report

July 12, 2024

1 Global Stock Market Analytics

in this project we will look at predicting the open direction of the Nifty 50 index by looking at other indices and indicators. We will break the project up into five phases:

- 1. preparing the master data from the global indices
- 2. preliminary analysis of data
- 3. predictive modelling of open direction of Nifty 50
- 4. comparing different models at prediction
- 5. sentiment analysis of X / Twitter data relating to Nifty 50

1.1 Phase 1 - Prepare the Master Data

We begin by preparing the data that we will be working with. We import the libraries we will be using, set some plotting configurations, and declare some constants. The indexes of interest are:

• NSEI: Nifty 50

• DJI: Dow Jones Index

IXIC: NasdaqHSI: Hang SengN225: Nikkei 225GDAXI: Dax

• VIX: Volatility Index

```
import os
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

import statsmodels.api as sm
import yfinance as yf
import ta

import nltk

from scipy import stats
```

```
from statsmodels.api import Logit
from statsmodels.stats.outliers_influence import variance_inflation_factor
from sklearn.metrics import classification_report, roc_curve, roc_auc_score
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.naive_bayes import GaussianNB
from sklearn.neighbors import KNeighborsClassifier
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier
from sklearn.svm import SVC
from sklearn.neural_network import MLPClassifier
from sklearn.preprocessing import MinMaxScaler, StandardScaler
import torch
import torch.nn as nn
import torch.utils.data as utils
from string import punctuation, digits
from nltk.corpus import stopwords
from nltk.tokenize import word_tokenize
from nltk.sentiment.vader import SentimentIntensityAnalyzer
from wordcloud import WordCloud
plt.style.use("ggplot")
sns.set_style("darkgrid")
sns.set_context("paper")
INDICES = ['NSEI', 'DJI', 'IXIC', 'HSI', 'N225', 'GDAXI', 'VIX']
COLUMNS = [f"{index}_DAILY_RETURNS" for index in INDICES]
```

Next, we declare a function that we will use to download the OHLC data, and make use of it:

```
def retrieve_data(index, start_date = '2017-12-1', end_date = '2024-1-31',
progress = False):
    data = yf.download(f'^{index}', start_date, end_date, progress = progress)

# create daily returns for each index
    data['Daily Returns'] = data.Close.pct_change() * 100

# rename columns - prefix with index name
```

```
data.columns = ["_".join(c.upper() for c in column.split()) for column in_
    data.columns]
    data.columns = [f"{index}_{column}" for column in data.columns]

    return data

data = [retrieve_data(index) for index in INDICES]
```

Next, we declare a function we will use to test the normality of the data we will be working with:

```
[]: def test_normality(data, column_name, index_name):
        print()
        print(f"\t Index {index_name}")
        print(f"\tColumn {column_name}")
        print()
        data = data[column_name].dropna()
        if data.shape[0] < 50:</pre>
            print("\t Shapiro-Wilks Test:")
            result = stats.shapiro(data)
            print("\tKolmogorov-Smirnov Test:")
            result = sm.stats.diagnostic.lilliefors(data)
                        p-value: {result[1]}")
        print(f"\t
        if result[1] < 0.05:</pre>
            print("\treject null hypothesis - data is not drawn from a normal ⊔
      ⇔distribution")
        else:
            print("\tfail to reject null hypothesis - data is drawn from a normal⊔
      print()
    for d, c, i in zip(data, COLUMNS, INDICES):
        test_normality(d, c, i)
```

Index DJI

Column DJI_DAILY_RETURNS

Kolmogorov-Smirnov Test:

reject null hypothesis - data is not drawn from a normal distribution

Index IXIC

Column IXIC_DAILY_RETURNS

Kolmogorov-Smirnov Test:

p-value: 0.000999999999998899

reject null hypothesis - data is not drawn from a normal distribution

Index HSI

Column HSI_DAILY_RETURNS

Kolmogorov-Smirnov Test:

reject null hypothesis - data is not drawn from a normal distribution

Index N225

Column N225_DAILY_RETURNS

Kolmogorov-Smirnov Test:

p-value: 0.000999999999998899

reject null hypothesis - data is not drawn from a normal distribution

Index GDAXI

Column GDAXI_DAILY_RETURNS

Kolmogorov-Smirnov Test:

p-value: 0.000999999999998899

reject null hypothesis - data is not drawn from a normal distribution

Index VIX

Column VIX_DAILY_RETURNS

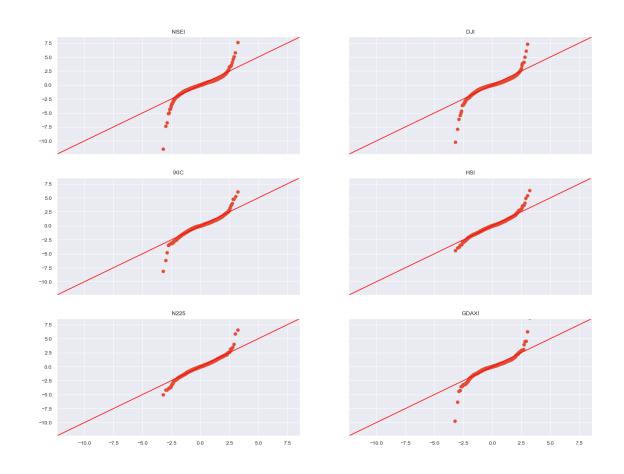
Kolmogorov-Smirnov Test:

p-value: 0.000999999999998899

reject null hypothesis - data is not drawn from a normal distribution

Next, we declare a function we will use to create qq-plots of the data we will be working with:

Q-Q Plots of Daily Returns



The daily returns do not appear to follow - or be drawn from - a Normal Distribution - specifically at the tails.

Next, we declare a function we will use to merge the data, using LOCF for missing data, and adding variables for MONTH, QUARTER, and YEAR:

```
[]: def merge_data(data, start_date = '2018-01-02', end_date = '2023-12-29'):
    # merge data with outer join
    merged = pd.concat(data, axis = 1)

# impute missing data using LOCF (forward fill)
merged.ffill(inplace = True)

# add indicators for MONTH, QUARTER, and YEAR
merged['MONTH'] = merged.index.month
merged['QUARTER'] = merged.index.quarter
merged['YEAR'] = merged.index.year

return merged[start_date:end_date]

master = merge_data(data)
```

1.2 Phase 2 - Preliminary Analysis

Now that we have our master data we perform some preliminary analysis hoping to answer the following questions:

- 1. Which index has given consistently good returns?
- 2. Which index was highly volatile?
- 3. How are global markets correlated during 6 years period and is the correlation structure similar in the recent year-2023?
- 4. Assuming primary target variable as "Nifty Opening Price Direction", what are preliminary insights?

Lets define a few functions we will use across our analysis - to begin with, a function that plots box plots of the daily returns by year:

```
[]: #/ label: box-plots-daily-returns-yearly

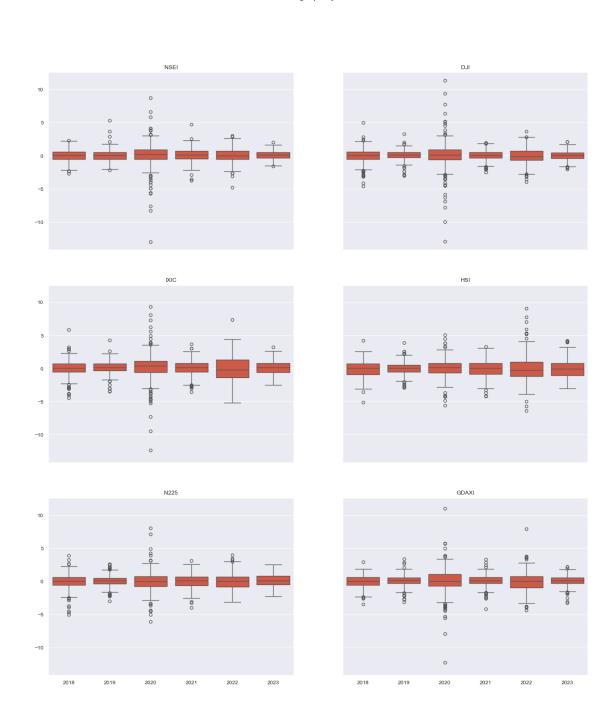
def performance_analytics_box_plots(data, group_by, title, count = 6):
    fig, axes = plt.subplots(3, 2, figsize = (16, 18), sharex = True, sharey = True)
    fig.suptitle(title)

for index in range(count):
    axes[index // 2, index % 2].set_title(INDICES[index])
```

```
sns.boxplot(x = data[group_by], y = data[COLUMNS[index]], ax =
axes[index // 2, index % 2])
axes[index // 2, index % 2].set_xlabel("")
axes[index // 2, index % 2].set_ylabel("")

performance_analytics_box_plots(master, "YEAR", "Box Plots grouped by Year")
```

Box Plots grouped by Year



All indexes seem pretty consistent - all years have similar spreads, and consistent medians, with one or two exceptions. All indexes for 2020 have more outliers than normal. But HSI seems to have more outliers in 2022 than in 2020.

Next, we declare a function that prints a table of summary statistics for the daily returns of each index:

```
[]: def performance_analytics_tables(data, group_by, count = 6):
    for index in range(count):
        table = round(data.groupby(group_by, observed = False)[COLUMNS[index]].
        agg(['count', 'mean', 'std', 'var']), 3)
        print(f"\n{INDICES[index]}\n\n{table}\n\n")

performance_analytics_tables(master, "YEAR")
```

NSEI

		count	mean	std	var
YE	AR				
20	18	260	0.012	0.804	0.647
20	19	260	0.062	0.862	0.744
202	20	262	0.059	2.004	4.015
202	21	261	0.094	0.980	0.960
202	22	260	0.055	1.096	1.202
202	23	260	0.079	0.620	0.384

DJI

	count	mean	std	var
YEAR				
2018	260	-0.035	1.143	1.306
2019	260	0.099	0.784	0.614
2020	262	0.057	2.277	5.186
2021	261	0.075	0.773	0.597
2022	260	-0.025	1.237	1.531
2023	260	0.060	0.709	0.503

IXIC

count mean std var

YEAR				
2018	260	-0.020	1.330	1.768
2019	260	0.133	0.975	0.950
2020	262	0.170	2.200	4.838
2021	261	0.096	1.124	1.262
2022	260	-0.124	2.000	4.001
2023	260	0.157	1.085	1.177

HSI

	count	mean	std	var
YEAR				
2018	260	-0.035	1.244	1.547
2019	260	0.033	0.981	0.962
2020	262	0.026	1.445	2.087
2021	261	-0.028	1.262	1.593
2022	260	-0.021	2.054	4.221
2023	260	-0.053	1.409	1.984

N225

	count	mean	std	var
YEAR				
2018	260	-0.047	1.198	1.436
2019	260	0.065	0.860	0.740
2020	262	0.013	1.615	2.608
2021	261	0.029	1.152	1.328
2022	260	-0.036	1.262	1.592
2023	260	0.105	0.999	0.997

GDAXI

	count	mean	std	var
YEAR				
2018	260	-0.056	0.975	0.951
2019	260	0.084	0.888	0.788
2020	262	0.043	2.064	4.260
2021	261	0.071	0.899	0.808
2022	260	-0.035	1.460	2.133
2023	260	0.082	0.809	0.655

The clear winner here is NSEI - not one year in the range has a negative mean return. And with the exception of 2020 and 2022, NSEI has low volatility (< 1) throughout all years.

Next, we declare a function that plots median daily returns by year:

```
[]: #/ label: bar-plots-daily-returns-yearly
     def performance_analytics_bar_plots(data, group_by, title, count = 6, aggfunc =_u

¬"median"):
         fig, axes = plt.subplots(3, 2, figsize = (16, 18), sharex = True, sharey = __
      →True)
         fig.suptitle(title)
         for index in range(count):
             axes[index // 2, index % 2].set_title(INDICES[index])
             table = data.groupby(group_by, observed = False)[COLUMNS[index]].
      →agg([aggfunc])
             sns.barplot(x = table.index, y = table[aggfunc], ax = axes[index // 2,
      →index % 2])
             axes[index // 2, index % 2].set_xlabel("")
             axes[index // 2, index % 2].set_ylabel("")
     performance_analytics_bar_plots(master, "YEAR", "Bar Plots of Median Returns_
      ⇒grouped by Year")
```

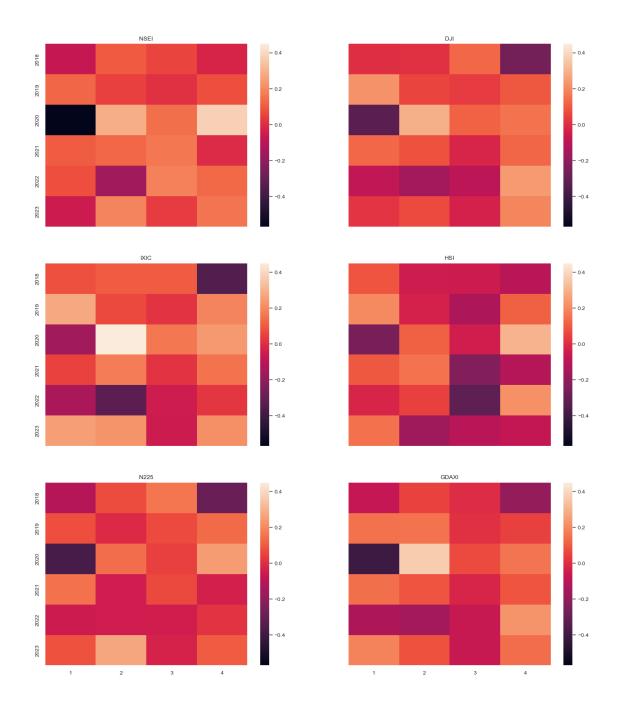


The clear winner here is NSEI - at no time is the median daily returns for any of the years below 0. IXIC has an unusually high 2020, but a bad 2022. HSI also has an unusually bad 2022.

