W9205/9206/9209/9210/103C

DUBLIN INSTITUTE OF TECHNOLOGY

School of Mathematical Sciences

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# DT9205 MSc Mathematical Physics
# DT9206 MSc Mathematical Physics
# DT9209 MSc Applied Mathematics
# DT9210 MSc Applied Mathematics

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**WINTER EXAMINATIONS 2015/2016**

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## MATH 9971: CLASSICAL MECHANICS AND THERMODYNAMICS

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DR EMIL PRODANOV

DR C HILLS

PROFESSOR E O'RIORDAN

9.30 – 11.30, Friday, 8 January 2016

Duration: 2 hours

Attempt three questions only

All questions carry equal marks

Approved calculators may be used

Mathematical tables are provided

**1.** a) Use the formalism of the Levi-Civita $\epsilon$ symbol and the Kronecker $\delta$ symbol to prove that

$$\text{curl (curl } \vec{F}) = \text{grad (div } \vec{F}) - \Delta \vec{F}.$$

(15)

b) Find expressions for the critical parameters $V_c, p_c$, and $T_c$ for a real Dieterici gas with equation

$$p(V - b) = RT\, e^{-a/RTV}.$$

(18)

**[33]**

**2.** a) Show how the isotropy of space leads to the conservation of the angular momentum.

(19)

b) Find the values of the first and second virial coefficients of a van der Waals' gas and the temperature at which the first virial coefficient is equal to zero (Boyle's temperature). The equation of state for a van der Waals' gas is:

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT.$$

(14)

**[33]**

**3.** Consider the motion of a particle of mass $m$ under the influence of a conservative central force.

a) Determine the magnitude of the vector of the angular momentum and the total energy and show that these are integrals of motion.

(16)

b) Starting from Newton's second law, derive the expression for the conserved Laplace–Runge–Lenz vector.

(17)

**[33]**

4. **a)** Starting from the Lagrangian $L(q^i, \dot{q}^i)$, $i = 1, 2, \ldots, D$ (where $D$ is the number of the degrees of freedom), introduce the Hamiltonian and derive the Hamilton equations. (16)

**b)** Consider a planar pendulum of mass $m$ attached to an ideal cord of length $l$. The suspension point of the pendulum has mass $M$ and can slide frictonlessly along a straight line lying in the plane of the pendulum. Write down the equations of motion for the system. Do not solve these equations. (17)

**[33]**