Part 1: Introduction

1. Find the general solution for the following:

   (a)
   $$u_{yy} + u = 0;$$

   (b)
   $$u_{xx} + u_x = 0;$$

   (c)
   $$u_{xx} - 4u_{xy} + 3u_{yy} \quad \text{with} \quad u(x, y) = f(\lambda x + y).$$

2. Show
   $$u(x, t) = f(x - ct) + g(x + ct)$$
   is the general solution to the wave equation
   $$u_{tt} = c^2 u_{xx}$$
   (where f and g are twice differentiable).

3. Assuming u is a homogeneous function of degree n, show
   $$xu_x + yu_y = nu$$
   .

4. Show
   $$u(x, y) = \mathrm{e}^{-x/b} f(ax - by)$$
   satisfies
   $$bu_x + au_y + u = 0$$
   .

5. Obtain a general solution to
   $$u_{tt} - c^2 u_{xx} + 2bu_t = 0$$
   assuming u of the form
   $$u(x, t) = (A \cos kx + B \sin kx)V(t)$$
   .

6. Show
$$u(r,t) = \frac{V(r)}{r}\cos(nct)$$
($n \in \mathbb{N}$) satisfies
$$c^2\left(u_{rr} + \frac{2}{r}u_r\right) - u_t t = 0$$
by finding a differential equation for V.

7. Solve
$$uu_x + u_y = 1 \quad x(s,0) = 2s^2,\ y(s,0) = 2s,\ u(s,0) = 0,\ s > 0.$$

Part 2: Separation of Variables

8. Solve
$$y^2u_x^2 + x^2u_y^2 = (xyu)^2$$

9. Solve
$$u_x^2 + u_y^2 = 1$$

10. Solve
$$u_x^2 + u_y + x^2 = 0$$

11. Solve
$$x^2u_x^2 + y^2u_y^2 = u^2$$