Heat Equation- FTCS in notes

March 8, 2017

1 Heat Equation

1.1 The Differential Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Initial Condition

$$u(x,0) = 2x, \;\; 0 \leq x \leq \frac{1}{2}$$

$$u(x,0) = 2(1-x), \;\; \frac{1}{2} \leq x \leq 1$$

1.2 Boundary Condition

$$u(0,t) = 0, u(1,t) = 0$$

1.3 The Explicit Forward Time Centered Space (FTCS) Difference Equation

$$w[i,j+1] = w[i,j+1] + \frac{k}{h^2}(w[i+1,j] - 2w[i,j] + w[i-1,j])$$

$$w[i,j+1] = rw[i-1,j] + (1-2r)w[i,j] + rw[i+1,j]$$

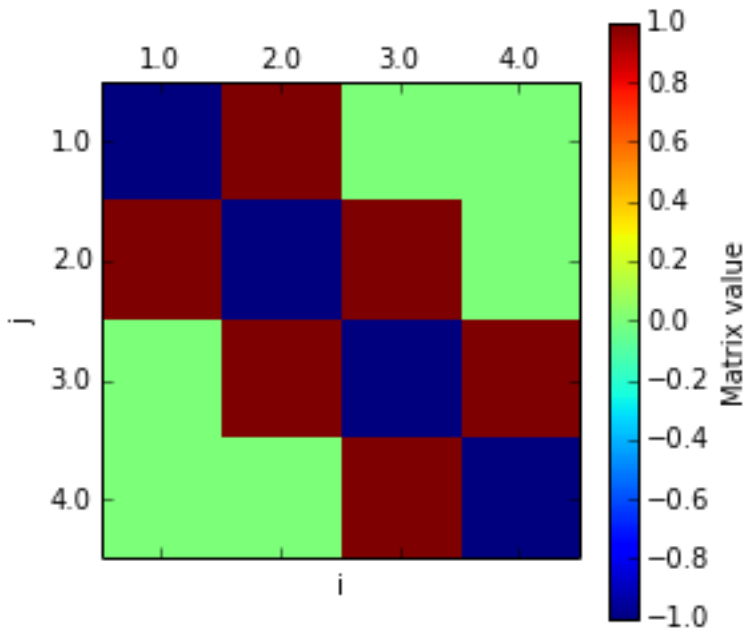
where $r = \frac{k}{h^2}$

In [2]:
```
# LIBRARY
# vector manipulation
import numpy as np
# math functions
import math

# THIS IS FOR PLOTTING

%matplotlib inline
import matplotlib.pyplot as plt # side-stepping mpl backend
import warnings
warnings.filterwarnings("ignore")
```

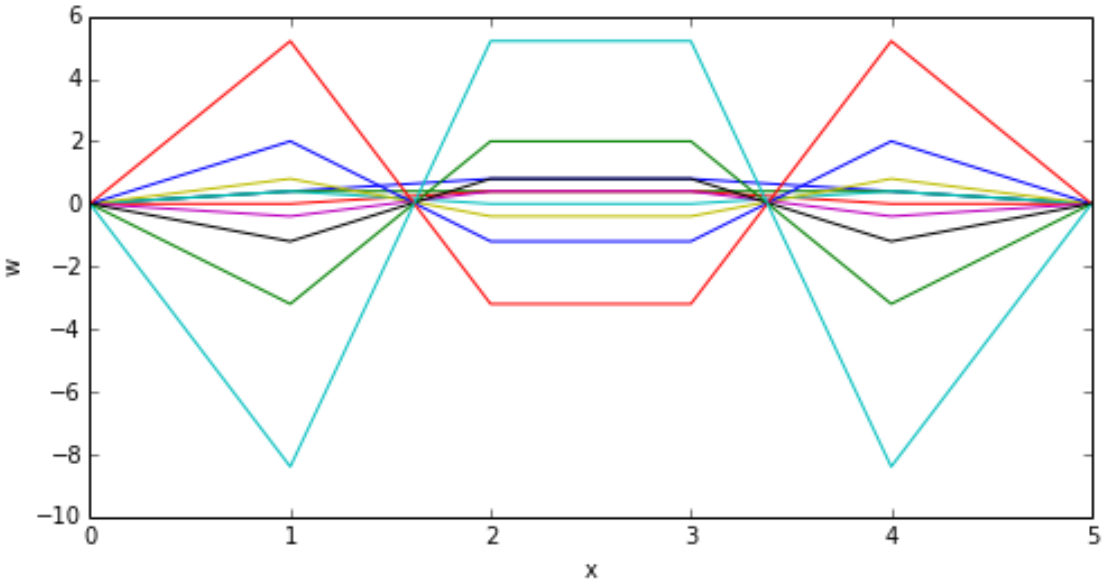
In [51]:
```
N=5
Nt=25
h=1/N
ht=1/Nt

time_iteration=10
time=np.arange(0,(time_iteration+.5)*ht,ht)

x=np.arange(0,1.0001,h)

w=np.zeros((N+1,time_iteration+1))
r=ht/(h*h)
A=np.zeros((N-1,N-1))
c=np.zeros(N-1)
b=np.zeros(N-1)
b[0]=0

# Initial Condition
for i in range (1,N):
    #w[0,i]=1-x[i]-1/np.pi*np.sin(2*np.pi*x[i])
    w[i,0]=2*x[i]
    if x[i]>0.5:
        w[i,0]=2*(1-x[i])


# Boundary Condition
for k in range (0,time_iteration):
    #w[k,0]=1
    w[0,k]=0
    w[N,k]=0


for i in range (0,N-1):
    A[i,i]=1-2*r

for i in range (0,N-2):
    A[i+1,i]=r
    A[i,i+1]=r


fig = plt.figure(figsize=(8,4))
plt.matshow(A)
plt.xlabel('i')
plt.ylabel('j')
plt.xticks(np.arange(N-1), np.arange(1,N-0.9,1))
plt.yticks(np.arange(N-1), np.arange(1,N-0.9,1))
clb=plt.colorbar()
clb.set_label('Matrix value')
```

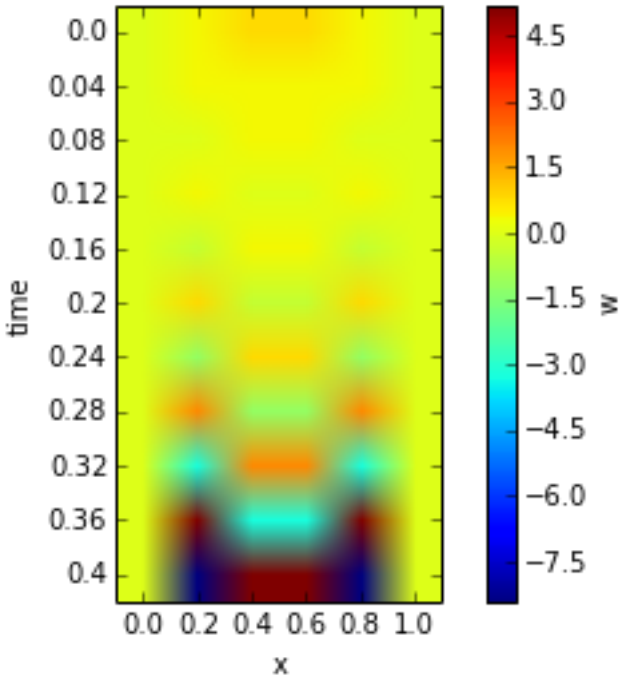
```
        clb.set_clim((-1,1))


        plt.show()




        for k in range (1,time_iteration+1):
            w[1:(N),k]=np.dot(A,w[1:(N),k-1])



        print(w[:,3])
        print(A)

        fig = plt.figure(figsize=(8,4))
        plt.plot(w)
        plt.xlabel('x')
        plt.ylabel('w')

        fig = plt.figure()
        plt.imshow(w.transpose())
        plt.xticks(np.arange(len(x)), x)
        plt.yticks(np.arange(len(time)), time)
        plt.xlabel('x')
        plt.ylabel('time')
        clb=plt.colorbar()
        clb.set_label('w')

        plt.show()
```

```
<matplotlib.figure.Figure at 0x7f6e94b00ef0>
```




```
[  0.00000000e+00   4.00000000e-01  -8.32667268e-16   1.16573418e-15
   4.00000000e-01   0.00000000e+00]
[[-1.  1.  0.  0.]
 [ 1. -1.  1.  0.]
 [ 0.  1. -1.  1.]
 [ 0.  0.  1. -1.]]
```




time
0.0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
0.4
0.0 0.2 0.4 0.6 0.8 1.0
x
4.5
3.0
1.5
0.0
−1.5
−3.0
−4.5
−6.0
−7.5
w

In [ ]: