Pairs trading - A quantitative approach

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Executive Summary

Pairs trading is an old and widely known investment strategy that focuses on profiting from the relative mispricing between assets. In pairs trading there are generally two steps to constructing a model. The first step determines which pairs of stocks are suited for pairs trading. The second step concerns how to trade the candidate pairs. In this thesis we vary various types of parameters to see which are optimal. We differentiate between the pairs trading strategies and the pairs trend strategies. The pairs trading strategies are based on a spread series calculated from a pair of stocks to signal trades, while the pairs trend strategies rely on a trend indicator to signal for trades. We use the variance ratio test as a trend indicator. To determine the candidate pairs we use the squared minimum distance, correlation and cointegration to rank pairs of stock and select the attractive pairs. We also modify the pairs trading strategies in different ways to enhance their performance. We find the best performing models in an in-sample period and use them in an out-of-sample period. We find the best performing models to be the minimum distance strategy filtered for volatility and the cointegration strategy, using only the entry type settings BEYOND and OUTWARDS.[1]

Before constructing the pairs trading and pairs trend strategies for analysis we first test for the persistence of cointegration in the years from 1988 to 2013 and investigate if there are statistical reasons for pairs trading to work. We find that their are statistical evidence in favor of persistence of cointegration, varies through time.

The analysis of the pairs trading and pairs trend models consisted of analyzing colored scale tables of average monthly returns and Sharpe ratios, varying the different parameter settings maximum holding period, trade entry type and standard deviation threshold. We also used graphs displaying the average equity curve after each trade signal to gauge the robustness of the strategies.

We find that the out-of-sample model struggles to generate a positive Sharpe ratio after transaction costs. The profits are modest at best and it is reasonable to infer that it could only be considered from the standpoint of an institutional investor. The Fama-French 3 factor regressions show no significant abnormal return before transaction costs, but not after transaction costs.

[1]To aquire the data and program please contact me at: kewo08ac@student.cbs.dk

Contents

Part I

Introduction

Pairs trading is a widely known investment strategy that focuses on profiting from the relative mispricing between assets. In pairs trading the focus is not on predicting the direction of price of an asset, but predicting the future direction of a relative difference in prices of a set of assets.

We differentiate between forecasting returns in absolute and relative terms. Predicting the future direction of a single given asset is to forecast in absolute terms. In relative terms the task is instead to forecast the value of one or more assets in comparison to one or more other assets. In our case, we only investigate univariate pair trading models. Multivariate models might have the potential to uncover more from the given data, they however also require an increased amount of computations. Pairs of stocks instead become baskets of stocks. In pairs trading unrestricted pairings of n stocks gives $\frac{n!}{(n-2)!2!}$ pairs. We let r be the size of the basket or pair. When r is 2 we have pairs and when r is greater than 2 we have baskets. The amount of possible basket combinations is $\frac{n!}{(n-r)!r!}$. Unrestricted pairs of 500 stocks gives 124,750 unique pairs. Unrestricted baskets of size 3 gives 20,708,500 unique baskets. A 166 fold increase. Examining multivariate basket strategies, increases the amount of computations exponentially as the size of the pairs increase and become baskets. In this thesis we exclusively focus on the univariate kind, pairs trading.

In essence, relative pricing means that two assets that are close substitutes, should be priced similarly. The price does not have to reflect the correct value of the assets, but there should not be a significant discrepancy between the two. Relative pricing theory therefore excludes the possibility of arbitrage, but does not exclude the possibility of market bubbles. The Law of One Price (LOP) implies that this should hold true even when the price is mispriced. [Chen and Knez (1995)] extended the proposition to "closely integrated markets should assign similar payoffs to prices that are close." A "near-LOP" implies that similar assets' with similar payoffs should have similar prices. While the condition here is weaker and allows for sparse departures from the stringent LOP, we still need to specify how we measure two assets similarity or dissimilarity, before we can examine the performance of the models empirically.

In pairs trading there are generally two steps to constructing a model. The first step determines which pairs of stocks are suited for pairs trading. The second step concerns how to trade the candidate pairs. In this thesis we vary various types of parameters to see which are optimal.

The academic literature on pricing anomalies often cite the momentum literature as being one of the most robust anomalies. An indicator that is related to the traditional momentum models, is the trend indicator. In classic technical analysis trend indicators often take the shape of simple moving averages or exponential moving averages. If we combine the trend indicator with the pairs trading models, then we can use the trend indicator to signal for trade entry. In this thesis we also combine a trend indicator with pairs trading and examine the results.

This thesis consists of eleven parts. Part I was the introduction. Part II provides the literature section. Part III is the problem identification and delimination. Part IV presents the theoretical foundation of the thesis. Part V presents the data specifications. Part VI presents the methodology used in the thesis. The analysis is found in Part VII, afterwards a discussion is carried out in Part VIII. The thesis ends with the conclusion in Part IX and Part X contains the appendix.

1 Background - origins

It is debatable when pairs trading was discovered. If we focus on pairs trading as described in this thesis then we can cite [Vidyamurthy 2004], which states it was Nunzio Tartaglia that could be accredited the discovery. Exactly who at Morgan Stanley should be accredited is blurred. Edward Thorp [Edward Thorp 2004] a famous quantitative investor states in an article to Wilmott[2] that:

> *"(...) in 1982 or 1983 an ingenious researcher at Morgan Stanley invented another statistical arbitrage scheme with characteristics like ours but with substantially less variability. His project probably began trading real time in 1983. As his confidence increased with experience, it expanded in size. By 1985 it was a significant profit center at Morgan Stanley but the credit for its discovery, and the rewards from the firm, reportedly did not attach to the discoverer, Jerry Bamberger. While his boss Nunzio Tartaglia continued to expand the operation with great initial success, a dissatisfied Bamberger chose to leave Morgan Stanley."*

[2] www.willmott.com

Edward Thorp also states that he also used similar methods in the late 70s. The exact time of invention and by whom is not an easy question to answer. There is also the issue with how to define pairs trading. One of the methods by Edward Thorp was based on a ranking of all stocks derived from their past performance and not pairwise combinations. While the place of origins is blurred we can at least say that the basic strategy is by no means new.

Part II

Literature

2 Pairs Trading: Performance of a Relative-Value Arbitrage Rule

One of the most cited papers in the academic field of pairs trading is [Gatev 2006]. They employ a simple pair trading strategy on a data stretching from 1962 to 2002 on the US stock market. The first draft was released in 1999, enabling them to create an out-of-sample analysis in the later 2006 publication.

[Gatev 2006] uses a simple pair trading strategy. First they normalize the return series for all stocks in the formation period, then they rank the pairs according to the sum of squared differences, seeking to trade the pairs of stocks with the smallest sum of squared distances. In effect implying that the smaller the distance the more attractive the pair is for pair trading. A trade is initiated when the distance exceeds two standard deviations and exits when the distance is zero. The formation period is 12 month and the trading period is 6 month. The first 12 months of the sample period is used as the formation period and subsequent to this the trading period begins. In the first month after the formation period only one strategy is active. For every month after the first 12 month a new strategy is initiated. After 6 months a total of 6 trading sessions will be active each lagging the foregoing by one month, as one session is dropped the other is initiated. Monthly returns are calculated by aggregating the 6 active sessions total return for the particular month.

[Gatev 2006] uses the return difference between strategy without signal delay and the strategy with one day signal delay to infer plausible trading cost. After transaction costs the 6 month return ranged from 113 to 225 basis-points. They concluded that the returns were still economically and statistically significant. By using proxies for short selling cost the performance is only slightly impacted. [Gatev 2006] also concluded that the profits were not driven by bankruptcy risk. The performance was best in the period from 1960s to the 1980s. From the 1990s and onwards the performance declined. The authors offered various explanations, one was a decrease in transaction cost, lessening the friction preventing rapid price convergence. Another explanation, was that competition was driving profits down, pointing to the growth of the hedge fund sector. Lastly the authors suggest that a common latent factor is driving the temporal variability of pairs trading profits and uncovering the latent factor could help explain the recent decades decline in pairs trading profits.

3 Does Simple Pairs Trading Still Work?

[Do and Faff 2010] replicated [Gatev 2006]'s methods and applied them to more recent data. They confirmed the last two decades lower profitability. They assumed one day delay between signal and entry, and did not account for transaction costs. They discovered, that during turbulent market conditions the profits of the pairs trading strategy rose considerably. To further investigate the possible causes of the declining profits, they divided all trading sessions into four types of pairs groups: Pairs that never traded (non-traded), pairs that traded, but did not converge (non-convergent), pairs that traded once (single round-trip pairs) and pairs that trade multiple times (multiple round-trip pairs). They found that in comparison to the pre-1988 period there was an increase in the proportion of non-convergent pairs from 26% to 40% and a decline in the proportion of multiple round-trip pairs, from 42% to 24%. They discovered that the convergence following the day after divergence had dropped significantly post-1988 across all four types of pairs groups. They also concluded that increased arbitrage risks were the main drivers and not increased market efficiency. Their results show that up until 2002 arbitrage risks had increased. After 2002 market efficiency rose with the exception of the 2000-2002 and 2007-2009 crashes, where market efficiency worsened dramatically and outweighed arbitrage risks, allowing pairs trading to generate positive returns. In a cross-sectional regression of the registered trades they showed that zero crossings (zero/mean-crossings), company-specific industry volatility and whether the company was from the same industry, all had statistically significant parameters, while the SSD[3] had not.

[3]sum of squared differences

4 Are Pairs Trading Profits Robust to Trading Costs?

[Do and Faff 2011] followed up on the research in [Do and Faff 2010] by carefully accounting for various cost incurred by the pairs trader. Highlighting the fact that [Gatev 2006] did not take into account commissions either, they also expanded on the findings in [Do and Faff 2010] by analyzing 29 different portfolio formations. These portfolios were based on combinations between their ranking deciles on the SSD and number of zero crossings (NZC) measures and whether or not the pairs were restricted to be formed within sectors or industries. Pairs trading remains profitable after adjusting for costs, but at much lower levels. The original strategy from [Gatev 2006] is unprofitable and only more refined versions remained profitable. The more refined versions was based on SSD, NZC, industry segmentation and sector segmentation.

5 Combining pairs trading and trend indicator

In Deutsche Banks Quantcraft article of 2012, entitled "Catching trends" the authors proposed to fusion pair trading and trend modeling. Their data set consisted of currencies, commodities and indexes. They used a trend indicator to generate signals that further required that the trend indicator was signaling simultaneously with a trend signal in another highly correlated asset. This fusion acted as an extra confirmation filter on top of the trend indicator. They reported impressive annualized Sharpe ratios ranging from 1.9 to 3.2 over the 5 month simulation period using 5 minute frequency data on futures and currencies. In this thesis we will name the combined models of pairs trading and trend indicators, pairs trend models.

Part III

Problem identification and delimitation

Over the last three decades quantitative hedge funds have emerged to become a significant force in the investment industry. One of the oldest known strategies with quantitative roots is pairs trading. To better understand whether the strategy still is a profitable venture, we ask the following question.

Does quantitative pairs trading yield economic significant returns on the S&P 500 and can they serve as an independently competitive investment strategy?

To answer this we first test for the persistence of cointegration in the years from 1988 to 2013 and investigate if there are statistical reasons for pairs trading to work. Secondly we backtest[4] a multiple of different pairs trading models in an in-sample period and form a final variation of the strategies for the out-of-sample period. The performance of the models in the out-of-sample period should tell us how the pairs trading phenomenon is presently doing. We choose the S&P 500 as it is the most liquid stocks in the world minimizing illiquidity effects.

The general outline of this thesis can be divided up into the following research questions:

- Does cointegration in one year persist into the following year?
- Is it possible to generate significant abnormal return using pairs trading strategies?
- What commonalities and differences does the different pairs trading strategies have?
- Are there signs of temporal factor driving the returns of the pairs trading strategies.

To answer these questions the analysis begins by examining the cointegration persistence from one year to next. After this several different pairs trading and pairs trend models are backtested and evaluated.

Part IV

Theory

In this thesis we examine two different ways of doing pairs trading. One is the traditional pairs trading the other is what we have chosen to call pairs trend models. The pairs trading models we investigate can be divided into three different types. The minimum distance strategy, the correlation strategy and the cointegration strategy, where minimum distance, correlation and cointegration is referring to how pairs of stocks are ranked among each other.

[4]Backtest means to simulate a trading strategy through time by using past price data.

The pairs trend models share the ranking systems of the pairs trading models, which is why the pairs trend models also have three different types of ranking. the pairs trend minimum distance strategy, the pairs trend correlation strategy, the pairs trend cointegration strategy. As mentioned earlier the pairs trend model and the pairs trading model do not differ in how pairs are formed, but in how trading signals are generated.

The pairs trading models constructs spread series through regressions and uses the spread to signal for trades. The pairs trend model evaluates whether it has detected two trends in opposite direction in each one of the stocks in the pair.

6 Statistical methods overview

In this part we will present the statistical methods used in the strategies. Section 7 defines the distance measure used in the minimum distance strategy. Section 8 presents the Augmented Dickey Fuller test and the Johansens Cointegration test. The Augmented Dickey Fuller test is used in the analysis of yearly persistence in cointegration. The Johansens Cointegration test is used to evaluate the cointegration of pairs in the cointegration strategy. Section 9 presents the Variance Ratio test, which is used as the trend indicator in the pairs trend model. Section 10 presents the Chi-squared test for use in the analysis of cointegration persistence. Section 11 presents the theory and reasoning for the use of correlation as a measure for the similarity between two stocks. The correlation measure is used in the correlation strategy. Lastly in section 12 we present the metric Sharpe ratio as our second performance measure, which unlike average monthly returns adjusts for risk.

The Augmented Dickey Fuller presented in section 8.1 is used to evaluate the cointegration persistence of stocks in year 1988 to 2013, both years included. The advantage of using the Augmented Dickey Fuller is twofold. First, it makes it simpler to compare with results in the literature. Secondly it makes the analysis work easier as retrieval of previously calibrated parameters are straightforward.

Johansen Cointegration test presented in section 8.2 is used in the pairs trading and pairs trend model to evaluate the statistical significance of whether a pair of stock is cointegrating. The degree of statistical significance in a pair of stock is used to score and rank the stocks in the cointegration strategy. We use the Johansen Cointegration test instead of the Augmented Dickey Fuller as the Johansen Cointegration test is independent of the order in which we choose the variables. The Johansen Cointegration test once implemented also opens up the possibility of multivariate pairs trading, although this thesis does not cover this.

The variance ratio test as presented in section 9 is the trend indicator in the pairs trend model. The pairs trading model uses a spread series in combination with various rules for how the spread series moves to signal for trades. In the pairs trend model the trend indicator signals a trend and a direction, when this occurs in both stocks of a pair this signals for a trade to be entered.

7 Distance measure

We define the average distance between two, A and B stocks as:

$$avg.distance_{A,B} = \frac{1}{T}\sum_{t=0}^{T}\left(p_t^A - p_t^B\right)^2$$

$p_t^A$and p_t^B is the normalized cumulative return of stock$A$ and B, respectively. T is the length of the two return vectors. We choose to take the average distance instead of simply the total distance in hope that it would be more intuitive when comparing between strategies. Even though the correlation and cointegration measure does not use the distance measure in their trading logic we still record the average distance for later comparison. If we drop taking the average the formula would be identical to the one used in [Gatev 2006], [Do and Faff 2010] and [Do and Faff 2011]. Taking the average does not change the ranking of the stocks, as such the ranking method is identical to the one in [Gatev 2006], [Do and Faff 2010] and [Do and Faff 2011]. We call the strategy minimum distance, shorthand for squared minimum distance and the word *minimum* is because the desirable pairs of stocks are the ones with the smallest distance.

8 Cointegration

8.1 Augmented Dickey Fuller Test

Below is an AR(1) model. A random walk with zero mean.

$$y_t = \rho y_{t-1} + u_t$$

y_t is the variable, u_t is the error term and ρ is the coefficient. When $\rho = 1$ the model is nonstationary. The first difference of the model can be written as

$$\triangle y_t = (\rho - 1)\, y_{t-1} + u_t = \delta y_{t-1} + u_t$$

where the goal is to estimate if $\delta = 0$, where δ is defined as $\delta \equiv \rho - 1$. There are three versions of the traditional Dickey-Fuller test. [5]

$$\triangle y_t = \delta y_{t-1} + u_t$$
$$\triangle y_t = a_0 + \delta y_{t-1} + u_t$$
$$\triangle y_t = a_0 + a_1 t + \delta y_{t-1} + u_t$$

The first equation tests for a unit root. The second tests for a unit root with drift and the final one tests for a unit root with drift and deterministic time trend. Each version has their specific distribution. We used the mathematical library [SuanShu] to calculate the critical values and statistical significances. The Augmented Dickey Fuller test expands on the traditional Dickey Fuller test by adjusting for auto-correlation in the first difference of the variables.[6]

$$\triangle y_t = \delta y_{t-1} + \gamma_1 \triangle y_{t-1} + \gamma_2 \triangle y_{t-2} + ... + \gamma_p \triangle y_{t-p} + u_t$$
$$\triangle y_t = a_0 + \delta y_{t-1} + \gamma_1 \triangle y_{t-1} + \gamma_2 \triangle y_{t-2} + ... + \gamma_p \triangle y_{t-p} + u_t$$
$$\triangle y_t = a_0 + a_1 t + \delta y_{t-1} + \gamma_1 \triangle y_{t-1} + \gamma_2 \triangle y_{t-2} + ... + \gamma_p \triangle y_{t-p} + u_t$$

[5] [Dickey Fuller]
[6] [Augmented Dickey Fuller]

To determine the p lag length we use the same formula as the `adf.test` function in the `tseries` package from the statistical programming language R. The formula is $p = \sqrt[3]{length\,(y) - 1}$ [7], where the length of y is the number of observations in the time series. For this thesis we test for a unit root, without drift and without deterministic time trend, using the Augmented Dickey Fuller test.

We will use the ADF test to examine the cointegration properties of the spread series as well as the individual level series (price series) of the stock. In both cases the setting is without intercept, unit root.

8.2 Johansens Cointegration Test

Johansens Cointegration test as used in this thesis is presented briefly below. Consider two variables each integrated by an order of 1.

$$X_t \sim I(1) \quad Y_t \sim I(1)$$

At most there will be one cointegrating vector between the two variables. For more than two variables there will be at most $n-1$ cointegrating vectors, with n denoting the number of variables.

From the Dicky Fuller test we have:

$$\triangle\, y_t = (a_1 - 1)\, y_{t-1} + \varepsilon_t$$

in the multivariate case we have:

$$X_t = A_t X_{t-1} + \varepsilon_t$$

X_t is a $n \times 1$ vector of variables and ε_t a $n \times 1$ vector of noise components. A_t is an $n \times n$ matrix of parameters

[7] [tseries]

$$\begin{aligned}\triangle X_t &= A_t X_{t-1} - X_{t-1} + \varepsilon_t \\ \triangle X_t &= (A_t - I) X_{t-1} + \varepsilon_t \\ \triangle X_t &= \pi X_{t-1} + \varepsilon_t\end{aligned}$$

If the rank of π is zero the equation becomes a first order VAR process:

$$\triangle X_t = \varepsilon_t$$

The difference of X_t is then $I(0)$. Each variable in X_t, $X_{i,t}$ is a unit process $X_{i,t} = X_{i,t-1} + \varepsilon_{i,t}$. No linear combination can make them stationary. In the opposite extreme, if π is full rank then the number of cointegrating vectors is equal to n and the n processes already are stationary.

The Johansens test for cointegration has two test statistics, the trace and maximum eigenvalue statistic:

$$\begin{aligned}\lambda_{trace}(r) &= -T \sum_{i=r+1}^{n} ln\left(1 - \hat{\lambda}_i\right) \\ \lambda_{max}(r, r+1) &= -T ln\left(1 - \hat{\lambda}_{r+1}\right)\end{aligned}$$

λ is the characteristics roots for the matrix π. $\hat{\lambda}$ is the estimate of λ. The order is as:

$$\lambda_1 > \lambda_2 > \lambda_3 > ... > \lambda_n$$

The trace statistic tests the null hypothesis that the number of distinct cointegrating vectors is equal to or less than r, against a general alternative. The second statistic is the one we use in this thesis. The maximum eigenvalue test statistic tests the null hypothesis that the number of cointegrating vectors is r, against the alternative of $r+1$. Critical values are obtained through simulation.[8] The main advantage of the Johansen cointegration as opposed to the Augmented Dicky Fuller is that it can handle multivariate models.

When using the Johansens test for cointegration we will be using the settings, 2 lags, with intercept.

9 Variance Ratio Test

The variance ratio test is commonly known for testing whether or not a time series follows a random walk. [Lo A. W. 1988] used this test to see if the stock market followed a random walk. The variance ratio is intuitive and based on the property that a random walks volatility is expected to increase linearly with time. The variance of a time series where every observation is used is denoted as $V\hat{ar}(1)$. Skipping every other observation gives $V\hat{ar}(2)$, skipping every third observation gives $V\hat{ar}(3)$ and so on. In general we can say that of period q-differenced is q times the variance of first difference. Where difference is meant to specify the increments between observations. In layman terms we say that in using every other observation we would expect the variance to be twice as large as if we had used every observation. The following formula is used to estimate the variance test ratio.

$$V\hat{R}(q) = \frac{V\hat{ar}(q)}{V\hat{ar}(1)\, q}$$

$V\hat{R}(q)$ is the estimate of the variance ratio test, $V\hat{ar}(1)$ is the estimate of the variance of the first difference, $V\hat{ar}(q)$ is the estimate of the variance of the q difference. If $V\hat{R}(q)$ is below 1 the time series is mean-reverting and if it is above 1 it is trending. To find the statistical significance of the variance ratio test, we need to find the test statistic $Z(q)$. Under the assumption of Identical and Independent Distributed (IID) observations the test statistic $Z(q)$ can be found as[9]:

$$Z(q) = \sqrt{n}\left(V\hat{R}(q) - 1\right) \sim N\left(0, \frac{2(2q-1)(q-1)}{3q}\right)$$

[8] [J. Hudson Johansen]

[9] [Lo A. W. 1988]

There is also a method to estimate when observations have heteroskedastic properties.[10] In this thesis the results did not differ significantly when using either. For our purposes the IID version will suffice.

10 Chi-squared Test

The first part of the analysis is evaluating if there is persistence in cointegration in our data set. To evaluate if there is cointegration persistence we first find all cointegrating pairs in period y and see how many of these pairs continue to cointegrate in $y+1$. If there is persistence in cointegration we would expect that the probability for cointegrating in pairs to continue to be a cointegrated pair in period $y+1$ to exceed that of simply cointegrating period $y+1$ irrespective of whether the pair cointegrated in period y. The hypothesis is then:

$$Pr\left(x \in CI\left(y+1\right) \mid x \in CI\left(y\right)\right) > Pr\left(x \in CI\left(y+1\right)\right)$$

where x is a pair of stocks, $x \in CI\left(y+1\right) \mid x \in CI\left(y\right)$ is the number of pairs in period $y+1$ that cointegrate in period $y+1$ given they cointegrated in period y. $Pr\left(x \in CI\left(y+1\right)\right)$ is the probability of cointegrating in period $y+1$. To evaluate this expression we use the Chi-square goodness of fit test, denoted as χ^2 test[11].

$$\frac{\left(observed - expected\right)^2}{expected} = test\ statistic$$

Observed would be the number of cointegrating pairs that cointegrated in both periods. Expected would be the percentage of cointegrating pairs in period $y+1$ multiplied by the number of pairs in period y. The test statistic under the null hypothesis is distributed as a chi-square distribution. For our purposes the number of degrees of freedom is 1, since our two proportions are year y and $y+1$. We use the one-sided upper tail test from [Chi-squared distribution table] to find the statistical significances.

[10] [Quantcraft 2012]
[11] [Chi-square test]

11 Motivation for use of correlation as similarity measure

In pairs trading we are betting that the spread between a pair of stocks converge. To calculate the spread, we first need to normalize the price series so that they can be compared. The spread is then measured as

$$x_t - \gamma y_t = spread$$

where x_t is the normalized cumulative return of stock x at time t and likewise y_t is the normalized cumulative return of stock y at time t. The aim is to have a spread that displays mean-reverting characteristics, which would be the case if the spread series was stationary. This implies that γ is the cointegrating coefficient.

When the spread series is stationary it becomes predictable. The expectation of the stationary spread series is the mean. If the spread series deviates from the mean, we will considered it to be noise and expect the series to revert back towards the mean.

While cointegration measures the mean-reverting characteristics, the computational cost of performing the Johansens Cointegration tests are substantial. Measuring cointegration on a daily basis with a large data set on numerous pairs of stocks is a very expensive operation. We therefore use a filter based on correlation to reduce the amount of pairs needed to compute, as suggested in [Vidyamurthy 2004]. The rationalization of using correlation as a filter comes from asset pricing theory.

The theoretical foundation for our chosen measures of similarity stems from [Vidyamurthy 2004]. The theory is rooted in arbitrage pricing theory (APT), which has the Law of One Price at its foundation. APT states that two assets with the same risk exposures should have the same expected return. Generally risk is categorized into two types, systemic risk and idiosyncratic risk. Systemic risks are the risk exposures that are common for both assets and those that determine the assets value. Idiosyncratic risk are risks specific for the given asset. Idiosyncratic risk can be mitigated through diversification, while systemic risks cannot. When we refer to risk exposures we are referring to systematic risk exposures. Following [Vidyamurthy 2004] we suppose that stock prices assume the following form:

$$log(price_t) = n_t + \varepsilon_t$$

Where,

- $price_t$ is the price of the stock at time t.
- n_t is a random walk, a nonstationary trend component. This can be thought of as systemic risk. Eg. unobserved economic factors.
- ε_t is a stationary noise component. This can be thought of as idiosyncratic risk. Eg. firm specific news.

Utilizing the commonly used method of taking logs[12] between succinct prices to approximate returns we get,

$$log(price_t) - log(price_{t-1}) = n_t - n_{t-1} + \varepsilon_t - \varepsilon_{t-1}$$

$$r_t = r_t^c + r_t^s$$

where r_t is the return of stock t. r_t^c is return due to nonstationary trend component and r_t^s is return due to the stationary noise component. Drawing from APT, if two stocks have the same risk factor exposures different only by a scalar, then they meet the requirements for cointegration.

Suppose we have two stocks A and B, with risk profiles γx and x . The risk factor exposures only differ by a scalar, γ. F_k are the unobserved economic factors (systemic risks). x_k is the sensitivity of the stock A to economic factor k and γx_k is the sensitivity of the stock B to economic factor k .

Stock A :

$$r_A = \gamma(x_1 F_1 + x_2 F_2 + ... + x_k F_k)$$

[12]Taking logs is the approximation of $\frac{price_t - price_{t-1}}{price_t}$, where $\frac{price_t - price_{t-1}}{price_t}$ is the actual return that the investor experiences.

Stock B :

$$r_B = x_1F_1 + x_2F_2 + ... + x_kF_k$$

The returns for A and B are

$$r_A = \gamma(x_1F_1 + x_2F_2 + ... + x_kF_k) + r_A^s$$

$$r_B = (x_1F_1 + x_2F_2 + ... + x_kF_k) + r_B^s$$

Where r_A^s are the stationary components, the idiosyncratic risks. The non-stationary, common factor components are

$$r_A^c = \gamma(x_1F_1 + x_2F_2 + ... + x_kF_k)$$

$$r_B^c = x_1F_1 + x_2F_2 + ... + x_kF_k$$

We can see that $r_A^c = \gamma r_B^c$. Thereby meeting the requirement for cointegration.

We now combine these two stocks into a portfolio, where we go long one share A and short γ shares B.

$$r_A - \gamma r_B = \left(r_A^{cf} - \gamma r_B^{cf}\right) + (\varepsilon_A - \varepsilon_B)$$

If stocks A and B are cointegrated as we previously stated, then $\left(r_A^{cf} - \gamma r_B^{cf}\right)$ is zero.

$$r_{port} = \left(r_A^{cf} - \gamma r_B^{cf}\right) + (\varepsilon_A - \varepsilon_B)$$

This also implies that the common factors are perfectly positively or negatively correlated. $(\varepsilon_A - \varepsilon_B)$ is zero as we have assumed they are uncorrelated and with zero mean. If we measure the correlation between r_A and r_B, it will most likely not be perfectly correlated. This is due to the required perfect alignment of risk exposures as well as the presence of noise from $(\varepsilon_A - \varepsilon_B)$. However on average stocks have common risk exposures, Eg. CAPM, correlation to the market. If the ε_A and ε_B are small compared to r_A^{cf} and r_B^{cf} in contributing to the return generating process, we should expect r_A and r_B to be more correlated than not.

The correlation measure is interesting, because it can indicate whether or not two time series share economic risk factors. If two stocks share common risk factors, we will be more inclined to say that they are similar stocks and have economic reason to co-move. In addition the correlation measure is also much faster to compute than estimating the cointegrating coefficient, allowing us to reduce the computational search space of our strategy.

Correlation is calculated as Pearson product-moment correlation coefficient[13].

$$\rho_{A,B} = \frac{cov(r_A, r_B)}{\sigma_A \sigma_B}$$

$cov(r_A, r_B)$ is the covariance between r_A and r_B. σ_A is the standard deviation of r_A and σ_B is the standard deviation of r_B .

12 Sharpe ratio

Sharpe ratio is performance measure ubiquitous in the investment realm. The Sharpe ratio measures the risk-adjusted returns of a given portfolio and is calculated as,

$$S = \frac{\bar{r}_p - r_f}{\sigma_p}$$

where S stands for Sharpe ratio, $\bar{r}_p$ mean return of the portfolio, r_f is the risk free rate and σ_p is the standard deviation of the portfolio. Sharpe ratio is often used to give a sense of the historical risk-adjusted performance of an investment vehicle. Returns are attractive, however if they are accompanied by an exorbitant amount of risk, they can become less attractive maybe even unattractive compared to alternatives. It is therefore important when we evaluate an investment choice that we also measure the

[13] [Wiki Correlation]

risk of the investment. This implies that we assume risk can be measured as the standard deviation of the historical returns, which of course is not always the case. However given our limited information we have no choice, but to rely on past returns as the only information we have and as such choose standard deviation as our proxy for risk. We will use the average monthly returns as a performance measure as it is intuitive. To control for risk we also use the Sharpe ratio as our risk-adjusted performance measure.

We annualize the daily Sharpe ratio by multiplying with $\sqrt{252}$, where 252 is the number of trading days in a year. Likewise convert the daily return to the monthly return by the formula

$$monthly\ ret = (1 + daily\, ret)^{21} - 1$$

where 21 is number of trading days in a month.

Part V

Data

13 Return calculation

The price data was retrieved from the Compustat database. The variables Price Close (`prccd`), Daily Total Return Factor (`trfd`) and daily adjustment factor (`ajexdi`) were used to calculate returns. The formula for calculating the returns was retrieved from the WRDS[14] database as.

$$return_t = \left(\frac{\frac{prccd_t \times trfd_t}{ajexdi_t}}{\frac{prccd_{t-1} \times trfd_{t-1}}{ajexdi_{t-1}}} - 1 \right) \times 100\%$$

From the the Compustat database we also retrieve the index constituents of the S&P 500 index from 1989 to 2013. Using the GVKEY stock identifier we link the index constituents from the Compustat database to the daily security files in the Compustat database. The advantage of using the Compustat

[14] Wharton Research Data Services

database for both index constituents and prices is the ease in which we can map the index constituents with stock price series using the GVKEY stock identifiers.

14 Data specifications

It was not always possible to obtain 500 time series on any given date, even though the S&P 500 constituents list had between 498 to 502 companies on any given day. The reasons for this was that we limited the price time series to only come from companies that were listed on a US exchange[15], with the issue IDD 01 and was designated share type 0 (ordinary shares). Since the details on Compustat were limited, we attempted to cross reference a few of these excluded stocks. Some of the reasons for odd registers in the Compustat database were companies not incorporated in US, such as Invesco (incorporated in Bermuda), that had prices for days the exchange was closed. Other companies had A and B shares classes, with different lengths of historical time series. Solving these and others issue were beyond the time horizon of this thesis, and as such left for further study. It should however be noted that it is considerably more work to use constituents list real-time. Some publications that use constituents list do not go to these lengths[16]. Some simply use a constituents list from a certain date and extrapolate this into the whole data set, thereby knowingly introducing a survivorship bias into their analysis.

Part VI

Methodology

15 Method of measuring cointegration persistence

The cointegration analysis procedure used here is from [Clegg 2014]. To calculate the cointegration persistence in any given two year period we begin by gathering all stocks that are: non-stationary,

[15]These exchange identification numbers were: 11, 12, 14, 15, 16, 17, 18

[16]Such as [Clegg 2014]

has continuous prices in the two year period, satisfy the data requirements mention in part V and is a member of the S&P500 in the last day of the two year period. The time series from each stock in pair are the normalized cumulative returns of each stock.

The calculations are split in two parts the formation period and the trading period. The formation period is the first year in a two year period and the trading period is the subsequent year, the second year in the two year period. The two year period acts as a rolling window that rolls from 1988 to 2013. We exclude stocks that are stationary. To do this we measure the stationarity by using the Augmented Dickey Fuller test on the normalized cumulative return of the given stock. First the stocks considered are the non-stationary stocks of formation period, the first year in the two year window. The resulting stocks are then formed into pairs. To measure whether a pair of stock is cointegrating in the formation period, we begin by taking the normalized cumulative return series of each stock in a given pair and regress one stock on the other to get the slope coefficient (cointegrating coefficient). We use the coefficient to construct the spread series. We use the Augmented Dickey Fuller test to see if the spread series is stationary. If it is then the coefficient is a cointegration coefficient and the two time series cointegrate. We say it cointegrates when the p-value is below the 0.05% significance level of the Augmented Dickey Fuller test. In the analysis we also supply the average of mean-reverting coefficients of the cointegrating spread series. The spread is mean-reverting when this coefficient, the autoregressive coefficient is less than one. From subsection 8.1 we borrow the first equation which was given as:

$$y_t = \rho y_{t-1} + u_t$$

where y_t is the spread value at time t, u_t is the error term and ρ is the autoregressive coefficient. This equation implies that the spread is a random walk when $\rho = 1$ and a stationary time series when $\rho < 1$. The average of autoregressive coefficients is also presented in the analysis.

It is important that we exclude stocks with stationary time series. Any pair of stocks where either time series is stationary will generate a stationary time series, as we can simply weight the non-stationary stock to zero. The resulting cointegrating time series will simply consist (almost) entirely of the stationary stock and not the non-stationary stock. We therefore filter out the stationary stocks.

16 Transaction costs and interest rates

The transaction cost estimates are retrieved from [Do and Faff 2011]. The costs are shown below in Table 1.

Year	Basispoints	Year	Basispoints	Year	Basispoints	Year	Basispoints
1988	19	**1995**	16	**2002**	10	**2009**	9
1989	20	**1996**	14	**2003**	10	**2010**	9
1990	20	**1997**	10	**2004**	10	**2011**	9
1991	21	**1998**	10	**2005**	10	**2012**	9
1992	19	**1999**	10	**2006**	9	**2013**	9
1993	17	**2000**	10	**2007**	9	**2014**	9
1994	17	**2001**	10	**2008**	9		

Table 1 – Table of transaction costs in the various year. Round trip transaction cost would be twice this. The cost estimates are in basis-points.

We have assumed a 1% constant yearly fee on the amount of stocks short sold. This is also known as the short selling loan fee. This estimate is also used in [Do and Faff 2011]. We will explain the assumptions we have made in the short selling procedure with an example.

Let us assume that we have 100 dollars, where we place 50 dollars to buy stock A and 50 dollars to short sell stock B. When we short sell stock B we receive the amount that we short sold for. So net-cash we have 100 dollars, plus 50 from the short sale, minus 50 from the buying of stock B. However our short selling transaction is not over, our broker demands all of the short sale amount plus 20% of the short sale amount as collateral. This means we have to deposit 50 plus 10 to our broker as collateral. We are then left with 40 dollars. These 40 dollars are then placed in to earn the risk-free rate for as long as the trade position is open. In effect we earn 80% risk-free interest rate on the short side of the pairs trade.

The risk-free interest rate is gathered from Kenneth French' data library[17].

17 Strategy construction

17.1 The traditional framework

The framework used in [Gatev 2006], [Do and Faff 2010] and [Do and Faff 2011] is similar to the framework used in the momentum literature. Pairs are formed in a formation period and subsequently traded

[17] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/

in a trading period. The strategy is then run every month. Since the trading period used is six month, the maximum amount of active strategies is six. As soon as the the formation period of 12 month and the trading period of 6 months has passed, there will be 6 active strategies running concurrently. When all strategies are running, the reported returns are calculated by aggregating the returns of the 6 strategies for the given month. Each strategies return is weighted equally. To arrive at after-cost returns the time-varying costs are simply subtracted from the aggregated return series. The average of the 6 simultaneous strategies can be thought of as mimicking the trades of 6 different traders.

17.2 Our framework

The framework used in this thesis resembles the challenges faced by the sophisticated real world investor. We simulate an investor who has a starting capital of 10 million dollars and has to allocate capital to each investment. This means that when a position is liquidated the investor immediately seeks out new investment opportunities. The search space consists of pairs who meet the specified requirements and has an entry signal. Thereby keeping the investors capital invested as much as possible. The traditional framework merely allocates a set amount of capital to each pair and only invests when the pair gives an entry signal. Our framework tries to overcome this limitation by continuously scanning for new pairs. In order to have a fair comparison of the strategies with and without transaction cost we do not require the cash to be positive in order to initiate a trade. This does not significantly effect the results, as the negative balance rarely dips below 5000 dollars, which when subtracting interest rate cost has minimal influence.

17.3 Advantages and disadvantages of the two frameworks

The traditional framework differentiates between calculating returns as committed capital or fully-employed capital. Committed capital is the capital allocated to each candidate pair, irrespective of whether the pair is trading or not. The fully-employed capital only calculates returns on the capital actively employed. The disadvantage of the committed capital approach is under-utilization of the capital allocated. The disadvantage of the fully-committed capital measure it that it is unrealistic. The advantage of the two methods is that they are robust to tweaks in the back-testing settings, ie. backtest start date or removal of outliers. Our framework is fully employed and sensitive to changes in start-date and starting capital. Small changes affect all subsequent trades. One trade that did occur in one back-test, but did not in the other, changes the portfolio composition for all other subsequent trades. These ripples effects makes replication of the backtest cumbersome. The computational cost of simulating a constant monitoring investor is also significant. The advantage of our framework is that it is closer to the return that could be expected from an investor who seeks to constantly has all his capital invested.

18 Strategy overview

There are steps in order to arrive at a given strategy with certain settings. Before constructing pairs we have to decide how pairs of stocks can be constructed. In this thesis we have chosen to limit the number of possible pair combinations to those where the stocks are from the same industry. Industry is defined as one of the 5 major SIC codes[18] (Sector Industry Classification). We also require that the stocks are members of the S&P 500.

We then differentiate between whether strategies are base models, levels adjusted models or volatility adjusted models. There is only one volatility adjusted model, which is the minimum distance strategy. In the base model and levels adjusted model there are three different strategies types, the minimum distance strategy, the correlation strategy and the cointegration strategy. The correlation strategy has four variations, SMALL, CLOSEONE, BEYOND and ONE. There are no variations to the minimum distance strategy and cointegration strategy.

Once model and strategy is chosen we need to specify the settings. There are three different types of settings:

1. Threshold crossing type, comes in three forms: BEYOND, OUTWARDS and INWARDS.
2. The standard deviation threshold comes in 6 different settings, from 0.5 to 3, in increments of 0.5.
3. The maximum holding period, comes in 7 different settings, from 1 to 7 months.

[18]The same SIC codes as Kenneth French uses , http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/changes_ind.html

The pairs trend models has the commonalities in that pairs are formed with similar requirements. The strategies also come in three different types, minimum distance strategy, correlation strategy and cointegration strategy. However as there are not generated any spread series, as there is with the traditional pairs trading models, we do not have variations of the correlation strategy. The three different strategies just determine how to score the pairs attractiveness before we calculate the trading signals on the pair. In the traditional trading signals we generate trading signals via a spread series. The pairs trend models use the trend indicator to signal for trading signals. Once a trend has been detected in both stocks of a pair and these trends are moving in opposite direction we give a trading signal. There are two different settings for generation of the trading signal.

1. The length at which the trend indicator is calibrated over. Varies from 1 to 12 months.
2. The maximum holding length of a trade. Varies from 1 to 6 month.

The pairs trend model is more naive than the traditional pairs trading model. The pairs trading model has a timed trade exit in the form of a maximum holding period and an exit signal for when the spread series converges to zero. The pairs trend model only has a timed exit via the maximum holding length. At any given moment no more then 20 pairs are invested. For nearly all of the backtested period there are almost always 20 active pairs.

The trend indicator used for the pairs trend strategy is the variance ratio test. The significance level of the variance ratio test has to be below 0.15 for us to consider it a trend. It is more preferable to set the significance level threshold to 0.05, but that would leave us with too few opportunities to trade.

18.1 Strategy setup and parameter settings

There are a huge number of possible combinations of pairs, conditions and strategy settings. Many more than my laptop can compute within reasonable time. We therefore had to leave some parameters fixed and only vary a few. The settings and conditions are as follows.

18.1.1 The backtest conditions

- The stock must have at least 252 days of trading with no missing prices.

- The stock must be a valid member of the S&P 500 as indicated by the Compustat index constituents list at the specific day of signal generation.
- If the stock has a missing price, it is exited at last observed price.

18.1.2 Strategy settings

We will briefly define the terminology used below by explaining how the pairs trading procedure works. We begin by selecting all pairs of stocks that on a given day is a member of the S&P 500 and has continuous price history from the last 252 trading days (equivalent to one year). The formation period is period from the given day and spanning back 252 trading days. The trading period is from the given day and forward. If the trading period length was 1 month the trading period would span from the given day and up to 1 month. If the backtest is set to run with a 1 day delay, then the trading period would shift forward 1 day and begin the day after the given day and end 1 day later.

In our setup a trading period is only initiated, because a trade was signaled in the formation period. This happens when the last observation in the formation period signals a trade entry. We call this signal the entry signal, as it signals the entry of a trade. In this thesis we use the term a *trade* interchangeably with a *position*. In principle, you have a position once a trade is entered. Once a trade is entered, the position is open and once a trade exited (sold) the position is closed. An entry signal is the signal to open a position and the exit signal is the signal to close the position.

When we use the term entry threshold we refer to the threshold with which the spread series needs to cross or exceed before an entry signal is generated. If the entry threshold is 2, it means the the spread series needs to exceed to 2 standard deviations (in either direction) before a trade entry signal is generated. Additional requirements on how the threshold is crossed are also specified. The way the threshold is crossed is called entry type. There are four scenarios that can occur regarding the latest observation of the spread series. First, the latest value of the spread series is within the threshold. Second, the latest value of the spread series has just crossed the threshold from the mean and outwards. Third the latest value has just crossed the threshold from the mean and inwards. Fourth the latest value is beyond the threshold, but it crossed the threshold 2 or more observations ago. Maximum holding period is the maximum time a trade can be kept open, exceeding this generates an exit signal.

1. We decide to fix the formation period length to 252 observations, so as to mimic one year of trading, identical to previous literature.

2. The maximum number of pairs that can be open at any one time is 20.

3. The entry signal is generated based on the spread value being more than a number of standard deviation away from the mean. We let this threshold vary from 0.5 to 3 std. dev. with 0.5 increments. This totals 6 different parameters settings. We call this parameter the **entry threshold** setting.

4. The entry signal has 3 different types of entry signals. The first is that we require the spread to cross the entry threshold OUTWARDS, going from a value close to the mean to one further away. Secondly, we have the opposite, namely INWARDS, going from a spread value away from the mean to one closer, while crossing the entry threshold. Lastly we can simply require the signal to be beyond the entry threshold at whatever time we observe the spread, namely the BEYOND setting. In total this is 3 different parameters settings. We call this the **entry type**.

5. The exit signal can come in various shapes. One is where the spread crosses the mean of the spread series. Another is based on how long the trade position has been open. The maximum trade length, before issuing an exit signal is varied from 21 to 147 trading days with increments of 21 days. This mimics holding periods of 1 to 7 months. Totaling 7 different parameters settings. We call this the **max. holding period**.

6. Pairs are ranked based on their measure of correlation, their average distance or their cointegration test statistic depending on the type of strategy being backtested. The correlation strategy ranks by the measure of correlation in a descending manner. The minimum distance strategy ranks the average distance in an ascending manner. The cointegration strategy ranks based on the cointegration test statistic in a descending manner (p-value/statistical significance is inversely related to the t statistic). [19]

In total a single strategy gives 3 different types of entry signals, 6 different entry threshold settings and 7 different maximum holding period lengths. The total amount of permutations is $3 \times 6 \times 7 = 126$.

[19]Of this we find maximum 22 that generated entry signals and use these to enter positions. This shortcut only makes difference if there is a delay between signal generation and the time a actual position is taken and the stock simultaneous displays missing prices, thereby making it impossible for a position to be taken. A rare occurrence.

A single strategy takes roughly 3 and a half hours. 126 permutations run sequentially, would require 126 separate strategy runs. To get all the different permutations, would take $3.5 \times 126 = 441$ hours. Not even including separate runs for with and without transactions cost, as transaction are varying through out time. Luckily many of the computations done are shared. By caching shared calculations and running all permutations simultaneously we can save an enormous amount of processing time. Implementing this reduces the total runtime to roughly 5 hours per 126 permutations.

18.1.3 Design of the program

There was significant effort invested into choosing the right programming language to create the back-testing platform. Initially higher level programming languages such as R and Matlab were considered. However given the scope of this backtest it was doubtful whether the higher level programming languages could achieve the required performance. It is not to say that it is impossible, it would however require significant expertise in Matlab or R and possibly some serious hardware to cope, neither of these are at this authors disposal. The lower level programming language Java was chosen as a compromise between development ease and performance. The mathematical library [SuanShu] was employed to perform the Johansens Cointegration tests.

The key to reduce the computational time is to reduce the amount of computations. This is done by never doing the same calculation twice. Each Calculation Object checks its own designated cache to see if a computation already has been computed. If it has then it simply retrieves the results from the cache. When a given trading day has passed in the loop, the cache is flushed to free memory for new calculation results. The program flow and design can been seen in Figure 1. The program begins by loading the data and instantiating the Simulator Object based on the specified parameters. The Simulator Object constructs a map with all the different strategy permutations. Each strategy permutation has its own Strategy Object (calculates the entry/exit signals), Account Object (handles the portfolio) and Printer Object (prints results). The Strategy Objects can reference the various shared Indicator Objects, eg. Johansen Indicator Object, Correlation Indicator Object, Minimum Distance Indicator Object, Standard Deviation Indicator Object, Mean Crossing Indicator Object, Variance Ratio Indicator Object, OLS Indicator Object. The Indicator Objects performs the calculations. Only the Signal Object is not shared between strategy permutation. At the end of each month the results

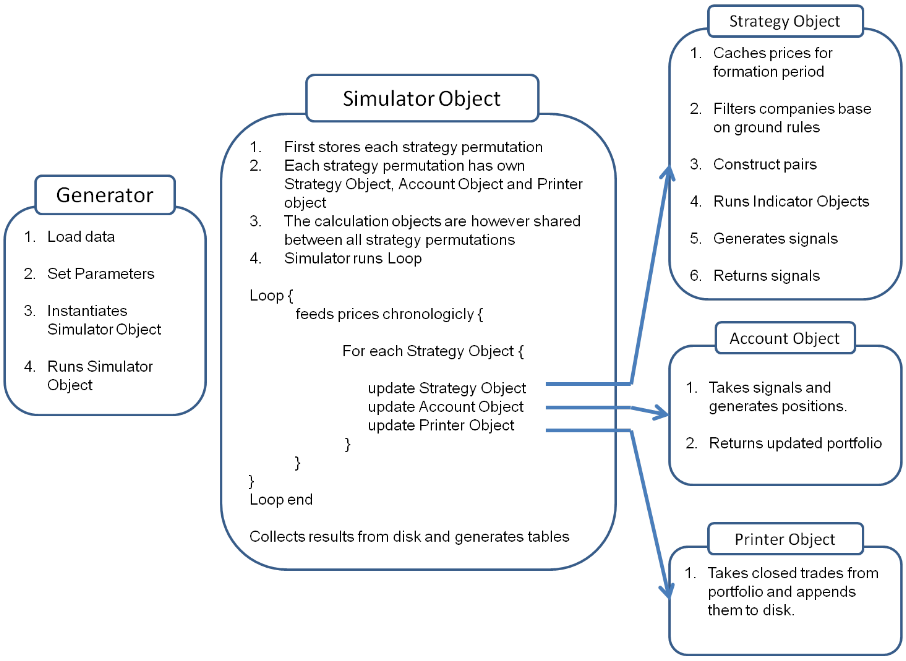


Figure 1 – The program outline.

are written to the hard drive in separate files to free memory. When all the prices for all the days have been looped through, the program collects the results from the hard disk and creates tables in Excel.

18.2 Strategies in depth - Trade examples

In order to fully understand all the intricacies for each primary strategies, we explicitly walk through a trade of the minimum distance strategy and the correlation strategy. Many strategies assume constant rebalancing, but ours do not. Constant rebalancing is used to keep the ratio of the two pairs of stocks constant throughout the trading period. When a position in a pair is taken, stock is bought (shorted) in one stock and shorted (bought) in the other. The value of the positions in the two stocks will change as the stock prices change. These changes will affect the original ratio between the two stocks. To keep the original ratio constant there has to be a constant rebalancing as the ratio between the two stocks change. We will now present examples that illustrates this.

