Chapter 6

Integration Essentials: Revision

{chap:6}

Swokowski Chapter 4

Some essential facts from earlier modules that you really should know. They are going to be useful in this part of the course.

6.1 Standard Integrals

You should know the standard integrals listed below. If you can't write the answers in immediately, go and find out the correct answers and **learn** them. They are essential and really knowing them will make a lot of your subsequent modules much easier!

In the following integrals a, b and n are constants.

1. $\int e^{ax} dx =$

2. $\int \sin ax dx =$

3. $\int \cos ax dx =$

4. $n \neq -1, \qquad \int (ax+b)^n \, dx =$

5. $\int \frac{1}{ax+b} dx =$

6. $\int \tan(ax) dx =$

7. $\int \sec^2 ax dx =$

8. $\int \frac{1}{a^2+x^2} dx =$

9. $\int \frac{1}{\sqrt{a^2-x^2}} dx =$

6.2 Definition of an Integral

There are many ways to think of an integral. In the mathematical sense, it is defined in terms of a limit as

$$I = \int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\delta x,$$

where

$$\delta x = \frac{b-a}{n}, \text{ and } x_i = a + \left(i - \frac{1}{2}\right)\delta x.$$

An alternative, but equivalent definition is

$$I = \int_a^b f(x)dx = \lim_{\delta x\to 0} \sum_{i=1}^{n} f(x_i)\delta x.$$

This definition allows us to determine the limits of double and triple integrals by adding up the contributions $f(x_i)dx$ to the integral.

Visualising the integral as a summation can help determine the limits of the integration, as we will see later.

6.3 Polar Coordinates

We are used to working with Cartesian coordinates but there are many situations where it is simpler to use (and indeed it is the only way to proceed) other coordinate systems. You have already seen *polar coordinates* for two-dimensional space, where the point described by the Cartesian coordinates (x, y) can be represented by the polar coordinates (R, ϕ). The relationship between Cartesian and polar coordinates (shown in Figure 6.1) is obtained by simple trigonometry and gives

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:6.2}
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:6.4}

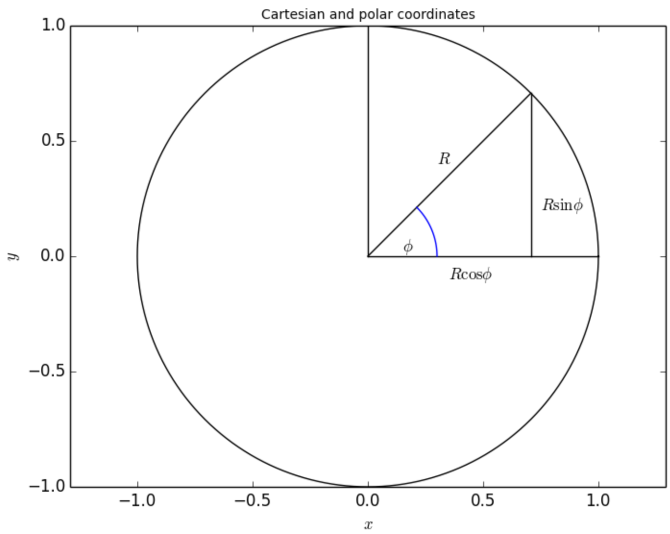
$$\begin{aligned} x &= R\cos\phi, && (6.1)\\ y &= R\sin\phi, && (6.2)\\ R &= \sqrt{x^2+y^2}, && (6.3)\\ \phi &= \tan^{-1}\left(\frac{y}{x}\right), && (6.4) \end{aligned}$$

If you understand these, you will understand cylindrical coordinates later on. In addition, spherical coordinates will also appear easier. Note that the notation used here is not that standard and is used specifically to ease use of other coordinates later. Normally, (r, θ) are used for polar coordinates. It is a fact of mathematical life that you need to use different notation for the same phenomena in different situations.

6.4 Curve Sketching

Remember all the ideas involved in curve sketching, e.g. locate turning points, determine how a function behaves for $x = 0$ and x large in negative and positive sense etc. We will need to sketch functions of two and three variables but a key feature is the ability to draw standard functions of one variable.

Can you sketch, for example, the following functions?



{fig:Cart} Figure 6.1: The relationship between Cartesian and polar coordinates.

- e^x
- $\log x$
- $1/(1+x^2)$