Introduction to population growth models

- Background - models
- What is a population?
- Exponential growth
  - Continuous model
  - Discrete model
- Stochastic (chance) effects
  - Demographic
  - Environmental

Why use population models?

- To address these types of questions:
  1. How do populations grow?
  2. What limits population growth?
  3. What factors affect population dynamics?
  4. What are the population consequences of species interactions?
  5. How do we manage ecological resources?

What is a population?

- Definition:
  - A group of individuals of the same species in the same location that could potentially interact

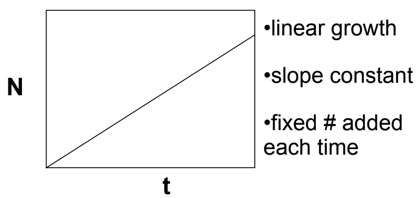
What is an individual?

- Two types:
  1.
  2.

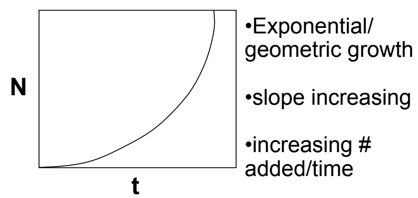
Why can population growth be geometric?

- Example with cell division:
  - Division of cells: 2 > 4 > 8 > 16 etc ….2^t
  - Don't add fixed amount each time step
  - Instead, population size is doubled at each time step

Not like population growth!



More like population growth!



Modeling geometric population growth

- We are interested in the relationship between population size and time – how does the size of the population vary with time?

- Tool Kit:

  N = population size

  t = time

  N_t = population size at time t

  N_0 = population size at start (t=0)

Modeling geometric population growth

- What factors determine what the population size will be from one time step to the next?
  - Initial population size, births, deaths, immigration, emigration

    $N_{t+1} = N_t + B – D + I – E$
- How much does the population size change from one time step to the next?

  $N_{t+1} – N_t = B – D + I – E = \mathbf{\Delta N}$

Continuous model of geometric population growth

- If we assume a closed population (no I or E):

  $\mathbf{\Delta N}$ = B – D = (rate of births-rate of deaths) *$\mathbf{\Delta t}$
- This is true for absolute changes over discrete time steps, but we can also model rates:
- dN/dt = rate of births - rate of deaths
- Both ΔN and dN/dt tell us how the population is growing:

  $\mathbf{\Delta N}$ or dN/dt > 0

  $\mathbf{\Delta N}$ or dN/dt < 0

  $\mathbf{\Delta N}$ or dN/dt = 0

Continuous model of geometric population growth

- Rates of birth and death depend on population size:

  Total birth rate = bN b is the per female (per capita) birth rate

  Total death rate = dN d is the per capita death rate

- We can get an index of population growth that is independent of population size if we use b and d (scaling B and D to individual females):

  dN/dt = total birth rate - total death rate

  = bN – dN

  = (b - d)N (Substitute r for b - d)

  dN/dt = rN (r - per capita growth rate)

Continuous model of geometric population growth

dN/dt = total birth rate - total death rate

= bN – dN

= (b - d)N (Substitute r for b - d)

dN/dt = rN (r - per capita growth rate)

(1/N)dN =r dt (divide both sides by N, multiply by dt)

$N_t = N_0e^{rt}$ (integrate both sides)

Continuous model of geometric population growth

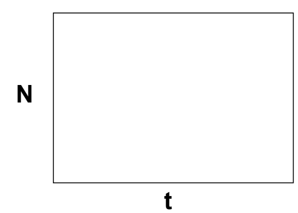
- Examples of per capita growth rates (r) in nature:

	r	Doubling Time
Virus	110,000	3.3 minutes
Bacteria	21,000	17 minutes
Hydra	124	2 days
Cow	0.365	1.9 years
Humans	0.013	50 years

Continuous model of geometric population growth

- We now have an equation that can describe how population size changes with time:

  dN/dt = rN (r - per capita growth rate)



Change in population size for any given time step: slope = dN/dt =

Population growth rate per individual: r =

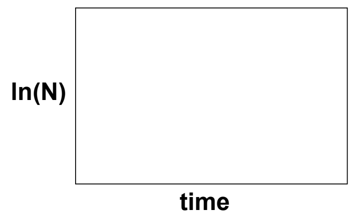
- What if we also want to be able to predict a future population size based on this growth rate?