All the strategies begin by normalizing the cumulative return for a pair of stocks in a given period. In our example the formation period span will be 252 trading days, corresponding to one year. Figure 2 illustrates the cumulative returns graph for the two stocks Boatmens Bancshares and BankAmerica in 1994 and 1995. In the mid nineteens both these stocks were some of the largest banking corporations

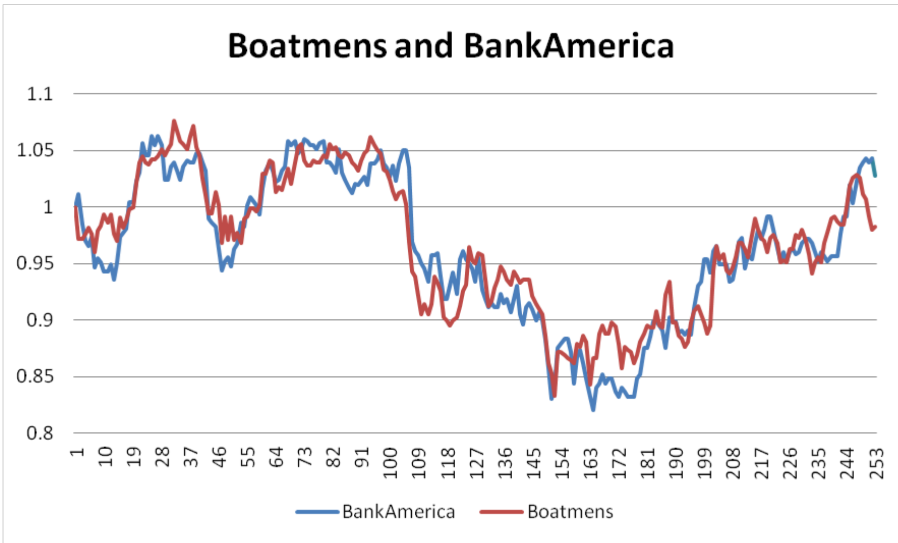


Figure 2 – The normalized cumulative returns for Boatmens and BankAmerica. The first observation is on the 20th of April 1994 and the last is on 19th of April 1995.

in the US.

18.2.1 Minimum distance - Trade example

The exact pair of stocks traded in this example are trades taken in the backtest of the minimum distance strategy. The settings for the strategy of the minimum distance strategy was as the following settings:

- 252 days formation period and max. holding period of 21 trading days, excluding signal-delay days.
- The delay between the signal generation and actual trade entry is 1 trading day.
- Entry threshold of 2 standard deviations.
- Entry type was INWARDS. Only triggering when the spread crosses the threshold inwards.
- The exit signal is when the two accumulated return series cross (when the spread series is zero or crosses zero value).
- The two stocks are weighted equally (dollar neutral).

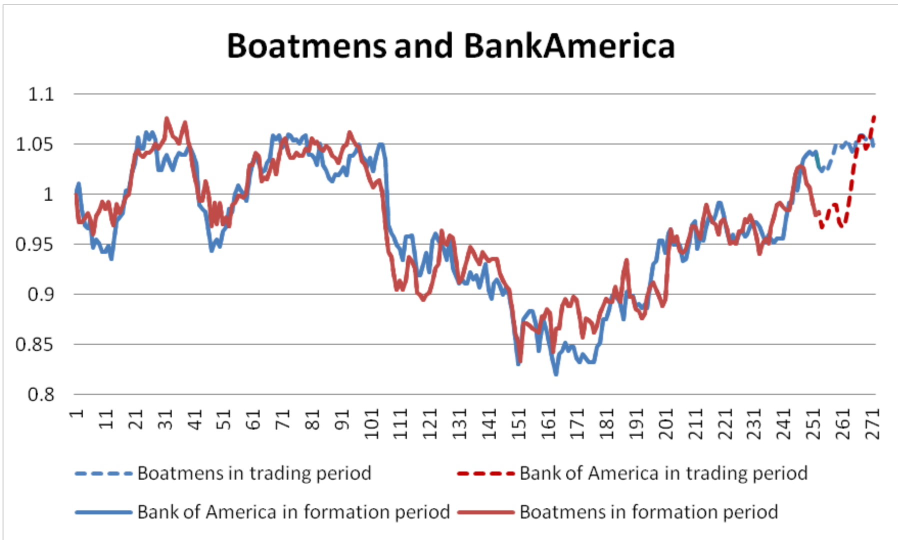


Figure 3 – Boatmens Bancshares was one of the largest banking corporations in US when it was acquired by NationsBank in 1996, which in 1998 acquired BankAmerica to become Bank of America. The accumulated return series of both stocks have been normalized to begin at 1. The first 252 observations are the formation period, the next 19 are in the trading period. The 253rd is the signal delay of 1 trading day. The subsequent observations denote the trading period, which is the dotted line. The exit signal is also delayed 1 day. The two delay periods are included in the trading period.

In Figure 3 we present two stocks that were traded by the minimum distance strategy on May 1995. On 16th of May 1995 (252nd observation) the trade is signaled as the spread series scrosses the 2 standard deviations threshold, inwards towards the mean of the spread series. The standard deviation threshold is calculated based on the spread values from the 1st observation to the 252nd observation. Figure 4 illustrates the spread between the two stocks calculated as $r_{Boatmens} - r_{BankAmerica}$. The dashed dots mark the period where the trade has been initiated, the trading period including the 2 delay days of the trade initiation and closure. In Figure 4 we can see that around the 252th observation the spread series has crossed the 2 standard deviation threshold outwards and subsequently barely crosses the threshold inwards.

The upper std. dev. threshold is 2 std. dev. from the mean which correspond to a spread value of 0.047. At the 251st observation the spread value was 0.063 and 0.045 on the 252nd observation. Thereby crossing the upper std. dev. threshold inwards towards zero, satisfying the entry type INWARDS. The trade was entered after the 1 day signal delay at the 253rd observation with a spread value of 0.056. In Figure 2, at 253rd observation the Boatmens stock had a cumulative normalized return of 1.023, while the BankAmerica stock had a cumulative normalized return of 0.968 when the trade initiated. Since the spread is calculated as $r_{Boatmens} - r_{BankAmerica} = spread$ we need to short the spread portfolio.

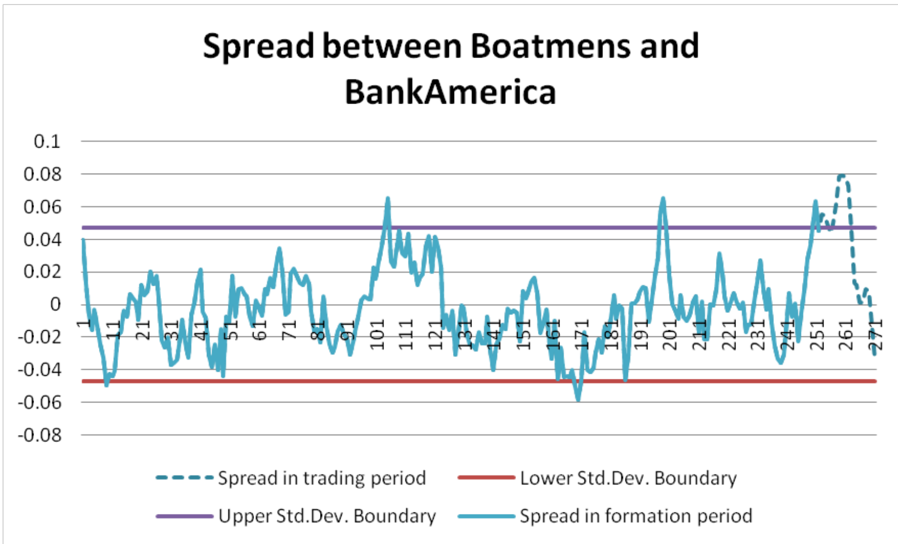


Figure 4 – The spread series between Boatmens and BankAmerica. The upper and lower std. dev. thresholds are calculated based on the first 252 observations and are 2 standard deviations from zero. The first 252 observations are the formation period, the next 19 are in the trading period. The 253rd is the signal delay of 1 trading day. The subsequent observations denote the trading period, which is the dotted line. The exit signal is also delayed 1 day. The two delay periods are included in the trading period.

The minimum distance strategy is inherently dollar neutral as the positions are equal in size. Dollar neutral means to say that we are long and short equal amounts. We short-sell Boatmens stock and buy BankAmerica stock. On the 270th observation the spread crosses the zero threshold and the exit signal is triggered. 1 trading day later the trade is liquidated at a spread value of -0.034, at which point the Boatmens stock rises to 1.043 and the BankAmerica to 1.077. We were short the Boatman stock and long the BankAmerica stock, so we had a loss on the Boatman stock and a gain on the BankAmerica stock. The loss on the Boatmens stock was $\left(-\frac{1.043}{1.023}+1\right) \times 100\% = -1.9\%$, while the gain on the BankAmerica stock was $\left(\frac{1.077}{0.968}-1\right) \times 100\% = 11.3\%$. The total return was 9.4%, excluding transaction cost and interest rate gains.

18.2.2 A non-equal weighted hedge-ratio - Trade example

The correlation strategy and cointegration strategy only differ in how they rank the candidate pairs and how the hedge ratio is calculated. The correlation strategy begins by ranking the stocks in a descending fashion, with the most correlating pairs being the most attractive. The cointegration strategy ranks by the t-statistic value, the higher the t-statistic value (the lower the p-value) the more attractive the pair is. The minimum distance strategy ranks the pairs in ascending fashion and determines the hedge ratio between the two stocks by simply weighting them equally. The general approach to pairs trading by [Vidyamurthy 2004] was to sort pairs by the most correlating, then to determine the hedge ratio by Ordinary Least Squares (OLS) procedure. The residuals from the OLS regression were used to test if the pair was stationary, using the Augmented Dicky Fuller test (ADF). In essence this is a double-sort method, first filtering out a subset of pairs with correlation then ranking using cointegration test statistics[20]. It is the most correlating pairs that are chosen for the cointegration test. Those pairs that are most statistically significant cointegrating are the ones we choose to trade. This means that the cointegration strategy relies not only on cointegration methods, but also the correlation measure. The correlation strategy does not use cointegration methods. It uses OLS to find the hedge ratio.

By using the OLS procedure we run into some small issues. The regression of y on x is not the inverse of regressing x on y. In regression terms $y = ax + b \not\Leftrightarrow x = y\frac{1}{a} - \frac{b}{a}$. The hedge ratios we retrieve from the regressions are not the inverse of each other, the order is dependent. The intuition can be illustrated as followed. The regression coefficient is defined as:

$$\frac{cov\left(x,y\right)}{var\left(x\right)}$$

$$cov\left(x,y\right) = cov\left(y,x\right)$$

Since covariance of x and y is the same as the covariance of y and x, we can conclude that the larger regression coefficient of two will be the one with the smaller variance. Vice versa the smaller regression coefficient will come from the time series with the larger variance. [Vidyamurthy 2004] suggests using the larger of the two regression coefficients to reduce precision error. However one could however also argue that the results should not differ in either way since the statistical significance difference is negligible.

We have chosen to focus on four different variations. Using the biggest coefficient, the smallest coefficient, the coefficient closest to 1 and simply weighting them equally as in the minimum distance strategy.

20

[Vidyamurthy 2004] also mentioned testing trade-ability by requiring a certain amount crossings around the mean of the spread series

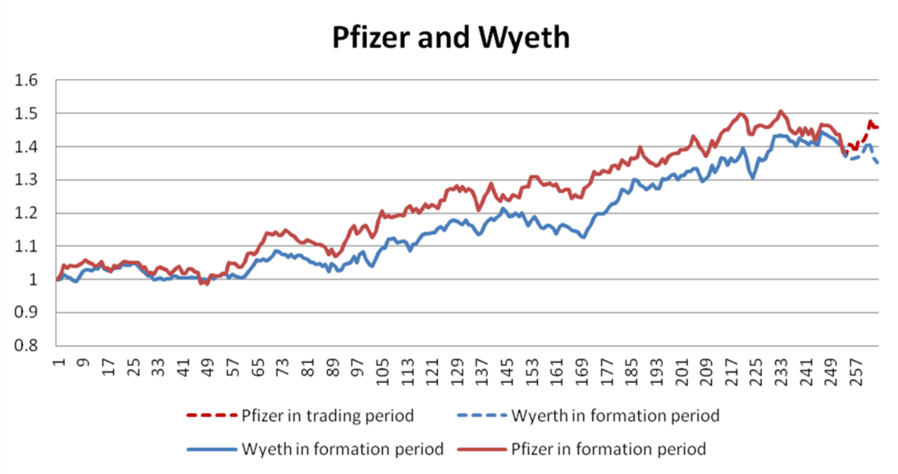


Figure 5 – The normalized cumulative return series of the two stock Pfizer and Wyeth.

The following example illustrates how a non-equal weighted pairs trade is conducted. The example was an actual simulated trade in the backtest of the correlation strategy. The settings for the correlation strategy was:

- 252 days formation period and max. holding period of 21 trading days, excluding signal-delay days.
- The delay between the signal trigger and actual trade entry is 1 trading day.
- Using the coefficient closest to 1, variation CLOSEONE.
- 2 standard deviation entry threshold.

The Figure 5 shows the two normalized cumulative returns for stock Pfizer and Wyeth. The two stocks do seem to follow each other in tandem, but their return curves do not seem to cross each other more than a handful of times. The intention of the correlation strategy is not to exit when the pairs cross each others curve, but rather to exit when the stocks return to the relative mean defined by the spread series. In contrast to the trade in the minimum distance strategy, the number of zero/mean crossings is fewer, 24 versus 52.

The spread series is shown Figure 5. The hedge ratio is 1.086 and the mean of the spread series -0.036. The lower std. dev. threshold is -0.11. At the 251th observation the spread value is -0.09 and -0.135 on the 252th observation. Thereby crossing the lower std. dev. threshold inwards towards the mean,

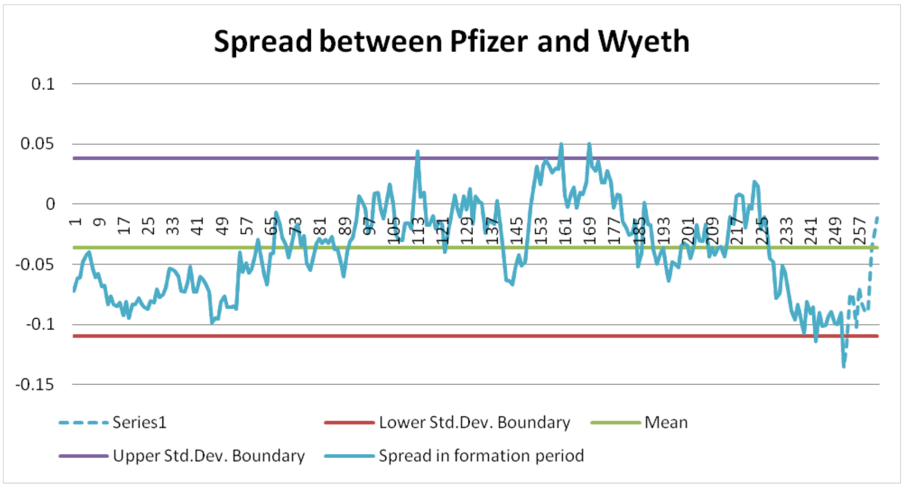


Figure 6 – The spread series between Pfizer and Wyeth.

satisfying the entry type INWARDS. The trade is entered after the 1 day signal-delay at the 253th observation of -0.118. In Figure 5 on the 253th observation the Pfizer stock is at 1.3704, while the Wyeth stock is at 1.3699 when the trade is initiated. We long the spread by buying the portfolio combination. We short-sell 1.086 Wyeth stock for every 1 Pfizer stock that we buy. On the 262th observation the spread series crosses the mean threshold and the exit signal is triggered. 1 trading day later the trade is liquidated at a spread value of -0.0115, at which point the Pfizer stock has risen to 1.459 and the Wyeth stock to 1.354. The Pfizer stock gains in value while the Wyeth stock loses in value. The gain on the Pfizer stock is $\left(\frac{1.459}{1.3704} - 1\right) \times 100\% = 6.5\%$, while the gain from shorting the Wyeth stock is $-\left(-\frac{1.37}{1.354} + 1\right) \times 100\% = 1.2\%$. The total return is $6.5\% \times \frac{1}{1+1.086} + 1.2\% \times \frac{1.086}{1+1.086} = 3.7\%$, excluding transaction cost and interest rate gains.

19 Argumentation for use of levelsadjusted models

When you do not continuously rebalance the portfolio there occurs an irregularity in the weighting scheme. The minimum distance strategy weighs the stocks in the proportion one to one. The correlation strategy uses OLS to determine the hedge. The cointegration strategy uses the cointegrating coefficient. There is a mismatch between the spread series and the actual value of the pairs trading portfolio.

In the case of the minimum distance strategy the exit signal is generated when the cumulative return series cross each other, which is when the spread crosses zero. Suppose stock A has the cumulative

Stock A	Stock B	Spread	Stock A return	Stock B return	Portfolio return
1.6	1.2	0.4	-33%	50%	8%
1.5	1.1	0.4	-25%	38%	6%
1.4	1	0.4	-17%	25%	4%
1.3	0.9	0.4	-8%	13%	2%
1.2	0.8	0.4	0%	0%	0%
1.1	0.7	0.4	8%	-13%	-2%
1	0.6	0.4	17%	-25%	-4%
0.9	0.5	0.4	25%	-38%	-6%
0.8	0.4	0.4	33%	-50%	-8%

Table 2 – Table of the hypothetical Stock A and Stock B values and various portfolio returns, while the spread value is constant.

return p_t^A. The 252th observation, (the most recent return in the formation period) is 1.2. For stock B p_t^B, the 252th observation is 0.8. Then we can calculate the spread as.

$$p_{252}^A - p_{252}^B = spread$$

$$1.2 - 0.8 = 0.4$$

Suppose an entry signal also is generated at the 252th observation. Assuming we have no delay, we short 1 monetary unit of A and buy 1 monetary unit of B. The exit signal is when the spread crosses zero. There are situations, where the spread value does not change, while the value of the portfolio does.

Then suppose that the cumulative return on stock A decreases to 1.1, a loss of 8% and stock B's cumulative return also decreases to 0.7, a loss of 13%. Since we are short stock A, the 8% loss is actually a gain. Our total portfolio can be calculated as $r^A \times w^A + r^B \times w^B = r^{port}$, with w denoting the weights and r the returns. The result is a total portfolio loss of 4%, even though the spread value still is 0.4, $1.1 - 0.7 = 0.4$. In Table 2 we can see that for that various values the portfolio value of the two pairs of stocks fluctuate, while the spread value is constant.

This shows that the spread value can become non-informative of portfolio value. Table 3 shows that there also are many different outcomes, for when the exit signal is generated.

Since the correlation strategy and the cointegration strategy allows for unequal weighted position sizes the outcome of an exit signal can even become negative, as shown in Table 5. In the example Table 5 we

Stock A	Stock B	Spread	Stock A return	Stock B return	Portfolio return
1.2	1.2	0	0%	50%	50%
1.1	1.1	0	8%	38%	46%
1	1	0	17%	25%	42%
0.9	0.9	0	25%	13%	38%
0.8	0.8	0	33%	0%	33%
0.7	0.7	0	42%	-13%	29%
0.6	0.6	0	50%	-25%	25%
0.5	0.5	0	58%	-38%	21%
0.4	0.4	0	67%	-50%	17%

Table 3 – Table of the hypothetical Stock A and Stock B values and various portfolio returns, when the spread value is zero (exit signal).

Stock A	Stock B	Spread	Stock A return	Stock B return	Portfolio return
1.6	1.6	0.8	-33%	100%	33%
1.5	1.4	0.8	-25%	75%	25%
1.4	1.2	0.8	-17%	50%	17%
1.3	1	0.8	-8%	25%	8%
1.2	0.8	0.8	0%	0%	0%
1.1	0.6	0.8	8%	-25%	-8%
1	0.4	0.8	17%	-50%	-17%
0.9	0.2	0.8	25%	-75%	-25%
0.8	0	0.8	33%	-100%	-33%

Table 4 – Table of the hypothetical Stock A and Stock B values and various resulting portfolio returns. All scenarios construct so that the spread value is same in all cases.

have assumed a hedge ratio of 0.5 (1:2) and a mean spread of 0.6. The correlation and cointegration are set to exit when the spread crosses the mean of the series. Similarly to the minimum distance strategy we assume the portfolio was bought at the levels of 1.2 for stock A and 0.8 for stock B. The resulting spread when signal is generated is then $1.2 - 0.8 \times 0.5 = 0.8$. In Table 4 we can see that the spread can stay at the same level, while the portfolio loses or gains in value.

Let us assume that spread drops to 0.6 and we exit our positions. In Table 5 we can see various exit scenarios some have large gains and others losses, though all are with the spread value of 0.6. In the case where we have non-equal weights in the pairs of stock and large fluctuations to the stock prices the portfolio value not only varies, but are in some cases also negative.

To get the spread values to be consistent with the portfolio return we need to let the relative value of the two stocks A and B be proportional to the the positions sizes in A and B along with the hedge ratio. Let us consider two different exit scenarios. The first is one where Stock A declines, while Stock B is flat. The second is where Stock A is flat, while Stock B rises. In equal weighted strategy, we enter at equal proportions on each stock. When the spread indicates a divergence we short Stock A and buy Stock B.

Stock A	Stock B	Spread	Stock A return	Stock B return	Portfolio return
1.4	1.6	0.6	-17%	100%	42%
1.3	1.4	0.6	-8%	75%	33%
1.2	1.2	0.6	0%	50%	25%
1.1	1	0.6	8%	25%	17%
1	0.8	0.6	17%	0%	8%
0.9	0.6	0.6	25%	-25%	0%
0.8	0.4	0.6	33%	-50%	-8%
0.7	0.2	0.6	42%	-75%	-17%
0.6	0	0.6	50%	-100%	-25%

Table 5 – Table of the hypothetical Stock A and Stock B values and various portfolio returns assuming a hedge ratio of 0.5. If we let a spread value of 0.6 be the mean and exit value.

Stock A	Stock B	Spread	Weight A	Weight B	Stock A return	Stock B return	Portfolio weighted return
1.6	0.8	0.8			-33%	0%	-17%
1.5	0.8	0.7			-25%	0%	-13%
1.4	0.8	0.6			-17%	0%	-8%
1.3	0.8	0.5			-8%	0%	-4%
1.2	0.8	0.4	0.5	0.5	0%	0%	0%
1.1	0.8	0.3			8%	0%	4%
1	0.8	0.2			17%	0%	8%
0.9	0.8	0.1			25%	0%	13%
0.8	0.8	0			33%	0%	17%

(a) Stock A declines. Stock B is flat.

Stock A	Stock B	Spread	Weight A	Weight B	Stock A return	Stock B return	Portfolio weighted return
1.2	1.2	0			0%	50%	25%
1.2	1.1	0.1			0%	38%	19%
1.2	1	0.2			0%	25%	13%
1.2	0.9	0.3			0%	13%	6%
1.2	0.8	0.4	0.5	0.5	0%	0%	0%
1.2	0.7	0.5			0%	-13%	-6%
1.2	0.6	0.6			0%	-25%	-13%
1.2	0.5	0.7			0%	-38%	-19%
1.2	0.4	0.8			0%	-50%	-25%

(b) Stock A is flat. Stock B rises.

Table 6 – Equal weighted scenarios.

The two scenarios illustrated in Table 6 show that when Stock B rises, the gain is 25% and only 17% when Stock A declines. This is because a 40 percentage point increase for Stock B is a 50% gain, while a 40% percentage point drop for Stock A is a mere 33% short-selling gain.

The two scenarios illustrated in Table 7 are identical to Table 6, the only difference is we weight the positions in Stock A and B by the levels instead of equal-weighting. The weight for Stock A is $\frac{1.2}{1.2+0.8} = 0.6$ and for Stock B $\frac{0.8}{1.2+0.8} = 0.4$. Thereby the returns on Stock A are leverage 3:2 compared to Stock B, resulting in a 20% gain if either Stock A drops 33% or Stock B gains 50%. Irrespective of Stock A declining or Stock B rising, the portfolio gains 20% when the spread goes to zero in either case.

Stock A	Stock B	Spread	Weight A	Weight B	Stock A return	Stock B return	Portfolio weighted return
1.6	0.8	0.8			-33%	0%	-20%
1.5	0.8	0.7			-25%	0%	-15%
1.4	0.8	0.6			-17%	0%	-10%
1.3	0.8	0.5			-8%	0%	-5%
1.2	0.8	0.4	0.6	0.4	0%	0%	0%
1.1	0.8	0.3			8%	0%	5%
1	0.8	0.2			17%	0%	10%
0.9	0.8	0.1			25%	0%	15%
0.8	0.8	0			33%	0%	20%

(a) Levels adjusted weighting. Stock A declines.

Stock A	Stock B	Spread	Weight A	Weight B	Stock A return	Stock B return	Portfolio weighted return
1.2	1.2	0			0%	50%	20%
1.2	1.1	0.1			0%	38%	15%
1.2	1	0.2			0%	25%	10%
1.2	0.9	0.3			0%	13%	5%
1.2	0.8	0.4	0.6	0.4	0%	0%	0%
1.2	0.7	0.5			0%	-13%	-5%
1.2	0.6	0.6			0%	-25%	-10%
1.2	0.5	0.7			0%	-38%	-15%
1.2	0.4	0.8			0%	-50%	-20%

(b) Levels adjusted weighting. Stock B rises.

Table 7 – Levels adjusted weighting.

Aside from the base models of minimum distance, correlation and cointegration, we will also investigate the levels adjusted models. We consider it to be interesting if the levelsadjusted models provided additional value to the traditional pairs trading models.

Part VII

Analysis

The pairs trading and pairs trend backtest analysis is split into an in-sample and an out-of-sample analysis. The in-sample spans 1st of Jan. 1988 to 1st of Jan. 2003, while the out-of-sample spans 1st of Jan. 2003 to 24th Oc. 2014. The in-sample period is chosen for easy comparison with [Gatev 2006], [Do and Faff 2010] and [Do and Faff 2011]. For the cointegration persistence analysis we use the full sample, but as the whole year of 2014 is not included in the data the cointegration persistence analysis only stretches from 1988 to 2013. Section 20 presents the cointegration persistence analysis and section 21 the in-sample analysis for the pairs trading strategies and pairs trend strategies. Section 22 presents the pairs trading strategies and pairs trend strategies out-of-sample results and analysis.

