ex 1

I.C.

$$\rho(x,0) = 0 \quad \forall x$$

$$\lim_{t \to 0} \int_0^\infty x\,\rho(x,t)\,dx = 1$$

$$\rho(x,t) = A(t)e^{-(x-1)t}$$

$$\frac{\partial \rho(x,t)}{\partial t} = -(x-1)\rho(x,t) + 2\int_{x+1}^\infty \rho(y,t)\,dy$$

$$\frac{\partial}{\partial t}\left(A(t)e^{-(x-1)t}\right) = -(x-1)A(t)e^{-(x-1)t}$$

$$+ 2\int_{x+1}^\infty A(t)e^{-(y-1)t}\,dy$$

$$A'(t)e^{-(x-1)t} + \cancel{A(t)\cdot-(x-1)e^{-(x-1)t}}$$

$$= -A(t)\cancel{(x-1)}e^{-(x-1)t} + 2A(t)\left(\frac{e^{-(y-1)t}}{-t}\right)\Bigg|_{x+1}^\infty$$

$$= 2A(t)\left(0 - \frac{e^{-(x-1)t-t}}{-t}\right)$$

$$= \frac{2A(t)}{t}e^{-(x-1)t}e^{-t}$$

Ex. 1

$$A'(t) = \frac{2A(t)}{t} e^{-t}$$

$$\rho(x,0) = A(0) e^{-(x-1).0}$$

$$= A(0)$$

$$A(0) = 0$$

$$\lim_{t \to 0} \int_0^\infty x \, \rho(x,t) \, dx = \lim_{t \to 0} \int_0^\infty x \, A(t) e^{-(x-1)t} dx$$

$$= \lim_{t \to 0} A(t) \int_0^\infty x e^{-(x-1)t} dx$$

$$\int u \, dv = uv - \int du \, v$$

$$\int x e^{-(x-1)t} dx = -\frac{x}{t} \cdot e^{-(x-1)t} \Big|_0^\infty - \int_0^\infty \frac{e^{-(x-1)t}}{-t} dx$$

$$= -\frac{e^{-(x-1)t}}{t^2} \Big|_0^\infty$$

$$= \frac{1}{t^2}$$

Ex 1

$$\lim_{t\to 0} \frac{A(t)}{t^2} = 1$$

Ex 2

$$A(t) = t^2 f(t)$$

$$A'(t) = 2t f(t) + t^2 f'(t)$$

$$A'(t) = \frac{2A(t)}{t} e^{-t}$$

$$2t f(t) + t^2 f'(t) = 2t f(t) e^{-t}$$

$$2f(t) + t f'(t) = 2 f(t) e^{-t}$$

$$t f'(t) = 2 f(t)\left(e^{-t} - 1\right)$$

$$f'(t) = \frac{2 f(t) \left(e^{-t} - 1\right)}{t}$$

$$f'(t) + f(t)\left\{2\left(\frac{1-e^{-t}}{t}\right)\right\} = 0$$

$$I = e^{\int_0^t 2\frac{(1-e^{-\tau})}{\tau} d\tau}$$

$$f(t) \cdot I = 1$$

$$f(t) = \exp\left(-\int_0^t 2\frac{(1-e^{-\tau})}{\tau} d\tau\right)$$

EX 3

$$\boxed{\begin{aligned}\cancel{\rho(x,t)} &\cancel{= A(t)\, e^{-(x-1)t}} \\ &= t^2 f(t)\, e^{-(x-1)t}\end{aligned}}$$

$$\cancel{\frac{\partial}{\partial t}\rho(x,t) = 2t f(t) e^{-(x-1)t} + t^2 f'(t) e^{-(x-1)t} - t^2 (x-1) f(t) e^{-(x-1)t}}$$

$$2\int_{x+1}^{\infty} \rho(y,t)\, dy = 2t^2 \frac{f(t)\, e^{-(y-1)t}}{-t}\Bigg|_{x+1}^{\infty}$$

$$= -2t f(t)\, e^{-(y-1)t}\Big|_{x+1}^{\infty}$$

$$\frac{\partial}{\partial t}\rho(x,t) = 2t f(t)\, e^{-xt}$$

$$\rho(x,t) = 2\int_0^t \tau f(t) e^{-x\tau}\, d\tau$$

EX 4

$$Q(t) = \int_0^{\infty} \left(2 \int_0^t \tau f(\tau) e^{-x\tau} d\tau \right) dx + \int_1^{\infty} t^2 f(t) e^{-(x-1)t} dx$$

$$\frac{dQ}{dt} = \int_0^1 \left(2t f(t) e^{-xt} \right) dx + \int_1^{\infty} \Big($$

$$= 2tf(t) \int_0^1 e^{-xt} dx$$

$$= 2t f(t) \left[\frac{e^{-xt}}{-t} \right]_0^1$$

$$= \frac{2t}{t} f(t) \left[-e^{-xt} \right]_0^1$$

$$= 2 f(t) \left(1 - e^{-t} \right)$$

$$\frac{d}{dt} \int_1^{\infty} t^2 f(t) e^{-(x-1)t} dx = \int_1^{\infty} \frac{d}{dt} \left(t^2 f(t) e^{-(x-1)t} \right) dx$$

$$= \left[2tf(t) e^{-(x-1)t} + t^2 f'(t) e^{-(x-1)t} \cancel{-} x t^2 f(t) e^{-(x-1)t} \right]$$

$$= \int^{\infty} \cancel{2t f(t) e^{-(x-1)t} + t^2 f'(t) e^{-(x-1)t} - \ldots e^{-(x-1)t}}$$

$$= \cancel{\frac{K_1}{-t} + K_2}$$

$$2tf(t)\int_1^\infty e^{-(x-1)t}\,dx + t^2 f'(t)\int_1^\infty e^{-(x-1)t}\,dx + t^2 f(t)\int_1^\infty -(x-1)e^{-(x-1)t}\,dt$$

$$2t\,f(t)\left[\frac{e^{-(x-1)t}}{-t}\right]_1^\infty + t^2 f'(t)\left[\frac{e^{-(x-1)t}}{-t}\right]_1^\infty$$

$$+\,t^2 f(t)\left[-\tfrac{1}{t^2}\right.$$

$$2tf(t)\left(0-\frac{1}{-t}\right) + t^2 f'(t)\cdot\frac{1}{t} - t^2 f(t)\cdot\frac{1}{t^2}$$

$$\left[\,f'(t) = f(t)\cdot -2\left(\frac{1-e^{-t}}{t}\right)\right]$$

$$\left[\int_1^\infty -(x-1)\,e^{-(x-1)t} = -\tfrac{1}{t^2}\right]$$

$$= 2f(t) - 2f(t)e^{-t} + 2f(t) - 2f(t) + 2f(t)e^{-t} - f(t)$$

$$\frac{d\theta}{dt} = f(t)$$

$$\therefore\ \theta(t) = \int_0^t f(\tau)\,d\tau$$