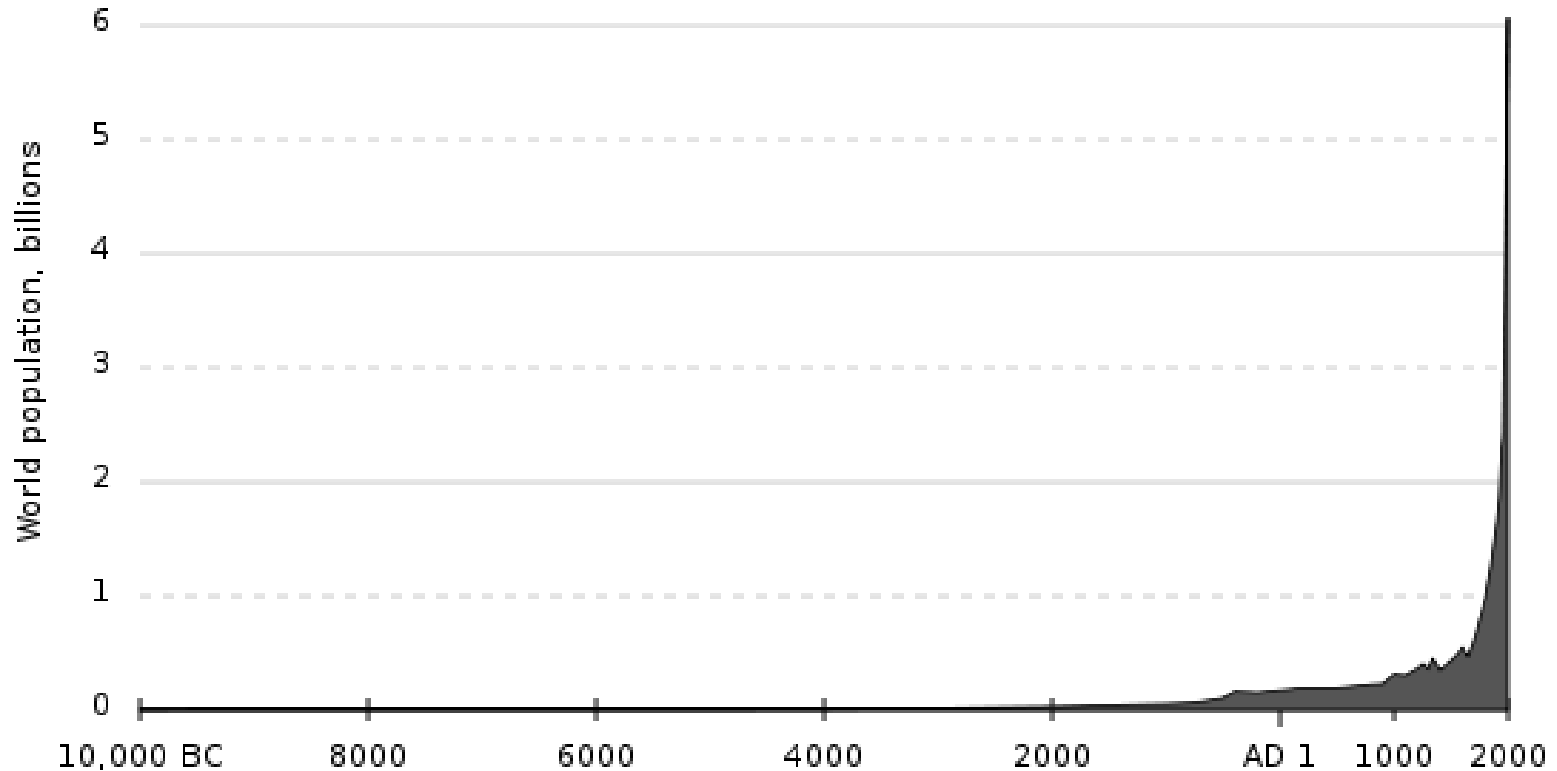


# Population Growth



**Reading: Molles & Cahill 2008, Chapter 12**

# Note to Students:

- Some examples in this lecture are drawn from two texts. Note that the form of the equations (and related examples) can vary, but the concepts are the same.

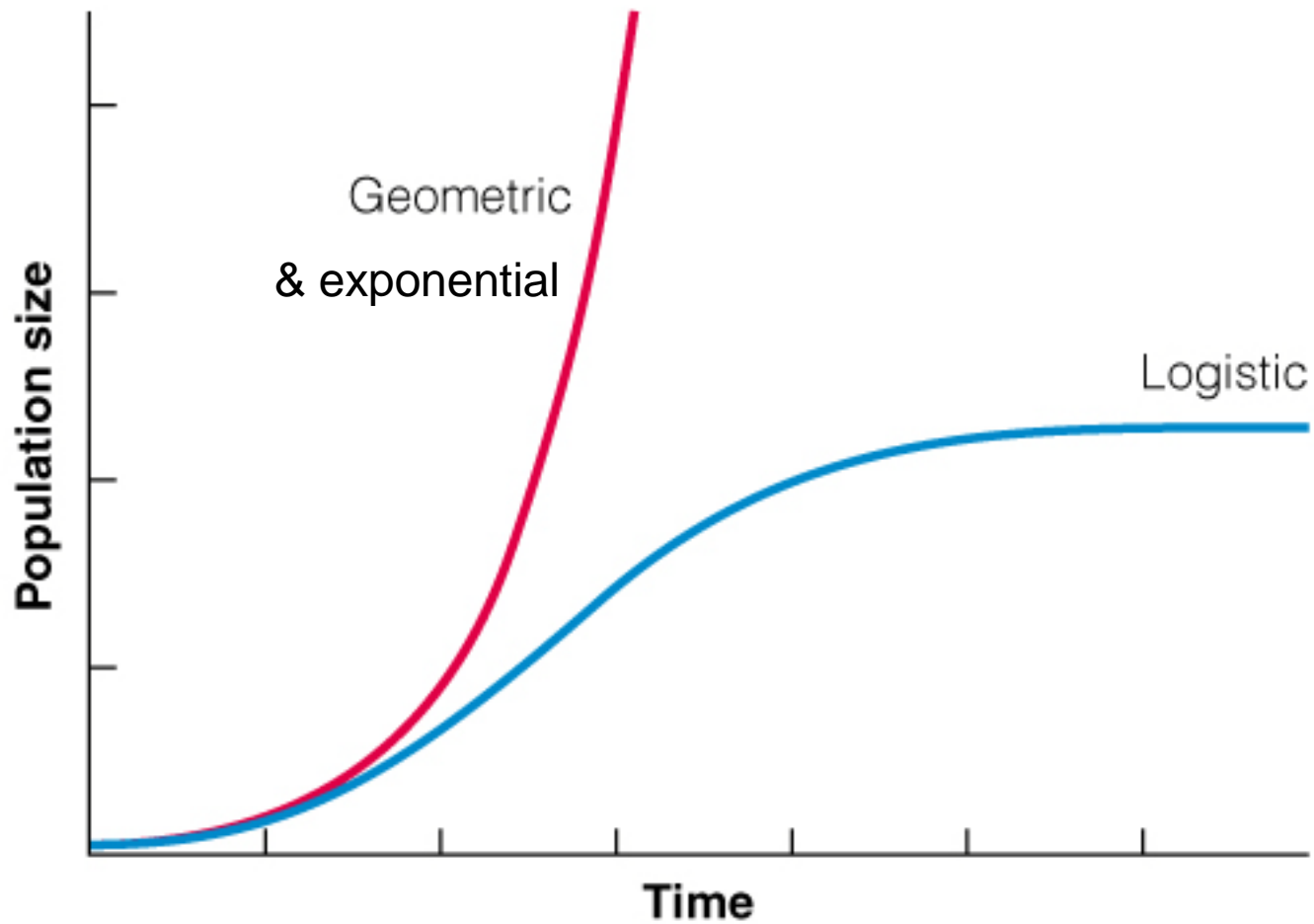
Molles & Cahill 2008, Chapter 12

Krebs\*, Chapters 9-11

\*Krebs CJ (2001) *Ecology, Fifth Edition*,  
Benjamin Cummings, San Francisco.

# Population Growth

- Two idealized patterns of population growth:
  - 1) Exponential (or “geometric” for discrete version)
  - 2) Logistic (s-shaped)
- Today, we will explore these population growth models.
- Both models imply certain key assumptions.
- Are these realistic? Where do they work and where do they fail?



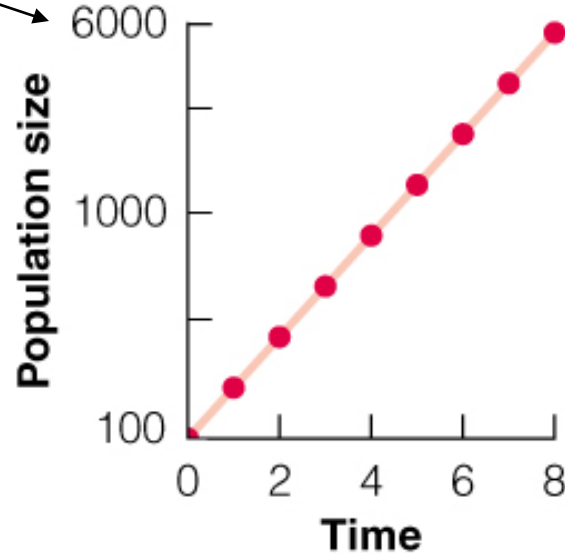
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Krebs 2001 – Figure 11.4

# 1) Exponential Growth

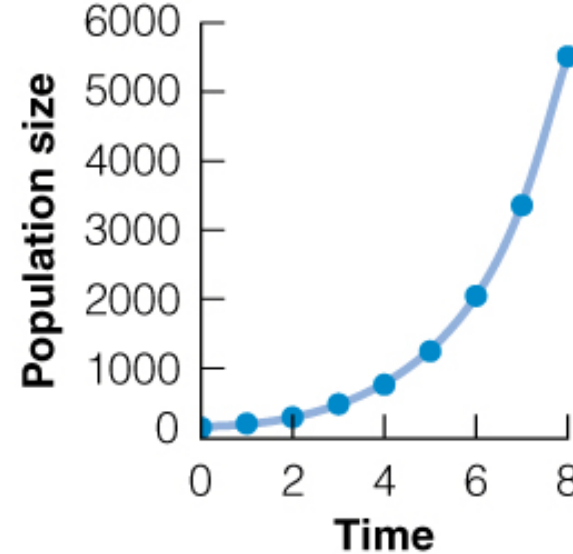
(note that a log transformation linearizes the plot, as illustrated on the left).