20 Cointegration persistence

In this section we will attempt to search for statistical evidence that cointegration persists. Traditional pairs trading as defined in this thesis and in [Gatev 2006], [Do and Faff 2011] and [Do and Faff 2010] relies on the hypothesis that cointegration persists through time. If so then it might be feasible to construct profitable investment strategy.

[Clegg 2014] investigated the persistence of cointegration in a pair of stock from one year to the next. His sample data was the S&P 500. [Clegg 2014] examined the cointegration persistence from 2002 to 2012 using Yahoo as the stock data source. [Clegg 2014] concluded that there was no significant persistence in the cointegration of stocks.

In light of [Clegg 2014]'s discoveries we will briefly examine the persistence of cointegration in our data. We begin by measuring the cointegration of all stock pairs in a given year and see how many continue to cointegrate in the subsequent year, using the parameters obtained in the previous year. We exclude stocks that are already stationary as the resulting cointegration will be nonsensical.

Before we measure the cointegration in a pair we need to sort for stationary stocks and make sure the stocks have continuous prices in the whole two year period and satisfy the data requirements in part V. Additionally we also make sure the stock is a member of the S&P 500 at the end of the trading period.

In Table 8 we can see the amount of cointegrating pairs in year y, the formation period and in $y+1$, the trading period. In the formation period column, $N(1)$ is the number of pairs that are non-stationary in the formation period. $x \in CI(y)$ is the number of cointegrating pairs. $Pr\left(x \in CI(y)\right)$ is calculated as $Pr\left(x \in CI(y)\right) = \frac{x \in CI(y)}{N(1)}$ and is the percentage of pairs that are cointegrating in the formation period for the given year y. The cointegration significance levels is 0.05%. The mean of ρ is the mean of the autoregressive coefficients of the spread series in all of the cointegrating pairs. The std. of ρ is the standard deviation of the autoregressive coefficients.

The mean of ρ is below one and ranges from 0.95 to 0.97. If we interpret the $Pr\left(x \in CI(y)\right)$ as the probability of a pair cointegrating in the respective period then we can say the probability to

	Excluding stocks that are I(0) in period y									
	Formation Period (y)					Trading Period (y+1)				
y	N(I1)	$x \in CI$ (y)	$Pr(x \in CI\ (y))$	mean of ρ	std. of ρ	N(I1)	$x \in CI$ (y+1)	$Pr(x \in CI\ (y+1))$	mean of ρ	std. of ρ
1988	88831	3614	4.1%	0.950	0.029	79003	1626	2.1%	0.980	0.017
1989	87990	3045	3.5%	0.963	0.028	86320	2604	3.0%	0.980	0.018
1990	97461	5280	5.4%	0.964	0.025	70876	3034	4.3%	0.976	0.021
1991	73536	4896	6.7%	0.963	0.027	68265	2075	3.0%	0.972	0.022
1992	92235	4081	4.4%	0.958	0.027	84666	1459	1.7%	0.973	0.019
1993	91378	3618	4.0%	0.958	0.025	81810	2227	2.7%	0.971	0.023
1994	88410	4746	5.4%	0.956	0.029	78606	1737	2.2%	0.982	0.018
1995	89253	3118	3.5%	0.960	0.028	81003	2839	3.5%	0.975	0.020
1996	93528	4181	4.5%	0.960	0.026	88410	1417	1.6%	0.980	0.020
1997	97020	2484	2.6%	0.963	0.028	93096	2196	2.4%	0.978	0.019
1998	98790	3540	3.6%	0.963	0.025	90525	1895	2.1%	0.977	0.021
1999	96141	2779	2.9%	0.961	0.026	83436	2096	2.5%	0.974	0.021
2000	91806	3696	4.0%	0.957	0.026	73920	2362	3.2%	0.970	0.019
2001	86736	6828	7.9%	0.956	0.024	81810	2012	2.5%	0.977	0.020
2002	106491	6472	6.1%	0.958	0.027	75466	2511	3.3%	0.980	0.018
2003	80200	3643	4.5%	0.963	0.024	72390	3296	4.6%	0.979	0.018
2004	101926	5341	5.2%	0.965	0.023	94830	2192	2.3%	0.977	0.018
2005	102831	3170	3.1%	0.962	0.023	97461	1869	1.9%	0.980	0.017
2006	102831	3640	3.5%	0.965	0.022	97020	2091	2.2%	0.979	0.020
2007	106953	4035	3.8%	0.959	0.028	102378	1689	1.6%	0.982	0.021
2008	112101	7705	6.9%	0.952	0.031	48205	2308	4.8%	0.975	0.021
2009	51360	2974	5.8%	0.956	0.027	48828	1098	2.2%	0.979	0.017
2010	110685	3799	3.4%	0.966	0.021	101926	3271	3.2%	0.977	0.019
2011	102831	5243	5.1%	0.959	0.024	93096	3590	3.9%	0.976	0.017
2012	100576	5393	5.4%	0.962	0.022	88410	2343	2.7%	0.980	0.018

Table 8 – This table shows the number of pairs that exhibit cointegration in the formation period and then in the trading period. Column $N(I1)$ is the number of pairs under examination in the given period. Column $x \in CI(y)$ and $x \in CI(y+1)$ are the number of pairs that cointegrate in period y and $y+1$, respectively. ρ is the autoregressive coefficient of the time series produced from the cointegrating pair.

cointegrate ranges from 2.6% to 7.9% in the formation period. In the trading period the number of pairs with no stationary stocks decreases from the formation period. This is because the pairs in the trading period is a subset of the pairs in the formation period. From when the pairs in the formation period are calculated and carried over to the trading period, some stocks are evaluated to be stationary in the trading period and are therefore excluded. Filtering from a subset further reduces the amount of pairs under consideration.

In Table 9a we can see the amount of eligible stock pairs in each period. Looking at the formation period column in table 9a we can see considerable drops in the amount of non-stationary stocks in year 1991, 2003 and 2009. [Clegg 2014]'s results also showed a large drop in number of non-stationary pairs in year 2009.

The number of stocks in the trading period are lower than the number of stocks in the formation period as only the ones computed in the formation period are carried over to the trading period. Between the calculations in the formation period and the calculations in the trading period some stocks are

y	Formation Period (y)	Trading Period (y+1)
1988	421	397
1989	419	415
1990	441	376
1991	383	369
1992	429	411
1993	427	404
1994	420	396
1995	422	402
1996	432	420
1997	440	431
1998	444	425
1999	438	408
2000	428	384
2001	416	404
2002	461	388
2003	400	380
2004	451	435
2005	453	441
2006	453	440
2007	462	452
2008	473	310
2009	320	312
2010	470	451
2011	453	431
2012	448	420

(a) This table shows the number of stocks eligible for calculation after sorting for stationary stocks on top of the normal requirements such as continuous time series and member of the S&P 500 at the end of the trading period.

y	Pr $(x \in CI(y+1) \mid x \in CI(y))$	$Pr(x \in CI\ (y+1))$
1988	2.9%	2.1% ***
1989	3.9%	3.0% ***
1990	3.6%	4.3% ***
1991	3.4%	3.0%
1992	2.4%	1.7% ***
1993	3.0%	2.7% *
1994	2.4%	2.2%
1995	3.3%	3.5%
1996	2.4%	1.6% ***
1997	3.8%	2.4% ***
1998	2.9%	2.1% ***
1999	3.4%	2.5% ***
2000	3.6%	3.2%
2001	4.6%	2.5% ***
2002	2.7%	3.3% ***
2003	4.4%	4.6%
2004	3.5%	2.3% ***
2005	2.7%	1.9% ***
2006	3.1%	2.2% ***
2007	2.7%	1.6% ***
2008	2.2%	4.8% ***
2009	2.3%	2.2%
2010	6.7%	3.2% ***
2011	4.1%	3.9% ***
2012	2.3%	2.7% ***

(b) This table shows the significance levels in the various y years. Statistical significance is marked with *, ** and *** for the significance levels, 10%, 5% and 1%, respectively

Table 9 – Cointegration persistence tables.

dropped. This is because they in the trading period have been evaluated to be stationary and therefore are excluded. Since the pairs of stock in the trading period are a subset of the pairs in the formation period, the amount of pairs in the trading period can at best be equal to the amount of pairs in the formation period. However because there always are some stocks that display stationary behavior in the trading period and as such are excluded, the trading period therefore often contains less pairs than the formation period.

Our aim is to see if the cointegrating pairs from the formation period are more likely to cointegrate in the trading period than the non-cointegrating pairs. We can formulate this as the following equation:

$$Pr\left(x \in CI\left(y+1\right) \mid x \in CI\left(y\right)\right) > Pr\left(x \in CI\left(y+1\right)\right)$$

The left side of the equation is the probability that a pair is cointegrating in the trading period $(y+1)$

given that it was cointegrating in formation period (y). The probability on the left side is calculated as the number of cointegrating pairs in the trading period that also are cointegrated in the formation period divided by the total number of cointegrating pairs in the formation period. The right side of the equation is the probability that a pair is cointegrating in the trading period. This is calculated as the number of cointegrating pairs in the trading period divided by the total number of pairs in the formation period. We evaluate this expression each year, spanning from year 1988 to 2013.

To test for the statistical significance we use the χ^2 test, Chi-square goodness of fit test, in similar fashion to [Clegg 2014]. For each year y we use χ^2 test with one degrees freedom to compute the significance levels.

The results are shown in Table 9b. The statistical significance is marked with *, ** and *** for the significance levels, 10%, 5% and 1%, respectively.

Of the various years sampled 18 out of the 25 years were statistically significant at the 5% level, but 3 of those years were significant in the wrong direction, instead supporting the hypothesis:

$$Pr\left(x \in CI\left(y+1\right) \mid x \in CI\left(y\right)\right) < Pr\left(x \in CI\left(y+1\right)\right)$$

The years 2002 and 2008 were followed by years where the markets rose after having fallen. In those years the probability of cointegrating (assuming the stock was not measured to be stationary) was higher than previous years despite using parameters calibrated for the previous year and being filtered for stationary stocks. This is likely to be due to the market behavior being dominated by a single or few determining factors. In the last five years many market gyrations have often been ascribed to a risk on/risk off type behavior[21]. In the early 2000s we had the dot-com bubble were markets tanked for almost three years[22]. In the early 1990s there was a recession, which was preceded by the 1987 stock market crash and the Savings and Loans Crisis, compounded by rising oil prices[23].

It might be that stocks cointegrate due to the common market risk factor. To proxy for this we control for beta exposure to the S&P 500. We regress stock prices to the S&P 500 to find the slope coefficient

[21] http://www.investopedia.com/terms/r/risk-on-risk-off.asp
[22] http://en.wikipedia.org/wiki/Dot-com_bubble
[23] http://bancroft.berkeley.edu/ROHO/projects/debt/1990srecession.html

(with intercept) and calculate the expected return of the given stock in each day of this two year span. We subtract the expected return from the actual return to get the beta adjusted return series of the stock. Controlling for this factor the amount of cointegrating pairs should be reduced. Table 37 shows the results. The average amount of cointegrating pairs in the formation period and trading period in the unadjusted case is, 4.60% and 2.78%. In the adjusted case the average is 4.67% and 2.11%.

Table 39 shows that adjusting for market exposure did not change the picture much. The number of significant years drop by two, but the number of cointegrating pairs remains high at 16 out of 25. This leads us to suspect that an alternative factor must be driving persistence in cointegration. However the purpose of this exercise was not to find the factor driving cointegration of stocks, but test if the effect was there.

[Clegg 2014] concludes that there was not any persistence in cointegration using this and other methods. We suspect that the difference in results come from how we determine our data. Firstly [Clegg 2014] had the issue of survivorship bias. Secondly the further back his data went the fewer stocks he had. This means that our calculations are not drawn from the exact same data and there in lies the crux of the problem. One could then suggest that the calculations are sensitive to changes in data. However the Chi-squared test statistics are quite significant, the majority of p-values are below 0.01%, although not all years were exhibiting persistence in cointegration.

One note to be made is that this analysis was done on the yearly frequency. The pairs trading models in this thesis show only up to seven months of trading periods. For some models the most profitable was very short holding periods, eg. a month.

21 In-sample

We begin our analysis by examining the performance of the 3 main strategies, minimum distance, correlation and cointegration. We define average return as the total return of equity and not excess return, where excess return is return discounted by the risk-free rate.

21.1 Base models

All the tables in the following subsection 21.1 have a common colored scale.

	Standard deviation					
				Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-0.10%	-0.03%	0.10%	0.12%	0.24%	0.25%
2	0.06%	0.13%	0.19%	0.26%	0.30%	0.26%
3	0.09%	0.14%	0.13%	0.22%	0.30%	0.28%
4	0.14%	0.15%	0.14%	0.17%	0.21%	0.22%
5	0.11%	0.16%	0.13%	0.27%	0.21%	0.22%
6	0.10%	0.13%	0.16%	0.21%	0.22%	0.27%
7	0.14%	0.11%	0.18%	0.23%	0.16%	0.16%
				Outwards		
1	-0.14%	0.04%	0.11%	0.20%	0.27%	0.33%
2	0.15%	0.16%	0.20%	0.27%	0.36%	0.36%
3	0.12%	0.18%	0.24%	0.27%	0.29%	0.35%
4	0.14%	0.22%	0.14%	0.32%	0.23%	0.36%
5	0.18%	0.17%	0.24%	0.33%	0.27%	0.25%
6	0.12%	0.25%	0.25%	0.30%	0.31%	0.33%
7	0.14%	0.18%	0.21%	0.26%	0.23%	0.28%
				Inwards		
1	-0.45%	-0.29%	0.03%	-0.05%	0.07%	0.18%
2	-0.22%	-0.09%	0.14%	0.18%	0.27%	0.10%
3	-0.07%	0.06%	0.19%	0.20%	0.21%	0.22%
4	-0.01%	0.08%	0.19%	0.14%	0.18%	0.27%
5	0.02%	0.11%	0.22%	0.16%	0.20%	0.27%
6	0.01%	0.08%	0.16%	0.14%	0.20%	0.19%
7	0.08%	0.15%	0.15%	0.23%	0.19%	0.23%

(a) Average monthly returns.

	Standard deviation					
				Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-1.46	-1.17	-0.57	-0.45	-0.02	0.00
2	-0.76	-0.48	-0.20	0.08	0.20	0.06
3	-0.64	-0.43	-0.44	-0.11	0.21	0.14
4	-0.42	-0.40	-0.42	-0.28	-0.14	-0.09
5	-0.56	-0.37	-0.45	0.11	-0.12	-0.09
6	-0.59	-0.45	-0.33	-0.14	-0.11	0.09
7	-0.42	-0.53	-0.26	-0.04	-0.33	-0.31
				Outwards		
1	-1.44	-0.82	-0.51	-0.17	0.08	0.26
2	-0.36	-0.33	-0.15	0.10	0.40	0.36
3	-0.50	-0.25	-0.01	0.08	0.15	0.35
4	-0.38	-0.08	-0.42	0.28	-0.06	0.40
5	-0.23	-0.28	-0.02	0.31	0.11	0.01
6	-0.48	0.03	0.03	0.20	0.23	0.31
7	-0.39	-0.27	-0.13	0.04	-0.04	0.13
				Inwards		
1	-2.76	-2.19	-0.84	-1.06	-0.59	-0.20
2	-1.84	-1.33	-0.41	-0.24	0.10	-0.45
3	-1.23	-0.74	-0.24	-0.15	-0.13	-0.09
4	-1.00	-0.65	-0.24	-0.38	-0.24	0.07
5	-0.89	-0.53	-0.10	-0.33	-0.15	0.09
6	-0.89	-0.65	-0.34	-0.40	-0.17	-0.20
7	-0.61	-0.37	-0.36	-0.06	-0.19	-0.03

(b) Annualized Sharpe ratio.

Table 10 – Minimum distance strategy.

In Table 10a we can see the average return per month for the minimum distance strategy. On average the OUTWARDS version of the strategy had the best results. The average of BEYOND, OUTWARDS and INWARDS minimum distance strategies was, 0.17%, 0.23% and 0.10%. There is a clustering of better performing parameters around the 2 to 3 standard deviations. If we set the entry parameter and maximum holding period too close to zero profits evaporate. Likewise extending the maximum holding period increases the average monthly returns. The average for the minimum distance strategy as a whole is 0.17% in average monthly return.

In Table10b the average Sharpe ratio across all permutations is -0.31. The average Sharpe ratio of the BEYOND, OUTWARDS and INWARDS is -0.30, -0.08 and -0.55, respectively. The INWARDS has the worst results just as with the average monthly returns.

For the correlation strategy we had four different ways of performing the correlation strategy. They were either using the smallest (SMALL) of the two possible slope coefficients (OLS hedge-ratio), the coefficients closest to one (CLOSEONE), the biggest coefficients (BIG) or simply weighting the stocks 1:1 (ONE) as in the minimum distance strategy.

From Table 11 we can see that CLOSEONE model variation is the better of the four models. The average monthly return for SMALL, BIG and ONE is 0.09%, 0.10% and 0.11%, respectively. The CLOSEONE model generates a positive return of 0.13% on average. As with the minimum distance strategy the INWARDS version had the lowest average return of 0.04% across the four model variations. The BEYOND and OUTWARDS had on average 0.12% and 0.16%, respectively.

In Table 12 we can see that the average Sharpe ratio for the four variations SMALL, CLOSEONE, BIG and ONE are -0.37, -0.31, -0.39 and -0.40. With CLOSEONE being the better one among the four. The average Sharpe ratio for the three entry types BEYOND, OUTWARDS and INWARDS are -0.31, -0.24 and -0.56. The better one of the three is OUTWARDS, which it also was in respect to average monthly returns. The average percent of positive Sharpe ratios for the four variations is 5%, which is lower than the minimum distance strategy, which had 24%.

From Table 13a and Table 13b we can see a stronger green color dispersed around the 1.0 to 2.5 std. dev. entry thresholds for the BEYOND and OUTWARDS variations of the cointegration strategy. 30% of all Sharpe ratios are positive. Likewise to the minimum distance and the correlation strategy the INWARDS setting is the poorest performing entry type, with average monthly return across permutations of 0.05%. Compared to the BEYOND and OUTWARDS entry type of 0.22% and 0.27%. Average Sharpe ratios are -0.07, 0.03 and -0.44 for the three different entry types.

21.1.1 Base models - summary

We average the various parameters; entry type, max. holding length and entry threshold across all models. We weight the various strategies minimum distance, correlation and cointegration equally. For the various correlation models, SMALL, CLOSEONE, BIG and ONE we use the average of them to represent the average of the correlation strategy. For the entry types BEYOND, OUTWARDS and INWARDS the average is 0.22%, 0.26% and 0.05%, respectively. On average OUTWARDS had the highest return, while INWARDS had the lowest. For Sharpe ratios the picture was similar, the average Sharpe ratios were -0.08, 0.02 and -0.45 for the entry types BEYOND, OUTWARDS and INWARDS.

Table 14 shows the maximum holding length parameters and their respective average monthly return and Sharpe ratios across all models. All parameter settings show negative Sharpe ratios. The average monthly returns range from 0.11% to 0.19%. The best parameters setting of the maximum holding length are 5 and 6 months.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	0.02%	-0.02%	-0.01%	0.05%	0.25%	0.19%
2	0.07%	0.11%	0.07%	0.13%	0.12%	0.10%
3	0.13%	0.16%	0.15%	0.15%	0.17%	0.02%
4	0.10%	0.19%	0.11%	0.05%	0.17%	0.19%
5	0.11%	0.17%	0.16%	0.14%	0.21%	0.29%
6	0.08%	0.12%	0.13%	0.11%	0.15%	0.11%
7	0.14%	0.11%	0.06%	0.02%	0.17%	0.23%
				Outwards		
1	0.02%	-0.02%	0.00%	0.11%	0.17%	0.26%
2	0.13%	-0.07%	0.14%	0.11%	0.24%	0.13%
3	0.15%	0.25%	0.13%	0.15%	0.28%	0.07%
4	0.11%	0.18%	0.20%	0.18%	0.17%	0.19%
5	0.03%	0.10%	0.19%	0.22%	0.28%	0.08%
6	0.16%	0.17%	0.12%	0.12%	0.25%	0.15%
7	0.14%	0.04%	0.07%	0.03%	0.10%	0.11%
				Inwards		
1	-0.37%	-0.26%	-0.25%	-0.18%	-0.13%	-0.06%
2	-0.12%	0.03%	-0.03%	0.06%	0.01%	0.12%
3	-0.08%	0.00%	-0.01%	0.09%	-0.03%	0.04%
4	-0.06%	0.08%	0.02%	0.12%	0.17%	0.09%
5	-0.07%	-0.01%	0.02%	0.08%	0.01%	0.13%
6	-0.16%	0.08%	0.13%	0.11%	0.03%	0.10%
7	-0.01%	0.23%	-0.02%	-0.01%	0.07%	0.00%

(a) Correlation strategy SMALL.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-0.01%	-0.01%	0.08%	0.18%	0.20%	0.18%
2	0.07%	0.17%	0.24%	0.23%	0.29%	0.20%
3	0.12%	0.14%	0.25%	0.14%	0.17%	0.13%
4	0.08%	0.13%	0.16%	0.19%	0.20%	0.22%
5	0.13%	0.05%	0.16%	0.07%	0.26%	0.21%
6	0.12%	0.12%	0.24%	0.03%	0.24%	0.21%
7	0.12%	0.16%	0.20%	0.12%	0.09%	0.17%
				Outwards		
1	0.18%	0.11%	0.14%	0.21%	0.19%	0.15%
2	0.22%	0.14%	0.21%	0.23%	0.21%	0.20%
3	0.15%	0.14%	0.13%	0.12%	0.22%	0.18%
4	0.13%	0.12%	0.16%	0.25%	0.16%	0.21%
5	0.13%	0.16%	0.17%	0.16%	0.24%	0.25%
6	0.23%	0.05%	0.22%	0.18%	0.22%	0.25%
7	0.16%	0.18%	0.16%	0.07%	0.18%	0.18%
				Inwards		
1	-0.34%	-0.15%	-0.22%	-0.04%	0.01%	0.01%
2	-0.08%	0.05%	0.08%	0.12%	0.14%	0.10%
3	-0.13%	0.01%	0.08%	0.10%	0.21%	0.12%
4	-0.12%	0.07%	0.01%	0.05%	0.21%	0.04%
5	-0.08%	0.08%	0.09%	0.09%	0.19%	0.26%
6	-0.12%	0.04%	0.09%	0.09%	0.13%	0.28%
7	-0.04%	0.01%	0.11%	0.12%	0.20%	0.16%

(b) Correlation strategy CLOSEONE.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-0.04%	-0.01%	0.03%	0.08%	0.09%	-0.03%
2	0.08%	0.08%	0.16%	0.16%	0.19%	0.13%
3	0.03%	0.05%	0.13%	0.13%	0.18%	0.15%
4	0.03%	0.16%	0.11%	0.16%	0.16%	0.09%
5	0.06%	0.06%	0.16%	0.19%	0.31%	0.27%
6	0.02%	0.06%	0.12%	0.08%	0.16%	0.15%
7	0.05%	0.05%	0.12%	0.12%	0.06%	-0.01%
				Outwards		
1	0.02%	-0.01%	0.13%	0.06%	0.11%	0.15%
2	0.24%	0.12%	0.24%	0.25%	0.27%	0.22%
3	0.13%	0.21%	0.13%	0.21%	0.19%	0.15%
4	0.21%	0.14%	0.11%	0.23%	0.26%	0.20%
5	0.12%	0.18%	0.17%	0.15%	0.20%	0.25%
6	0.17%	0.14%	0.07%	0.09%	0.15%	0.08%
7	0.23%	0.20%	0.13%	0.02%	0.20%	0.32%
				Inwards		
1	-0.24%	-0.23%	-0.14%	0.04%	-0.09%	-0.05%
2	-0.09%	-0.03%	-0.02%	0.05%	0.09%	0.14%
3	-0.09%	-0.03%	0.03%	0.07%	0.21%	0.16%
4	-0.11%	0.05%	-0.03%	0.18%	0.19%	0.23%
5	-0.04%	0.09%	0.14%	0.17%	0.15%	0.19%
6	-0.05%	-0.05%	0.12%	0.20%	0.09%	0.16%
7	-0.05%	0.09%	0.04%	0.13%	0.14%	0.15%

(c) Correlation strategy BIG.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-0.10%	-0.05%	0.04%	0.05%	0.04%	0.13%
2	0.03%	0.07%	0.13%	0.19%	0.23%	0.24%
3	0.05%	0.14%	0.10%	0.14%	0.15%	0.15%
4	0.05%	0.14%	0.16%	0.16%	0.18%	0.19%
5	0.09%	0.10%	0.21%	0.21%	0.14%	0.25%
6	0.07%	0.09%	0.17%	0.08%	0.12%	0.19%
7	0.05%	0.10%	0.16%	0.11%	0.13%	0.09%
				Outwards		
1	-0.03%	0.07%	0.03%	0.18%	0.11%	0.16%
2	0.06%	0.15%	0.17%	0.33%	0.13%	0.20%
3	0.13%	0.05%	0.26%	0.28%	0.18%	0.18%
4	0.10%	0.05%	0.20%	0.18%	0.19%	0.27%
5	0.15%	0.10%	0.20%	0.30%	0.16%	0.09%
6	0.15%	0.11%	0.12%	0.18%	0.18%	0.31%
7	0.13%	0.14%	0.13%	0.29%	0.07%	0.16%
				Inwards		
1	-0.45%	-0.16%	-0.14%	-0.10%	0.01%	-0.04%
2	-0.13%	0.05%	0.03%	0.08%	0.14%	0.24%
3	-0.05%	0.05%	0.00%	0.11%	0.18%	0.05%
4	-0.01%	0.04%	0.08%	0.16%	0.15%	0.36%
5	-0.03%	0.10%	0.03%	0.19%	0.15%	0.28%
6	-0.01%	0.11%	0.11%	0.20%	0.22%	0.23%
7	-0.02%	0.12%	0.01%	0.17%	0.13%	0.12%

(d) Correlation strategy ONE.

Table 11 – Mean monthly returns for the correlation strategy.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.50	-0.56	-0.50	-0.38	-0.02	-0.12
2	-0.39	-0.29	-0.36	-0.24	-0.26	-0.29
3	-0.25	-0.20	-0.22	-0.21	-0.17	-0.44
4	-0.33	-0.13	-0.28	-0.41	-0.16	-0.13
5	-0.30	-0.18	-0.20	-0.22	-0.09	0.06
6	-0.37	-0.27	-0.24	-0.29	-0.22	-0.29
7	-0.23	-0.31	-0.38	-0.49	-0.16	-0.05
					Outwards	
1	-0.58	-0.65	-0.56	-0.28	-0.17	0.00
2	-0.29	-0.78	-0.25	-0.30	-0.03	-0.25
3	-0.23	0.00	-0.27	-0.21	0.05	-0.35
4	-0.33	-0.16	-0.12	-0.15	-0.16	-0.13
5	-0.52	-0.36	-0.12	-0.07	0.05	-0.36
6	-0.20	-0.19	-0.27	-0.28	-0.01	-0.21
7	-0.25	-0.49	-0.40	-0.50	-0.32	-0.29
					Inwards	
1	-1.73	-1.35	-1.19	-0.88	-0.77	-0.57
2	-0.94	-0.56	-0.65	-0.41	-0.51	-0.26
3	-0.85	-0.61	-0.57	-0.34	-0.59	-0.43
4	-0.77	-0.40	-0.51	-0.29	-0.18	-0.31
5	-0.75	-0.62	-0.50	-0.37	-0.50	-0.25
6	-0.97	-0.39	-0.25	-0.29	-0.48	-0.31
7	-0.62	-0.05	-0.56	-0.52	-0.38	-0.48

(a) Correlation strategy SMALL.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.65	-0.63	-0.40	-0.15	-0.10	-0.15
2	-0.45	-0.21	-0.03	-0.05	0.09	-0.12
3	-0.32	-0.27	-0.01	-0.24	-0.19	-0.25
4	-0.41	-0.28	-0.22	-0.15	-0.12	-0.06
5	-0.30	-0.48	-0.21	-0.42	0.01	-0.09
6	-0.31	-0.31	-0.03	-0.52	-0.02	-0.09
7	-0.33	-0.21	-0.12	-0.30	-0.38	-0.18
					Outwards	
1	-0.18	-0.38	-0.28	-0.10	-0.13	-0.21
2	-0.07	-0.29	-0.10	-0.04	-0.09	-0.11
3	-0.27	-0.28	-0.30	-0.33	-0.07	-0.16
4	-0.32	-0.35	-0.22	0.01	-0.22	-0.09
5	-0.34	-0.23	-0.21	-0.22	-0.03	0.00
6	-0.04	-0.48	-0.08	-0.17	-0.08	0.00
7	-0.24	-0.17	-0.22	-0.39	-0.16	-0.16
					Inwards	
1	-1.78	-1.19	-1.22	-0.70	-0.56	-0.50
2	-0.97	-0.55	-0.46	-0.33	-0.27	-0.33
3	-1.10	-0.66	-0.45	-0.38	-0.09	-0.30
4	-1.02	-0.50	-0.60	-0.48	-0.09	-0.48
5	-0.90	-0.45	-0.41	-0.38	-0.14	0.01
6	-0.99	-0.51	-0.39	-0.38	-0.30	0.05
7	-0.77	-0.60	-0.37	-0.30	-0.11	-0.21

(b) Correlation strategy CLOSEONE.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.80	-0.69	-0.54	-0.41	-0.38	-0.59
2	-0.45	-0.44	-0.24	-0.23	-0.14	-0.29
3	-0.57	-0.52	-0.30	-0.30	-0.19	-0.24
4	-0.59	-0.24	-0.35	-0.24	-0.21	-0.37
5	-0.50	-0.47	-0.24	-0.14	0.14	0.05
6	-0.60	-0.49	-0.34	-0.41	-0.21	-0.24
7	-0.52	-0.52	-0.34	-0.33	-0.49	-0.59
					Outwards	
1	-0.69	-0.75	-0.34	-0.50	-0.35	-0.24
2	-0.03	-0.37	-0.01	0.01	0.06	-0.06
3	-0.36	-0.12	-0.33	-0.09	-0.16	-0.25
4	-0.12	-0.31	-0.38	-0.04	0.03	-0.11
5	-0.39	-0.19	-0.21	-0.26	-0.12	0.00
6	-0.22	-0.29	-0.48	-0.40	-0.26	-0.41
7	-0.06	-0.13	-0.31	-0.56	-0.13	0.18
					Inwards	
1	-1.60	-1.53	-1.10	-0.55	-0.83	-0.69
2	-1.10	-0.83	-0.77	-0.53	-0.39	-0.26
3	-1.04	-0.85	-0.60	-0.45	-0.10	-0.23
4	-1.13	-0.58	-0.78	-0.17	-0.16	-0.06
5	-0.87	-0.46	-0.30	-0.20	-0.26	-0.15
6	-0.88	-0.81	-0.34	-0.14	-0.41	-0.20
7	-0.87	-0.45	-0.56	-0.32	-0.28	-0.23

(c) Correlation strategy BIG.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-1.07	-0.88	-0.59	-0.54	-0.52	-0.29
2	-0.66	-0.51	-0.33	-0.17	-0.04	-0.03
3	-0.58	-0.30	-0.41	-0.29	-0.27	-0.25
4	-0.56	-0.32	-0.26	-0.25	-0.19	-0.15
5	-0.47	-0.42	-0.11	-0.11	-0.27	0.00
6	-0.54	-0.44	-0.22	-0.45	-0.36	-0.15
7	-0.57	-0.41	-0.24	-0.37	-0.30	-0.39
					Outwards	
1	-0.96	-0.57	-0.68	-0.20	-0.37	-0.20
2	-0.65	-0.34	-0.25	0.23	-0.32	-0.12
3	-0.40	-0.62	0.04	0.12	-0.20	-0.16
4	-0.47	-0.64	-0.16	-0.21	-0.17	0.07
5	-0.31	-0.44	-0.14	0.16	-0.24	-0.39
6	-0.30	-0.40	-0.37	-0.20	-0.20	0.16
7	-0.36	-0.31	-0.35	0.13	-0.49	-0.24
					Inwards	
1	-2.44	-1.40	-1.23	-1.05	-0.64	-0.70
2	-1.25	-0.68	-0.67	-0.49	-0.32	-0.03
3	-1.00	-0.64	-0.77	-0.42	-0.18	-0.49
4	-0.88	-0.67	-0.52	-0.24	-0.26	0.28
5	-0.91	-0.47	-0.66	-0.16	-0.28	0.08
6	-0.83	-0.41	-0.42	-0.13	-0.07	-0.06
7	-0.83	-0.39	-0.68	-0.22	-0.32	-0.33

(d) Correlation strategy ONE.

Table 12 – Sharpe ratio for the correlation strategy.

Month	Standard deviation 0.5	1	1.5	2	2.5	3
				Beyond		
1	0.26%	0.37%	0.30%	0.27%	0.28%	0.24%
2	0.25%	0.38%	0.35%	0.20%	0.17%	0.20%
3	0.23%	0.15%	0.21%	0.06%	0.22%	0.06%
4	0.17%	0.24%	0.21%	0.15%	0.18%	0.17%
5	0.26%	0.29%	0.28%	0.22%	0.22%	0.18%
6	0.18%	0.27%	0.22%	0.26%	0.24%	0.30%
7	0.16%	0.33%	0.13%	0.17%	0.11%	0.23%
				Outwards		
1	0.35%	0.43%	0.36%	0.35%	0.52%	0.19%
2	0.22%	0.49%	0.50%	0.22%	0.32%	0.22%
3	0.23%	0.32%	0.34%	0.21%	0.32%	0.07%
4	0.29%	0.27%	0.26%	0.26%	0.29%	0.15%
5	0.23%	0.18%	0.24%	0.27%	0.34%	0.20%
6	0.27%	0.17%	0.27%	0.27%	0.39%	0.24%
7	0.21%	0.08%	0.20%	0.03%	0.30%	0.20%
				Inwards		
1	-0.34%	-0.08%	-0.04%	0.06%	-0.13%	-0.04%
2	-0.22%	-0.11%	0.09%	0.15%	0.12%	0.15%
3	-0.15%	0.00%	0.05%	0.14%	0.27%	0.07%
4	-0.06%	-0.06%	0.06%	0.07%	0.17%	0.16%
5	-0.01%	-0.01%	0.02%	0.19%	0.26%	0.18%
6	-0.02%	0.01%	0.05%	0.15%	0.20%	0.17%
7	-0.01%	-0.03%	0.04%	0.09%	0.11%	0.24%

(a) Average monthly returns for the cointegration strategy.

Month	Standard deviation 0.5	1	1.5	2	2.5	3
				Beyond		
1	0.00	0.22	0.07	0.03	0.04	-0.04
2	0.00	0.26	0.19	-0.11	-0.17	-0.11
3	-0.04	-0.20	-0.10	-0.42	-0.07	-0.40
4	-0.17	-0.04	-0.08	-0.23	-0.17	-0.17
5	0.00	0.07	0.05	-0.06	-0.08	-0.15
6	-0.16	0.02	-0.08	0.02	-0.03	0.08
7	-0.18	0.14	-0.28	-0.20	-0.31	-0.06
				Outwards		
1	0.19	0.35	0.22	0.19	0.58	-0.13
2	-0.07	0.50	0.52	-0.06	0.14	-0.07
3	-0.04	0.15	0.19	-0.08	0.15	-0.38
4	0.09	0.03	0.03	0.02	0.07	-0.21
5	-0.04	-0.15	-0.03	0.04	0.20	-0.11
6	0.05	-0.17	0.04	0.04	0.30	-0.03
7	-0.08	-0.39	-0.12	-0.49	0.10	-0.12
				Inwards		
1	-1.26	-0.74	-0.63	-0.40	-0.83	-0.68
2	-1.00	-0.79	-0.38	-0.23	-0.28	-0.22
3	-0.80	-0.52	-0.45	-0.27	0.03	-0.39
4	-0.68	-0.66	-0.44	-0.39	-0.17	-0.19
5	-0.56	-0.54	-0.51	-0.15	0.02	-0.15
6	-0.58	-0.52	-0.46	-0.23	-0.11	-0.19
7	-0.56	-0.57	-0.48	-0.36	-0.31	-0.03

(b) Sharpe ratios of the cointegration strategy.

Table 13 – Cointegration strategy.

Months	1	2	3	4	5	6	7
Avg. Ret.	0.11%	0.18%	0.17%	0.17%	0.19%	0.19%	0.16%
Avg. Sharpe	-0.48	-0.21	-0.22	-0.22	-0.17	-0.17	-0.24

Table 14

Std. Dev.	0.5	1	1.5	2	2.5	3
Avg. Ret.	0.07%	0.14%	0.18%	0.19%	0.23%	0.21%
Avg. Sharpe	-0.57	-0.38	-0.22	-0.15	-0.05	-0.07

Table 15

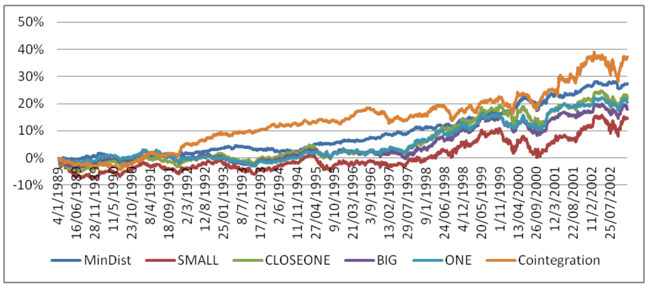


Figure 7 – The equity curve of all the strategies.

Across all models the performance of the std. dev. entry threshold is illustrated in Table 15. There are no positive Sharpe ratios and the average monthly return range from 0.07 to 0.23%, the best setting is 2.5 standard deviations.

Of the three strategies the minimum distance strategy, the correlation strategy and the cointegration strategy, the correlation strategy performed the poorest. The average across all permutations in each strategy was, 0.17%, 0.11% and 0.18% for the three strategies, minimum distance, correlation and cointegration, respectively. Where 0.11% is the average of all four correlation models. The Sharpe ratios are similarly -0.37, -0.31 and -0.16. The Sharpe ratios are negative, because the cumulative return on the risk-free rate exceeds the returns of the strategies in this period. Figure 7 shows the equity curves for the various strategies. There is a similar behavior among the six strategies, since the cointegration strategy is a subset of the correlation strategy they have the greatest similarity. The minimum distance strategy has the starkest contrast yet also experiences the same dip in performance around the year 2000 internet bubble.

21.2 Levels adjusted models

All the tables in the following subsection 21.2 have a common colored scale.

Table 16a shows average return per month for the minimum distance strategy. On average the OUTWARDS version of the strategy had the best results. The average of BEYOND, OUTWARDS and INWARDS was, 0.14%, 0.20% and 0.08%, respectively. There is a clustering of better performing

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.12%	-0.05%	0.07%	0.09%	0.19%	0.20%
2	0.04%	0.11%	0.16%	0.23%	0.26%	0.21%
3	0.07%	0.12%	0.11%	0.18%	0.26%	0.24%
4	0.13%	0.13%	0.12%	0.13%	0.17%	0.17%
5	0.09%	0.14%	0.11%	0.24%	0.17%	0.16%
6	0.08%	0.11%	0.14%	0.18%	0.17%	0.22%
7	0.13%	0.09%	0.16%	0.20%	0.11%	0.11%
					Outwards	
1	-0.16%	0.02%	0.08%	0.16%	0.22%	0.28%
2	0.14%	0.14%	0.18%	0.24%	0.32%	0.30%
3	0.11%	0.16%	0.22%	0.23%	0.25%	0.30%
4	0.13%	0.21%	0.11%	0.29%	0.19%	0.32%
5	0.17%	0.15%	0.22%	0.29%	0.23%	0.20%
6	0.12%	0.23%	0.22%	0.26%	0.27%	0.29%
7	0.13%	0.15%	0.18%	0.22%	0.20%	0.23%
					Inwards	
1	-0.46%	-0.30%	0.02%	-0.06%	0.04%	0.15%
2	-0.24%	-0.10%	0.13%	0.16%	0.23%	0.07%
3	-0.09%	0.05%	0.17%	0.18%	0.18%	0.19%
4	-0.03%	0.07%	0.17%	0.12%	0.14%	0.23%
5	0.01%	0.10%	0.20%	0.13%	0.17%	0.23%
6	0.00%	0.07%	0.14%	0.11%	0.16%	0.16%
7	0.08%	0.14%	0.13%	0.20%	0.15%	0.20%

(a) Mean monthly return of the levels adjusted minimum distance strategy.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-1.54	-1.25	-0.68	-0.58	-0.17	-0.16
2	-0.87	-0.56	-0.32	-0.06	0.05	-0.10
3	-0.70	-0.51	-0.53	-0.24	0.06	-0.03
4	-0.50	-0.49	-0.52	-0.42	-0.29	-0.25
5	-0.63	-0.45	-0.56	-0.02	-0.28	-0.27
6	-0.65	-0.54	-0.42	-0.26	-0.26	-0.07
7	-0.47	-0.61	-0.35	-0.16	-0.49	-0.47
					Outwards	
1	-1.50	-0.91	-0.60	-0.31	-0.07	0.10
2	-0.41	-0.41	-0.25	-0.01	0.27	0.17
3	-0.53	-0.33	-0.11	-0.06	0.02	0.18
4	-0.42	-0.15	-0.51	0.15	-0.18	0.24
5	-0.27	-0.35	-0.10	0.18	-0.05	-0.13
6	-0.51	-0.04	-0.07	0.06	0.09	0.14
7	-0.44	-0.35	-0.22	-0.08	-0.16	-0.04
					Inwards	
1	-2.81	-2.23	-0.90	-1.13	-0.71	-0.32
2	-1.92	-1.39	-0.47	-0.32	-0.04	-0.58
3	-1.29	-0.78	-0.29	-0.25	-0.25	-0.19
4	-1.06	-0.69	-0.29	-0.47	-0.36	-0.07
5	-0.92	-0.57	-0.16	-0.43	-0.25	-0.06
6	-0.93	-0.68	-0.40	-0.49	-0.28	-0.29
7	-0.65	-0.42	-0.42	-0.15	-0.33	-0.15

(b) Annualized Sharpe ratio of the levels adjusted minimum distance strategy.

Table 16 – Levels adjusted minimum distance strategy.

parameters around the 2 to 3 standard deviations. The Sharpe ratios for entry types BEYOND, OUTWARDS and INWARDS are -0.42, -0.19 and -0.63. In comparison to the traditional base models the types BEYOND, OUTWARDS and INWARDS all uniformly showed a worsened performance in both average monthly return and Sharpe ratio.

From Table 17 we can see that, identical to the base models, the CLOSEONE model is the better of the four variations of the correlation strategy. However none of them generate a positive return. The average monthly return for SMALL, CLOSE, BIG and ONE is 0.04%, 0.07%, 0.04% and 0.04% respectively. There are no positive Sharpe ratios in any of the four variations, showed in Table 18. Similarly to the base models the CLOSEONE model is the best of a poor performing bunch, with a Sharpe ratio of -0.42, compared to the SMALL, BIG and ONE with -0.45, -0.53 and -0.57, respectively.

In Table 19a we can see similarly to the minimum distance strategy and the correlation strategy the INWARDS setting is the poorest performing setting, with average monthly return across the permutations of -0.01%. Compared to the BEYOND and OUTWARDS settings of 0.05 and 0.10%, respectively, the Sharpe ratios show similar conclusion with -0.46, -0.36 and -0.66.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.02%	-0.05%	-0.06%	-0.04%	0.16%	0.12%
2	0.05%	0.08%	0.04%	0.06%	0.02%	0.02%
3	0.11%	0.12%	0.10%	0.08%	0.09%	-0.04%
4	0.07%	0.13%	0.06%	-0.03%	0.10%	0.13%
5	0.07%	0.13%	0.08%	0.07%	0.12%	0.23%
6	0.06%	0.08%	0.07%	0.04%	0.07%	0.04%
7	0.08%	0.06%	0.00%	-0.07%	0.12%	0.16%
					Outwards	
1	0.03%	-0.01%	0.01%	0.06%	0.10%	0.21%
2	0.13%	-0.06%	0.12%	0.03%	0.15%	0.05%
3	0.14%	0.24%	0.11%	0.12%	0.18%	-0.01%
4	0.08%	0.13%	0.17%	0.12%	0.09%	0.13%
5	0.01%	0.10%	0.18%	0.15%	0.21%	0.03%
6	0.13%	0.15%	0.10%	0.09%	0.18%	0.08%
7	0.09%	0.04%	0.05%	-0.01%	0.01%	0.04%
					Inwards	
1	-0.37%	-0.27%	-0.27%	-0.30%	-0.18%	-0.14%
2	-0.11%	0.00%	-0.05%	0.01%	-0.05%	0.03%
3	-0.07%	-0.01%	-0.05%	0.04%	-0.09%	-0.04%
4	-0.06%	0.05%	-0.01%	0.08%	0.10%	0.00%
5	-0.06%	-0.04%	-0.01%	0.03%	-0.06%	0.06%
6	-0.16%	0.05%	0.08%	0.06%	-0.02%	0.01%
7	-0.03%	0.20%	-0.05%	-0.04%	0.05%	-0.10%

(a) Correlation SMALL.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.07%	-0.04%	0.02%	0.08%	0.14%	0.11%
2	0.05%	0.11%	0.18%	0.09%	0.21%	0.09%
3	0.12%	0.09%	0.21%	0.06%	0.10%	0.04%
4	0.05%	0.07%	0.09%	0.11%	0.10%	0.13%
5	0.09%	0.00%	0.09%	-0.02%	0.19%	0.12%
6	0.08%	0.08%	0.18%	-0.05%	0.15%	0.15%
7	0.07%	0.11%	0.14%	0.03%	0.04%	0.08%
					Outwards	
1	0.19%	0.11%	0.14%	0.16%	0.13%	0.09%
2	0.22%	0.13%	0.19%	0.19%	0.12%	0.12%
3	0.13%	0.12%	0.07%	0.06%	0.16%	0.09%
4	0.12%	0.06%	0.13%	0.21%	0.10%	0.12%
5	0.11%	0.13%	0.16%	0.10%	0.15%	0.16%
6	0.19%	-0.01%	0.18%	0.11%	0.16%	0.18%
7	0.13%	0.19%	0.15%	0.02%	0.12%	0.07%
					Inwards	
1	-0.34%	-0.14%	-0.23%	-0.09%	-0.06%	-0.06%
2	-0.09%	0.03%	0.06%	0.05%	0.07%	0.00%
3	-0.14%	0.00%	0.07%	0.06%	0.14%	0.05%
4	-0.13%	0.04%	-0.05%	0.01%	0.14%	-0.02%
5	-0.10%	0.06%	0.05%	0.04%	0.11%	0.15%
6	-0.13%	0.02%	0.05%	0.05%	0.06%	0.19%
7	-0.08%	0.00%	0.09%	0.09%	0.16%	0.07%

(b) Correlation CLOSEONE.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.12%	-0.06%	-0.03%	0.02%	0.01%	-0.13%
2	0.05%	0.03%	0.07%	0.09%	0.09%	0.00%
3	-0.03%	-0.02%	0.06%	0.05%	0.09%	0.04%
4	-0.01%	0.10%	0.03%	0.06%	0.05%	0.00%
5	0.00%	0.00%	0.10%	0.11%	0.19%	0.14%
6	-0.04%	0.01%	0.06%	0.01%	0.05%	0.02%
7	-0.02%	-0.03%	0.05%	0.03%	-0.02%	-0.15%
					Outwards	
1	0.00%	-0.04%	0.11%	-0.02%	0.03%	0.06%
2	0.21%	0.09%	0.18%	0.19%	0.17%	0.13%
3	0.13%	0.17%	0.08%	0.14%	0.11%	0.04%
4	0.19%	0.11%	0.04%	0.17%	0.17%	0.08%
5	0.10%	0.15%	0.13%	0.08%	0.08%	0.17%
6	0.17%	0.14%	-0.01%	0.04%	0.06%	-0.03%
7	0.20%	0.15%	0.07%	0.00%	0.11%	0.19%
					Inwards	
1	-0.28%	-0.23%	-0.19%	-0.05%	-0.19%	-0.15%
2	-0.10%	-0.05%	-0.05%	-0.03%	0.00%	-0.01%
3	-0.11%	-0.04%	0.00%	0.00%	0.12%	0.05%
4	-0.12%	0.02%	-0.08%	0.12%	0.10%	0.13%
5	-0.03%	0.04%	0.09%	0.14%	0.05%	0.09%
6	-0.05%	-0.07%	0.08%	0.16%	0.01%	0.04%
7	-0.06%	0.06%	-0.02%	0.07%	0.07%	0.05%

(c) Correlation BIG.

	Standard deviation				Beyond	
Month	0.5	1	1.5	2	2.5	3
1	-0.17%	-0.12%	-0.04%	-0.03%	-0.06%	0.02%
2	-0.02%	0.01%	0.04%	0.09%	0.08%	0.11%
3	-0.01%	0.09%	0.02%	0.05%	0.05%	0.02%
4	-0.02%	0.07%	0.06%	0.06%	0.03%	0.05%
5	0.02%	0.04%	0.12%	0.13%	0.01%	0.16%
6	0.01%	0.02%	0.10%	0.01%	0.00%	0.08%
7	-0.02%	0.00%	0.09%	0.01%	0.00%	-0.04%
					Outwards	
1	-0.04%	0.06%	-0.01%	0.12%	-0.01%	0.04%
2	0.04%	0.12%	0.13%	0.24%	0.01%	0.10%
3	0.09%	0.05%	0.21%	0.21%	0.08%	0.02%
4	0.07%	0.00%	0.16%	0.10%	0.08%	0.16%
5	0.11%	0.06%	0.13%	0.23%	0.04%	-0.05%
6	0.12%	0.10%	0.05%	0.12%	0.09%	0.17%
7	0.10%	0.12%	0.08%	0.21%	-0.04%	0.03%
					Inwards	
1	-0.50%	-0.20%	-0.21%	-0.16%	-0.10%	-0.16%
2	-0.15%	0.01%	-0.01%	0.01%	0.01%	0.10%
3	-0.10%	0.00%	-0.06%	0.04%	0.05%	-0.07%
4	-0.04%	0.01%	0.01%	0.09%	0.06%	0.25%
5	-0.05%	0.03%	-0.03%	0.12%	0.03%	0.17%
6	-0.05%	0.07%	0.06%	0.13%	0.12%	0.13%
7	-0.06%	0.09%	-0.06%	0.12%	0.05%	0.03%

(d) Correlation ONE.

Table 17 – The mean monthly return of the levels adjusted correlation strategies.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-0.59	-0.62	-0.61	-0.54	-0.17	-0.24
2	-0.45	-0.36	-0.42	-0.38	-0.44	-0.43
3	-0.30	-0.28	-0.31	-0.34	-0.31	-0.55
4	-0.37	-0.25	-0.38	-0.56	-0.29	-0.25
5	-0.38	-0.25	-0.33	-0.36	-0.26	-0.05
6	-0.41	-0.36	-0.36	-0.42	-0.36	-0.40
7	-0.35	-0.42	-0.49	-0.64	-0.27	-0.18
				Outwards		
1	-0.53	-0.61	-0.54	-0.40	-0.30	-0.09
2	-0.30	-0.75	-0.28	-0.45	-0.20	-0.39
3	-0.25	-0.01	-0.30	-0.28	-0.14	-0.48
4	-0.39	-0.28	-0.17	-0.27	-0.32	-0.25
5	-0.54	-0.37	-0.15	-0.21	-0.09	-0.44
6	-0.28	-0.25	-0.30	-0.34	-0.14	-0.34
7	-0.36	-0.49	-0.42	-0.56	-0.49	-0.41
				Inwards		
1	-1.67	-1.35	-1.24	-1.15	-0.88	-0.71
2	-0.92	-0.63	-0.68	-0.52	-0.62	-0.41
3	-0.82	-0.62	-0.67	-0.42	-0.68	-0.56
4	-0.74	-0.45	-0.58	-0.36	-0.30	-0.46
5	-0.72	-0.67	-0.55	-0.47	-0.61	-0.36
6	-0.95	-0.45	-0.37	-0.38	-0.58	-0.44
7	-0.65	-0.12	-0.62	-0.56	-0.42	-0.60

(a) Correlation SMALL.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-0.77	-0.71	-0.52	-0.36	-0.23	-0.27
2	-0.50	-0.33	-0.16	-0.35	-0.10	-0.35
3	-0.32	-0.38	-0.10	-0.43	-0.33	-0.43
4	-0.47	-0.42	-0.35	-0.31	-0.33	-0.26
5	-0.38	-0.59	-0.36	-0.59	-0.14	-0.28
6	-0.39	-0.39	-0.17	-0.68	-0.24	-0.22
7	-0.42	-0.33	-0.25	-0.48	-0.48	-0.34
				Outwards		
1	-0.14	-0.37	-0.27	-0.21	-0.27	-0.35
2	-0.07	-0.32	-0.15	-0.15	-0.29	-0.28
3	-0.31	-0.33	-0.43	-0.45	-0.22	-0.32
4	-0.33	-0.49	-0.28	-0.10	-0.35	-0.28
5	-0.40	-0.30	-0.23	-0.34	-0.23	-0.18
6	-0.14	-0.60	-0.17	-0.31	-0.21	-0.16
7	-0.31	-0.15	-0.26	-0.50	-0.28	-0.37
				Inwards		
1	-1.72	-1.13	-1.21	-0.83	-0.71	-0.64
2	-0.97	-0.61	-0.50	-0.48	-0.42	-0.53
3	-1.09	-0.70	-0.46	-0.45	-0.27	-0.46
4	-1.00	-0.57	-0.73	-0.57	-0.26	-0.56
5	-0.91	-0.51	-0.51	-0.50	-0.31	-0.23
6	-1.00	-0.56	-0.49	-0.46	-0.43	-0.13
7	-0.82	-0.63	-0.42	-0.36	-0.20	-0.38

(b) Correlation CLOSEONE.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-1.01	-0.81	-0.67	-0.54	-0.55	-0.78
2	-0.53	-0.57	-0.43	-0.40	-0.36	-0.55
3	-0.71	-0.69	-0.46	-0.47	-0.37	-0.46
4	-0.67	-0.36	-0.52	-0.43	-0.44	-0.55
5	-0.64	-0.62	-0.37	-0.33	-0.13	-0.25
6	-0.71	-0.59	-0.45	-0.55	-0.44	-0.47
7	-0.68	-0.66	-0.48	-0.51	-0.62	-0.79
				Outwards		
1	-0.73	-0.84	-0.39	-0.69	-0.52	-0.42
2	-0.11	-0.44	-0.19	-0.15	-0.19	-0.26
3	-0.35	-0.23	-0.45	-0.28	-0.33	-0.50
4	-0.16	-0.39	-0.54	-0.19	-0.20	-0.39
5	-0.43	-0.26	-0.32	-0.39	-0.39	-0.18
6	-0.23	-0.30	-0.65	-0.49	-0.46	-0.58
7	-0.15	-0.28	-0.44	-0.56	-0.33	-0.13
				Inwards		
1	-1.66	-1.46	-1.24	-0.74	-1.04	-0.89
2	-1.05	-0.87	-0.85	-0.71	-0.59	-0.56
3	-1.09	-0.82	-0.67	-0.61	-0.30	-0.46
4	-1.10	-0.65	-0.89	-0.33	-0.36	-0.28
5	-0.79	-0.58	-0.42	-0.26	-0.48	-0.35
6	-0.86	-0.84	-0.42	-0.23	-0.58	-0.43
7	-0.85	-0.52	-0.70	-0.43	-0.42	-0.43

(c) Correlation BIG.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	-1.21	-1.03	-0.76	-0.69	-0.72	-0.50
2	-0.77	-0.66	-0.54	-0.40	-0.40	-0.32
3	-0.71	-0.43	-0.60	-0.51	-0.49	-0.53
4	-0.73	-0.47	-0.48	-0.48	-0.50	-0.45
5	-0.62	-0.56	-0.33	-0.29	-0.55	-0.20
6	-0.66	-0.59	-0.37	-0.57	-0.60	-0.39
7	-0.71	-0.62	-0.40	-0.53	-0.58	-0.61
				Outwards		
1	-0.95	-0.60	-0.77	-0.34	-0.65	-0.46
2	-0.68	-0.42	-0.33	-0.02	-0.61	-0.35
3	-0.50	-0.61	-0.12	-0.09	-0.44	-0.51
4	-0.55	-0.73	-0.26	-0.39	-0.44	-0.22
5	-0.43	-0.54	-0.33	-0.05	-0.52	-0.66
6	-0.39	-0.42	-0.52	-0.34	-0.39	-0.17
7	-0.42	-0.35	-0.45	-0.09	-0.67	-0.51
				Inwards		
1	-2.43	-1.46	-1.38	-1.16	-0.88	-0.93
2	-1.26	-0.77	-0.77	-0.65	-0.62	-0.37
3	-1.10	-0.76	-0.89	-0.58	-0.52	-0.72
4	-0.92	-0.72	-0.70	-0.42	-0.46	-0.01
5	-0.94	-0.65	-0.76	-0.34	-0.54	-0.18
6	-0.90	-0.51	-0.52	-0.29	-0.32	-0.28
7	-0.89	-0.47	-0.79	-0.33	-0.47	-0.51

(d) Correlation ONE.

Table 18 – Annualized Sharpe ratio of the levels adjusted correlation strategies.

	Standard deviation					
			Beyond			
Month	0.5	1	1.5	2	2.5	3
1	0.32%	0.34%	0.31%	0.25%	0.29%	0.08%
2	0.37%	0.39%	0.35%	0.19%	0.20%	0.16%
3	0.29%	0.27%	0.29%	0.15%	0.09%	0.04%
4	0.31%	0.33%	0.31%	0.08%	0.11%	0.11%
5	0.34%	0.26%	0.30%	0.16%	0.11%	0.13%
6	0.35%	0.34%	0.26%	0.24%	0.16%	0.16%
7	0.16%	0.26%	0.21%	0.15%	0.09%	0.12%
			Outwards			
1	0.34%	0.39%	0.36%	0.27%	0.43%	0.10%
2	0.25%	0.48%	0.48%	0.24%	0.31%	0.17%
3	0.25%	0.34%	0.29%	0.19%	0.29%	0.01%
4	0.30%	0.30%	0.17%	0.20%	0.22%	0.03%
5	0.36%	0.22%	0.18%	0.34%	0.18%	0.18%
6	0.33%	0.28%	0.31%	0.24%	0.32%	0.10%
7	0.31%	0.14%	0.18%	0.04%	0.23%	0.17%
			Inwards			
1	-0.35%	-0.11%	-0.04%	-0.03%	-0.06%	-0.09%
2	-0.20%	-0.03%	0.09%	0.08%	0.13%	0.05%
3	-0.09%	-0.01%	0.16%	0.14%	0.09%	0.03%
4	-0.04%	0.02%	0.02%	0.09%	0.15%	0.00%
5	0.10%	-0.01%	-0.02%	0.14%	0.17%	0.11%
6	0.10%	0.01%	0.10%	0.17%	0.20%	0.12%
7	0.07%	-0.03%	-0.01%	0.07%	0.14%	0.16%

(a) Mean monthly return of the levels adjusted cointegration strategy.

	Standard deviation					
			Beyond			
Month	0.5	1	1.5	2	2.5	3
1	0.13	0.15	0.08	-0.02	0.06	-0.32
2	0.22	0.26	0.19	-0.14	-0.11	-0.18
3	0.06	0.04	0.08	-0.20	-0.33	-0.42
4	0.10	0.15	0.12	-0.36	-0.29	-0.28
5	0.16	0.01	0.10	-0.21	-0.28	-0.24
6	0.19	0.18	0.01	-0.02	-0.19	-0.19
7	-0.18	0.01	-0.10	-0.21	-0.33	-0.25
			Outwards			
1	0.18	0.25	0.21	0.04	0.38	-0.31
2	0.00	0.44	0.47	-0.03	0.13	-0.16
3	-0.01	0.18	0.07	-0.14	0.09	-0.48
4	0.10	0.09	-0.18	-0.12	-0.06	-0.45
5	0.21	-0.07	-0.15	0.16	-0.16	-0.15
6	0.13	0.05	0.11	-0.03	0.15	-0.32
7	0.13	-0.24	-0.15	-0.46	-0.05	-0.17
			Inwards			
1	-1.32	-0.82	-0.58	-0.61	-0.64	-0.79
2	-0.97	-0.61	-0.35	-0.38	-0.26	-0.42
3	-0.70	-0.58	-0.20	-0.26	-0.34	-0.44
4	-0.65	-0.51	-0.51	-0.35	-0.21	-0.51
5	-0.33	-0.51	-0.57	-0.26	-0.16	-0.28
6	-0.33	-0.54	-0.33	-0.17	-0.12	-0.27
7	-0.42	-0.57	-0.55	-0.41	-0.22	-0.18

(b) Annualized Sharpe ratio of the levels adjusted cointegration strategy.

Table 19 – Levels adjusted cointegration strategy.

The cointegration strategy is the only strategy among the levels adjusted strategies that show any sign of improvement. 32% positive Sharpe ratios as opposed to the 30% for the base model. However the average monthly returns of the levels adjusted are worse, with 0.22%, 0.25% and 0.04% for the BEYOND, OUTWARDS and INWARDS, respectively. The Sharpe ratios paints the same picture with, -0.06, -0.01 and -0.46 for the BEYOND, OUTWARDS and INWARDS, respectively.

21.2.1 Levels adjusted models - summary

We average the various parameters; entry type, maximum trade length and entry threshold across all models. We weight the various strategies minimum distance, correlation and cointegration equally. For the various correlation models, SMALL, CLOSEONE, BIG and ONE we use the average of them to represent the average of the correlation strategy. For the entry types BEYOND, OUTWARDS and INWARDS the average is 0.22%, 0.25% and 0.04%, respectively. On average OUTWARDS had the highest return, while INWARDS had the lowest and the average was negative across all models. For Sharpe ratios the picture was similar, the average Sharpe ratios were -0.08, 0.02 and -0.47 for the entry types BEYOND, OUTWARDS and INWARDS.

Months	1	2	3	4	5	6	7
Avg. Ret.	0.08%	0.17%	0.15%	0.15%	0.17%	0.18%	0.14%
Avg. Sharpe	-0.56	-0.27	-0.28	-0.30	-0.24	-0.22	-0.30

Table 20

Std. Dev.	0.5	1	1.5	2	2.5	3
Avg. Ret.	0.07%	0.14%	0.18%	0.19%	0.23%	0.21%
Avg. Sharpe	-0.57	-0.38	-0.22	-0.15	-0.05	-0.07

Table 21

Table 20 shows the maximum trade length parameters average monthly return across all models. The Sharpe ratios are negative for all settings, 1 to 7 months. The average monthly return ranges from 0.08% to 0.18, the better settings for maximum holding period are similar to the base models with 0.17% and 0.18% for the 5 and 6 month holding period.

Across all models the performance of the std. dev. setting is illustrated in Table 21. All settings have a negative Sharpe ratio. However as with the monthly parameters, a higher value of maximum holding period gives better Sharpe ratios, albeit still negative Sharpe ratios . The average monthly returns range from 0.07% to 0.23%.

Figure 8 shows the average equity curves of the various strategies. The equity curves show similar fluctuations as in the base models average equity curves.

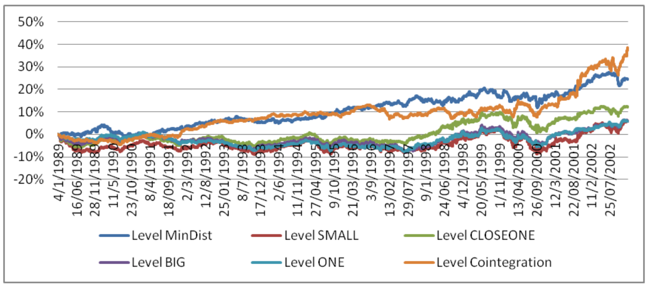


Figure 8 – The equity curve of all the strategies.

21.3 Motivation for sorting on volatility

The average performance of the permutations within each strategy is displayed in Table 22. Win %, is defined as the percentage of trades that results in a positive gain, the win percentages are around 57% and upwards to around 63%. The magnitudes of the positive trades are substantially lower than the negative trades. This can be a problem if transaction costs and fees are a substantial percent of the

Base	Minimum distance	Correlation SMALL	Correlation CLOSEONE	Correlation BIG	Correlation ONE	Cointegration
Win %	60.84%	57.16%	59.19%	57.63%	57.23%	62.57%
Positive Mag.	2.65%	4.26%	3.74%	3.85%	3.65%	3.44%
Negative Mag.	4.04%	6.21%	5.72%	5.62%	5.14%	5.88%
Ratio Pos./Neg.	0.66	0.69	0.65	0.69	0.71	0.58
Distance	0.56%	2.52%	2.52%	2.58%	2.56%	2.80%
Correlation	46.09%	65.41%	66.03%	65.54%	65.35%	47.06%
Cointegration	25.55%	36.07%	35.01%	35.57%	36.49%	9.89%

Table 22 – Table of detailing the results of the base models backtest. Win % is the percentage of trades that resulted in a gain. Positive magnitude is the average gain of winning trades. The negative magnitude is the average loss on losing trades. Distance is the average distance on trades, as measured in section 7. Correlation is the average correlation of pairs in the trades. Cointegration is the average statistical significance of the Johansen cointegration test on the pairs traded.

	Standard deviation			Beyond		
Month	0.5	1	1.5	2	2.5	3
1	4541	4013	3691	3484	3393	3315
2	2871	2411	2167	2014	1903	1802
3	2229	1872	1635	1462	1374	1294
4	1868	1568	1351	1192	1089	1008
5	1649	1373	1134	1037	931	868
6	1437	1240	1019	911	830	763
7	1321	1098	962	819	734	657
				Outwards		
1	5779	4420	3857	3555	3417	3329
2	3891	2848	2332	2089	1944	1827
3	3078	2207	1773	1558	1404	1331
4	2655	1914	1433	1302	1155	1053
5	2416	1666	1304	1135	959	888
6	2168	1527	1151	999	855	776
7	2030	1365	1013	869	781	693
				Inwards		
1	8715	5496	4389	3795	3536	3394
2	6133	3633	2746	2282	2034	1879
3	4924	2885	2145	1731	1484	1390
4	4333	2451	1791	1429	1214	1104
5	3818	2208	1598	1203	1054	931
6	3538	1976	1416	1110	932	828
7	3332	1884	1345	1016	857	730

Table 23 – The minimum strategy and number of trades of each permutation.

average trade return. The problem with low magnitude gains in trades, is that they have to overcome trading costs.

In Table 23 the total number of trades for each minimum distance strategy permutation is shown. The shorter the holding period the faster you close trades. The small the std. dev. threshold the more likely trades are to close quicker. More trades also mean more transaction cost.

It might be that selecting the pairs that are only slight worse in terms of minimizing the distance, but offer greater gains when the trades succeed. Given minimum distance has the lowest magnitudes on both winning and losing trades it would be interesting to see how it fared if we could increase the magnitudes of the winning and losing trades to sizes similar to the other strategies.

For all strategies only the top 2000 pairs are included for search in entry signals. We modify these by filtering out the 1000 pairs with the least volatile spread series. Volatility is measured as standard

	Standard deviation					
Month	0.5	1	1.5	2	2.5	3
					Beyond	
1	0.19%	0.29%	0.34%	0.21%	0.44%	0.31%
2	0.19%	0.30%	0.27%	0.34%	0.33%	0.29%
3	0.32%	0.26%	0.29%	0.35%	0.34%	0.31%
4	0.21%	0.24%	0.38%	0.37%	0.34%	0.27%
5	0.35%	0.27%	0.37%	0.40%	0.47%	0.48%
6	0.33%	0.23%	0.32%	0.29%	0.32%	0.38%
7	0.28%	0.23%	0.39%	0.32%	0.34%	0.43%
					Outwards	
1	-0.04%	0.21%	0.21%	0.05%	0.29%	0.48%
2	0.03%	0.35%	0.26%	0.29%	0.29%	0.31%
3	0.08%	0.27%	0.33%	0.25%	0.31%	0.37%
4	0.15%	0.27%	0.30%	0.33%	0.35%	0.53%
5	0.27%	0.25%	0.29%	0.21%	0.37%	0.43%
6	0.23%	0.27%	0.31%	0.25%	0.33%	0.30%
7	0.23%	0.34%	0.21%	0.28%	0.14%	0.39%
					Inwards	
1	-0.46%	-0.19%	-0.02%	-0.01%	0.03%	0.09%
2	-0.14%	0.08%	0.11%	0.24%	0.12%	0.17%
3	0.01%	-0.01%	0.14%	0.23%	0.09%	0.30%
4	-0.04%	0.06%	0.15%	0.23%	0.32%	0.39%
5	0.07%	0.19%	0.16%	0.23%	0.24%	0.32%
6	0.03%	0.25%	0.25%	0.15%	0.32%	0.40%
7	0.05%	0.17%	0.18%	0.29%	0.16%	0.26%

(a) Average monthly returns.

	Standard deviation					
Month	0.5	1	1.5	2	2.5	3
					Beyond	
1	-0.17	0.10	0.23	-0.09	0.43	0.13
2	-0.16	0.14	0.06	0.24	0.20	0.09
3	0.19	0.02	0.11	0.28	0.24	0.13
4	-0.11	-0.01	0.37	0.31	0.23	0.04
5	0.28	0.06	0.32	0.39	0.59	0.55
6	0.22	-0.04	0.19	0.11	0.18	0.33
7	0.09	-0.05	0.37	0.19	0.24	0.43
					Outwards	
1	-0.81	-0.11	-0.10	-0.49	0.08	0.50
2	-0.59	0.28	0.04	0.11	0.10	0.14
3	-0.46	0.06	0.20	-0.01	0.16	0.28
4	-0.26	0.07	0.14	0.21	0.25	0.69
5	0.05	0.01	0.11	-0.09	0.30	0.45
6	-0.05	0.05	0.18	0.01	0.19	0.12
7	-0.05	0.23	-0.09	0.08	-0.29	0.34
					Inwards	
1	-2.04	-1.24	-0.72	-0.68	-0.56	-0.37
2	-1.08	-0.45	-0.37	-0.02	-0.35	-0.20
3	-0.66	-0.70	-0.30	-0.05	-0.41	0.13
4	-0.78	-0.52	-0.26	-0.06	0.19	0.36
5	-0.49	-0.16	-0.23	-0.06	-0.03	0.17
6	-0.59	0.00	-0.01	-0.28	0.17	0.38
7	-0.53	-0.19	-0.18	0.12	-0.23	0.03

(b) Annualized Sharpe ratio.

Table 24 – Minimum distance strategy filtered on volatility of the spread.

deviation of the spread series. While this approach is sensible when all spread series are created equal, such as equal weighting, the story is different for hedge ratios that vary. We therefore only filter for spread volatility on the minimum distance strategy.

21.4 Minimum distance strategy filtered on volatility

All the tables in the following subsection 21.4 have a common colored scale.

Table 24a shows mean return per month for the minimum distance strategy. Out of 126 Sharpe ratios 77 were positive (58%), as apposed to the 30 (24%) in the non-filtered version. The BEYOND version of the strategy had the best results. The average of BEYOND, OUTWARDS and INWARDS minimum distance strategies was, 0.32%, 0.27% and 0.13%, respectively. In comparison to the non-filtered which was 0.17%, 0.23% and 0.10%. The average of the volatility filtered is 0.24% the base model was 0.17%. The average Sharpe is -0.03 and the BEYOND, INWARDS and OUTWARDS are 0.18, 0.05 and -0.32.

Months	1	2	3	4	5	6	7
Avg. Ret.	0.13%	0.21%	0.24%	0.27%	0.30%	0.28%	0.26%
Avg. Sharpe	-0.33	-0.10	-0.04	0.05	0.12	0.06	0.03

Table 25 – Minimum distance strategy filtered on volatility of the spread. Average monthly return for different max. holding length.

Std. Dev.	0.5	1	1.5	2	2.5	3
Avg. Ret.	0.11%	0.21%	0.25%	0.25%	0.28%	0.34%
Avg. Sharpe	-0.38	-0.12	0.00	0.01	0.08	0.22

Table 26

21.4.1 Base models volatility filtered - summary

We average the parameters; max. holding length and entry threshold across all permutations and types.

Table 25 shows the maximum trade length parameters average monthly return across all models. On average a max. holding length of 5 and 6 generate the best Sharpe ratios and average monthly returns. From a holding period of 4 months and up there is positive Sharpe ratios.

Across all permutations and types the performance of the std. dev. entry threshold is illustrated in Table 26. A std. dev. of 1.5 and upwards generates on average positive Sharpe ratios with the highest Sharp ratio of 0.22 at std. dev. of 3. The pattern is the same for average returns.

Overall there is a significant improvement to the minimum distance strategy when filtering the pairs based on the volatility of the spread.

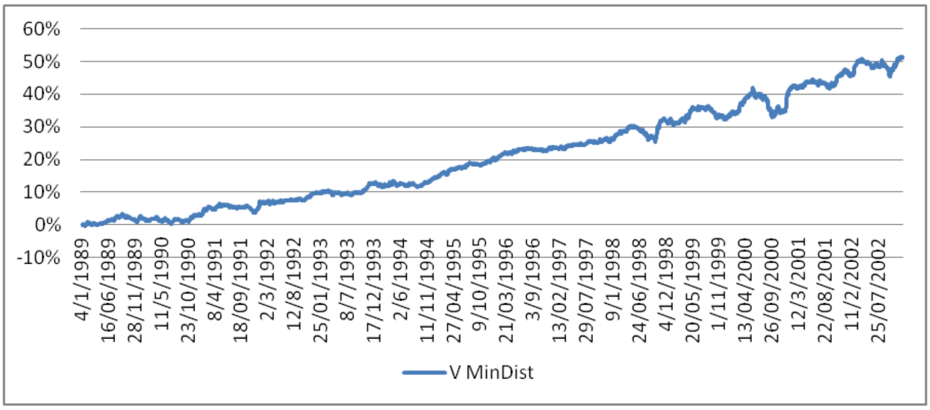


Figure 9 – The minimum distance strategies filtered for volatility average equity curve.

Month	1	2	3	4	5	6
1	-0.03%	0.19%	0.16%	-0.08%	-0.01%	0.12%
2	-0.17%	-0.03%	0.17%	0.03%	0.01%	0.05%
3	-0.12%	0.07%	0.15%	0.23%	0.21%	0.11%
4	-0.02%	-0.11%	0.22%	0.26%	0.00%	0.19%
5	0.01%	-0.08%	0.12%	0.28%	0.27%	0.22%
6	-0.18%	-0.27%	0.08%	0.13%	-0.05%	0.09%
7	-0.37%	-0.09%	-0.08%	-0.11%	-0.07%	0.04%
8	-0.24%	-0.19%	-0.19%	0.08%	0.01%	0.03%
9	-0.44%	-0.18%	-0.16%	-0.17%	0.02%	0.08%
10	-0.33%	-0.18%	-0.19%	-0.06%	-0.04%	-0.12%
11	-0.27%	-0.25%	-0.29%	-0.23%	-0.01%	0.00%
12	-0.37%	-0.21%	-0.25%	-0.22%	0.09%	0.01%

(a) The trend model with transaction cost.

