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ORIGINAL ARTICLE

Statistical inferences of competing risks generalized half-logistic lifetime populations in presence of generalized type-I hybrid censoring scheme

Abdulaziz S. Alghamdi

Department of Mathematics, College of Science & Arts, King Abdulaziz University, P. O. Box 344, Rabigh 21911, Saudi Arabia

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Abstract The reliability analysis is usually concerned with failures in the time domain which show the difference between the quality control and reliability engineering. Also, It is very common in engineering or biological science units are falling under different causes of failure, which is known by competing risks model. In this paper, we have adopted the competing risks model when the lifetime of units distributed with generalized half-logistic (GHL) distribution. This model is formulated under consideration the failure done with two independent causes of failure. Also, to balance in statistical inference between the minimum number of failure and ideal test time, we are using, the type-I generalized hybrid censoring scheme (GHCS). The model parameters are estimated by maximum likelihood (ML) and Bayes approaches. The approximate confidence intervals, bootstrap confidence intervals and credible intervals are constructed. The developed results are discussed and assessed through Monte Carlo simulation study. A real data set is used and analyzed to report and illustrate the developed results.

1. Introduction

In the literature of statistics, censoring was described as a condition in which the value of observations is only partially known. Also, censoring is common phenomena in life testing experiments and the early common ones are called type-I and type-II censoring schemes. In type-I censoring scheme, we determine the ideal test time, but number of failures is random. This scheme has a luck of memory that, number of failures may be small or zero. In type-II censoring scheme, we determine number of failures which is needing in statistical inference, but the test time is random. This scheme has a luck of memory that, a large test time may by infinity. The experimenters, under consideration of cost and time lamination may be need to run the experiment under joint case of type-I and type-II censoring schemes which known in literature by the hybrid censoring scheme (HCS). Therefore, in HCS (τ, m) the ideal test time and the number of failures needing in statistical inference are proposed. The experiment terminated at min (τ, T_m) in type-I HCS, where T_m is the failure of m-th unit. But, the experiment terminated at $\max(\tau, T_m)$ in type-II HCS. Statistically, the last two type of HCSs have the same luck of momery of type-I and type-II censoring schemes,

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smaller number of failures or larger test time. For the extensive review of HSCs, see Gupta and Kundu [1], Kundu and Pradhan [2] and Ghazal [3]. As given in Chandrasekar et al. [4], this problems overcome by proposing the generalized hybrid censoring scheme (GHCS). Therefore, GHCS defined under type-I and type-II censoring schemes as follows.

Type-I GHCS: Suppose that, n units are randomly selected from a life population. Prior the experiment is running, two integers s, m and the ideal test time τ are proposed to satisfies $1 \leqslant s < m \leqslant n$. If $T_S < \tau$ then, the experiment terminated at min (T_m, τ). But, if $T_S > \tau$ the experiment terminated at T_S. Therefore, the observed data under type-I GHCS is defined by $\underline{t} = \{t_{i:n}\}, i = 1, 2, \ldots, r$, where

$$r = \begin{cases} s, & \text{if } \ T_S > \tau \\ s < r < m, & \text{if } \ T_S < \tau < T_m \\ m & \text{if } \ T_S < T_m < \tau \end{cases}. \tag{1}$$

Also, the corresponding test terminated time η is defined by

$$\eta = \begin{cases} T_S, & \text{if } \ T_S > \tau \\ \tau, & \text{if } \ T_S < \tau < T_m \\ T_m & \text{if } \ T_S < T_m < \tau \end{cases}. \tag{2}$$

Under consideration the joint type-I GHC sample the likelihood function is given by

$$f_{1,2,\ldots,r}(\underline{t}) = \frac{n!}{(n-r)!}(S(\eta))^{n-r}\prod_{i=1}^{r} f(t_i), \tag{3}$$

where, $f(.)$ and $S(.)$ are the density and survival function, for more detail, see Algarni et al. [5] and Tahani et al. [6].

Type-II GHCS: For a randomly selected n units, propose two prior timesτ_1 and τ_2 as well as a suitable number of failure m to satisfy $\tau_1 < \tau_2$ and $1 < m < n$. If $T_m < \tau_1$ then, the experiment terminated at τ_1. But, if $\tau_1 < T_m < \tau_2$ the experiment terminated at T_m. Finally, if $\tau_1 < \tau_2 \leqslant T_m$ the experiment terminated at τ_2. Hence, the maximum time of the experiment is τ_2. Suppose, the observed data under type-II GHCS define by $\underline{t} = \{t_{i:n}\}, i = 1, 2, \ldots, r$, where

$$r = \begin{cases} r < m, & \text{if } \ \tau_1 < \tau_2 < T_m \\ m, & \text{if } \ \tau_1 < T_m < \tau_2 \\ r > m, & \text{if } \ T_m < \tau_1 \end{cases}. \tag{4}$$

Also, the corresponding test terminated time $\eta$is defined by

$$\eta = \begin{cases} \tau_2, & \text{if } \ \tau_1 < \tau_2 < T_m \\ T_m, & \text{if } \ \tau_1 < T_m < \tau_2 \\ \tau_1, & \text{if } \ T_m < \tau_1 \end{cases}. \tag{5}$$

Also, for given type-II GHC sample the likelihood function formulated as given in (3). The

The common phenomenon in life populations that, units or material under testing fails with respected to a several risks which is defined in leterture by competing risks model. These models our objective is assessment the effect of any risk factor in the presence of other risks. The statistical models under competig risks model discussed early by Cox [7], David and Moeschberger [8], Crowder [9], Balakrishnan [10], Ganguly and Kundu [11], Bakoban and Abd-Elmougod [12] and Modhesh and Abd-Elmougod [13]. Also, for the recently works about this models, see Algarni et al. [14], Almarashi et al. [15], Tahani el al.[16], Alghamdi [17] Alghamdia et al.[18] and Alghamdia et al. [19]. For modeling the competing risks model with type-I GHCS, we consider that, the failure done under two independent causes of failure with density and survival functions $f_1(.)$, $S_1(.)$ and $f_2(.), S_2(.)$, respectively. Then, the joint likelihood function (3) is reduced to

$$f_{1,2,\ldots,r}(\underline{t}) = \frac{n!}{(n-r)!}(S_1(\eta)S_2(\eta))^{n-r} \times \prod_{i=1}^{r}[f_1(t_i)S_2(t_i)]^{I(\delta_i=1)}[f_2(t_i)S_1(t_i)]^{I(\delta_i=2)}, \tag{6}$$

where

$$I(\delta_i = j) = \begin{cases} 1, & \delta_i = j \\ 0, & \delta_i \neq j \end{cases} \tag{7}$$

and

$$t_i = t_{i:n}, \ 0 < t_1 < t_2 < \ldots < t_m < \infty.$$

Early, the half logistic (HL) distribution introduced by Balakrishnan [20] to model the absolute standard logistic random variable. Also, Balakrishnan and Hossain [21] presented the generalized version of half logistic distribution to called generalized half-logistic (GHL) distribution. If the random variable T has two parameters GHL distribution then, the cmulative distribution function CDF given by

$$F(t) = 1 - \left(\frac{2e^{-\frac{t}{\beta}}}{1+e^{-\frac{t}{\beta}}}\right)^{\alpha}, t > 0, \ \alpha, \beta > 0. \tag{8}$$

Also, the corresponding probability density function (PDF), survival function $S(t)$ and hazard rate function $H(t)$ are given by

$$f(t) = \frac{\alpha}{\beta(1+e^{-\frac{t}{\beta}})}\left(\frac{2e^{-\frac{t}{\beta}}}{1+e^{-\frac{t}{\beta}}}\right)^{\alpha}, \tag{9}$$

$$S(t) = \left(\frac{2e^{-\frac{t}{\beta}}}{1+e^{-\frac{t}{\beta}}}\right)^{\alpha}, \tag{10}$$

and

$$H(t) = \frac{\alpha}{\beta(1+e^{-\frac{t}{\beta}})}. \tag{11}$$

Different works are presented of GHL distribution, see Ramakrishnan [22] for the parameter of stress–strength reliability under type-I progressive censoring scheme, Arora et al. [23] presented the estimator of the shape parameter and Kim et al. [24], presented the estimators under Bayes method. The reliability functions discussed by Chaturvedi et al. [25] and the parameters are estimated under constant-stress by Almarashi [26].

In this paper, we are discuss some important aspects and impacts of various classical and Bayesian approaches towards the competing risk model in survival analysis. The problem of modeling competing risks model under type-I GHCS when the failure time of units distributed by GHL distribution is our aim in this paper. Building the model and analysis sets of real data under proposed model are developed. The model parameters are estimated under ML and Bayes approach and the corresponding interval estimators are formulated. All results are discussed and compared through Monte Carlo study. We analyzed the mortality data with help of Mathematica software for illustration of competing risks application. Finally, we have given discussion and concluding remarks on the advantages and disadvantages of these methods.

The paper is organized as follows: The model and its assumptions in Section 2. The estimators of model parameters and the corresponding asymptotic confidence intervals under ML methods are discussed in Section 3. Approximate confidence interval with bootstrap technique proposed in Section 4. The Bayes estimation with credible intervals are reported in Section 5. The problem of assessment of performance estimators done through Monte Carlo study in Section 6. A real data set is analyzed in Section 7.

2. Model assumptions

Let, a random sample size n of identical independent units is selected from population with PDF and CDF given by $f(t)$ and $F(t)$. The prior integers s and m are proposed to satisfy that $1 \leqslant s < m \leqslant n$ with the corresponding test time τ. When the experiment running the unit failure time T_i and unit cause of failure δ_i are reported, $i = 1, 2, \ldots, r$ and $\delta_i = \{1, 2\}$. If the failure time T_s is observed before the time τ then, the experiment terminated at min (τ, T_m). But, if the failure time T_s is observed after the time τ then, the experiment terminated at T_s. Suppose that, the observed number of failure r and the corresponding terminated time η are reported. Hence, the type-II GHCS sample under competing risks model is defined by $\underline{t} = \{(t_i, \delta_i)\}, i = 1, 2, \ldots, r,$ where $t_i = T_{i:n}$. The proposed model satisfies the following assumptions.

1. The i-failure time of unit $T_i = \min\{T_{i1}, T_{i2}\}$, where T_{ij} is i-failure time under cause $j, j = 1, 2$.
2. The i-failure time under cause j has GHL distribution with shape parameter $\alpha_j, j = 1, 2$ and $i = 1, 2, \cdots, r$. Therefore, the PDF and CDF of GHL distribution are given by

$$f_j(t) = \frac{\alpha_j}{\beta(1 + e^{-\frac{t}{\beta}})} \left(\frac{2e^{-\frac{t}{\beta}}}{1 + e^{-\frac{t}{\beta}}} \right)^{\alpha_j}, \ t > 0, \ \beta, \alpha_j > 0, \tag{12}$$

and

$$F_j(t) = 1 - \left(\frac{2e^{-\frac{t}{\beta}}}{1 + e^{-\frac{t}{\beta}}} \right)^{\alpha_j}, \tag{13}$$

where, α_j is a shape parameters and $\beta$is scale parameter. Also, $S_j(t)$ and $H_j(t)$ of GHL distributions, respectively given by

$$S_j(t) = \left(\frac{2e^{-\frac{t}{\beta}}}{1 + e^{-\frac{t}{\beta}}} \right)^{\alpha_j}, \tag{14}$$

$$H_j(t) = \frac{\alpha_j}{\beta(1 + e^{-\frac{t}{\beta}})}. \tag{15}$$

3. The latent failure time T_i distributed as GHL distribution with CDF given by

$$F(t) = 1 - \left(\frac{2e^{-\frac{t}{\beta}}}{1 + e^{-\frac{t}{\beta}}} \right)^{\alpha_1 + \alpha_2}, \tag{16}$$

4. Suppose that, r_1 and r_2 are the numbers of failure under cause $j = 1, 2$. Therefor, r_j have the binomial distribution with parameters r_j and $\frac{\alpha_j}{\alpha_1 + \alpha_2}$.

3. Estimation under Maximum likelihood Method

In this section, we consider two independent causes of failure and type-I GHCS is applied on the competing risks model. The observed data are used to obtain the point and interval ML estimators of the model parameters as follows.

3.1. MLEs

For given competing risks type-II GHCS sample $\underline{t} = \{(t_i, \delta_i)\}, \ i = 1, 2, \ldots, r$ the joint likelihood function (6) is reduced to

$$L(\alpha_1, \alpha_2, \beta | \underline{t}) \propto \alpha_1^{r_1} \alpha_2^{r_2} \beta^{-r} \left(\frac{2e^{-\frac{\eta}{\beta}}}{1 + e^{-\frac{\eta}{\beta}}} \right)^{(n-r)(\alpha_1 + \alpha_2)} \prod_{i=1}^{r} \frac{1}{1 + e^{-\frac{t_i}{\beta}}} \left(\frac{2e^{-\frac{t_i}{\beta}}}{1 + e^{-\frac{t_i}{\beta}}} \right)^{\alpha_1 + \alpha_2}. \tag{17}$$

Hence, the natural logarithms of the likelihood function given by

$$\begin{aligned} \ell(\alpha_1, \alpha_2, \beta | \underline{t}) = {} & r_1 \log \alpha_1 + r_2 \log \alpha_2 * -r \log \beta \\ & + (n - r)(\alpha_1 + \alpha_2) \log \left[\frac{2e^{-\frac{\eta}{\beta}}}{1 + e^{-\frac{\eta}{\beta}}} \right] \\ & - \sum_{i=1}^{r} \log \left[1 + e^{-\frac{t_i}{\beta}} \right] + (\alpha_1 + \alpha_2) \sum_{i=1}^{r} \log \left[\frac{2e^{-\frac{t_i}{\beta}}}{1 + e^{-\frac{t_i}{\beta}}} \right]. \end{aligned} \tag{18}$$

We search of the zero values of the first partial derivatives of log-likelihood function (18) with respected the model parameters which give the mines values of the second partial derivatives of (18) called the ML estimate of the model parameters. The first partial derivatives of (18) with respected to α_1 and α_2 are reduced to

$$\widehat{\alpha}_j = \frac{-r_j}{(n - r) \log \left[\frac{2e^{-\frac{\eta}{\beta}}}{1 + e^{-\frac{\eta}{\beta}}} \right] + \sum_{i=1}^{r} \log \left[\frac{2e^{-\frac{t_i}{\beta}}}{1 + e^{-\frac{t_i}{\beta}}} \right]}. \tag{19}$$

Also, The partial derivatives of (18) with respected to βreduced to

$$\begin{aligned} & \frac{-r}{\beta} + \frac{(n - r)(\alpha_1 + \alpha_2)\eta}{\beta^2 \left(1 + e^{-\frac{\eta}{\beta}} \right)} - \frac{1}{\beta^2} \sum_{i=1}^{r} \frac{t_i e^{-\frac{t_i}{\beta}}}{\left(1 + e^{-\frac{t_i}{\beta}} \right)} \\ & \quad + \frac{(\alpha_1 + \alpha_2)}{\beta^2} \sum_{i=1}^{r} \frac{t_i}{\left(1 + e^{-\frac{t_i}{\beta}} \right)} \\ & \quad = 0. \end{aligned} \tag{20}$$

which is reduced to

$$\begin{aligned} & -r\beta + \frac{(n - r)(\alpha_1 + \alpha_2)\eta}{\left(1 + e^{-\frac{\eta}{\beta}} \right)} - \sum_{i=1}^{r} \frac{t_i e^{-\frac{t_i}{\beta}}}{\left(1 + e^{-\frac{t_i}{\beta}} \right)} \\ & \quad + (\alpha_1 + \alpha_2) \sum_{i=1}^{r} \frac{t_i}{\left(1 + e^{-\frac{t_i}{\beta}} \right)} = 0 \end{aligned} \tag{21}$$

The equations obtained from likelihood function are reduced to one non-linear Eq. (21) solve numerical with Newton Raphson or fixed point techniques. Hence, the estmate values $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ are obtained from (19). The Eq. (18) after replacing α_j by (19), we have the profile log-likelihood function which has presented the initial value used in iteration Newton Raphson or fixed point methods.

3.2. Interval estimation

In statistical literature, the minus expectation of second derivatives of the log-likelihood function (18) known by Fisher information matrix of the model parameters. Let, $\Omega(\alpha_1,\alpha_2,\beta)$ denote to the Fisher information matrix which is defined by

$$\Omega(\alpha_1,\alpha_2,\beta)=-E\left(\frac{\partial^2\ell(\alpha_1,\alpha_2,\beta|\underline{t})}{\partial\theta_k\partial\theta_l}\right),k,l=1,2,3, \tag{22}$$

where $\theta=\{\alpha_1,\alpha_2,\beta\}$, The second derivatives of log-likelihood function formulate by

$$\frac{\partial^2\ell(\alpha_1,\alpha_2|\underline{t})}{\partial\alpha_1^2}=\frac{-r_1}{\alpha_1^2}, \tag{23}$$

$$\frac{\partial^2\ell(\alpha_1,\alpha_2|\underline{t})}{\partial\alpha_2^2}=\frac{-r_2}{\alpha_2^2}, \tag{24}$$

$$\begin{aligned}\frac{\partial^2\ell(\alpha_1,\alpha_2|\underline{t})}{\partial\beta^2}=&\frac{r}{\beta^2}-\frac{(n-r)(\alpha_1+\alpha_2)\eta}{\beta^4\left(1+e^{-\frac{\eta}{\beta}}\right)^2}\left(2\beta+(2\beta+\eta)e^{-\frac{\eta}{\beta}}\right)-\frac{1}{\beta^4}\sum_{i=1}^{r}\frac{t_i^2e^{-\frac{t_i}{\beta}}}{\left(1+e^{-\frac{t_i}{\beta}}\right)^2}\\&-\frac{(\alpha_1+\alpha_2)}{\beta^4}\sum_{i=1}^{r}\frac{t_i\left(2\beta+(2\beta+t_i)e^{-\frac{t_i}{\beta}}\right)}{\left(1+e^{-\frac{t_i}{\beta}}\right)^2},\end{aligned} \tag{25}$$

$$\frac{\partial^2\ell(\alpha_1,\alpha_2|\underline{t})}{\partial\alpha_1\partial\alpha_2}=\frac{\partial^2\ell(\alpha_1,\alpha_2|\underline{t})}{\partial\alpha_2\partial\alpha_1}=0, \tag{26}$$

and

$$\begin{aligned}\frac{\partial^2\ell(\alpha_1,\alpha_2,\beta|\underline{t})}{\partial\alpha_1\partial\beta}=&\frac{\partial^2\ell(\alpha_1,\alpha_2,\beta|\underline{t})}{\partial\beta\partial\alpha_1}\frac{\partial^2\ell(\alpha_1,\alpha_2,\beta|\underline{t})}{\partial\alpha_2\partial\beta}=\frac{\partial^2\ell(\alpha_1,\alpha_2,\beta|\underline{t})}{\partial\beta\partial\alpha_2}=\\&\frac{(n-r)\eta}{\beta^2\left(1+e^{-\frac{\eta}{\beta}}\right)}+\frac{1}{\beta^2}\sum_{i=1}^{r}\frac{t_i}{\left(1+e^{-\frac{t_i}{\beta}}\right)}.\end{aligned} \tag{27}$$

In several cases the expectation of the second partially derivative more serous hence, Fisher information matrix is replaced by approximate information matrix computed by

$$\Omega(\alpha_1,\alpha_2,\beta)=-\left(\frac{\partial^2\ell(\alpha_1,\alpha_2|\underline{t})}{\partial\theta_k\partial\theta_l}\right),k,l=1,2,3, \tag{28}$$

Under the property of normality distribution of ML estimators with mean $\theta=\{\alpha_1,\alpha_2,\beta\}$ and variance obtained from the diagonal of variance–covariance matrix $\Omega_0^{-1}(\widehat{\alpha}_1,\widehat{\alpha}_2,\widehat{\beta})$, where.$\Omega_0^{-1}$ is denoted to the inverse of approximate information matrix at ML estimate of the model parameters. The approximate $(1-\nu)100\%$ confidence intervals (CIs) of the model parameters $\theta=\{\alpha_1,\alpha_2,\beta\}$ is given by

$$\widehat{\alpha}_1\mp z_{\frac{\nu}{2}}\zeta_1,\widehat{\alpha}_2\mp z_{\frac{\nu}{2}}\zeta_2\text{and}\widehat{\beta}\mp z_{\frac{\nu}{2}}\zeta_3, \tag{29}$$

where values ζ_1,ζ_2 and ζ_3 present the diagonal of approximate variance–covariance matrix $\Omega_0^{-1}(\widehat{\alpha}_1,\widehat{\alpha}_2,\widehat{\beta})$ with percentile standard normal tabulated value $z_{\frac{\nu}{2}}$ at the confidence level $\frac{\nu}{2}$.

