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Conference Paper

Stock Price Modeling with Geometric Brownian Motion and Value with Risk PT Ciputra Development TBK

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ABSTRACT

Financial sector investment is an activity that attracts a lot of public interest. One of them is investing funds in purchasing the company's shares. Profit received from stock investment activity can be seen from the value of stock returns. While, if the previous stock returns to Normal distribution, the future stock price can be predicted by Geometric Brownian Motion Method. Based on the stock price prediction, can also be measured an estimated value of the investment risk. The result of data processing shows that the stock price prediction of PT. Ciputra Development Tbk period December 1, 2016, until January 31, 2017, has very good accuracy, based on the value of MAPE 1.98191%. Further, the Value Risk Method of Monte Carlo Simulation with α = 5% significance level was used to measure the share investment risk of PT.Ciputra Development Tbk. Thus, this method is only useful if it can be used to predict accurately. Therefore, backtesting is needed. Based on the processing obtained data, backtesting generates the value of violation ratio at 0, it means that at significance level α = 5%, the Value at Risk Method of Monte Carlo Simulation can be used at all levels of probability violation.

Keywords: Geometric Brownian motion, risk, value at risk, backtesting

Introduction

Investment is one of the activities that are quite attractive to the people of Indonesia. Investments that attract a lot of interest from investors, especially in the financial sector, are investments in the capital market. One of the most widely traded securities in the capital market is shares.

The decision of investors to invest in stocks is based on the desire to make a profit. The advantages of investing in stocks can be seen from the number of stock returns. Stock prices often experience changes that are difficult to predict, resulting in uncertain stock return values, for that we need a mathematical model to predict future stock prices based on past stock price data. Stock investment in addition to providing benefits, also contains an element of risk. One method that can be used to measure risk is Value at Risk, Value at Risk can be defined as an estimate of the maximum loss that will be obtained over a certain period.

This study discusses the application of the Geometric Brownian Motion (GBM) method to predict stock prices based on the stock return value of the previous period and to measure the value at risk of stock price predictions using the Monte Carlo simulation method. Then to test the accuracy of the resulting VaR value, a backtesting test was carried out by calculating the violation ratio

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value. In this study, the stocks analyzed were shares of PT. Ciputra Development Tbk period 4 January 2016 to 31 January 2017.

Material and Methods

Shares and share prices

Shares are securities as evidence of the participation or ownership of individuals or institutions in a company (Hadi, 2013). Based on Anoraga and Pakarti (2001), the stock price is defined as the price in the real market. The price of shares traded in the capital market follows a Lognormal distribution.

Stock returns

By Ruppert (2011), *return* is the rate of return on the results obtained as a result of investing. Security analysis generally uses *geometric returns*. The *geometric return* method is formulated as follows:

$$R_t = \ln\left(\frac{S(t_i)}{S(t_{i-1})}\right) \tag{1}$$

where R_t represents the stock return, $S(t_i)$ states the stock price in the period t_i, and $S(t_{i-1})$ states the stock price in the period t_{i-1}.

Normal distribution

According to Bain and Engelhardt (1992), a random variable X is said to follow a Normal Distribution with mean μ and variance σ^2 if it has a probability density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{2}$$

for $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $0 < \sigma < \infty$, can be denoted by $X \sim N(\mu, \sigma^2)$.

Lognormal distribution

Based on Ginos (2009), the probability density function for this lognormal distribution can be expressed as follows:

$$f(y) = \frac{1}{\sqrt{(2\pi\sigma^2)}\; y} \exp\left[-\frac{(\ln(y)-\mu)^2}{2\sigma^2}\right] \tag{3}$$

For $y > 0$, $-\infty < \mu < \infty$ and $0 < \sigma < \infty$, can be denoted by $Y \sim LN\ (\mu, \sigma^2)$.

Kolmogorov – Smirnov normality test

According to Daniel (1989), the Kolmogorov-Smirnov test procedure to test the normality of data is as follows:

1. Hypothesis:
   H_0 : $F(x) = F_0(x)$
   H_1 : $F(x) \neq F_0(x)$
2. Significance level: α
3. Test Statistics:
   $D = \sup_x |F_0(x) - F(x)|$
   where : $F_0(x)$ = Normal distribution cumulative probability.

$F(x)$ = Cumulative probability of empirical distribution.

4. Criteria for rejection:
   is rejected if the value of D is greater than the quantile 1- α Kolmogorov Smirnov test table, or the *p-value* < α.

