School of Mathematics and Statistics

MT5836 Galois Theory

Handout 0: Course Information

Lecturer: Martyn Quick, Room 326.

Prerequisite: MT3505 (or MT4517) Rings & Fields

Lectures: Mon (odd), Wed & Fri 11am, Maths Room 1A

Tutorials: The tutorials begin in Week 2. There are two choices of tutorial time:

Mon 2pm (Maths Room 1C), Fri 2pm (Maths Room 1C)

Assessment: 100% exam

Fire Safety: If the fire alarm were to be sounded during a lecture, the route to exit from Room 1A is to leave via the back-door of the Maths Institute.

Recommended Texts:

- Ian Stewart, *Galois Theory*, Chapman & Hall; 3rd Edition, 2004 in the library; 4th Edition, 2015.
- John M. Howie, *Fields and Galois Theory*, Springer Undergraduate Mathematics Series, Springer, 2006.
- P. M. Cohn, *Algebra, Vol. 2*, Wiley, 1977, Chapter 3. [Out of print, but available in the library.]

Support Materials: All handouts, problem sheets and solutions will be available in PDF format from MMS. Solutions will only be posted in MMS after all relevant tutorials have happened. The handouts should contain all information appearing on slides.

Course Content:

- **Basic facts about fields and polynomial rings:** Mostly a review of material from MT3505, but some new information about irreducible polynomials.
- **Field extensions:** Terminology and basic properties about the situation of two fields with $F \subseteq K$.
- **Splitting fields & normal extensions:** Field extensions constructed by adjoining the roots of a polynomial, constructed so that this polynomial factorizes into linear factors over the larger field.
- **Basic facts about finite fields:** Existence and uniqueness of fields of order p^n, plus the fact that the multiplicative group of a finite field is cyclic.
- **Separable extensions & the Theorem of the Primitive Element:** Separability: A technical condition to avoid repeated roots of irreducible polynomials. The Theorem of the Primitive Element applies in this circumstance and allows us to assume that our field extensions have a specific form and hence to simplify various proofs.
- **Galois groups & the Fundamental Theorem of Galois Theory:** The definition of the Galois group as the collection of invertible structure preserving maps of a field extension. The Fundamental Theorem of Galois Theory states that the structure of the Galois group corresponds to the structure of the field extension.
- **Applications:** Specifically the link between solution of the polynomial equation by radicals to solubility of the Galois group.