TIME SERIES REGRESSION

Contents

Linear Regression Model for Time Series Data

$$Y_t = b_0 + b_1 X_{1t} + b_2 X_{2t} + \ldots + b_p X_{pt} + e_t$$

Where,

Y_t	:	Dependent Variable **at time t**
$X_{1t}, X_{2t}, \ldots, X_{pt}$	:	Independent Variables Measured **at time t**
$b_0, b_1, \ldots, b_p$	:	Parameters of Model
	:	Random Error Component **at time t**

Variables can either be **Continuous or Categorical**

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The Concept of Autocorrelation

- Autocorrelation is a **correlation between a time series (Y_t) and another time series representing lagged values of the same time series(Y_{t-k}).**

- Autocorrelation refers to the **correlation of a time series with its own past values**

t	Y_t
1	Y_1
2	Y_2
"	"
"	"
10	Y_{10}

t	Y_t	Y_{t-1}
2	Y_2	Y_1
3	Y_3	Y_2
"	"	"
"	"	"
10	Y_{10}	Y_9

- Correlation between Y_t and Y_{t-1} is **"first order autocorrelation"**

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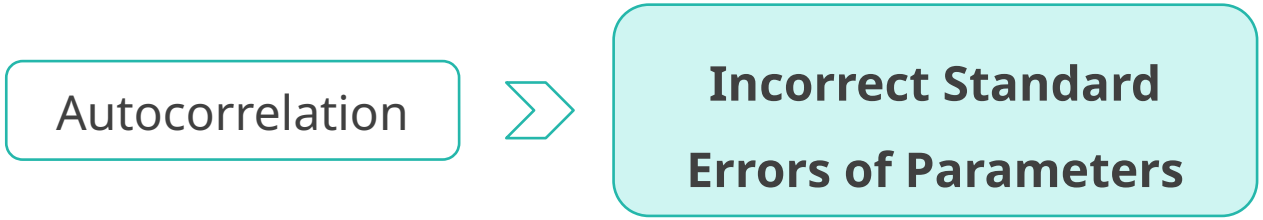
Autocorrelation in Multiple Linear Regression

- Autocorrelation is observed only when the data is a time series or panel data
- In regression model, absence **of autocorrelation among errors is one of the key assumptions**



If we obtain the OLS estimators of the regression model without correcting error autocorrelation, the model may yield

- **Underestimation of True Error Variance**
- **Overestimation of R^2**
- **Misleading Results of t & F Tests**

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Case Study – Predicting Sales

Background

- A retail store undertakes two-fold marketing campaign, one for print media and the other for digital. The company also collects information on yearly increments (price adjusted) in sales. The company wishes to check if the marketing expenses have any bearing on the sales

Objective

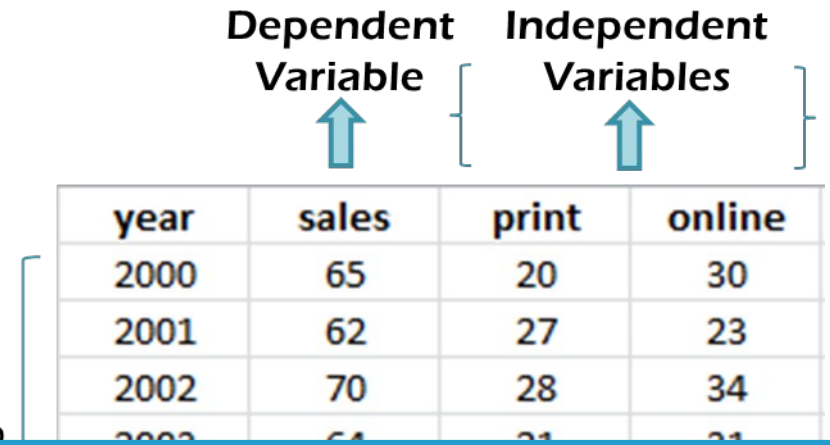
- To predict price adjusted incremental sales based on expenses on marketing campaigns

Available Information

- Yearly time series data, of 11 years
- Independent Variables: Marketing expenses for – Print Media and Online
- Dependent Variable: Price Adjusted Incremental Sales

Data Snapshot

SALES VS MARKETING COSTS



year	sales	print	online
2000	65	20	30
2001	62	27	23
2002	70	28	34

Columns0	Description	Type	Measurement	Possible values
Year	Year	Numeric	2000 to 2010	11
Sales	Price Adjusted Incremental Sales	Numeric	in INR Million	Positive values
Print	Expenses on Print Marketing	Numeric	in INR Million	Positive values
online	Expenses on Online Marketing	Numeric	in INR Million	Positive values

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Detecting Presence of Autocorrelation – Durbin Watson Test

Objective	To check the assumption of 'No Autocorrelation'

Null Hypothesis H_0: Autocorrelation is not present among errors

Alternate Hypothesis H_1: Not H0

Test Statistic	d ≅ 2 (1- r) Where r is sample autocorrelation based on residuals obtained in regression model. Ideal Value of d = 2 (taking r = 0)
Decision Criteria	Reject the null hypothesis if p-value < 0.05

Durbin Watson Test in R

```
# Importing the Data and Fitting Linear Model
sales<-read.csv("SALES VS MARKETING COSTS.csv",header=TRUE)
salesmodel<-lm(sales~print+online,data=sales)

summary(salesmodel)
```

```
# Output
```

```
Call:
lm(formula = sales ~ print + online, data = sales)

Residuals:
     Min       1Q   Median       3Q      Max
-2.16109 -1.00608  0.07342  0.85923  2.32128

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 30.05815    3.78520   7.941 4.61e-05 ***
print        0.48154    0.14275   3.373  0.00974 **
online       0.76633    0.06828  11.224 3.56e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.486 on 8 degrees of freedom
Multiple R-squared:  0.952,     Adjusted R-squared:   0.94
F-statistic:  79.3 on 2 and 8 DF,  p-value: 5.317e-06
```

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Durbin Watson Test in R

```
# Install & Load package “car”
# Durbin Watson Test
install.packages("car")
library(car)

durbinWatsonTest(salesmodel)
```

durbinWatsonTest() in package car carries out the Durbin Watson test. Output gives the d-w statistic as well as p-value

```
# Output
> durbinWatsonTest(salesmodel)
 lag Autocorrelation D-W Statistic p-value
   1       0.4804249     0.7259509   0.004
 Alternative hypothesis: rho != 0
```

Interpretation :

- Reject H0, p-value<0.05.
- Errors have significant autocorrelation.

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Error Process in the Presence of Autocorrelation

Presence of autocorrelation implies that, **errors at time t are related to errors at previous time points**

is coefficient of autocorrelation of lag k between &

is such that,

E()=0

V()= Constant

Cov(,)=0 s≠0

Note: Typically in time series regression we consider autocorrelation of order 1 only

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Consequences of Ignoring Autocorrelation

If we obtain the OLS estimators of the model, without correcting error autocorrelation, the model may yield

- Underestimation of True Error Variance
- Overestimation of R^2
- Misleading Results of t & F Tests

Autocorrelation – Remedial Measures

The following three methods can be used to estimate regression parameters when there exists autocorrelation in the time series data. We will focus on MLE method.

- Cochrane-Orcutt Method
- Praise-Winsten Method
- Maximum Likelihood Estimation Method

Maximum Likelihood Estimation Method

- Maximum likelihood estimator (MLE) can be obtained by maximizing the log likelihood function with respect to b, σ^2_u, ρ

- Log likelihood function is given as follows:

$$\ln L =$$

Parameter Estimation – Maximum Likelihood Estimation Method

```
# Parameter Estimation (Maximum Likelihood Estimation)
salests<-ts(sales$sales,start=2000,end=2010)

xvar<-subset(sales,select=c("online","print"))

salesmodel<-arima(salests,order=c(1,0,0),xreg=xvar)

coef(salesmodel)
      ar1 intercept      onl
0.7774653 32.0288203
```

ts() converts a column from a data frame to a simple time series object.

subset() creates a subset of all the

arima() estimates model parameters. Subset of independent variables (which is a vector or matrix of external regressors) is used in **xreg=**

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Parameter Estimation – Maximum Likelihood Estimation Method

```
#Parameter Estimation (Maximum Likelihood Estimation)
install.packages("lmtest")
library(lmtest)

coeftest(salesmodel)
```

arima() does not give p-values, hence **coeftest()** from package **lmtest()** is used. It performs z and (quasi-)t tests of estimated coefficients.

```
# Output
z test of coefficients:

           Estimate Std. Error z value  Pr(>|z|)
ar1        0.777465   0.171698  4.5281 5.952e-06 ***
intercept 32.028820   2.137725 14.9827 < 2.2e-16 ***
online     0.811782   0.033897 23.9486 < 2.2e-16 ***
print      0.362140   0.065133  5.5600 2.697e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Interpretation :

- Both print and online are significant variables in this output as well

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Predictions

```
# Predictions (For Next Two Years)
online<-c(40,45)

print<-c(50,55)

xvarnew<-data.frame(online,pri



predict(salesmodel, n.ahead=2, newxr
```

Expected Values of online and print marketing expenses in next 2 years.

data.frame() combines the future expenses into a single dataframe object, to be used for predictions.

predict() uses the MLE regression model, **n.ahead=2** ensures next two forecasts are generated, **newxreg=** specifies the new values of independent variables.

```
# Output
$pred
Time Series:
Start = 2011
End = 2012
Frequency = 1
[1] 82.80867 88.63343

$se
Time Series:
Start = 2011
End = 2012
Frequency = 1
[1] 0.8509076 1.0778190
```

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Quick Recap

Statistical Model	• $Y_t=b_0 + b_1x_{1t} + b_2x_{2t} + - - - - + b_px_{pt} + e_t$ • Here, t – the time element is added to each term
What is Autocorrelation	• Autocorrelation refers to the correlation of a time series with its own past values
Consequences	• Presence of Autocorrelation results in incorrect standard errors of model parameters
Test	• The Durbin Watson test is used to check autocorrelation • **`durbinwatsonTest()`** in package **car** performs the D-W Test in R
Maximum Likelihood Estimation Method	• Method of correcting autocorrelation problem

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THANK YOU!!

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