CONIC SECTIONS

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OVERVIEW / TIME TABLE

L1 (MON)

SKETCHING CONIC SECTIONS
- eq^{n} of ellipse
  hyperbola
  parabola

P1 (WED)

Examples class based on L1

L2 (FRI) + L3 (WED)

GEOMETRIC DEF^{n} OF CONIC SECTIONS
- parabola
- ellipse
- hyperbola

ECCENTRICITY + DIRECTRICES

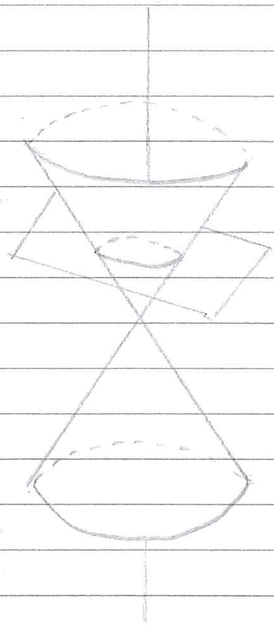
P2 (FRI)

Problems based on L2 + L3.

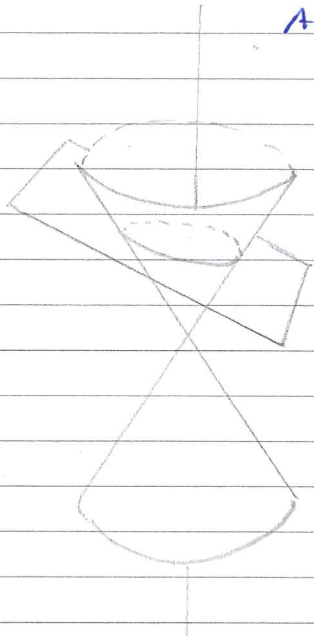
T1 : Practice in section conic sections
- based on L1 + P1.

T2 : Practice in geometric interpretation
based on L2 + L3.

