DUBLIN INSTITUTE OF TECHNOLOGY

School of Mathematical Sciences

Structured PhD

Assignment 1 2016/2017

MATH 9973: NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS

DR. JOHN S. BUTLER

DR C HILLS

External examiner missing!

Due: 24, March 2017

Duration:

Attempt all questions

All questions carry equal marks

Approved calculators may be used

Mathematical tables are provided

New Cambridge Statistical Tables are NOT permitted

1. **a)** Use the Euler's method to estimate the solution of the system of first order initial value problems

$$\frac{du}{dt} = u^2 + v, \quad u(0) = 1,$$
$$\frac{dv}{dt} = u - v, \quad v(0) = -1,$$

using $h = 0.25$, approximate the value of the solution at $t = 0.5$. (18 marks)

b) Consider the differential equation

$$y' + 10y + x = 0, \;\; 2 \leq x \leq 3, \;\; y(2) = 1.$$

Apply the midpoint method to approximate the solution at $y(3.0)$ using $h = 0.25$

(11 marks)

c) Suggest two ways to improve the numerical solution. (4 marks)

2. **a)** Derive the Adams-Bashforth two step method which is of the form

$$w_0 = \alpha_0 \quad w_1 = \alpha_1$$

$$w_{n+1} = w_n + \frac{h}{2}[3f(t_n, w_n) - f(t_{n-1}, w_{n-1})]$$

for the initial value problem

$$y' = f(x, y),$$

$$y(x_0) = w_0.$$

(15 marks)

b) Show that the local truncation error of the 2 step Adams-Bashforth method is

$$\tau_{n+1}(h) = \frac{5h^2}{12} y^{(3)}(\mu_n).$$

(8 marks)

c) Define what it means for a linear multistep method to be strongly stable, weakly stable and unstable.

(6 marks)

d) Show that the three step Adams-Bashforth method is stable.

(4 marks)

3. **a)** Use the Midpoint method to approximate the solutions to the following initial value problem

$$y' = 2ty, \ (0 \le t \le 1), \ y(0) = 1,$$

with $h = 0.2$ with and compare with the exact solution $y(t) = e^{t^2}$.
Calculate the theoretical upper limit of error for the above initial value problem for the midpoint method.

(18 marks)

b) Apply the two step Adams-Moulton method to approximate the solution of the initial value problem:

$$y' = \frac{y}{t} - y, \ (2 \le t \le 3) \ \ y(3) = 0.$$

Using $N = 5$ steps, given that $y(2.2) = 0.2442$.

(15 marks)