Logarithmic  
scale



(a) Logarithmic scale

Linear (arithmetic) scale



(b) Arithmetic scale

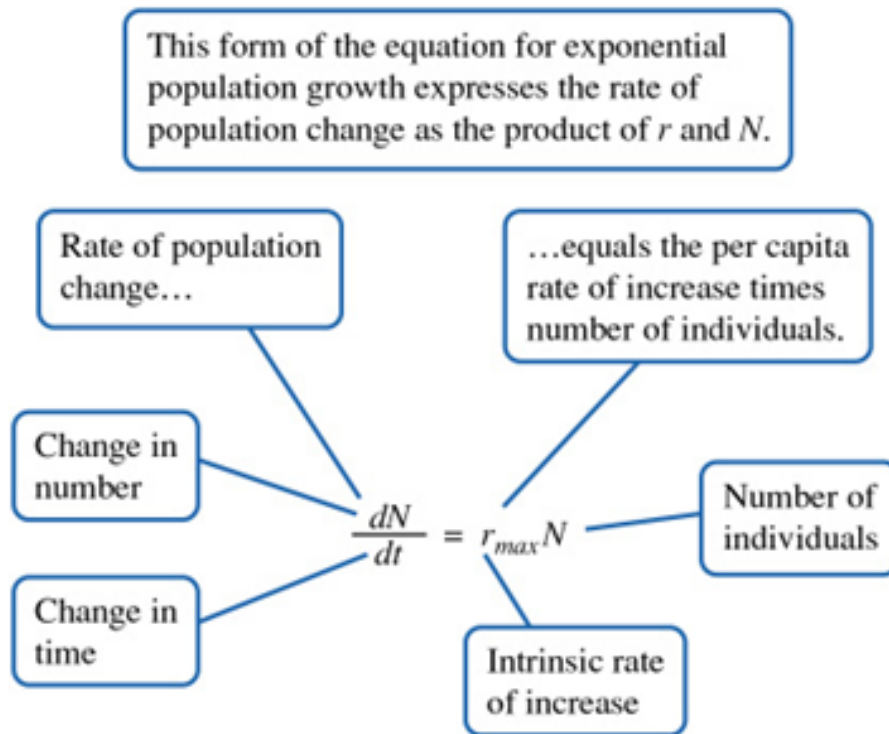
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Krebs 2001 – Figure 10.7

# Exponential Growth – *Instantaneous* form

In this case, the intrinsic rate of increase “ $r_{\max}$ ” is assumed to be constant

Expresses rate of change:



Calculates population size:

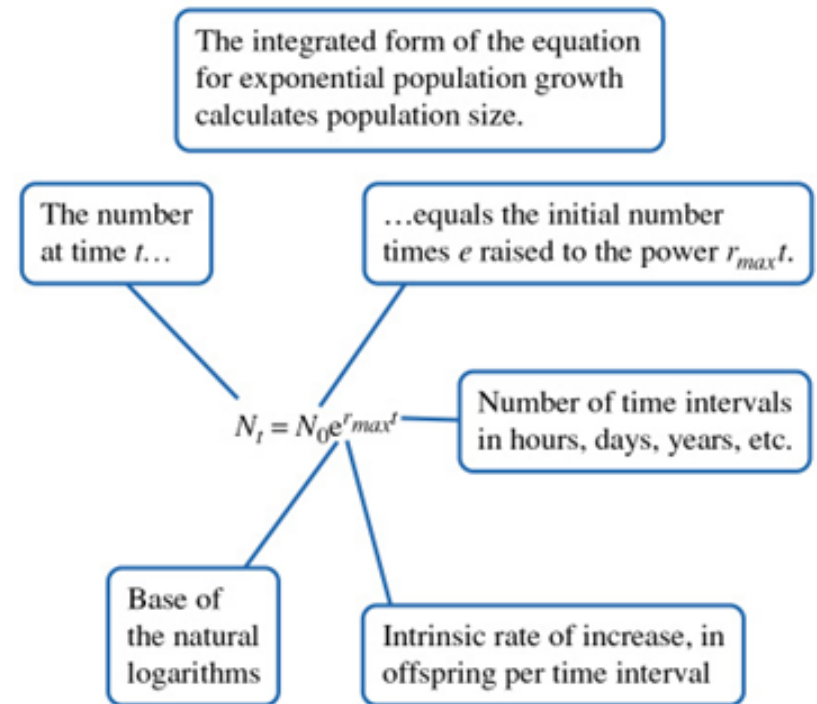


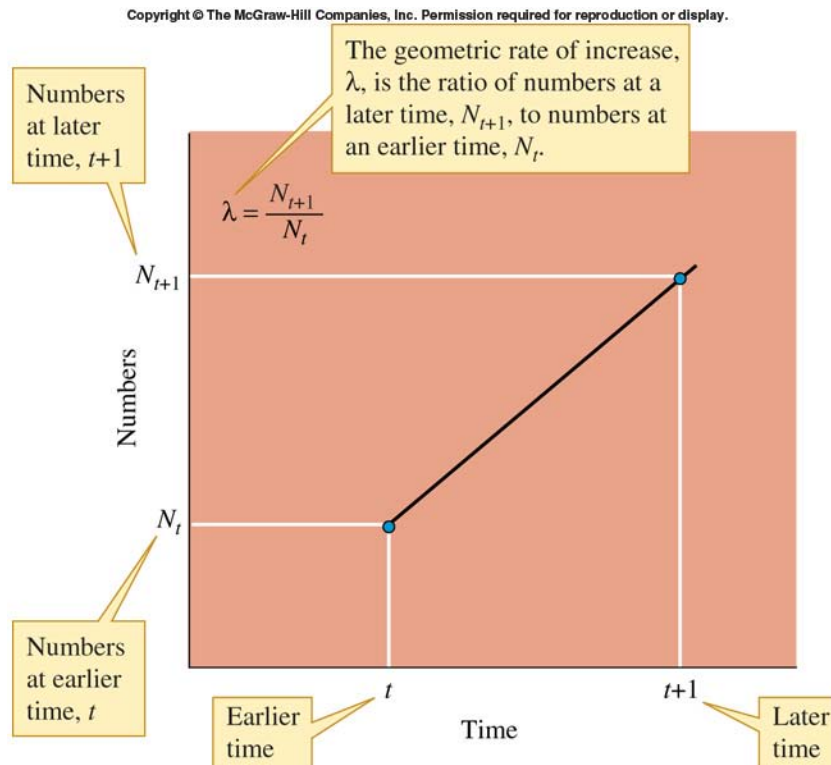
Fig. 12.4  
Molles & Cahill  
2008, p. 313

These examples represent *continuous* growth

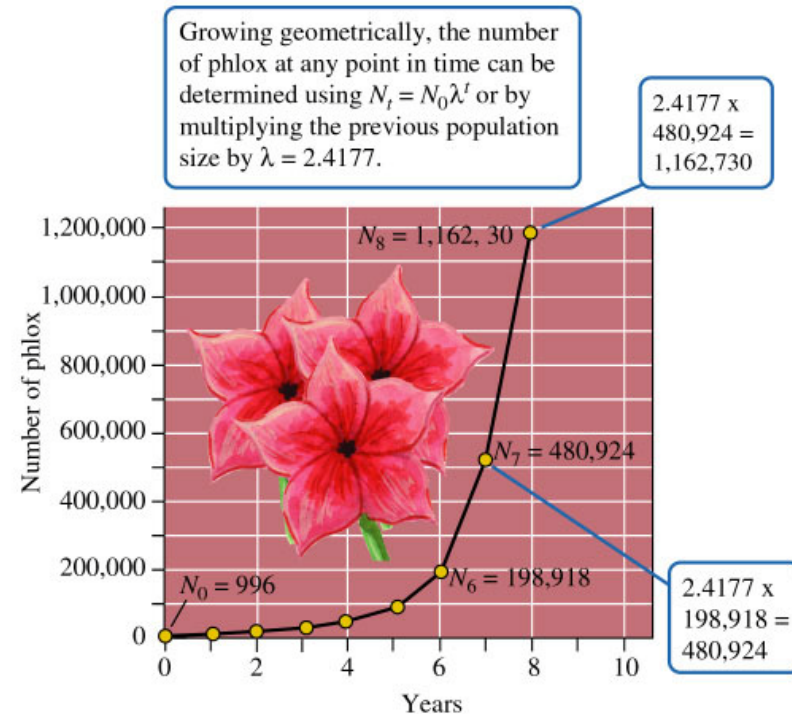
What about species that exhibit *discrete* growth patterns – e.g. annual plants populations that reproduce once each year?

In *discrete* cases, we call exponential growth “geometric growth” (see pp. 295-296, Molles & Cahill 2008)

# Graphical representation of population growth for an organism with discrete reproduction (e.g. annual plants)



$\lambda$  = geometric rate of increase





# Estimating net reproductive rate ( $R_0$ ) for *Phlox drummondii* from survivorship and seed production data.

Combining survivorship with seed production by *P. drummondii* to estimate net reproductive rate,  $R_0$

Age (days)	Number surviving to day $x$	Proportion surviving to day $x$	Average number of seeds per individual during time interval	Multiplication of $l_x$ and $m_x$
$x$	$n_x$	$l_x$	$m_x$	$l_x m_x$
0–299	996	1.0000	0.0000	0.0000
299–306	158	0.1586	0.3394	0.0532
306–13	154	0.1546	0.7963	0.1231
313–20	151	0.1516	2.3995	0.3638
320–27	147	0.1476	3.1904	0.4589
327–34	136	0.1365	2.5411	0.3470
334–41	105	0.1054	3.1589	0.3330
341–48	74	0.0743	8.6625	0.6436
348–55	22	0.0221	4.3072	0.0951
355–62	0	0.0000	0.0000	0.0000

Data from Leverich and Levin 1979.

$R_0 = \sum l_x m_x = 2.4177$

Each individual leaves an average of 2.4177 offspring.

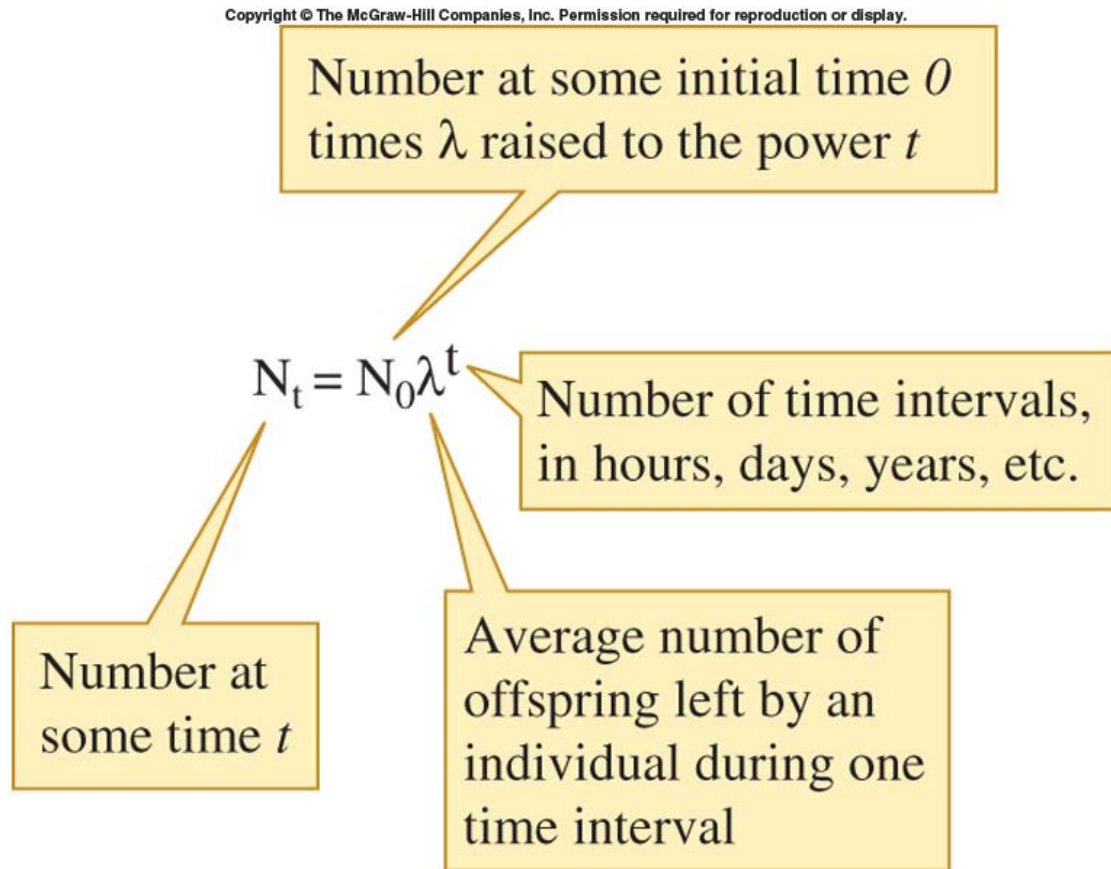
The value of  $R_0$ , which is greater than 1.0, indicates that this population of *P. drummondii* is growing.