  N_t =

Continuous model of geometric population growth

- Recall: e is the base of natural logarithm
  - $\ln(e^x) = x$
- $N_t = N_0 e^{rt}$
  - Take natural log of both sides:

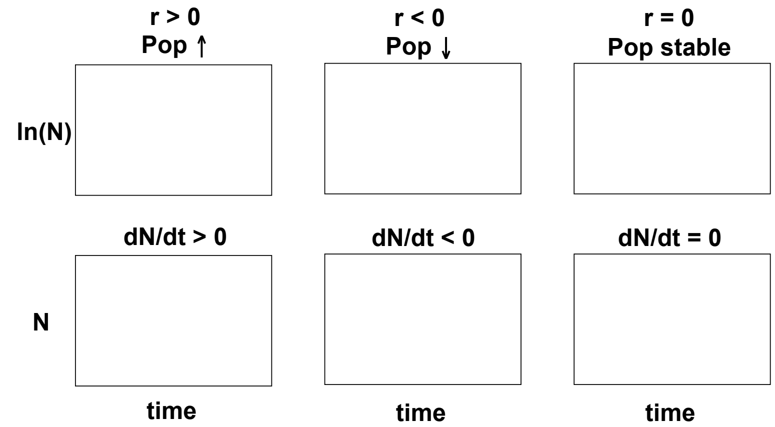
$$\begin{aligned} \ln(N_t) &= \ln(N_0 e^{rt}) \\ &= \ln(N) + \ln(e^{rt}) \\ &= \ln(N) + rt \\ y &= b + mx \end{aligned}$$



Note: semi-log plot because x-axis is not logged

Continuous model of geometric population growth

- r describes how the population is growing:



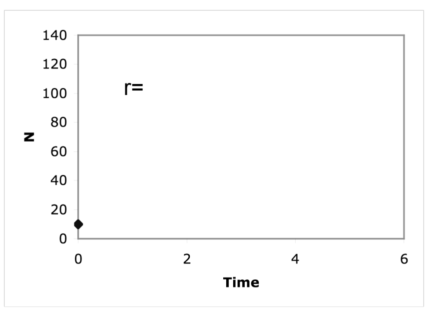
Continuous model of geometric population growth

- $N_t = N_0 e^{rt}$
- Example:

  b = 0.75 (3 kids/4 yrs)

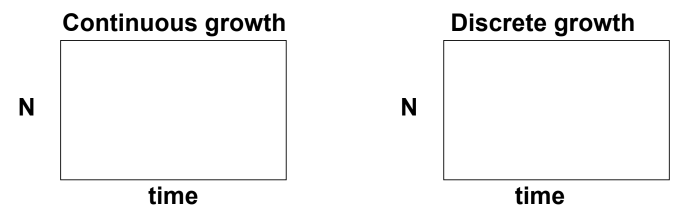
  d = 0.25 (1 death/4yrs)

  $N_0 = 10$



Discrete model of geometric population growth

- Continuous population models assume that the population is always growing at every time step
- However, we know that in nature this is not always true
  - e.g. some organisms reproduce only during a distinct season



Discrete model of geometric population growth

- The discrete model:
  - $N_{t+1} = N_t + r_D N_t$
    $= N_t(1 + r_D)$ (Substitute λ for $1 + r_D$)
    $N_{t+1} = N_t \lambda$

    $N_1 = N_0 \lambda$
    $N_2 = N_1 \lambda = N_0 \lambda \lambda = N_0 \lambda^2$
    $N_t = N_0 \lambda^t$

Discrete model of geometric population growth

- How does the discrete per capita growth rate (λ) compare to the continuous per capita growth rate (r)?

