Heat Equation- BTCS in notes

March 8, 2017

1 Heat Equation

1.1 The Differential Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Initial Condition

$$u(x,0) = 2x, \;\; 0 \leq x \leq \frac{1}{2}$$

$$u(x,0) = 2(1-x), \;\; \frac{1}{2} \leq x \leq 1$$

1.2 Boundary Condition

$$u(0,t) = 0, u(1,t) = 0$$

1.3 The Implicit Backward Time Centered Space (BTCS) Difference Equation

$$w[i,j+1] = w[i,j] + \frac{k}{h^2}(w[i+1,j+1] - 2w[i,j+1] + w[i-1,j+1])$$

$$-rw[i-1,j+1] + (1+2r)w[i,j+1] - rw[i+1,j+1] = w[i,j]$$

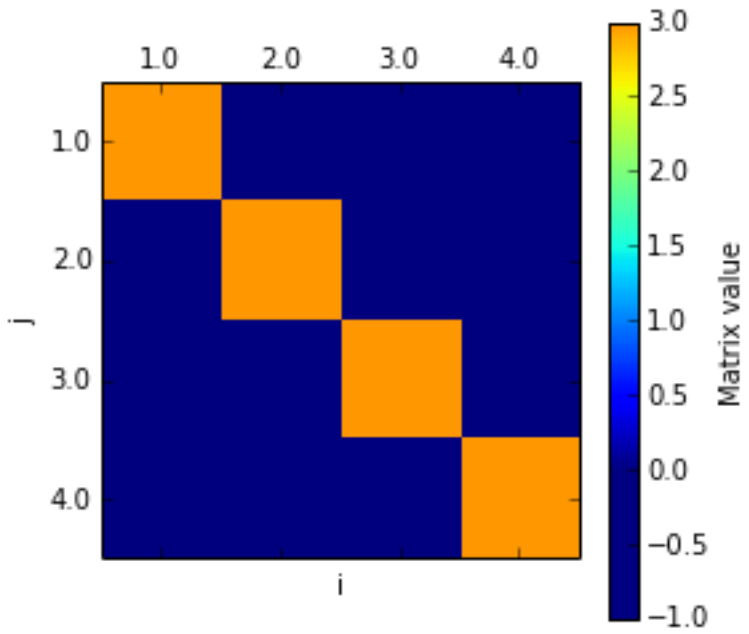
where $r = \frac{k}{h^2}$

In [2]:
```
# LIBRARY
# vector manipulation
import numpy as np
# math functions
import math

# THIS IS FOR PLOTTING

%matplotlib inline
import matplotlib.pyplot as plt # side-stepping mpl backend
import warnings
warnings.filterwarnings("ignore")
```

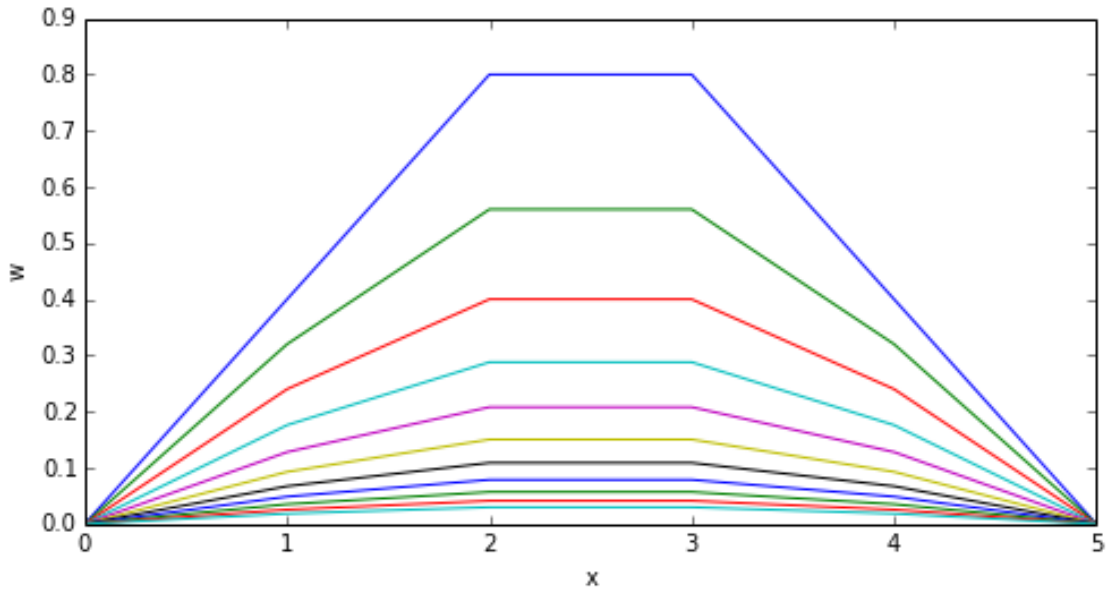
In [13]:
```
N=5
Nt=25
h=1/N
ht=1/Nt

time_iteration=10
time=np.arange(0,(time_iteration+.5)*ht,ht)

x=np.arange(0,1.0001,h)

w=np.zeros((N+1,time_iteration+1))
r=ht/(h*h)
A=np.zeros((N-1,N-1))
c=np.zeros(N-1)
b=np.zeros(N-1)
b[0]=0

# Initial Condition
for i in range (1,N):
    #w[0,i]=1-x[i]-1/np.pi*np.sin(2*np.pi*x[i])
    w[i,0]=2*x[i]
    if x[i]>0.5:
        w[i,0]=2*(1-x[i])


# Boundary Condition
for k in range (0,time_iteration):
    #w[k,0]=1
    w[0,k]=0
    w[N,k]=0


for i in range (0,N-1):
    A[i,i]=1+2*r

for i in range (0,N-2):
    A[i+1,i]=-r
    A[i,i+1]=-r


fig = plt.figure(figsize=(8,4))
plt.matshow(A)
plt.xlabel('i')
plt.ylabel('j')
plt.xticks(np.arange(N-1), np.arange(1,N-0.9,1))
plt.yticks(np.arange(N-1), np.arange(1,N-0.9,1))
clb=plt.colorbar()
clb.set_label('Matrix value')
```