Month	1	2	3	4	5	6
1	-0.54	-0.12	-0.18	-0.63	-0.51	-0.27
2	-0.76	-0.53	-0.17	-0.4	-0.4	-0.42
3	-0.7	-0.34	-0.2	-0.06	-0.09	-0.28
4	-0.48	-0.65	-0.07	0.01	-0.48	-0.13
5	-0.45	-0.6	-0.24	0.03	0.03	-0.07
6	-0.78	-0.96	-0.32	-0.22	-0.57	-0.31
7	-1.08	-0.6	-0.58	-0.62	-0.57	-0.4
8	-0.86	-0.77	-0.79	-0.32	-0.44	-0.41
9	-1.15	-0.75	-0.69	-0.72	-0.39	-0.32
10	-0.97	-0.75	-0.75	-0.55	-0.52	-0.65
11	-0.88	-0.9	-0.94	-0.8	-0.46	-0.45
12	-1.07	-0.77	-0.86	-0.81	-0.3	-0.42

(b) Sharpe ratios of the trend model.

Table 27 – Trend model.

21.5 Pairs trend models

21.5.1 Trend model

In order to properly evaluate the pair-trend models we first have to establish the performance of the stand-alone trend model. To better compare the stand-alone trend model with the pair trading models we structure the trend portfolio to consist of 20 long positions and 20 short position. We vary the formation period for the trend indicator from 1 to 12 month and vary the trading period 1 to 6 month. We will call the formation period of the trend indicator the lookback length parameter of the pairs trend model. Stocks that are a member of the S&P 500 index and have at least 252 trading days of prices are used in the strategy. Stocks are ranked by the statistical significances of the variance ratio test. The direction of the stock is determined by whether the stock has risen or fallen during the formation period. If the stock has trending behavior and the direction is upwards (downwards) a long (short) entry signal is generated for that stock. The strategy attempts to maintain a equivalent size long and short portfolio by constantly monitoring for stocks.

In Table 27a we can see that the performance of the long/short trend model is primarily in the red. The average of 72 Sharpe ratios is -0.52, with only 3 out of 72 permutations generating positive return. It is possible that the trend signal is very weak and drowns in the transaction cost. The average monthly return is -0.04%. Despite the lackluster result we continue with this trend indicator in the pair-trend models.

21.5.2 Pairs trend models

In constructing the pair trading model we have to choose a method for selecting candidate pairs of stocks and how to trade them. Since we already have selected the trend model as the method of entering and exiting trades we still need to decide on the method of sorting among the pairs of stocks and ranking them. We choose 3 different ranking methods, minimum distance, correlation and cointegration. Namely those used in the base models.

In Table 28 we see backtest results of the pairs trend models, with different ranking systems.

The strategies show no clear plateau of green cells in the tables that could indicate areas of robust parameters, but on average they outperform the pairs trading models with the exception of the minimum distance strategy filtered for volatility. The average monthly return is 0.20% for the minimum distance strategy, 0.17% for the correlation strategy and 0.19% for the cointegration strategy. The Sharpe ratios are similarly -0.11, -0.18 and -0.14 for the minimum distance strategy, the correlation strategy and the cointegration strategy, respectively.

21.5.3 Pairs trend models - summary

Table 29 shows the lookback length parameter of the pairs trend model and the respective average monthly return and Sharpe ratios across all settings. All parameter settings show negative Sharpe ratios. The average monthly returns range from 0.11% to 0.29%. The best parameters setting of the maximum holding length are 8 months, but the worst settings is 9. This does not indicate that the parameters have any robust settings in the form of plateaus of steady performance. The average of all the pairs trend strategies, excluding the stand-alone trend strategy is 0.19% per month and a Sharpe ratio of -0.13.

Month	1	2	3	4	5	6
1	0.23%	0.24%	0.24%	0.18%	0.22%	0.13%
2	0.26%	0.17%	0.17%	0.17%	0.13%	0.31%
3	0.15%	0.14%	0.13%	0.25%	0.21%	0.26%
4	0.23%	0.22%	0.16%	0.26%	0.19%	0.24%
5	0.20%	0.14%	0.13%	0.22%	0.28%	-0.03%
6	0.23%	0.17%	0.30%	0.19%	0.21%	0.27%
7	0.23%	0.20%	0.22%	0.29%	0.22%	0.30%
8	0.15%	0.24%	0.35%	0.34%	0.32%	0.28%
9	0.15%	0.15%	0.26%	0.21%	0.28%	0.11%
10	0.32%	0.25%	0.27%	0.29%	0.29%	0.16%
11	0.13%	0.19%	-0.13%	0.13%	0.11%	0.13%
12	0.15%	0.09%	0.13%	0.21%	0.27%	0.18%

(a) Average monthly return for the pairs trend model with minimum distance formation criteria.

Month	1	2	3	4	5	6
1	-0.04	-0.03	-0.01	-0.18	-0.07	-0.33
2	0.04	-0.20	-0.20	-0.22	-0.34	0.18
3	-0.26	-0.30	-0.31	0.01	-0.10	0.03
4	-0.06	-0.08	-0.24	0.02	-0.16	-0.03
5	-0.13	-0.28	-0.28	-0.09	0.09	-0.63
6	-0.06	-0.22	0.13	-0.15	-0.10	0.07
7	-0.04	-0.13	-0.06	0.11	-0.07	0.13
8	-0.28	-0.02	0.25	0.25	0.19	0.09
9	-0.28	-0.26	0.02	-0.10	0.10	-0.36
10	0.20	-0.01	0.06	0.10	0.10	-0.21
11	-0.29	-0.16	-0.77	-0.20	-0.31	-0.30
12	-0.23	-0.38	-0.26	-0.09	0.04	-0.17

(b) Sharpe ratios for the pairs trend model with minimum distance formation criteria.

Month	1	2	3	4	5	6
1	0.08%	0.09%	0.17%	0.11%	0.11%	0.15%
2	0.08%	0.05%	0.11%	0.23%	0.20%	0.17%
3	0.36%	0.26%	0.43%	0.30%	0.25%	0.19%
4	0.09%	0.22%	0.13%	0.10%	0.05%	0.02%
5	0.17%	0.22%	0.26%	0.14%	0.07%	0.10%
6	0.05%	0.12%	0.16%	0.25%	0.08%	0.11%
7	0.11%	0.19%	0.26%	0.16%	0.24%	0.33%
8	0.27%	0.31%	0.27%	0.15%	0.22%	0.39%
9	0.07%	-0.03%	0.00%	-0.06%	0.16%	-0.07%
10	0.17%	0.27%	0.08%	0.29%	0.14%	0.17%
11	0.23%	0.31%	0.03%	0.21%	0.27%	0.11%
12	0.35%	0.21%	0.13%	0.01%	0.18%	0.11%

(c) Average monthly return for the pairs trend model with correlation formation criteria.

Month	1	2	3	4	5	6
1	-0.42	-0.41	-0.19	-0.35	-0.38	-0.28
2	-0.32	-0.47	-0.31	-0.06	-0.11	-0.17
3	0.26	0.02	0.44	0.15	-0.01	-0.15
4	-0.41	-0.08	-0.26	-0.33	-0.42	-0.43
5	-0.17	-0.08	0.01	-0.24	-0.28	-0.26
6	-0.46	-0.27	-0.23	0	-0.35	-0.26
7	-0.27	-0.12	0.01	-0.19	-0.03	0.14
8	0.03	0.1	0.03	-0.19	-0.07	0.28
9	-0.3	-0.51	-0.47	-0.54	-0.18	-0.61
10	-0.18	0.03	-0.34	0.08	-0.25	-0.17
11	-0.06	0.11	-0.41	-0.09	0	-0.26
12	0.04	-0.08	-0.18	-0.38	-0.13	-0.3

(d) Sharpe ratios for the pairs trend model with correlation formation criteria.

Month	1	2	3	4	5	6
1	0.04%	-0.02%	0.16%	0.17%	0.12%	0.13%
2	0.21%	0.28%	0.22%	0.29%	0.25%	0.36%
3	0.18%	0.26%	0.25%	0.20%	0.36%	0.08%
4	0.14%	0.19%	0.24%	0.29%	0.22%	0.16%
5	0.16%	0.22%	0.23%	0.23%	0.28%	0.23%
6	0.19%	0.05%	0.22%	0.16%	0.09%	0.18%
7	0.24%	0.12%	0.14%	0.15%	0.25%	0.17%
8	0.30%	0.32%	0.40%	0.27%	0.40%	0.24%
9	0.23%	0.11%	0.16%	0.01%	0.09%	0.12%
10	0.05%	0.42%	0.14%	0.13%	0.27%	0.18%
11	0.03%	0.23%	0.24%	0.17%	0.26%	0.24%
12	-0.01%	0.10%	0.19%	0.15%	-0.01%	0.25%

(e) Average monthly return for the pairs trend model with cointegration formation criteria.

Month	1	2	3	4	5	6
1	-0.56	-0.71	-0.23	-0.22	-0.33	-0.31
2	-0.1	0.08	-0.07	0.1	-0.01	0.25
3	-0.17	0.03	0.01	-0.13	0.24	-0.37
4	-0.29	-0.14	-0.03	0.08	-0.08	-0.18
5	-0.18	-0.08	-0.05	-0.06	0.05	-0.04
6	-0.14	-0.46	-0.07	-0.21	-0.35	-0.15
7	-0.02	-0.27	-0.24	-0.24	0	-0.18
8	0.09	0.14	0.25	0.04	0.23	-0.02
9	-0.05	-0.3	-0.2	-0.5	-0.33	-0.25
10	-0.44	0.32	-0.24	-0.23	0.03	-0.16
11	-0.42	-0.05	-0.02	-0.15	0.01	-0.03
12	-0.51	-0.3	-0.13	-0.18	-0.48	-0.01

(f) Sharpe ratios for the pairs trend model cointegration formation criteria.

Table 28 – Sharpe ratios and average monthly returns of the pairs trend models.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Avg. Ret.	0.14%	0.20%	0.24%	0.17%	0.18%	0.17%	0.21%	0.29%	0.11%	0.22%	0.16%	0.15%
Avg. Sharpe	-0.28	-0.11	-0.03	-0.17	-0.15	-0.18	-0.08	0.08	-0.28	-0.07	-0.19	-0.21

Table 29 – The average monthly return and Sharpe ratio of the various lookback length of the variance ratio trend indicator.

Month	1	2	3	4	5	6
Avg. Ret.	0.17%	0.19%	0.19%	0.19%	0.20%	0.18%
Avg. Sharpe	-0.18	-0.15	-0.13	-0.12	-0.11	-0.15

Table 30 – The average monthly return and Sharpe ratio of the various maximum holding length.

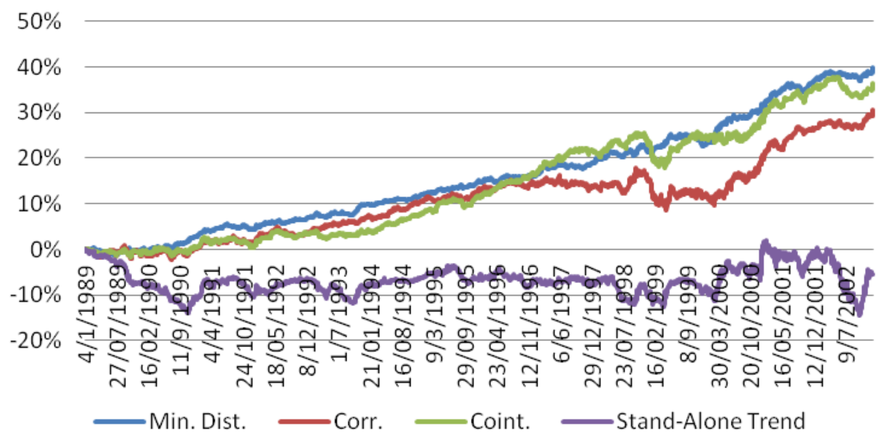


Figure 10 – Trend and pairs trend strategies average equity curves.

Table 30 shows the various maximum holding length of the pairs trend strategies and the corresponding average return and Sharpe ratio. There does not seem to a be signs of a performance plateau as the average returns only vary from 0.17% to 0.20%. The Sharpe ratios also only range from -0.11 to -0.18. Both show the 5 month holding period as the most optimal.

The average equity curves of the various pairs trend and trend strategies are shown in Figure 10. The stand-alone trend model' struggles are clear, as the average performance of the pairs trend models distance themselves from the stand-alone trend model.

21.6 Models signal strength

When we were looking at the tables of the performance measures average monthly return and annualized Sharpe ratio, we were not only interested in finding the highest possible values, but also high values that were surrounded by other high values. We were looking for plateaus of high, but stable values. Plateaus give confidence that the parameters are robust around several parameter settings and therefore perhaps also across time, from in-sample period to out-of-sample period.

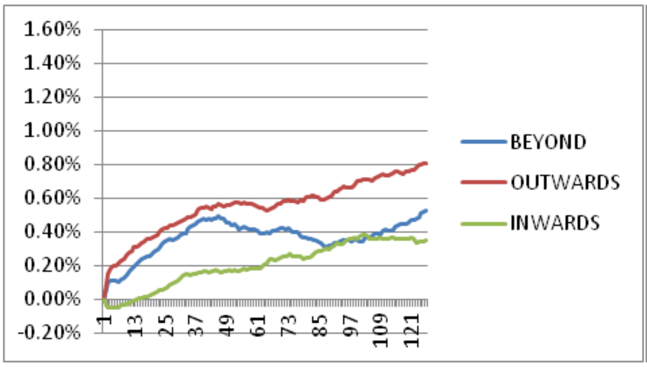
To further examine the element of robustness we also examine the signal quality of the various strategies using under settings. By signal quality we mean how the hypothetical trade positions gain or lose in value from when a signal is generated. The value of the position over time is illustrated as the equity curve from 0 to 126 trading days after signal generation, where 126 days correspond to half a year. The equity curves are calculated excluding by gains from investments in the risk free rate on the short side of the pair trade and excluding all transaction costs, both short selling fees and commissions. Note there is no signal delay in the graphs displayed here. Our hope is to find smooth equity curves that rise without jagged behavior.

We illustrate the differences in robustness between the various models by looking into shape of the equity curve after an entry signal. We do this by collecting all the trades made by the base models, base models with filtering permutations, pair-trend models and the stand-alone trend model. We have excluded the levelsadjusted models for to keep the section brief and concise. The total amount of trades aggregated here is 6,351,310 trades. For the pair trading models we aggregate the trades based on entry type (BEYOND, OUTWARDS, INWARDS) and std. dev. (0.5, 1.0, 1.5 ... 3). The pair-trends and trend model are categorized by the number of observation used to calibrate the trend indicator. The number of past observations in the past are in categories of, one, two or more months up to twelve months past, where twelve months is equivalent to the full formation period. Among the correlation strategies we select the CLOSEONE variation to represent the correlation strategy.

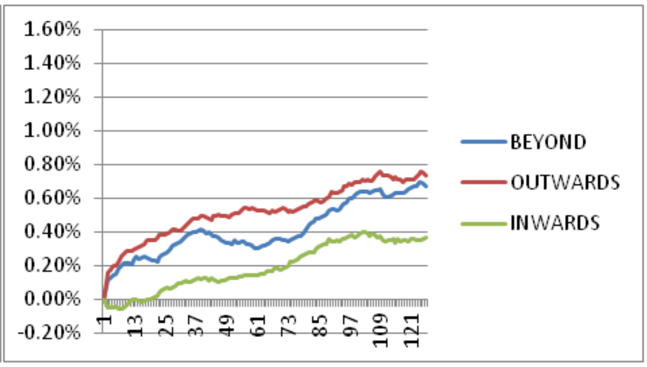
21.6.1 Base models

In Figure 11 we can see the equity curve of after entry signal by the various types BEYOND, OUTWARDS, INWARDS. All three base models show INWARDS as the worst performing one, typically dipping below zero for the first month or two and the slowly moving upwards to positive returns, nearly always saying below the OUTWARDS and INWARDS entry types. In the minimum distance and correlation strategy the OUTWARDS setting generates the most return, while in the cointegration strategy the OUTWARDS setting is edged out by the BEYOND setting. There is a particular sharp rise in the first few days after an entry signal is generated for in the BEYOND and OUTWARDS settings. For these settings the cost of entering the position with one trading day delay omits on average for all three models what would otherwise have been a potential 0.1% gain.

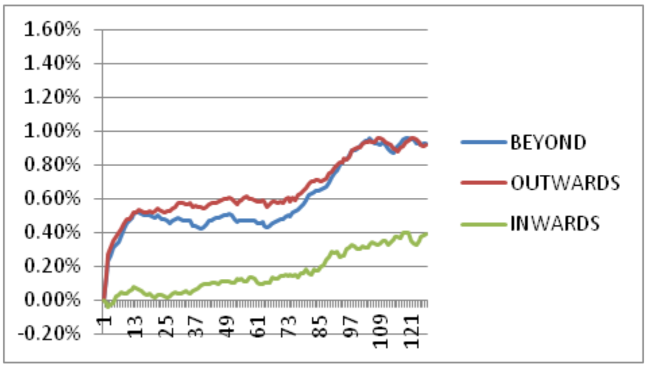
In Figure 12 we can see the signal paths after entry signal at the various std. dev. thresholds. The pattern is almost uniform. As the std. dev. threshold goes from 0.5 to 3.0 the average total return



(a) Equity curves after signal generation. Minimum distance strategy. Average path of each signal type.

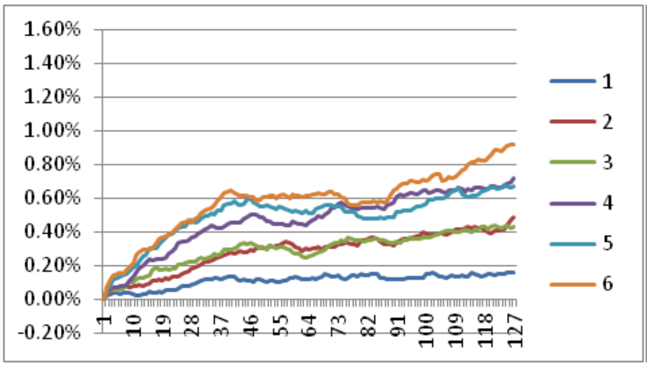


(b) Equity curves after signal generation. Correlation strategy variation CLOSEONE. Average path of each signal type.

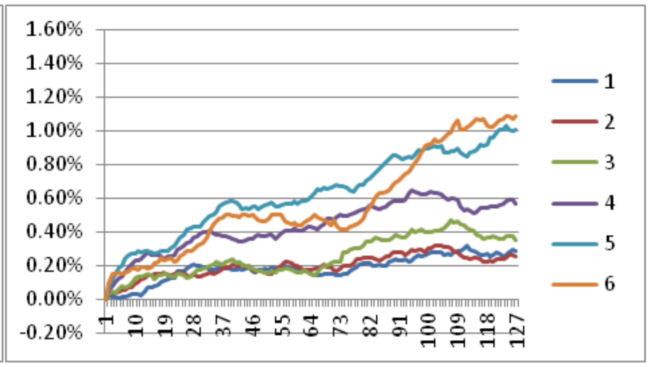


(c) Equity curves after signal generation. Cointegration strategy. Average path of each signal type.

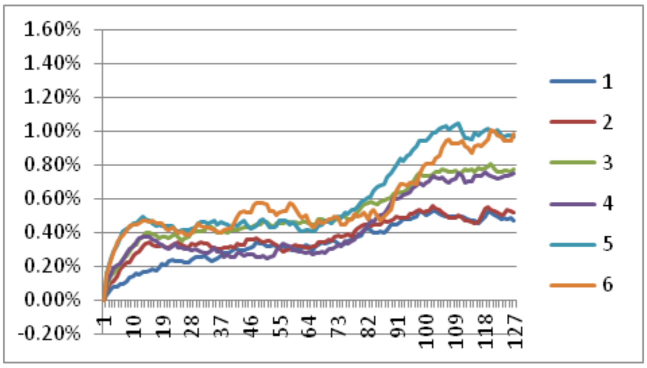
Figure 11 – Base models signal path by type.



(a) Equity curves after signal generation. Minimum distance strategy. Average path after entry signal at various std. dev. thresholds.



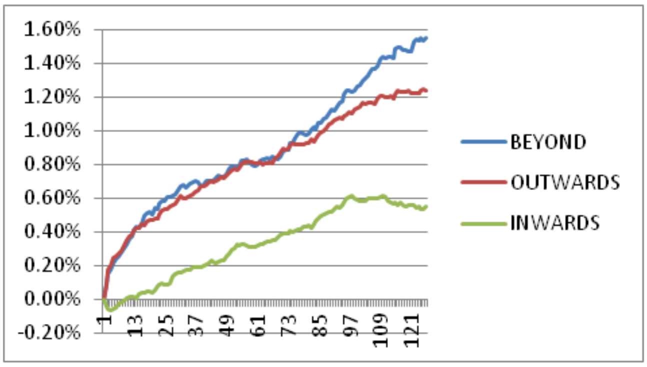
(b) Equity curves after signal generation. Correlation strategy variation CLOSEONE. Average path after entry signal at various std. dev. thresholds.



(c) Equity curves after signal generation. Cointegration strategy. Average path after entry signal at various std. dev. thresholds.

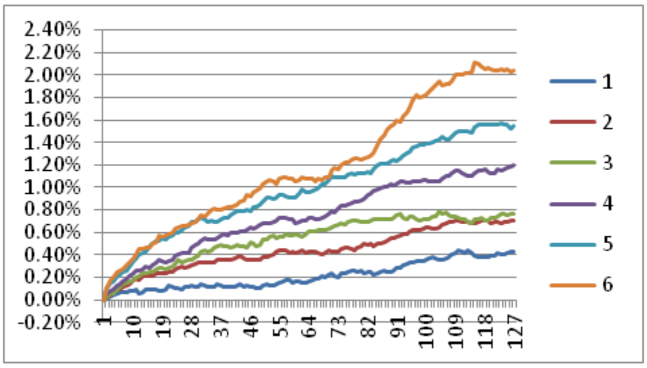
Figure 12 – Base model signal path by std. dev. entry threshold.

of each trade increases. While this is clear for the minimum distance strategy it is slightly less clear for the correlation and more so for the cointegration strategy. For the correlation strategy the lower half of the signal thresholds 0.5, 1.0 and 1.5 signal curves are entangled and it is unclear which has better performance. For the cointegration strategy the upper half of the signal curves 2.0, 2.5 and 3.0 signal curves are entangled and none show consistent higher return than the other. There is a slight decay in signal strength around 37 days of trading. The slope of the equity curves stagnates and picks up again around the 70 to 80 days of trading. This effect is most prevalent in the minimum distance strategy and the cointegration strategy. The cointegration strategy has the most sharpe upswings in the beginning compared to the others.



(a) Equity curves after signal generation. Minimum distance strategy volatility filtered. Average path of each signal type.

Figure 13 – Base models signal path by type.



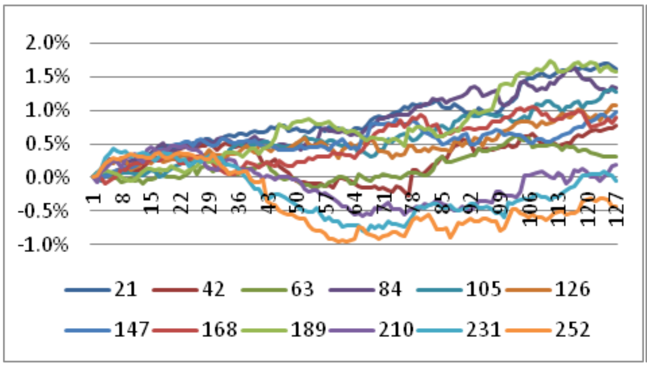
(a) Equity curves after signal generation. Minimum distance strategy. Average path after entry signal at various std. dev. thresholds.

Figure 14 – Base model signal path by std. dev. entry threshold.

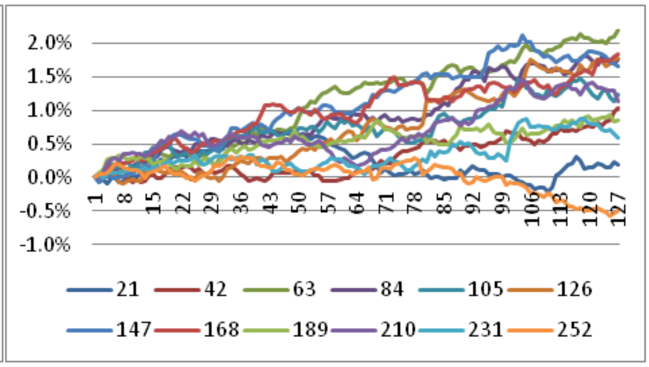
21.6.2 Minimum distance strategy with volatility filter

In Figure 13 we can observe the improved performance of the minimum distance strategy for all three types BEYOND, OUTWARDS and INWARDS. The signal curves look considerably more straight and the INWARDS entry type have a distinctive kink after around 110 trading days. Their performance is considerably improved and it is hard to distinguish between the BEYOND and OUTWARDS signal curves as the performance is similar. The equity curves are higher in all types compared to the base models after about a month of trading.

