

Time Series Modeling

Seasonal ARIMA Model

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Case Study

Background

- Annual Sales for a specific company from year 1961 to 2017

Objective

- To assess stationarity of time series

Available Information

- Number of cases: 57
- Variables: Year, sales(in 10's GBP)

Data Snapshot

turnover_annual data

Variables

Observations on Discrete Time Scale

Year	sales
1961	224786
1962	230034
1963	236562
1964	250960
1965	261615
1966	268316
1967	283589
1968	280160
1969	301422
1970	308018
1971	320035

Columns	Description	Type	Measurement	Possible values
Year	Financial Year	Numeric	-	-
sales	sales(in 10's GBP)	Numeric	In British Pound	Positive values
		1974	364834	
		1975	392503	

Creating and Plotting Time Series in Python

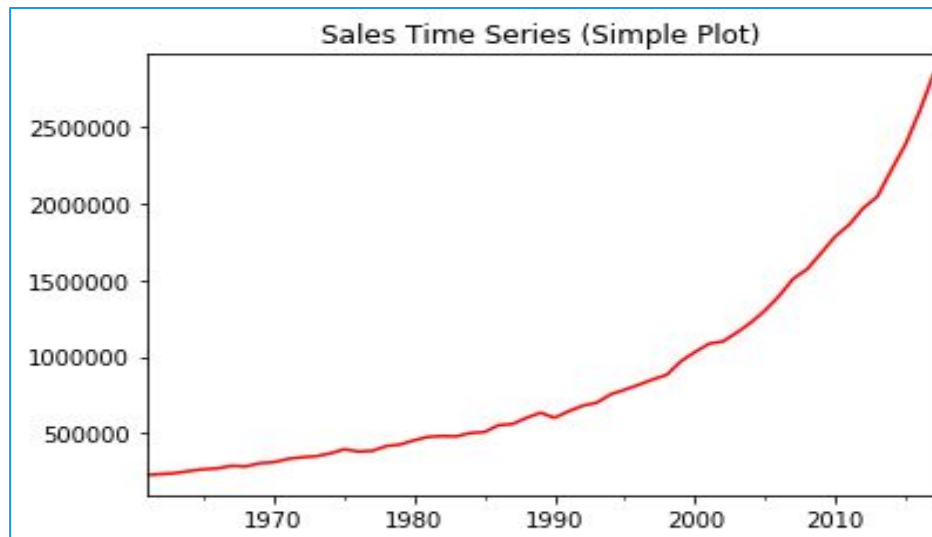
#Importing turnover_annual data

```
import pandas as pd
salesdata=pd.read_csv('turnover_annual.csv')
```

#Creating and Plotting a Time Series Object

```
rng = pd.date_range('01-01-1961','31-12-2017',freq='Y')
s = salesdata.sales.values
salesseries = pd.Series(s, rng)
```

```
salesseries.plot(color='red', title ="Sales Time Series  
(Simple Plot)")
```



- **date_range()**
creates pandas date object.
- **freq='Y'**
indicates yealy data
- **pd.Series()**
creates time series object
- Plot function gives line chart

Interpretation :

- The time-series clearly shows a positive trend.

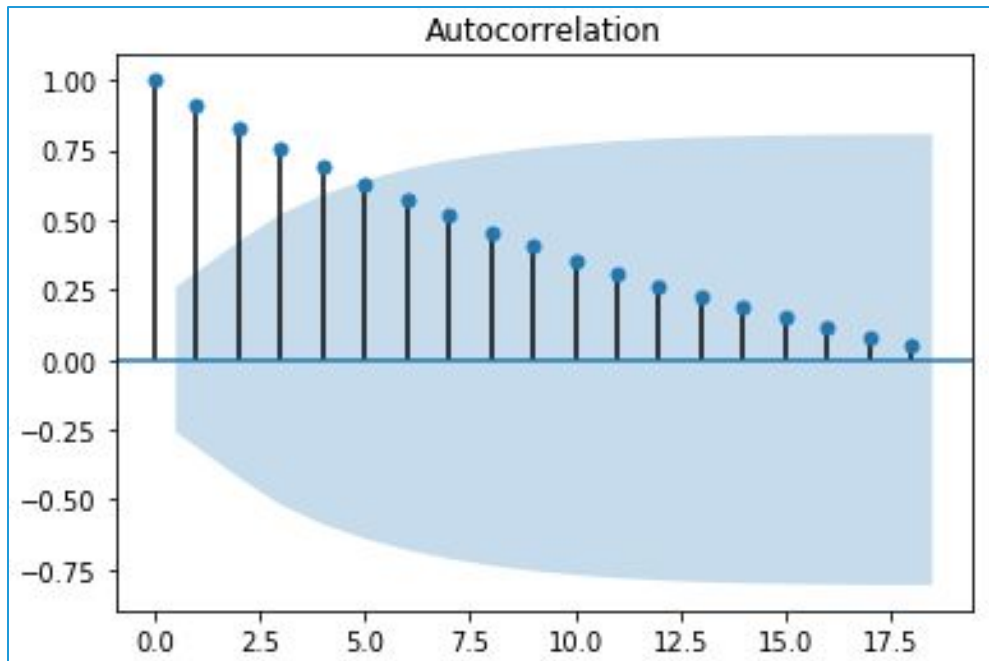
Checking Stationarity – Correlogram

ACF Plot

```
import matplotlib.pyplot as plt
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
plot_acf(salesseries)
```

□ **plot_acf()** returns an ACF (Auto Correlation Function) plot.

Output



Interpretation :

□ We can observe that there is a very slow decay which is a sign of Non-stationarity.

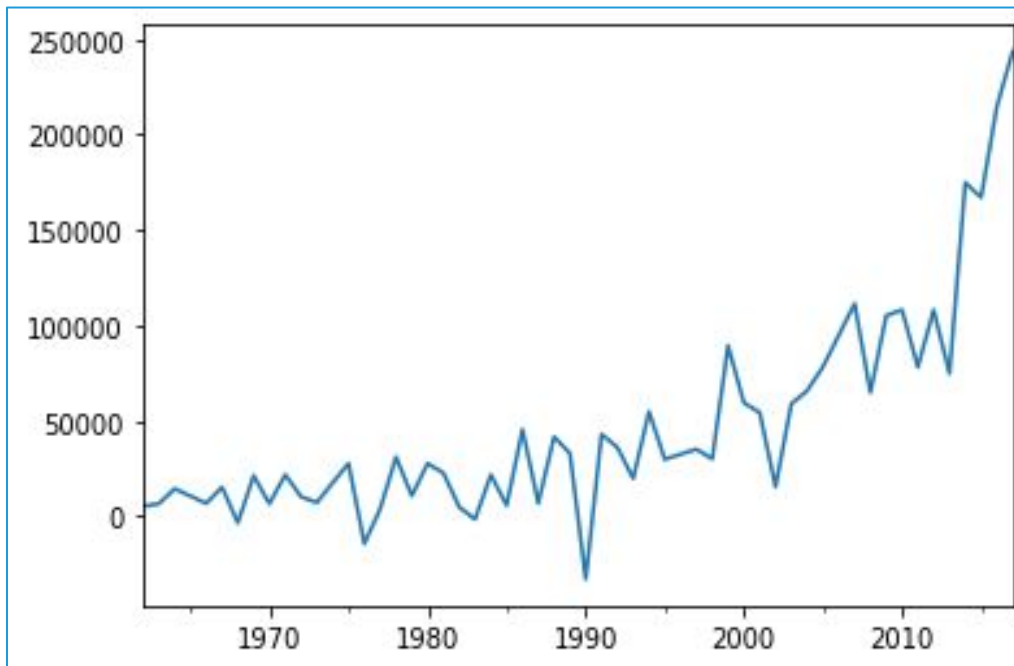
Plot of 1st Order Differenced Time Series

Creating and Plotting a Difference Series

```
from statsmodels.tsa.statespace.tools import diff
salesdiff = diff(salesseries)
salesdiff.plot()
```

- ❑ `diff()` gives 1st order differences
- ❑ `plot` function gives line chart for differenced series

Output



Interpretation :

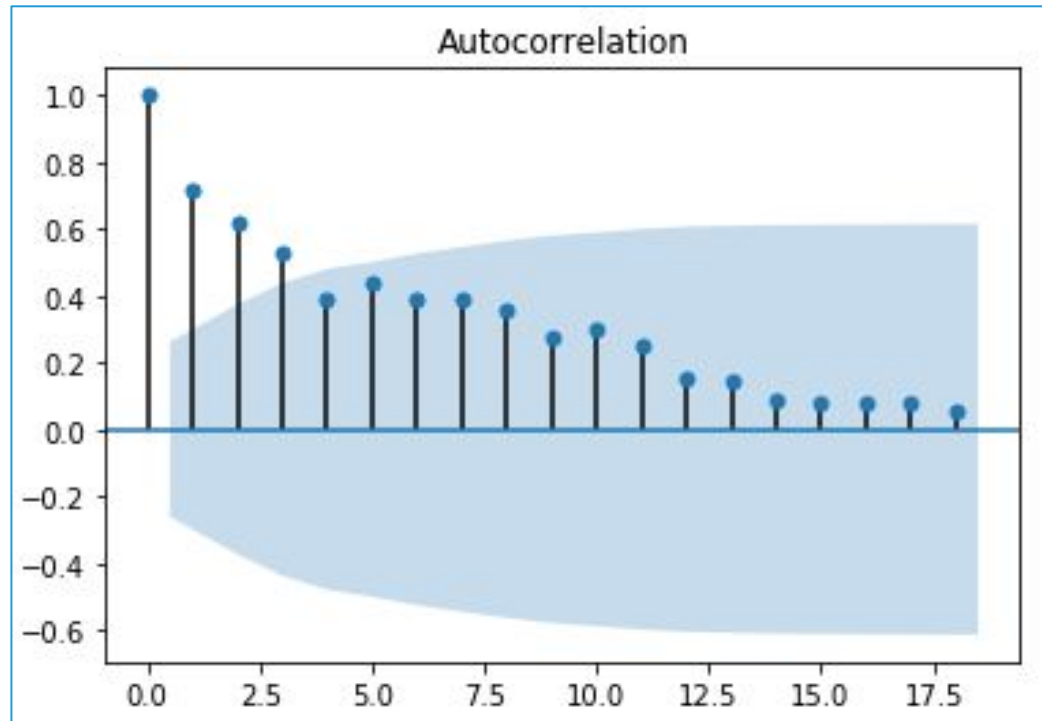
- ❑ Even after first order differencing, the series looks non-stationary.

Correlogram for 1st Order Differenced Time Series

ACF Plot

```
plot_acf(salesdiff)
```

Output



Interpretation :

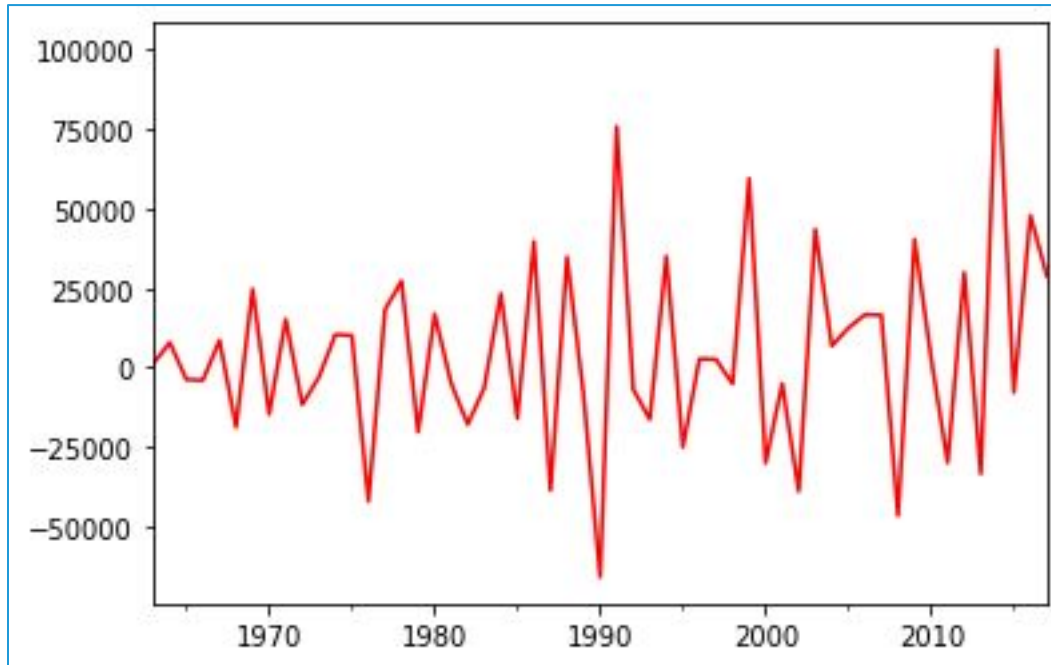
- ACF plot shows slow decay
- Stationarity is not achieved with first difference.

Plot of 2nd Order Differenced Time Series

#Creating and Plotting 2nd Difference Series

```
salesdiff2 = diff(salesdiff)  
salesdiff2.plot(color='red')
```

Output



Interpretation :

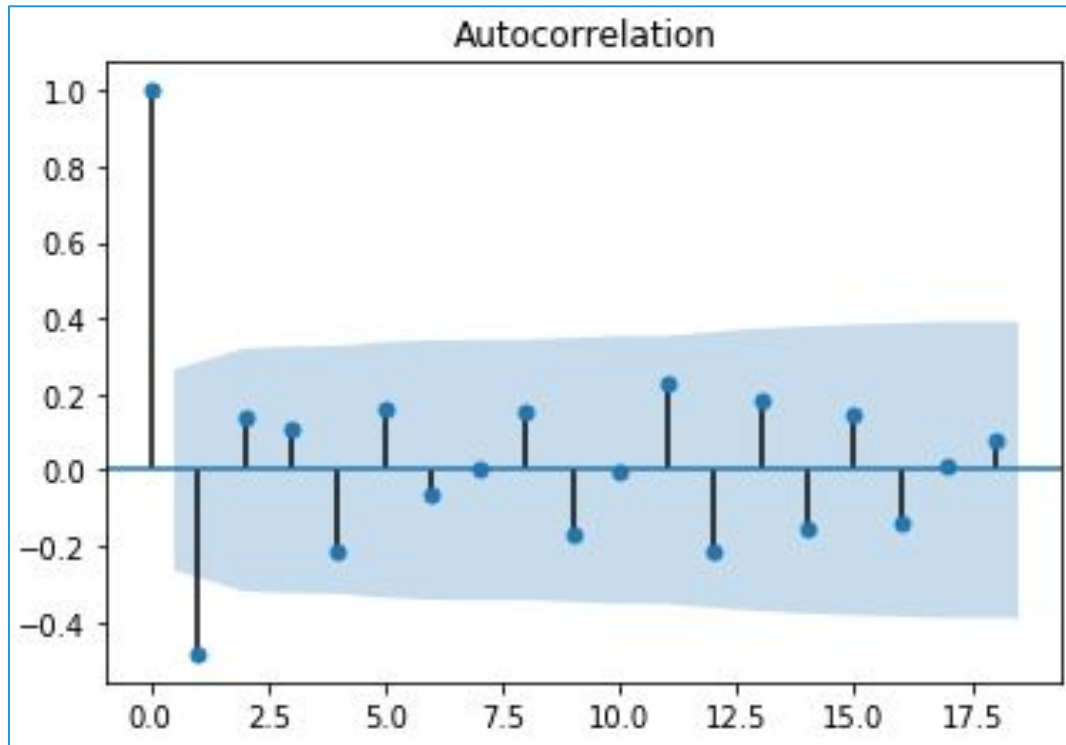
- After 2nd order differencing, the series looks **stationary**.

Correlogram for 2nd Order Differenced Time Series

ACF Plot

```
plot_acf(salesdiff2)
```

Output



Interpretation :

- Stationarity is achieved with 2nd order difference.

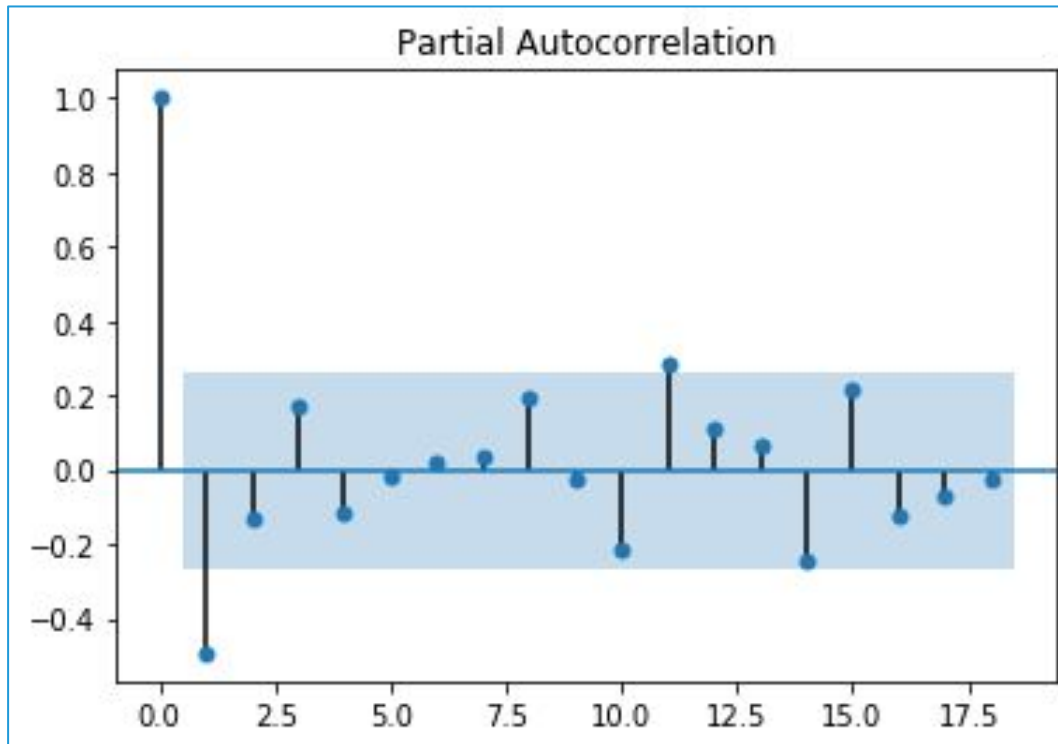
Partial Autocorrelations for 2nd Order Differenced Time Series

PACF Plot

```
plot_pacf(salesdiff2)
```

Output

- **plot_pacf()** returns an PACF (Partial Auto Correlation Function) plot.



Interpretation :

- Stationarity is achieved with 2nd order difference.

Analytical Method – Dickey Fuller (DF) Test

- A linear stochastic process has a unit root if 1 is the root of the process's characteristic equation. Such a process is non-stationary.
- Dickey and Fuller pioneered idea of testing for unit roots for stationarity checking.

Consider X_t ($t=1,2,3,\dots$) is a time series of the form

$$X_t = \rho X_{t-1} + U_t \dots\dots\dots (1)$$

If $\rho=1$ then X_t becomes a random walk

- We assume that $U_t \sim \text{IID} (0, \sigma^2)$, i.e U_t is a white noise
- Therefore, we are interested in testing for $\rho=1$



IID means independent and identically distributed

Dickey Fuller (DF) Unit Root Test

Objective	To test the null hypothesis that time series is not stationary
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$$\begin{aligned} \text{Null Hypothesis } H_0: \rho &= 1 \\ \text{Alternate Hypothesis } H_1: \rho &< 1 \\ (X_t - X_{t-1}) = \Delta X_t &= (\rho - 1) X_{t-1} + U_t \dots \dots \dots \text{from (1)} \\ H_0: \rho^* &= 0, H_1: \rho^* < 0, \rho^* = (\rho - 1) \end{aligned}$$

Test Statistic	$(\rho^* / SE(\rho^*))$ Test statistic follows DF distribution under null
Decision Criteria	Reject the null hypothesis $t_{cal} < DF \text{ table value}$

Dickey Fuller Test

```
# Install "arch"
```

```
pip install arch
```

```
# Import "ADF" from library "arch"
```

```
from arch.unitroot import ADF
```

```
adf = ADF(salesseries, lags=0, trend='nc')  
adf.summary()
```

- ❑ **ADF()** performs a Dickey Fuller unit root test on time series data.
- ❑ **lags=** allows to mention the number of lags to use in the ADF regression. We have used zero.
- ❑ **trend='nc'** specifies no trend and constant in regression

```
# Output
```

```
Augmented Dickey-Fuller Results  
=====
```

Test Statistic	19.275
P-value	1.000
Lags	0

```
-----
```

```
Trend: No Trend  
Critical Values: -2.61 (1%), -1.95 (5%), -1.61 (10%)  
Null Hypothesis: The process contains a unit root.  
Alternative Hypothesis: The process is weakly stationary.
```

Interpretation :

- ❑ Time series is non-stationary as value of test statistic is greater than 5% critical value.

Dickey Fuller Test

Checking stationarity for series with difference of order 2

```
adf = ADF(salesdiff2,lags=0,trend='nc')  
adf.summary()
```

Output

```
Augmented Dickey-Fuller Results  
=====
```

Test Statistic	-11.908
P-value	0.000
Lags	0

```
-----  
  
Trend: No Trend  
Critical Values: -2.61 (1%), -1.95 (5%), -1.61 (10%)  
Null Hypothesis: The process contains a unit root.  
Alternative Hypothesis: The process is weakly stationary.
```

Interpretation :

- Time series is stationary as value of test statistic is less than 5% critical value.

Quick Recap

Correlograms

- **plot_acf()** & **plot_pacf()** function in Python generate Correlograms

Differencing a Time Series

- Simple numeric function **diff()** can be used to difference a series

Dickey Fuller Test

- **ADF()** function from the package **arch** performs a Dickey Fuller test
- The output gives test statistic and critical values for the test statistic