

2-3

$$(2) \quad \forall x (P(x) \wedge P(y) \wedge \neg E(x, y)) \rightarrow$$

$$\forall x \forall y (P(x) \wedge P(y) \wedge \neg E(x, y)) \rightarrow$$

$$(\exists z)(L(z) \wedge R(x, y, z)).$$

$$(3) \quad (\forall x \forall y (P(x) \wedge L(x, y) \wedge R(x, y) \rightarrow Q(x, y))) \text{ 表示 } x \text{ 是奇数,}$$

x 与 y 之和, x 与 y 之积. x 大于 y .

$$\exists x \exists y \exists z (Q(L(x, y), R(x, z))).$$

$$P(x): x \text{ 是奇数} \quad G(x, y), x \text{ 大于 } y$$

$$(\exists x)(\exists y)(\exists z) (P(x) \wedge P(y) \wedge R(z) \wedge G(x+y, x \cdot z))$$

2-4.

$$(4) \quad (c). (\forall x) (P(x) \rightarrow Q(x)) \Leftrightarrow$$

$$(\forall x) (\neg P(x) \vee Q(x)).$$

$$(P(a) \rightarrow Q(a)) \wedge (P(b) \rightarrow Q(b)) \wedge (P(c) \rightarrow Q(c))$$

$$(3) \quad (b). (P \rightarrow Q(2)) \wedge (P \rightarrow Q(3)) \wedge (P \rightarrow Q(6)) \vee R(5).$$

$$(T \rightarrow T) \wedge (T \rightarrow T) \wedge (T \rightarrow F) \vee F \Leftrightarrow F.$$

$$1.4) (b) (\forall u) (P(u) \rightarrow (R(u) \vee Q(u))).$$

$$\exists v R(v) \rightarrow (\exists z) S(x, z),$$

2-5.

$$1.6) (\forall x) (\forall y) (P(x, y) \rightarrow P(f(x), f(y))) \Leftrightarrow$$

$$(\forall x) (P(x, 1) \rightarrow P(f(x), 2)) \Leftrightarrow$$

$$(P(1, 1) \rightarrow P(2, 2)) \wedge (P(1, 2) \rightarrow P(2, 1))$$

$$\wedge (P(2, 1) \rightarrow P(1, 2)) \wedge (P(2, 2) \rightarrow P(1, 1)).$$

$$\Leftrightarrow (T \rightarrow F) \wedge (T \rightarrow F) \wedge (F \rightarrow T) \wedge (F \rightarrow T)$$

$$\Leftrightarrow F \wedge F \wedge T \wedge T \Leftrightarrow F.$$

$$4 \text{ 证明: } (\exists x) (A(x) \rightarrow B(x)) \Leftrightarrow (\exists x) (\neg A(x) \vee B(x))$$

$$\Leftrightarrow (\exists x) \neg A(x) \vee (\exists x) B(x). \Leftrightarrow \neg (\forall x) A(x) \vee (\exists x) B(x)$$

$$\Leftrightarrow (\forall x) A(x) \rightarrow (\exists x) B(x).$$

$$7. (\forall x) (\forall y) (P(x) \rightarrow Q(y)) \Leftrightarrow$$

$$(\forall x) (\forall y) (\neg P(x) \vee Q(y)) \Leftrightarrow$$

$$\forall x \neg P(x) \vee \forall y Q(y) \Leftrightarrow$$

$$\neg Q(\exists x) P(x) \vee \forall y Q(y) \Leftrightarrow$$

$$(\exists x) P(x) \rightarrow (\forall y) Q(y).$$

2-b.