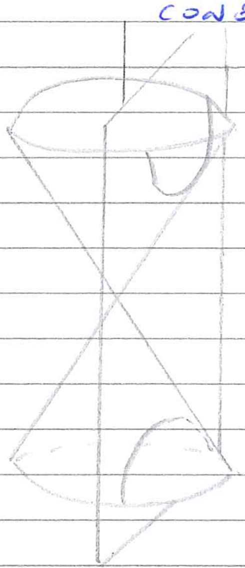
STANDARD CONIC SECTIONS : CURVES IN WHICH A PLANE CUTS A DOUBLE CONE



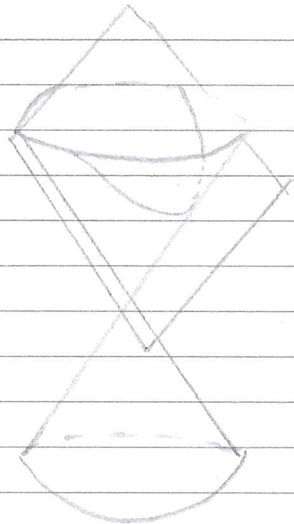
CIRCLE : PLANE ⊥ TO CONE AXIS



ELLIPSE : PLANE NOT ⊥ TO CONE AXIS

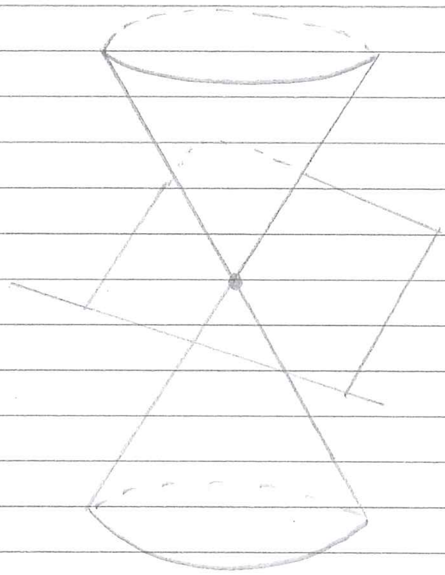


HYPERBOLA : PLANE PARALLEL TO CONE AXIS

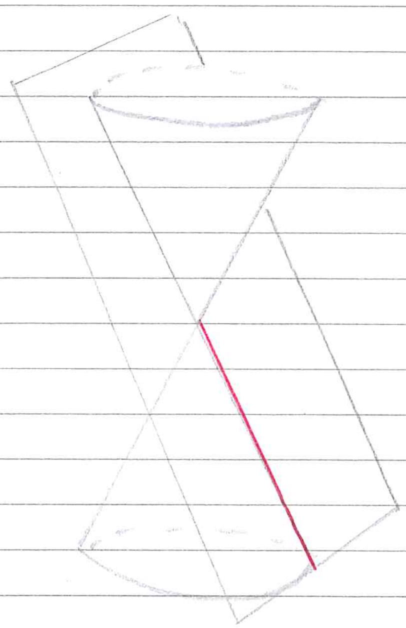


PARABOLA : PLANE PARALLEL TO SIDE OF CONE

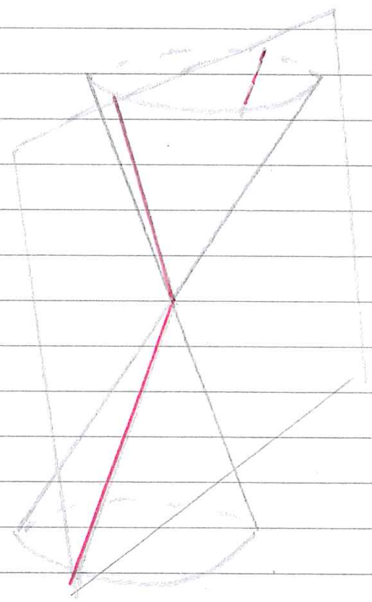
DEGENERATE CONIC SECTIONS : THE POINT AND LINES OBTAINED BY PASSING A PLANE THROUGH THE CONES VERTEX.



POINT : PLANE THRU CONE VERTEX ONLY



STRAIGHT LINE: PLANE TANGENT TO CONE



PAIR OF INTERSECTING LINES

CONIC SECTIONS

PRELIMINARIES

The standard conic sections are the curves in which a plane cuts a double cone. (circle, ellipse, hyperbola, parabola)
Degenerate conic sections are the point and lines obtained by passing a plane through the cones vertex.

- see handout

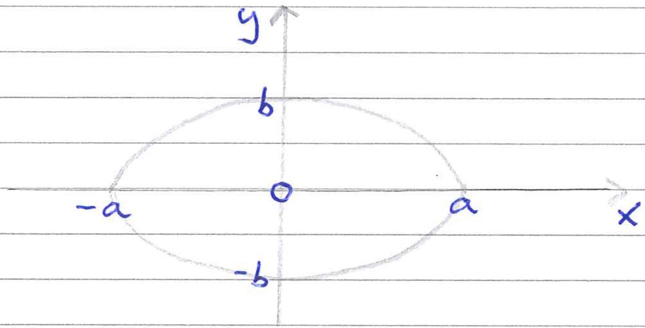
1) SKETCHING CONIC SECTIONS

Let's look at how to sketch conic sections starting with their eq$^{\underline{ns}}$ w.r.t carefully chosen x and y axes in the plane of intersection.

1.1) THE ELLIPSE

The eq$^{\underline{n}}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (1)$$



describes an ellipse.

Letting $x = 0$ we see that the curve meets the y-axis at $y = \pm b$, similarly $y = 0 \Rightarrow$ the curve intercepts the x-axis at $x = \pm a$

* Note if $a = b$ we have a circle of radius a.
$(x^2 + y^2 = a^2)$

- Suppose we choose translated (shifted) $\bar{x}, \bar{y}$ axes at a new origin (h, k) then

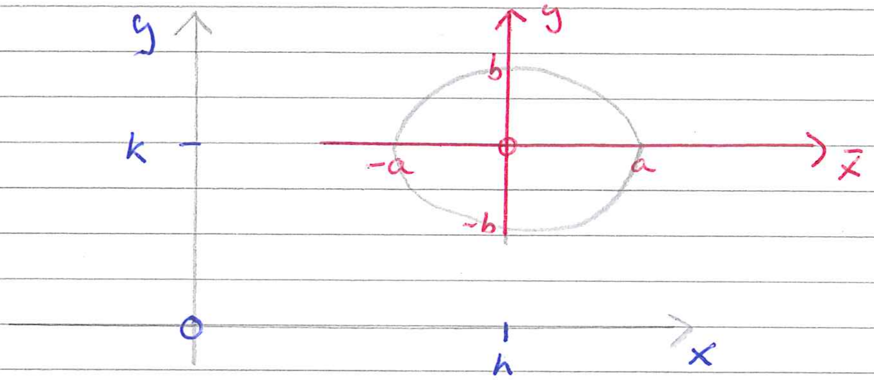
$$\bar{x} = x - h \quad , \quad \bar{y} = y - k \qquad \text{and}$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad (2)$$

can be written as

$$\frac{\bar{x}^2}{a^2} + \frac{\bar{y}^2}{b^2} = 1$$ which is an ellipse in the $\bar{x}$-$\bar{y}$ plane.

