Hyperbolic functions formula sheet.

1 Definitions.

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}, \quad x \neq 0, \quad \sec hx = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}, \quad \csc hx = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}, \quad x \neq 0$$

2. Identities

Hyperbolic	Trigonometric
$\cosh^2 x - \sinh^2 x = 1$	$\sin^2 x + \cos^2 x = 1$
$1 - \tanh^2 x = \sec h^2 x$	$\sec^2 x = 1 + \tan^2 x$
$\coth^2 x - 1 = \csc h^2 x$	$\mathrm{cosec}^2 x = 1 + \cot^2 x$

3. Derivatives

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cosh x) = \sinh x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tanh x) = \sec h^2 x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

4. Inverses

$$\sinh^{-1} x = \ln\{x + \sqrt{x^2+1}\}$$

$$\cosh^{-1} x = \ln\{x + \sqrt{x^2-1}\}$$

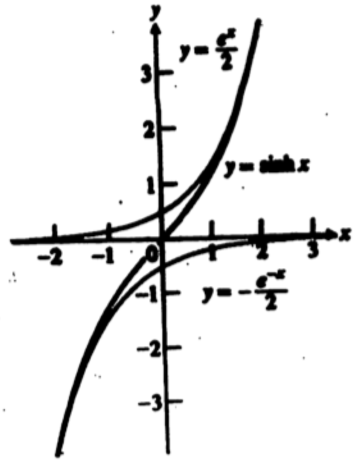
$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}, \quad |x| < 1$$

5. Derivatives

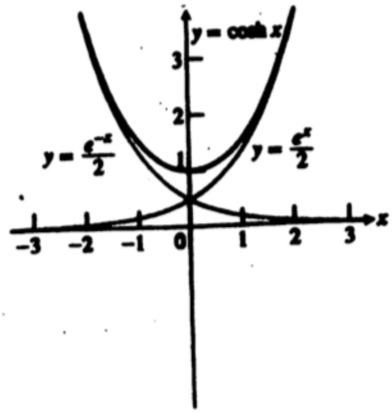
$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}} \qquad \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, \quad x > 1 \qquad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}, \quad |x| < 1$$

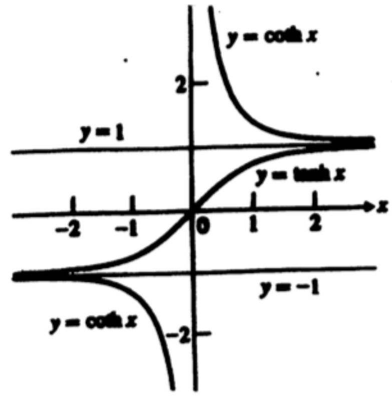
$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, \quad |x| < 1 \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, \quad x \in \mathbb{R}$$



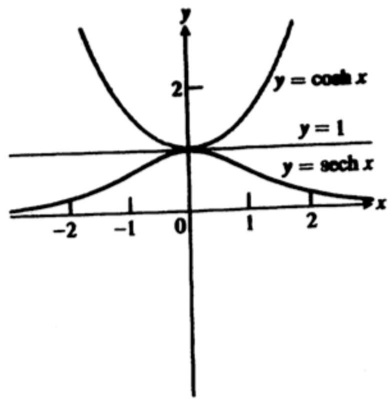
(a) The hyperbolic sine and its component exponentials.



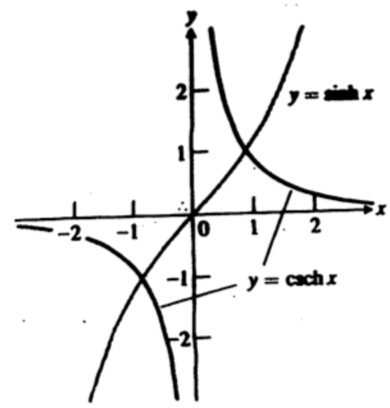
(b) The hyperbolic cosine and its component exponentials.



(c) The graphs of $y = \tanh x$ and $y = \coth x = 1/\tanh x$.

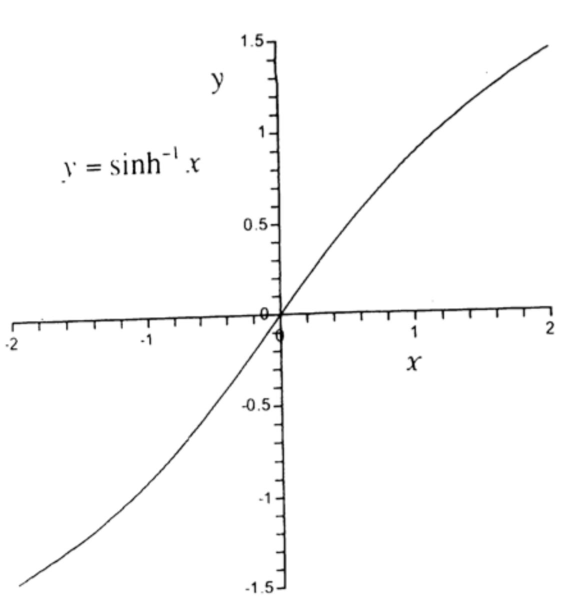


(d) The graphs of $y = \cosh x$ and $y = \operatorname{sech} x = 1/\cosh x$.



(e) The graphs of $y = \sinh x$ and $y = \operatorname{csch} x = 1/\sinh x$.

Figure 1. Graphs of the indicated hyperbolic functions



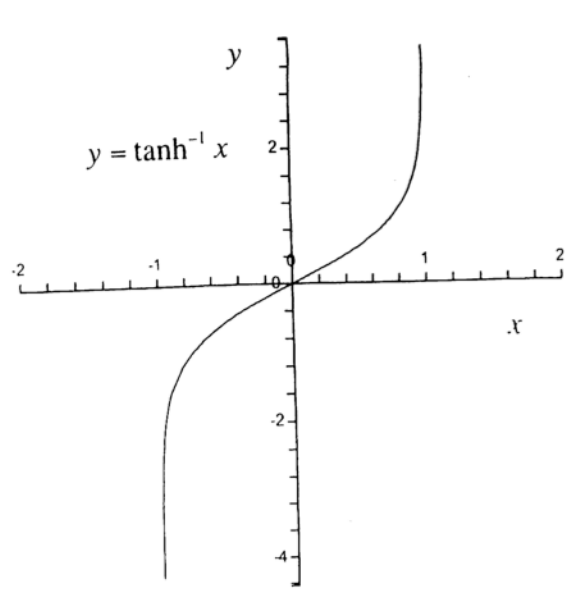


Figure 2. Graphs of the indicated inverse hyperbolic functions