Problem Sheet 3

PhD students: solutions are to be handed in by Tuesday 25th of April

(1) Consider the vector space $\mathbb{R}^2$ of ordered pairs of real numbers (x, y) with the norm $\|(x, y)\|_1 = |x| + |y|$.

(a) Show that this norm is not strictly convex.

(b) Let $a = (2, 0)$, find all points on the unit ball of V which are minimal distance from a.

(c) Do the same as in part (b) if the norm is $\|(x, y)\|_2 = \sqrt{x^2 + y^2}$.

(2) (a) Find the Chebyshev approximation to $f(x) = x^4$ out of the space $P_3[-1, 1]$.

(b) In $C[-1, 1]$ find the best approximation to $g(x) = x^3 + x^2$ by a quadratic polynomial.

(3) Show that the subset

$$M = \{(x_1, \dots x_n) : \sum_{i=1}^{n} x_i = 1\}$$

of the Euclidean space $\mathbb{R}^n$ is convex. Find the vector of minimum norm in M.

(4) Let V be a vector space, $b \in V$ and W be a finite-dimensional subspace of V with orthonormal basis $e_1, ..., e_n$.

(a) Let x be the least square approximation of b out of W, show that $b - x \perp e_i$ for $i = 1, ..., n$.

(b) Show that $b - x \perp w$ for all $w \in W$.

(5) Consider the data $(-1, -1)$, $(0, 3)$, $(2, 11)$ and $(3, 27)$. Construct an interpolating polynomial $p(x)$ for this data by using:

(a) Lagrange's formula;

(b) the method of undetermined coefficients;

(c) Newton's method.