# Heat Equation- Crank-Nicholson in notes

March 8, 2017

## 1 Heat Equation

### 1.1 The Differential Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

## Initial Condition

$$u(x,0) = 2x, \;\; 0 \leq x \leq \frac{1}{2}$$

$$u(x,0) = 2(1-x), \;\; \frac{1}{2} \leq x \leq 1$$

### 1.2 Boundary Condition

$$u(0,t) = 0, u(1,t) = 0$$

### 1.3 The Difference Equation

$w[i,j+1] = w[i,j+1] + \frac{1}{2}\left(\frac{k}{h^2}(w[i+1,j+1] - 2w[i,j+1] + w[i-1,j+1]) + \frac{k}{h^2}(w[i+1,j] - 2w[i,j] + w[i-1,j]\right.$

$-rw[i-1,j+1] + (2+2r)w[i,j+1] - rw[i+1,j+1] = rw[i-1,j] + (2-2r)w[i,j] + rw[i+1,j]$

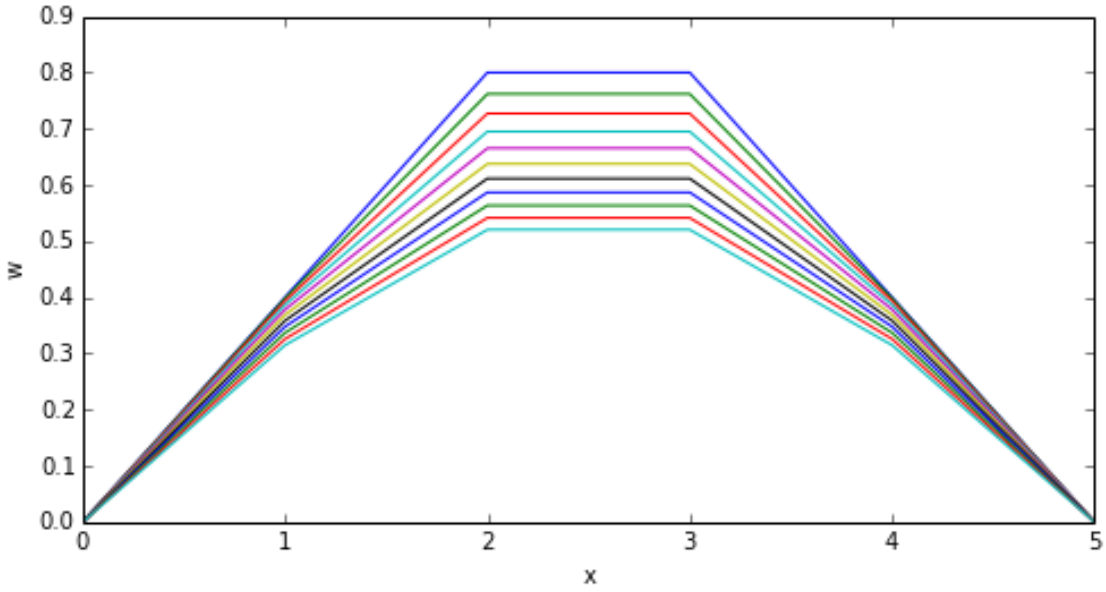
where $r = \frac{k}{h^2}$

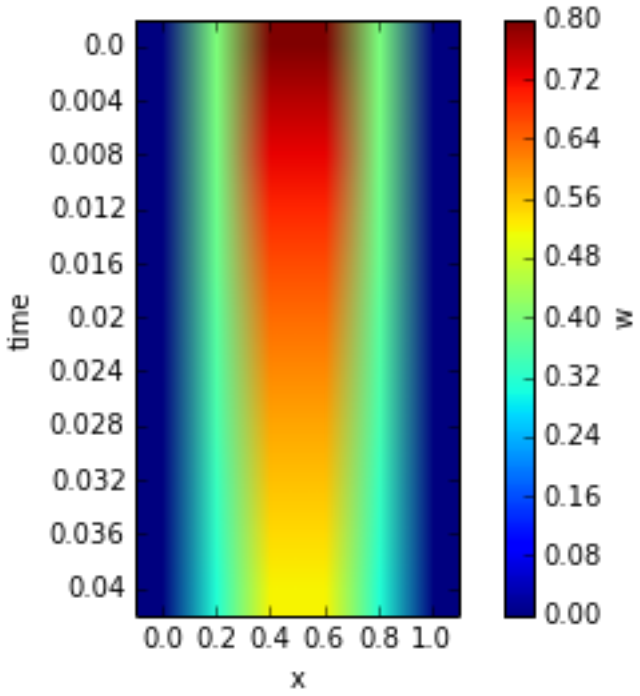
```
In [1]: # LIBRARY
        # vector manipulation
        import numpy as np
        # math functions
        import math

        # THIS IS FOR PLOTTING

        %matplotlib inline
        import matplotlib.pyplot as plt # side-stepping mpl backend
        import warnings
        warnings.filterwarnings("ignore")
```

In [17]:
```
N=5
Nt=250
h=1/N
ht=1/Nt

time_iteration=10
time=np.arange(0,(time_iteration+.5)*ht,ht)

x=np.arange(0,1.0001,h)

w=np.zeros((N+1,time_iteration+1))
r=ht/(h*h)
A=np.zeros((N-1,N-1))
B=np.zeros((N-1,N-1))
c=np.zeros(N-1)
b=np.zeros(N-1)
b[0]=0

# Initial Condition
for i in range (1,N):
    #w[0,i]=1-x[i]-1/np.pi*np.sin(2*np.pi*x[i])
    w[i,0]=2*x[i]
    if x[i]>0.5:
        w[i,0]=2*(1-x[i])


# Boundary Condition
for k in range (0,time_iteration):
    #w[k,0]=1
    w[0,k]=0
    w[N,k]=0


