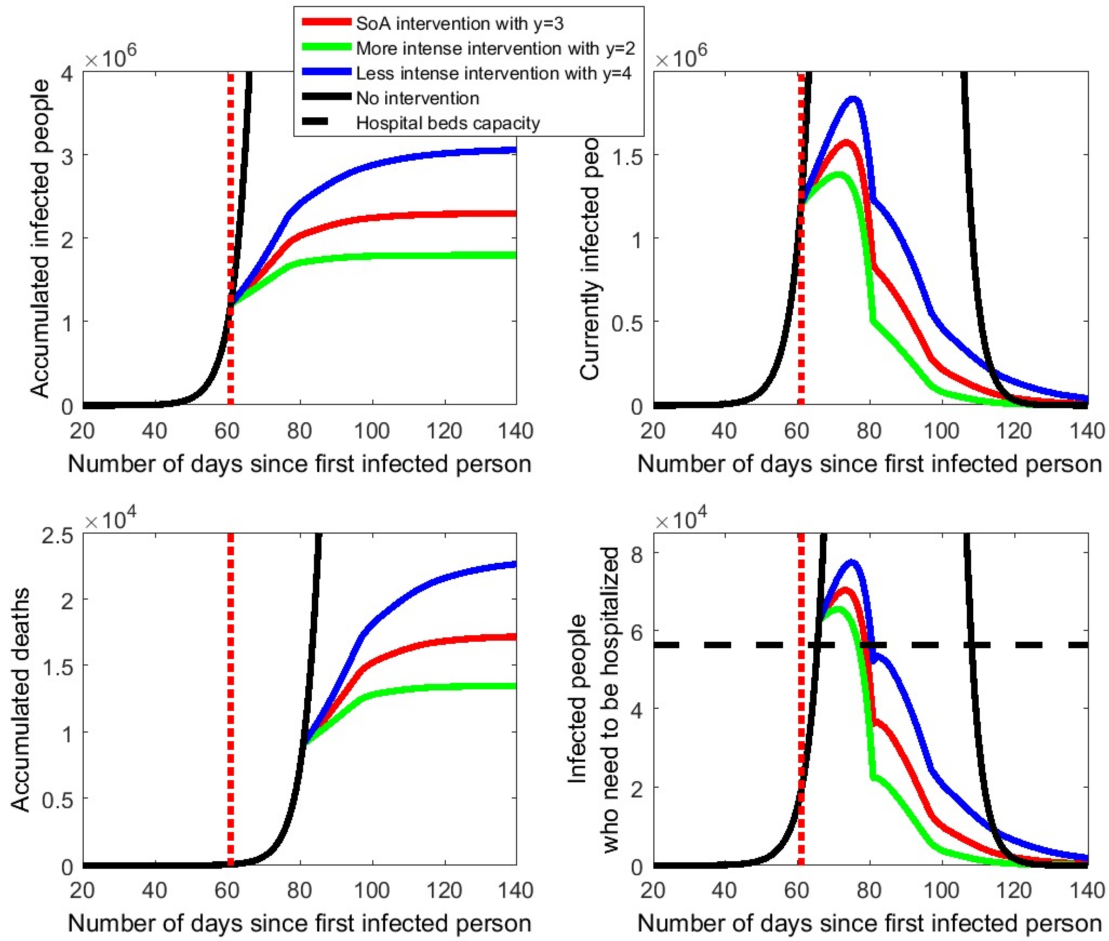


Table 4: Simulation results of the intensity of social distancing in Spain

	No intervention	$y = 2$	$y = 3$ (SoA)	$y = 4$
• Accumulated infected people, millions	46.89	1.80	2.30	3.08
• Deaths, thousands	351.7	13.5	17.2	23.1
• Daily peak of hospitalized people, thousands	1906.5	65.5	70.4	77.6
Peak day	85	70	72	74

A caveat to bear in mind is that the quantitative results obtained from the simulations of the model are sensitive to the choice of the calibrated parameters.[13] Overall, together with the quantitative effects under alternative scenarios, the qualitative results of our empirical simulations clearly show the significant effects of different decisions of social interaction and policy actions to contain the COVID-19 spread.

5 Conclusions

We presented a dynamic discrete-time model of the COVID-19 spread. The model provides information on six variables relevant for the quantitative analysis of many ongoing containment efforts. Three general results emerge from the simulations. First, isolation significantly slows down the speed of the contagion. Second, isolation reduces both the total number of people infected and deaths. Third, isolation exhibits increasing returns. The model provides a clear interpretation of the forces that produce the "flattening of the curve" of infected people towards the maximum capacity of the health care system.

In the simulation exercises conducted to examine the COVID-19 spread in Spain, the calibrated model shows that the actions of social distancing have tremendous effects on the evolution of the disease. According to the model results, no intervention to reduce people face-to-face interactions would condemn almost all the population to get infected and more than 350,000 Spanish people would die. The State of Alarm, characterized in the model by a reduction in the number of face-to-face contacts among individuals from 25 to 3 per day (and further down to 1.8 encounters with tighter enforcement passed 15 days after the initial declaration), is estimated to cut the number of deaths by 95% and the number of hospital beds needed by 96%. Remarkably, even significantly larger cuts would have been found with a tighter social distancing action, and especially, with an earlier policy intervention.

Our model can be calibrated to other countries' data to quantify the impacts of isolation measures in the face of the COVID-19 pandemic.

[13] For example, there might be changes in the primary contagion probability, α, which could be capturing the response of Spanish citizens to SoA health recommendations to wear protective gear (masks, gloves, washing hands) when exposed to physical contacts.

References

Anderson, R. M., Heersterbeek, H., Klinkenberg, D. and Hollingsworth, T. D.: 2020, How will country-based mitigation measures influence the course of the COVID-19 epidemic?, *The Lancet* .

Atkeson, A.: 2020, What will be the economic impact of COVID-19 in the US? rough estimates of disease scenarios, *NBER Working Paper 26867*, UCLA.

Eichenbaum, M. S., Rebelo, S. and Trabandt, M.: 2020, The macroeconomics of epidemics, *NBER Working Paper 26882*, Northwestern University.

Ferguson, N. M., Laydon, D., Nedjati-Gilani, G., Imai, N., Ainslie, K., Baguelin, M., Bhatia, S., Boonyasiri, A., Cucunubá, Z., Cuomo-Dannenburg, G., Dighe, A., Dorigatti, I., Fu, H., Gaythorpe, K., Green, W., Hamlet, A., Hinsley, W., Okell, L. C., van Elsland, S., Thompson, H., Verity, R., Volz, E., Wang, H., Wang, Y., Walker, P. G. T., Walters, C., Winskill, P., Whittaker, C., Donnelly, C. A., Riley, S. and Ghani, A. C.: 2020, Impact of non-pharmaceutical interventions (NPIs) to reduce covid-19 mortality and healthcare demand, *Technical report*, Imperial College COVID-19 Response Team, Imperial College, London.

Roser, M., Ritchie, H. and Ortiz-Ospina, E.: 2020, Coronavirus disease (COVID-19) – Statistics and Research, *Our World in Data* . https://ourworldindata.org/coronavirus.

Wang, H., Wang, Z., Dong, Y., Chang, R., Xu, C., Yu, X., Zhang, S., Tsamlag, L., Shang, M., Huang, J., Wang, Y., Xu, G., Shen, T., Zhang, X. and Cai, Y.: 2019, Phase-adjusted estimation of the number of coronavirus disease 2019 cases in Wuhan, China., *Cell Discovery* **6**(1).

Wu, J. T., Leung, K., Bushman, M., Kishore, N., Niehus, R., de Salazar, P. M., Cowling, B. J., Lipsitch, M. and Leung, G. M.: 2020, Estimating clinical severity of COVID-19 from the transmission dynamics in Wuhan, China., *Nature Medicine* .