Naive Bayes Classifier - I

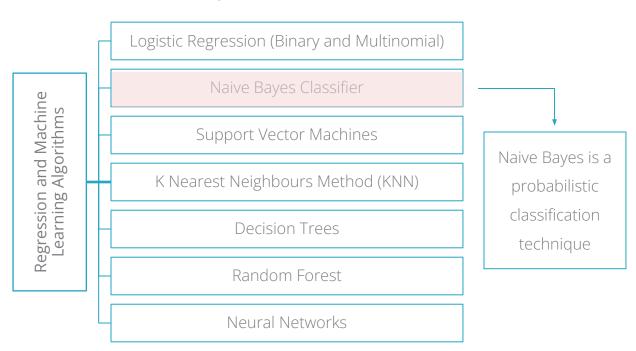
Classifier Based on Bayes' Theorem

Contents

- 1. Classification Methods
- 2. Introduction to Naive Bayes Classifier
- 3. Conditional Probability and Bayes' Theorem
- 4. Classification Rule
- 5. Expected Output
- 6. Advantages and Limitations of Naive Bayes Method

Classification Methods

Apart from logistic regression, several types of machine learning algorithms are effective in classification and prediction.



About Naive Bayes Classifier

- Simple probabilistic classifier based on Bayes Theorem.
- It can be used as an alternative method to logistic regression (Binary or Multinomial).
- It assumes conditional independence among the predictors.
- It is particularly suited when the dimensionality of the inputs is high.

Despite its simplicity, Naive Bayes can often outperform more sophisticated classification methods.

Conditional Probability

The conditional probability of an event B is the probability that event B will occur given the knowledge that an event A has already occurred.

This probability is written as P(B|A).

• If A and B are independent events then

$$P(B|A) = P(B)$$

An unbiased die, with numbers 1-6 is tossed

A: Getting a number greater than 1

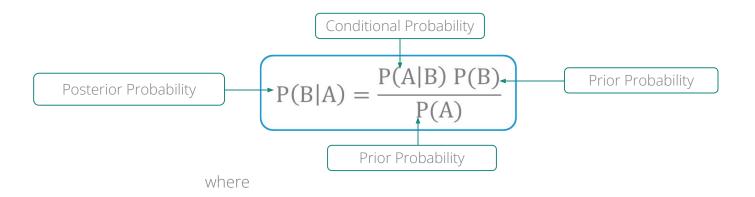
B: Getting an even number

$$P(A) = 5/6$$

 $P(B) = 3/6$
 $P(B|A) = 3/5$

Here the sample space has 5 points given A has occurred.

Bayes Theorem



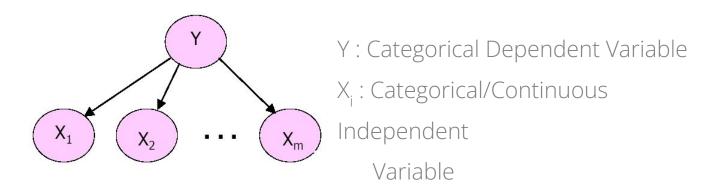
P(A): Prior probability or marginal probability of A

P(A | B): Conditional probability of A given B

P(B|A): Conditional probability of B given A

P(B): Prior or marginal probability of B

Naive Bayes Framework



Objective: To estimate Y given the values of X_i's or

To estimate $P(Y|X_1, X_2, ..., X_m)$ using the Naïve Bayes Classifier

Assumption: All X_i's are conditionally independent of each other

Naive Bayes Framework - Example

Consider a simple example where Y is binary (response to a certain question) with 2 independent categorical variables X_1 and X_2

We classify	Y = 1 "Buyer"
	Y = 0 "Non-Buyer"
Let X ₁ denote age of the	$X_1 = 0$ for age group 25-30 years
individual	$X_1 = 1$ for age group 31-40 years
Let X ₂ denote gender	$X_2 = 0$ if Gender=female
	$X_2 = 1$ if Gender=male

Classification Rule

For the given values of X_1 and X_2 we want to know if the individual will be a potential buyer or not. Using Naive Bayes classifier we estimate:

$$P(Y = 0|X_1 = a_1, X_2 = a_2)$$
 &
$$P(Y = 1|X_1 = a_1, X_2 = a_2)$$

where a₁ and a₂ are values of X₁ and X₂ for a particular respondent

We classify Y = 0 if
$$P(Y = 0 | X_1 = a_1, X_2 = a_2) > 0.5$$
 OR
Y = 1 if $P(Y = 1 | X_1 = a_1, X_2 = a_2) > 0.5$

In the general case i.e. when Y has more than 2 categories we compare $P(Y=y_k \mid X)$ for all values of y_k and classify $Y=y_k$ for which $P(Y=y_k \mid X)$ is the maximum

Expected Output

Once the classification rule is applied the output can be shown as follows:

Case#	X1	X2	P(Y=1/X ₁ ,X ₂)	P(Y=0/X ₁ ,X ₂)	Y classified as
1 2	1 1	0 1 .	0.44 0.7	0.56 0.3	0 1
240	0		0.2	0.8	0

Advantages of Naive Bayes Method

- Classification rule is simple to understand.
- •The method requires a small amount of training data to estimate the parameters necessary for classification.
- •The evaluation of the classifier is quick and easy.
- •The method can be a good alternative to logistic regression.

Limitations of Naive Bayes Method

- Assumption of conditional independence of the independent variables is highly impractical.
- In case of continuous independent variables the density function must be known or assumed to be normal.
- In case of categorical independent variables the probabilities cannot be calculated if the count in any conditional category is zero. For instance: If there are no respondents in the age group 25-30 yrs. then $P(X_1=0 \mid Y=1)=0$

Quick Recap

In this session, we learnt Naive Bayes Classification technique:

Conditional Probability and Bayes' Theorem

- The conditional probability of an event B is the probability that event B will occur given the knowledge that an event A has already occurred.
- P(B|A) = P(A|B) P(B) / P(A)

Naive Bayes Classifier

- To estimate Y given the values of X_i 's or $P(Y|X_1, X_2, ..., X_m)$ using the Naïve Bayes Classifier.
- Assumption: All X_i's are conditionally independent of each other.
- Advantages: Simple classification rule, requires a small amount of training data to estimate the parameters necessary for classification, Evaluation of the classifier is quick and easy, Good alternative to logistic regression.
- Major drawback: Assumption of conditional independence of the independent variables is highly impractical.

Naive Bayes Classifier - II

Classifier Based on Bayes' Theorem

Contents

- 1. Naive Bayes Classifier in Python
 - i. For Continuous predictors
 - ii. For Categorical predictors
- 1. Laplace Smoothing

Case Study – Modeling Loan Defaults

Background

 A bank possesses demographic and transactional data of its loan customers. If the bank has a model to predict defaulters it can help in loan disbursal decision making.

Objective

• To predict whether the customer applying for the loan will be a defaulter or not.

Available Information

- Sample size is 700
- Independent Variables: Age group, Years at current address, Years at current employer, Debt to Income Ratio, Credit Card Debts, Other Debts. The information on predictors was collected at the time of loan application process.
- **Dependent Variable**: Defaulter (=1 if defaulter ,0 otherwise). The status is observed after loan is disbursed.

Data Snapshot

Independent Variables

Dependent Variable

SN	ı	AGE	EMPLOY	ADDRESS	DEBTINC	CREDDEBT	OTHDEBT	DEFAULTE	
	1	3	17	12		10,700,000,000,000	5.01	1	
	2	1		6			4	0	
	3	2	15 15	14 14		100000000000000000000000000000000000000	2.17 0.82	0	
Column		Descript		Тур	No. 10-	/leasure	10,000,000,000	Possible	Values
SN		erial Nui		nume		-		-	
						1(<28 ye	ars),		
AGE		Age Groups		Integ	eger 2(28-40 years), 3				
			•			3(>40 ye	ears)		
	Nι	ımber of	fyears						
EMPLOY	cust	tomer working at		Integ	er	-		Positive value	
	CUI	rent em	ıployer	oloyer					
	Nι	Number of years							
ADDRESS	cust	omer st	aying at	Integ	Integer			Positive value	value
	CL	irrent ad	ldress						
DEBTINC	Debt	to Incor	address come Ratio Cont		ıous	-		Positive	value
CREDDEBT	Cred	dit to Del	bit Ratio	Continu	uous	-		Positive	value
OTHDEBT		Other D	ebt	Continu	Jous	-		Positive	value
DEFAULTER	<i>?</i>	ether cu faulted c		Integ	er nı	1(Defaul Non-Def		2	

Naive Bayes Method in Python

Importing required Libraries

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.naive_bayes import GaussianNB, MultinomialNB

from sklearn.metrics import confusion_matrix, f1_score,
precision_score, recall_score, accuracy_score,
roc_curve, roc_auc_score
```

- Naive Bayes methods differ based on the type of predictors- continuous or categorical
- Python's sklearn has various methods available and the two methods explored hereon are Gaussian Naive Bayes and Multinomial Naive Bayes.

Importing and Readying the Data for Modeling

```
bankloan = pd.read_csv("BANK LOAN.csv")
bankloan1 = bankloan.drop(['SN','AGE'], axis = 1)
bankloan1.head()
```

- drop() is used to remove unwanted variables. AGE is removed because it is a categorical variable.
- axis = 1 drops columns.

Output

	EMPLOY	ADDRESS	DEBTINC	CREDDEBT	OTHDEBT	DEFAULTER
0	17	12	9.3	11.36	5.01	1
1	10	6	17.3	1.36	4.00	0
2	15	14	5.5	0.86	2.17	0
3	15	14	2.9	2.66	0.82	0
4	2	0	17.3	1.79	3.06	1

Creating Train and Test Data Sets

- train_test_split() from sklearn.model_selection is used to split dataset into random train and test sets.
- test_size represents the proportion of dataset to be included in the test set.
- random_state sets the seed for the random number generator.

```
# Model Fitting
NBmodel = GaussianNB()
NBmodel.fit(X train, y train)
```

- GaussianNB() fits a Gaussian Naive Bayes algorithm for classification.
- This model is suitable for continuous predictors and assumes the likelihood of predictors to be normal.

```
# Predicted Probabilities
```

```
predprob test = NBmodel.predict proba(X test)
predprob test
```

Output

```
array([[0.96499091, 0.03500909],
       [0.86941828, 0.13058172],
       [0.90585744, 0.09414256],
       [0.97398393, 0.02601607],
       [0.99549445, 0.00450555],
       [0.52978724, 0.47021276],
```

predict_proba() returns predicted probabilities for the test data.

Custom Cutoff Value for Prediction Labels

```
cutoff = 0.3
pred_test = np.where(predprob_test[:,1] > cutoff, 1, 0)
pred_test

The output is an array of binary
labels.
```

Output

```
# Confusion Matrix
confusion_matrix(y_test, pred_test, labels=[0, 1])
                                     accuracy_score() = number of correct
array([[135, 22],
                                         predictions out of total predictions
       [ 26, 27]])
                                        precision score() = true positives / (true
accuracy score(y test, pred test)
                                         positives + false positives)
0.7714285714285715
                                        recall_score() also known as 'Sensitivity' =
precision_score(y test, pred test)
                                         true positives / (true positives + false
0.5510204081632653
                                         negatives)
recall score(y test, pred test)
0.5094339622641509
```

```
# Area Under ROC Curve

auc = roc_auc_score(y_test, predprob_test[:,1])

print('AUC: %.3f' % auc)

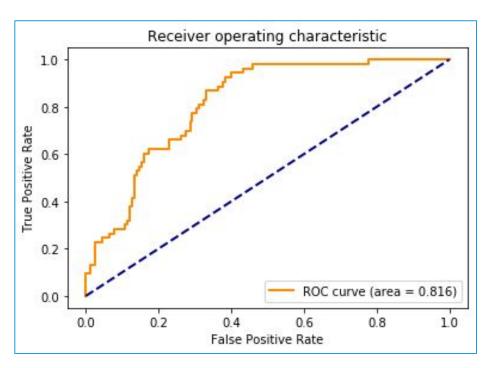
AUC: 0.816

roc_auc_score computes Area Under the ROC curve.
```



```
# ROC Curve
NBfpr, NBtpr, thresholds = roc curve(y test, predprob test[:,1])
                                       roc_curve is used to Compute
# plot the roc curve for the model
                                        Receiver operating characteristic.
plt.figure()
1w = 2
plt.plot(NBfpr, NBtpr, color='darkorange',lw=lw, label='ROC curve
(area = \%0.3f)' \% auc)
plt.plot([0, 1], [0, 1], color='navy', lw=lw, linestyle='--')
plt.axis('tight')
plt.xlabel('False Positive Rate');plt.ylabel('True Positive Rate')
plt.title('Receiver operating characteristic')
plt.legend(loc="lower right")
plt.show()
```

Output:



Case Study – Employee Churn Model

Background

 A company has comprehensive database of its past and present workforce, with information on their demographics, education, experience and hiring background as well as their work profile. The management wishes to see if this data can be used for predictive analysis, to control attrition levels.

Objective

• To develop an Employee Churn model via Naive Bayes

Available Information

- Sample size is 83
- Gender, Experience Level (<3, 3-5 and >5 years), Function (Marketing, Finance, Client Servicing (CS)) and Source (Internal or External) are independent variables
- Status is the dependent variable (=1 if employee left within 18 months from joining date)

Data Snapshot

EMPLOYEE CHURN DATA

		Dependent Independent Variable Variables sn status function exp gender source		}				
		sn 1	status 1	function CS	exp <3	gender M	source external	
Columns	De	Description		Type		Measure	ement	Possible values
sn	Seri	Serial Number		Integer		-		-
status	Within	1		Integ	ger	1,0		2
function	Employee Job Profile		Chara	Character		ance, Ting	3	
exp	Experi	Experience in Years		Chara	cter	<3,3-5	5,>5	3
gender	Gender o	Gender of the Employee		Character		M,F	=	2
source	was A	Whether the Employee was Appointed via Internal or External Links		Chara	cter	exterr interr	•	2

Importing and Readying the Data for Modeling, Model Fitting
empdata = pd.read_csv("EMPLOYEE CHURN DATA.csv")
empdata1 = empdata.loc[:, empdata.columns != 'sn']

loc() is used to create a subset of the data frame using column name. Removing column with serial numbers.

Output

empdata1.head()

source	gender	exp	function	status	
external	M	<3	CS	1	0
external	M	<3	CS	1	1
internal	M	>=3 and <=5	CS	1	2
internal	F	>=3 and <=5	CS	1	3
internal	M	<3	CS	1	4

Creating Dummy Variables

```
empdata2 = pd.get_dummies(empdata1)
empdata2.head()
```

pd.get_dummies() converts categorical variables into dummy variables. This step is crucial because the naive Bayes function used for categorical variables requires this format.

Output

source_internal	source_external	 function_CS	status	
0	1	 1	1	0
0	1	 1	1	1
1	0	 1	1	2
1	0	 1	1	3
1	0	 1	1	4

Creating Data Partitions

X_emp = empdata2.loc[:,empdata2.columns != 'status']
y_emp = empdata2.loc[:, 'status']

Model Fitting

MNBmodel = MultinomialNB(alpha = 0)

MNBmodel.fit(X emp, y emp)

- MultinomialNB() fits a Multinomial Naive Bayes algorithm for classification. This model is suitable for categorical predictors.
- alpha = 0 ensures the model doesn't apply any smoothing on the data.

Predicted Probabilities

```
predprob_MNB = MNBmodel.predict_proba(X_emp)
predprob_MNB

# Output

array([[0.06419224, 0.93580776],
```

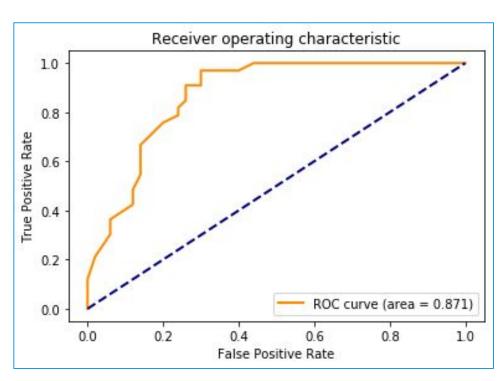
Custom Cutoff Value for Prediction Labels

```
cutoff = 0.3
pred_test = np.where(predprob_MNB[:,1] > cutoff, 1, 0)
```

```
# Confusion Matrix
confusion matrix(y emp, pred test, labels=[0, 1])
                                      accuracy_score() = number of
array([[37, 13],
                                      correct predictions out of total
       [ 3, 30]])
                                      predictions
accuracy score(y emp, pred test)
                                      precision_score() = true positives /
0.8072289156626506
                                      (true positives + false positives)
precision_score(y emp, pred test)
0.6976744186046512
                                      recall score() also known as
                                      'Sensitivity' = true positives / (true
recall_score(y emp, pred test)
0.9090909090909091
                                      positives + false negatives)
# Area Under ROC Curve
auc = roc_auc_score(y_emp, predprob_MNB[:,1])
print('AUC: %.3f' % auc)
AUC: 0.871
```

ROC Curve MNBfpr, MNBtpr, thresholds = roc curve(y emp, predprob MNB[:,1]) # plot the roc curve for the model plt.figure() 1w = 2plt.plot(MNBfpr, MNBtpr, color='darkorange',lw=lw, label='ROC curve (area = %0.3f)' % auc)plt.plot([0, 1], [0, 1], color='navy', lw=lw, linestyle='--') plt.axis('tight') plt.xlabel('False Positive Rate');plt.ylabel('True Positive Rate') plt.title('Receiver operating characteristic') plt.legend(loc="lower right") plt.show()

Output



Laplace Smoothing

$$P(x = x_i | y = y_i) = f_i/N_i$$

This prob will be 0 if numerator count (f_i) is 0

Laplace smoothing will replace this probability with a value obtained by the formula:

$$\widehat{\theta}_{i} = \frac{f_{i} + \alpha}{N_{j} + \alpha d_{i}}$$

where

α : Smoothing Parameter

 Y_j : Number of observations for $Y = y_j$

 d_i : Number of classes of x

Quick Recap

Naive Bayes in Python

• GaussianNB for continuous variables, MultinomialNB for categorical variables in library sklearn.naive_bayes

Laplace Smoothing

- If a given class and feature value never occur together in the training data, then the frequency-based probability estimate will be zero.
- A pseudo-count is incorporated, in all probability estimates such that no probability is ever set to be exactly zero.
- This way of regularizing naive Bayes is called Laplace Smoothing

K Nearest Neighbours Classifier

Learn how a Simple Lazy Learning Algorithm Works

Contents

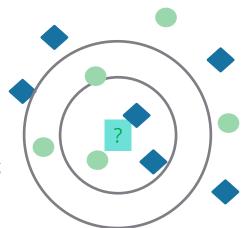
- 1. Introduction to K Nearest Neighbours (KNN) Algorithm
- 2. KNN for Classification
 - i. Measuring Distance
 - ii. Distance Based on Standardised Variables
- 3. Selection of K
- 4. Voting Rules in KNN Classification
- 5. KNN Classification in PYTHON
- 6. KNN for Regression

KNN for Classification

- Training dataset has 11 observations belonging to two categories.
- 12th observation is introduced, class of which is not known.
- Nearest neighbour algorithm classifies
 new observation to the class of the training
 observation closest to it.

When K=1, nearest one case is considered
As we go on increasing K, classification may vary

K	
1	Blue
3	Blue
5	Orange



Three most important components of this method are **Distance** between cases, **Value of K** and **Voting** criteria.

Simple Example To Understand KNN Method

Age	Current Debt Default		
25	40,000	N	
35	60,000	N	
45	80,000	N	
20	20,000	N	
35	120,000	N	
52	18,000	N	
23	95,000	Y	
40	62,000	Y	
60	100,000	Y	
48	220,000	Y	
33	150,000	Y	
48	142,000	?	

New observation
will be
classified as "N"
or "Y" based on
KNN method

Distance Based on Standardised Variables

$$\mathbf{X_s} = \frac{\mathbf{X} - \mathbf{Min}}{\mathbf{Max} - \mathbf{Min}}$$
 Alternatively, $\frac{(\mathbf{X} - \mathbf{Mean})}{\mathbf{SD}}$ can also be

Age	Current Debt	Default	Distance
0.125	0.11	N	0.7652
0.375	0.21	N	0.5200
0.625	0.31	N	0.3160
0	0.01	N	0.9245
0.375	0.50	N	0.3428
0.8	0.00	N	0.6220
0.075	0.38	Y	0.6669
0.5	0.22	Y	0.4437
1	0.41	Y	0.3650
0.7	1.00	Y	0.3861
0.325	0.65	Y	0.3771
0.7	0.61	?	

New observation will be classified as "N"

Selection of K

The second component of KNN model is selecting the appropriate value for K

- If K = 1, the case is classified using the nearest neighbour
- However, K is usually greater than 1. Consider the following when choosing K:
 - Mostly odd numbered K is preferred to avoid tie.
 - For a very large K the classifier may result in misclassification, as group of nearest neighbours may include data points which are actually located far away from it.

Thumb Rule:

K = sqrt(n)

n is the number of observations in training data

Voting Criteria

Most common criteria for classification decision is Majority Voting.

Frequency of each class in K instances is measured. Class having the highest frequency is attributed to the new case.

Eg. Suppose for K = 7, 4 cases belong to class A and 3 to class B. New case is given class A

Drawback:

Classification is inappropriate when the class distribution is skewed. That is, examples of a more frequent class tend to dominate the prediction of the new example, because they tend to be common among the k nearest neighbors due to their large number.

Is there a way to correct this?

?

One option to remove this drawback is to create training data with equal class frequency. However, this is possible only if data is very large.

Case Study – Predicting Loan Defaulters

Background

• The bank possesses demographic and transactional data of its loan customers. If the bank has a robust model to predict defaulters it can undertake better resource allocation.

Objective

• To predict whether the customer applying for the loan will be a defaulter

Available Information

- Sample size is 389
- Age group, Years at current address, Years at current employer, Debt to Income Ratio, Credit Card Debts, Other Debts are the independent variables
- **Defaulter** (=1 if defaulter, 0 otherwise) is the dependent variable

Data Snapshot

BANK LOAN KNN

Variables

	SN	AGE	EMPLOY	ADDRESS	DEBTING	CREDDEBT	OTHDEBT	DEFAULTER	
Column	Des	cription		Type	N	leasuren	nent	Possible '	Values
SN	Seria	l Numbe	er			-		=	
AGE	Age	Groups		Categorio		1(<28 ears),2(28 rs),3(>40		3	
EMPLOY	custome	er of yea er workin It emplo	ng at	Continuc	us	-		Positive	value
ADDRESS	custom	er of yea er stayir nt addre	ng at	Continuc	us	-		Positive	value
DEBTINC	Debt to	Income	Ratio	Continuc	us	-		Positive	value
CREDDEBT	Credit	Card De	ebt	Continuc	us	-		Positive	value
OTHDEBT	Oth	ner Debt	(Continuc	us	-		Positive	value
DEFAULTER		er custoi ted on la		Binary		1(Default Non-Defa		2	-

Importing the Libraries

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.neighbors import KNeighborsClassifier

from sklearn.metrics import confusion_matrix, f1_score,
precision_score, recall_score, accuracy_score,
roc_curve, roc_auc_score
```