(1) b) 化为前束范式.

$$(\exists x) (\neg ((\exists y) P(x, y)) \rightarrow ((\exists z) Q(z) \rightarrow R(x)). \Leftrightarrow$$

$$(\exists x) (\exists y) P(x, y) \vee ((\exists z) Q(z) \rightarrow R(x)) \Leftrightarrow$$

$$(\exists x) (\exists y) P(x, y) \vee (\neg (\exists z) Q(z) \vee R(x)) \Leftrightarrow$$

$$(\exists x) (\exists y) P(x, y) \vee ((\forall z) \neg Q(z) \vee R(x)) \Leftrightarrow$$

$$(\exists x) (\exists y) (\forall z) (P(x, y) \vee \neg Q(z) \vee R(x))$$

$$(2) c) (\forall x) P(x) \rightarrow (\exists x) ((\forall z) Q(x, z) \vee (\forall z) R(x, y, z))$$

$$\Leftrightarrow \neg (\forall x) P(x) \vee (\exists x) ((\forall z) Q(x, z) \vee (\forall z) R(x, y, z))$$

$$\Leftrightarrow (\exists x) \neg P(x) \vee (\exists x) ((\forall z) Q(x, z) \vee (\forall z) R(x, y, z))$$

$$\Leftrightarrow (\exists x) \neg P(x) \vee (\exists x) ((\forall z) Q(x, z) \vee (\forall z) R(x, y, z))$$

$$\Leftrightarrow (\exists x)(\forall z)(\forall r_1)(\neg P(x) \vee Q(x, z) \vee R(x, y, r_1))$$

即前束合取范式。接下来换成前束析取范式：

$$\begin{aligned} \Leftrightarrow (\exists x)(\forall z)(\forall r_1) & (P(x) \wedge Q(x, z) \wedge R(x, y, r_1)) \vee \\ & (P(x) \wedge Q(x, z) \wedge \neg R(x, y, r_1)) \vee (P(x) \wedge \neg Q(x, z) \wedge \\ & R(x, y, r_1)) \vee (P(x) \wedge \neg Q(x, z) \wedge \neg R(x, y, r_1)) \vee \\ & (\neg P(x) \wedge Q(x, z) \wedge \neg R(x, y, r_1)) \vee \\ & (\neg P(x) \wedge \neg Q(x, z) \wedge R(x, y, r_1)) \vee \\ & (\neg P(x) \wedge \neg Q(x, z) \wedge \neg R(x, y, r_1)) \end{aligned}$$

2=7

(1) c) 证明: (1) $(\forall x)(A(x) \rightarrow B(x))$

P

(2) $(A(u) \rightarrow B(u))$

~~USE~~ USU

(3) $\neg B(u) \rightarrow \neg A(u)$

T(2) E

(4) $(\forall x)(C(x) \rightarrow \neg B(x))$

P

(5) $C(u) \rightarrow \neg B(u)$

~~USE~~ USU

(6) ~~$(\forall x)(C(x) \rightarrow \neg A(x))$~~ $C(u) \rightarrow \neg A(u)$ T(3) U1

(7) $(\forall x)(C(x) \rightarrow \neg A(x))$

UG(6).

d) 证明: (1) $(\forall x)(A(x) \vee B(x))$ P

(2) $A(u) \vee B(u)$ $US(1)$

(3) $(\forall x)(B(x) \rightarrow \neg C(x))$ P

(4) $B(u) \rightarrow \neg C(u)$ $US(3)$

(5) $(\forall x) C(x)$ P

(6) $C(u)$ $US(5)$

(7) $\neg B(u)$ $T(4)(6)I$

(8) $A(u)$ $T(4)(7)I$

(9) $(\forall x) A(x)$ $UG(8)$

(2) b) 证明: $(\forall x) P(x) \vee (\exists x) Q(x) \Leftrightarrow$

$\neg(\forall x) P(x) \rightarrow (\exists x) Q(x)$ ~~\Leftrightarrow~~

$(\exists x) \neg P(x) \rightarrow (\exists x) Q(x)$ ~~\Leftrightarrow~~

(3)

(1) $(\exists x) \neg P(x)$

P (附加条件)

(2) $\neg P(u)$

~~$ES(1)$~~

(3) $(\forall x)(P(x) \vee Q(x))$

P

(4) $P(u) \vee Q(u)$

$ES(3)$

$$(5) \quad Q(u)$$

$$\neg(\neg(\neg I)$$

$$(6) \quad (\exists x) Q(x)$$

$$\neg G U$$

$$(7) \quad (\exists x) \neg P(x) \rightarrow (\exists x) Q(x) \quad CP.$$