Summing the final column yields  $R_0$ , the net reproductive rate per individual.

Note that  $R_0$  (net reproductive rate) can also be called  $\lambda$  (geometric rate of increase)

# Exponential growth - *Discrete* Form

(called “geometric” growth in Molles & Cahill, 2008, p 312)



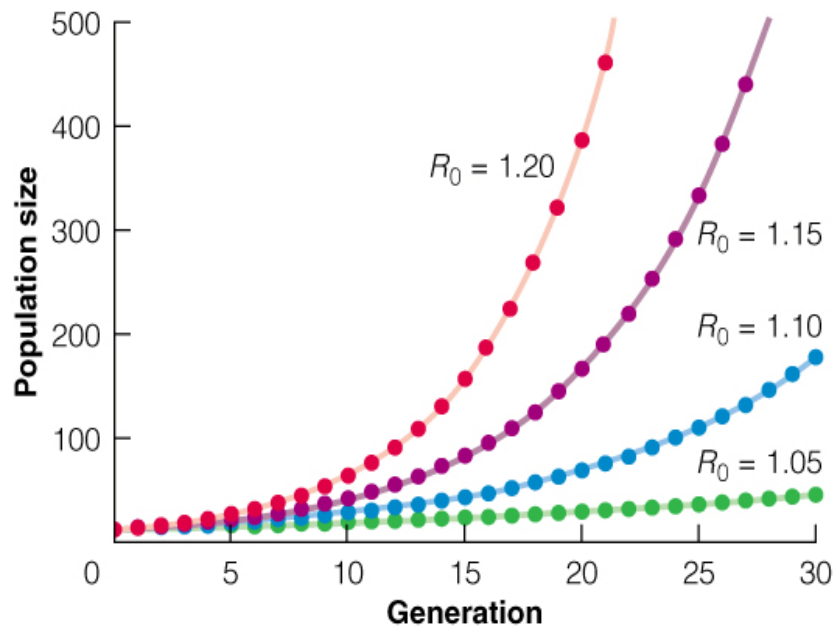
**Note that  $R_0 = \lambda$  (Lambda) in Molles (2008)**

# Exponential vs. Geometric Growth Explained:

- The *exponential* form is used when populations reproduce continually (e.g. humans). In this case, we use a small “ $r$ ” (intrinsic rate of increase).
- The *discrete* (geometric) form of the equation is applied when populations have discrete generations (no overlap in reproduction) – e.g. *Phlox drummondii* (see Molles 2008 & Cahill, chapters 11&12). In this case we use a capital “ $R$ ” (net reproductive rate).
- *Both* models have the same shape when plotted as population ( $N$ ) versus time ( $t$ ) – i.e. both have an “exponential” (or “geometric”) form.

# Variations on the Geometric model

- Vary the reproductive rate (R):



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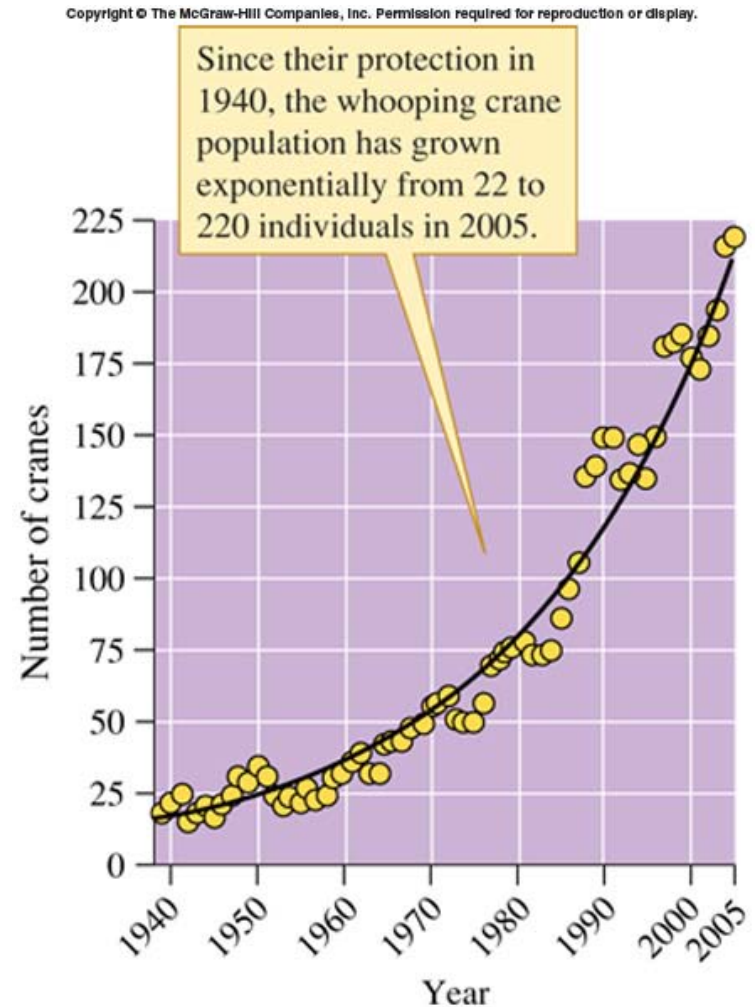
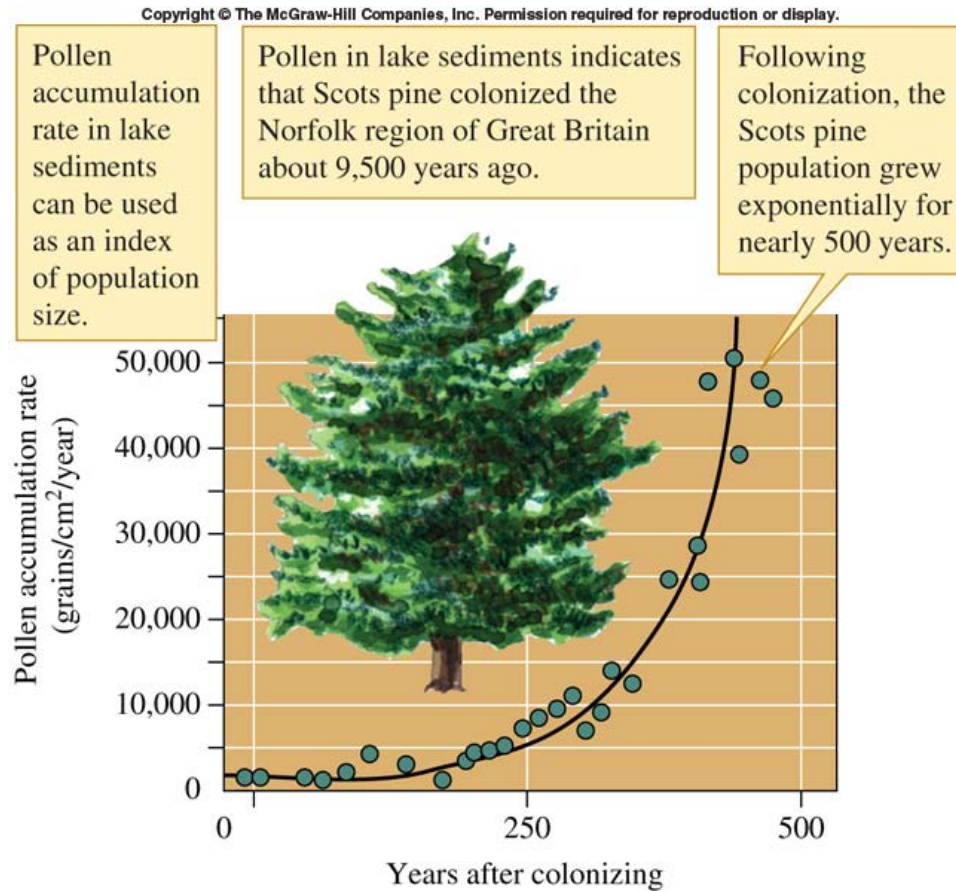
A higher  $R$  leads to a more rapid population increase.

$$N_{t+1} = R_0 N_t$$

(Note that we get similar effects if we vary “ $r$ ” in the exponential model)

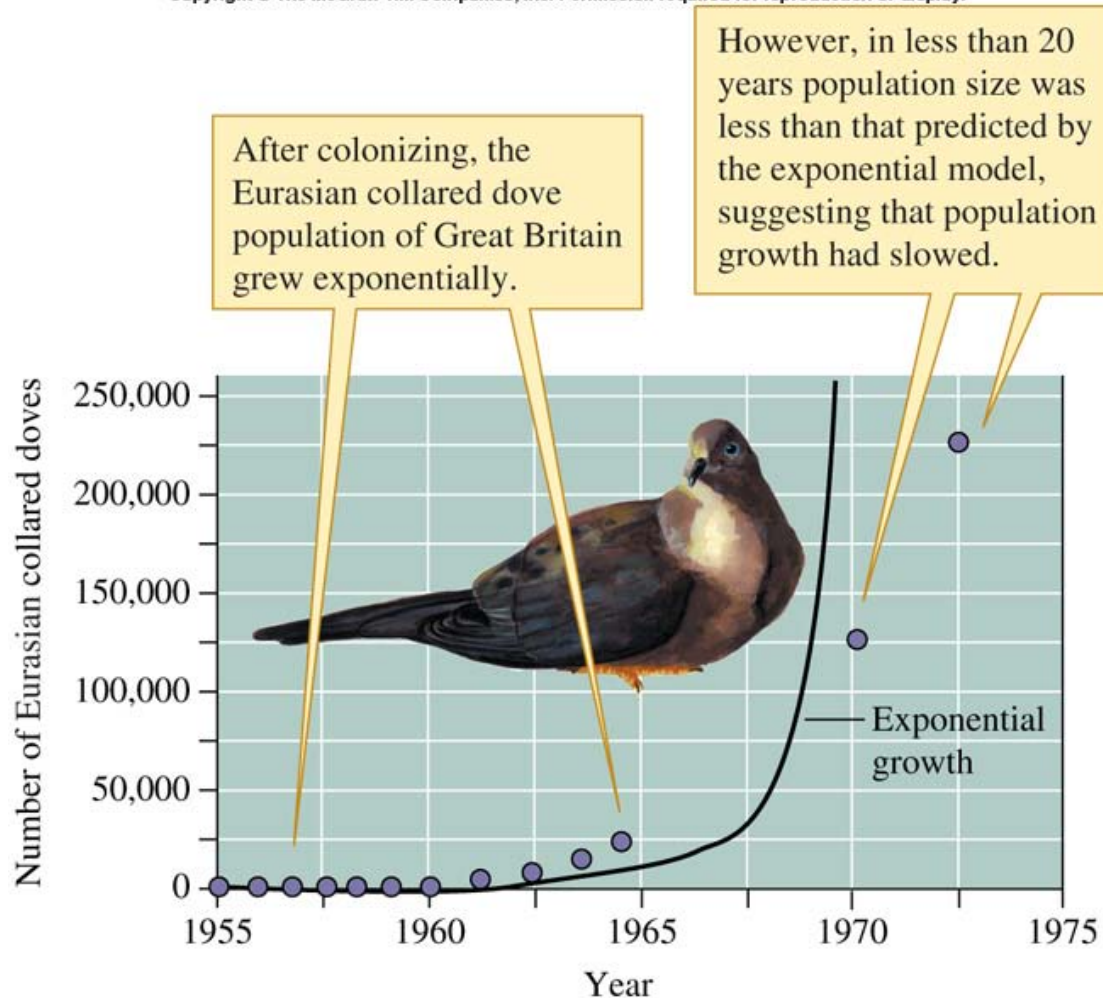
Krebs 2001 – Fig. 11.1

# Examples of populations in nature exhibiting exponential (or geometric) increases

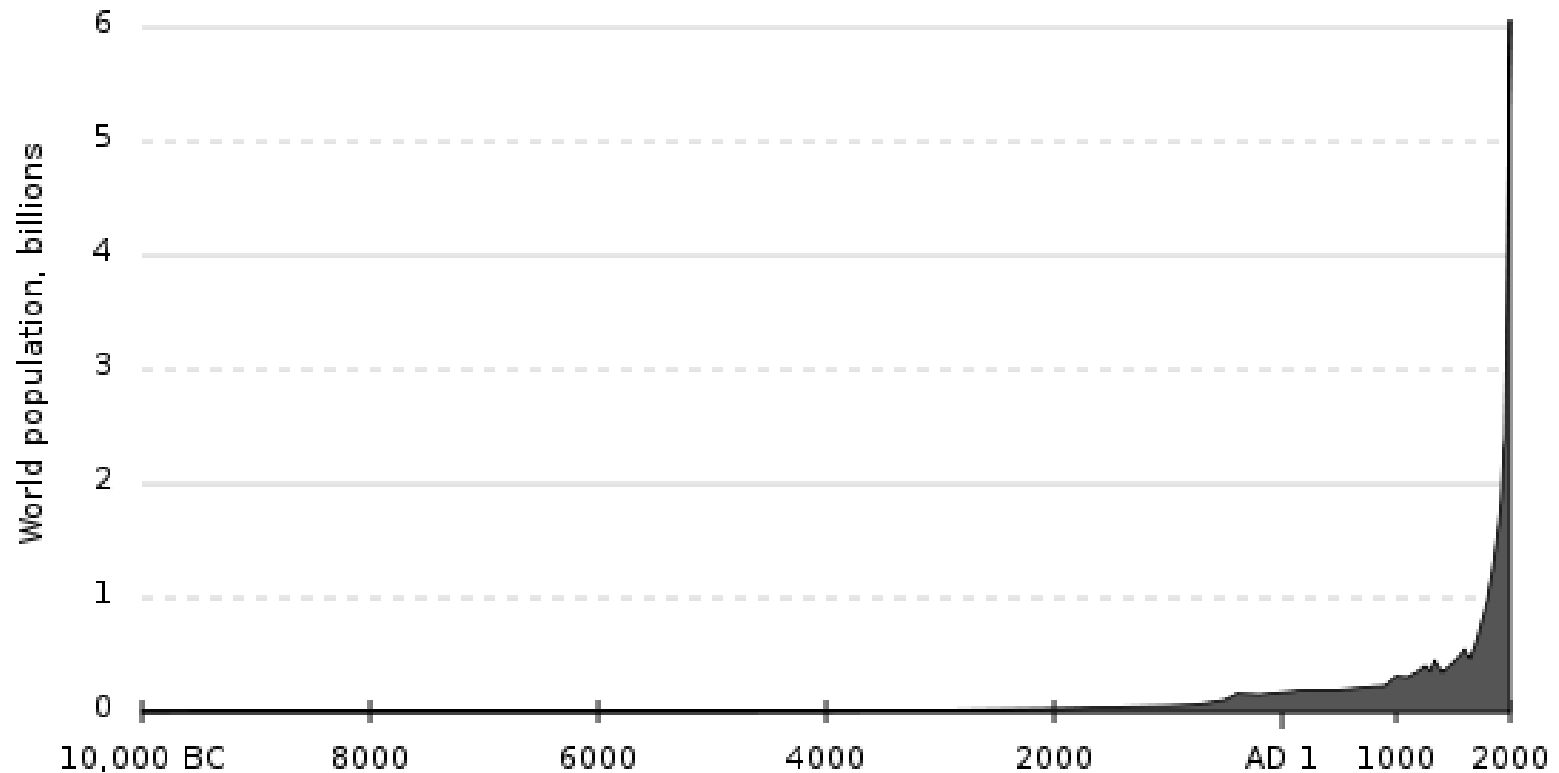


# Many populations exhibit an initial exponential phase followed by slowed growth

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# What about humans?



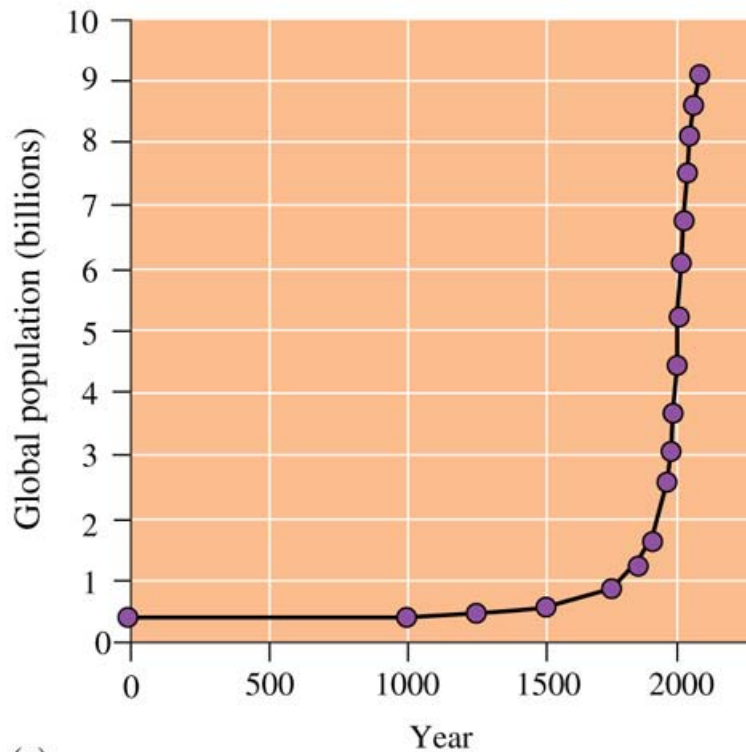


Actually, the human population has shown faster-than-exponential growth, but is now exhibiting a gradual slowing of the growth rate (analogous to the collared dove in Great Britain)

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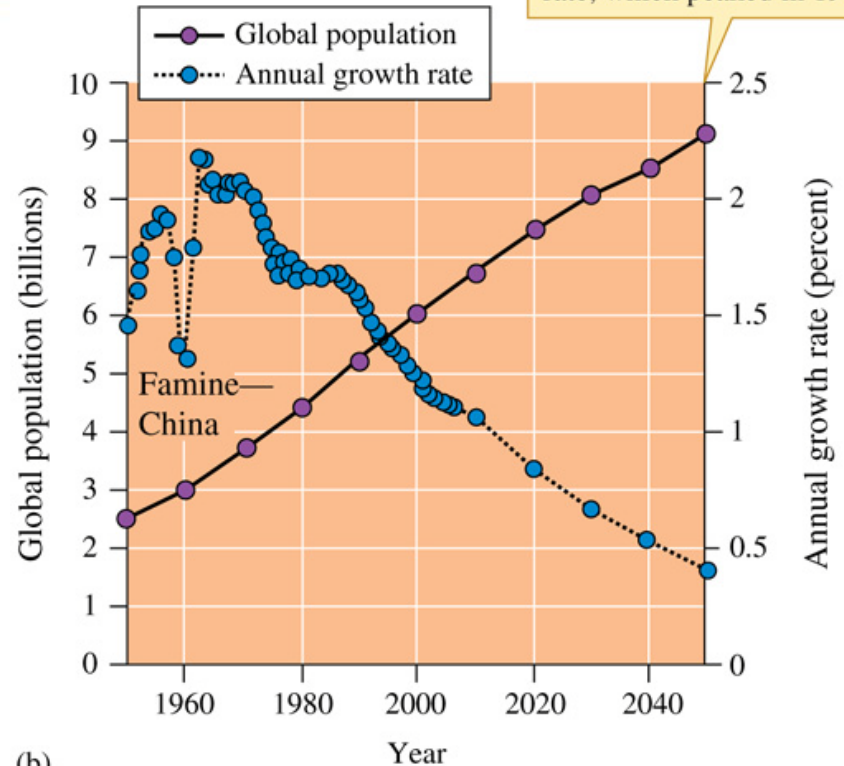
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During the past 2,000 years, the global population has grown exponentially!



(a)

... however, if plotted over the single century from 1950 to 2050, our population shows clear signs of reduced growth rate, which peaked in 1962–63.



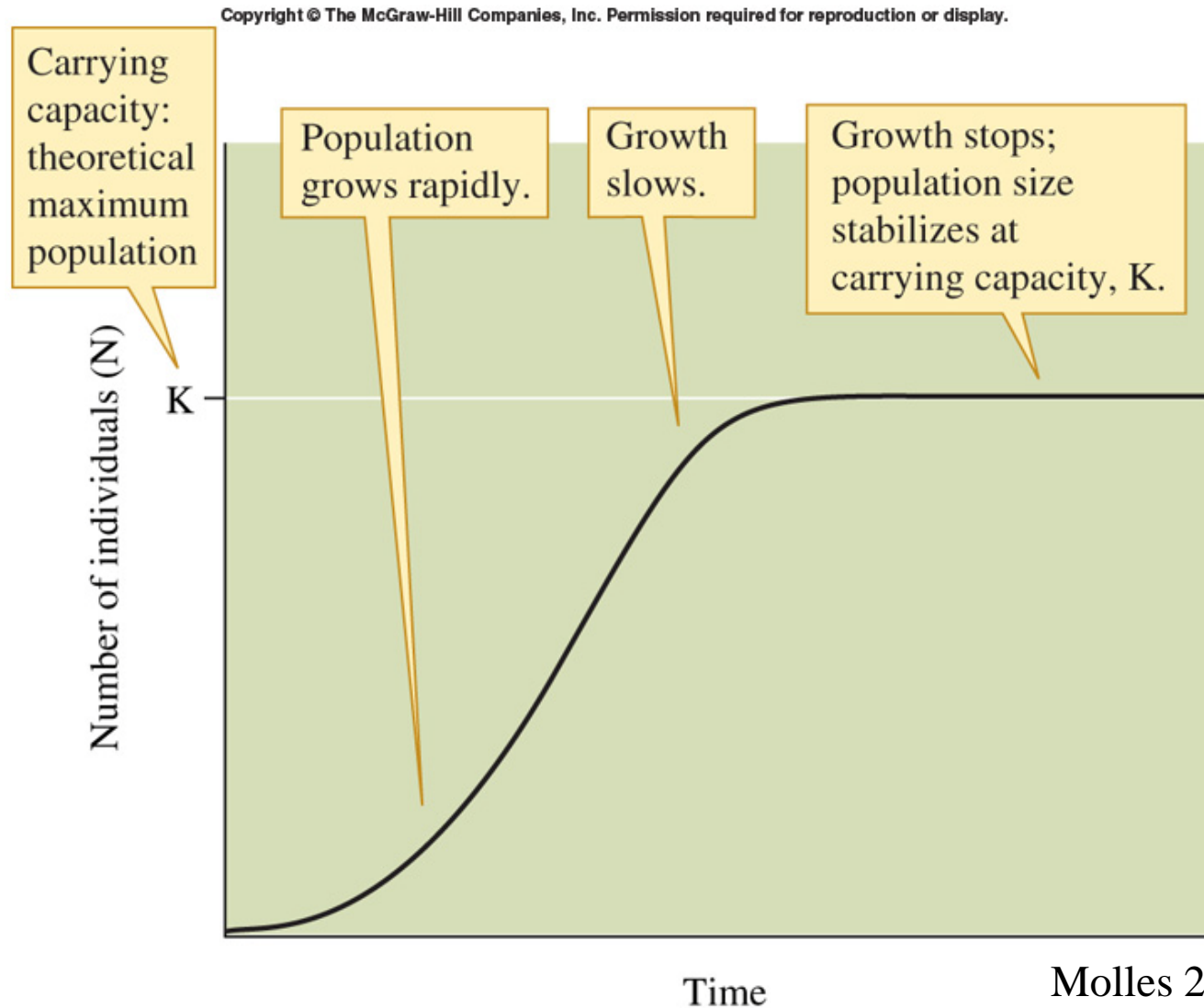
(b)



# What causes growth to slow?

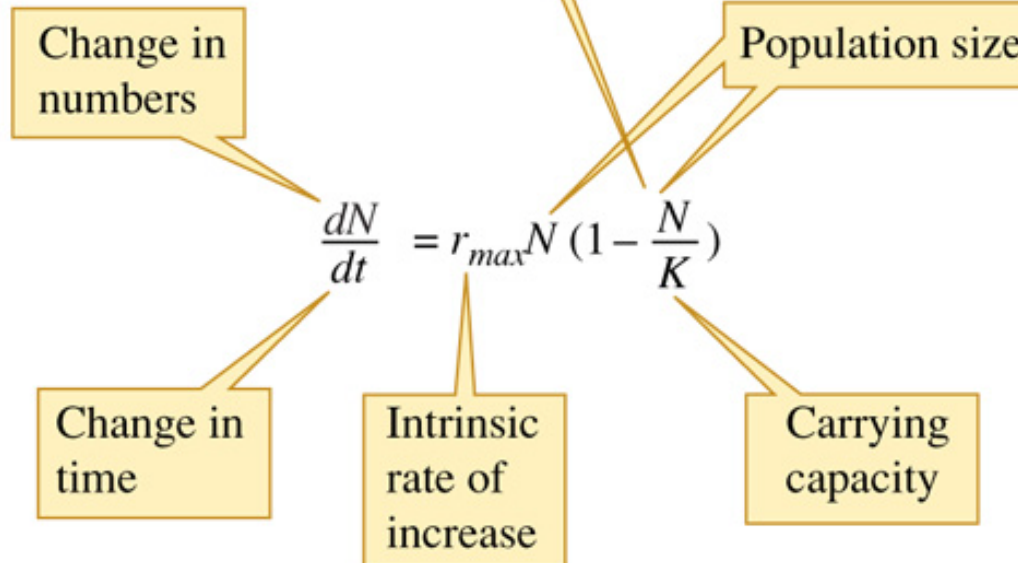
- Environmental limitations (captured in the term “carrying capacity,”  $K$ )
- How do we represent environmental limitations to growth?
  - Logistic growth model (a simple modification of our exponential model)
- The *Demographic Transition Model* (see last lecture) has also been invoked to explain why the growth rates of human populations slow

# Idealized Logistic Model of Population Growth



The logistic equation gives the rate of population change as a function of  $r_{max}$ ,  $N$ , and  $K$ .

As the ratio  $\frac{N}{K}$  increases, population growth slows.



Molles 2008  
(page 261)  
cf. Molles & Cahill,  
2008, p. 316

# Constants (parameters) in these equations:

$$r_{\max} = \text{intrinsic rate of increase}^* = b - d$$

(= birth minus death rate)

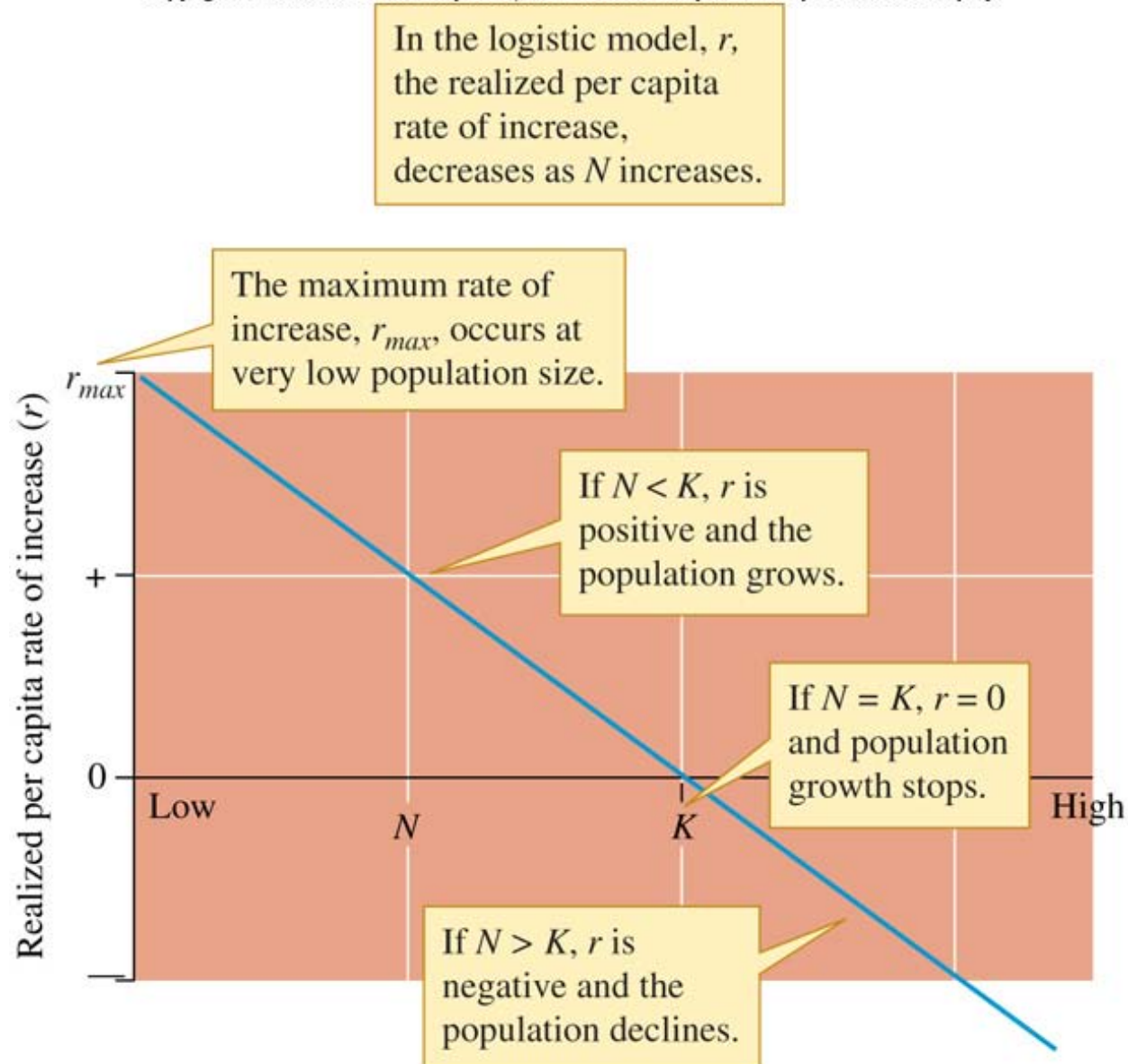
equals the realized, instantaneous rate of population growth *only under idealized conditions* of exponential increase

$K$  = carrying capacity

- the maximum population that a habitat can sustain)
- reflects environmental limitations to growth due to competition (space, resources, etc.), predation, disease, etc.

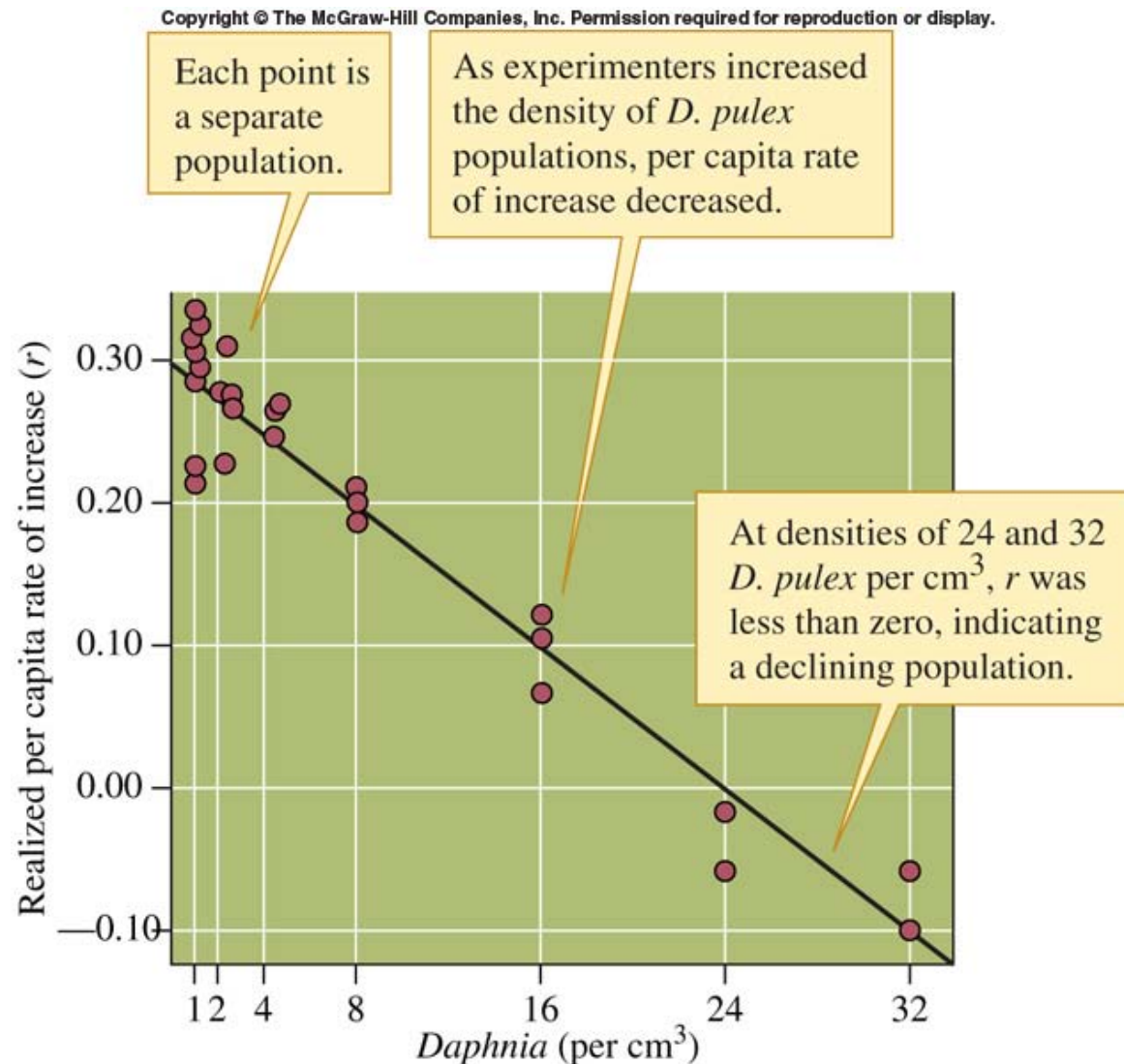
In reality,  $r$  is density-dependent (declines from  $r_{\max}$  as density increases).

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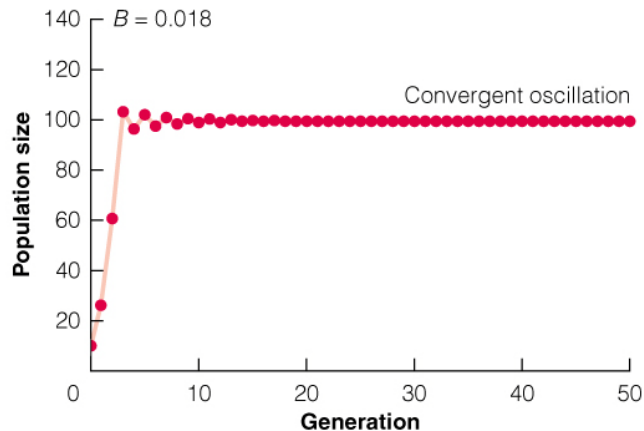
Molles & Cahill  
2008, fig. 12.15

# Experimental evidence for the density-dependent behavior of $r$ :



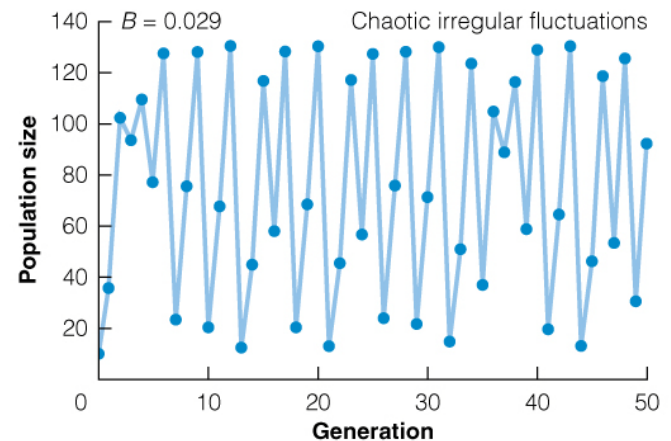
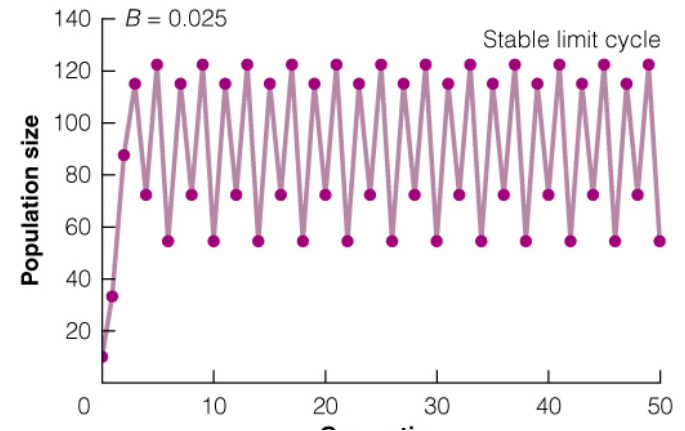
# Variations on the Logistic Model:

(allowing  $B$  to vary causes varying degrees in oscillation of the asymptotic region)



(a)

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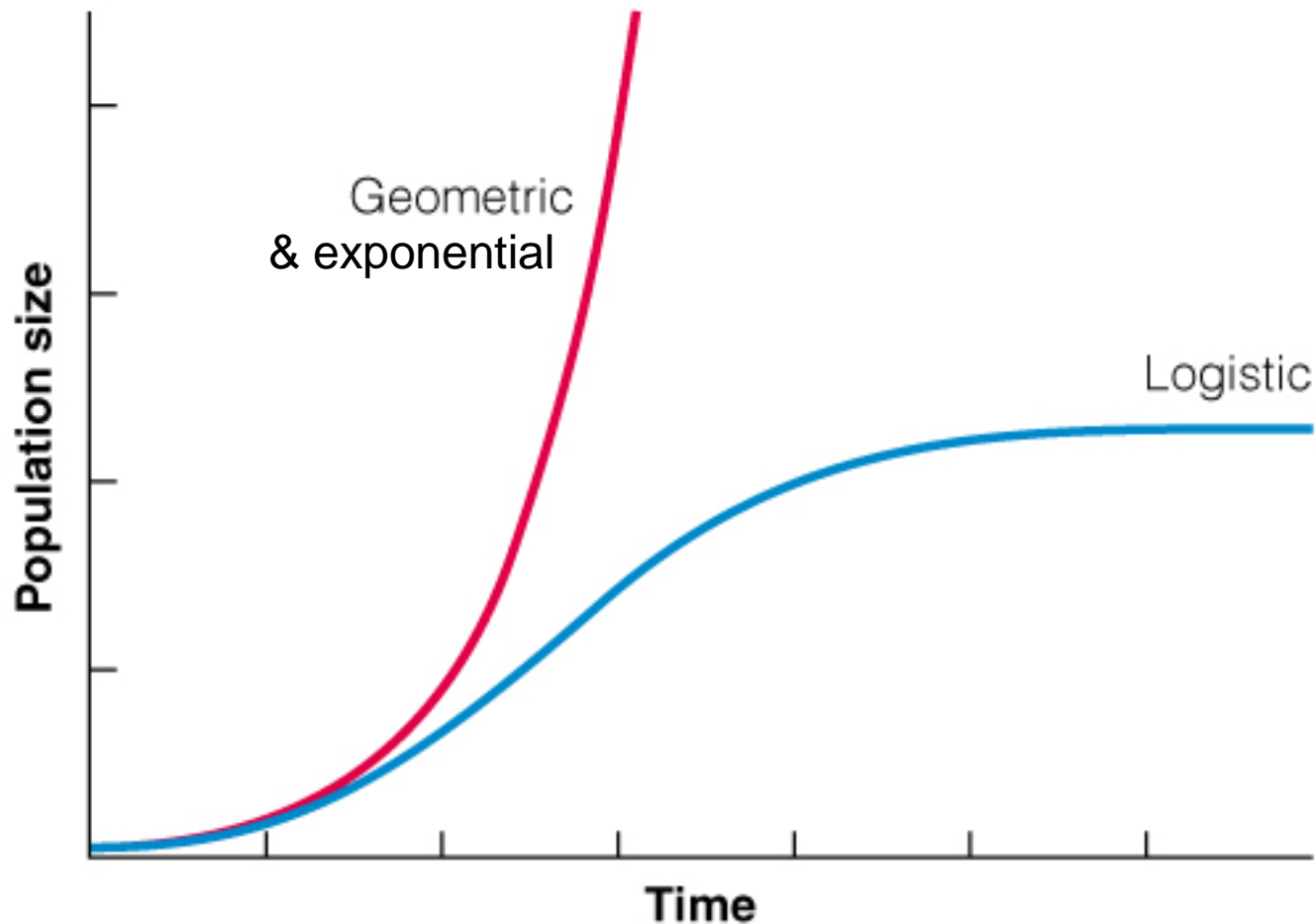


(c)

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Krebs 2001 – Fig. 11.3

# Do real populations actually behave like this?

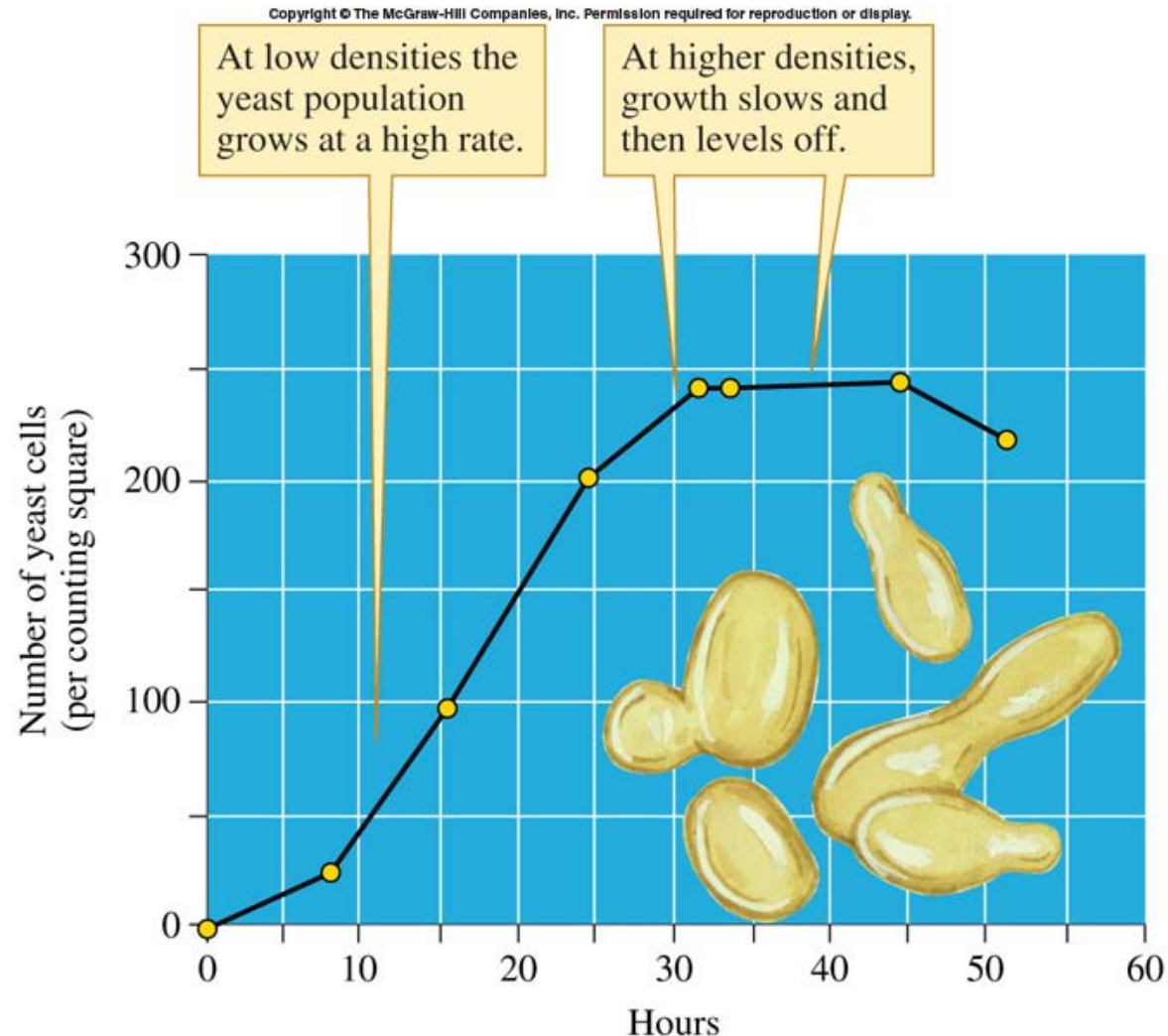


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Krebs 2001 – Figure 11.4



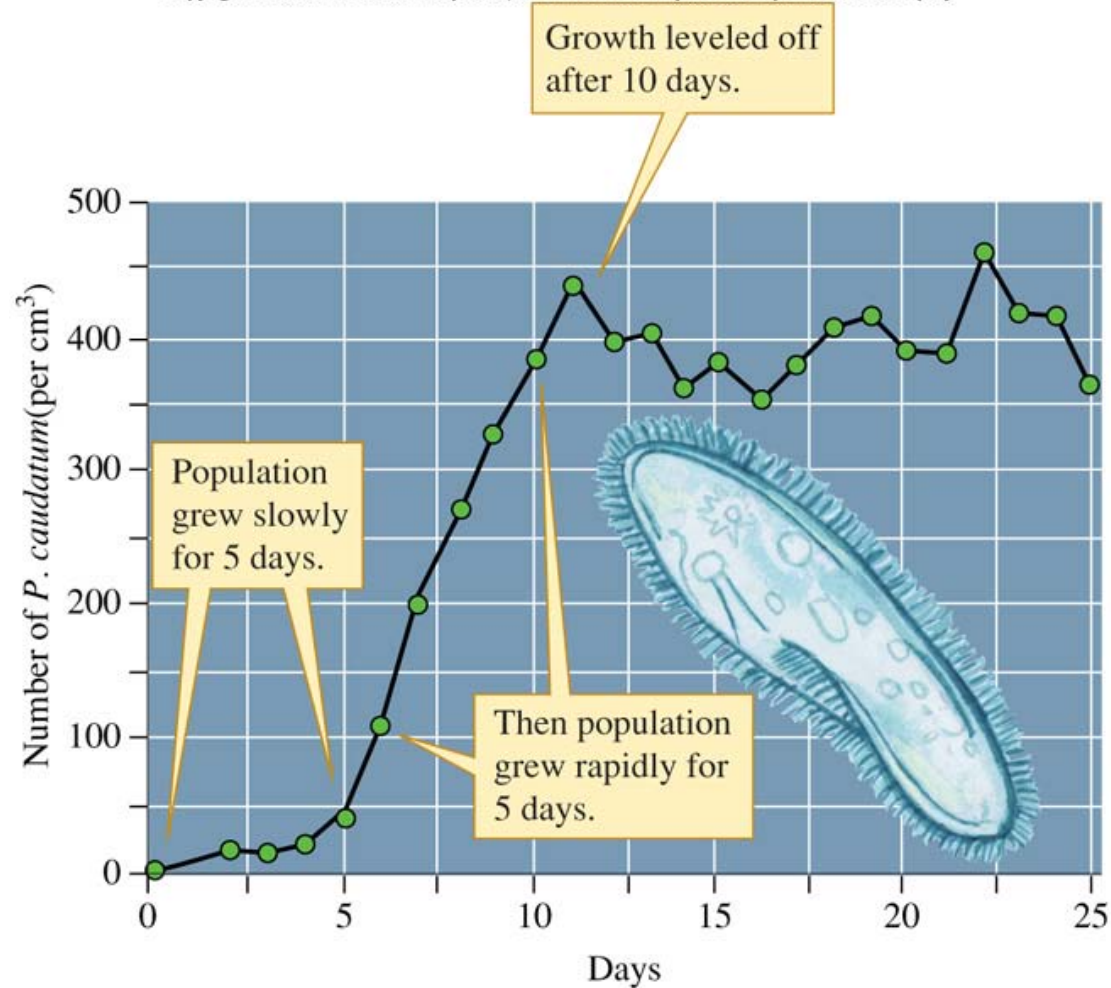
# Experimental populations of yeast (poisoned by their own production of alcohol)



Gause (1934), cited in Molles 2008, fig. 11.9

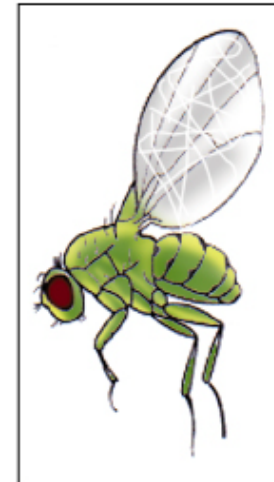
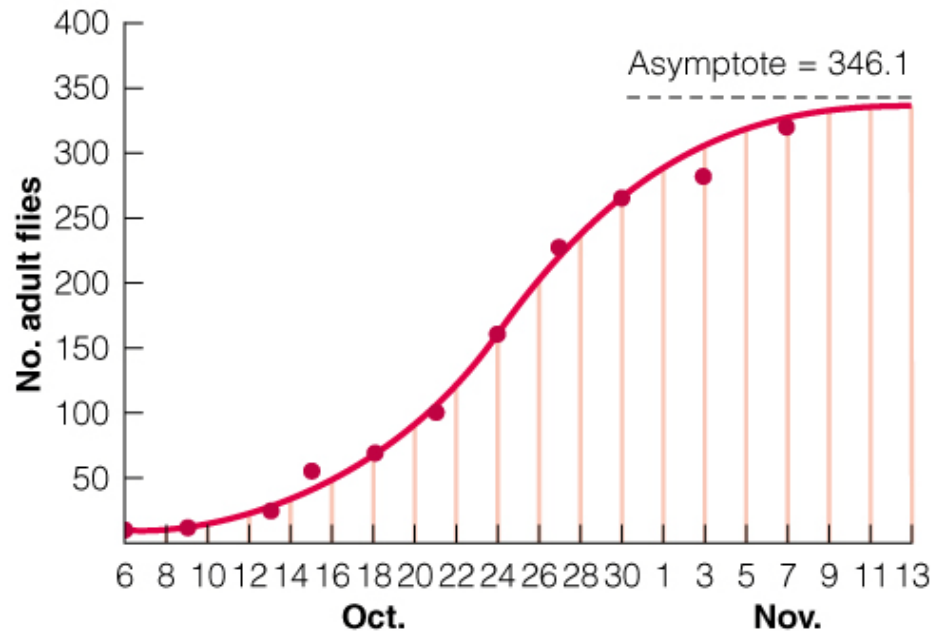
# Experimental populations of *Paramecium caudatum*

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Gause (1934), cited in Molles (2008) fig. 11.10

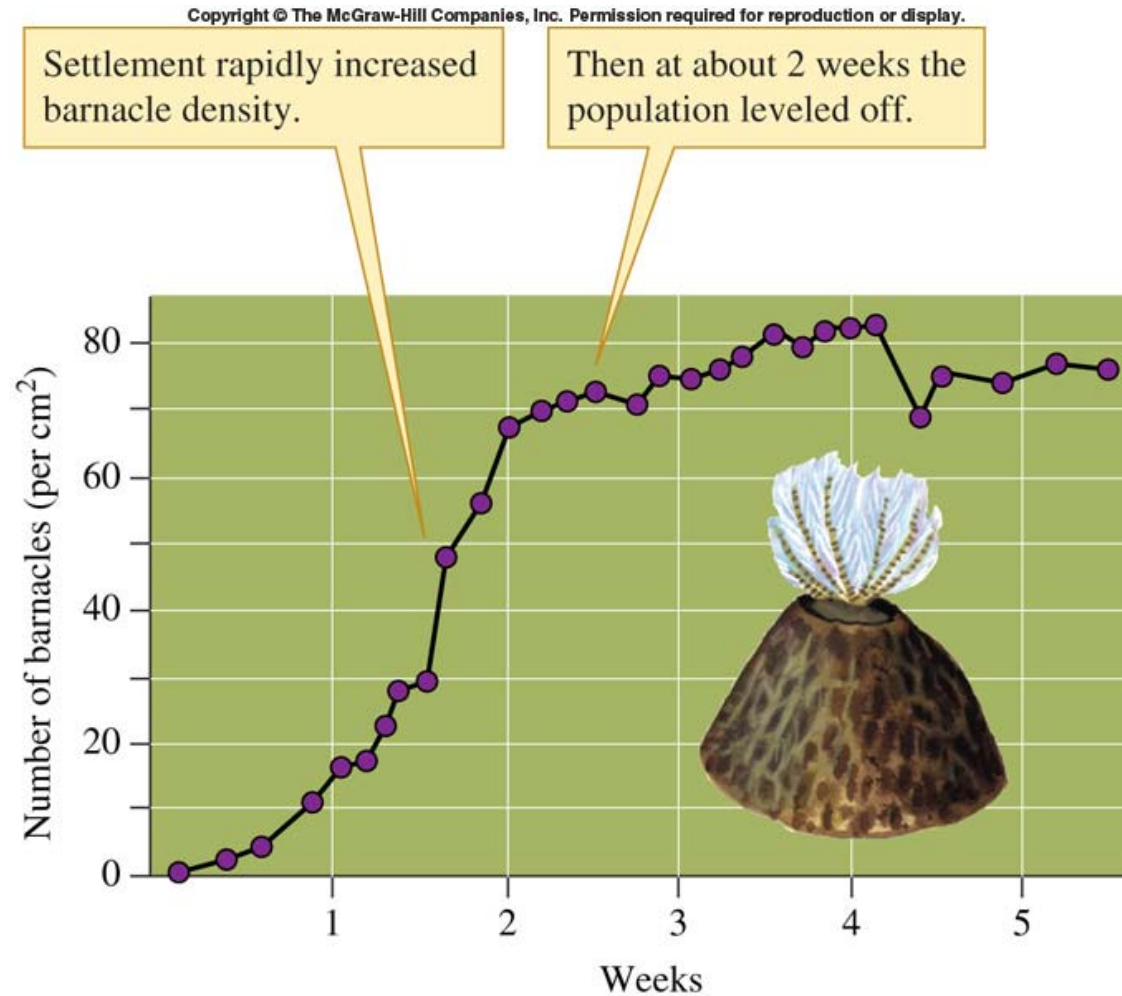
# Laboratory studies of *Drosophila melanogaster*



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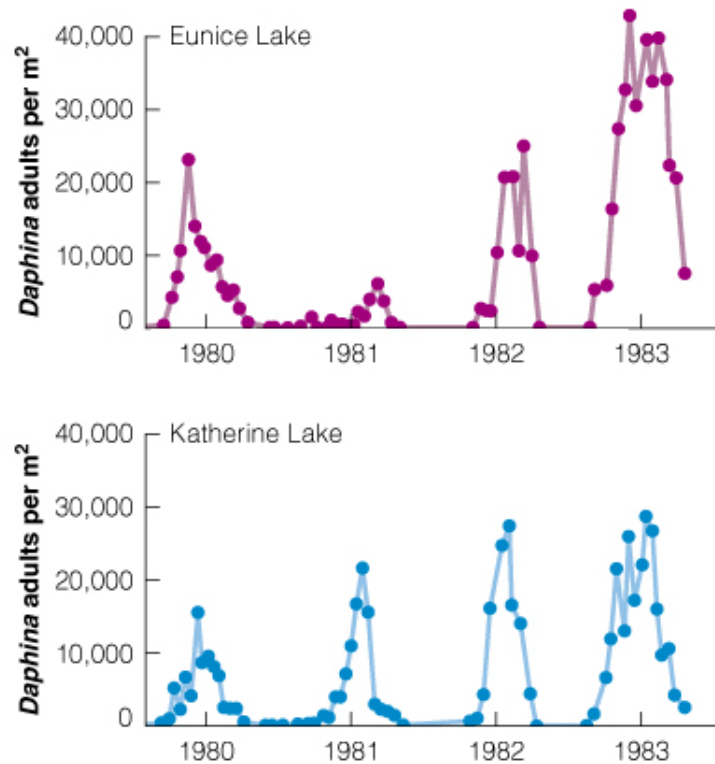
Krebs 2001 – Figure 11.6

# Studies of barnacle larvae settlement in the intertidal.



# But ... some populations don't behave!

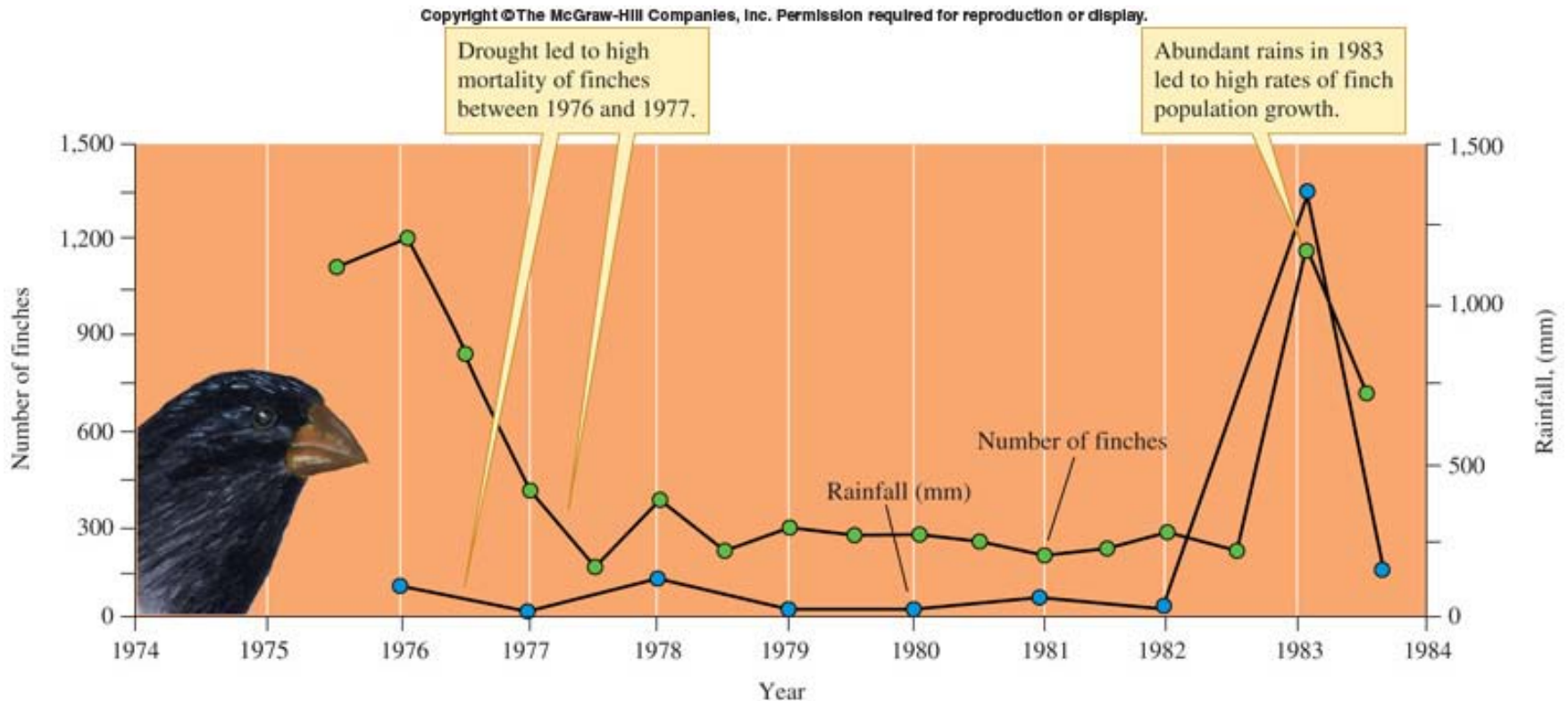
## *Daphnia rosea* in Canadian lakes



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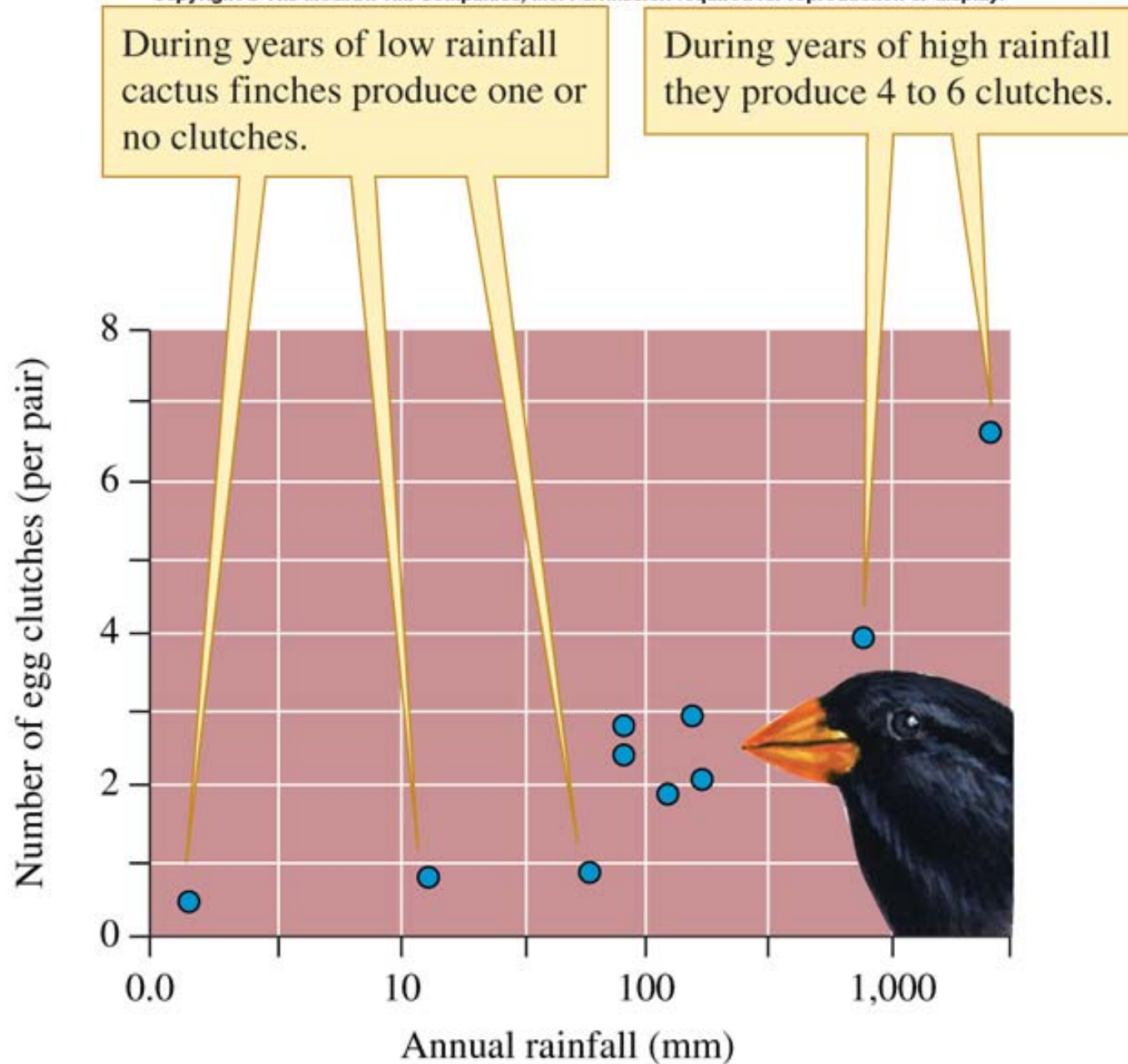
Krebs 2001 – Figure 11.11

In nature, carrying capacity ( $K$ ) varies with environmental conditions, leading to dynamic population patterns.



Gibbs & Grant (1987), cited in Molles (2008), fig. 11.17





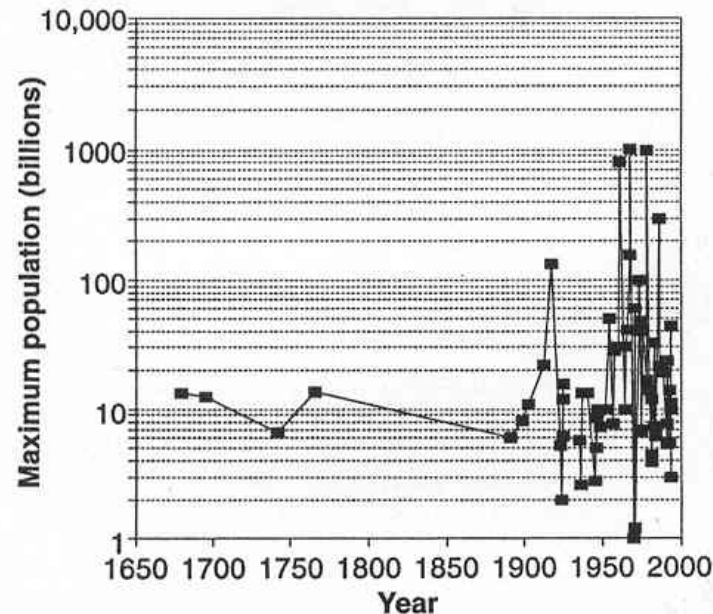
Molles (2008) fig 11.19

# Conclusions

- Exponential (geometric) models work well under certain limited conditions (e.g. population released from environmental constraints,  $K$ )
- Logistic model works well for many idealized (e.g. laboratory) populations, and less well in nature.
- Complicating factors include varying  $K$ ,  $r$ , immigration, emigration, biotic interactions (e.g. predator-prey interactions) – other models (not discussed here) include these factors.
- We will consider biotic interactions later.

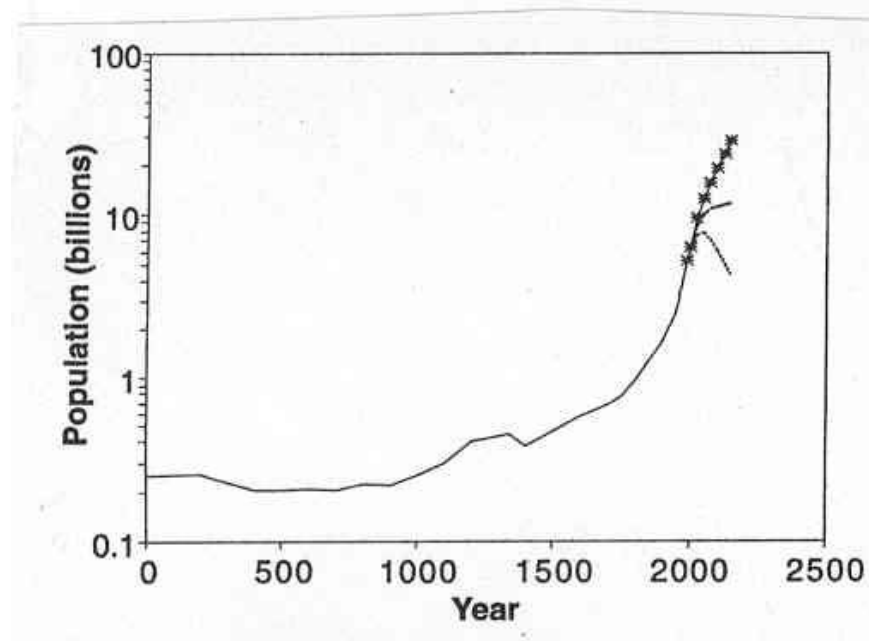


# Estimates of K for human population



**Fig. 3.** Estimates of how many people Earth can support, by the date at which the estimate was made. When an author gave a range of estimates or indicated only an upper bound, the highest number stated is plotted here (55).

# Various scenarios for future human population growth



**Fig. 1.** Recent world population history A.D. 1 to 1990 (solid line) (53) and 1992 population projections of the UN (11) from 1990 to 2150: high (solid line with asterisks); medium (dashed line); and low (dotted line). Population growth was faster than exponential from about 1400 to 1970.