4. Confidence Intervals under Bootstrap Technique

Commonly in statistical literature, not only interval estimation but estimate bias and variance of an estimator with bootstrap technique. Therefore, bootstrap technique is widely used specially in calibrate hypothesis tests. In this section, we applied parametric percentile bootstrap technique to formulate confidence intervals of the model parameters. For more detail about bootstrap technique, see Davison and Hinkley [27], Efron and Tibshirani[28], Efron [29] and Hall [30]. The following algorithms is used to formulate two type of bootstrap confidence interval percentile bootstrap$-p$ and percentile bootstrap$-t$ as follows.

Step 1: Under consideration the original type-I GHC sample under competing risks model $\underline{t}=\{(t_i,\delta_i)\}$, $i=1,2,\ldots,r$, caculate the estimate vector $\widehat{\theta}=\{\widehat{\alpha}_1,\widehat{\alpha}_2,\widehat{\beta}\}$.

Step 2: For given censoring parameters n,s,m and τ, generate bootstrap random variable $\underline{t}^*=\{t_i^*\}$, $i=1,2,\ldots,r$, from GHL distribution with shape parameter $\widehat{\alpha}_1+\widehat{\alpha}_2$ and scale parameter $\widehat{\beta}$.

Step 3: The two integers r_1 and r_2 are generated from binomail distribution with parameters r_j and $\frac{\widehat{\alpha}_j}{\widehat{\alpha_1+\alpha_2}}$.

Step 4: The bootstrap sample estimate $\widehat{\theta}^*=\{\widehat{\alpha}_1^*,\widehat{\alpha}_2^*,\widehat{\beta}^*\}$.

Step 5: Steps from 2 to 4 are repeated $\mathbf{M}$ times.

Step 6: The bootstrap sample estimate $\widehat{\theta}^*=\{\widehat{\alpha}_1^*,\widehat{\alpha}_2^*,\widehat{\beta}^*\}$ put in aseding order as

$$(\widehat{\theta}_i^{*(1)},\widehat{\theta}_i^{*(2)},\ldots,\widehat{\theta}_i^{*(\mathbf{M})}),i=1,2,3. \tag{30}$$

4.1. Confidence intervals under percentile bootstrap technique (BCI-p)

Suppose that, the empirical CDF of each values $\widehat{\theta}_i^*$ is defined by $G(x)=P(\widehat{\theta}_i^*\leqslant x), i = 1, 2, 3$. Therefore, the bootstrap point estimate of model parameters is given by

$$\widehat{\theta}_i(\text{Boot})=\frac{1}{\mathbf{M}}\sum_{i=1}^{\mathbf{M}}\widehat{\theta}_i^{*(i)}. \tag{31}$$

Also, the corresponding $(1-\nu)100\%$ BCI-ps is given by

$$(\widehat{\theta}^*_{i(\mathbf{M}\frac{\nu}{2})},\widehat{\theta}^*_{i(\mathbf{M}(\mathbf{1}-\frac{\nu}{2}))}), \tag{32}$$

where of $\widehat{\boldsymbol{\theta}}_i^*=G^{-1}(x)$.

4.2. Confidence intervals under percentile bootstrap technique (BCI-t)

From the aseding order values (30), we formulate the aseding order statistics define by $\Psi_i^{*(1)}<\Psi_i^{*(2)}<\ldots<\Psi_i^{*(\mathbf{M})}$, where

$$\Psi_i^{*(k)}=\frac{\widehat{\theta}_i^{*(k)}-\widehat{\theta}_i}{\sqrt{\text{var}\left(\widehat{\theta}_i^{*(k)}\right)}},\ k=1,2,\ldots,\mathbf{M},i=1,2,3. \tag{33}$$

Hence, the $(1-\nu)100\%$ BCI-t are given by

$$\left(\tilde{\Psi}^{*}_{i\mathrm{Boot-t}(\frac{\nu}{2})},\ \tilde{\Psi}^{*}_{i\mathrm{Boot-t}(1-\frac{\nu}{2})}\right), \tag{34}$$

where, $\tilde{\Psi}^{*}_{i\mathrm{Boot-t}}$ is value given by

$$\tilde{\Psi}^{*}_{i\mathrm{Boot-t}} = \hat{\theta}^{*}_{i} + \sqrt{\mathrm{Var}(\hat{\theta}_i)}G^{-1}(x), \tag{35}$$

and $G(x) = P(\tilde{\Psi}^{*}_{i} \leqslant x)$ be the CDF of θ^{*}_{i}.

5. Estimation under Bayesian Approach

In section, we estimate the model parameter under consideration the available prior information formulate in the form of independent gamma prior. Therefore, the independent gamma priors can be defined by

$$P^{*}_{i}(\theta_i) \propto \theta_i^{a_i-1}e^{-b_i\theta_i}, i=1,2,3, \text{and} a_i, b_i > 0, \tag{36}$$

and the joint prior density formulated by

$$P^{*}(\theta) \propto \prod_{i=1}^{3} P^{*}_{i}(\theta_i) \propto \prod_{i=1}^{3} \theta_i^{a_i-1}e^{-b_i\theta_i}, \tag{37}$$

where, $\theta = (\alpha_1, \alpha_2, \beta)$. The joint posterior density function of the model parameters $\theta$is formulated by

$$P(\theta|\underline{t}) = \frac{P^{*}(\theta)L(\alpha_1, \alpha_2, \beta|\underline{t})}{\int_\theta P^{*}(\theta)L(\alpha_1, \alpha_2, \beta|\underline{t})d\theta}, \tag{38}$$

and the corresponding estimator of any function $z(\theta)$ with respected squared error loss function also is formulated by

$$\hat{z}_B(\theta) = \frac{\int_\theta z(\theta)P^{*}(\theta)L(\alpha_1, \alpha_2, \beta|\underline{t})d\theta}{\int_\theta P^{*}(\theta)L(\alpha_1, \alpha_2, \beta|\underline{t})d\theta}. \tag{39}$$

Each of (38) and (39) have shown that, the posterior closed form or the closed from of ratio of two integral specially in a high dimensional case are more serous to obtained. Different methods can be used to approximate these integrals such as numerical integration, Lendly approximation and MCMC methods. In this section, we adopted MCMC methods as follows.

5.1. Full conditional distribution

From (17) and (39) the posterior distribution (36) is reduced to

$$\begin{aligned} &P(\alpha_1, \alpha_2, \beta|\underline{t}) \propto \alpha_1^{r_1+a_1-1}\alpha_2^{r_2+a_2-1}\beta^{r+a_3-1} \\ &\exp\left\{(n-r)(\alpha_1+\alpha_2)\log\left[\frac{2e^{-\frac{\eta}{\beta}}}{1+e^{-\frac{\eta}{\beta}}}\right]\right. \\ &\left.-\sum_{i=1}^{r}\log\left[1+e^{-\frac{t_i}{\beta}}\right] + (\alpha_1+\alpha_2)\sum_{i=1}^{r}\log\left[\frac{2e^{-\frac{t_i}{\beta}}}{1+e^{-\frac{t_i}{\beta}}}\right] - b_1\alpha_1 - b_2\alpha_2 - b_2\beta\right\}. \end{aligned} \tag{40}$$

Hence from the joint posterior distribution the conditional PDFs formulated by

$$\begin{aligned} P(\alpha_1|\alpha_2, \beta, \underline{t}) \quad &\propto \quad \alpha_1^{r_1+a_1-1}\exp\left\{(n-r)\alpha_1\log\left[\frac{2e^{-\frac{\eta}{\beta}}}{1+e^{-\frac{\eta}{\beta}}}\right]\right. \\ &\left.+\alpha_1\sum_{i=1}^{r}\log\left[\frac{2e^{-\frac{t_i}{\beta}}}{1+e^{-\frac{t_i}{\beta}}}\right] - b_1\alpha_1\right\}. \end{aligned} \tag{41}$$

$$\begin{aligned} P(\alpha_2|\alpha_1, \beta, \underline{t}) \quad &\propto \quad \alpha_2^{r_1+a_2-1}\exp\left\{(n-r)\alpha_2\log\left[\frac{2e^{-\frac{\eta}{\beta}}}{1+e^{-\frac{\eta}{\beta}}}\right]\right. \\ &\left.+\alpha_2\sum_{i=1}^{r}\log\left[\frac{2e^{-\frac{t_i}{\beta}}}{1+e^{-\frac{t_i}{\beta}}}\right] - b_2\alpha_2\right\}. \end{aligned} \tag{42}$$

and

$$\begin{aligned} &P(\beta|\alpha_1, \alpha_2, \underline{t}) \propto \beta^{r+a_3-1}\exp\left\{(n-r)(\alpha_1+\alpha_2)\log\left[\frac{2e^{-\frac{\eta}{\beta}}}{1+e^{-\frac{\eta}{\beta}}}\right]\right. \\ &\left.-\sum_{i=1}^{r}\log\left[1+e^{-\frac{t_i}{\beta}}\right] + (\alpha_1+\alpha_2)\sum_{i=1}^{r}\log\left[\frac{2e^{-\frac{t_i}{\beta}}}{1+e^{-\frac{t_i}{\beta}}}\right] - b_2\beta\right\}. \end{aligned} \tag{43}$$

There for the joint posterior distribution is reduced to two conditional gamma functions (40) and (41) and more general function its properties sammilar to normal distribution (42).

5.2. MCMC method

The full conditional distributions given by (40)–(42) shown that Gibbs algorithms and more general case MH under Gibbs are suitable for MCMC scheme, see Metropolis et al. [31]. Hence, the problem of generation from posterior distribution can be described by the following algorithms.

MH under Gibbs Algorithm:

2. Put $\kappa = 1$.
3. The iterative values $\alpha_1^{(\kappa)}$ and $\alpha_2^{(\kappa)}$ generate from gamma densities (41) and (42).
4. The value $\beta^{(\kappa)}$ generate from conditional density (43) by MH algorithms with normal proposal distributions.
5. Report the generated vector $\theta^{(\kappa)} = (\alpha_1^{(\kappa)}, \alpha_2^{(\kappa)}, \beta^{(\kappa)})$.
6. Putt $\kappa = 1 + 1$ and repeat Steps from 3 to 5 N times.
7. The MCMC estimators of the model parameters are given by

$$\hat{\theta}_{i\mathrm{B}} = \frac{1}{N-\mathbf{M}^{*}}\sum_{k=\mathbf{M}^{*}+1}^{N}\theta_i^{(k)}, i=1,2,3, \tag{44}$$

where, $\mathbf{M}^{*}$ denote to number of iteration need to reach the stationary distribution. Also, the corresponding Bayes variance is given by

$$\mathrm{Var}(\hat{\theta}_{i\mathrm{B}}) = \frac{1}{N-\mathbf{M}^{*}}\sum_{i=\mathbf{M}^{*}+1}^{N}\left(\hat{\theta}_{i\mathrm{B}} - \theta_i^{(i)}\right)^2. \tag{45}$$

8. After put $\theta_i^{(k)}, i=1,2,3$, in asseding order $\theta_{i(k)}, i=1,2,3$, the Bayes $(1-\nu)100\%$ credible intervals can be formulated by

$$\left(\theta_{i(\frac{\nu}{2}(N-\mathbf{M}^{*}))}, \theta_{i((1-\frac{\nu}{2})(N-\mathbf{M}^{*}))}\right), i=1,2,3. \tag{46}$$

6. Simulation Studying

The developed results in this paper under different methods of estimation are assessed and compared through building Monte Carlo simulation study. The simulation results are reported for different choices of the censoring parameters, n, s, m and τ. Also, different choices of the parameter values and different prior informations. Through the numerical computations, we are assessed the effect of changing different combinations and results of numerical study are reported in Tables 1–4. Therefore, we are adopted two sets of the parameters values $\theta = \{\alpha_1, \alpha_2, \beta\} = \{0.2, 0.5, 2.5\}$ and $\{0.5,\ 1.0,\ 1.5\}$. The point estimators, are assessed by using the mean squared errors (MSEs). The interval estimtors assessed by using average interval lengths (AILs) and probability coverage (PC). The prior information are selected to satisfies $E(\theta_i) = \frac{a_i}{b_i} \cong \theta_i, i = 1, 2, 3$. Therefor, the non-informative prior information mean that $a_i = b_i = 0.0001$ and denoted by P^0 and informative prior information mean that, a_i and b_i are selected almost to satisfies $\frac{a_i}{b_i} \cong \theta_i$ and denoted by P^1. For the algorithms of MCMC (Gibbs and more MH under Gibbs) has chan with 11000 iteration termenated the first 1000 iteration as burn-in. In all cases the integers values (n, s, m) is defined by

$$\begin{cases} I_1 = (40, 15, 25) \\ I_2 = 40, 20, 25 \\ I_3 = 40, 20, 30 \end{cases} \text{and} \begin{cases} I_4 = (60, 30, 45) \\ I_5 = (60, 40, 45) \\ I_6 = (60, 40, 50) \end{cases}$$

From the numerical results presented in Tables 1–4, we have observed the following points

1. The proposed type-I GHCS competing risks model serve well for statistical inference of the two parameter generalized half-logistic lifetime distribution.
2. The MLEs and non-informative Bayes estimate are more closed for each.
3. Informative Bayes estimate serve well than maximum likelihood and bootstrap and non-informative Bayes estimators.
4. When affect sample size has increase MSE and MIL are decreases and CP more closed to proposed one.
5. The large value of τ serve well of model parameter.

7. Real Data Analysis

In this section, we consider laboratory experiment for obtaining a real survival times presented by Hoel [32]. The lifetime data obtained from a radiation male mice over the period 5–6 weeks. The real data set divided into two sets to exposed two causes of failure. the first set present the first cause of failure by other causes. Also, the second set present the second cause of failure by Thymic Lymphoma and reported in Table 5. For simplicity and to be more sensitivity to GHL distribution data transformed by $Y = \frac{T}{10}$. For $n = 61$, $s = 25$, $m = 40$ and $\tau = 4$ the type-I GHC sample given by Table 6. From the real data, we are observed that $r = 35$, $r_1 = 18$ and $r_2 = 17$. The profile log likelihood function (20) show that, iteration begin with initial value of $\beta = 1$. For Bayesian approach, we adopted the non-informative information $a_i = b_i = 0.0001, i = 1, 2, 3,$. Also, MCMC approach run with 11000 chan terminated the first 1000 chan as burn-in. The simulated number generated from posterior distribution described by Figs. listed in 2 and 3. Hence, the results of estimation reported in Table 7.

8. Conclusions

Generally, the experimenters search for the censoring scheme which have the ability to balance between minimum number of failure needing in statistical inference and the ideal test time. Type-I GHCS serve this problem very well. Also, the problem of statical inference of a life populations when its units fail by different causes of failure discussed in this paper very well specially when the life of units have GHL distribution. The model parameters are estimated with MLE, bootstrap and Bayes methods. The numerical results which reported in Tables 1–4 and the results proposed in numerical example show that, the performance of proposed model serve well. Also, the meth-

Table 1 MSEs of MLE, bootstrap and Bayes estimtors at $\theta = \{0.2, 0.5, 2.5\}$.

		MLE			Boot			BayesP^0			Bayes P^1		
τ		α_1	α_2	β	α_1	α_2	β	α_1	α_2	β	α_1	α_2	β
2	I_1	0.142	0.232	0.455	0.161	0.259	0.469	0.135	0.228	0.418	0.098	0.200	0.385
	I_2	0.127	0.215	0.415	0.145	0.242	0.428	0.118	0.211	0.418	0.071	0.182	0.351
	I_3	0.089	0.187	0.352	0.108	0.211	0.367	0.100	0.181	0.387	0.059	0.150	0.314
	I_4	0.071	0.171	0.322	0.087	0.198	0.341	0.068	0.164	0.317	0.051	0.132	0.288
	I_5	0.065	0.158	0.301	0.084	0.184	0.314	0.059	0.152	0.292	0.048	0.125	0.261
	I_6	0.057	0.142	0.292	0.081	0.171	0.308	0.051	0.137	0.279	0.041	0.111	0.245
4	I_1	0.131	0.211	0.437	0.151	0.239	0.451	0.126	0.203	0.418	0.098	0.175	0.371
	I_2	0.115	0.202	0.400	0.138	0.228	0.414	0.111	0.198	0.404	0.064	0.171	0.344
	I_3	0.074	0.162	0.335	0.091	0.190	0.352	0.092	0.157	0.372	0.051	0.129	0.302
	I_4	0.059	0.145	0.304	0.080	0.171	0.318	0.061	0.138	0.308	0.04	0.111	0.269
	I_5	0.049	0.139	0.283	0.071	0.165	0.299	0.042	0.133	0.277	0.040	0.104	0.243
	I_6	0.041	0.122	0.277	0.058	0.151	0.281	0.030	0.118	0.264	0.037	0.091	0.239
6	I_1	0.114	0.192	0.441	0.132	0.222	0.459	0.109	0.187	0.427	0.079	0.160	0.379
	I_2	0.101	0.184	0.409	0.119	0.205	0.424	0.091	0.183	0.413	0.049	0.152	0.340
	I_3	0.059	0.150	0.339	0.081	0.181	0.355	0.084	0.147	0.381	0.037	0.120	0.315
	I_4	0.042	0.131	0.311	0.063	0.154	0.331	0.045	0.124	0.319	0.025	0.095	0.275
	I_5	0.037	0.124	0.281	0.061	0.152	0.298	0.029	0.117	0.284	0.021	0.089	0.249
	I_6	0.029	0.109	0.271	0.044	0.141	0.288	0.018	0.103	0.271	0.019	0.081	0.243

Table 2 AILs(CP) of 95% MLE, boot-p, boot-t and Bayes interval estimtors at $\theta = \{0.2, 0.5, 2.5\}$.

τ		MLE			Boot-p			Boot-t			BayesP[1]		
		α_1	α_2	β	α_1	α_2	β	α_1	α_2	β	α_1	α_2	β
2	I_1	0.57(88)	1.22(89)	4.78(87)	0.64(89)	1.27(89)	4.77(89)	0.46(91)	1.07(91)	4.35(88)	0.43(90)	1.00(90)	4.27(90)
	I_2	0.55(89)	1.15(89)	4.73(88)	0.57(89)	1.19(89)	4.73(89)	0.41(90)	0.99(89)	4.29(91)	0.39(89)	0.95(91)	4.23(91)
	I_3	0.52(89)	1.11(90)	4.70(90)	0.55(90)	1.14(90)	4.69(89)	0.38(92)	0.95(92)	4.28(92)	0.36(91)	0.92(93)	4.21(90)
	I_4	0.49(90)	1.08(90)	4.66(90)	0.51(91)	1.11(90)	4.67(90)	0.34(88)	0.92(91)	4.24(92)	0.33(92)	0.87(93)	4.17(96)
	I_5	0.46(91)	1.00(91)	4.64(93)	0.49(90)	1.05(91)	4.65(92)	0.30(91)	0.87(92)	4.18(92)	0.30(93)	0.82(96)	4.14(92)
	I_6	0.44(92)	0.99(93)	4.64(90)	0.46(92)	1.00(93)	4.64(91)	0.31(92)	0.86(92)	4.17(93)	0.29(92)	0.80(92)	4.13(94)
4	I_1	0.56(89)	1.14(90)	4.77(87)	0.57(89)	1.18(90)	4.75(90)	0.43(92)	1.05(92)	4.24(93)	0.41(90)	0.99(92)	4.26(93)
	I_2	0.54(90)	1.07(89)	4.71(89)	0.53(90)	1.11(89)	4.69(90)	0.41(90)	0.96(92)	4.25(93)	0.38(90)	0.92(91)	4.22(91)
	I_3	0.51(89)	1.03(92)	4.69(92)	0.51(90)	1.07(92)	4.67(91)	0.37(93)	0.94(92)	4.21(90)	0.35(91)	0.90(92)	4.18(96)
	I_4	0.47(91)	099(90)	4.65(90)	0.48(92)	1.02(90)	4.65(92)	0.34(92)	0.87(95)	4.19(95)	0.31(93)	0.84(93)	4.16(95)
	I_5	0.45(91)	097(91)	4.63(90)	0.48(90)	0.99(91)	4.63(94)	0.31(95)	0.83(95)	4.17(94)	0.29(95)	0.80(95)	4.13(92)
	I_6	0.43(90)	0.94(92)	4.62(92)	0.46(94)	0.98(92)	4.61(93)	0.32(93)	0.81(93)	4.14(93)	0.29(92)	0.77(92)	4.11(93)
6	I_1	0.54(90)	1.08(90)	4.78(89)	0.54(91)	1.11(90)	4.76(88)	0.39(90)	0.99(92)	4.28(91)	0.38(91)	0.95(92)	4.27(90)
	I_2	0.52(93)	1.02(90)	4.74(89)	0.50(90)	1.06(90)	4.71(90)	0.37(91)	0.93(92)	4.25(91)	0.36(90)	0.90(91)	4.22(91)
	I_3	0.49(92)	097(92)	4.69(90)	0.49(93)	097(92)	4.67(90)	0.35(92)	0.91(92)	4.23(92)	0.33(92)	0.88(92)	4.20(90)
	I_4	0.46(91)	091(91)	4.66(90)	0.45(94)	091(91)	4.67(92)	0.33(92)	0.84(91)	4.19(92)	0.30(94)	0.81(93)	4.17(92)
	I_5	0.42(95)	088(91)	4.64(91)	0.46(92)	088(91)	4.63(91)	0.29(92)	0.77(90)	4.18(93)	0.27(92)	0.76(95)	4.15(92)
	I_6	0.40(92)	0.86(91)	4.63(90)	0.44(95)	0.86(91)	4.62(90)	0.28(93)	0.73(94)	4.14(91)	0.28(94)	0.72(92)	4.12(90)

Table 3 MSEs of MLE, bootstrap and Bayes estimtors at$\theta = \{1.5, 1.2, 1.5\}$.

τ		MLE			Boot			BayesP[0]			Bayes P[1]		
		α_1	α_2	β	α_1	α_2	β	α_1	α_2	β	α_1	α_2	β
0.5	I_1	0.324	0.278	0.388	0.161	0.295	0.469	0.313	0.271	0.365	0.256	0.225	0.300
	I_2	0.303	0.255	0.369	0.145	0.277	0.428	0.294	0.252	0.348	0.242	0.211	0.277
	I_3	0.291	0.239	0.351	0.108	0.254	0.367	0.279	0.229	0.331	0.219	0.204	0.254
	I_4	0.277	0.224	0.338	0.087	0.242	0.341	0.255	0.222	0.314	0.202	0.191	0.237
	I_5	0.252	0.211	0.324	0.084	0.222	0.314	0.241	0.207	0.300	0.189	0.182	0.219
	I_6	0.238	0.203	0.307	0.081	0.219	0.308	0.219	0.198	0.284	0.168	0.174	0.204
1.5	I_1	0.313	0.267	0.375	0.151	0.281	0.451	0.302	0.262	0.351	0.242	0.214	0.288
	I_2	0.291	0.248	0.361	0.138	0.269	0.414	0.279	0.244	0.333	0.229	0.200	0.262
	I_3	0.284	0.231	0.337	0.091	0.248	0.352	0.260	0.227	0.319	0.208	0.189	0.238
	I_4	0.259	0.215	0.319	0.080	0.228	0.318	0.245	0.211	0.300	0.189	0.176	0.218
	I_5	0.238	0.204	0.309	0.071	0.217	0.299	0.228	0.201	0.287	0.172	0.162	0.204
	I_6	0.229	0.194	0.291	0.058	0.205	0.281	0.209	0.191	0.269	0.155	0.154	0.190
2.5	I_1	0.302	0.254	0.381	0.132	0.272	0.459	0.291	0.252	0.364	0.231	0.203	0.291
	I_2	0.277	0.237	0.371	0.119	0.254	0.424	0.264	0.233	0.342	0.217	0.187	0.269
	I_3	0.270	0.224	0.342	0.081	0.239	0.355	0.2432	0.219	0.324	0.200	0.173	0.245
	I_4	0.242	0.204	0.324	0.063	0.219	0.331	0.232	0.199	0.309	0.154	0.166	0.229
	I_5	0.224	0.197	0.311	0.061	0.205	0.298	0.208	0.192	0.291	0.149	0.154	0.213
	I_6	0.220	0.182	0.298	0.044	0.191	0.288	0.201	0.180	0.277	0.129	0.142	0.199

Table 4 AILs(CP) of 95% MLE, boot-p, boot-t and Bayes interval estimtors at $\theta = \{1.5, 1.2, 1.5\}$.

		MLE			Boo-p			Boo-t			Bayes[1]		
τ		α_1	α_2	β	α_1	α_2	β	α_1	α_2	β	α_1	α_2	β
0.5	I_1	3.57(89)	3.18(89)	3.89(88)	3.71(90)	3.38(89)	3.94(88)	3.62(89)	3.33(90)	3.91(90)	3.43(90)	3.17(91)	3.76(91)
	I_2	3.55(89)	3.12(90)	3.84(89)	3.68(90)	3.31(90)	3.91(90)	3.55(90)	3.28(92)	3.85(92)	3.40(91)	3.12(91)	3.74(92)
	I_3	3.52(91)	3.04(91)	3.81(90)	3.62(91)	3.25(90)	3.88(92)	3.47(93)	3.25(90)	3.79(91)	3.37(93)	3.06(92)	3.70(90)
	I_4	3.48(90)	3.00(92)	3.76(92)	3.54(93)	3.19(93)	3.82(92)	3.42(92)	3.19(93)	3.74(94)	3.33(93)	3.02(93)	3.64(94)
	I_5	3.44(92)	2.91(92)	3.73(92)	3.49(91)	3.12(92)	3.79(93)	3.37(93)	3.12(92)	3.69(93)	3.30(92)	2.97(94)	3.61(96)
	I_6	3.42(94)	2.84(92)	3.70(90)	3.45(93)	3.07(93)	3.74(92)	3.31(92)	3.05(94)	3.64(93)	3.27(94)	2.91(94)	3.58(92)
1.5	I_1	3.55(90)	3.12(89)	3.86(89)	3.63(91)	3.31(89)	3.97(90)	3.57(89)	3.28(91)	3.86(91)	3.40(93)	3.12(91)	3.74(91)
	I_2	3.53(89)	3.04(92)	3.83(90)	3.59(90)	3.28(89)	3.91(90)	3.51(90)	3.22(92)	3.81(92)	3.38(91)	3.07(92)	3.72(92)
	I_3	3.50(92)	3.00(91)	3.79(92)	3.54(94)	3.21(92)	3.88(93)	3.44(92)	3.18(92)	3.74(93)	3.33(90)	3.01(92)	3.68(93)
	I_4	3.45(90)	2.95(92)	3.74(90)	3.50(93)	3.14(93)	3.82(92)	3.39(92)	3.14(93)	3.71(94)	3.31(93)	2.97(94)	3.62(94)
	I_5	3.42(93)	2.82(96)	3.71(93)	3.48(92)	3.09(90)	3.76(94)	3.34(92)	3.08(94)	3.62(92)	3.28(95)	2.91(94)	3.59(94)
	I_6	3.40(91)	2.80(94)	3.69(92)	3.44(93)	3.03(94)	3.73(95)	3.28(93)	3.01(90)	3.58(94)	3.23(94)	2.84(95)	3.57(95)
2.5	I_1	3.53(90)	3.03(90)	3.87(89)	3.53(92)	3.26(90)	3.91(89)	3.51(90)	3.21(91)	3.79(92)	3.38(92)	3.03(90)	3.75(91)
	I_2	3.52(92)	2.94(90)	3.85(91)	3.51(90)	3.21(91)	3.88(90)	3.46(93)	3.16(92)	3.74(92)	3.36(91)	2.95(91)	3.72(93)
	I_3	3.48(92)	2.81(92)	3.78(92)	3.46(94)	3.15(92)	3.84(91)	3.41(92)	3.12(92)	3.71(93)	3.31(93)	2.89(92)	3.69(93)
	I_4	3.43(94)	2.69(92)	3.75(90)	3.42(94)	3.11(93)	3.79(92)	3.33(92)	3.07(92)	3.65(92)	3.28(93)	2.82(93)	3.63(90)
	I_5	3.40(93)	2.60(93)	3.72(92)	3.41(92)	3.04(92)	3.77(94)	3.29(94)	3.03(94)	3.61(92)	3.25(92)	2.75(94)	3.60(94)
	I_6	3.37(96)	2.52(92)	3.69(93)	3.37(95)	3.00(93)	3.71(92)	3.25(94)	2.97(95)	3.54(93)	3.22(91)	2.71(93)	3.59(91)

Table 5 Data obtained from laboratory experiment by Hoel [32].

Other causes													
40	42	51	62	163	179	206	222	228	252	249	282	324	333
341	366	385	407	420	431	441	461	462	482	517	517	524	564
567	586	619	620	621	622	647	651	686	761	763			
Thymic Lymphoma													
159	189	191	198	200	207	220	235	245	250	256	261	265	266
280	343	356	383	403	414	428	432						

Table 6 Type-I GHC sample.

(0.40,2)	(0.42,2)	(0.51,2)	(0.62,2)	(1.59,1)	(1.63,2)	(1.79,2)	(1.89,1)	(1.91,1)
(1.98,1)	(2.00,1)	(2.06,2)	(2.07,1)	(2.20,1)	(2.22,2)	(2.28,2)	(2.35,1)	(2.45,1)
(2.49,2)	(2.50,1)	(2.52,2)	(2.56,1)	(2.61,1)	(2.65,1)	(2.66,1)	(2.80,1)	(2.82,2)
(3.24,2)	(3.33,2)	(3.41,2)	(3.43,1)	(3.56,1)	(3.66,2)	(3.83,1)	(3.85,2)	

Table 7 The point and 95% interval estimate.

	ML	Boot	Bayes	95% ACI	Boot-p	Boot-t	95% CI
α_1	0.1424	0.1621	0.1330	(-0.0718, 0.3566)	(0.0421, 0.5412)	(0.0475, 0.3562)	(0.0751, 0.2059)
α_2	0.1345	0.1500	0.1332	(-0.0684, 0.3374)	(0.0541, 0.5823)	(0.0624, 0.4210)	(0.0763, 0.2074)
β	1.1471	1.1555	1.1379	(-0.2140, 2.5082)	(0.3214, 2.8541)	(0.714, 1.5142)	(0.9948, 1.2721)

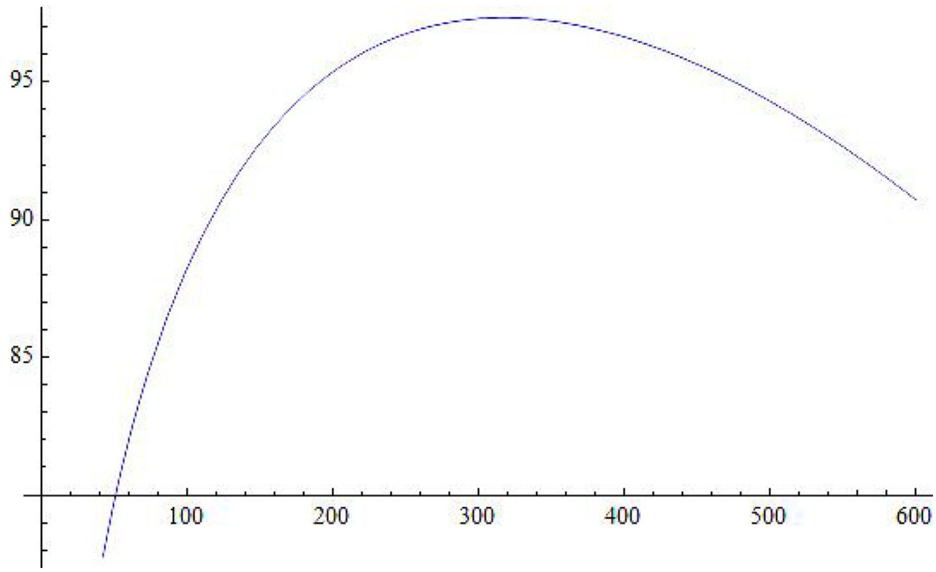

Fig. 1 Profile log-likelihood function of β.

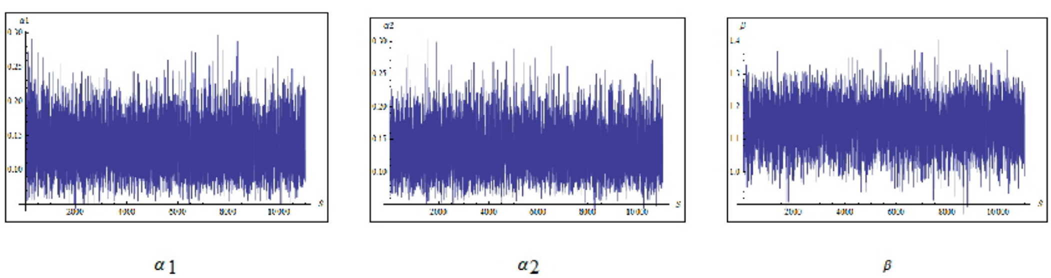

Fig. 2 Simulation number of generated by important sample method.

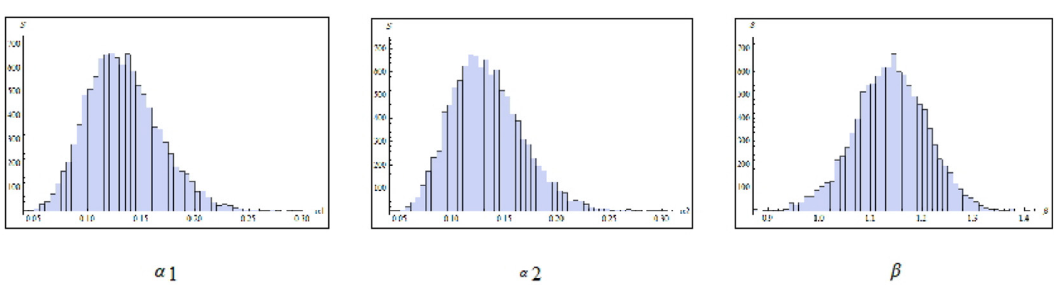

Fig. 3 Histogram of parameter generated by MCMC method.

ods used in parameters estimation are more acceptable for all choices of parameters values and different scheme values. (See Figs. 1–3).

Declaration of Competing Interest

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