This shows that ② describes an ellipse with centre (h, k)

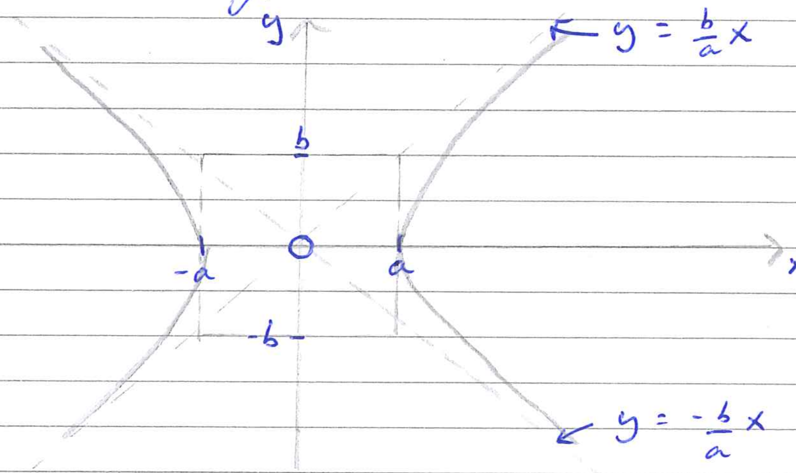


1.2) THE HYPERBOLA

The eq^{n}

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad ③$$

describes a hyperbola. To sketch it :-



- mark $\pm a$ on the x-axis, $\pm b$ on the y axis.
- draw a rectangle using the 4 points.
- draw and extend diagonals of rectangle.
- the hyperbola has these diagonals $y = \pm \frac{b}{a} x$ as asymptotes.

- $y = 0 \Rightarrow x = \pm a$ (curve crosses x-axis at $\pm a$)

- $x = 0 \Rightarrow y^2 = -b^2$ which has no real sol^{n} i.e. curve does not cross the y-axis.

Note that

$$③ \Rightarrow \quad y = \pm b\sqrt{\frac{x^2}{a^2} - 1}$$

- $$\lim_{x \to \infty} \left[\pm b\sqrt{\frac{x^2}{a^2} - 1} \mp \frac{b}{a}x \right] = 0$$

i.e $y = \pm \frac{bx}{a}$ are indeed asymptotes of the curve.

- If the -ve sign in ③ appears with the x term i.e

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad ④$$

then the asymptotes are still $y = \pm \frac{bx}{a}$ but the hyperbola crosses the y axis at $\pm b$ and it does not cross the x axis.

~~4) THE PARABOLA~~

- Equations of the form

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \text{or}$$

$$-\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

describe hyperbolas with centre (h, k). To sketch them draw the rectangle with centre (h, k) crossing at right angles the $\bar{x}$-axis at $\bar{x} = \pm a$ and the $\bar{y}$-axis at $\pm b$.

$\bar{x} = x - h$
$\bar{y} = y - h$

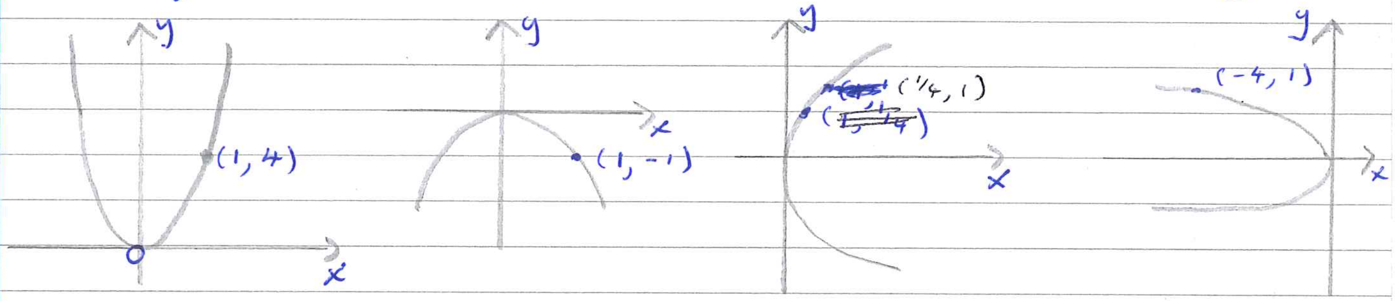
Draw + extend the diagonals of the rectangle + proceed as above.

1.4) THE PARABOLA

The curves

$$y = 4x^2, \quad y = -x^2, \quad x = \frac{1}{4}y^2, \quad x = -4y^2$$

are parabolas with vertices at the origin.



$y = 4x^2$ $\qquad$ $y = -x^2$ $\qquad$ $x = \frac{1}{4}y^2$ $\qquad$ $x = -4y^2$

~~The size of the coeff of the quadratic term controls how fast the parabola "opens".~~

If the vertex of a parabola $y = cx^2$ (c = const) is moved to (h, k) the eq^n describing it is given by

$$y - k = c(x - h)^2$$

[Similarly $x = y^2 \rightarrow x - h = c(y - k)^2$]

The sign of c determines the dir^n in which the parabola "opens" and $|c|$ controls how quickly it opens.

Any polynomial eq^n in x and y that is quadratic in one of the variables + linear in the other describes a parabola.