$M(x) \qquad 0 \leq x < 1$

$= 0$

$M(1) = 1$

$M(x) \qquad 1 \leq x < 2$

$$= \frac{2}{x-1} \int_0^{x-1} M(t)\,dt + 1$$

$= 0 + 1$

$= 1$

$$M(2) = 2\int_0^1 M(t)\,dt + 1$$

$$= 2\int_0^{1-\epsilon} M(t)\,dt + 2\int_{1-\epsilon}^{1+\epsilon} M(t)\,dt + 1$$

$$= 0 + 2\int_{1-\epsilon}^{1+\epsilon} dt + 1$$

$$= 2 \cdot t\Big|_{1-\epsilon}^{1+\epsilon} + 1$$

$$= 2\left(1+\epsilon - (1-\epsilon)\right) + 1$$

$$= 4\epsilon + 1$$

$$= 1 \quad \text{as } \epsilon \to 0$$

$M(x) \qquad 2 \le x \le 3$

$$= \frac{2}{x-1} \int_0^{x-1} M(t)\, dt + 1$$

$$= \frac{2}{x-1} \int_0^{1} M(t)\, dt + \frac{2}{x-1} \int_1^{x-1} M(t)\, dt + 1$$

$$= 0 + \frac{2}{x-1} \int_1^{x-1} M(t)\, dt + 1$$

$$= \frac{2}{x-1}\, t \Big|_1^{x-1} + 1$$

$$= \frac{2}{x-1} \left(x-1 - 1 \right) + 1$$

$$= 2 - \frac{2}{x-1} + 1$$

$$= 3 - \frac{2}{x-1}$$

$$M(3) = \frac{2}{2}\int_0^2 M(t)\,dt + 1$$

$$= \int_0^1 M(t)\,dt + \int_1^{2-\epsilon} M(t)\,dt + \int_{2-\epsilon}^{2+\epsilon} M(t)\,dt + 1$$

$$= \cancel{0} + \int_1^{2-\epsilon} 1\,dt + \int_{2-\epsilon}^{2+\epsilon} 1\,dt + 1$$

$$= t\Big|_1^{2-\epsilon} + t\Big|_{2-\epsilon}^{2+\epsilon} + 1$$

$$= \left[2-\epsilon-1\right] + \left[2+\epsilon-2+\epsilon\right] + 1$$

$$= 1 - \epsilon + 2\epsilon + 1$$

$$= 2 + \epsilon$$

$$= 2 \quad \text{as } \epsilon \to 0$$

$M(x) \quad 3 \le x < 4$

$$= \frac{2}{x-1}\int_0^{x-1} M(t)\,dt + 1$$

$$= \frac{2}{x-1}\int_0^{2} M(t)\,dt + \int_2^{x-1} M(t)\,dt + 1$$

$$= \frac{2}{x-1}\int_0^{1} M(t)\,dt + \frac{2}{x-1}\int_1^{2} M(t)\,dt + \frac{2}{x-1}\int_2^{x-1} M(t)\,dt + 1$$

$$\underset{0}{\downarrow} \qquad \frac{2}{x-1}\int_1^{2} 1\,dt + \frac{2}{x-1}\int_2^{x-1}\left(3 - \frac{2}{t-1}\right)dt + 1$$

$$= \frac{2}{x-1} + \frac{2}{x-1}\int_2^{x-1} 3\,dt - \frac{2}{x-1}\int_2^{x-1}\frac{2}{t-1}\,dt + 1$$

$$= \frac{2}{x-1} + \frac{2}{x-1}\left[3t\Big|_2^{x-1}\right] - \frac{4}{x-1}\left[\ln(t-1)\Big|_2^{x-1}\right] + 1$$

$$= \frac{2}{x-1} + \frac{2}{x-1}\left[3x-3-6\right] - \frac{4}{x-1}\left[\ln(x-2) - 0\right] + 1$$

$$= \frac{2}{x-1} + \frac{2}{x-1}\cdot 3\cdot(x-3) - \frac{4\ln(x-2)}{x-1} + 1$$

$$= \frac{2+6x-18}{x-1} - \frac{4\ln(x-2)}{x-1} + 1$$

$$= \frac{6x-6}{x-1} + \frac{-10}{x-1} - \frac{4\ln(x-2)}{x-1} + 1$$

$$= 7 - \frac{10}{x-1} - \frac{4\ln(x-2)}{x-1}$$

$$M(4) = \frac{2}{3}\int_0^3 M(t)\,dt + 1$$

$$= \frac{2}{3}\int_0^1 M(t)\,dt + \frac{2}{3}\int_1^2 M(t)\,dt + \frac{2}{3}\int_2^{3-\epsilon} M(t)\,dt + \frac{2}{3}\int_{3-\epsilon}^{3+\epsilon}$$

$$= 0 + \frac{2}{3}\int_1^2 dt + \frac{2}{3}\int_2^{3-\epsilon}\left(3 - \frac{2}{t-1}\right)dt$$

$$+ \frac{2}{3}\int_{3-\epsilon}^{3+\epsilon} 2\,dt + 1$$

$$= \frac{2}{3} + \frac{4}{3}\cdot 2\epsilon + \frac{2}{3}\int_2^{3-\epsilon} 3\,dt - \frac{4}{3}\int_2^{3-\epsilon}\frac{1}{t-1}\,dt + 1$$

$$= 1 + \frac{2}{3} + \frac{8}{3}\epsilon + \frac{2}{3}\left[3t\Big|_2^{3-\epsilon}\right] - \frac{4}{3}\left[\ln(t-1)\Big|_2^{3-\epsilon}\right]$$

$$= 1 + \frac{2}{3} + \frac{8}{3}\epsilon + \frac{2}{3}\left[9 - 3\epsilon - 6\right] - \frac{4}{3}\ln(2-\epsilon)$$

$$= \frac{5}{3} + \frac{6}{3} + \frac{8}{3}\epsilon - \frac{6}{3}\epsilon - \frac{4}{3}\ln(2-\epsilon)$$

$$= \frac{11}{3} - \frac{4}{3}\ln 2 \quad \text{as } \epsilon \to 0$$

$$= \frac{11 - 4\ln 2}{3}$$

$M(x) \qquad 4 \le x < 5$

$$= \frac{2}{x-1}\int_0^{x-1} M(t)\,dt + 1$$

$$= \frac{2}{x-1}\int_0^1 M(t)\,dt + \frac{2}{x-1}\int_1^2 M(t)\,dt + \frac{2}{x-1}\int_2^3 M(t)\,dt$$

$$+ \frac{2}{x-1}\int_3^{x-1} M(t)\,dt + 1$$

$$= \cancel{7} \;\cancel{+}\; \frac{10}{x-1} \;\cancel{+}\; -\frac{4}{x-1}\left[\ln(t-1)\Big|_2^3\right]$$

$$+ \frac{2}{x-1}\left[\int_3^{x-1}\left(7 - \frac{10}{t-1} - \frac{4\ln(t-2)}{t-1}\right)dt\right] + 1$$

yack!

I'll finish this when I have more time and not falling asleep