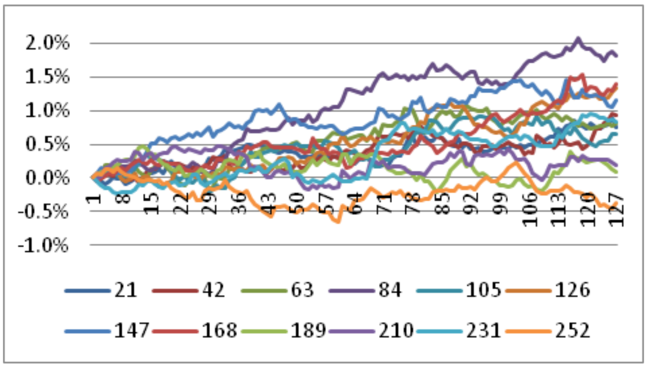
In Figure 14 std. dev. threshold settings the improvements are less clear. Table **??** shows the minimum distance strategy shows consistent improvement with greater spread between the different equity curves.



(a) Equity curves after signal generation. Pair-trend model ranked by minimum distance. Average signal path of each setting of the number of most recent days in the formation period used to calibrate the trend indicator.



(b) Equity curves after signal generation. Pair-trend model ranked by correlation. Average signal path of each setting of the number of most recent days in the formation period used to calibrate the trend indicator.



(c) Equity curves after signal generation. Pair-trend model ranked by cointegration. Average signal path of each setting of the number of most recent trading days in the formation period used to calibrate the trend indicator.

Figure 15 – Pairs trend models by the number of trading days used to calibrate the trend indicator .

21.6.3 Pair-trend models

In Figure 15 the picture is more erratic compared to the pairs trading strategies. Most equity curves have a positive drift, but it is hard to infer anything from the intertwined equity curves. This erratic behavior is was also present in the tables of the average month returns and Sharpe ratios. There was little sense of steady performance in parameters. That being said all strategies have on average positive performance. The pattern they display however does not give us any hints as to what characteristics the better performing strategies have or how to improve them.

The stand-alone trend model in Figure 16 has lower magnitudes and less volatile equity curves. However as we saw in the tables of average monthly returns and Sharpe ratios the increase in the equity curve

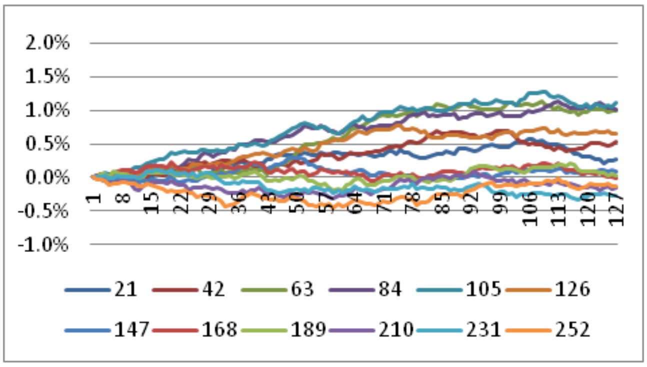


Figure 16 – Equity curves after signal generation. Stand-alone trend model ranked by cointegration. Average signal path of each setting of the number of most recent trading days in the formation period used to calibrate the trend indicator.

	Min. Dist.	SMALL	CLOSEONE	BIG	ONE	Coint.
Base	0.17%	0.09%	0.13%	0.10%	0.11%	0.18%
Levels.	0.14%	0.04%	0.07%	0.04%	0.04%	0.17%
Vol.	0.24%					

(a) Average monthly returns.

	Min. Dist.	SMALL	CLOSEONE	BIG	ONE	Coint.
Base	-0.31	-0.37	-0.31	-0.39	-0.40	-0.16
Levels.	-0.41	-0.45	-0.42	-0.53	-0.57	-0.18
Vol.	-0.03					

(b) Annualized Sharpe ratio.

Table 31 – Average monthly return of each strategy variation, minimum distance, correlation and cointegration.

was not enough to overcome trading costs.

21.7 In-sample summary

Instead of choosing the best model for the out-of-sample we will choose by method of elimination. In other words we will exclude models, and settings that did not work and use the remaining models as the out-of-sample model.

Table 31 shows the average monthly returns and Sharpe ratios for all the strategies in the in-sample analysis. In both Sharpe ratio and average monthly return the levelsadjusted performed poorer then the base models.

Among the base models and the levelsadjusted models the minimum distance strategy and the cointegration strategy performed the best. The average monthly return for the base cointegration strategy and the base minimum distance strategy was, 0.17% and 0.18%, respectively. We therefore excluded all of the varies correlation models from the final out-of-sample model.

We attempted to boost returns on the minimum distance strategy by filtering the least volatile 1000 pairs out of the top ranking 2000 pairs. The resulting 1000 pairs were used to trade. The minimum distance strategy improved significantly, going from 0.003% to 0.08%, see Table 31. Among the three different variations of the minimum distance strategy the volatility filtered performed the best. The tables of average monthly return and Sharpe ratios showed areas of better performance around certain parameters settings namely the higher std. dev. threshold settings.There was also a quite consistent behavior with the signal equity curves, where higher std. dev. showed higher returns. Without considering specific settings the minimum distance strategy filtered for volatility is the most attractive strategy.

The cointegration strategy did not have as nice signal equity curves as the minimum distance strategy, but the performance in the average monthly return tables and the Sharpe ratio tables showed superior performance to the base minimum distance strategy. It would therefore be hard to exclude the cointegration model based on inferior returns and Sharpe ratios.

The stand-alone trend model did not show any strong performance . The initial impression was that the variance ratio test as a trend indicator for a long short portfolio was ineffective. In combination with pair trading the picture changed. The returns for the three pairs trend strategies was 0.20%, 0.17% and 0.19%, minimum distance, correlation and cointegration, respectively. The tables of average monthly return and Sharpe ratios were a bit scattered and hard to see if there was areas of robust settings. It is therefore hard to improve upon the pairs trend strategies.

We argued that the levels adjusted models should work as they better follow the spread that signals entry and exit. They are however worse on every parameter and model considered. On all of the pairs trading strategies we used the INWARDS entry type was the worst. The signal equity curves also showed clear underperformance of the settings compared to the BEYOND and OUTWARDS setting. The BEYOND and OUTWARDS setting were more evenly matched. Most of the time the OUTWARDS setting was best, but for the minimum distance strategy filtered for volatility the BEYOND setting was best. Based on these arguments we exclude the INWARDS setting from inclusion

	Min. Dist.	SMALL	CLOSEONE	BIG	ONE	Coint.
Base	0.20%	0.13%	0.16%	0.13%	0.14%	0.25%
Levels.	0.17%	0.08%	0.11%	0.06%	0.06%	0.24%
Vol.	0.29%					

(a) Average monthly returns.

	Min. Dist.	SMALL	CLOSEONE	BIG	ONE	Coint.
Base	-0.19	-0.26	-0.21	-0.30	-0.32	-0.02
Levels.	-0.30	-0.36	-0.33	-0.45	-0.50	-0.03
Vol.	0.11					

(b) Annualized Sharpe ratio.

Table 32 – Average monthly return of each strategy variation, minimum distance, correlation and cointegration without the INWARDS entry type setting.

to final out-of-sample strategy.

Table 32 shows the improved performance in all strategies from the exclusion of the INWARDS setting. The minimum distance strategy filtered for volatility goes from 0.24% to 0.29% in average monthly return and goes from -0.03 to 0.11 in Sharpe ratio. The cointegration strategy goes from 0.18% to 0.25% in average monthly return and the Sharpe ratio goes from -0.16 to -0.02. These improvements distances the strategies from the pairs trend strategies. We choose the minimum distance strategy filtered for volatility and the base cointegration strategy as the combined out-of-sample model.

22 Out-of-sample

The out-of-sample strategy is calculated as the equally weighted average of the two strategies, the minimum distance strategy filtered for volatility and the base cointegration strategy using only the INWARDS and OUTWARDS entry type setting. The out-of-sample stretches from 1. January 2003 to 22. October 2014.

Figure 17 shows the out-of-sample equity curve. Without transaction cost the combined strategy shows an up sloping equity curve, albeit with a falling equity curve from 2005 to late 2008. The Sharpe ratio for the model without transaction cost in this period is 0.02. The average monthly return is 0.18%.

Figure 18 shows the performance when we take into account transaction cost. The Sharpe ratio is -0.01. The average monthly return is 0.09%.

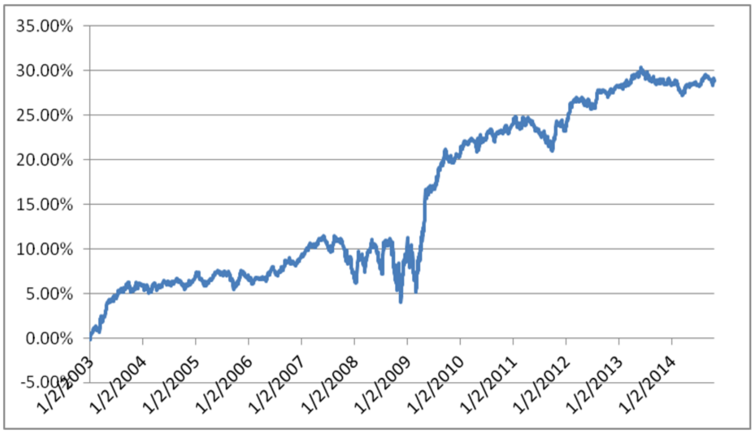


Figure 17 – The combined out-of-sample model, without transaction cost.

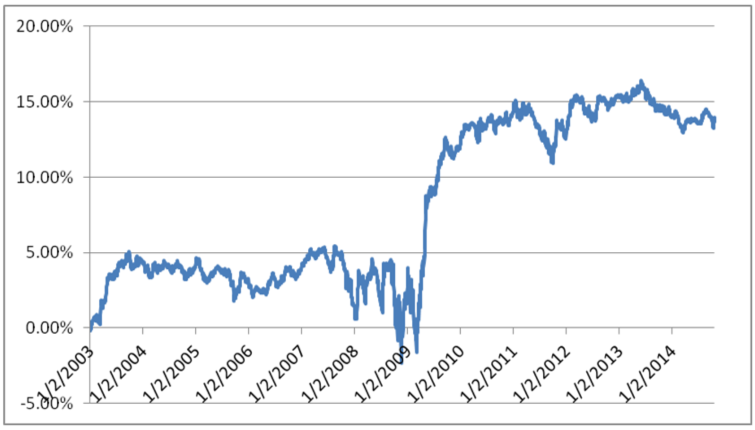


Figure 18 – The combined out-of-sample model, with transaction cost.

```
No cost
Estimated Coefficients:
                   Estimate       SE           tStat      pValue
    (Intercept)    0.0063385     0.0030761    2.0606       0.039432
    x1              0.054395     0.0027106    20.067     4.2256e-84
    x2             -0.031186     0.0056251    -5.544     3.2151e-08
    x3              0.075015     0.0061103    12.277     7.8783e-34


Number of observations: 2972, Error degrees of freedom: 2968
Root Mean Squared Error: 0.168
R-squared: 0.223,  Adjusted R-Squared 0.222
F-statistic vs. constant model: 283, p-value = 8.64e-162
```

Table 33 – Fama-French regression between the factors market return (x1), SML (x2), HML (x3) for the combined out-of-sample model without transaction cost. Note the estimates are in percentage.

```
With Cost
Estimated Coefficients:
                  Estimate      SE           tStat      pValue
    (Intercept)   0.0021852     0.0030118    0.72555       0.46817
    x1              0.05175     0.0026539       19.5      8.85e-80
    x2            -0.031045     0.0055074    -5.6369    1.8939e-08
    x3             0.077581     0.0059825     12.968     1.894e-37


Number of observations: 2972, Error degrees of freedom: 2968
Root Mean Squared Error: 0.164
R-squared: 0.223,  Adjusted R-Squared 0.222
F-statistic vs. constant model: 284, p-value = 6.08e-162
```

Table 34 – Fama-French regression between the factors market return (x1), SML (x2), HML (x3) for the combined out-of-sample model with transaction cost.

Next we regress the returns to examine whether the returns can be attributed to the risk factors included in the Fama-French three factor model. Table 33 shows the results. x1 is the market factor. x2 is the size-factor, SML. x3 is the book-factor, HML. Regression on the returns without transaction cost show that the intercept is the only significant factor at the 5% significance levels. The intercept can be interpreted of as the abnormal return. The return which is not attributed to any of the Fama-French risk factors. The intercept is 0.006% and the observation frequency is daily. This translates to roughly 0.13% at the monthly frequency. As we can see this is notably higher than the mean return from the portfolio. This can be interpreted as the portfolio having some potential for reducing risk by hedging this pairs portfolio with portfolios constructed on the aforementioned risk factors.

When we take into account the transaction cost, the intercept is no longer significant and the risk factors are all still highly significant. Table 34 shows thatT the returns or more likely to be explained by the three risk factors than by the intercept. This indicates that after taking transaction costs into account the combined pairs trading portfolio does not produce abnormal return.

During the financial crisis there were short-selling bans on many different stocks and mostly on financial stocks. To see if the short selling bans would affect our results we exclude the 5th SIC code containing finanical stocks from the out-of-sample data and run the models again. The 5th SIC sector is as defined in section 18. The results can be found section 23 in appendix and are almost identical to those above.

Part VIII

Discussion

That the pairs trend model worked well compared to the stand-alone trend model. The average monthly return of the stand-alone trend model was -0.04%, while it was 0.20%, 0.17% and 0.19% for the three pairs trend models, minimum distance, correlation and cointegration. The results here support the merits of the strategy from Deutsche Bank. The intuition of the strategy was that two highly correlated assets that both exhibited trends in opposite direction would be likely to persist than that of non- or less correlating assets. The results can only confirm this hypothesis, but a deeper analysis would be required to correctly determine what the cause is of this behavior. The tables of average monthly return and Sharpe ratios were poor in giving us hints how the strategy worked, the signal equity curves also had a large spread leaving us little clue, as to how the strategy functioned. Further analysis is needed.

The pairs trend models barely outperformed the pairs trading models, but once the pairs trading models were optimized by excluding underperforming settings the pairs trading models outperformed the pairs trend models. We did not include the pairs trend models in the out-of-sample period. It might have been a good idea from the perspective of diversification, since the pairs trend models were constructed differently from the pairs trading models.

The levelsadjusted models were clear underperformers to the base models. All models were slightly worse than the base models. In our argumentation of the levelsadjusted models we made the point that the levelsadjusted models had a identical payoff whether it was the better performing stock in the pair that favorably lost in value (and thereby resulting in a short-sale gain) or the worse performing stock that favorably gained in value. The examples in section 19 showed that for the base model to outperform the levelsadjusted model, the base model had to experience more scenarios where the underperforming stock was the one to gain most in value. Whether it is this scenarios that contributes to the base models outperformance or different mechanism is left for further research.

There was success in eliminating the INWARDS entry type from all models and choosing the minimum distance strategy filtered for volatility over the base minimum distance strategy. The INWARDS entry type was clearly inferior to the BEYOND and OUTWARDS entry type and filtering for volatility on the minimum distance strategy also showed clear improvements.

We could have optimized the std. dev. setting and the maximum holding period, but we chose not to. The cointegration strategy was bias towards low std. dev. thresholds and the minimum distance strategy was biased towards higher std. dev. thresholds. We therefore chose not to optimize, in fear of further data snooping.

The Fama-French regressions showed that the out-of-sample after transaction cost could be entirely explained by the risk factors, market, size and book-to-equity. Before transaction cost the alpha (intercept) was significant, suggesting the strategy only being fitting for an investor facing low transaction costs. However even before transaction cost the Sharpe ratio of 0.02 is not overly impressive, and only attractive from the perspective of diversification.

The stocks under investigation in this thesis was the S&P 500. These are some of the most liquid stocks in the world. It might not be unreasonable to believe that the transaction costs applied here were too harsh as the transaction costs from [Do and Faff 2011] was the average of all stocks. This means the transaction costs used in this thesis were conservative in comparison to the estimates in [Do and Faff 2011]. Institutional investors might be able to get much lower costs than what was given in [Do and Faff 2011].

It might be possible to increase the performance if we skipped the 1 day signal delay. This would be akin to calculating the signal and executing the trades immediately. In reality one might have to do this 15 minutes before the close, that would however give some discrepancy between the backtest and the reality of trading it.

Another way to look at it is that the quantitative approach used in this thesis, cannot be used by on it own exclusively, but should be used in conjunction with a qualitative analysis of each signal. Our information is only price, but as price ultimately is driven by fundamentals, it would be obvious to filter out false signals. More often than not large swings in price come from news, which the algorithm cannot see. When a pharmaceutical company suddenly has success with a new drug and a competitor does not, there is bound to be differences in stock price movements. For the qualitative investor it is obvious that the price changes stem from a change expectations of the growth in the two companies, but the purely quantitative approach does not filter for this. Corporate actions such as tender offers, potential takeovers are not accounted for. Neither are disappointing sales statistics and sudden macroeconomic events influencing export markets. The methods in this thesis are only truly attractive in the hands of a knowledgeable qualitative investor.

We ran the out-of-sample on data without financial stocks to see if the massive shorts-selling bans would have an effect on our results. The results were almost identical and we therefore conclude that the short-sale bans did not effect the effectiveness of the pairs trading strategies.

The in-sample period in general showed a better performance than the out-of-sample period. While we did put in significant effort into choosing the robust strategies and settings we cannot exclude that we have been data-snooping. It is also likely that the performance of pairs trading declined over the years as was suggested in [Do and Faff 2010] and [Do and Faff 2011].

Part IX

Conclusion

For the out-of-sample model we choose the minimum distance strategy filtered for volatility and the cointegration strategy, using only the BEYOND and OUTWARDS entry settings. We argued that the std. dev. threshold and maximum holding period did not show the same clear pattern of performance as the entry settings and therefore did not optimize on them.

In the problem identification and delimitation, Part III, we ask whether pairs trading could generate economic significant returns. As can be seen the out-of-sample pairs trading strategy did generate return, but only modestly so. Transaction cost almost halved the total cumulative return and the Sharpe ratios before and after were not impressive at 0.02 before transaction costs and -0.01 after transaction cost. In the discussion we suggested various ways how the strategies might generate more value. One was with investors that faced lower transaction costs, the other as a filter for the qualitative pairs trader. The quantitative pairs trading strategy as presented here, does not represent an independently competitive investment strategy.

One of the sub-questions that the we asked was whether cointegration persisted from one year to the next. From the results in, section 20, we can say that it does, but it varies through time. In 15 out of 25 years the persistence of cointegration was statistically significant. [Clegg 2014] arrived at a different conclusion, but we attributed this to differences in data.

We also showed that the pairs trend models in principle worked as a investment strategy, albeit not very well. It was hard to examine the results of the pairs trend models and ultimately we excluded it from the out-of-sample model. It could have been interesting to compare the pairs trading and pairs trend models to see how their investment choices overlapped or differed.

The persistence in cointegration analysis gave us the expectation that there would be impactful events in 1991, 2003 and 2009. For the out-of-sample model 2009 was an extraordinary good year were nearly all of the strategies return came from. From the persistence in cointegration analysis we can see that there was a large increase in the amount of cointegrating pairs. However this might not be the only explanation it could also be that the average profitable trade increased, in other words in 2009 the divergences in cointegrating pairs might also have been large. Further research could answer these questions.

Part X

Appendix

23 Fama-french regressions on non-financial stocks

```
No cost
Estimated Coefficients:
                   Estimate      SE           tStat      pValue
    (Intercept)    0.0060699     0.0029997     2.0235      0.043112
    x1              0.039421     0.0026431     14.915    1.4527e-48
    x2             -0.024248     0.0054854    -4.4204    1.0205e-05
    x3               0.03089     0.0059585     5.1841    2.3158e-07


Number of observations: 2972, Error degrees of freedom: 2968
Root Mean Squared Error: 0.163
R-squared: 0.107,  Adjusted R-Squared 0.106
F-statistic vs. constant model: 119, p-value = 1.6e-72
```

Table 35 – Fama French regression on the out-of-sample model restricted to sectors excluding financial assets and without transaction costs.

```
With Cost
Estimated Coefficients:
                   Estimate      SE           tStat      pValue
    (Intercept)    0.0023008     0.0029041    0.79226       0.42827
    x1              0.036699     0.0025591     14.341    3.7941e-45
    x2             -0.023448     0.0053106    -4.4154    1.0442e-05
    x3              0.031629     0.0057687      5.483    4.5337e-08


Number of observations: 2972, Error degrees of freedom: 2968
Root Mean Squared Error: 0.158
R-squared: 0.103,  Adjusted R-Squared 0.102
F-statistic vs. constant model: 114, p-value = 8.55e-70
```

Table 36 – Fama French regression on the out-of-sample model restricted to sectors excluding financial assets and with transaction costs.

	Formation Period (y)					Trading Period (y+1)				
y	N(I1)	$x \in CI$ (y)	$Pr(x \in CI$ (y))	mean of ρ	std. of ρ	N(I1)	$x \in CI$ (y+1)	$Pr(x \in CI$ (y+1))	mean of ρ	std. of ρ
1988	88410	3616	4.1%	0.949	0.029	79401	1305	1.6%	0.981	0.016
1989	88831	3222	3.6%	0.963	0.027	86320	1705	2.0%	0.982	0.015
1990	97020	5342	5.5%	0.963	0.025	79800	2681	3.4%	0.975	0.022
1991	83028	5311	6.4%	0.962	0.026	76636	1751	2.3%	0.972	0.022
1992	92665	4074	4.4%	0.957	0.028	85491	1381	1.6%	0.971	0.019
1993	92235	3816	4.1%	0.957	0.025	82215	1819	2.2%	0.971	0.021
1994	89253	5133	5.8%	0.956	0.028	79800	1544	1.9%	0.982	0.018
1995	90100	3193	3.5%	0.960	0.028	82215	2352	2.9%	0.977	0.019
1996	94395	4169	4.4%	0.961	0.026	89676	1124	1.3%	0.981	0.018
1997	97020	2459	2.5%	0.963	0.028	92665	1544	1.7%	0.979	0.018
1998	99681	3754	3.8%	0.963	0.025	92235	1457	1.6%	0.977	0.020
1999	96580	3056	3.2%	0.961	0.026	87571	1761	2.0%	0.972	0.020
2000	93528	3874	4.1%	0.955	0.026	74305	2157	2.9%	0.966	0.025
2001	86736	6847	7.9%	0.953	0.025	75855	1041	1.4%	0.974	0.022
2002	107880	6962	6.5%	0.957	0.027	90100	2570	2.9%	0.981	0.022
2003	94830	4274	4.5%	0.963	0.024	86736	2616	3.0%	0.979	0.017
2004	103285	5492	5.3%	0.965	0.023	95703	1643	1.7%	0.977	0.017
2005	102378	3285	3.2%	0.962	0.023	98346	1624	1.7%	0.980	0.016
2006	104196	3684	3.5%	0.964	0.023	97020	1526	1.6%	0.980	0.019
2007	106030	4008	3.8%	0.958	0.029	101475	1443	1.4%	0.974	0.028
2008	112101	8058	7.2%	0.947	0.035	41616	1604	3.9%	0.965	0.032
2009	41328	2562	6.2%	0.957	0.026	39340	642	1.6%	0.979	0.018
2010	111156	3645	3.3%	0.966	0.022	103285	2285	2.2%	0.976	0.018
2011	103285	5299	5.1%	0.959	0.024	94395	1799	1.9%	0.975	0.017
2012	102378	4838	4.7%	0.963	0.022	91806	1995	2.2%	0.982	0.017

Table 37 – This table shows number of pairs that exhibit cointegration in the formation period and in the trading period for the beta adjusted returns. Column named $N(I1)$ are the number of pairs under examination. Column $x \in CI(y)$ and $x \in CI(y+1)$ are the number of pairs that cointegrate in period y and $y+1$, respectively. ρ is the autoregressive coefficient of the time series produced from the cointegrating pair.

24 Cointegration persistence tables - control for beta exposure

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y	Formation Period (y)	Trading Period (y+1)
1988	420	398
1989	421	415
1990	440	399
1991	407	391
1992	430	413
1993	429	405
1994	422	399
1995	424	405
1996	434	423
1997	440	430
1998	446	429
1999	439	418
2000	432	385
2001	416	389
2002	464	424
2003	435	416
2004	454	437
2005	452	443
2006	456	440
2007	460	450
2008	473	288
2009	287	280
2010	471	454
2011	454	434
2012	452	428

Table 38 – This table shows the number of stocks eligible for calculation after sorting for stationary stocks on top of the normal requirements such as continuous time series and mamber of the S&P 500 at the end of the trading period.

y	Pr $(x \in CI(y+1) \mid x \in CI(y))$	$Pr(x \in CI\ (y+1))$
1988	1.7%	1.6%
1989	3.3%	2.0% **
1990	3.0%	3.4%
1991	2.7%	2.3% *
1992	2.5%	1.6% ***
1993	1.7%	2.2% *
1994	1.9%	1.9%
1995	3.1%	2.9%
1996	1.5%	1.3% ***
1997	2.7%	1.7% ***
1998	1.7%	1.6% ***
1999	2.0%	2.0% ***
2000	3.1%	2.9%
2001	1.8%	1.4% ***
2002	2.5%	2.9% ***
2003	2.2%	3.0%
2004	1.9%	1.7% ***
2005	2.3%	1.7% ***
2006	2.1%	1.6% ***
2007	1.8%	1.4% ***
2008	1.9%	3.9% ***
2009	1.8%	1.6%
2010	2.6%	2.2% ***
2011	2.5%	1.9% ***
2012	1.8%	2.2% ***

Table 39 – This table shows the significance of the hypothesis in the various y years.

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