Next, we plot of heatmaps by year and quarter showing mean returns:

```
[]: #/ label: heat-maps-mean-daily-returns-yearly
     def performance_analytics_heat_maps(data, group_by, title, column = "QUARTER",_
      ⇔count = 6, aggfunc = "median"):
         fig, axes = plt.subplots(3, 2, figsize = (16, 18), sharex = True, sharey = __
      →True)
         fig.suptitle(title)
         tables = []
         values = []
         for index in range(count):
             table = pd.pivot_table(data, values = COLUMNS[index], index =_
      →[group_by], columns = [column], aggfunc = aggfunc, observed = False)
             tables.append(table)
             values.extend(table.values.ravel())
         vmax = max(values)
         vmin = min(values)
         for index in range(count):
             axes[index // 2, index % 2].set_title(INDICES[index])
             sns.heatmap(tables[index], ax = axes[index // 2, index % 2], vmin =
      →vmin, vmax = vmax)
             axes[index // 2, index % 2].set_xlabel("")
             axes[index // 2, index % 2].set_ylabel("")
     performance_analytics_heat_maps(master, "YEAR", "Heat Maps of Mean Returns_

¬grouped by Year", aggfunc = "mean")
```



With the exception of the 1st quarter in 2020, NSEI has pretty consistent daily returns - where most cells sre pretty bright, denoting above 0. Most of the other indexes have a blend of light and dark, which would indicate more volatile behaviour over the quarters.

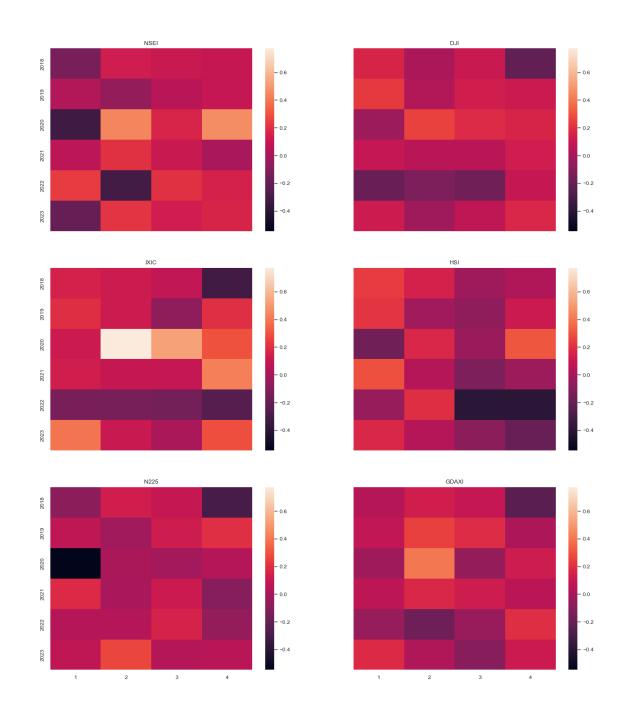
Next we look at heatmaps by year and quarter showing median returns:

[]: #/ label: heat-maps-median-daily-returns-yearly

performance_analytics_heat_maps(master, "YEAR", "Heat Maps of Median Returns⊔

⇔grouped by Year")

Heat Maps of Median Returns grouped by Year



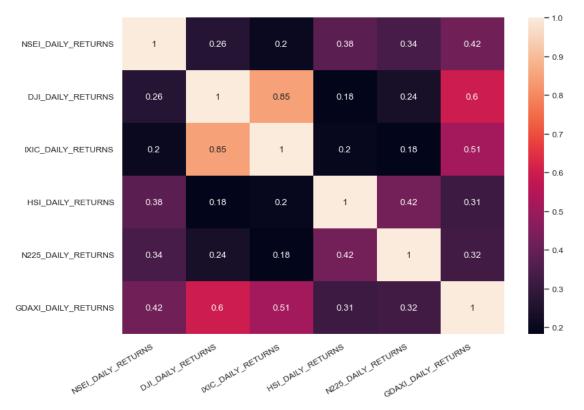
On the other hand, when looking at median returns across quarters NSEI seems pretty average there does not seem to be a clear winner here.

Lets look at a correlation matrix of the 6 years daily returns:

```
def correlation_matrix(data):
    plt.figure(figsize = (9, 6))
    matrix = data[COLUMNS[:-1]].corr()

ax = sns.heatmap(matrix, annot = True)
    ax.set_xticklabels(
        ax.get_xticklabels(),
        rotation = 30,
        horizontalalignment = "right"
    )

correlation_matrix(master)
```



It looks like strong correlation between daily returns of IXIC and DJI, and some correlation between GDAXI and DJI. These indexes are likely to result in multicolinearity at the regression stage.

Next, we look at the same correlation matrix for the 2023 year alone to see if there are similar correlations:

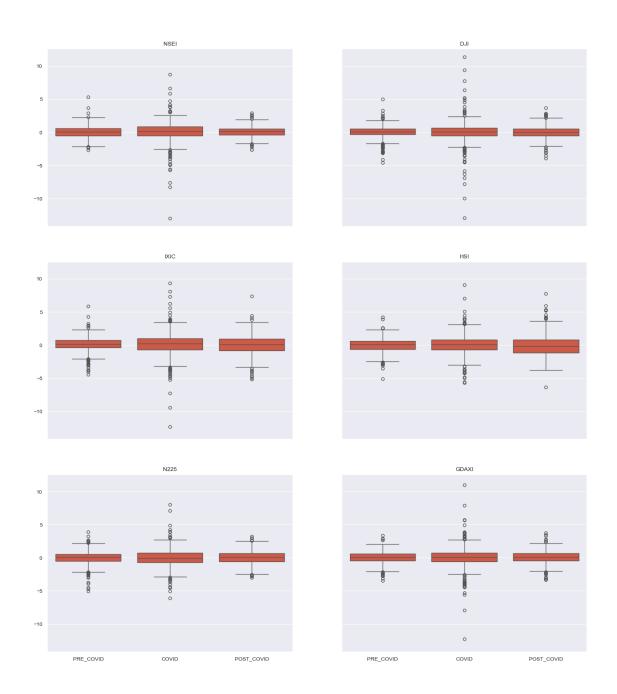
```
[]: #/ label: correlation-matrix-daily-returns-last-year correlation_matrix(master['2023-01-02':'2023-12-29'])
```



We can see similar - but slightly weaker - correlations exist between the same indexes for 2023. Next, we do similar analysis for the Pandemic period - Pre Covid, Covid, and Post Covid:

performance_analytics_box_plots(master, "PANDEMIC", "Box Plots grouped by →Pandemic Period")

Box Plots grouped by Pandemic Period



We can see that the spreads of each index over the Pandemic are consistent, with the Covid period itself having more outliers - which of course you might expect.

Next we look at summary statistics for the Pandemic:

[]: performance_analytics_tables(master, "PANDEMIC")

NSEI

	count	mean	std	var
PANDEMIC				
PRE_COVID	542	0.033	0.830	0.690
COVID	589	0.069	1.563	2.442
POST COVID	432	0.082	0.777	0.604

DJI

	count	mean	std	var
PANDEMIC				
PRE_COVID	542	0.034	0.969	0.938
COVID	589	0.043	1.657	2.745
POST_COVID	432	0.038	0.981	0.962

IXIC

count	mean	std	var
542	0.062	1.153	1.330
589	0.077	1.812	3.285
432	0.065	1.525	2.326
	542 589	589 0.077	count mean std 542 0.062 1.153 589 0.077 1.812 432 0.065 1.525

HSI

	count	mean	std	var
PANDEMIC				
PRE_COVID	542	-0.013	1.126	1.268
COVID	589	0.012	1.508	2.274
POST COVID	432	-0.047	1.664	2.769

N225

count mean std var

PANDEMIC PRE_COVID 0.000 1.046 542 1.095 COVID 589 0.017 1.417 2.009 POST_COVID 432 0.055 1.066 1.136

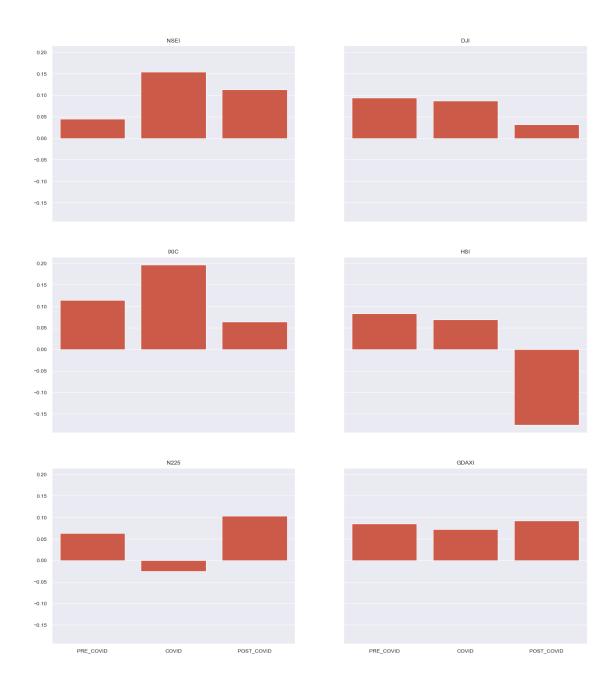
GDAXI

	count	mean	std	var
PANDEMIC				
PRE_COVID	542	0.011	0.935	0.875
COVID	589	0.035	1.634	2.671
POST_COVID	432	0.052	1.027	1.055

All indexes had higher volatility over the Covid period.

- NSEI performed reasonably well over the Covid period, with an increase in volatility in the Covid period, and with a significant bump in the Post Covid period.
- DJI seemed consisten over the three periods, with an increase in volatility in the Covid period.
- IXIC looked pretty good over the three period, but maybe slightly more volatile overall, and in particular in the Covid period.
- HSI has performed poorely in general, with negative returns in the pre and post Covid periods, and with consistently greater volatility than most.
- N225 appears to perform not so well, and with relatively high volatility.
- GDAXI also appears to perform not so well in general, and with relatively high volatility.

Next, we we look at median daily returns by Pandemic period:



We can see that IXIC looks like the clear winner here, with NSEI in second place, and DJI and GSAXI in a fight for third place. HSI appears to have had a terrible Post Covid period, and N225 appears to have had a pretty bad Covid period.

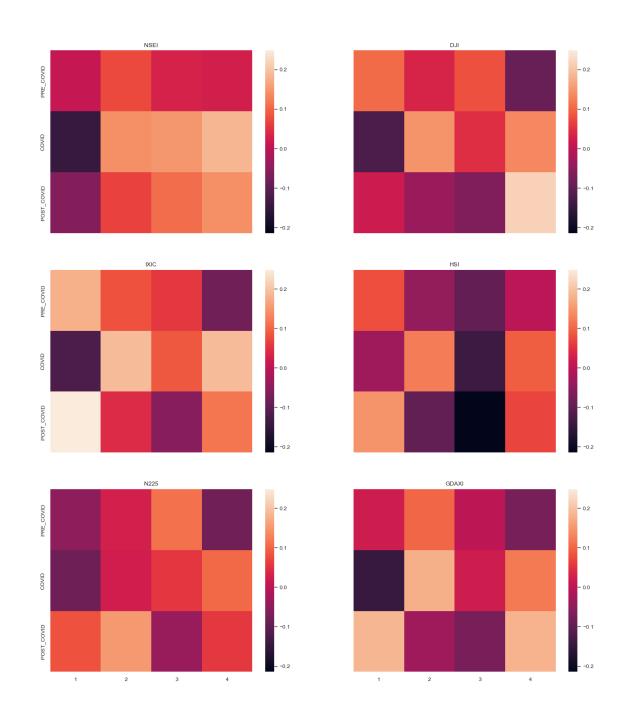
Next, we plot of heatmaps by Pandemic period and quarter showing mean returns:

[]: #/ label: heat-maps-mean-daily-returns-covid

performance_analytics_heat_maps(master, "PANDEMIC", "Heat Maps of Mean Returns□

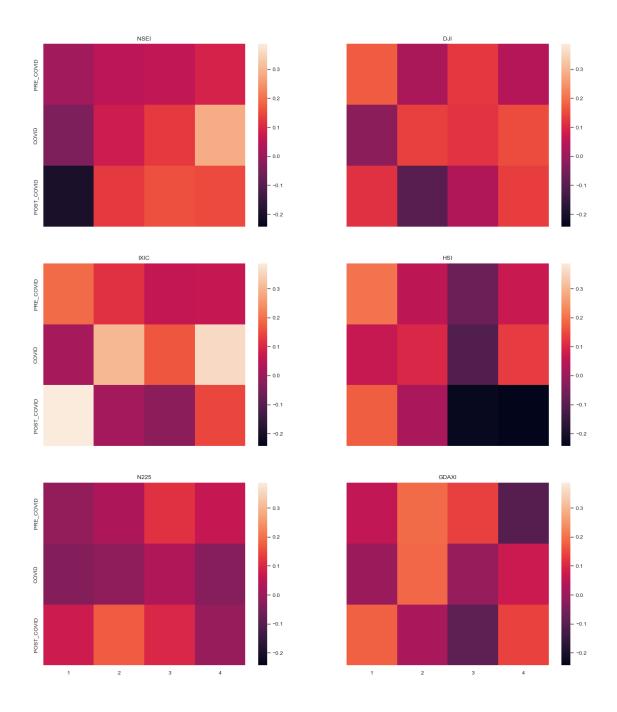
⇔grouped by Pandemic Period", aggfunc = "mean")

Heat Maps of Mean Returns grouped by Pandemic Period



Again, NSEI appears to be the most consistent of all indexes. All indexes have bad first quarters during the Pandemic, but improve post Covid.

Next we look at heatmaps by Pandemic period and quarter showing median returns:



Median returns tells a similar story over the Pandemic period - HSI in particular appears to have had the worst recovery.

Next, we try to estimate the time taken for each of the indexes to return to the Pre Covid levels - my approach is to find how many days it takes for each index to reach a value greater than or

equal to the Pre Covid mean returns value:

```
[]: #/ label: return-to-pre-covid-levels
     for i in range(6):
         pre_covid = master.loc[(master['PANDEMIC'] == 'PRE_COVID'), [COLUMNS[i]]]
         post_covid = master.loc[(master['PANDEMIC'] == 'POST_COVID'), [COLUMNS[i]]]
         mean pre = pre covid.values.mean()
         post_count = np.where(post_covid[COLUMNS[i]].ge(mean_pre).values ==_u
      →True) [0] [0]
         post_date = post_covid.index[post_covid[COLUMNS[i]].ge(mean_pre)][0].date()
         print(f"{INDICES[i].rjust(5)} returned to pre-covid levels (mean fmean_pre: u
      ⇒2.4f}) on {post_date} after {post_count} trading day(s)")
     NSEI returned to pre-covid levels (mean 0.0334) on 2022-05-16 after 7 trading
    day(s)
      DJI returned to pre-covid levels (mean 0.0337) on 2022-05-13 after 6 trading
    day(s)
     IXIC returned to pre-covid levels (mean 0.0619) on 2022-05-10 after 3 trading
    day(s)
      HSI returned to pre-covid levels (mean -0.0126) on 2022-05-11 after 4 trading
    day(s)
     N225 returned to pre-covid levels (mean 0.0001) on 2022-05-06 after 1 trading
    GDAXI returned to pre-covid levels (mean 0.0114) on 2022-05-10 after 3 trading
    day(s)
    Interestingly, N225 returned to it's Pre Covid level after just 1 day.
    Lets define NSEI OPEN DIR as 1 if NSEI Open at time t > NSEI Close at time t - 1, and 0
    otherwise:
[]: master["NSEI_OPEN_DIR"] = np.where(master["NSEI_OPEN"] > master["NSEI_CLOSE"].
      ⇒shift(), 1, 0)
    Lets look at the percentages of NSEI OPEN DIR = 1 by year:
[]: table1 = master.groupby("YEAR", observed = False)[["NSEI_OPEN_DIR"]].sum()
     table2 = master.groupby("YEAR", observed = False)[["NSEI_OPEN_DIR"]].count()
     table = ((table1["NSEI_OPEN_DIR"] / table2["NSEI_OPEN_DIR"]) * 100).round(2)
     print("\nNifty Fifty Daily Movement\n")
     print(f"\n{table}\n")
```

Nifty Fifty Daily Movement

```
YEAR
2018 70.38
2019 69.23
2020 70.61
2021 71.65
2022 59.23
2023 67.31
Name: NSEI_OPEN_DIR, dtype: float64
```

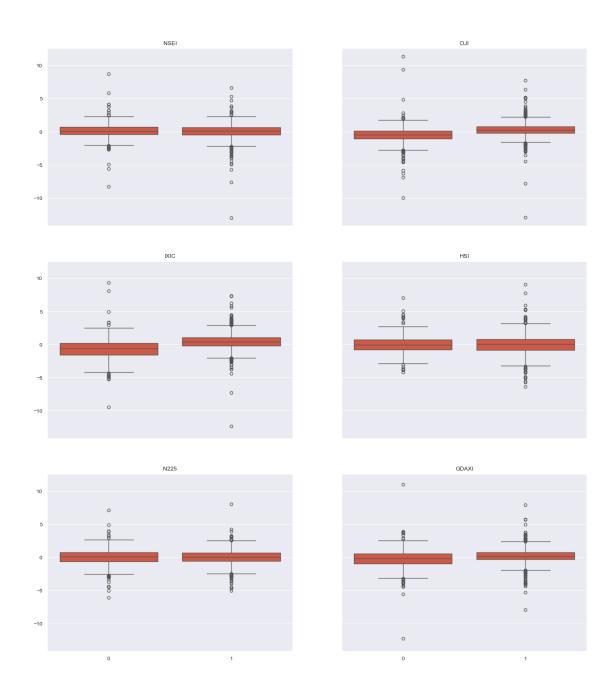
With the exception of 2022, every year has around 70% where NSEI_OPEN_DIR = 1.

Next we look at the indices for each category of NSEI OPEN DIR:

```
[]: #/ label: box-plots-daily-returns-grouped-by-open-dir

fig, axes = plt.subplots(3, 2, figsize = (16, 18), sharex = True, sharey = True)
fig.suptitle("Box Plots grouped by NSEI Open Direction")

for index in range(6):
    axes[index // 2, index % 2].set_title(INDICES[index])
    sns.boxplot(x = master["NSEI_OPEN_DIR"], y = master[COLUMNS[index]].
    shift(), ax = axes[index // 2, index % 2])
    axes[index // 2, index % 2].set_xlabel("")
    axes[index // 2, index % 2].set_ylabel("")
```



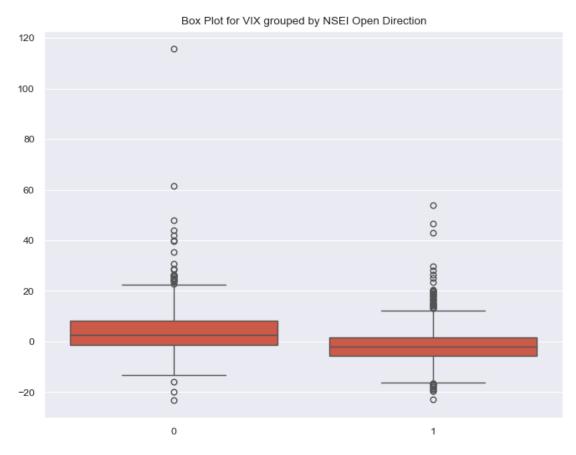
we look at VIX separately - as it requires a different scale:

```
[]: #/ label: box-plots-vix-daily-return-grouped-by-open-dir
plt.figure(figsize = (8, 6))
```

```
sns.boxplot(x = master["NSEI_OPEN_DIR"], y = master[COLUMNS[6]].shift())

plt.title(f"Box Plot for {INDICES[6]} grouped by NSEI Open Direction")
plt.xlabel("")
plt.ylabel("")

plt.show()
```



All of the box plots look consistent across each categroy of NSEI_OPEN_DIR, with the exceptions of IXIC and VIX.

1.3 Phase 3 - Training a Logistic Model

Before proceeding with modelling NSEI_OPEN_DIR, lets define, and add, some indicators and ratios:

```
[]: RATIOS = ["NSEI_HL_RATIO", "DJI_HL_RATIO"]
INDICATORS = ["NSEI_RSI", "DJI_RSI", "NSEI_TSI", "DJI_TSI"]
ALL_COLS = COLUMNS + RATIOS + INDICATORS
```

Lets add NSEI_HL_RATIO and DJI_HL_RATIO:

```
[]: master["NSEI_HL_RATIO"] = master["NSEI_HIGH"] / master["NSEI_LOW"]
master["DJI_HL_RATIO"] = master["DJI_HIGH"] / master["DJI_LOW"]
```

Lets add some technical indicators - RSI and TSI for NSEI and DJI:

```
[]: master["NSEI_RSI"] = ta.momentum.rsi(master["NSEI_CLOSE"])
master["DJI_RSI"] = ta.momentum.rsi(master["DJI_CLOSE"])

master["NSEI_TSI"] = ta.momentum.tsi(master["NSEI_CLOSE"])
master["DJI_TSI"] = ta.momentum.tsi(master["DJI_CLOSE"])
```

Lets create a data frame containing all the data we will be working with:

	data.nead()					
[]:		NSEI_OPEN_DIR N	SEI_DAILY	_RETURNS D	JI_DAILY_RETURNS	\
	Date					
	2018-02-22	1.0	=	-0.141862	0.664177	
	2018-02-23	1.0		1.043559	1.392128	
	2018-02-26	1.0		0.872647	1.577556	
	2018-02-27	0.0	-	-0.267418	-1.163939	
	2018-02-28	0.0	-	-0.582229	-1.498739	
		IXIC_DAILY_RETUR	NS HSI_I	DAILY_RETURN	S N225_DAILY_RETU	RNS \
	Date					
	2018-02-22	-0.1127	72	-1.48324	2 -1.066	738
	2018-02-23	1.765	85	0.97362	7 0.719	252
	2018-02-26	1.1457	73	0.74016	8 1.191	496
	2018-02-27	-1.2276	354	-0.72999	9 1.066	320
	2018-02-28	-0.7822	232	-1.35579	7 -1.436	450
		GDAXI_DAILY_RETU	JRNS VIX	_DAILY_RETUR	NS NSEI_HL_RATIO	\
	Date					
	2018-02-22	-0.068	3803	-6.4935	1.005502	
	2018-02-23	0.175	5574	-11.9123	91 1.009854	
	2018-02-26	0.346	3449	-4.1843	52 1.006915	
	2018-02-27	-0.289	9850	17.6582	27 1.008959	
	2018-02-28	-0.439	373	6.7778	39 1.007069	
		DJI_HL_RATIO N	ISEI_RSI	DJI_RSI	NSEI_TSI DJI_TS	I
	ъ.				_	

Lets split our data into features (X) and labels (y):

```
[ ]: X = data[ALL_COLS]
y = data['NSEI_OPEN_DIR']
```

And lets add an intercept to the features:

```
[]: X.insert(loc = 0, column = "Intercept", value = 1)
```

Now we spit our data into training and testing sets - with an 80 / 20 train / test split:

```
[]: X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.2, userandom_state = 1337)
```

We define a function that will prune any features that are either found to be insignificant, or that are found to be collinear:

```
[]: def prune(X, y, verbose = True):
         dropped = []
         while True:
            model = Logit(y, X).fit(disp = 0)
             insignificant = [p for p in zip(model.pvalues.index[1:], model.
      \Rightarrowpvalues[1:]) if p[1] > 0.05]
                     = [variance_inflation_factor(model.model.exog, i) for i in_
      →range(1, model.model.exog.shape[1])]
             colinear = [val for val in zip(model.model.exog names[1:], values) if =
      \rightarrowval[1] > 5]
             if insignificant:
                 insignificant.sort(key = lambda p: -p[1])
                 if verbose:
                     print(f"dropping {insignificant[0][0]} with p-value_
      X = X.drop([insignificant[0][0]], axis = 1)
                 dropped.append(insignificant[0][0])
             elif colinear:
                 colinear.sort(key = lambda c: -c[1])
                 if verbose:
                     print(f"dropping {colinear[0][0]} with vif {colinear[0][1]}")
```

```
X = X.drop([colinear[0][0]], axis = 1)
    dropped.append(colinear[0][0])

else:
    return model, dropped

model, dropped = prune(X_train, y_train)
```

```
dropping DJI_DAILY_RETURNS with p-value 0.7234766099770011 dropping GDAXI_DAILY_RETURNS with p-value 0.6162105670376612 dropping NSEI_HL_RATIO with p-value 0.4277618505298021 dropping DJI_HL_RATIO with p-value 0.1563055988923202 dropping NSEI_DAILY_RETURNS with p-value 0.13281329048460666 dropping NSEI_TSI with vif 5.865700460659149 dropping NSEI_RSI with p-value 0.7783762272653001
```

The function outputs a list of pruned features, together with the associated p-value or vif value. The function returns the pruned model, together with a list of pruned feature names.

Lets look at the summary of the model returned:

```
[]: #/ label: logistic-model-summary

model.summary()
```

[]:

Dep. Variable:	NSEI_OPEN_DIR		No. Observations:		1220	
Model:	Logit		Residu	als:	1213	
Method:	MLE		Df Model:		6	
Date:	Wed, 10 Jul 2024		eudo R	-squ.:	0.1375	
Time:	19:08:10		g-Likeli	hood:	-660.02	
${f converged:}$	True		-Null:		-765.23	
Covariance Type:	nonrobust		R p-val	lue:	1.141e-42	
	coef	std err	${f z}$	$\mathbf{P} > \mathbf{z} $	[0.025]	0.975]
Intercept	-1.4041	0.656	-2.139	0.032	-2.690	-0.118

				1 1	-	
Intercept	-1.4041	0.656	-2.139	0.032	-2.690	-0.118
IXIC_DAILY_RETURNS	0.4552	0.075	6.093	0.000	0.309	0.602
$HSI_DAILY_RETURNS$	-0.1395	0.053	-2.632	0.008	-0.243	-0.036
N225_DAILY_RETURNS	-0.1960	0.068	-2.897	0.004	-0.329	-0.063
VIX_DAILY_RETURNS	-0.0397	0.013	-3.054	0.002	-0.065	-0.014
$\mathrm{DJI}\mathrm{_RSI}$	0.0447	0.013	3.415	0.001	0.019	0.070
DJI_TSI	-0.0205	0.008	-2.660	0.008	-0.036	-0.005

And now lets look at the list of dropped features:

```
[]: #/ label: logistic-model-dropped-features
print("\n".join(dropped))
```

```
DJI_DAILY_RETURNS
GDAXI_DAILY_RETURNS
NSEI_HL_RATIO
DJI_HL_RATIO
NSEI_DAILY_RETURNS
NSEI_TSI
NSEI_RSI
```

And lets also look at the variance inflation factors for each of the retained features:

```
[]: Feature VIF
0 IXIC_DAILY_RETURNS 2.073867
1 HSI_DAILY_RETURNS 1.244922
2 N225_DAILY_RETURNS 1.353286
3 VIX_DAILY_RETURNS 1.994009
4 DJI_RSI 4.850250
5 DJI_TSI 4.379409
```

As we can see, all retained features have a vif of less than 5.

Let us define a function to plot ROC curves which we will use throughout this report:

```
[]: def performance_analytics_roc_curve(fpr, tpr, title = "ROC Curve"):
    plt.figure(figsize = (8, 6))

    plt.plot(fpr, tpr, label = 'ROC Curve')
    plt.plot([0, 1], [0, 1], 'k--', label = 'Random Guess')

    plt.xlim([0.0, 1.0])
    plt.ylim([0.0, 1.05])

    plt.title(title)
    plt.xlabel('FPR')
    plt.ylabel('TPR')

    plt.legend(loc = 'lower right')
    plt.show()
```

Let us plot the ROC curve for the training data:

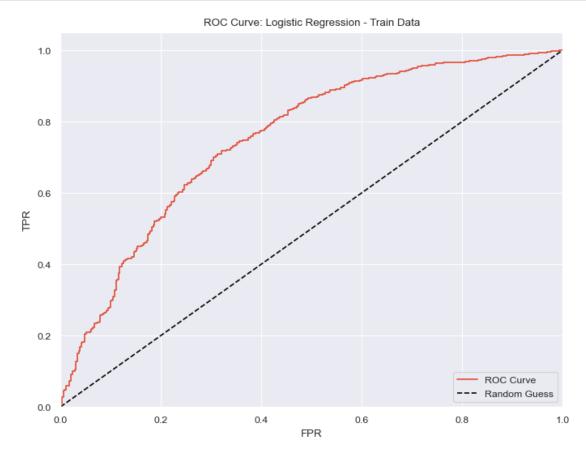
```
[]: #/ label: logistic-model-roc-curve-train-data

y_pred = model.predict(X_train.drop(dropped, axis = 1))

fpr, tpr, thresholds = roc_curve(y_train, y_pred)

performance_analytics_roc_curve(fpr, tpr, title = "ROC Curve: Logistic"

→Regression - Train Data")
```



Let us find the AUC for the training data:

```
[]: #/ label: logistic-model-auc-train-data

train_auc_roc = roc_auc_score(y_train, y_pred)
print(f'Train Data - AUC ROC: {train_auc_roc}')
```

Train Data - AUC ROC: 0.7529115595469844

Now we find the optimal threshold that balances sensitivity and specificity:

```
[]: optimal_threshold = round(thresholds[np.argmax(tpr - fpr)], 3)
    print(f'Optimal Threshold: {optimal_threshold}')
```

Optimal Threshold: 0.684

Next we generate a classification report for the training data with the optimal threshold:

```
[]: #/ label: logistic-model-classification-report-train-data

y_pred_class = np.where(y_pred <= optimal_threshold, 0, 1)
print(classification_report(y_train, y_pred_class))</pre>
```

	precision	recall	f1-score	support
0.0	0.53	0.68	0.60	391
1.0	0.83	0.72	0.77	829
accuracy			0.70	1220
macro avg	0.68	0.70	0.68	1220
weighted avg	0.73	0.70	0.71	1220

Next we generate a confusion matrix for the training data with the optimal threshold:

```
[]: #/ label: logistic-model-confusion-matrix-train-data

table_train = pd.crosstab(y_pred_class, y_train)
print(table_train)
```

```
NSEI_OPEN_DIR 0.0 1.0
row_0
0 265 234
1 126 595
```

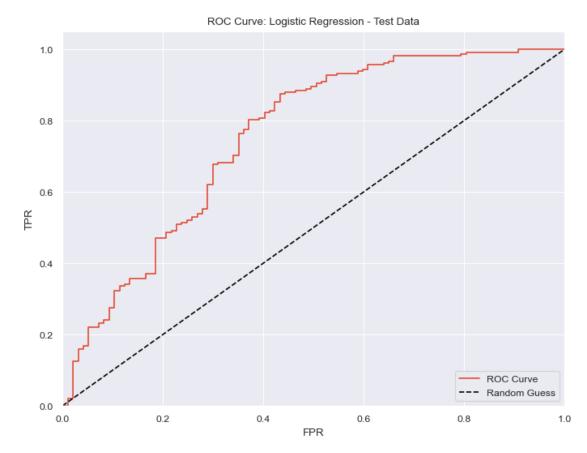
And finally, we obtain the sensitivity and specificity for the training data:

```
Train Data - Sensitivity: 71.77%
Train Data - Specificity: 67.77%
```

Now we do the same for the test data - starting with obtaining the ROC curve:

```
[]: #/ label: logistic-model-roc-curve-test-data

y_test_pred = model.predict(X_test.drop(dropped, axis = 1))
```



Next, we obtain the AUC for the test data:

```
[]: #/ label: logistic-model-auc-test-data

test_auc_roc = roc_auc_score(y_test, y_test_pred)
print(f'Test_Data - AUC_ROC: {test_auc_roc}')
```

Test Data - AUC ROC: 0.7520816812053925

Next we generate a classification report for the test data (with the previously calculated optimal threshold):

```
[]: #/ label: logistic-model-classification-report-test-data

y_test_pred_class = np.where(y_test_pred <= optimal_threshold, 0, 1)
print(classification_report(y_test, y_test_pred_class))</pre>
```

	precision	recall	f1-score	support
0.0	0.53	0.65	0.58	97
1.0	0.82	0.73	0.77	208
accuracy			0.70	305
macro avg	0.67	0.69	0.67	305
weighted avg	0.72	0.70	0.71	305

Next we generate a confusion matrix for the test data (with the previously calculated optimal threshold):

```
[]: #/ label: logistic-model-confusion-matrix-test-data

table_test = pd.crosstab(y_test_pred_class, y_test)
print(table_test)

NSEI_OPEN_DIR   0.0   1.0
row_0
0      63   57
1      34   151
```

And finally, we obtain the sensitivity and specificity for the test data:

```
Test Data - Sensitivity: 72.6%
Test Data - Specificity: 64.95%
```

Reviewing the AUC for both train and test data - we see they are very close implying consistency in the performance of the models:

```
[]: #/ label: logistic-model-auc-comparison

print(f'Train Data - AUC ROC: {train_auc_roc}')
print(f' Test Data - AUC ROC: {test_auc_roc}')
```

```
Train Data - AUC ROC: 0.7529115595469844
Test Data - AUC ROC: 0.7520816812053925
```

But the accuracy of the model is not great. We could obtain better results by selecting a different classification model.

1.4 Phase 4 - Compare Models

Next, we compare the performance of a number of different models. We first implement two functions that will help us:

```
[]: def model_metrics_plots(X_train, X_test, y_train, y_test, model, name =_u

¬"MODEL"):
         model.fit(X_train, y_train)
         y_train_pred_prob = model.predict_proba(X_train)
         train_fpr, train_tpr, = roc_curve(y_train, y_train_pred_prob[:, 1])
         y_test_pred_prob = model.predict_proba(X_test)
         test_fpr, test_tpr, _ = roc_curve(y_test, y_test_pred_prob[:, 1])
         fig, axes = plt.subplots(1, 2, figsize = (16, 6))
         fig.suptitle("ROC Curves")
         axes[0].set_title(f"{name} - Train Data")
         axes[0].plot(train_fpr, train_tpr, label = 'ROC Curve')
         axes[0].plot([0, 1], [0, 1], 'k--', label = 'Random Guess')
         axes[0].legend(loc = 'lower right')
         axes[0].set_xlabel("FPR")
         axes[0].set_ylabel("TPR")
         axes[0].set_xlim([0.0, 1.0])
         axes[0].set_ylim([0.0, 1.05])
         axes[1].set title(f"{name} - Test Data")
         axes[1].plot(test_fpr, test_tpr, label = 'ROC Curve')
         axes[1].plot([0, 1], [0, 1], 'k--', label = 'Random Guess')
         axes[1].legend(loc = 'lower right')
         axes[1].set_xlabel("FPR")
         axes[1].set_ylabel("TPR")
         axes[1].set_xlim([0.0, 1.0])
         axes[1].set ylim([0.0, 1.05])
     def model_metrics_data(X_train, X_test, y_train, y_test, model, name = "MODEL"):
         model.fit(X_train, y_train)
         y_train_pred_prob = model.predict_proba(X_train)
         train_fpr, train_tpr, thresholds = roc_curve(y_train, y_train_pred_prob[:,_
      →1])
         y_test_pred_prob = model.predict_proba(X_test)
         optimal_threshold = round(thresholds[np.argmax(train_tpr - train_fpr)], 3)
```

```
train_auc_roc = roc_auc_score(y_train, y_train_pred_prob[:, 1])
  y_train_pred_class = np.where(y_train_pred_prob[:, 1] <= optimal_threshold,__
\rightarrow 0, 1)
  train_table = pd.crosstab(y_train_pred_class, y_train)
  train_sensitivity = round((train_table.iloc[1, 1] / (train_table.iloc[0, 1]
train_specificity = round((train_table.iloc[0, 0] / (train_table.iloc[0, 0]_u
test_auc_roc = roc_auc_score(y_test, y_test_pred_prob[:, 1])
  y_test_pred_class = np.where(y_test_pred_prob[:, 1] <= optimal_threshold, __</pre>
\hookrightarrow 0, 1)
  test_table = pd.crosstab(y_test_pred_class, y_test)
  test_sensitivity = round((test_table.iloc[1, 1] / (test_table.iloc[0, 1] +
→test_table.iloc[1, 1])) * 100, 2)
  test_specificity = round((test_table.iloc[0, 0] / (test_table.iloc[0, 0] +
→test_table.iloc[1, 0])) * 100, 2)
  print()
  print(f"{name}")
  print()
  print(f"\nTrain Data - Classification Report:\n")
  print(classification_report(y_train, y_train_pred_class))
  print(f"\nTrain Data - Confusion Matrix:\n\n{train_table}\n")
  print(f"\n Test Data - Classification Report:\n")
  print(classification_report(y_test, y_test_pred_class))
  print(f"\n Test Data - Confusion Matrix:\n\n{test_table}\n")
  print()
  print(f"Train Data - Sensitivity for cut-off {optimal_threshold}:

√{train_sensitivity}%")

  print(f" Test Data - Sensitivity for cut-off {optimal threshold}:

√{test_sensitivity}%\n")

  print(f"Train Data - Specificity for cut-off {optimal_threshold}:
print(f" Test Data - Specificity for cut-off {optimal_threshold}:
print(f"Train Data - AUC ROC: {train_auc_roc}")
  print(f" Test Data - AUC ROC: {test_auc_roc}\n")
```

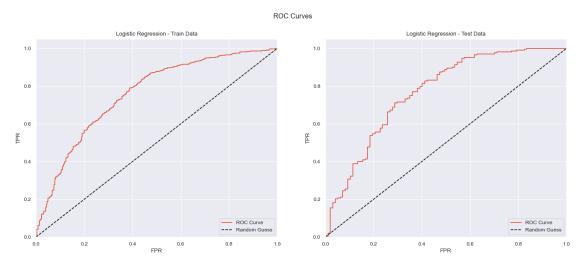
We prepare our data:

We once again look at a Logistic model, which will become our baseline:

```
[]: #/ label: model-metrics-plots-logistic

model_log = LogisticRegression(max_iter = 1000, random_state = 1337)

model_metrics_plots(X_train, X_test, y_train, y_test, model_log, name = U_test = U_tes
```



and the corresponding data:

```
[]: #/ label: model-metrics-data-logistic

model_metrics_data(X_train, X_test, y_train, y_test, model_log, name =

□

□

□

"Logistic Regression")
```

Logistic Regression

Train Data - Classification Report:

	precision 1		f1-score	support
0.0	0.58	0.61	0.59	391
1.0	0.81	0.79	0.80	829

accuracy			0.73	1220
macro avg	0.70	0.70	0.70	1220
weighted avg	0.74	0.73	0.73	1220

Train Data - Confusion Matrix:

col_0 0.0 1.0 row_0 0 239 174 1 152 655

Test Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.56	0.63	0.59	97
1.0	0.82	0.77	0.79	208
accuracy			0.72	305
macro avg	0.69	0.70	0.69	305
weighted avg	0.73	0.72	0.73	305

Test Data - Confusion Matrix:

col_0 0.0 1.0 row_0 0 61 48 1 36 160

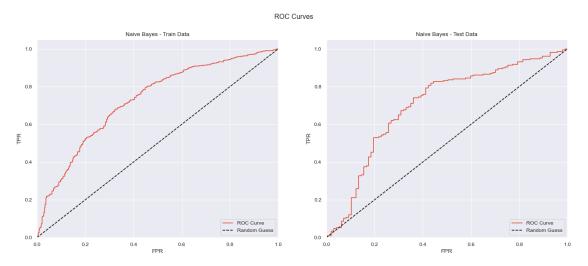
Train Data - Sensitivity for cut-off 0.654: 79.01% Test Data - Sensitivity for cut-off 0.654: 76.92%

Train Data - Specificity for cut-off 0.654: 61.13% Test Data - Specificity for cut-off 0.654: 62.89%

Train Data - AUC ROC: 0.7584554774340638 Test Data - AUC ROC: 0.767248215701824

We notice that the scikit-learn LogisticRegression model slightly outperforms the Statsmodels Logit model. We use the Statsmodels Logit model when we need to perform analysis of features. Moving to scikit-learn's LogisticRegression model after the model has been finalised is acceptable.

Next we look at Naive Bayes:



and the corresponding data:

```
[]: #/ label: model-metrics-data-naive-bayes

model_metrics_data(X_train, X_test, y_train, y_test, model_nb, name = "Naive_
→Bayes")
```

Naive Bayes

Train Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.50	0.68	0.57	391
1.0	0.82	0.68	0.74	829
accuracy			0.68	1220
macro avg	0.66	0.68	0.66	1220
weighted avg	0.71	0.68	0.69	1220

Train Data - Confusion Matrix:

```
col_0 0.0 1.0
row_0 0 264 266
1 127 563
```

Test Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.47	0.70	0.56	97
1.0	0.82	0.62	0.71	208
2.couracu			0.65	305
accuracy macro avg	0.64	0.66	0.63	305
weighted avg	0.71	0.65	0.66	305

Test Data - Confusion Matrix:

```
col_0 0.0 1.0
row_0
0 68 78
1 29 130
```

```
Train Data - Sensitivity for cut-off 0.916: 67.91%
Test Data - Sensitivity for cut-off 0.916: 62.5%

Train Data - Specificity for cut-off 0.916: 67.52%
Test Data - Specificity for cut-off 0.916: 70.1%

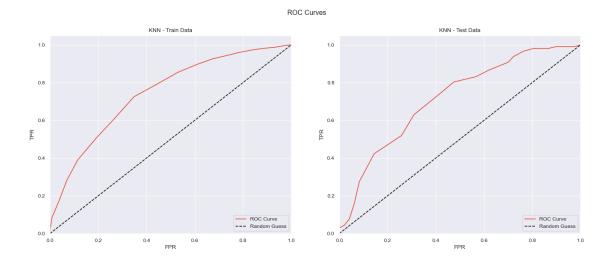
Train Data - AUC ROC: 0.7282400451658085
Test Data - AUC ROC: 0.7022204599524188
```

The Naive Bayes model clearly performs worse than the Logistic model. Next, we look at a KNN classifier:

```
[]: #/ label: model-metrics-plots-knn

scaler = StandardScaler()
scaler.fit(X_train)

model_knn = KNeighborsClassifier(leaf_size = 10, n_neighbors = 30)
model_metrics_plots(X_train, X_test, y_train, y_test, model_knn, name = "KNN")
```



and the corresponding data:

KNN

Train Data - Classification Report:

support	f1-score	recall	precision	
391	0.57	0.74	0.47	0.0
829	0.70	0.60	0.83	1.0
1220	0.65			accuracy
1220	0.64	0.67	0.65	macro avg
1220	0.66	0.65	0.71	weighted avg

Train Data - Confusion Matrix:

col_0 0.0 1.0 row_0 0 288 328 1 103 501

Test Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.47	0.69	0.56	97
0.0	0.47	0.09	0.50	91
1.0	0.81	0.63	0.71	208
accuracy			0.65	305
macro avg	0.64	0.66	0.63	305
weighted avg	0.70	0.65	0.66	305

Test Data - Confusion Matrix:

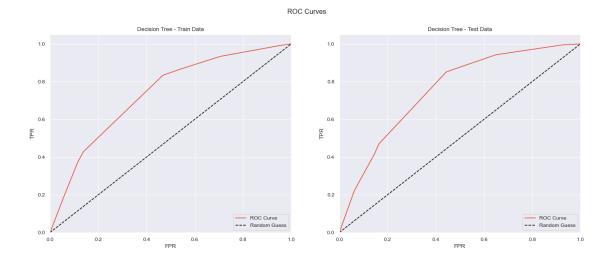
```
col_0 0.0 1.0
row_0
0 67 77
1 30 131
```

```
Train Data - Sensitivity for cut-off 0.7: 60.43%
Test Data - Sensitivity for cut-off 0.7: 62.98%

Train Data - Specificity for cut-off 0.7: 73.66%
Test Data - Specificity for cut-off 0.7: 69.07%

Train Data - AUC ROC: 0.7450939257540746
Test Data - AUC ROC: 0.7160735527359239
```

The KNN model also performs badly when compared to the Logistic model. Next, we look at a Decision Tree classifier:



and the data:

Decision Tree

Train Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.42	0.86	0.56	391
1.0	0.87	0.43	0.57	829
accuracy			0.57	1220
macro avg	0.64	0.65	0.57	1220
weighted avg	0.72	0.57	0.57	1220

Train Data - Confusion Matrix:

Test Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.42	0.84	0.56	97
1.0	0.86	0.47	0.61	208
accuracy			0.59	305
macro avg	0.64	0.65	0.59	305
weighted avg	0.72	0.59	0.59	305

Test Data - Confusion Matrix:

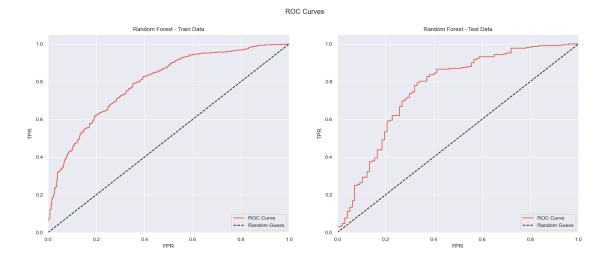
```
col_0 0.0 1.0
row_0
0 81 110
1 16 98
```

```
Train Data - Sensitivity for cut-off 0.723: 42.82%
Test Data - Sensitivity for cut-off 0.723: 47.12%

Train Data - Specificity for cut-off 0.723: 86.19%
Test Data - Specificity for cut-off 0.723: 83.51%

Train Data - AUC ROC: 0.7342235892626312
Test Data - AUC ROC: 0.7527260111022998
```

The Decision Tree classifier performs very badly - with an accuracy of around 55%. Next, we look at a Random Forest classifier:



and the data:

Random Forest

Train Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.59	0.65	0.62	391
1.0	0.83	0.79	0.81	829
accuracy			0.74	1220
macro avg	0.71	0.72	0.71	1220
weighted avg	0.75	0.74	0.75	1220

Train Data - Confusion Matrix:

col_0 0.0 1.0 row_0 0 253 175 1 138 654

Test Data - Classification Report:

support	f1-score	recall	precision	
97	0.63	0.67	0.60	0.0
208	0.81	0.79	0.84	1.0
305	0.75			accuracy
305	0.72	0.73	0.72	macro avg
305	0.76	0.75	0.76	weighted avg

Test Data - Confusion Matrix:

```
col_0 0.0 1.0
row_0
0 65 43
1 32 165
```

```
Train Data - Sensitivity for cut-off 0.638: 78.89%
Test Data - Sensitivity for cut-off 0.638: 79.33%

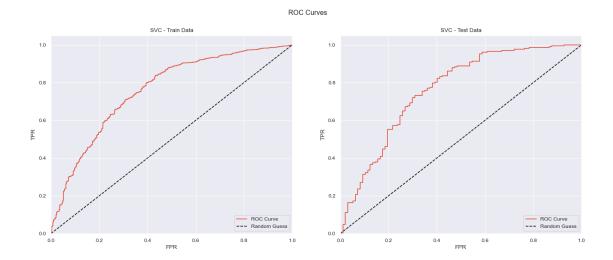
Train Data - Specificity for cut-off 0.638: 64.71%
Test Data - Specificity for cut-off 0.638: 67.01%

Train Data - AUC ROC: 0.7938461585924558
Test Data - AUC ROC: 0.7607057890563046
```

The Random Forest classifier performs reasonably well, but not as well as the baseline Logistic model. Next, we look at Support Vector Machine classifier:

```
[]: #/ label: model-metrics-plots-sum

model_svm = SVC(C = 1, kernel = "linear", probability = True, random_state = 1337)
model_metrics_plots(X_train, X_test, y_train, y_test, model_svm, name = "SVC")
```



and the data:

SVC

Train Data - Classification Report:

support	f1-score	recall	precision	
391	0.59	0.56	0.62	0.0
829	0.82	0.84	0.80	1.0
1220	0.75			accuracy
1220	0.70	0.70	0.71	macro avg
1220	0.75	0.75	0.74	weighted avg

Train Data - Confusion Matrix:

col_0 0.0 1.0 row_0 0 219 133 1 172 696

Test Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.61	0.59	0.60	97
1.0	0.81	0.82	0.82	208
accuracy			0.75	305
macro avg	0.71	0.70	0.71	305
weighted avg	0.75	0.75	0.75	305

Test Data - Confusion Matrix:

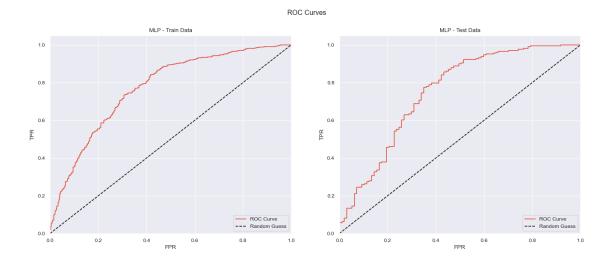
```
col_0 0.0 1.0
row_0
0 57 37
1 40 171
```

```
Train Data - Sensitivity for cut-off 0.642: 83.96%
Test Data - Sensitivity for cut-off 0.642: 82.21%

Train Data - Specificity for cut-off 0.642: 56.01%
Test Data - Specificity for cut-off 0.642: 58.76%

Train Data - AUC ROC: 0.759672547888406
Test Data - AUC ROC: 0.763332672482157
```

This is the best performing model yet with a consistent accuracy of 75% and consistent AUC of ~ 0.76 . Next, we move on to Multilayer Perceptron classifier:



and the data:

MLP

Train Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.55	0.69	0.61	391
1.0	0.84	0.73	0.78	829
accuracy			0.72	1220
macro avg	0.69	0.71	0.70	1220
weighted avg	0.74	0.72	0.73	1220

Train Data - Confusion Matrix:

```
col_0 0.0 1.0
row_0
0 271 221
1 120 608
```

Test Data - Classification Report:

	precision	recall	f1-score	support
0.0	0.52	0.66	0.58	97
1.0	0.82	0.71	0.76	208
accuracy			0.70	305
macro avg	0.67	0.69	0.67	305
weighted avg	0.72	0.70	0.70	305

Test Data - Confusion Matrix:

```
col_0 0.0 1.0
row_0
0 64 60
1 33 148
```

```
Train Data - Sensitivity for cut-off 0.712: 73.34%
Test Data - Sensitivity for cut-off 0.712: 71.15%

Train Data - Specificity for cut-off 0.712: 69.31%
Test Data - Specificity for cut-off 0.712: 65.98%

Train Data - AUC ROC: 0.7711321377557159
Test Data - AUC ROC: 0.7492069785884219
```

The Multilayer Perceptron model does reasonably well, but not as well as the SVM classifier. Overall, the SVM classifier performs the best.

Lets see if we can improve on the SVM classifier with a deep learning model implemented using PyTorch. First, lets check to see if cuda is available:

```
[]: device = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")
print(device)
```

cuda:0

Next we create a deep learning model:

```
[]: class DL(nn.Module):
    def __init__(self, input_dim, hidden_dim, output_dim):
        super(DL, self).__init__()
    self.input = nn.Linear(input_dim, hidden_dim)
```

```
self.act_in = nn.ReLU()
    self.hidden1 = nn.Linear(hidden_dim, hidden_dim)
    self.act1 = nn.ReLU()
    self.hidden2 = nn.Linear(hidden_dim, hidden_dim)
    self.act2 = nn.ReLU()
    self.hidden3 = nn.Linear(hidden_dim, hidden_dim)
    self.act3 = nn.ReLU()
    self.hidden4 = nn.Linear(hidden dim, hidden dim)
    self.act4 = nn.ReLU()
    self.hidden5 = nn.Linear(hidden dim, hidden dim)
    self.act5 = nn.ReLU()
    self.hidden6 = nn.Linear(hidden_dim, hidden_dim)
    self.act6 = nn.ReLU()
    self.hidden7 = nn.Linear(hidden_dim, hidden_dim)
    self.act7 = nn.ReLU()
    self.output = nn.Linear(hidden_dim, output_dim)
    self.act_out = nn.Sigmoid()
def forward(self, x):
    x = self.act_in(self.input(x))
    x = self.act1(self.hidden1(x))
    x = self.act2(self.hidden2(x))
    x = self.act3(self.hidden3(x))
    x = self.act4(self.hidden4(x))
    x = self.act5(self.hidden5(x))
    x = self.act6(self.hidden6(x))
    x = self.act7(self.hidden7(x))
    return self.act_out(self.output(x))
```

Next we scale the data, and create a train test split:

Next we convert the data into tensors, and move them onto the cuda device if available:

```
[]: X_train = torch.from_numpy(X_train).type(torch.Tensor)
X_test = torch.from_numpy(X_test).type(torch.Tensor)
y_train = torch.from_numpy(y_train).type(torch.Tensor)
```

```
if torch.cuda.is_available():
    X_train = X_train.cuda()
    X_test = X_test.cuda()
    y_train = y_train.cuda()
```

Next define our model parameters, create an instance of our model, and define our loss criterion and optimiser:

```
[]: input_dim = X.shape[1]
hidden_dim = 64
output_dim = 1

model = DL(input_dim, hidden_dim, output_dim)
model = model.to(device)

criterion = nn.MSELoss()
optimiser = torch.optim.Adam(model.parameters(), lr = 0.0001)
```

Next we create functions to train our model, with and without batches, and make use of it:

```
[]: def train(X, y, model, criterion, optimiser, epochs = 500):
         losses = []
         for epoch in range(epochs):
             out = model(X)
             loss = criterion(out, y)
             losses.append(loss.item())
             optimiser.zero_grad()
             loss.backward()
             optimiser.step()
             if epoch \% 10 == 9:
                 print(f"Epoch {epoch + 1:>3} - MSE: {loss.item()}")
         return losses
     def train_batched(X, y, model, criterion, optimiser, epochs = 500, batch_size = __
      \Rightarrow20, shuffle = False):
         losses = []
                  = utils.TensorDataset(X, y)
         dataloader = utils.DataLoader(dataset, batch_size = batch_size, shuffle = u
      ⇔shuffle)
         for epoch in range(epochs):
```

```
for batch, (X_batch, y_batch) in enumerate(dataloader):
             out = model(X_batch)
            loss = criterion(out, y_batch)
            losses.append(loss.item())
            optimiser.zero_grad()
            loss.backward()
            optimiser.step()
             if batch % 10 == 9:
                print(f"Epoch {epoch + 1:>3} - Batch {batch + 1:>3} - MSE:__
 →{loss.item()}")
    return losses
# losses = train(X_train, y_train[:, None], model, criterion, optimiser)
losses = train_batched(X_train, y_train[:, None], model, criterion, optimiser, __
  ⇒batch_size = 100, shuffle = True)
       1 - Batch 10 - MSE: 0.24581755697727203
Epoch
Epoch
        2 - Batch 10 - MSE: 0.2428935021162033
```

```
Epoch
      3 - Batch 10 - MSE: 0.24090971052646637
Epoch 4 - Batch 10 - MSE: 0.2452872395515442
Epoch 5 - Batch 10 - MSE: 0.24358420073986053
Epoch 6 - Batch 10 - MSE: 0.2402580976486206
Epoch 7 - Batch 10 - MSE: 0.23894134163856506
Epoch 8 - Batch 10 - MSE: 0.23425158858299255
Epoch 9 - Batch 10 - MSE: 0.22734546661376953
Epoch 10 - Batch 10 - MSE: 0.24289806187152863
Epoch 11 - Batch 10 - MSE: 0.22648021578788757
Epoch 12 - Batch 10 - MSE: 0.2319982498884201
Epoch 13 - Batch 10 - MSE: 0.221490740776062
Epoch 14 - Batch 10 - MSE: 0.2225533127784729
Epoch 15 - Batch 10 - MSE: 0.2453363686800003
Epoch 16 - Batch 10 - MSE: 0.22490368783473969
Epoch 17 - Batch 10 - MSE: 0.22733283042907715
Epoch 18 - Batch 10 - MSE: 0.21217261254787445
Epoch 19 - Batch 10 - MSE: 0.20337630808353424
Epoch 20 - Batch 10 - MSE: 0.2104637324810028
Epoch 21 - Batch 10 - MSE: 0.22966529428958893
Epoch 22 - Batch 10 - MSE: 0.21050739288330078
Epoch 23 - Batch 10 - MSE: 0.2088465839624405
Epoch 24 - Batch 10 - MSE: 0.22618086636066437
Epoch 25 - Batch 10 - MSE: 0.2149863839149475
Epoch 26 - Batch 10 - MSE: 0.22453854978084564
Epoch 27 - Batch 10 - MSE: 0.1945880502462387
Epoch 28 - Batch 10 - MSE: 0.20430682599544525
```

```
Epoch 29 - Batch 10 - MSE: 0.2090795785188675
Epoch
      30 - Batch
                 10 - MSE: 0.23998676240444183
Epoch 31 - Batch
                  10 - MSE: 0.23788504302501678
Epoch
      32 - Batch
                  10 - MSE: 0.21982870995998383
Epoch
     33 - Batch
                  10 - MSE: 0.22588509321212769
Epoch 34 - Batch
                  10 - MSE: 0.23842665553092957
Epoch
                  10 - MSE: 0.19864657521247864
      35 - Batch
Epoch 36 - Batch 10 - MSE: 0.2045675367116928
Epoch 37 - Batch 10 - MSE: 0.20656293630599976
Epoch 38 - Batch 10 - MSE: 0.21950800716876984
      39 - Batch 10 - MSE: 0.2241199016571045
Epoch
Epoch 40 - Batch 10 - MSE: 0.18328139185905457
Epoch 41 - Batch
                  10 - MSE: 0.19495323300361633
Epoch
      42 - Batch
                  10 - MSE: 0.2197721302509308
                  10 - MSE: 0.2022537887096405
Epoch 43 - Batch
Epoch 44 - Batch
                  10 - MSE: 0.21757613122463226
Epoch 45 - Batch
                  10 - MSE: 0.21592575311660767
Epoch
      46 - Batch
                  10 - MSE: 0.18501685559749603
Epoch 47 - Batch 10 - MSE: 0.2049361616373062
                 10 - MSE: 0.17691770195960999
Epoch 48 - Batch
Epoch
      49 - Batch
                  10 - MSE: 0.19991381466388702
Epoch 50 - Batch 10 - MSE: 0.19326888024806976
Epoch 51 - Batch 10 - MSE: 0.23071834444999695
Epoch 52 - Batch 10 - MSE: 0.2128244787454605
Epoch 53 - Batch 10 - MSE: 0.19277410209178925
Epoch 54 - Batch 10 - MSE: 0.20953021943569183
Epoch 55 - Batch 10 - MSE: 0.19199645519256592
Epoch 56 - Batch
                  10 - MSE: 0.1940612494945526
Epoch 57 - Batch
                  10 - MSE: 0.1877998262643814
Epoch 58 - Batch
                  10 - MSE: 0.19676573574543
Epoch 59 - Batch
                  10 - MSE: 0.21654623746871948
                  10 - MSE: 0.16983123123645782
Epoch 60 - Batch
Epoch 61 - Batch 10 - MSE: 0.17954587936401367
Epoch 62 - Batch
                  10 - MSE: 0.22203460335731506
                  10 - MSE: 0.17913733422756195
Epoch 63 - Batch
Epoch 64 - Batch 10 - MSE: 0.15494060516357422
Epoch 65 - Batch 10 - MSE: 0.1639593094587326
Epoch 66 - Batch 10 - MSE: 0.18090836703777313
Epoch 67 - Batch 10 - MSE: 0.17307689785957336
Epoch 68 - Batch 10 - MSE: 0.17274436354637146
Epoch 69 - Batch 10 - MSE: 0.13173899054527283
Epoch
     70 - Batch 10 - MSE: 0.1719854772090912
Epoch
      71 - Batch
                  10 - MSE: 0.17228080332279205
Epoch
     72 - Batch
                 10 - MSE: 0.197896346449852
                  10 - MSE: 0.15217575430870056
Epoch 73 - Batch
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     74 - Batch
                  10 - MSE: 0.16725614666938782
     75 - Batch 10 - MSE: 0.20702368021011353
Epoch
Epoch 76 - Batch 10 - MSE: 0.1669454276561737
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Epoch 77 - Batch 10 - MSE: 0.13878539204597473
Epoch
      78 - Batch
                  10 - MSE: 0.20258475840091705
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     79 - Batch
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Epoch 81 - Batch
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      83 - Batch
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Epoch 85 - Batch 10 - MSE: 0.18528254330158234
Epoch 86 - Batch 10 - MSE: 0.18323619663715363
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      87 - Batch
                  10 - MSE: 0.1791437268257141
Epoch 88 - Batch 10 - MSE: 0.14909659326076508
Epoch 89 - Batch
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      90 - Batch
                  10 - MSE: 0.1458713710308075
                  10 - MSE: 0.1595243215560913
Epoch 91 - Batch
Epoch 92 - Batch
                  10 - MSE: 0.16912803053855896
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Epoch 123 - Batch 10 - MSE: 0.16471973061561584
Epoch 124 - Batch 10 - MSE: 0.16528114676475525
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Epoch 171 - Batch 10 - MSE: 0.16080260276794434
Epoch 172 - Batch 10 - MSE: 0.1632588505744934
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Epoch 194 - Batch 10 - MSE: 0.12598194181919098
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Epoch 204 - Batch
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Epoch 205 - Batch 10 - MSE: 0.1506449282169342
Epoch 206 - Batch
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Epoch 207 - Batch
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Epoch 208 - Batch
                  10 - MSE: 0.17314788699150085
Epoch 209 - Batch
                  10 - MSE: 0.19067241251468658
Epoch 210 - Batch 10 - MSE: 0.15235209465026855
Epoch 211 - Batch 10 - MSE: 0.13596954941749573
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Epoch 214 - Batch 10 - MSE: 0.14556676149368286
Epoch 215 - Batch 10 - MSE: 0.16942687332630157
Epoch 216 - Batch 10 - MSE: 0.14981865882873535
Epoch 217 - Batch 10 - MSE: 0.14419466257095337
Epoch 218 - Batch
                  10 - MSE: 0.16640594601631165
Epoch 219 - Batch 10 - MSE: 0.15924663841724396
Epoch 220 - Batch 10 - MSE: 0.16166435182094574
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Epoch 221 - Batch 10 - MSE: 0.1643018275499344
Epoch 222 - Batch
                  10 - MSE: 0.18999503552913666
Epoch 223 - Batch
                  10 - MSE: 0.16552573442459106
Epoch 224 - Batch
                  10 - MSE: 0.1627187281847
Epoch 225 - Batch
                  10 - MSE: 0.15122252702713013
Epoch 226 - Batch
                  10 - MSE: 0.15112082660198212
Epoch 227 - Batch
                  10 - MSE: 0.18435458838939667
Epoch 228 - Batch 10 - MSE: 0.19059886038303375
Epoch 229 - Batch 10 - MSE: 0.15306374430656433
Epoch 230 - Batch 10 - MSE: 0.15099579095840454
Epoch 231 - Batch 10 - MSE: 0.18324565887451172
Epoch 232 - Batch 10 - MSE: 0.15665270388126373
Epoch 233 - Batch 10 - MSE: 0.15289010107517242
Epoch 234 - Batch 10 - MSE: 0.17920516431331635
Epoch 235 - Batch 10 - MSE: 0.17003324627876282
Epoch 236 - Batch 10 - MSE: 0.18504157662391663
Epoch 237 - Batch 10 - MSE: 0.2127557098865509
Epoch 238 - Batch
                 10 - MSE: 0.15622863173484802
Epoch 239 - Batch 10 - MSE: 0.1524023413658142
Epoch 240 - Batch 10 - MSE: 0.15379540622234344
Epoch 241 - Batch
                  10 - MSE: 0.16103637218475342
Epoch 242 - Batch 10 - MSE: 0.15858229994773865
Epoch 243 - Batch 10 - MSE: 0.13737520575523376
Epoch 244 - Batch 10 - MSE: 0.18123428523540497
Epoch 245 - Batch 10 - MSE: 0.16798816621303558
Epoch 246 - Batch 10 - MSE: 0.20221315324306488
Epoch 247 - Batch 10 - MSE: 0.15459442138671875
Epoch 248 - Batch 10 - MSE: 0.14411002397537231
Epoch 249 - Batch 10 - MSE: 0.17188438773155212
Epoch 250 - Batch 10 - MSE: 0.19021667540073395
Epoch 251 - Batch 10 - MSE: 0.1403995305299759
Epoch 252 - Batch
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Epoch 253 - Batch 10 - MSE: 0.16594621539115906
Epoch 254 - Batch 10 - MSE: 0.1805490106344223
Epoch 255 - Batch
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Epoch 256 - Batch 10 - MSE: 0.17116805911064148
Epoch 257 - Batch 10 - MSE: 0.16077306866645813
Epoch 258 - Batch 10 - MSE: 0.20374706387519836
Epoch 259 - Batch 10 - MSE: 0.17594262957572937
Epoch 260 - Batch 10 - MSE: 0.21873033046722412
Epoch 261 - Batch 10 - MSE: 0.166362926363945
Epoch 262 - Batch 10 - MSE: 0.2162914276123047
Epoch 263 - Batch 10 - MSE: 0.14846499264240265
Epoch 264 - Batch 10 - MSE: 0.1909022480249405
Epoch 265 - Batch 10 - MSE: 0.16099385917186737
Epoch 266 - Batch
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Epoch 267 - Batch 10 - MSE: 0.1613456755876541
Epoch 268 - Batch 10 - MSE: 0.15146027505397797
```

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Epoch 270 - Batch
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Epoch 271 - Batch
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Epoch 272 - Batch
                  10 - MSE: 0.16022613644599915
Epoch 273 - Batch
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Epoch 274 - Batch
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Epoch 275 - Batch
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Epoch 276 - Batch 10 - MSE: 0.13526256382465363
Epoch 277 - Batch 10 - MSE: 0.17133808135986328
Epoch 278 - Batch 10 - MSE: 0.1700066477060318
Epoch 279 - Batch 10 - MSE: 0.22967484593391418
Epoch 280 - Batch 10 - MSE: 0.15682096779346466
Epoch 281 - Batch 10 - MSE: 0.16341932117938995
Epoch 282 - Batch 10 - MSE: 0.15198618173599243
Epoch 283 - Batch 10 - MSE: 0.1858779937028885
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Epoch 287 - Batch 10 - MSE: 0.1974947452545166
Epoch 288 - Batch 10 - MSE: 0.12630191445350647
Epoch 289 - Batch
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Epoch 290 - Batch 10 - MSE: 0.19950221478939056
Epoch 291 - Batch 10 - MSE: 0.1946037858724594
Epoch 292 - Batch 10 - MSE: 0.1881369799375534
Epoch 293 - Batch 10 - MSE: 0.17554813623428345
Epoch 294 - Batch 10 - MSE: 0.15931373834609985
Epoch 295 - Batch 10 - MSE: 0.1587013304233551
Epoch 296 - Batch 10 - MSE: 0.16550859808921814
Epoch 297 - Batch 10 - MSE: 0.15632322430610657
Epoch 298 - Batch 10 - MSE: 0.1313316822052002
Epoch 299 - Batch 10 - MSE: 0.15559357404708862
Epoch 300 - Batch
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Epoch 301 - Batch 10 - MSE: 0.1734093576669693
Epoch 302 - Batch
                  10 - MSE: 0.18473312258720398
Epoch 303 - Batch
                  10 - MSE: 0.1618439257144928
Epoch 304 - Batch 10 - MSE: 0.1380319744348526
Epoch 305 - Batch 10 - MSE: 0.15244251489639282
Epoch 306 - Batch 10 - MSE: 0.17342250049114227
Epoch 307 - Batch 10 - MSE: 0.15935753285884857
Epoch 308 - Batch 10 - MSE: 0.15146155655384064
Epoch 309 - Batch 10 - MSE: 0.1372508853673935
Epoch 310 - Batch 10 - MSE: 0.20402874052524567
Epoch 311 - Batch 10 - MSE: 0.14825542271137238
Epoch 312 - Batch 10 - MSE: 0.1762375384569168
Epoch 313 - Batch 10 - MSE: 0.1544989049434662
Epoch 314 - Batch
                  10 - MSE: 0.13760623335838318
Epoch 315 - Batch 10 - MSE: 0.12614087760448456
Epoch 316 - Batch 10 - MSE: 0.15417271852493286
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Epoch 318 - Batch
                  10 - MSE: 0.2364005446434021
Epoch 319 - Batch
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Epoch 320 - Batch
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Epoch 321 - Batch
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Epoch 322 - Batch
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Epoch 323 - Batch
                  10 - MSE: 0.14749981462955475
Epoch 324 - Batch 10 - MSE: 0.17428840696811676
Epoch 325 - Batch 10 - MSE: 0.2226262092590332
Epoch 326 - Batch 10 - MSE: 0.1523682326078415
Epoch 327 - Batch 10 - MSE: 0.19328582286834717
Epoch 328 - Batch 10 - MSE: 0.13918346166610718
Epoch 329 - Batch 10 - MSE: 0.1825328916311264
Epoch 330 - Batch 10 - MSE: 0.19007842242717743
Epoch 331 - Batch 10 - MSE: 0.1857742965221405
Epoch 332 - Batch 10 - MSE: 0.1521233171224594
Epoch 333 - Batch
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Epoch 334 - Batch
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Epoch 335 - Batch 10 - MSE: 0.15520402789115906
Epoch 336 - Batch 10 - MSE: 0.17119815945625305
Epoch 337 - Batch
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Epoch 338 - Batch 10 - MSE: 0.16575591266155243
Epoch 339 - Batch 10 - MSE: 0.20302550494670868
Epoch 340 - Batch 10 - MSE: 0.2177649587392807
Epoch 341 - Batch 10 - MSE: 0.1436559110879898
Epoch 342 - Batch 10 - MSE: 0.140285924077034
Epoch 343 - Batch 10 - MSE: 0.17153039574623108
Epoch 344 - Batch 10 - MSE: 0.19042198359966278
Epoch 345 - Batch 10 - MSE: 0.182703897356987
Epoch 346 - Batch 10 - MSE: 0.14573052525520325
Epoch 347 - Batch 10 - MSE: 0.19810999929904938
Epoch 348 - Batch
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Epoch 349 - Batch 10 - MSE: 0.1996491253376007
Epoch 350 - Batch
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Epoch 351 - Batch
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Epoch 352 - Batch
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Epoch 353 - Batch 10 - MSE: 0.17313043773174286
Epoch 354 - Batch 10 - MSE: 0.2001902014017105
Epoch 355 - Batch 10 - MSE: 0.1643482893705368
Epoch 356 - Batch 10 - MSE: 0.15921786427497864
Epoch 357 - Batch 10 - MSE: 0.16756701469421387
Epoch 358 - Batch 10 - MSE: 0.19492404162883759
Epoch 359 - Batch 10 - MSE: 0.19567203521728516
Epoch 360 - Batch 10 - MSE: 0.14397959411144257
Epoch 361 - Batch 10 - MSE: 0.18612071871757507
Epoch 362 - Batch
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Epoch 363 - Batch 10 - MSE: 0.17043355107307434
Epoch 364 - Batch 10 - MSE: 0.18462105095386505
```

```
Epoch 365 - Batch 10 - MSE: 0.14405560493469238
Epoch 366 - Batch
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Epoch 367 - Batch
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Epoch 368 - Batch
                  10 - MSE: 0.16479556262493134
Epoch 369 - Batch
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Epoch 370 - Batch
                  10 - MSE: 0.17983844876289368
Epoch 371 - Batch
                  10 - MSE: 0.19866906106472015
Epoch 372 - Batch 10 - MSE: 0.11989164352416992
Epoch 373 - Batch 10 - MSE: 0.16393031179904938
Epoch 374 - Batch 10 - MSE: 0.18946319818496704
Epoch 375 - Batch 10 - MSE: 0.17762227356433868
Epoch 376 - Batch 10 - MSE: 0.17559918761253357
Epoch 377 - Batch
                  10 - MSE: 0.18278899788856506
Epoch 378 - Batch
                  10 - MSE: 0.17486608028411865
Epoch 379 - Batch 10 - MSE: 0.15818098187446594
Epoch 380 - Batch 10 - MSE: 0.13261620700359344
Epoch 381 - Batch
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Epoch 382 - Batch
                  10 - MSE: 0.18351227045059204
Epoch 383 - Batch 10 - MSE: 0.17257969081401825
Epoch 384 - Batch
                  10 - MSE: 0.16452795267105103
Epoch 385 - Batch
                  10 - MSE: 0.1973210871219635
Epoch 386 - Batch 10 - MSE: 0.13632801175117493
Epoch 387 - Batch 10 - MSE: 0.1781628131866455
Epoch 388 - Batch 10 - MSE: 0.192433163523674
Epoch 389 - Batch 10 - MSE: 0.16165708005428314
Epoch 390 - Batch 10 - MSE: 0.21402208507061005
Epoch 391 - Batch 10 - MSE: 0.15525858104228973
Epoch 392 - Batch
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Epoch 393 - Batch 10 - MSE: 0.16705450415611267
Epoch 394 - Batch 10 - MSE: 0.14892394840717316
Epoch 395 - Batch 10 - MSE: 0.15407489240169525
Epoch 396 - Batch
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Epoch 397 - Batch 10 - MSE: 0.1635637879371643
Epoch 398 - Batch
                  10 - MSE: 0.13302522897720337
Epoch 399 - Batch
                  10 - MSE: 0.1494939923286438
Epoch 400 - Batch
                  10 - MSE: 0.1897820681333542
Epoch 401 - Batch
                  10 - MSE: 0.12411243468523026
Epoch 402 - Batch 10 - MSE: 0.14890716969966888
Epoch 403 - Batch 10 - MSE: 0.14228606224060059
Epoch 404 - Batch 10 - MSE: 0.16451892256736755
Epoch 405 - Batch 10 - MSE: 0.13171614706516266
Epoch 406 - Batch 10 - MSE: 0.16266539692878723
Epoch 407 - Batch
                  10 - MSE: 0.1650329977273941
Epoch 408 - Batch 10 - MSE: 0.1385434865951538
Epoch 409 - Batch 10 - MSE: 0.1806800812482834
Epoch 410 - Batch
                  10 - MSE: 0.12339024990797043
Epoch 411 - Batch 10 - MSE: 0.16778400540351868
Epoch 412 - Batch 10 - MSE: 0.16393402218818665
```

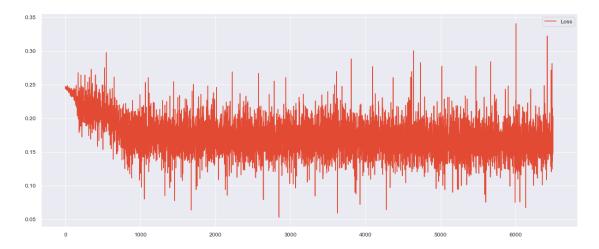
```
Epoch 413 - Batch 10 - MSE: 0.20435860753059387
Epoch 414 - Batch
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Epoch 415 - Batch
                  10 - MSE: 0.15043427050113678
Epoch 416 - Batch
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Epoch 417 - Batch
                  10 - MSE: 0.16187235713005066
Epoch 418 - Batch
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Epoch 419 - Batch
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Epoch 420 - Batch 10 - MSE: 0.18533802032470703
Epoch 421 - Batch 10 - MSE: 0.13929328322410583
Epoch 422 - Batch 10 - MSE: 0.18716473877429962
Epoch 423 - Batch
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Epoch 424 - Batch 10 - MSE: 0.2104954719543457
Epoch 425 - Batch
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Epoch 428 - Batch
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Epoch 429 - Batch
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Epoch 430 - Batch
                  10 - MSE: 0.15354381501674652
Epoch 431 - Batch
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Epoch 432 - Batch
                  10 - MSE: 0.18092654645442963
Epoch 433 - Batch
                  10 - MSE: 0.1344960480928421
Epoch 434 - Batch 10 - MSE: 0.19490645825862885
Epoch 435 - Batch 10 - MSE: 0.1887393742799759
Epoch 436 - Batch 10 - MSE: 0.11276905983686447
Epoch 437 - Batch 10 - MSE: 0.15347358584403992
Epoch 438 - Batch 10 - MSE: 0.17881205677986145
Epoch 439 - Batch 10 - MSE: 0.1661592572927475
Epoch 440 - Batch
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Epoch 441 - Batch
                  10 - MSE: 0.18148116767406464
Epoch 442 - Batch 10 - MSE: 0.18597754836082458
Epoch 443 - Batch
                  10 - MSE: 0.14472268521785736
Epoch 444 - Batch
                  10 - MSE: 0.12602879106998444
Epoch 445 - Batch 10 - MSE: 0.15830697119235992
Epoch 446 - Batch
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Epoch 447 - Batch
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Epoch 448 - Batch
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Epoch 449 - Batch
                  10 - MSE: 0.141123428940773
Epoch 450 - Batch 10 - MSE: 0.1559642255306244
Epoch 451 - Batch 10 - MSE: 0.18704497814178467
Epoch 452 - Batch 10 - MSE: 0.13671107590198517
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Epoch 455 - Batch
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Epoch 456 - Batch
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Epoch 457 - Batch
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Epoch 458 - Batch
                  10 - MSE: 0.1562102735042572
Epoch 459 - Batch 10 - MSE: 0.1761079877614975
Epoch 460 - Batch 10 - MSE: 0.17323213815689087
```

```
Epoch 461 - Batch 10 - MSE: 0.1777808666229248
Epoch 462 - Batch 10 - MSE: 0.22641809284687042
Epoch 463 - Batch 10 - MSE: 0.17013618350028992
Epoch 464 - Batch 10 - MSE: 0.1422959566116333
Epoch 465 - Batch 10 - MSE: 0.16917571425437927
Epoch 466 - Batch 10 - MSE: 0.12414183467626572
Epoch 467 - Batch 10 - MSE: 0.1702871322631836
Epoch 468 - Batch 10 - MSE: 0.17252278327941895
Epoch 469 - Batch 10 - MSE: 0.1521681696176529
Epoch 470 - Batch 10 - MSE: 0.17496119439601898
Epoch 471 - Batch 10 - MSE: 0.13557679951190948
Epoch 472 - Batch 10 - MSE: 0.16271933913230896
Epoch 473 - Batch 10 - MSE: 0.16245132684707642
Epoch 474 - Batch 10 - MSE: 0.17371013760566711
Epoch 475 - Batch 10 - MSE: 0.16650693118572235
Epoch 476 - Batch 10 - MSE: 0.13814926147460938
Epoch 477 - Batch 10 - MSE: 0.13412192463874817
Epoch 478 - Batch 10 - MSE: 0.1884339451789856
Epoch 479 - Batch 10 - MSE: 0.12401027232408524
Epoch 480 - Batch 10 - MSE: 0.1507880538702011
Epoch 481 - Batch 10 - MSE: 0.180315762758255
Epoch 482 - Batch 10 - MSE: 0.17684558033943176
Epoch 483 - Batch 10 - MSE: 0.16967007517814636
Epoch 484 - Batch 10 - MSE: 0.20554262399673462
Epoch 485 - Batch 10 - MSE: 0.18705759942531586
Epoch 486 - Batch 10 - MSE: 0.14628174901008606
Epoch 487 - Batch 10 - MSE: 0.20378321409225464
Epoch 488 - Batch 10 - MSE: 0.18998713791370392
Epoch 489 - Batch 10 - MSE: 0.14322997629642487
Epoch 490 - Batch 10 - MSE: 0.1907835751771927
Epoch 491 - Batch 10 - MSE: 0.1915583312511444
Epoch 492 - Batch 10 - MSE: 0.19672508537769318
Epoch 493 - Batch 10 - MSE: 0.1801026165485382
Epoch 494 - Batch 10 - MSE: 0.16596324741840363
Epoch 495 - Batch 10 - MSE: 0.13894709944725037
Epoch 496 - Batch 10 - MSE: 0.1928841769695282
Epoch 497 - Batch 10 - MSE: 0.16051974892616272
Epoch 498 - Batch 10 - MSE: 0.16116826236248016
Epoch 499 - Batch 10 - MSE: 0.1638757735490799
Epoch 500 - Batch 10 - MSE: 0.14822082221508026
Lets plot the losses:
```

```
[]: #/ label: model-metrics-plots-dl-loss

plt.figure(figsize = (15, 6))
plt.plot(losses, label = "Loss")
plt.legend()
```

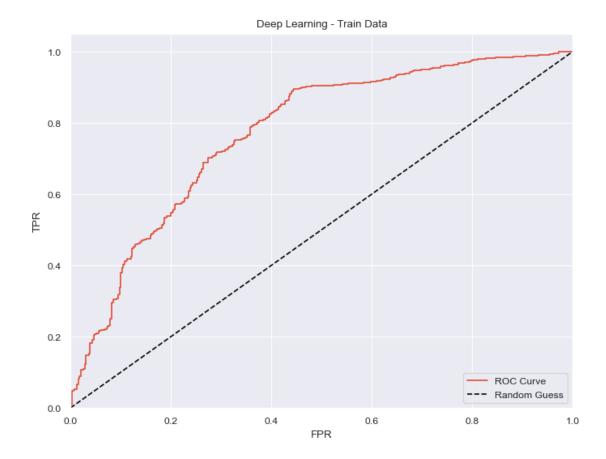
plt.show()



Next lets plot the ROC curve for the training data:

```
[]: y_train_pred_prob = model(X_train).detach().cpu().numpy()

if torch.cuda.is_available():
    y_train = y_train.cpu()
```



Lets calculate the optimal threshold:

```
[]: optimal_threshold = round(train_thresholds[np.argmax(train_tpr - train_fpr)], 3)
    print(f'Deep Learning - Optimal Threshold: {optimal_threshold}')
```

Deep Learning - Optimal Threshold: 0.36000001430511475

With the optimal threshold we calculate the AUC for the train data:

```
[ ]: train_auc_roc = roc_auc_score(y_train, y_train_pred_prob)
print(f'Deep Learning - AUC ROC: {train_auc_roc}')
```

Deep Learning - AUC ROC: 0.7720391560410812

Lets look at the classification report for the train data:

```
[]: y_train_pred_class = np.where(y_train_pred_prob <= optimal_threshold, 0, 1)
print(classification_report(y_train, y_train_pred_class))</pre>
```

	precision	recall	11-score	support
0.0	0.71	0.56	0.62	391
1.0	0.81	0.89	0.85	829

```
accuracy 0.79 1220 macro avg 0.76 0.73 0.74 1220 weighted avg 0.78 0.79 0.78 1220
```

The accuracy seems good. Lets look at the confusion matrix for the train data:

```
[]: table = pd.crosstab(y_train_pred_class[:, 0], y_train)
print(table)
```

```
col_0 0.0 1.0
row_0
0 218 89
1 173 740
```

And lets look at the sensitivity and specificity for the train data:

```
Deep Learning - Sensitivity for cut-off 0.36000001430511475 is : 89.26% Deep Learning - Specificity for cut-off 0.36000001430511475 is : 55.75%
```

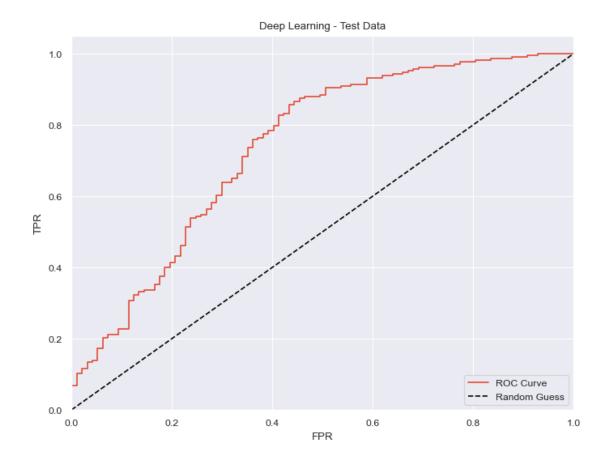
Now lets compare with the test data - we begin with the ROC curve:

```
[]: #/ label: model-metrics-plots-dl-roc-test

y_test_pred_prob = model(X_test).detach().cpu().numpy()

test_fpr, test_tpr, test_thresholds = roc_curve(y_test, y_test_pred_prob)

performance_analytics_roc_curve(test_fpr, test_tpr, "Deep Learning - Test Data")
```



Lets calculate the AUC for the test data:

```
[ ]: test_auc_roc = roc_auc_score(y_test, y_test_pred_prob)
print(f'Deep Learning - AUC ROC: {test_auc_roc}')
```

Deep Learning - AUC ROC: 0.7385507533703409

Lets look at the classification report for the test data:

```
[]: y_test_pred_class = np.where(y_test_pred_prob <= optimal_threshold, 0, 1)
print(classification_report(y_test, y_test_pred_class))</pre>
```

support	f1-score	recall	precision	
97	0.60	0.55	0.66	0.0
208	0.84	0.87	0.80	1.0
305	0.77			accuracy
305	0.72	0.71	0.73	macro avg
305	0.76	0.77	0.76	weighted avg

The accuracy seems good. Lets look at the confusion matrix for the test data:

```
[]: table = pd.crosstab(y_test_pred_class[:, 0], y_test)
print(table)
```

```
col_0 0.0 1.0
row_0
0 53 27
1 44 181
```

And lets look at the sensitivity and specificity for the test data:

```
Deep Learning - Sensitivity for cut-off 0.36000001430511475 is : 87.02% Deep Learning - Specificity for cut-off 0.36000001430511475 is : 54.64%
```

The results of the PyTorch deep learning model shows potential. The train and test AUC values are slightly inconsistent, but the accuracy values are reasonably good for little effort. It might be worth the effort to investigate this approach further.

Now we look to see if we can engineer a feature to improve the outcome.

1.5 Phase 5 - Sentiment Analysis

We now turn to Twitter / X data relating to the Nifty 50 index to see if we can mine some sentiment. First we load the tweets and create a data frame:

[]: Tweets

- 0 #bankNifty 50100 ce looks good at 70+-2 for a ...
- 1 "#market #banknifty #OptionsTrading #optionbuy...
- 2 PENNY STOCK MADHUCON PROJECTS LTD cmp-11 FOLLO...
- 3 #Nifty50 has been in a healthy uptrend since t...
- 4 #Gravita #livetrading #stockstowatch #stocksin...

Next, we do some basic pre-processing of the data to:

- 1. transform all words to lowercase
- 2. remove all punctuation
- 3. remove all digits
- 4. remove stopwords

[]: 0 banknifty ce looks good target nifty nifty
1 market banknifty optionstrading optionbuying t...
2 penny stock madhucon projects ltd cmp followht...
3 nifty healthy uptrend since beginning year did...
4 gravita livetrading stockstowatch stocksinfocu...
Name: Tweets, dtype: object

Next we look at the top 20 words by frequency:

```
[]: tweet_words = cleaned.str.cat(sep = " ")

freq_dist = nltk.FreqDist(tweet_words.split())
word_freq = pd.DataFrame(freq_dist.most_common(30), columns=["Word", "Freq"])
word_freq.head(30)
```

```
[]:
                      Word Freq
     0
                     nifty
                              399
     1
                bankniftv
                             104
     2
              stockmarket
                              71
     3
                 niftybank
                               45
     4
         stockmarketindia
                               44
     5
                    sensex
                               43
     6
                    stocks
                               38
     7
           optionstrading
                               36
     8
                               34
     9
           breakoutstocks
                               31
     10
                   trading
                               30
                    market
     11
                               29
     12
                     india
                               26
     13
                               25
                       ipm
```

```
14
                           24
                 good
15
                           24
                  nse
16
               growth
                           24
17
             nseindia
                           23
18
                           23
                  may
19
     pharmaceuticals
                           23
20
         indianpharma
                           23
          stockstobuy
21
                           22
22
          sharemarket
                           21
23
         stockmarkets
                           20
24
                   amp
                           19
25
               points
                           19
26
        stocksinfocus
                           19
27
                 time
                           19
28
                 bank
                           18
29
                 today
                           18
```

Lets add "nifty" to the list of stop words and create the cleaned tweets:

```
[]: stop_words = set(stopwords.words('english')) | set(["nifty"])

data["Cleaned_Tweets"] = data["Tweets"].apply(preprocess_tweet)
data.head()
```

```
[]: Tweets \
```

- 0 #bankNifty 50100 ce looks good at 70+-2 for a ...
- 1 "#market #banknifty #OptionsTrading #optionbuy...
- 2 PENNY STOCK MADHUCON PROJECTS LTD cmp-11 FOLLO...
- 3 #Nifty50 has been in a healthy uptrend since t...
- 4 #Gravita #livetrading #stockstowatch #stocksin...

Cleaned_Tweets

- 0 banknifty ce looks good target
- 1 market banknifty optionstrading optionbuying t...
- 2 penny stock madhucon projects ltd cmp followht...
- 3 healthy uptrend since beginning year didnt bre...
- 4 gravita livetrading stockstowatch stocksinfocu...

We look again at the top 20 words by frequency:

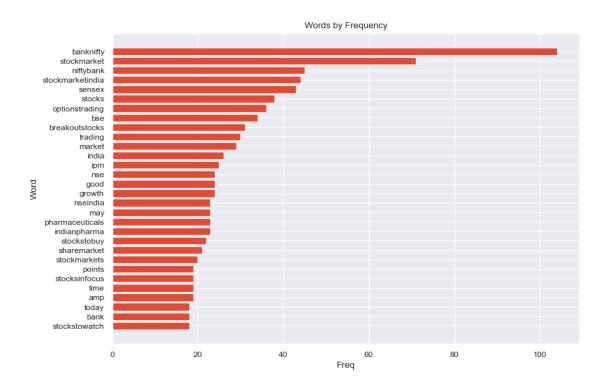
```
[]: tweet_words = data["Cleaned_Tweets"].str.cat(sep = " ")

freq_dist = nltk.FreqDist(tweet_words.split())
word_freq = pd.DataFrame(freq_dist.most_common(30), columns=["Word", "Freq"])
word_freq.head(30)
```

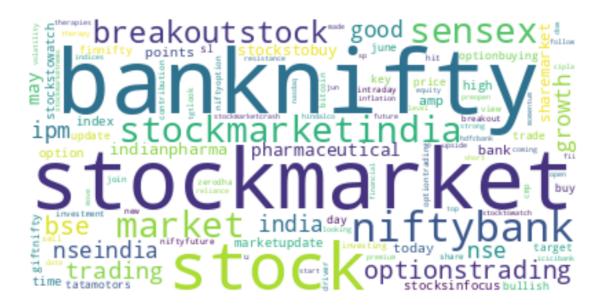
```
[]: Word Freq
0 banknifty 104
```

```
71
1
         stockmarket
2
            niftybank
                          45
3
                          44
    stockmarketindia
4
                          43
               sensex
5
               stocks
                          38
6
                          36
      optionstrading
7
                  bse
                          34
8
      breakoutstocks
                          31
9
              trading
                          30
10
               market
                          29
11
                india
                          26
12
                  ipm
                          25
13
                 good
                          24
14
                          24
                  nse
15
                          24
               growth
                          23
16
             nseindia
17
                          23
                  may
18
     pharmaceuticals
                          23
19
        indianpharma
                          23
20
                          22
         stockstobuy
21
         sharemarket
                          21
                          20
22
        stockmarkets
23
                          19
                  amp
24
               points
                          19
25
       stocksinfocus
                          19
26
                 time
                          19
27
                          18
                 bank
28
                today
                          18
29
       stockstowatch
                          18
```

We visualise these top 20 words by frequency:



Next, we create a word cloud:



Next, we extract sentiment scores for the tweets:

```
[]: sia = SentimentIntensityAnalyzer()

polarity_scores = data["Cleaned_Tweets"].apply(lambda x: sia.polarity_scores(x))

data["Positive_Score"] = polarity_scores.apply(lambda x: x["pos"])
   data["Negative_Score"] = polarity_scores.apply(lambda x: x["neg"])
   data["Neutral_Score"] = polarity_scores.apply(lambda x: x["neu"])
   data["Compound_Score"] = polarity_scores.apply(lambda x: x["compound"])

data.head()
```

```
[]: Tweets \
```

- 0 #bankNifty 50100 ce looks good at 70+-2 for a ...
- 1 "#market #banknifty #OptionsTrading #optionbuy...
- 2 PENNY STOCK MADHUCON PROJECTS LTD cmp-11 FOLLO...
- 3 #Nifty50 has been in a healthy uptrend since t...
- 4 #Gravita #livetrading #stockstowatch #stocksin...

	Cleaned_Tweets	Positive_Score	/
0	banknifty ce looks good target	0.420	
1	market banknifty optionstrading optionbuying t	0.075	
2	penny stock madhucon projects ltd cmp followht	0.155	
3	healthy uptrend since beginning year didnt bre	0.100	
4	gravita livetrading stockstowatch stocksinfocu	0.262	

Negative_Score Neutral_Score Compound_Score

0	0.000	0.580	0.4404
1	0.145	0.780	-0.3400
2	0.000	0.845	0.2960
3	0.198	0.702	-0.3935
4	0.000	0.738	0.5994

We look at the summary statistics for the sentiment scores:

```
[]: scores = data[["Positive_Score", "Negative_Score", "Neutral_Score", use of "Compound_Score"]]
scores.describe()
```

```
[]:
            Positive_Score
                             Negative_Score
                                              Neutral_Score
                                                             Compound_Score
                245.000000
                                 245.000000
                                                 245.000000
                                                                  245.000000
     count
    mean
                  0.116216
                                   0.028490
                                                   0.855314
                                                                    0.172913
                  0.146029
                                   0.071052
                                                   0.156892
     std
                                                                    0.343955
                  0.000000
                                   0.000000
                                                   0.213000
                                                                   -0.807400
    min
     25%
                  0.000000
                                   0.00000
                                                                    0.00000
                                                   0.742000
     50%
                  0.053000
                                   0.000000
                                                   0.868000
                                                                    0.000000
     75%
                  0.194000
                                   0.000000
                                                   1.000000
                                                                    0.440400
                  0.787000
                                   0.405000
                                                   1.000000
                                                                    0.928700
    max
```

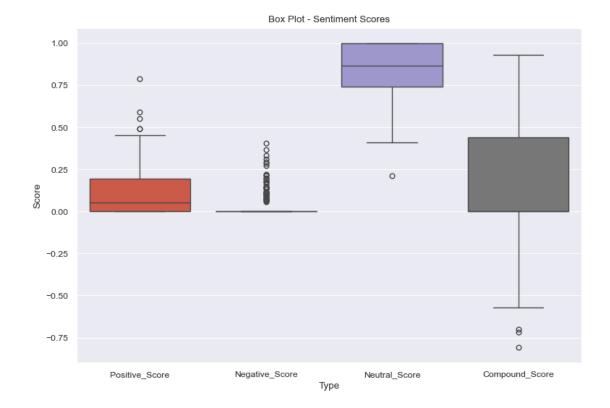
The compound score has a reasonable spread with a median of 0. We plot the sentiment scores below:

```
[]: #/ label: tweets-words-sentiment-scores-box-plot

plt.figure(figsize = (9, 6))
sns.boxplot(data = scores)

plt.title("Box Plot - Sentiment Scores")
plt.xlabel("Type")
plt.ylabel("Score")

plt.show()
```



It looks like the sentiment scores could be useful as an extra feature - but without access to historical tweets, it would be impossible to tell conclusively, but it could be worth investigating further.

1.6 Conclusion

predicting NSEI open direction looks very promissing. We have shown the ability to train models with accuracy of around 75% in the case of SVM, and 76% in the case of PyTorch deep learning model - I am confident that with enhanced model tuning, and by adding extra features such as sentiment, we can improve accuracy significantly.