Statistical Inference

Non-Parametric Tests II

Contents

Kruskal Wallis test

- The Kruskal Wallis test is considered a non-parametric alternative to one way analysis of variance (ANOVA).

- The Kruskal Wallis test is used to compare differences between more than two independent groups when the dependent variable is either ordinal or continuous, but not normally distributed.

- H0: K samples come from the same population
  H1: Not H0.

Kruskal Wallis test procedure

- Combine all the observations from k samples into a single sample of size n and arrange them in ascending order .
- Assign ranks to them from smallest to largest as 1 to n. if there is a tie at two or more places, each observation is given the mean of the ranks for which it is tied.
- The ranks assigned to observations in each of the k groups are added separately to give k rank sums.
- The test statistic is

$$H = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_j^2}{n_j} - 3(n+1)$$

$$n_j = number\ of\ observations\ in\ j^{th}\ sample$$

$$n = number\ of\ observations\ in\ the\ combined\ sample$$

$$R_j = sum\ of\ the\ ranks\ in\ the\ j^{th}\ sample.$$

- H follows Chi Square Distribution with k-1 df

Case Study - 1

Background

The data consists of the aptitude scores of 3 groups of employees.

Objective

To check whether there is difference in scores among the three groups.

Sample Size

Sample size: 20
Variables: aptscore, Group

Data Snapshot

Kruskal Wallis Test

Variables

Observations

aptscore	Group
11.5	GroupI
12.1	GroupI
14.5	GroupI
16.1	GroupI
19.8	GroupI
23.6	GroupI
11	GroupII
12.2	GroupII
15	GroupII
15.7	GroupII
18.5	GroupII

Combined sample arranged from smallest to largest

Ordered sample	Group	Rank
11	GroupII	1
11.1	GroupIII	2
11.5	GroupI	3
12.1	GroupI	4
12.2	GroupII	5
14.5	GroupI	6
15	GroupII	7
15.7	GroupII	8
16.1	GroupI	9
18.5	GroupII	10
19.1	GroupIII	11
19.8	GroupI	12
21	GroupIII	13
22.2	GroupIII	14

Rank ordered sample

Columns	Description	Type	Measurement	Possible values
aptscore	Aptitude score	Numeric	-	Positive values
Group	Group of employees	character	GroupI, GroupII, GroupIII	3

Kruskal Wallis test

Testing distribution of more than two samples

Objective	To test the **null hypothesis** that all the samples came from same population

Null Hypothesis (H_0): The three samples are from the same population
Alternate Hypothesis (H_1): The three samples do not come from the same population

Test Statistic	$$H = \frac{12}{n(n+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(n+1)$$ n_j = number of observations in j^{th} sample n = number of observations in the combined sample R_j = sum of the ranks in the j^{th} sample.
Decision Criteria	Reject the null hypothesis if the p-value < 0.05

Kruskal Wallis test example

Calculations :

	Value
Sample size	$n_1 = 6$ $n_2 = 7$ $n_3 = 7$
R_1	50
R_2	68
R_3	92
H	2.2309
p-value	0.3278

Kruskal Wallis test in R

```
# Import the CSV file
data<-read.csv("Kruskal Wallis Test.csv",header=TRUE)

# Kruskal walis test
kruskal.test(formula=aptscore~Group,data=data)
```

- *kruskal.test performs the Kruskal waliss test on the data.*
- *aptscore is the analysis variable.*
- *Group is the factor variable.*

Kruskal Walis test in **R**

```
# Output:
```

```
        Kruskal-Wallis rank sum test

data:  aptscore by Group
Kruskal-Wallis chi-squared = 2.2309, df = 2, p-value = 0.3278
```

Interpretation :

- *Since the p-value is >0.05, do not reject H0. Aptitude score is the same for all three groups of employees.*

Chi-square test of Association

- The chi-square test for independence, also called as Pearson's chi-square test or the chi-square test of association, is used to test if there is a relationship between two categorical variables.

- The two categorical variables can be nominal or ordinal.

- H0: Two attributes are independent (not associated)
  H1: Not H0.

Chi-square test procedure

- Assume that there are ‘r’ categories of attribute A and ‘c’ categories of attribute B. Therefore, we have a cross table of r*c (r rows and c columns).
- Let Ri be the total of the ith row and Cj be the total of the jth column.
- Observed frequencies are calculated from the data.
  Oij: Observed frequency in ith row and jth column.
- Expected frequencies are given by Eij = (Ri * Cj)/ n where n is total sample size.
  Expected frequencies are computed under the null hypothesis.
- Test statistic

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where Oij are the observed frequencies in the ith row and jth column.
Eij are the expected frequencies in the ith row and jth column.

- χ^2 follows a Chi-Square Distribution with (r-1)(c-1) degrees of freedom.

Case Study - 2

Background

The data consists of information regarding the Performance & Recruitment Source of employees.

Objective

To check whether Performance & Source of Recruitment are associated.

Sample Size

Sample size: 870
Variables: sn, performance, source

Data Snapshot

chi square test of association

Variables

Observations

sn	performan	source
1	Excellent	Internal
2	Excellent	Internal
3	Excellent	Internal
4	Excellent	Internal
101	Excellent	Campus
102	Excellent	Campus
251	Excellent	Jobportal
252	Excellent	Jobportal
253	Excellent	Jobportal
254	Excellent	Jobportal
291	Good	Internal
292	Good	Internal
293	Good	Internal
491	Good	Jobportal
492	Good	Jobportal
493	Good	Jobportal
591	Poor	Internal

Columns	Description	Type	Measurement	Possible values
sn	Serial number	Numeric	-	-
performance	Employee performance	character	Excellent, Good,Poor	3
source	Source of recruitment	Character	Campus, Internal, Jobportal	3

Get the observed frequency (count) table from this data.

Chi-square test of Association

Testing association between two categorical variables

Objective	To test the **null hypothesis** that two categorical variables are **independent**

Null Hypothesis (H_0): performance and source are not associated

Alternate Hypothesis (H_1): performance and source are associated

Test Statistic	$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$ **Oij = observed frequencies in the ith and jth column.** **Eij = expected frequencies in the ith row and jth column.**
Decision Criteria	Reject the null hypothesis if the p-value < 0.05

Chi-square test example

Observed Frequency table

	Recruitment Source			
Performance	Campus	Internal	Jobportal	Total
Excellent	150	100	40	290
Good	100	100	100	300
Poor	80	50	150	280
Total	330	250	290	870

Expected Frequency table

	Recruitment Source			
Performance	Campus	Internal	Jobportal	Total
Excellent	=(330*290)/870	83	97	290
Good	114	=(250*300)/870	100	300
Poor	106	80	=(290*280)/870	280
Total	330	250	290	870

	Value
r	3
c	3
χ^2	107.3786

Chi-Square test in R

```
# Import the CSV file
data<-read.csv("chi square test of association.csv", header=TRUE)
```

```
# Install and use the package "gmodels"
install.packages("gmodels")
library(gmodels)
```

"gmodels" is needed for the contingency table. The table displays frequencies, relative frequencies of two categorical variables.

```
# Chi-square test of association
CrossTable(data$performance, data$source, chisq=TRUE)
```

CrossTable function performs Chi-square test of association when chisq=TRUE.

Chi-Square test in R

```
# Output:
```

```
   Cell Contents
|-------------------------|
|                       N |
| Chi-square contribution |
|           N / Row Total |
|           N / Col Total |
|         N / Table Total |
|-------------------------|


Total Observations in Table:  870
```

```
                 | data$source
data$performance |    Campus |  Internal | Jobportal | Row Total |
-----------------|-----------|-----------|-----------|-----------|
       Excellent |       150 |       100 |        40 |       290 |
                 |    14.545 |     3.333 |    33.218 |           |
                 |     0.517 |     0.345 |     0.138 |     0.333 |
                 |     0.455 |     0.400 |     0.138 |           |
                 |     0.172 |     0.115 |     0.046 |           |
-----------------|-----------|-----------|-----------|-----------|
            Good |       100 |       100 |       100 |       300 |
                 |     1.672 |     2.207 |     0.000 |           |
                 |     0.333 |     0.333 |     0.333 |     0.345 |
                 |     0.303 |     0.400 |     0.345 |           |
                 |     0.115 |     0.115 |     0.115 |           |
-----------------|-----------|-----------|-----------|-----------|
            Poor |        80 |        50 |       150 |       280 |
                 |     6.467 |    11.531 |    34.405 |           |
                 |     0.286 |     0.179 |     0.536 |     0.322 |
                 |     0.242 |     0.200 |     0.517 |           |
                 |     0.092 |     0.057 |     0.172 |           |
-----------------|-----------|-----------|-----------|-----------|
    Column Total |       330 |       250 |       290 |       870 |
                 |     0.379 |     0.287 |     0.333 |           |
-----------------|-----------|-----------|-----------|-----------|


Statistics for All Table Factors


Pearson's Chi-squared test
------------------------------------------------------------
Chi^2 =  107.3786     d.f. =  4     p =  2.635987e-22
```

Interpretation :

- *Since the p-value is <0.05, reject H0. Recruitment source and employee performance are associated.*

Quick Recap

Kruskal Wallis test	• Nonparametric alternative to one way ANOVA.
Chi-Square test	• Also called Pearson's chi-square test or the chi-square test of association. It is used to test if there is a relationship between two categorical variables (nominal or ordinal).