An Introduction to Free and Moving Boundary Problems

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Outline

Introduction

- physical situations: changes occur across a region which is quite thin to be modelled as a sharp boundary
- position of the moving boundary is to be determined as part of the solution

Free and moving boundary problems induced by the physical unacceptability of solution. Shock wave.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

$$u(x,0) = u_0(x);$$

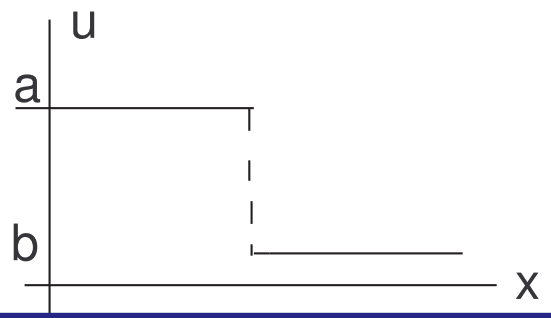
Free and moving boundary problems induced by the physical unacceptability of solution. Shock wave.

Example: Burgers' equation:

$$\frac{\partial u}{\partial t} + \frac{\partial \left(0.5u^2\right)}{x} = 0$$

$$u(x,0) = \begin{cases} \alpha & x < 0 \\ \beta & x > 0 \end{cases}$$

$s = \frac{\left[0.5u^{(2)}\right]_-^+}{[u]_-^+} = \frac{1}{2}(\alpha + \beta)$ - shock speed



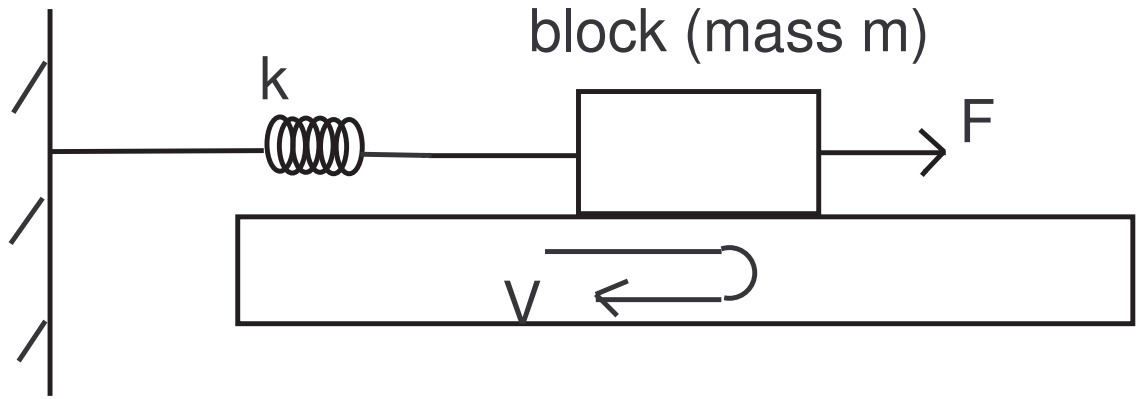
Problems induced by a sudden change in physical properties

- models whose physical predictions might be unacceptable
- macroscopic models in which the changes can be modelled by sharp discontinuities

Problems:

- solid mechanics
- heat conduction
- molecular diffusion

Problem in Solid Mechanics



Problems in Heat Conduction

- Stefan problems of the melting/freezing

$$\frac{\partial}{\partial x}(k\frac{\partial u}{\partial x}) = \rho c\frac{\partial u}{\partial t}$$

$u = u(x,t)$ - temperature, k - thermal conductivity, p - density, c - specific heat

the phase change: u=0, x=s(t):

conservation energy at the boundary

$$\left[k\frac{\partial u}{\partial x}\right]_{solid}^{liquid} = -\rho L\frac{\partial s}{\partial t}$$

- extra condition is specified at the unknown boundary
- analytical solution (Neumann solution). Material has its phase changed: $s = 2a\sqrt{t}$

Problem in Molecular Diffusion

Gas is diffusing through a biological cell. Concentration $c = c(x, t)$

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}(\gamma \frac{\partial c}{\partial x}) - R,$$

and BC

- realistic values for c

A region of zero concentration bounded by a moving boundary $x = s(t)$:

- $c \to 0$ as $x \to s|_{-0}$
- $\frac{\partial c}{\partial x}|_{x=s_{-0}} = 0$

Conclusions

- Models of different physical situations were considered
- Position of the moving boundary is to be determined as a part of the solution