W9205/9206/9209/9210/101C

**DUBLIN INSTITUTE OF TECHNOLOGY**

**School of Mathematical Sciences**

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# DT9205 MSc Mathematical Physics
# DT9206 MSc Mathematical Physics
# DT9209 MSc Applied Mathematics
# DT9210 MSc Applied Mathematics

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**WINTER EXAMINATIONS 2015/2016**

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## MATH 9972: Special Relativity and Tensor Calculus

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Dr R Ivanov

Dr C Hills

Professor E O'Riordan


9.30 – 11.30 am, Monday, 4 January 2016

Duration: 2 hours

Attempt three questions only

All questions carry equal marks

Approved calculators may be used

Mathematical tables are provided

New Cambridge Statistical Tables are NOT permitted

**1.** A rod of rest length $l_0$ (measured by 'co-moving' observer $S'$) moves with speed $v$ along the horizontal direction towards a 'stationary' observer $S$. The rod makes an angle $\theta_0$ with the horizontal direction according to $S'$.

**a)** Determine the length of the rod as measured by the stationary observer $S$. (15)

**b)** Determine the angle, $\theta$, the rod makes with the horizontal direction as measured by the stationary observer $S$. (8)

**c)** Determine the velocity $v$ if $\theta_0 = 30°$ and $\theta = 45°$. (10)

**[33]**

**2.** An electron $e^-$ with kinetic energy $K = 1.000$ MeV makes a head-on collision with a positron $e^+$ at rest. (A positron is an antimatter particle that has the same mass as the electron but opposite charge.) In the collision the two particles annihilate each other and are replaced by two photons of equal energy, travelling at angles $\pm\theta$ with respect to the electron's direction of motion.

The reaction is

$$e^- + e^+ \to 2\gamma.$$

**a)** Given that the rest energy of the electron is $m_0c^2 = 0.511$ MeV, where $m_0$ is the rest mass of the electron, find the momentum of the electron. Determine the energy, $E$, and the momentum, $p$, of each photon. (20)

**b)** Determine the angle of emission, $\theta$. (13)

**[33]**

**3.** **a)** Demonstrate that under coordinate transformations every symmetric contravariant tensor $A^{ij}$ of rank 2 transforms into a symmetric tensor of the same type. (9)

**b)** Write explicitly the Lorentz transformation of coordinates. Find the transformation law of the components of a **symmetric tensor** $A^{ij}$ under the Lorentz transformation. (24)

**[33]**

**4.** The length element $dl$ on a two-dimensional manifold is given by the following metric:

$$dl^2 = (1 + \alpha^2 r^2)dr^2 + r^2 d\theta^2,$$

where $\alpha$ is a constant parameter and $(r, \theta)$ are coordinates on the manifold.

**a)** Write the metric components $g_{ab}$ and $g^{ab}$. (5)

**b)** Calculate all Christoffel symbols

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad}(\partial_b g_{dc} + \partial_c g_{db} - \partial_d g_{bc}),$$

for the given metric. (10)

**c)** Calculate all the components of the Riemann tensor

$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{fc}\Gamma^f_{bd} - \Gamma^a_{fd}\Gamma^f_{bc}.$$

(11)

**d)** Calculate the scalar curvature of the manifold. (7)

**[33]**