Pop Growth	r	λ
Stable		
Increasing		
Decreasing		

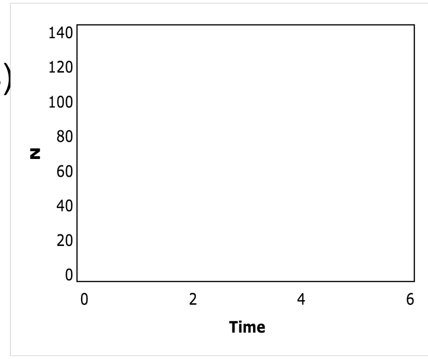
Discrete vs continuous growth

- Remember: $\lambda = 1 + r_D = N_{t+1}/N_t$ for discrete growth
- r for continuous growth, $N_t = N_0 e^{rt}$
- Can also calculate N_{t+1}/N_t for continuous growth:

  $N_0 e^{r(t+1)} / N_0 e^{rt} = e^r$
  $\approx (1+r)$ for r near 0
  (but slightly greater)

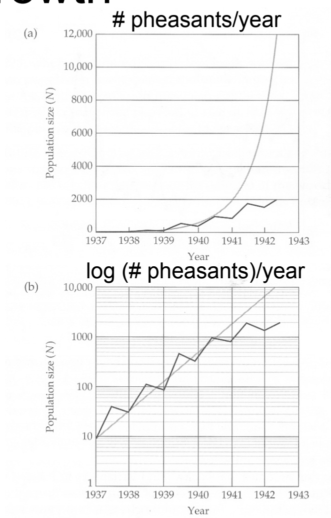
Discrete model of geometric population growth

- $N_t = N_0 \lambda^t$, where $\lambda = 1 + r_D$
- Example:
  b = 0.75 (3 kids/4 yrs)
  d = 0.25 (1 death/4yrs)
  $N_0 = 10$



Discrete model of geometric population growth

- Example of a population with discrete growth:
  - Pheasants introduced to Protection Island
  - Nearly discrete generations



Incorporating stochastic effects into population growth models

- Up until now we have been considering models that are deterministic
  -
  -
- However, we know that chance effects can alter birth and death rates
  -
  -
  -

Incorporating stochastic effects into population growth models

- Let's consider how stochasticity can affect population growth
- Consider 2 different populations with discrete growth: $N_t = N_0 \lambda^t$

Pop 1: λ always = 3 (deterministic)

Pop 2: λ variable = 1, 3, 5 (stochastic)

- Arithmetic mean λ of both populations is 3
- Does the arithmetic mean accurately describe the average population growth of both types of models?

Incorporating stochastic effects into population growth models

- However, the arithmetic mean does not accurately describe the average growth:
  - Pop 1: 3*3*3 = 27 $\sqrt[3]{3*3*3} = 3$
  - Pop 2: 1*3*5 = 15 $\sqrt[3]{1*3*5} = 2.46$
  - Are the two populations actually growing at the same average rate? If not, which one is growing faster?
- Is there better way to describe stochastic growth?
- Geometric mean: $\sqrt[n]{\lambda_1 \lambda_2 \lambda_3 \ldots \lambda_n}$

Incorporating stochastic effects into population growth models

- Example of an environmentally stochastic model of population growth:
  - Good years: $\lambda = 1.2$
  - Bad years: $\lambda = 0.8$
  - Equal chance of either event (like flipping a coin – heads means good year, tails means bad year)
  - Start $N_0 = 100$

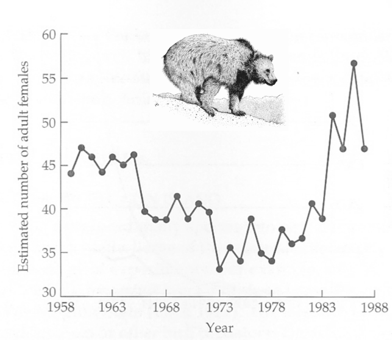
100 > h > 120 > h > 144 > h > 173 > t > 138 > h > 166 > t > 133

100 > t > 80 > h > 96 > h > 115 > t > 92 > h > 110 > t > 88

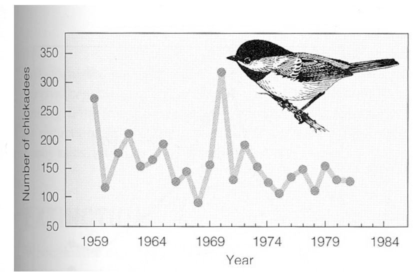
The effect of stochasticity on population dynamics

- Examples:

Grizzly Bears
- # adult females/year



Chickadees
chickadees/year

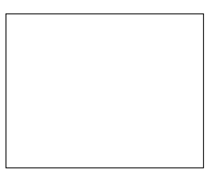


Incorporating density-dependence into population growth models

- The logistic model of population growth
  - Continuous, builds on exponential $dN/dt = rN$
  - Idea: per capita growth (r) rate should
  - Carrying capacity (K)

Per capita growth rate

0



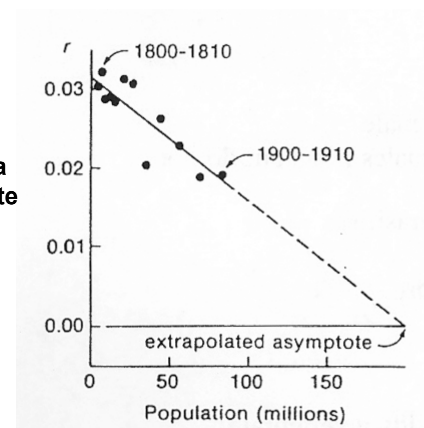
N

Intraspecific competition and density-dependence

- Example in humans:

Per capita growth rate

r(1-N/K)



Incorporating density-dependence into population growth models

- The logistic model of population growth
  - Slope: (0-r)/(K-0) = -r/K
  - Equation of the line (y = mx +b):
    per capita growth rate = -r/K*N + r = r(1-N/K)

Per capita growth rate 0

N

–Plug density-dependent per capita growth rate into exponential growth equation:
dN/dt = r N(1-N/K)

The logistic model of population growth

- Our new population growth equation which incorporates density-dependence:
  dN/dt = r N(1-N/K)

r(1-N/K)

N

	N	Per capita population growth rate
Tiny population		
Large population		
Population above K		

The logistic model of population growth

- What does this type of population growth look like when you compare population size over time?
  dN/dt = r N(1-N/K)

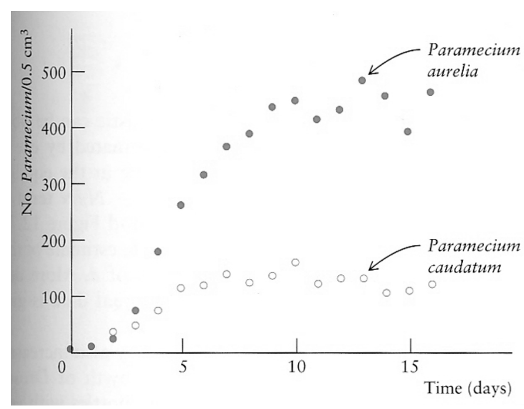
- Actual equation of the line:
  $N_t = K/(1+((K-N_0)/N_0)e^{-rt})$

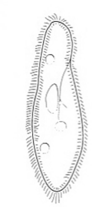
N

t

The logistic model of population growth

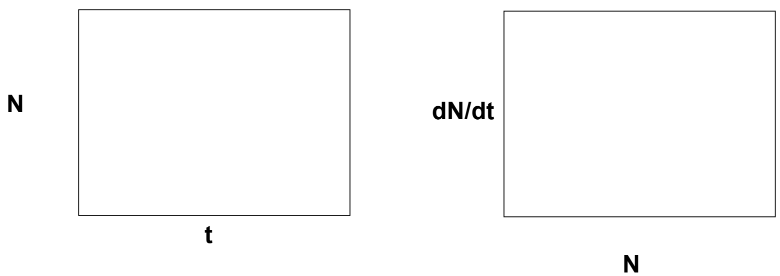
- Example with Paramecium:





The logistic model of population growth

- How does the rate of change of population size vary with time (i.e. the slope)?



The discrete logistic model of population growth

- So far we have incorporated intraspecific competition into a continuous model of population growth
- Let's now incorporate it into a discrete model
  - What is "discrete" population growth?

The discrete logistic model of population growth

- Recall that for a discrete model:

  $\lambda = r_D + 1$

  $N_{t+1} = N_t \lambda$

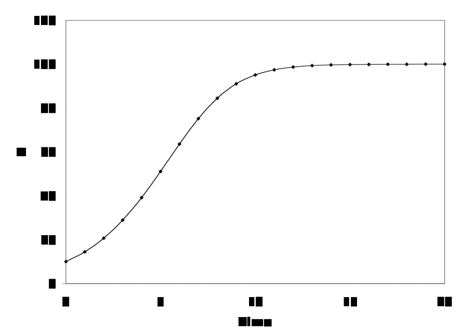
  $= N_t + r_D N_t$
- The discrete logistic model is therefore:

  $N_{t+1} = N_t + r_D N_t(1\text{-}N_t/K)$

To incorporate density-dependence, multiply r_D by $(1\text{-}N_t/K)$

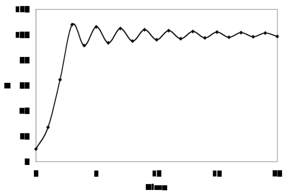
Example of discrete logistic population dynamics

- $N_{t+1} = N_t + r_D N_t(1\text{-}N_t/K)$
- Example: $b = 0.75$, $d = 0.25$, $N_0 = 10$, $K = 100$

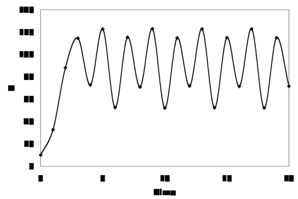


Interesting population dynamics and the discrete logistic model

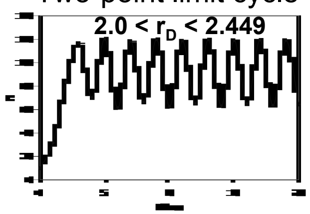
Damped oscillations
$r_D < 2.0$



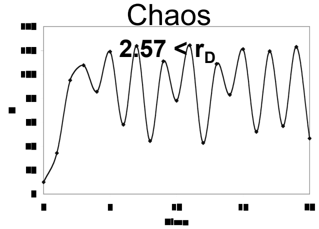
More complex limit cycles
$2.449 < r_D < 2.57$



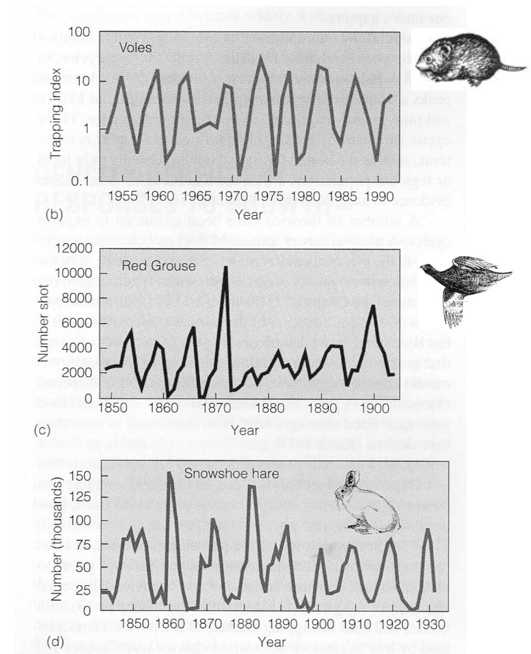
Two-point limit cycle
$2.0 < r_D < 2.449$



Chaos
$2.57 < r_D$



Examples of discrete logistic population dynamics



Important point about chaotic population dynamics

- If we run the same model repeatedly, with the same initial conditions (i.e. same N_0)
  - Chaos:
  - Stochastic dynamics:
- However, very small differences in N_0 will lead to different population trajectories

Supplemental Notes on website

- There is a typo on page 1, near the bottom
- After 10 generations we have 2^{10} individuals, not 12^{10}