Volatility

According to Maruddani and Purbowati (2009), volatility is the amount of fluctuation in the price of an asset. If there are n (number of observations) returns, then the expected return value can be estimated by the sample average (sample mean) returns:

$$\bar{R} = \frac{1}{n}\sum_{t=1}^{n} R_t \tag{4}$$

The average return is then used to estimate the variance for each period, namely:

$$s^2 = \hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left(R_t - \bar{R}\right)^2 \tag{5}$$

The root of the variance (standard deviation) is the estimated value of stock price volatility.

Stochastic process

According to Taylor and Karlin (1998), a stochastic process is a set of random variables $\{X(t), t \in T\}$ by t stating the time and $X(t)$ stating the process at a time t. The set T is called the index set of a stochastic process. If the set T is a time interval $t \in [0, \infty)$, then the stochastic process is said to be a continuous time stochastic process and is expressed in the form $\{X(t); t \geq 0\}$.

Brownian motion

1. Brownian Motion
   According to Dmouj (2006), a stochastic process is called Brownian motion if it meets the following criteria:
   i. $W(0) = 0$ (with probability 1).
   ii. For $0 \leq s \leq t \leq T$, random variable given by the change in $W(t) - W(s)$ Normal distribution with mean 0 and variance $\sigma^2(t-s)$.
   iii. For $0 \leq s < t < u < v \leq T$, changes in $W(t) - W(s)$ and $W(v) - W(u)$ are independent.
2. Standard Brownian Motion
   According to Higham (2001), A stochastic process $\{W(t), t \in T\}$ is said to be standard Brownian motion if the process satisfies the following criteria:
   i. $W(0) = 0$ (with probability 1).
   ii. For $0 \leq s \leq t \leq T$, random variable given by the change in $W(t) - W(s)$ Normal distribution with mean 0 and variance $t - s$, or equivalently $W(t) - W(s) \sim \sqrt{t-s}\ N(0,1)$, with $N(0,1)$ denotes a Normal distribution with mean 0 and variance 1.
   iii. For $0 \leq s < t < u < v \leq T$, changes in $W(t) - W(s)$ and $W(v) - W(u)$ are independent.
3. Brownian Motion with Drift Tribe
   Based on Dmouj (2006), Brownian Motion with drift tribe has the following equation :

$$B(t) = \mu(t) + \sigma\, W(t) \tag{6}$$

where $\mu(t)$ is the average value and σ is process standard deviation value *t*. $W(t) = Z\sqrt{t}$, Z is random numbers from the standard Normal distribution.

4. Geometric Brownian Motion

According to Dmouj (2006) If given a Brownian motion model with a drift term $B(t) = \mu^*(t) + \sigma\, W(t)$; $t \geq 0$ with drift parameters $\mu^* = \mu - \frac{1}{2}\sigma^2$, variance parameter σ^2, and $W(t)$ is Brownian motion process which starts at $W(0) = 0$. On stock price modeling, the stochastic process $\{P(t); t \geq 0\}$ is called Geometric Brownian Motion if $B(t) = \ln \frac{P(t)}{P(t-1)}$, where $B(t)$ is the stock return at time *t*.

Stochastic differential equation

According to Higham (2001), The Stochastic Differential Equation has the following form:

$$dX(t) = f(X(t))dt + g(X(t))dW(t) \tag{7}$$

for $t \in [0, T]$, initial value $X(0) = X_0$. The term $f(X(t))dt$ is the drift term, where $f(X(t))$ is drift coefficient. While $g(X(t))dW(t)$ is the diffusion term, where $g(X(t))$ is the diffusion coefficient. $W(t)$ is standard Brownian motion.

Teorema Itô

Based on Hull (2009), if there are variables $X(t)$ that follow the $It\hat{o}$ process with the equation :

$$dX(t) = \mu\,(X,t)\; dt + \sigma\,(X,t)\; dW(t) \tag{8}$$

where $W(t)$ is Brownian Motion and the value of μ and σ is a parameter of *X* and *t*, $It\hat{o}$ theorem states that, if there is a function $G = G(X,t)$, then the function G will follow the following equation:

$$dG = \left(\frac{\partial G}{\partial X(t)}\mu + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X(t)^2}\sigma^2 \right) dt + \frac{\partial G}{\partial X(t)}\sigma\; dW(t) \tag{9}$$