for i in range (0,N-1):
    A[i,i]=2+2*r
    B[i,i]=2-2*r

for i in range (0,N-2):
    A[i+1,i]=-r
    A[i,i+1]=-r
    B[i+1,i]=r
    B[i,i+1]=r



plt.show()

Ainv=np.linalg.inv(A)
```

```
C=np.dot(Ainv,B)

for k in range (1,time_iteration+1):
    w[1:(N),k]=np.dot(C,w[1:(N),k-1])



print(w[:,1])
print(A)
print(B)
print(w[:,2])
print(w[:,3])
print(w[:,4])
print(w[:,5])
print(time)
fig = plt.figure(figsize=(8,4))
plt.plot(w)
plt.xlabel('x')
plt.ylabel('w')

fig = plt.figure()
plt.imshow(w.transpose())
plt.xticks(np.arange(len(x)), x)
plt.yticks(np.arange(len(time)), time)
plt.xlabel('x')
plt.ylabel('time')
clb=plt.colorbar()
clb.set_label('w')

plt.show()
```

```
[ 0.          0.39826464  0.76182213  0.76182213  0.39826464  0.        ]
[[ 2.2 -0.1  0.   0. ]
 [-0.1  2.2 -0.1  0. ]
 [ 0.  -0.1  2.2 -0.1]
 [ 0.   0.  -0.1  2.2]]
[[ 1.8  0.1  0.   0. ]
 [ 0.1  1.8  0.1  0. ]
 [ 0.   0.1  1.8  0.1]
 [ 0.   0.   0.1  1.8]]
[ 0.          0.39352535  0.72697192  0.72697192  0.39352535  0.        ]
[ 0.          0.38660517  0.69488557  0.69488557  0.38660517  0.        ]
[ 0.          0.37813185  0.66512204  0.66512204  0.37813185  0.        ]
[ 0.          0.36858319  0.63733495  0.63733495  0.36858319  0.        ]
[ 0.     0.004  0.008  0.012  0.016  0.02   0.024  0.028  0.032  0.036
  0.04 ]
```


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
w
0
1
2
3
4
5
x


0.0
0.004
0.008
0.012
0.016
0.02
0.024
0.028
0.032
0.036
0.04
time
0.0 0.2 0.4 0.6 0.8 1.0
x
0.80
0.72
0.64
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00
w

In [ ]: