Statistical Inference

Non Parametric Tests - 1

Contents

Non-parametric statistical test

- Tests based on t and F distributions assume that populations are normally distributed.
- A large body of statistical methods is available which do not make assumptions about the nature of the distribution(e.g. normality)
- These testing procedures are termed nonparametric tests or distribution-free tests.
- If the underlying assumptions of the parametric test are met, then a parametric test will be more powerful than a non-parametric test.

Note : Always check for normality assumptions using the test explained earlier and then decide which hypothesis test is more accurate, depending upon the problem statement.

Mann-Whitney test

- The Mann-Whitney test is considered as a non-parametric alternative to t test for independent samples.
- The Mann-Whitney U test is used to compare differences between two independent groups when the dependent variable is either ordinal or continuous, but not normally distributed.
- The test is equivalent to Wilcoxon rank-sum test (WRS).
- The null hypothesis is that the distributions of both groups are identical, so that there is a 50% probability that an observation randomly selected from one population exceeds an observation randomly selected from another population.

Mann-Whitney test

- **Steps to follow :**
  - Combine the two samples.
  - Rank all the observations from smallest to largest.
  - Keep track of the group to which each observation belongs.

- Tied observations (observations with same value) are assigned a rank equal to the mean of the rank positions for which they are tied.

- The test statistic is

$$U = T - \frac{m(m+1)}{2}$$

Where T is the sum of the ranks of the first sample in the combined ordered sample, m and n are sample sizes.

$$E(U) = \frac{mn}{2} \qquad V(U) = \frac{mn(m+n+1)}{12}$$

- Standardized U is assumed to follow normal distribution.

- Compare the p-value with the level of significance & conclude.

Case Study - 1

Background

Data consists of the aptitude scores of 2 groups of employees.

Objective

To compare aptitude scores of the two groups and test if they come from the same population.

Sample Size

Sample size: 13
Variables: aptscore, Group

Data Snapshot

Mann Whitney test

Variables

Observations

aptscore	Group
11.5	G1
23.6	G1
12.1	G1
14.5	G1
16.1	G1
19.8	G1
11	G2
25	G2

Combine the two sample from smallest to largest

Ordered Sample	Group	Rank
11	GroupII	1
11.5	GroupI	2
12.1	GroupI	3
12.2	GroupII	4
14.5	GroupI	5
15	GroupII	6
15.7	GroupII	7
16.1	GroupI	8
18.5	GroupII	9

Rank ordered samples

Columns	Description	Type	Measurement	Possible values
aptscore	Aptitude score	Numeric	-	Positive values
Group	group of employees	character	G1,G2	2

- T is the sum of the ranks of the first sample in the combined ordered sample. m and n are sample sizes.
  T=39 , m=6, n= 7
  U=18 , E(U)=21 ,V(U)= 49

Mann-Whitney test

Testing distribution of two samples

Objective	To test the **null hypothesis** that **the median** of both samples is the same

Null Hypothesis (H_0): The two samples come from the same population

Alternate Hypothesis (H_1): The two samples do not come from the same population

Test Statistic	$U = T - \frac{m(m+1)}{2}$	**Where T is the sum of the ranks of first sample in the combined ordered sample, m and n are sample sizes**
Decision Criteria	Reject the null hypothesis if the p-value < 0.05	

Mann-Whitney test in R

```
# Import the CSV file
data<-read.csv("Mann Whitney test.csv", header=TRUE)
# Mann-Whitney test
wilcox.test(formula=aptscore~Group,data=data)
```

- *The Mann-whitney test is also known as the Wilcoxon Rank Sum test.*
- *The wilcox.test function gives the value of U(as W) and p-value.*
- *aptscore is the analysis variable.*
- *group is the factor.*

Mann-Whitney test in R

```
# Output:

        Wilcoxon rank sum test

data:  aptscore by Group
W = 18, p-value = 0.7308
alternative hypothesis: true location shift is not equal to 0
```

Interpretation :

- *Since p-value is >0.05, do not reject H0. aptitude score is same for both the groups i.e. samples come from the same population.*

Wilcoxon Signed Rank Test for paired data

- The Wilcoxon Signed Rank test is considered as a nonparametric alternative to paired t test .

- The Wilcoxon Signed Rank test is used to compare differences between two related or paired groups when the variable is either ordinal or continuous, but not normally distributed.

- H0: The median of difference in the population is zero
  H1: Not H0.

Wilcoxon Signed Rank Test for paired data

- **Steps to follow :**
  - Define Di = Xi- Yi, which are the differences between two values for each pair.
  - Obtain |Di|, which are absolute values of differences.
  - Rank all |Di| from smallest to largest.
  - Define Ri = rank of |Di|.
  - Obtain ‘W’, which is the sum of the ranks associated with positive Di.

- The test statistic is W, which is the sum of the ranks associated with positive Di. n is the sample size.

$$E(W) = \frac{n(n+1)}{4}$$

$$V(W) = \frac{n(n+1)(2n+1)}{24}$$

- Standardized W is assumed to follow normal distribution.

- Compare the p-value with the level of significance & conclude.

Case Study - 2

Background

A company organized a training program and the scores before and after training were recorded.

Objective

To test whether the median of paired samples is same.

Sample Size

Sample size: 12
Variables: Before, After

Data Snapshot

- A company organized a training program and the scores before and after training were recorded.

Variables

Observations

Before	After
58	74
52	65
61	60
48	45
50	58
39	53

Combine the two sample to get Di

Di=Before-After	Abs(Di)	Rank (Di)
-16	16	12
-13	13	10
1	**1**	**1**
3	**3**	**3**
-8	8	5
-14	14	11
-9	9	6

Rank associated with positive Di

Columns	Description	Type	Measurement	Possible values
Before	Score before training	Numeric	-	Positive values
After	Score after training	Numeric	-	Positive values

- W is sum of the ranks associated with positive Di. n is sample size.
  W=4 , n= 12
  E(W)=39 , V(W)= 162.5

Wilcoxon Signed Rank Test for paired data

Testing distribution of paired samples

Objective	To test the **null hypothesis** that **median** of paired samples is same.

Null Hypothesis (H_0): **The median of the difference in the population is zero**
Alternate Hypothesis (H_1): **The median of the difference in the population is less than zero.**

Test Statistic	**w=sum of the ranks associated with positive Di.** **Di = Xi- Yi which are the differences between data and specified median value.**
Decision Criteria	Reject the null hypothesis if p-value < 0.05

Wilcoxon Signed Rank Test for paired data in R

```
# Import the CSV file
data<-read.csv("Wilcoxon Signed Rank test for paired  data.csv",
               header=TRUE)
# Wilcoxon Signed Rank test
wilcox.test(data$Before, data$After, paired=TRUE,
            alternative = "less")
```

- *wilcox.test function gives the value of W (as V) and p-value.*
- *wilcox.test function performs Wilcox signed rank test for paired data when paired=TRUE is specified.*
- *Before and After are the paired observations.*
- *alternative=less specifies one tail test .since, score will be more if training program is effective.*

Wilcoxon Signed Rank Test for paired data in R

```
# Output:

        Wilcoxon signed rank test

data:  data$Before and data$After
V = 4, p-value = 0.001709
alternative hypothesis: true location shift is less than 0
```

Interpretation :

- *Since the p-value is <0.05, reject H0. The training program is effective as the score after training is more than before training.*

Quick Recap

Non Parametric Test	• Non parametric tests are performed if the normality assumption is not satisfied.
Mann-Whitney test	• Nonparametric alternative to the t test for independent samples.
Wilcoxon Signed Rank test	• Nonparametric alternative to the t test for paired samples.