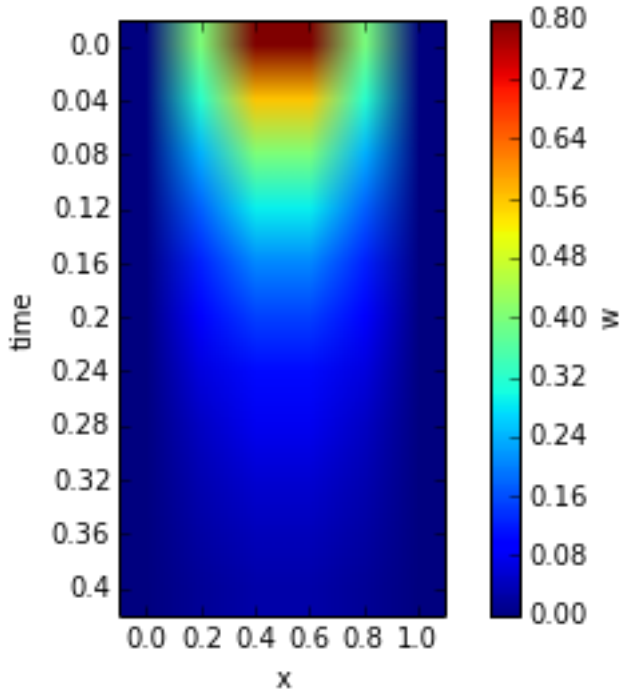
```
        clb.set_clim((0,4))


        plt.show()

        Ainv=np.linalg.inv(A)


        for k in range (1,time_iteration+1):
            w[1:(N),k]=np.dot(Ainv,w[1:(N),k-1])



        print(w[:,1])
        print(A)
        print(w[:,2])
        print(w[:,3])
        print(w[:,4])
        print(w[:,5])

        fig = plt.figure(figsize=(8,4))
        plt.plot(w)
        plt.xlabel('x')
        plt.ylabel('w')

        fig = plt.figure()
        plt.imshow(w.transpose())
        plt.xticks(np.arange(len(x)), x)
        plt.yticks(np.arange(len(time)), time)
        plt.xlabel('x')
        plt.ylabel('time')
        clb=plt.colorbar()
        clb.set_label('w')

        plt.show()
```

```
<matplotlib.figure.Figure at 0x7fc82dcdb5f8>
```




```
[ 0.    0.32  0.56  0.56  0.32  0.  ]
[[ 3. -1.  0.  0.]
 [-1.  3. -1.  0.]
 [ 0. -1.  3. -1.]
 [ 0.  0. -1.  3.]]
[ 0.    0.24  0.4   0.4   0.24  0.  ]
[ 0.     0.176  0.288  0.288  0.176  0.   ]
[ 0.     0.128  0.208  0.208  0.128  0.   ]
[ 0.      0.0928  0.1504  0.1504  0.0928  0.    ]
```


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
w
0
1
2
3
4
5
x


0.0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
0.4
time
0.0 0.2 0.4 0.6 0.8 1.0
x
0.80
0.72
0.64
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00
w

In [ ]: