

## Finite Mathematics Problem Set 2

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### 1 EXERCISE 1

Prove that

$$(p-1)! + 1 \equiv 0 \pmod{p}$$

### 2 EXERCISE 2

Let  $\mathcal{F}$  be a field of characteristic  $p$ . Prove that for any  $a, b \in \mathcal{F}$ ,

$$(a+b)^p = a^p + b^p$$

### 3 EXERCISE 3

How does the previous exercise not contradict the fact that a polynomial of degree  $p$  has at most  $p$  roots over a field?

### 4 EXERCISE 4

- Compute  $\phi(1728)$ .
- Prove that  $\sum_{d|n} \phi(d) = n$  by using the multiplicativity of  $\phi$ .