MATH 346
Assignment #1: Non-linear Difference Equations Models
Due: Friday, January 18, 2008

Problem 1. Complete problem 1.4.1 from Chapter One of the text
Problem 2. Complete problem 2.4.1 from Chapter Two of the text
Problem 3. Complete problem 2.4.2 from Chapter Two of the text

Problem 4. Find all equilibria of the given difference equation

$$X_{n+1} = \frac{rX_n^2}{X_n^2 + A} \quad \text{(r, A are nonnegative constants)}$$

and determine whether it is asymptotically stable or unstable

Problem 5. Consider an ecological model described by

$$X_{n+1} = CX_n^2\left(2 - X_n\right) \quad for \quad 0 \le X_n \le 2$$

where X_n is population density in year n and C=25/16.

a) Sketch the graph of the right hand side of this equation. Indicate extreme and inflection points.
b) Determine the fixed points of this system
c) Determine the stability of each fixed point and describe the dynamics in the neighbourhood of the fixed points
d) Use the cobwebbing method to sketch the approximate behaviour of solutions from some starting point X_0.

Problem 6. Solve the following difference equation

$$X_n - 5X_{n-1} + 6X_{n-2} = 0$$

subject to the IVP $\quad X_0 = 2, \quad X_1 = 5 \quad$ and sketch the solution.

MATH 346
Assignment #2: Non-linear System of Difference Equations Models
Due: Friday, February 2, 2008

Problem 1. Complete problem 2.4.13 from Chapter Two of the text
Problem 2. Complete problem 2.4.18 (part a) and b) only!) from Chapter Two of the text

Problem 3. For each of the given the two first order equations
a) Find the general solution of system
b) Graph the solution of system for various choices of the initial conditions.
c) Discuss your findings in part b)

3a). $$\begin{cases} X_{n+1} = 3X_n + 2Y_n \\ Y_{n+1} = X_n + 4Y_n \end{cases}$$

3b). $$\begin{cases} X_{n+1} = 0.25X_n + Y_n \\ Y_{n+1} = 0.1875X_n - 0.25Y_n \end{cases}$$

Problem 4. The following system of difference equations represents two species x and y competing for a common resource

$$X_{n+1} = \frac{(\alpha_1 + 1) X_n}{1 + X_n + \beta_1 Y_n} \quad , \quad Y_{n+1} = \frac{(\alpha_2 + 1) Y_n}{1 + \beta_2 X_n + Y_n} \qquad (1) \quad (\alpha_i, \beta_i > 0)$$

Find all of the equilibria for (1).

Problem 5. The following epidemic model is referred to as an SIS epidemic model:

$$S_{n+1} = S_n - \frac{\beta}{N} I_n S_n + (\gamma + b) I_n \ , \qquad I_{n+1} = I_n (1 - \gamma - b) + \frac{\beta}{N} I_n S_n$$

Assume that $\quad 0 < \beta < 1, \quad 0 < b + \gamma < 1, \quad S_0 + N_0 = N, \quad S_0, I_0 > 0$

a) Show that $\quad S_n + I_n = N \quad for \quad n = 1, 2, ...$
b) Show that there exist two equilibria and they are both nonnegative if

$$\frac{\beta}{(b+\gamma)} \geq 1$$