MT5823 Semigroups: Project Information

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In this module, 25% of the assessment is for a self-study project. This sheet contains the guidelines for your project work.

Your project will be marked out of 25, subdivided into four parts:

1. **Scope:** what was covered, and whether or not this was appropriate. [6 marks]
2. **Writing:** for the readability and overall structure of the project [6 marks]
3. **Mathematical rigour:** the quality and accuracy of the mathematics in the project [7 marks]
4. **Originality and initiative:** how much the project differed from the references, how many sources were combined into the project, original solutions to exercises, the inclusion of examples [6 marks]

Projects will be based on selected topics from the following monographs:

[**Bo**] R. V. Book and F. Otto, *String-rewriting systems*, Springer-Verlag, New York, Texts and Monographs in Computer Science, 1993.

[**Ho**] J. M. Howie, *Fundamentals of Semigroup Theory*, London Mathematical Society Monographs **12**, Clarendon Press, Oxford, 1995.

[**Ro**] J. J. Rotman, *An introduction to the theory of groups*, Springer-Verlag, New York, Graduate Texts in Mathematics, 1995.

They are all available through the Mathematics/Physics Library. It is within the scopes of copyright law for you to photocopy your chosen section, and some further surrounding relevant material. University guidelines on the Copyright Law can be inspected here:

http://www.st-andrews.ac.uk/library/information/furtherhelp/copyright/

Your project should consist of a brief presentation of the theoretical material, preferably with some key proofs included, followed by attempts at specified exercises. Additional illustrative examples, or references to further literature would enhance the project.

Topics

Congruences on 0-simple semigroups: [Ho], Sections 3.5 and/or 3.7, Exercises: 13, 14, 15.

E-unitary inverse semigroups: [Ho], Section 5.9, Exercises: 20, 21, 33, 34, and 35.

Free inverse semigroups: [Ho], Section 5.10, Exercises: 41 and 42.

Undecidability of the word problem for semigroup presentations: [Ro], Chapter 12, Section entitled "The Markov-Post Theorem". There are no exercises in this section, so instead try to give some examples of Turing machines that are not found in [Ro], perhaps try to find (but maybe not prove) further examples of semigroups with undecidable word problem.

The Knuth-Bendix algorithm: [Bo], Section 2.4. This book has no exercises, so instead you can try to look at the semigroup and monoid presentations we have seen throughout the course, and perform the algorithm on those examples.

Submission

Projects should be submitted by the end of Friday in Week 12.

It is absolutely fine to discuss your project with me, or your fellow students. The final project, however, ***must*** be written solely by you, and be a record of your understanding of the material.