***Geometric Brownian Motion* (GBM) stock price model**

Based on Brigo et al. (2008), the Geometric Brownian Motion stock price model assumes that past stock returns are normally distributed. The Brownian Motion Geometric Model is defined as follows:

$$d\,S(p) = \mu S(p)\; dP + \sigma\; S(p)\; dW(p) \tag{10}$$

where *S(p)* is the stock price at time *p*. *W* is the standard Brownian motion. Then the solution of the stochastic differential equation to obtain a Geometric Brownian Motion stock price model can be obtained through the $It\hat{o}$ theorem. If there is an equation:

$$dS(p) = \mu S(p)dp + \sigma\, S(p)dW(p)$$

So based on the $It\hat{o}$ theorem, the function $G = G(s, p)$ is as follows :

$$dG = \left(\frac{\partial G}{\partial S(p)} \mu S(P) + \frac{\partial G}{\partial p} + \frac{1}{2} \frac{\partial^2 G}{\partial S(p)^2} \sigma^2 S(p)^2 \right) dp + \frac{\partial G}{\partial S(p)} \sigma \; S(p) \; dW(p)$$

For example the function $G = \ln S(p)$, with the condition that $\frac{\partial G}{\partial S(p)} = \frac{1}{S(p)}$, $\frac{\partial^2 G}{\partial S(p)^2} = -\frac{1}{S(p)^2}$, and $\frac{\partial G}{\partial p} = 0$, from equation (10) obtained :

$$dG = \left(\mu - \frac{\sigma^2}{2} \right) \; dp + \sigma \; dW(p) \tag{11}$$

According to Abidin dan Jaffar (2014), if the change in the current period's stock price from the previous period's stock price differs by one day, where $p_0 < p_1 < p_2 \ldots < p_n$, Geometric Brownian Motion stock price model written:

$$\int_{p_{i-1}}^{p_i} d \; G = \int_{p_{i-1}}^{p_i} \left(\mu - \frac{\sigma^2}{2} \right) \; dp + \int_{p_{i-1}}^{p_i} \sigma \; dW(p)$$

$$\Leftrightarrow \ln(S(p_i)) - \ln(S(p_{i-1})) = \left(\mu - \frac{\sigma^2}{2} \right) \; (p_i - p_{i-1}) + \sigma \; (W(p_i) - W(p_{i-1}))$$

$$\Leftrightarrow \ln \frac{S(p_i)}{S(p_{i-1})} = \left(\mu - \frac{\sigma^2}{2} \right) \; (p_i - p_{i-1}) + \sigma \; (W(p_i) - W(p_{i-1}))$$

$$\Leftrightarrow \frac{S(p_i)}{S(p_{i-1})} = \exp\left(\left(\mu - \frac{\sigma^2}{2} \right) \; (p_i - p_{i-1}) + \sigma \; (W(p_i) - W(p_{i-1})) \right)$$

$$\hat{S}(p_i) = \hat{S}(p_{i-1}) \; \exp\left(\left(\hat{\mu} - \frac{\hat{\sigma}^2}{2} \right) \; (p_i - p_{i-1}) + \hat{\sigma} \sqrt{p_i - p_{i-1}} Z_{i-1} \right) \tag{12}$$

MAPE (*Mean Absolute Percentage Error*)

According to Abidin dan Jafar (2014), the calculation of the MAPE value is as follows:

$$MAPE = \frac{\sum_{p=1}^{n} \left| \frac{Y_p - F_p}{Y_p} \right|}{n} \; x \; 100\% \tag{13}$$

where Y_p is the actual value at time *p*. F_p is the forecast value at time *p*. n is the number of observations.

Table 1. MAPE accuracy rating scale

MAPE Value	Forecasting Accuracy
< 10%	Forecasting accuracy is very good
11% - 20%	Forecasting accuracy is good
21% - 50%	Forecasting accuracy is still within reasonable limits
>51%	Inaccurate forecasting accuracy

Sumber: Lawrence et al., 2009

Value at Risk monte carlo simulation method

Value at Risk is a statistical risk measurement method to estimate the maximum possible loss on a portfolio at a certain level of confidence. Based on Maruddani and Purbowati (2009), Value at Risk with the Monte Carlo simulation method on a single asset assumes that the return on assets is normally distributed. In general, a simple algorithm for calculating Value at Risk using the Monte Carlo simulation method on a single asset is as follows:

1. Determine the parameter value of a single asset return. Return is assumed to follow Normal distribution with mean μ and variance σ^2.
2. Simulating the return value by randomly generating a single asset return with the parameters obtained from step (1) as many as n pieces so that an empirical distribution of the return simulation results is formed.
3. Finding the maximum estimated loss at the confidence level $(1-\alpha)$ namely as the α quantile value of the empirical distribution of returns obtained in step (2), denoted by R^*
4. Calculate the value of VaR at the level of confidence $(1-\alpha)$ in a period *r* days i.e :

$$VaR_{(1-\alpha)}(r) = W_0 \ R^* \ \sqrt{r} \tag{14}$$

   where
   W_0 = asset or portfolio initial investment fund.
   R^* = value of the α quantile of the return distribution.
   $\sqrt{r}$ = period.
   The VaR value obtained is the maximum loss that will be suffered by a single asset.
5. Repeat step (2) to step (4) as many as m so that it reflects the various possible VaR values of a single asset i.e VaR_1, VaR_2,..., VaR_m.
6. Calculate the average of the results from step (5) to stabilize the value because the VaR value generated by each simulation is different.

Backtesting

According to Danielsson (2011), *backtesting* is a procedure for testing the accuracy of the calculated Value at Risk value. To perform backtesting, the sample with size *K* will be divided into two groups, namely the estimation window (K_E) and the test window (K_U). An estimation window (K_E) is a group of observations used to calculate the Value at Risk. Meanwhile, the test window (K_U) is a sample of the period (K_{E+1}) up to period *K*.

Violation ratio

In the period (K_{E+1}) up to period *K* (length of the test window), the violation is symbolized by η_k .

$$\eta_k = \begin{cases} 1 & if \ R_t \le -VaR_k \\ 0 & if \ R_t > -VaR_k \end{cases} \tag{15}$$

$$VR = \frac{v_1}{m_0 \times K_U} \tag{16}$$

With *VR* is the magnitude of the offense ratio, v_1 is the number of η_k that is worth 1 (number of days the violation occurred), m_0 is the probability of the alleged violation.

Results and Discussion

Determination of data in sample and data out sample

The data in the sample is determined as much as 219 data starting from 4/1/2016 to 30/11/2016. Out sample data is determined as many as 42 data starting from 1/31/2016 to 31/01/2017. The in-sample data is used to build a stock price model, while the out-sample data is used to validate the model.

Data Normality test in stock sample return

Hypothesis :

H_0 : The data in the stock return sample is normally distributed.

H_1 : The data in the stock return sample is not normally distributed.

Significance level : α = 5%

Test Statistics :

$D = \sup_x \left|S(x) - F_0(x)\right|$ = 0,047 atau *p-value* = 0,727

Conclusion :

H_0 is accepted because of the *p-value* (0,727) > α (0,05). Then the data in the stock return sample is normally distributed.

Parameter estimation and determination of geometric Brownian motion stock price model PT. Ciputra Development Tbk

Parameters in the Geometric Brownian Motion stock price model include the expected value of stock returns ($\hat{\mu}$), stock return variance ($\hat{\sigma}^2$), and stock volatility value ($\hat{\sigma}$). Value of $\hat{\mu}$ is –0,00025, value of $\hat{\sigma}^2$ is 0,00072, and value of $\hat{\sigma}$ is 0,02677.

Based on Equation (12), then the stock price model Geometric Brownian Motion PT. Ciputra Development Tbk are as follows:

$$\hat{S}(p_i) = \hat{S}(p_{i-1}) \exp\left(\left((-0{,}00025) - \frac{0{,}00072}{2}\right)(p_i - p_{i-1}) + 0{,}02677\sqrt{p_i - p_{i-1}}\,Z_{i-1}\right)$$

Stock Price Prediction of PT. Ciputra Development Tbk

Stock price predictions are carried out to find out the estimated stock prices for the next 42 periods, starting from 1/12/2016 to 31/1/2017. Through the help of software R 3.3.2, the stock price prediction value is obtained as follows:

Table 2. Actual and Predicted Share Prices of PT. Ciputra Development Tbk

k	Date	Actual Price	Predictive Price	*k*	Date	Actual Price	Predictive Price
1	01-Des-16	1380	1292	22	03-Jan-17	1300	1333
2	02-Des-16	1365	1329	23	04-Jan-17	1335	1298
3	05-Des-16	1360	1333	24	05-Jan-17	1310	1289
4	06-Des-16	1370	1333	25	06-Jan-17	1305	1335
5	07-Des-16	1360	1330	26	09-Jan-17	1300	1300
6	08-Des-16	1410	1323	27	10-Jan-17	1300	1307
7	09-Des-16	1415	1318	28	11-Jan-17	1275	1277
8	13-Des-16	1375	1325	29	12-Jan-17	1270	1267
9	14-Des-16	1355	1330	30	13-Jan-17	1270	1284
10	15-Des-16	1350	1331	31	16-Jan-17	1250	1270

11	16-Des-16	1365	1362	32	17-Jan-17	1265	1275
12	19-Des-16	1350	1370	33	18-Jan-17	1285	1281
13	20-Des-16	1330	1329	34	19-Jan-17	1325	1290
14	21-Des-16	1300	1390	35	20-Jan-17	1315	1300
15	22-Des-16	1295	1326	36	23-Jan-17	1280	1298
16	23-Des-16	1250	1316	37	24-Jan-17	1305	1286
17	27-Des-16	1275	1276	38	25-Jan-17	1340	1302
18	28-Des-16	1330	1336	39	26-Jan-17	1325	1278
19	29-Des-16	1330	1323	40	27-Jan-17	1315	1298
20	30-Des-16	1335	1327	41	30-Jan-17	1325	1325
21	02-Jan-17	1335	1350	42	31-Jan-17	1320	1320

MAPE value determination

MAPE (*Mean Absolute Percentage Error*) is a method that can be used to evaluate the forecast value. Through the help of software R 3.3.2, obtained a MAPE of 1.98191% (forecasting accuracy is in the very good category).

Normality test of predicted stock returns

Hypothesis :

H_0: Predicted stock price returns Normal distribution.

H_1: Predicted stock price return is not normally distributed.

Significance Level: α = 5%

Test statistics:

$D = \sup_x \left| S(x) - F_0(x) \right|$ = 0,047 or *p-value* = 0,727

Conclusion :

H_0 is accepted because of the *p-value* (0,727) > α (0,05). So, the data in the sample stock return is normally distributed.

Value at Risk stock price prediction with monte carlo simulation method

Because after calculating the Value at Risk backtesting will be carried out, so before determining the value of the stock return parameter, the length of the estimation window (K_E) and test window (K_U). In this study, the estimation window length is 37 data and the estimation window is 5 data.

Table 3. Estimation window and test window

Estimation Window		Test Window
k	***k* + *K_E* - 1**	***VaR* (*k* + *K_E*)**
1 (01/12/2016)	37 (24/12/2016)	*VaR*(38) (25/01/2017)
2 (02/12/2016)	38 (25/01/2017)	*VaR*(39) (26/01/2017)
3 (05/12/2016)	39 (26/01/2017)	*VaR*(40) (27/01/2017)
4 (06/12/2016)	40 (27/01/2017)	*VaR*(41) (30/01/2017)
5 (07/01/2017)	41 (30/01/2017)	*VaR*(42) (31/01/2017)

At the 95% confidence level with 5000 repetitions, the average VaR value is as follows:

Table 4. Value at risk stock price prediction test window

Test Window to -	k	Date	VaR
1	38	25/01/2017	-0,03350
2	39	26/01/2017	-0,03063
3	40	27/01/2017	-0,03097
4	41	30/01/2017	-0,03076
5	42	31/01/2017	-0,03085

If taken for example on 25/01/2017, it can be said that there is a 95% confidence that within 1 day after January 25, 2017, the losses received will not exceed 3.350%.

Backtesting

The main purpose of backtesting is to determine the value of the violation ratio. The calculation of the Monte Carlo simulation VaR violation ratio at a 95% confidence level is simulated on several values of alleged violation probability (m_0), namely 0.1 %, 0.5%, 1%, 2%, 3%, 4%, and 5%. With the help of software *R 3.3.2* the value of the violation ratio is obtained as follows:

Table 5. Violation ratio value at risk monte carlo simulation

***Value at Risk* Monte Carlo Simulation**	
$(\boldsymbol{m_0})$	**Violation Ratio**
0,1%	0
0,5%	0
1%	0
2%	0
3%	0
4%	0
5%	0

Conclusion

Based on the problems raised in this journal, the following conclusions can be drawn:

1. By using the Geometric Brownian Motion method to predict the stock price of PT Ciputera Development Tbk, the following model is obtained:
$$\hat{S}(p_i) = \hat{S}(p_{i-1}) \exp\left(\left((-0{,}00025) - \frac{0{,}00072}{2}\right)(p_i - p_{i-1}) + 0{,}02677\sqrt{p_i - p_{i-1}} Z_{i-1}\right)$$
with a prediction error value of only 1.98191% which means that the prediction accuracy is very good.
2. Based on the results of the Monte Carlo simulation VaR calculation with a 95% confidence level, the VaR value for January 25, 2017, is -0,03350, the VaR value for the period January 26, 2017 is -0,03063, the VaR value for the period January 27, 2017 is -0,03097, the VaR value for the period January 30, 2017 is -0,03076, the VaR value for the period January 31, 2017, is -0,03085.
3. Based on *backtesting* for evaluating the *Value at Risk*, no violations were found so in this study the Monte Carlo simulation method can be used for all probability values of violations.

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