Output

	EMPLOY	ADDRESS	DEBTINC	CREDDEBT	OTHDEBT	DEFAULTER
0	17	12	9.3	11.36	5.01	1
1	2	0	17.3	1.79	3.06	1
2	12	11	3.6	0.13	1.24	0
3	3	4	24.4	1.36	3.28	1
4	24	14	10.0	3.93	2.47	0

```
# Creating Train and Test Datasets
X = bankloan1.loc[:,bankloan1.columns != 'DEFAULTER']
y = bankloan1.loc[:, 'DEFAULTER']
X train, X test, y train, y test = train test split(X, y,
                                                  test size=0.30,
                                                  random state = 999)
                    train_test_split() from
                       sklearn.model selection is used to split
                       dataset into random train and test sets.
                      test_size represents the proportion of dataset
                       to be included in the test set
                      random_state sets the seed for the random
                       number generator
```

```
# Preparing Variables
scaler = StandardScaler() 
scaler.fit(X_train)

X_train = scaler.transform(X_train)
X_test = scaler.transform(X_test)
X_train

# Output
StandardScaler() from sklearn.preprocessing is a generic function used for centering or scaling columns of a numeric matrix. The default method for scaling is (X-Mean)/SD.
```

- All the continuous predictors are now scaled to mean=0 and sd=1.
- Note: Test data is transformed using the train data parameters

Output

```
# Predictions on Test Data
y pred = KNNclassifier.predict(X test)
                        predict() predicts the class labels of the provided
                        data. The default threshold is 0.5.
# Confusion Matrix
confusion matrix(y test, y pred, labels=[0, 1])
array([[49, 7],
                                    accuracy score() = number of correct
      [24, 37]])
                                      predictions out of total predictions
accuracy_score(y test, y pred)
                                      precision score() = true positives / (true
0.7350427350427351
                                      positives + false positives)
precision_score(y_test, y_pred)
                                     recall score() also known as 'Sensitivity'
0.8409090909090909
                                      = true positives / (true positives + false
recall score(y test, y pred)
0.6065573770491803
                                      negatives)
```



KNN for Regression

KNN algorithm can also be extended to regression problems, i.e. when the dependent variable is continuous

Process flow for classification and regression is the same, except for the last step



Average value of the response variable for k neighbours is calculated and assigned to the new case.

KNeighborsRegressor() from sklearn.neighbors can be used to run k-nearest neighbour regression in Python.

Get an Edge!

- KNN can be used for categorical variables as well.
- Before executing knn on train-test data, categorical variables have to be converted to numeric variables by creating dummy variables.

Quick Recap

In this session, we learnt about KNN Classifier :L

KNN for Classification Three most important components of this method are Distance between cases, Value of K and Voting criteria.

KNN for Classification in Python

• KNeighborsClassifier() from sklearn.neighbors.

KNN for Regression

 KNN algorithm can also be extended to regression problems when the dependent variable is continuous.

KNN for Regression in Python

• KNeighborsRegressor() from sklearn.neighbors.

Support Vector Machines in Python

Contents

- 1. Understanding Association Rules
- 2. Introduction to Market Basket Analysis
 - i. Uses
 - ii. Definitions and Terminology
- 3. Rule Evaluation
 - i. Support
 - ii. Confidence
 - iii. Lift
- 4. Market Basket Analysis in Python

Contents

- 1. Introduction to Support Vector Machine (SVM)
- 2. Understanding Hyper Planes
 - i. What is a Hyper Plane
 - ii. Hyper Plane Separation
- 3. Linear Separators
 - i. Classification Margin
- 4. Mathematical Approach to Linear SVM
- 5. Non-Linear SVM
- 6. About the Kernel Function
- 7. SVM in Python
 - i. SVM Modeling
 - ii. Confusion Matrix and Area Under ROC Curve

Introduction to Support Vector Machines

- Support Vector Machines (SVM's) are a relatively new learning method generally used for classification problem.
- Although the first paper dates way back to early 1960's it is only in 1992-1995 that
 this powerful method was universally adopted as a mainstream machine learning
 paradigm

The basic idea is to find a hyper plane which separates the d-dimensional data perfectly into its classes. However, since training data is often not linearly separable, SVM's introduce the notion of a "Kernel-induced Feature Space" which casts the data into a higher dimensional space where the data is separable.

What is a Hyper Plane

In two dimensions, a hyper plane is defined by the equation:

$$W_1 X_1 + W_2 X_2 + b = 0$$

This is nothing but equation of line.

The above equation can be easily extended to the p-dimensional setting:

$$W_1X_1 + W_2X_2 + \dots + W_pX_p + b = 0$$

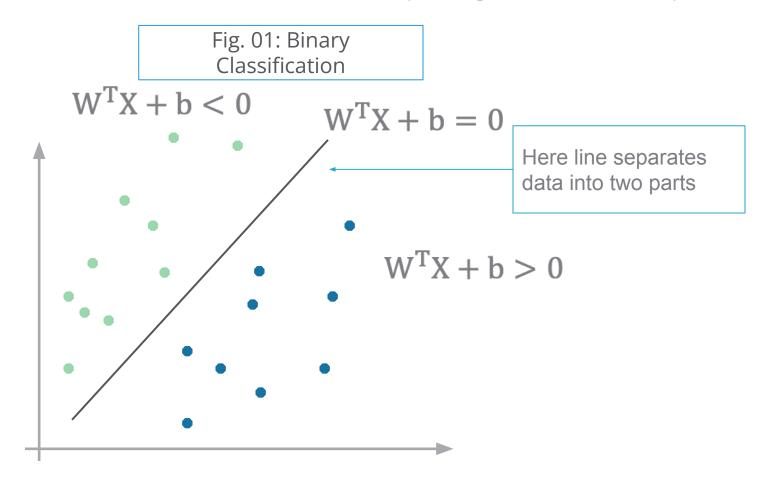
In short,

$$\mathbf{W}^{\mathrm{T}}\mathbf{X} + \mathbf{b} = \mathbf{0}$$

In p > 3 dimensions, it can be hard to visualize a hyper planes.

Separating a Hyper Plane

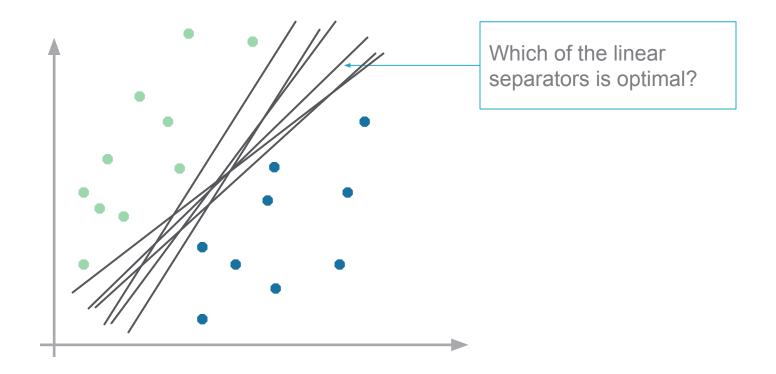
• Binary classification can be viewed as the task of separating classes in feature space:



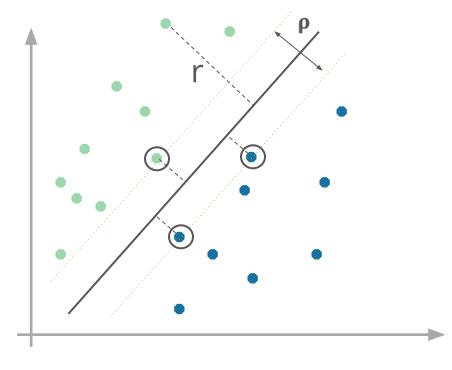
Linear Separators

The objective in SVM is to find optimum separator

Fig. 02: Linear Separators



Classification Margin



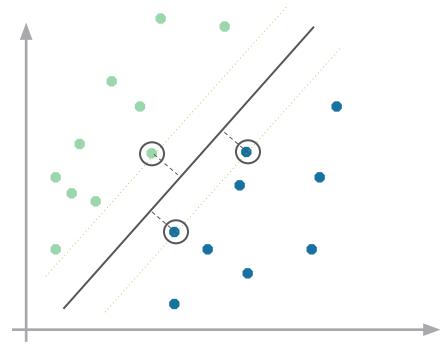
Distance from case x_i to the separator is

$$r = \frac{w^T x_i + b}{\parallel w \parallel}$$

Here | w | is length of a vector given by sqrt(sum(W^2))

- Cases closest to the hyper plane are Support Vectors
- Margin ρ of the separator is the distance between support vectors

Maximum Margin Classification



- The objective is now to maximize the margin ρ of the separator
- The focus is on 'Support Vectors'
- Other cases are not considered in the algorithm

Mathematical Approach to Linear SVM

Let training set be separated by a hyper plane with margin ρ . Then for each training observation

$$w^{T}x_{i} + b \leq -\rho/2 \quad \text{if } y_{i} = -1$$

$$w^{T}x_{i} + b \geq \rho/2 \quad \text{if } y_{i} = 1$$

$$y_{i}(w^{T}x_{i} + b) \geq \rho/2$$

For every support vector \mathbf{x}_s the above inequality is an equality

After rescaling w and b by $\rho/2$ in the equality, we obtain that distance between each x_s and the hyper plane is

$$r = \frac{y_i(w^Tx_s + b)}{\|w\|} = \frac{1}{\|w\|}$$

Margin can be expressed through (rescaled) w and b as:

$$\rho = 2r = \frac{2}{\parallel w \parallel}$$

Mathematical Approach to Linear SVM

Quadratic Optimisation problem is:

Find w and b such that

$$\rho = \frac{2}{\|\mathbf{w}\|} \text{ is maximised}$$
 and

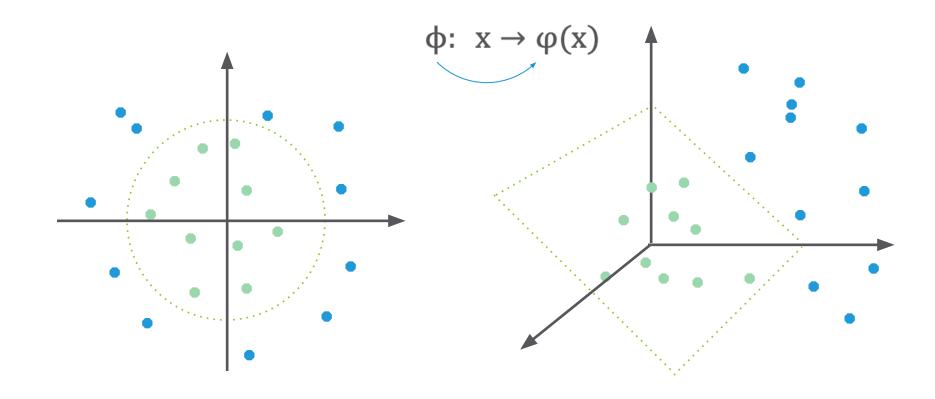
$$y_i(w^Tx_i + b) \ge 1$$

which can be reformulated as:

Find w and b such that $\phi(w) = w^T w \text{ is minimised}$ and $y_i(w^T x_i + b) \ge 1$

Non-Linear SVMs – Feature Spaces

General idea: The original feature space can always be mapped to some higher-dimensional feature space where the training set is separable



The "Kernel Trick"

The linear classifier relies on inner product between vectors

$$K(x_i, x_j) = x_i^T x_j$$

If every data point is mapped into high-dimensional space via some transformation $\phi\colon\thinspace x\to\phi(x)$ then the inner product becomes

$$K(x_i, x_j) = \varphi(x_i)^T \varphi(x_j)$$

A kernel function is a function that is equivalent to an inner product in some feature space

The "Kernel Trick"

Example:

2-dimensional vector $\mathbf{x} = [\mathbf{x}_1 \ \mathbf{x}_2];$

Let
$$K(x_i, x_j) = (1 + x_i^T x_j)^2$$

Need to show that $K(x_i, x_i) = \phi(x_i)^T \phi(x_i)$:

$$\begin{split} &K\big(x_i,x_j\big) = (1+x_i{}^Tx_j)^2 \\ &= 1+x_{i1}{}^2x_{j1}{}^2+2x_{i1}x_{j1}x_{i2}x_{j2}+x_{i2}{}^2x_{j2}{}^2+2x_{i1}x_{j1}+2x_{i2}x_{j2} \\ &= [1 \quad x_{i1}{}^2\sqrt{2}x_{i1}x_{i2} \quad x_{i2}{}^2\sqrt{2}x_{i1} \quad \sqrt{2}x_{i2}] \ T \ [1 \\ &x_{j1}{}^2\sqrt{2}x_{j1}x_{j2} \quad x_{j2}{}^2\sqrt{2}x_{j1} \quad \sqrt{2}x_{j2}] \\ &= \phi(x_i)^T\phi(x_j) \ \text{where} \ \phi(x) = [1 \ x_1{}^2\sqrt{2}x_1x_2 \ x_2{}^2\sqrt{2}x_1\sqrt{2}x_2] \end{split}$$

Thus, a kernel function implicitly maps data to a high-dimensional space (Without the need to compute each $\phi(x)$ explicitly)

Examples of Kernel Functions

Linear

$$K(x_i, x_j) = x_i^T x_j$$

Mapping φ

 $x \to \phi(x)$ where $\phi(x)$ is x itself

Polynomial of power ρ

$$K(x_i, x_j) = (1 + x_i^T x_j)^{\rho}$$

Gaussian (Radial basis function)

$$K(x_i, x_j) = e^{-\frac{\|x_i - x_j\|^2}{2\sigma^2}}$$

Case Study – Predicting Loan Defaulters

Background

• The bank possesses demographic and transactional data of its loan customers. If the bank has a robust model to predict defaulters it can undertake better resource allocation.

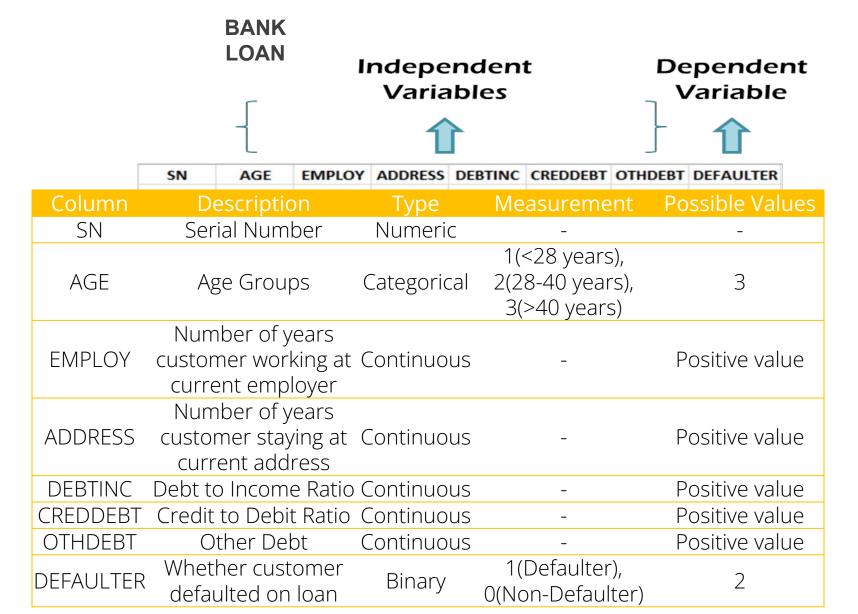
Objective

 To predict whether the customer applying for the loan will be a defaulter

Available Information

- Sample size is 700
- Age group, Years at current address, Years at current employer, Debt to Income Ratio, Credit Card Debts, Other Debts are the independent variables
- **Defaulter** (=1 if defaulter, 0 otherwise) is the dependent variable

Data Snapshot



SVM in Python

Importing the Libraries

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from sklearn.model_selection import train_test_split
from sklearn.svm import SVC

from sklearn.metrics import confusion_matrix, f1_score,
precision_score, recall_score, accuracy_score,
roc_curve, roc_auc_score,auc
```

Importing and Readying the Data

```
bankloan = pd.read_csv("BANK LOAN.csv")

bankloan['AGE'] = pd.Categorical(bankloan['AGE'])

bankloan.info()
bankloan1 = bankloan.drop(['SN','AGE'], axis = 1)
pd.Categorical()
changes age from an integer to a factor variable.
```

info() is used to check if the conversion to category has taken place and if all other variable formats are appropriate, before moving to SVM modeling.

SVM in Python

Output

Creating Train and Test Data Sets

SVM in Python

Model fitting svclassifier = SVC(kernel='linear',probability=True) svclassifier.fit(X train, y train) **svc()** trains a support vector machine. **kernel=** specifies the kernel type to be used in the algorithm'(linear', 'poly', 'rbf', 'sigmoid', 'precomputed'). # Output SVC(kernel='linear', probability=True) # Predicted Probabilities predprob test = svclassifier.predict_proba(X test) **predict proba()** returns predicted probabilities for the test data.

Predictions Based on SVM

Custom Cutoff Value for Prediction Labels

Output

Confusion Matrix and Area Under ROC Curve

```
# Confusion Matrix
confusion matrix(y test, pred_test, labels=[0, 1])
array([[118, 36],
                                            accuracy_score() = number of correct
                                            predictions out of total predictions
        [ 13, 43]])
                                            precision_score() = true positives /
accuracy_score(y test, pred test)
                                            (true positives + false positives)
0.7666666666666667
                                            recall_score() also known as
precision_score(y test, pred test)
                                            'Sensitivity' = true positives / (true
0.5443037974683544
                                            positives + false negatives)
recall score(y test, pred test)
0.7678571428571429
```

Area Under ROC Curve
auc = roc_auc_score(y_test, predprob_test[:,1])
print('AUC: %.3f' % auc)
AUC: 0.847



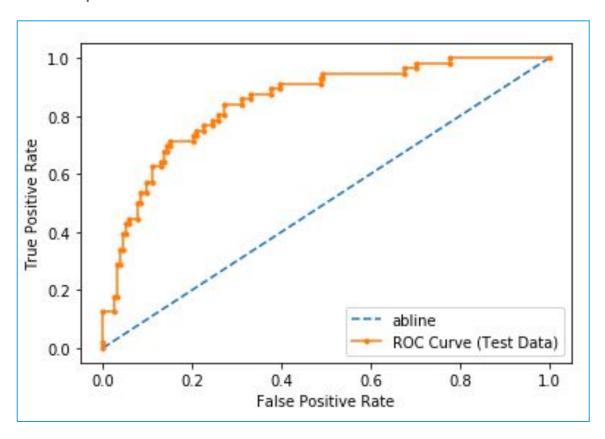
ROC Curve and Area Under ROC Curve

ROC Curve

```
fpr, tpr, thresholds = roc_curve(y test, predprob test[:,1])
#Compute AUC using 'auc' function
roc auc = auc(fpr, tpr)
#Plot the curve for model
plt.plot(fpr, tpr, color='darkorange', lw=2, label='ROC curve (area =
%0.2f)' % roc auc)
plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--')
plt.xlim([0.0, 1.0])
plt.ylim([0.0, 1.05])
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Receiver operating characteristic example')
plt.legend(loc="lower right")
plt.show()
```

ROC Curve and Area Under ROC Curve

Output:



Quick Recap

In this session, we learnt about **Support Vector Machines**:

Support Vector Machines

- SVMs find a hyper plane which separates the d-dimensional data perfectly into its classes
- Since training data is often not linearly separable, SVM's introduce the notion of a "Kernel-induced Feature Space" which casts the data into a higher dimensional space where the data is separable

SVM in Python

- Library "sklearn.svm" has SVC() that trains a support vector machine
- The function takes arguments to specify whether SVC()
 is to be used for classification or regression; if
 probabilities are to be returned and which kernel to
 use for training and predicting