Introduction to Predictive Modelling

Contents

What is Predictive modelling?

Statistical model created **to best predict**

the outcome

OR

probability of an outcome

Models **developed using**

Historical data

OR

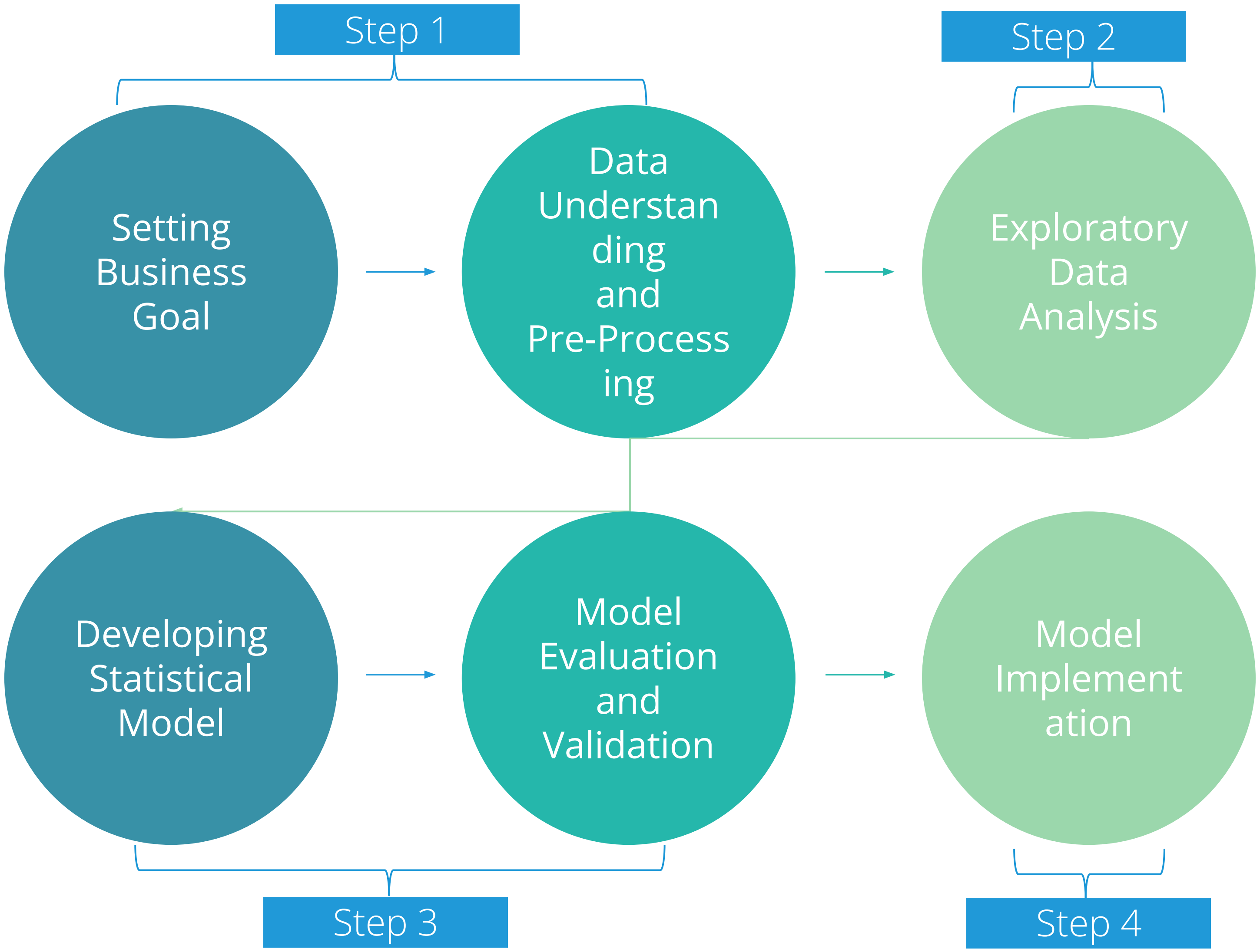
Purposely collected data

Predictive analytics is used in **financial services, insurance, telecommunications, retail, travel, healthcare, pharmaceuticals, sports and several other fields**

Predictive modelling – Important Techniques

- Regression
  - Linear Models
    - Simple Linear Regression
    - Multiple Linear Regression
  - Non Linear Models
    - Binary Logistic Regression
    - Ordinal Logistic Regression
    - Poisson Regression
    - Multinomial Logistic Regression
    - Cox Regression
- Time Series
  - ARIMA Models (Seasonal / Non Seasonal)
  - ARCH and GARCH Models
  - Time Series Regression
  - VAR Models

Predictive modelling – General Approach



Step 1 – Data Understanding and Pre-processing

Data Understanding and Pre Processing

- Data Understanding
  – Understanding data dimension, variable types, variable relationships
- Converting raw data to usable data

  Data cleaning by checking for and handling:
  - Missing values
  - Inconsistencies
- Transforming variables
- Feature engineering
  - Using domain knowledge to create new features or variables which can be used in the model

Pre-Processing:

- Grouping / Factoring / Segmentation / Reduction

Step 2 – Exploratory Data Analysis

Exploratory Data Analysis

- Performing exploratory data analysis using:
  - Frequency tables
  - Cross tables
  - Descriptive statistics
  - Visualizations
  - Correlation matrix

Step 3 – Model Identification , Selection and Validation

Model Selection and Validation

- Model identification and selection is based on:
  - Study objective
  - Type of dependent variable
  - Checking different statistics / decision criteria based on models (Eg. p-value, R^2, AIC, etc.) i.e diagnostic checks
  - Automatic search procedures
- Cross Validation
  - Splitting data into training data and test data
  - Checking predictive ability of model on new data
  - Comparison of results with theoretical expectations, empirical results and simulation results

Step 4 – Model Implementation

Model Implementation

- Drawing inferences from the model results by :
  - Building equations using only the coefficients of significant variables
  - Mapping the model chosen with the existing system
- Fitting the model on new data and generating predictions

- Observing values of Predictors :
  - In a spreadsheet or
  - Web application or any other user interface or
  - Integrating the current systems

Get an Edge!

Any predictive model is developed on historical data. Sample size and data dimension are key determinants for a good model

- If sample size is too small, model may not give good insight about the relationship among the variables.
- Also, if the data has large number of variables (columns) but few observations (rows), we are essentially trying to learn too much from a small sample. Results from models developed using such data will be erratic. **The rule of thumb for appropriate sample dimension is that observations should be 10 times the number of variables.** For instance, if we wish to study the relationship of 8 variables, then we must have more than 80 observations.

Quick Recap

In this session, we gained knowledge on the concept of **predictive modelling** :

Predictive modelling	• Used to predict the outcome or the probability of an outcome • Models developed using historical data or purposely collected data
Important Statistical models	• Regression Models- Linear and non-linear • Time Series Models
General Approach in Predictive modelling	• Data understanding and pre-processing • Exploratory data analysis • Model selection and validation • Model Implementation

Introduction to Multiple

Linear Regression - I

Content

1. Introduction to MLR
2. Statistical Model of MLR
3. Case Study
4. Model Fitting
5. Parameter Estimation – Ordinary Least Squares Method
6. Interpretation of Partial Regression Coefficients

Multiple Linear Regression

- Multiple linear regression is used to explain the relationship between one continuous dependent variable and two or more independent variables.
- The independent variables can be continuous or categorical.
- Multiple Linear Regression is used when we want to predict the value of a variable based on the values of two or more other variables.
- The variable we want to predict is called the dependent variable
- The variables used to predict the value of dependent variable are called independent variables (or explanatory variables/predictors).
- Multiple linear regression requires the model to be linear in the parameters.
- Example: The price house in USD can be a dependent variable and sze of house, location of house , air quality index in the area, distance from airport etc. can be independent variables.

Statistical Model

$$Y = b_0 + b_1X_1 + b_2X_2 + \ ... \ + b_pX_p + e$$

where,

Y	:	Dependent Variable
$X_1, X_2, \ldots, X_p$	:	Independent Variables
$b_0, b_1, \ldots, b_p$	:	Parameters of Model
e	:	Random Error Component

- Independent variables can either be **Continuous or Categorical**
- Multiple linear regression **requires the model to be linear in the parameters**
- Parameters of the model are estimated by Least Square Method.
- The **least squares (LS)** criterion states that the **sum of the squares of errors** (or residuals) **is minimum**.
- Mathematically, the following quantity is minimized to estimate parameters using the least square method.
- Error ss= $\Sigma (Y_i - \hat{Y}_i)^2$

Case Study – Modeling Job Performance Index

Background

- A company conducts different written tests before recruiting employees. The company wishes to see if the scores of these tests have any relation with post-recruitment performance of those employees.

Objective

- To predict employees' job performance index after a probationary period, based on test scores conducted at the time of recruitment

Available Information

- Sample size is 33
- Independent Variables: Test scores conducted before recruitment on the basis of four criteria – Aptitude, Test of Language, Technical Knowledge, General Information
- Dependent Variable: Job Performance Index, calculated after an employee finishes a probationary period (6 months)

Data Snapshot

Performance Index

Dependent Variable

4 Independent Variables

Observations

empid	jpi	aptitude	tol	technical	general
1	45.52	43.83	55.92	51.82	43.58
2	40.1	32.71	32.56	51.49	51.03
3	50.61	56.64	54.84	52.29	52.47

Columns	Description	Type	Measurement	Possible values
empid	Employee ID	integer	-	-
jpi	Job performance Index	numeric	-	positive values
aptitude	Aptitude score	numeric	-	positive values
tol	Test of Language	numeric	-	positive values
technical	Technical Knowledge	numeric	-	positive values
general	General Information	numeric	-	positive values

Graphical Representation of Data

- It is always recommended to have a general look at your data and behavior of all variables before moving to modelling.

- This helps you to make intuitive inferences about the data, which can be statistically validated by your final model.

- The simplest way of doing this is to create a scatter plot matrix, which will give bivariate relationships between variables.

```
#Importing the Data
```

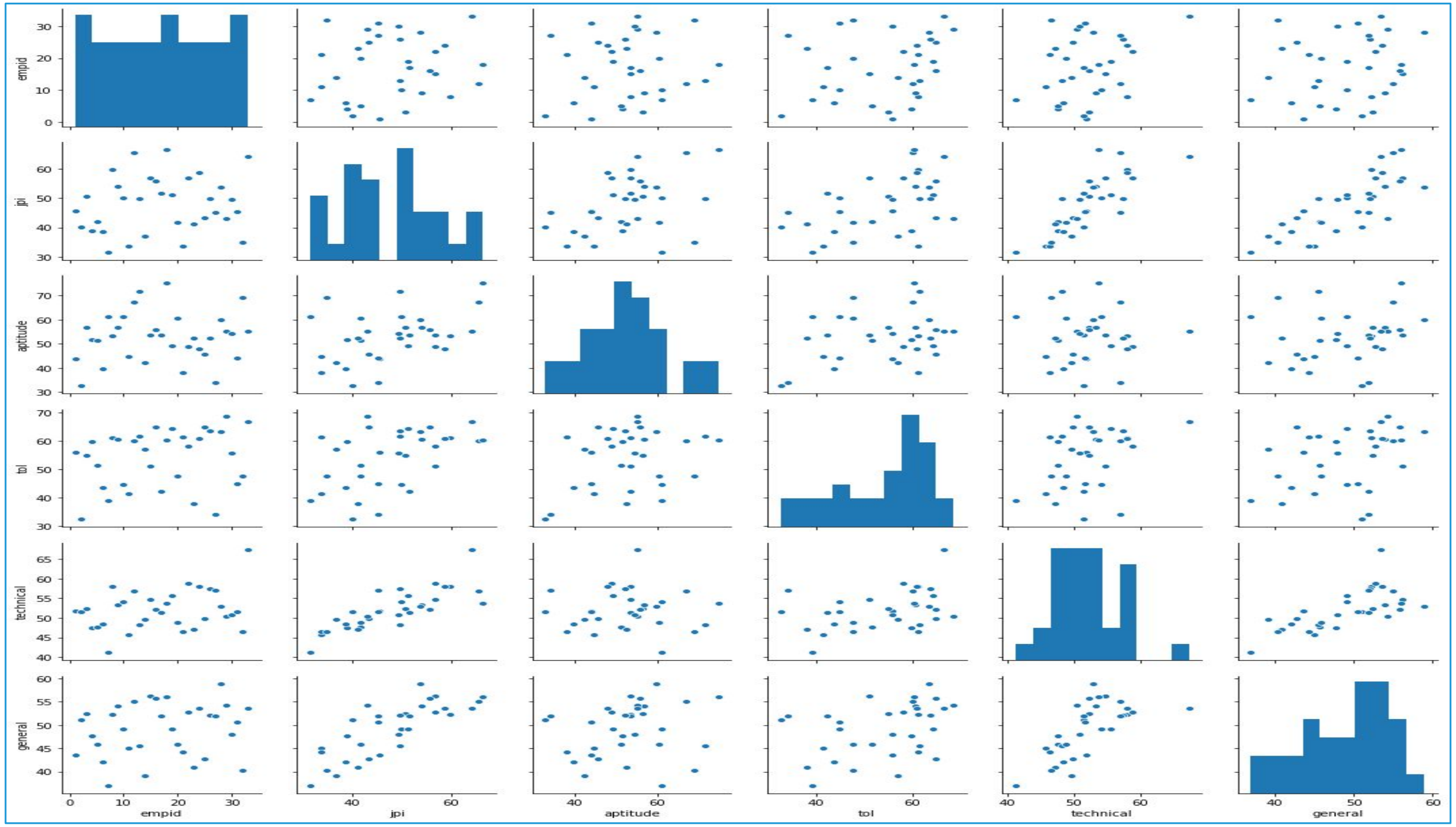
```
import pandas as pd
perindex = pd.read_csv("Performance Index.csv")
```

```
#Graphical Representation of the Data
```

```
import seaborn as sns
sns.pairplot(perindex)
```

Scatter Plot Matrix

The `pairplot()` function in the **`seaborn library`** gives a scatter plot matrix and distribution of all variables using histograms.



Model for the Case Study

Dependent Variable

Independent Variables

$$jpi = b_0 + b_1(aptitude) + b_2(tol) + b_3(technical) + b_4(general) + e$$

Parameter Estimates

Intercept / Constant

Error Term

Parameter Estimation using Least Square Method

Parameters	Coefficients
Intercept	-54.2822
aptitude	0.3236
tol	0.0334
technical	1.0955
general	0.5368

$$E(jpi) = -54.2822 + 0.3236\,(aptitude) + 0.0334\,(tol) + 1.0955\,(technical) + 0.5368\,(general)$$

Parameter Estimation Using ols() function in Python

#Model Fit

```
import statsmodels.formula.api as smf

jpimodel=smf.ols('jpi ~ tol + aptitude + technical +general', data=perindex).fit()

jpimodel.params
```

- *ols() fits a linear regression.*
- *~ separates dependent and independent variables*
- *Left hand side of tilde(~) represents the dependent variable and right-hand side shows independent variables*
- *+ separates multiple independent variables.*

#Output

```
Intercept    -54.282247
tol            0.033372
aptitude       0.323562
technical      1.095467
general        0.536834
dtype: float64
```

Interpretation :

- *jpimodel.params gives the model parameters.*
- *Signs of each parameter represent their relationship with the dependent variable.*

Interpretation of Partial Regression Coefficients

- For every unit increase in the independent variable (X), the expected value of the dependent variable (Y) will change by the corresponding parameter estimate (b), keeping all other variables constant

Parameters	Coefficients
Intercept	-54.2822
aptitude	0.3236
tol	0.0334
technical	1.0955
general	0.5368

- From the parameter estimates table, we observe that the parameter estimate for Aptitude Test is 0.3236

  We can infer that for one unit increase in aptitude test score, the expected value of job performance index will increase by 0.3236 units

Quick Recap

In this session we have covered the **basics of multiple linear regression using Python**.

Follow these simple steps to carry out your first analysis:

Understand the Data	• Ensure the data is complete and consistent • Identify dependent and independent variables
Data Visualization	• **`pairplot()`** function from **`seaborn`** library gives scatter plot matrix
Fit a Model	• **ols()** function from library **statsmodels** fits a linear regression model

Introduction to

Multiple Linear Regression II

Content

1. Global Testing – ANOVA
2. Individual Testing – t Test
3. Measure of Goodness of Fit – R Squared
4. Fitted values and Residuals
5. Predictions for New Dataset
6. Standardizing Coefficients

Partitioning Total Variance

- Total Variation in dependent variables Y can be split into two: Explained and Unexplained.
- Explained variation is summation of the squared difference between estimated values of Y and the mean value of Y. Whereas, the sum of the squared difference between the actual values of Y and estimated values is considered to be unexplained.

Total Variation

$$\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

Explained Variation

$$\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$$

Unexplained Variation

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

Global Testing – Using F Test

Testing whether at least one variable is significant

Objective	To test the **null hypothesis** that **all the parameters are simultaneously equal to zero**

Null Hypothesis (H_0): $b_1 = b_2 = \ldots = b_p = 0$

Alternate Hypothesis (H_1): At least one coefficient is not zero

Test Statistic	$F = \dfrac{\text{Mean Square of Regression}}{\text{Mean Square of Error}}$
Decision Criteria	Reject the null hypothesis if p-value < 0.05

Individual Testing – Using t Test

Testing which variable is significant

Objective	To test the **null hypothesis** that **parameters of individual variables are equal to zero**

Null Hypothesis (H_0): $b_i = 0$

Alternate Hypothesis (H_1): $b_i \neq 0$

where $i = 1,2,\ldots,p$

Test Statistic	$t = \dfrac{\text{Estimated } b_i}{\text{Standard Error of Estimated } b_i}$
Decision Criteria	Reject the null hypothesis if p-value < 0.05

Measure of Goodness of Fit – R Squared

- R^2 is the proportion of variation in a dependent variable which is explained by independent variables. Note that R^2 always increases if variable is added in the mode

$$R^2 = \frac{\text{Explained Variation}}{\text{Total Variation}} = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$$

The adjusted R-squared is a modified version of R-squared that has been adjusted for the number of predictors in the model

$$R_a^2 = 1 - \frac{n-1}{n-p-1}(1-R^2)$$

The adjusted R-squared is a modified version of R-squared that has been adjusted for the number of predictors in the model. Normally, R^2 greater than 0.7 is considered as the benchmark for accepting the goodness of fit of a model.

Understanding Summary Output

```
#Model Summary
jpimodel.summary()
```

summary() generates a detailed description of the model.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    jpi   R-squared:                       0.877
Model:                            OLS   Adj. R-squared:                  0.859
Method:                 Least Squares   F-statistic:                     49.81
Date:                Wed, 23 Oct 2019   Prob (F-statistic):           2.47e-12
Time:                        14:01:20   Log-Likelihood:                -85.916
No. Observations:                  33   AIC:                             181.8
Df Residuals:                      28   BIC:                             189.3
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    -54.2822      7.395     -7.341      0.000     -69.429     -39.135
tol            0.0334      0.071      0.468      0.643      -0.113       0.179
aptitude       0.3236      0.068      4.774      0.000       0.185       0.462
technical      1.0955      0.181      6.039      0.000       0.724       1.467
general        0.5368      0.158      3.389      0.002       0.212       0.861
==============================================================================
Omnibus:                        2.124   Durbin-Watson:                   1.379
Prob(Omnibus):                  0.346   Jarque-Bera (JB):                1.944
Skew:                          -0.544   Prob(JB):                        0.378
Kurtosis:                       2.518   Cond. No.                     1.25e+03
==============================================================================
```

Interpretation :

- *Reject Global Testing null hypothesis that no variables are significant as p-value is<0.05*
- *Intercept, aptitude, technical, general are significant variables (p-values<0.05)*
- *tol is not significant (p-value>0.05)*

Summary of Findings

Significant variables → **Aptitude**, **Technical knowledge**, **General information**

Out of four dependent variables, **three affect job performance index positively**

R^2 → **0.88**

88% of the variation in job performance index is explained by the model & 12% is unexplained variation

Fitted Values and Residuals

```
#Model Fitting after eliminating the insignificant variable
```

```
jpimodel_new=smf.ols('jpi ~ aptitude + technical +general',
data=perindex).fit()
jpimodel_new.params
```

The insignificant variable tol is not included in the new model

```
#Output
```

```
Intercept    -54.406443
aptitude       0.333346
technical      1.116627
general        0.543157
dtype: float64
```

Estimated values of the model parameters using the new model

* **To get fitted values and residuals values, the model should include significant variables only**

Fitted Values and Residuals

```
#Adding Fitted Values and Residuals to the Original Dataset
perindex=perindex.assign(pred=pd.Series(jpimodel_new.fittedvalues))
perindex=perindex.assign(res=pd.Series(jpimodel_new.resid))
perindex.head()
```

fittedvalues() and resid() fetch fitted values and residuals respectively.

```
#Output
```

	empid	jpi	aptitude	tol	technical	general	pred	res
0	1	45.52	43.83	55.92	51.82	43.58	41.738503	3.781497
1	2	40.10	32.71	32.56	51.49	51.03	41.709731	-1.609731
2	3	50.61	56.64	54.84	52.29	52.47	51.362151	-0.752151
3	4	38.97	51.53	59.69	47.48	47.69	41.691486	-2.721486
4	5	41.87	51.35	51.50	47.59	45.77	40.711451	1.158549

Interpretation :

- *pred values are calculated based on the values of the model parameters*
- *res is the difference between the actual jpi values and the pred values.*
- *Lower the residuals, lesser is the difference between fitted and observed and better is the model.*

Predictions for a New Dataset

- A new data set should have all the independent variables used in the model
- Column names of all common variables in the new and old datasets should be identical
- Note that missing values will be taken as 0 (which can be incorrect)

```
#Importing New Dataset
perindex_new=pd.read_csv("Performance Index new.csv")
perindex_new=perindex_new.assign(pred=pd.Series(jpimodel_new.predict(p
erindex_new)))
```

predict() returns predicted values. The fitted model is the first argument and new dataset object is the second argument. This ensures Python uses parameters from the fitted model for predictions on new data.

```
perindex_new.head()
```

	empid	jpi	tol	technical	general	aptitude	pred
0	34	66.35	59.20	57.18	54.98	66.74	61.552576
1	35	56.10	64.92	52.51	55.78	55.45	53.008978
2	36	48.95	63.59	57.76	52.08	51.73	55.621537
3	37	43.25	64.90	50.13	42.75	45.09	39.820600
4	38	41.20	51.50	47.89	45.77	50.85	40.879766

Predictions with Confidence Interval

```
#Predictions with Confidence Interval

result = jpimodel_new.get_prediction(perindex_new)
result.conf_int()
```

- *conf_int() generates 95% confidence intervals by default.*
- *Left hand side values in array gives lower confidence interval values, right gives upper.*

```
#Output

array([[59.00955719, 64.09559387],
       [50.67791702, 55.34003898],
       [53.65401364, 57.58906082],
       [37.73389546, 41.90730465],
       [39.23363549, 42.52589584],
       [45.41626758, 47.98650295]])
```

Q. Why are confidence intervals needed for predictions?
A. The point estimate is the best guess of the true value of the parameter, while the interval estimate gives a measure of accuracy of that point estimate by providing an interval that contains plausible values.

* **If you wish to specify the level of tolerance/confidence, use alpha= argument in the conf_int() function. For example, to calculate 90% confidence intervals,** alpha = 0.1

Standardized Coefficients

How to determine relative importance of predictors?

One possible answer is standardized regression coefficient

Predictors can have very different types of units, which make comparing regression coefficients meaningless. One solution is to standardize all variables before performing regression analysis.

standardization refers to the process of subtracting the mean (μ) from each value and dividing by the standard deviation (σ).

$$Z = \frac{X - \mu}{\sigma}$$

	X1	X2	Standardized X1	Standardized X2
	32	1052	-0.20	-1.74
	37	1237	0.46	-1.06
	25	1672	-1.12	0.54
	39	1724	0.72	0.74
	23	1555	-1.38	0.11
	41	1423	0.99	-0.37
	43	1870	1.25	1.27
	28	1661	-0.72	0.50
Mean	33.5	1524.25		
SD	7.60	271.69		

Standardized Coefficient - Python code

Generation of standardized parameter estimate

```
import pandas as pd
import numpy as np
from scipy import stats
import statsmodels.formula.api as smf

# standardizing dataframe
df_z =
perindex.select_dtypes(include=[np.number]).dropna().apply(stats.zscore)

# fitting regression
formula = 'jpi ~ aptitude + technical + general'
std_coef = smf.ols(formula, data=df_z).fit()
std_coef.params
```

- *stats.zscore standardizes the specified variables.*
- *.dropna(), Otherwise, stats.zscore will return all NaN for a column if it has any missing values.*
- *.select_dtypes(include=[np.number]) selects the numeric columns from data frame*
- *.params gives the standardized coefficients.*

Standardized Coefficient - Python code

#Output

```
Intercept   -9.072604e-16
aptitude     3.543742e-01
technical    5.880966e-01
general      3.236793e-01
dtype: float64
```

Interpretation:

- *technical has highest impact on job performance index followed by aptitude*

Quick Recap

Till now, we learnt the **basics of multiple linear regression**. Follow these simple steps to carry out your first analysis:

Check Variable Significance	• Undertake global and individual testing
Measure Goodness of Fit	• Check R-squared, Adjusted R-squared to see how much variation is explained by the model • Generally, R-squared greater than 0.7 is considered to be a good indicator
Summary Output	• Summary of **ols()** output is exhaustive and gives t statistics, p-value, R^2 to draw fundamental conclusions about the model

Quick Recap

In this session, we learnt how to **perform basic multiple linear regression in R**:

Fitted Values and Errors	• **`fitted()`** and **`resid()`** are used to fetch fitted values and residuals respectively
Predictions	• **`predict()`** function predicts values for new data • Predictions can be obtained as either point estimates or as confidence intervals
Standardizing Coefficients	• **`stats.zscore`** function in package **scipy** gives the standardized coefficients. • It is used to compare the relative importance of independent variables when the variables are in different metric units

Multiple Linear Regression

Using Categorical Variables

Content

Case Study – Predicting Restaurant Sales

Background

- A city-based association of restaurants and cafes records all sorts of transactions and descriptive data for the purpose of industry-level analysis. The association wishes to find out if this data can be used to determine sales of restaurants.

Objective

- To predict sales of restaurants

Available Information

- Sample size is 16
- Independent Variables: Location of the Restaurant – Categorical Variable with 3 Categories – Mall, Street and Highway and Number of Households in the Area
- Dependent Variable: Sales of the Restaurant

Data Snapshot

RESTAURANT SALES DATA

Independent Variables | **Dependent Variable**

RESTAURANT	NOH	LOCATION	SALES
1	155	highway	131.27
2	93	highway	68.14

Columns	Description	Type	Measurement	Possible values
RESTAURANT	Restaurant Number	numeric	-	-
NOH	Number of Households in the Vicinity of the Restaurant	numeric	-	positive values
LOCATION	Whether the Restaurant is Situated in a Mall, on a Street or on a Highway	Categorical	highway, mall, street	3
SALES	Annual Sales of the Restaurant	numeric	-	positive values

MLR using Categorical Independent Variables

If there is a categorical independent variable with K categories we must define only K – 1 dummy variables

In the case study,

Dependent Variable	Sales of a Restaurant
Independent Variables	1. Location of the Restaurant 2. Number of Households in the Area

- Location is a categorical variable with 3 categories – Mall, Street and Highway
  Therefore, there will be 3-1=2 dummy variables
- The category for which dummy variable is not defined is called 'Base Category'

Data Snapshot – With Dummy Variables

Y : Sales of a Restaurant

X1 : No. of Households

X2 : 1 if location is 'Mall' and 0 otherwise

X3 : 1 if location is 'Street' and 0 otherwise

- The first six rows have 0 under MALL and STREET columns
- This clearly implies that the restaurant is located neither in mall nor on street
- It is located on HIGHWAY

RESTAURANT	NOH	LOCATION	SALES	MALL	STREET
1	155	highway	135.27	0	0
2	93	highway	72.74	0	0
3	128	highway	114.95	0	0
4	114	highway	102.93	0	0
5	158	highway	131.77	0	0
6	183	highway	160.91	0	0
7	178	mall	179.86	1	0
8	215	mall	220.14	1	0
9	172	mall	179.64	1	0
10	197	mall	185.92	1	0
11	207	mall	207.82	1	0
12	95	mall	113.51	1	0
13	224	street	203.98	0	1
14	199	street	174.48	0	1
15	240	street	220.43	0	1
16	100	street	93.19	0	1

Why Not K Dummy Variables?

Can there be as many dummy variables as categories?

- If k dummy variables are created for k categories, there will be perfect multicollinearity – The Dummy Variable Trap
- In order to avoid falling into this trap, model with k categories and k dummy variables must have no intercept
- In such a model, coefficients will directly represent mean value of that variable

However, it is desirable to stick to the rule of k categories = k – 1 Dummy Variables

Statistical Model Using Dummy Variables

Basic Multiple Linear Regression Model

$$\boxed{Y = b_0 + b_1X_1 + b_2X_2 + \ldots + b_pX_p + e}$$

where,

- Y : Dependent Variable
- $X_1, X_2, \ldots, X_p$: Independent Variables
- $b_0, b_1, \ldots, b_p$: Parameters of Model
- e : Random Error Component

Intercept

Parameter Estimate for X_2 and X_3

Restaurant Sales

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + e$$

Random Error

Parameter Estimate for X_1

No of Households

Location Mall

Location Street

Interpretation of Results

Regression Coefficients of categorical dummy variables are interpreted **relative to the base category**

- The positive beta coefficient of mall (b_2) implies that if the restaurant is located in a mall, the sale amount will be higher than sale amount of restaurant on highway by $b_{2.}$ units.
- If the coefficient is negative b_2 then it implies that restaurant located in mall will have lower sales than restaurant on highway by b_2 units.
- The same applies to street v/s highway
- Remember, dummy variable inferences are useful only if the variable is significant

MLR with Categorical Dummy Variables in Python

#Importing the Data

```
import pandas as pd
restaurantsales = pd.read_csv("RESTAURANT SALES DATA.csv")
restaurantsales.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 16 entries, 0 to 15
Data columns (total 4 columns):
RESTAURANT    16 non-null int64
NOH           16 non-null int64
LOCATION      16 non-null object
SALES         16 non-null float64
dtypes: float64(1), int64(2), object(1)
memory usage: 640.0+ bytes
```

```
restaurantsales['LOCATION']=restaurantsales['LOCATION'].astype('catego
ry')
restaurantsales['LOCATION'].cat.categories
```

```
Index(['highway', 'mall', 'street'], dtype='object')
```

- *info() shows class and levels of variables in the data.*
- *.astype is used to convert object type into "category"*
- *cat.categories() is used to check categorical variable's levels and their order.*

MLR with Categorical Dummy Variables in Python

```
#Fitting Multiple Linear Regression Model
import statsmodels.formula.api as smf
salesmodel=smf.ols('SALES~NOH+LOCATION',data=restaurantsales).fit()

salesmodel.summary()
```

summary() generates a detailed description of the model.

MLR with Categorical Dummy Variables in Python

#Output

OLS Regression Results

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2.1892	8.592	0.255	0.803	-16.531	20.910
LOCATION[T.mall]	37.0524	5.814	6.373	0.000	24.385	49.720
LOCATION[T.street]	7.1537	6.731	1.063	0.309	-7.513	21.820
NOH	0.8383	0.056	14.920	0.000	0.716	0.961

Omnibus:	1.196	Durbin-Watson:	2.713
Prob(Omnibus):	0.550	Jarque-Bera (JB):	0.403
Skew:	-0.387	Prob(JB):	0.817
Kurtosis:	3.071	Cond. No.	637.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Interpretation:

- *Python orders factor levels alphabetically and takes the first level as the base category by default*
- *NOH and Mall are significant variables*
- *Beta coefficient for mall is 37.0524. This implies that if a restaurant is located in a mall, its sales will be more than the restaurant located on highway by 37.0524.*
- *Street too has a positive coefficient, implying that sales of restaurant located on street will be 7.1537 times higher than highway.*

Changing the Base Category in Python

```
#Importing the required library
from patsy.contrasts import Treatment
```

```
#Fitting Model on Data with Reordered Levels
salesmodel=smf.ols("SALES ~ C(LOCATION, Treatment(reference='mall')) +
NOH",data=restaurantsales).fit()
salesmodel.summary()
```

- *Treatment() reorders levels of a factor variables.*
- *reference= is used to specify changed reference (base) level.*

Changing the Base Category in Python

#Output

```
==============================================================================================================
                                                      coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------------------------
Intercept                                          39.2416     10.502      3.737      0.003      16.360      62.124
C(LOCATION, Treatment(reference='mall'))[T.highway] -37.0524     5.814     -6.373      0.000     -49.720     -24.385
C(LOCATION, Treatment(reference='mall'))[T.street]  -29.8987     6.123     -4.883      0.000     -43.239     -16.558
NOH                                                 0.8383      0.056     14.920      0.000       0.716       0.961
==============================================================================
Omnibus:                        1.196   Durbin-Watson:                   2.713
Prob(Omnibus):                  0.550   Jarque-Bera (JB):                0.403
Skew:                          -0.387   Prob(JB):                        0.817
Kurtosis:                       3.071   Cond. No.                         812.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
"""
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  SALES   R-squared:                       0.970
Model:                            OLS   Adj. R-squared:                  0.963
Method:                 Least Squares   F-statistic:                     130.0
Date:                Wed, 24 Feb 2021   Prob (F-statistic):           2.05e-09
```

Interpretation:

- *Mall has now become the base category*
- *Coefficient of highway is just negative of coefficient of mall observed in previous model (Degree of association between mall and highway is the same, changed sign indicates that relativity has reversed)*
- *Note that street is also significant*

Quick Recap

In this session, we learnt how to **handle categorical variables in multiple linear regression by introducing Dummy Variables**

Number of Dummy Variables	• The number of dummy variables must be one less than the number of levels in the categorical variable
Interpretation	• The coefficient attached to the dummy variables must always be interpreted in relation to the base or reference group.
Dummy Variables in Python	• Python automatically assigns dummies to categorical variables in `ols()` • Use `Treatment()` to change the base category for modeling

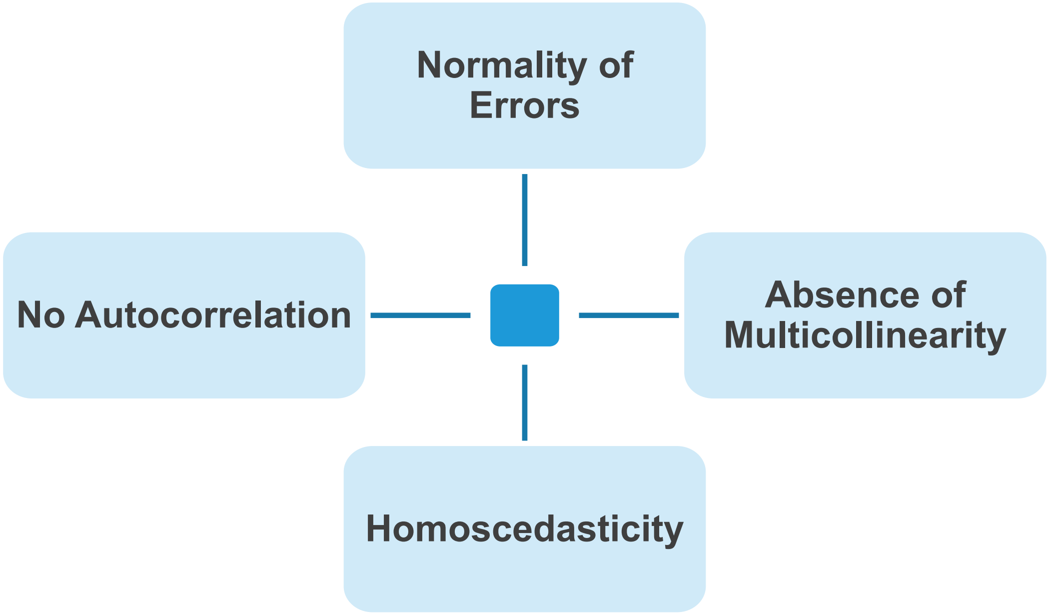
Multiple Linear Regression

Multicollinearity problem

Contents

Key Assumptions of Multiple Linear Regression

Multiple Linear Regression makes four key assumptions



Violations of these assumptions may result in biased variable relationships, over or under-estimation of parameters (i.e., biased standard errors), and unreliable confidence intervals and significance tests

Problem of Multicollinearity

Multicollinearity exists if there is strong linear relationship among the independent variables

Multicollinearity has two serious consequences:

1. Highly Unstable Model Parameters

As standard errors of their estimates are inflated

2. Model Fails to Accurately Predict for Out of Sample Data

Therefore, it is important to check for Multicollinearity in regression analysis

Indicators

- High Pairwise Correlation
  Among independent variables
- Significant F Value but Very Few Significant t Values for global testing and individual testing respectively

Detecting Multicollinearity Through VIF

VIF (Variance Inflation Factor) Method:

Dependent Variable : Y

Independent variables : X1, X2, X3, X4

Dependent Variable	Independent Variables	R^2	$1 - R^2 =$ Tolerance	VIF = 1/(Tolerance)
X1	X2, X3, X4			
X2	X1, X3, X4			
X3	X1, X2, X4			
X4	X1, X2, X3			

Any VIF > 5, indicates presence of Multicollinearity

Detecting Multicollinearity in Python

```
#Importing the Data, Fitting Linear Model
import pandas as pd
perindex=pd.read_csv("Performance Index.csv")

import statsmodels.formula.api as smf
jpimodel=smf.ols('jpi~aptitude+tol+technical+general',data=perindex).fit()
```

```
#Variance Inflation Factor
from patsy import dmatrices
from statsmodels.stats.outliers_influence import variance_inflation_factor

# Break data into left and right hand side; y and X
y, X = dmatrices('jpi ~ aptitude + tol +  technical +general',
data=perindex, return_type="dataframe")
```

- *patsy is a library that helps in converting data frames into design matrices.*
- *dmatrices Construct two design matrices using specified formula.By convention, the first matrix is the "y" data, and the second is the "x" data.*
- *variance_inflation_factor() requires a design matrix as input to calculate vif.*

* We use the same dataset "Performance Index" which was used in previous ppt

Detecting Multicollinearity in Python

```
# Calculating VIF & getting vif with their corresponding variable
# name

vif = pd.Series([variance_inflation_factor(X.values, i)for i in
range(X.shape[1])],index=X.columns)

vif
```

variance_inflation_factor() calculates VIFs.

```
# Output
```

```
Intercept    143.239081
aptitude       1.179906
tol            1.328205
technical      2.073907
general        2.024968
dtype: float64
```

Interpretation :
All VIFs are less than 5, Multicollinearity is not present.

Multicollinearity – Remedial Measures

The problem of Multicollinearity can be solved by different approaches:

- Drop one of the independent variables, which is explained by others
- Use Principal Component Regression in case of severe Multicollinearity
- Use Ridge Regression

*

Dropping a variable may not be a good idea if many VIFs are large.
Principal Component Method will be discussed in detail under Data Reduction and Segmentation

Case Study - Modelling Resale Price of Cars

Background

- A car garage has old cars for resale. They keep records for different models of cars and their specifications.

Objective

- To predict the resale price based on the information available about the engine size, horse power, weight and years of use of the cars

Available Information

- Records -26
- Independent Variables: **engine size**, **horse power**, **weight** and **years**
- Dependent Variable: **resale price**

Data Snapshot

ridge regression data

Dependent variable

Independent variables

Observations

MODEL	RESALE PRICE	ENGINE SIZE	HORSE POWER	WEIGHT	YEARS
Daihatsu Cuore	3870	846	32	650	2.9
Suzuki Swift 1.0 GL	4163	993	39	790	2.9
Fiat Panda Mambo L	3490	899	29	730	3.1

Columns	Description	Type	Measurement	Possible values
MODEL	Model of the car	character	-	-
RESALE PRICE	Resale price	numeric	Euro	positive values
ENGINE SIZE	Size of the engine	numeric	cc	positive values
HORSE POWER	Power of the engine	numeric	kW	positive values
WEIGHT	Weight of the car	numeric	kg	positive values
YEARS	Number of years in use	numeric	-	positive values

Correlation Matrix

```
# Importing the Data
ridgedata=pd.read_csv("ridge regression data.csv")
```

```
# Graphical representation of data
# Install and load package “seaborn”
import seaborn as sns
import matplotlib.pyplot as plt

sns.pairplot(ridgedata[['MODEL', 'RESALE PRICE', 'ENGINE SIZE', 'HORSE POWER','YEARS']]);plt.title('Fig.No. 2: ScatterPlot Matrix')
```

pairplot() in the package seaborn is used to plot the scatter plot matrix

Scatter Plot Matrix

Output

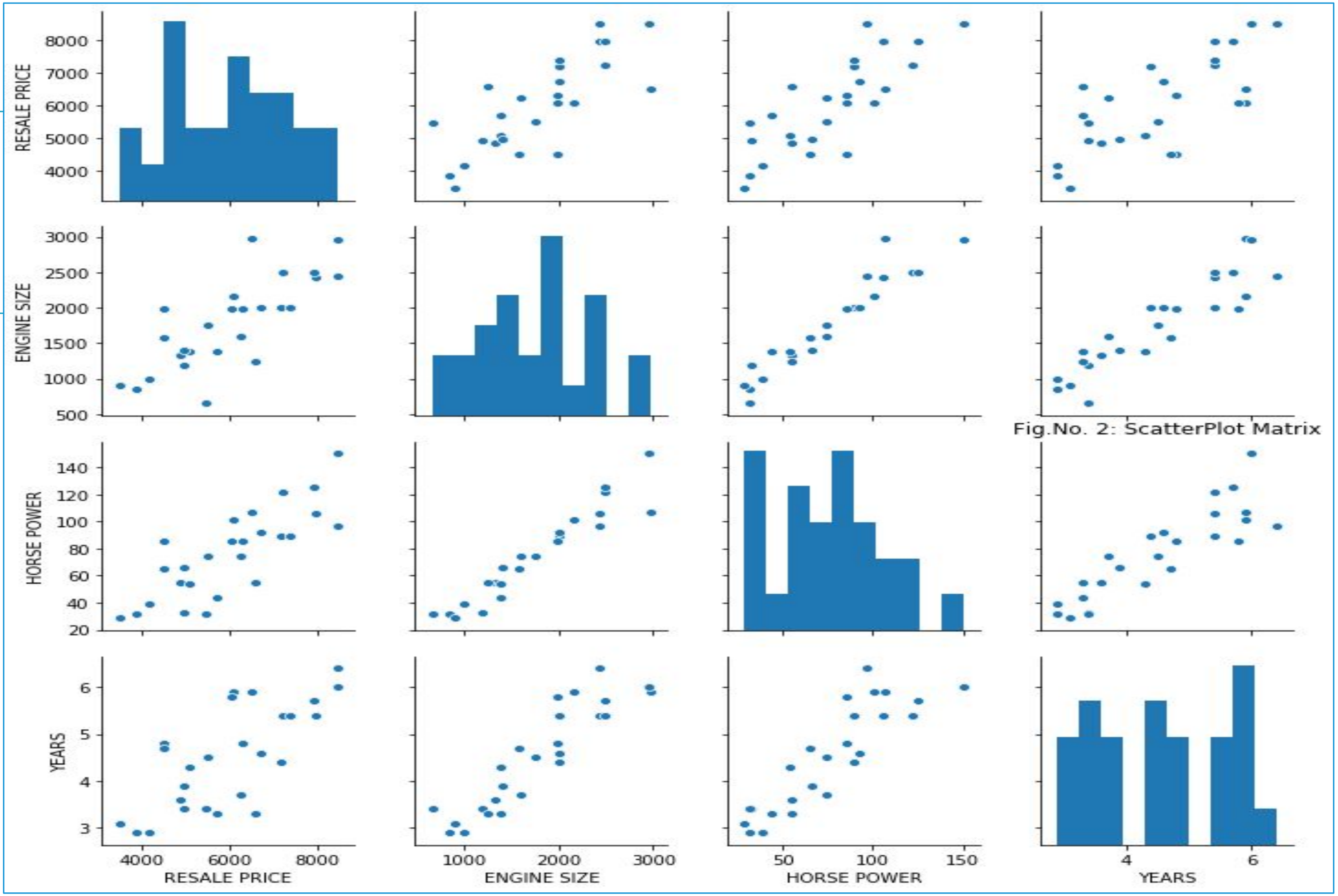


Fig.No. 2: ScatterPlot Matrix

Interpretation :

- *The independent variables have high positive correlation among themselves .*

Detecting Multicollinearity in Python

```
#Importing the Data, Fitting Linear Model
```

```
ridgedata.columns = [c.replace(' ', '_') for c in ridgedata.columns]
model = smf.ols('RESALE_PRICE~ENGINE_SIZE+ HORSE_POWER + WEIGHT + YEARS',
data = ridgedata).fit()
```

In pandas, the column names cannot contain spaces in between. Hence, before applying ols() remove spaces from column names wherever required.

```
#Variance Inflation Factor
```

```
y, X = dmatrices('RESALE_PRICE~ENGINE_SIZE+ HORSE_POWER + WEIGHT +
YEARS', data=ridgedata, return_type="dataframe")
vif = pd.Series([variance_inflation_factor(X.values, i)for i in
range(X.shape[1])],index=X.columns)
vif
```

```
# Output
```

```
Intercept      26.193279
ENGINE_SIZE    15.759113
HORSE_POWER    12.046734
WEIGHT          9.113045
YEARS          13.978640
dtype: float64
```

Interpretation:
VIF values for all the variables are greater than 5, hence we can conclude that there exist Multicollinearity between the independent variables.

Quick Recap

This session explained the **problem of Multicollinearity, along with its consequences and remedial measures:**

Multicollinearity Exists	• When independent variables have strong linear relationship
Results in	• Unstable model parameters • Inaccurate predictions for out of sample data
Indicators	• High pairwise correlation • Significant F value but very few significant t values
Checking in Python	• Variance Inflation Factor **`variance_inflation_factor()`** function in package **statsmodel**
Remedial Measures	• Drop variables • Use Principal Component Regression • Ridge regression

Normality and

Homoscedasticity Assumptions

Influential Observations

Contents

Normality and Homoscedasticity

- The errors in Multiple Linear Regression are assumed to follow Normal Distribution.

- If Normality of Errors is not true then statistical tests and associated P values based on F and t distribution are not reliable.

- **Homoscedasticity** describes a situation in which variance of error term is same across all values of the independent variables.

- In the absence of Homoscedasticity (Or presence of Heteroscedasticity) the standard errors of parameter estimates are incorrect.

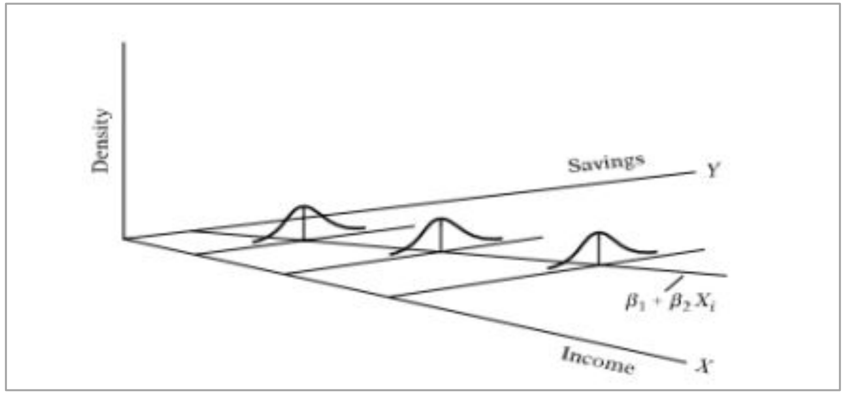
Assumption of Homoscedasticity

- Variance of error term must be constant across the independent variables (defined by X values)

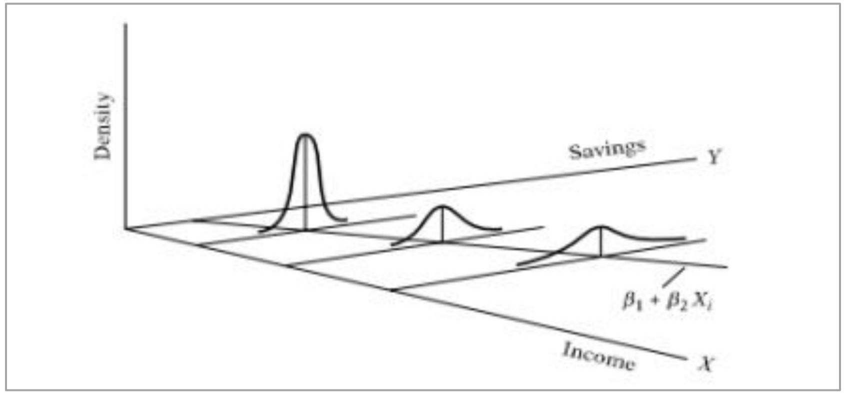
$$V\left({e_i}/{x_i}\right) = \sigma^2 \;\; indicates\ homoscedasticity$$

$$V\left({e_i}/{x_i}\right) = \sigma_i^2 \;\; indicates\ heteroscedasticity$$

Homoscedastic Errors



Heteroscedastic Errors



Residual Analysis

Observed Value – Predicted value = Residual

Residual Analysis

- Residuals v/s Predicted Plot
- Quantile-Quantile Plot
- Shapiro Wilk Test
- Kolmogorov-Smirnov (KS) Test

Checks randomness of errors

Checks distribution of errors

Residual Analysis for Performance Index Data

Continuing with the "**Performance Index** " data,

- **Model** job performance index (**jpi**) based on aptitude score (**aptitude**), test of language (**tol**), technical knowledge (**technical**) and general information (**general**)
- Get fitted values and residuals.
- Analyse the distribution of residuals

Residual v/s Predicted Plot in Python

```
#Importing the Data, Fitting Linear Model and Calculating Fitted
Values and Residuals
```

```
import pandas as pd
perindex= pd.read_csv("Performance Index.csv")

import statsmodels.formula.api as smf
jpimodel = smf.ols('jpi ~ tol + aptitude + technical +general',
data=perindex).fit()

perindex = perindex.assign(pred=pd.Series(jpimodel.fittedvalues))
perindex = perindex.assign(res=pd.Series(jpimodel.resid))
```

- *ols() fits a linear regression.*
- *fittedvalues() and resid() fetch fitted values and residuals*

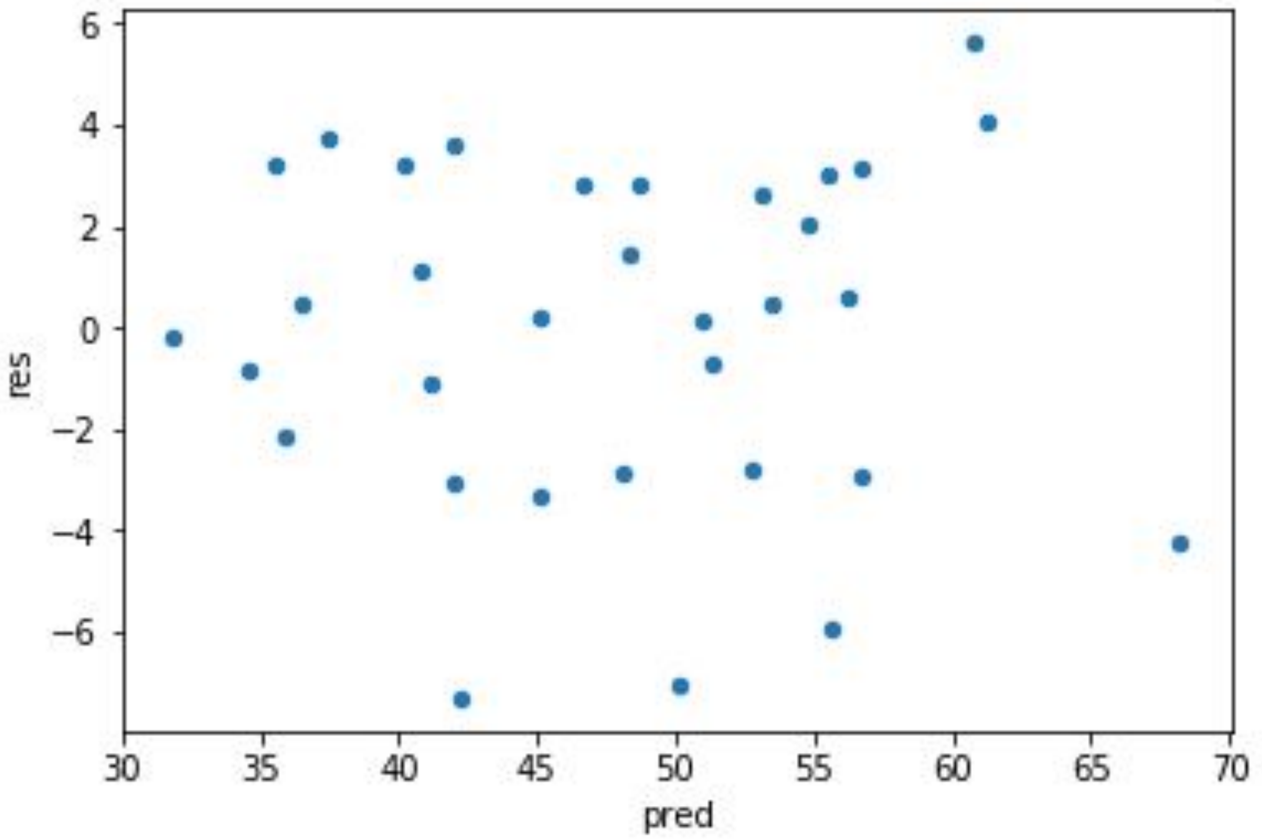
```
#Residuals v/s Predicted Plot
```

```
perindex.plot.scatter(x='pred', y='res')
```

.plot.scatter() is used to obtain scatter plot of predicted values against residuals.

Residual v/s Predicted Plot in Python

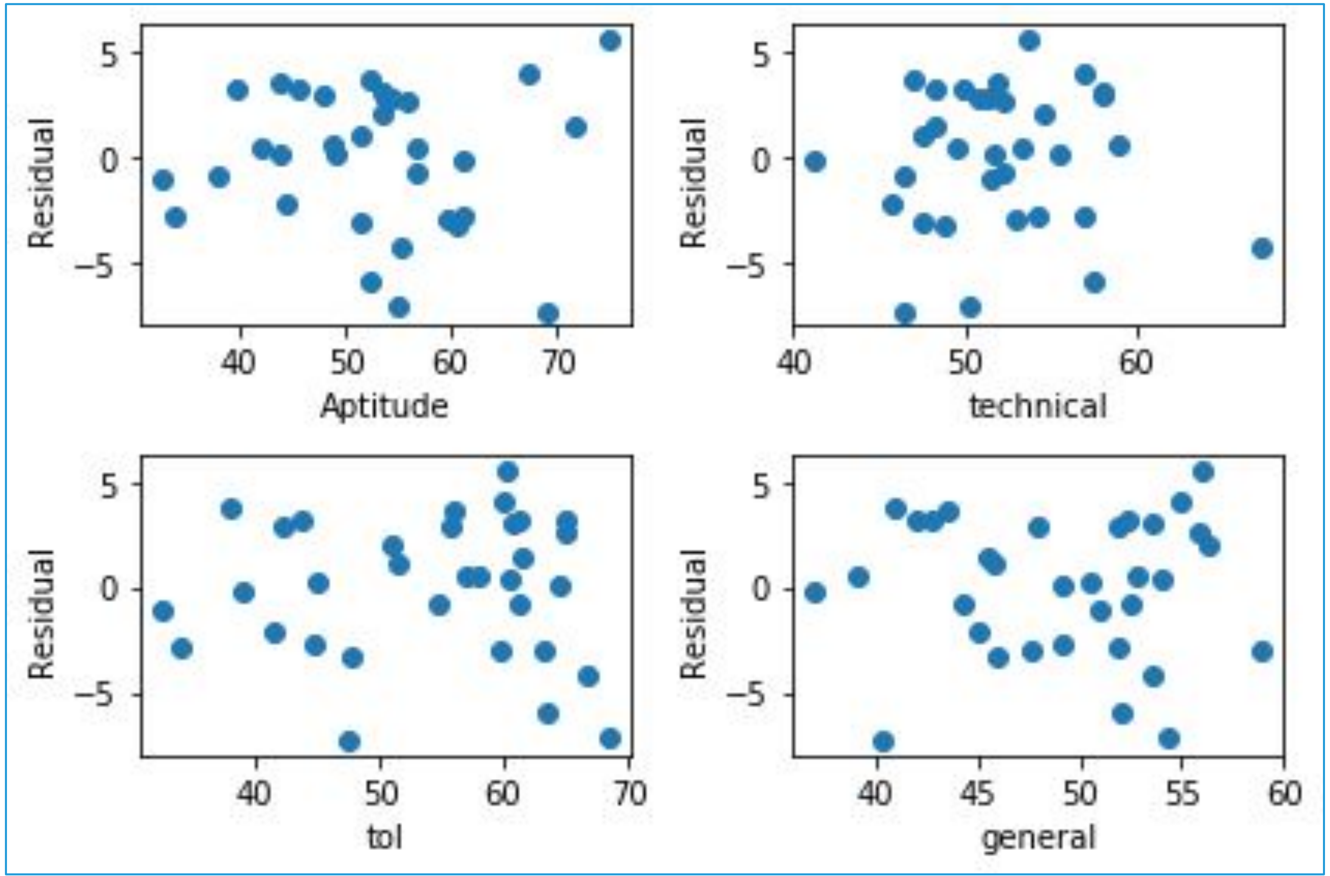
```
# Output
```



Interpretation:

- *Residuals in our model are randomly distributed which indicates presence of Homoscedasticity*

Residual v/s Independent variables Plot in Python



Interpretation:

- *Residuals in our model are randomly distributed which indicates presence of Homoscedasticity*

QQ Plot

- The Quantile-Quantile (QQ) Plot is a powerful graphical tool for assessing normality.
- Quantiles are calculated using sample data and plotted against expected quantiles under Normal distribution.

High Correlation between Sample Quantiles and Theoretical Quantiles → Normality

- If the data are truly sampled from a Gaussian (Normal) distribution, the QQ plot will be linear.

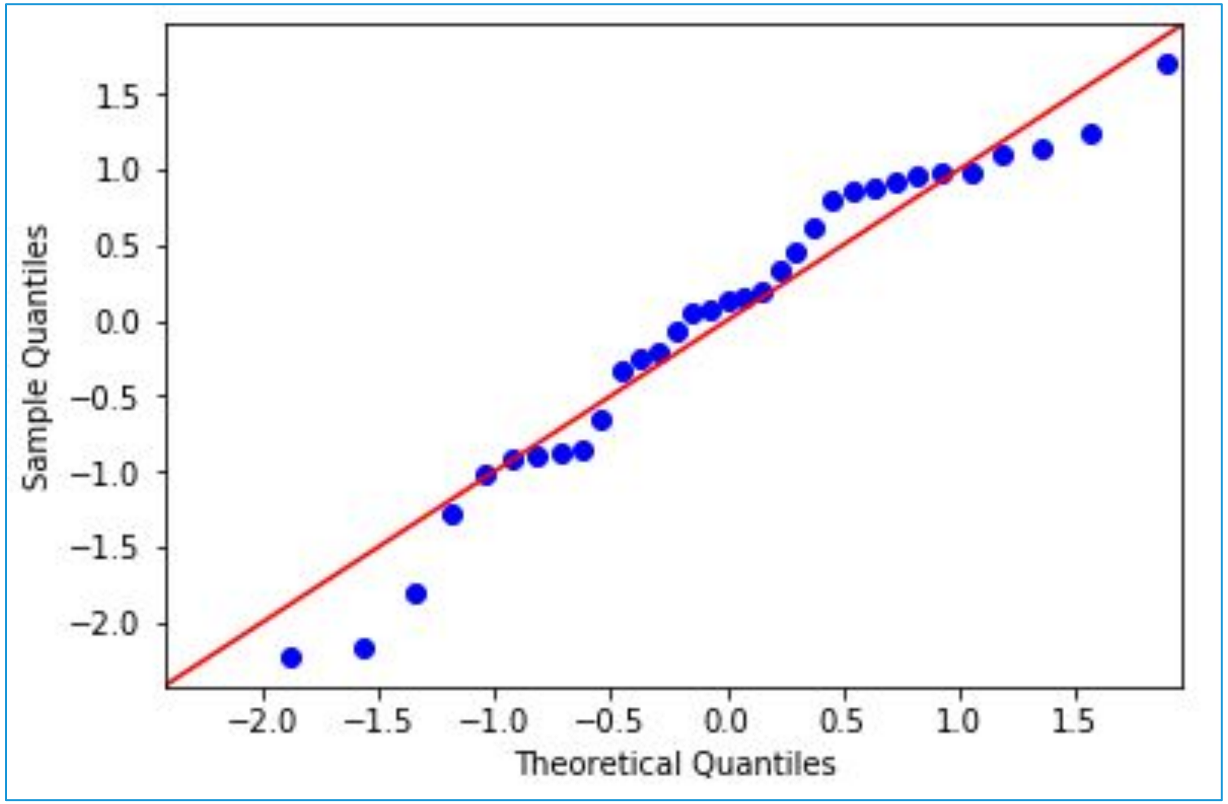
QQ Plot in Python

```
#QQ Plot
import statsmodels.api as sm
fig = sm.graphics.qqplot(perindex.res, line='45', fit=True)
```

- *qqplot() produces a plot with theoretical quantiles on x axis against the sample quantiles on y axis. Column for which normality is being tested is specified in the first argument.*
- *line= is an argument that adds reference line to the qqplot. Here it adds a 45-degree line*
- *fit=True indicates, parameters are fit using the distribution's fit() method.*

QQ Plot in Python

```
# Output
```



Interpretation:

- *Most of these points are close to the line except few values indicating no serious deviation from Normality.*

Shapiro Wilk Test

Objective	To **correlate**, sample ordered values with expected Normal scores in order **to test normality of the sample**

Null Hypothesis (H_0): Sample is drawn from Normal Population

Alternate Hypothesis (H_1): Not H_0

Test Statistic	
Decision Criteria	Reject the null hypothesis if p-value < 0.05

Shapiro Wilk Test in Python

```
# Shapiro Wilk Test
import scipy as sp
sp.stats.shapiro(perindex.res)
```

shapiro() from scipy package, returns correlation coefficient w and p-value.

```
# Output
(0.9498621821403503, 0.1318102478981018)
```

Interpretation:
p-value>0.05, Do not reject H_0. Normality can be assumed.

Absence of Normality – Remedial Measure

Mathematical Transformation of the dependent variable is used as a remedial measure in case of serious departure from Normality.

Typically Log Transformation is used. However, there is general transformation called as Box Cox Transformation given as :

- Box Cox transformation

$$Y^* = \frac{Y^\lambda - 1}{\lambda} \qquad \lambda \neq 0$$

$$= \log Y \qquad \lambda = 0$$

Where Y is the response variable

- R can automatically detect the optimum λ using **`boxcox()`** in package MASS

Influential Observation

- An **influential observation** is an observation whose deletion from the dataset would noticeably change the result of the calculation.
- In particular, in regression analysis an influential point is one whose deletion has a large effect on the parameter estimates.

Cook’s Distance Method

Cook's distance **measures the effect of deleting a given observation.**

Let Di be the Cook's distance for observation i.

$$D_i = \frac{\sum_{j=1}^{n} (\hat{Y}_j - \hat{Y}_{j(i)})^2}{p \text{ MSE}}$$

$\hat{Y}_j$ = prediction from the full regression model for observation j

$\hat{Y}_{j(i)}$ = prediction of j^{th} observation from a refitted model after removing i^{th} observation

MSE = mean square error of the regression model

p = number of fitted parameters in the model

Cut off to indicate influential observation,
- Simple operational guideline $D_i>1$
- Alternative $D_i >4/n$, where n is the number of observations

* **It is recommended to check model performance by excluding highly influential observation**

DFBETAs

DFBETA Statistics

DFBETA measures the difference in each parameter estimate with and without a specific observation. There is a DFBETA for each data point and for each parameter estimate.

Large values of DFBETAs

Indicate

Observations are influential in estimating a given parameter

Cut off to indicate influential observation,

- general cut off value recommended is **2**
- size adjusted cut off is taken to be **$2/\sqrt{n}$**

Finding Influential Observations in Python

#Importing the Data

```
import pandas as pd
perindex=pd.read_csv("Performance Index.csv")

import statsmodels.formula.api as smf
jpimodel=smf.ols('jpi ~ aptitude + tol + technical +general',
data=perindex).fit()
```

#Finding Influential Observations

```
influence = jpimodel.get_influence()
influence.summary_frame()
```

- *Influence is an object calling the method get_influence() which in turn allows us to call various measures of influence.*
- *summary_frame() calls a dataframe of 6 influence measures - Cook's Distance, Standardized residuals, dffits, dfbetas among others.*

Finding Influential Observations in Python

```
# Output
```

	dfb_Intercept	dfb_aptitude	dfb_tol	dfb_technical	dfb_general	cooks_d	standard_resid	hat_diag	dffits_internal	student_resid	dffits
0	0.122740655	-0.148903135	0.12930015	0.111295503	-0.231928116	0.027053557	1.070156154	0.105636531	0.367787693	1.073046027	0.368780874
1	-0.06974523	0.116652494	0.133587819	0.02982789	-0.09548941	0.010912673	-0.364255404	0.291400351	-0.233588024	-0.358542217	-0.229924297
2	0.007297141	-0.008657864	0.004774056	0.014949118	-0.026376846	0.000463024	-0.206575027	0.051460374	-0.048115671	-0.203007405	-0.047284697
3	-0.156960588	0.094733386	-0.173852663	0.214043541	-0.081100685	0.018150707	-0.899495717	0.100854509	-0.301253273	-0.896332474	-0.30019386
4	0.053857876	-0.010940236	0.00867167	-0.040889003	0.003662146	0.001250165	0.323269275	0.056438877	0.079062145	0.31803818	0.077782773
5	0.244563668	-0.167648781	-0.058829828	0.006424002	-0.120351526	0.0247544	0.95727578	0.118994501	0.351812453	0.9557968	0.351268907
6	-0.02187633	-0.017309924	0.012271264	0.006428915	0.013869655	0.000302058	-0.06455514	0.266006198	-0.03886248	-0.063396607	-0.038165038
7	-0.168200899	0.013234244	0.04743142	0.175275244	-0.069100347	0.01545173	0.918453917	0.083902352	0.277954402	0.915804589	0.277152627
8	-0.00893895	0.000748261	0.010029123	-0.012775933	0.020594356	0.000259695	0.13315947	0.068233348	0.03603436	0.130801426	0.035396249
9	0.112048775	-0.224748965	0.240141498	-0.17548137	0.078322088	0.023475703	-0.842281862	0.141964282	-0.342605483	-0.837785996	-0.340776751
10	-0.180550626	0.079534465	0.074017536	0.120829488	-0.057492908	0.01106923	-0.644822905	0.117472155	-0.235257629	-0.63795804	-0.232753046
11	-0.340527019	0.322925945	-0.062977416	0.148776604	0.044546954	0.044366369	1.221464622	0.129438005	0.470990281	1.232747401	0.475340861
12	-0.002789222	0.152126452	0.050719785	-0.020241173	-0.060819239	0.009463569	0.461241654	0.181948526	0.217526652	0.454660862	0.214423078
13	0.035353405	-0.027136734	0.033243364	0.016690271	-0.055540496	0.001256856	0.156009829	0.20521207	0.079273454	0.153265238	0.077878842
14	-0.060996651	1.62E-05	-0.079422031	-0.034363374	0.147870676	0.008619132	0.603984002	0.105654499	0.207594942	0.597002274	0.205195257
15	-0.026013807	-0.056282646	0.145739562	-0.191264061	0.22252013	0.020491421	0.791281772	0.140624982	0.320089214	0.78585952	0.317895805
16	0.005763587	0.05225852	-0.223037193	-0.045632185	0.148853441	0.018578284	0.85294471	0.113226147	0.30478094	0.848673035	0.303254552
17	-0.470812698	0.716128082	-0.106108271	-0.101654484	0.324161247	0.172591001	1.78156995	0.213764291	0.928953715	1.8579384	0.968774074
18	-0.002557978	-0.004065261	0.008450592	0.005075285	-0.005409883	4.13E-05	0.044548271	0.094144224	0.014361446	0.043747084	0.01410316
19	-0.052126913	-0.182788497	0.123094282	7.18E-05	0.058590983	0.01661995	-0.969160992	0.081281261	-0.288270271	-0.968072934	-0.287946635
20	-0.097589025	0.108404591	-0.09966008	0.077516309	-0.005991789	0.004816919	-0.272865	0.244414631	-0.155192119	-0.268305076	-0.152598659

Interpretation
One can use the threshold of 4/n where n is sample size and check cases with Cook's distance greater than the threshold.

Finding Influential Observations in Python

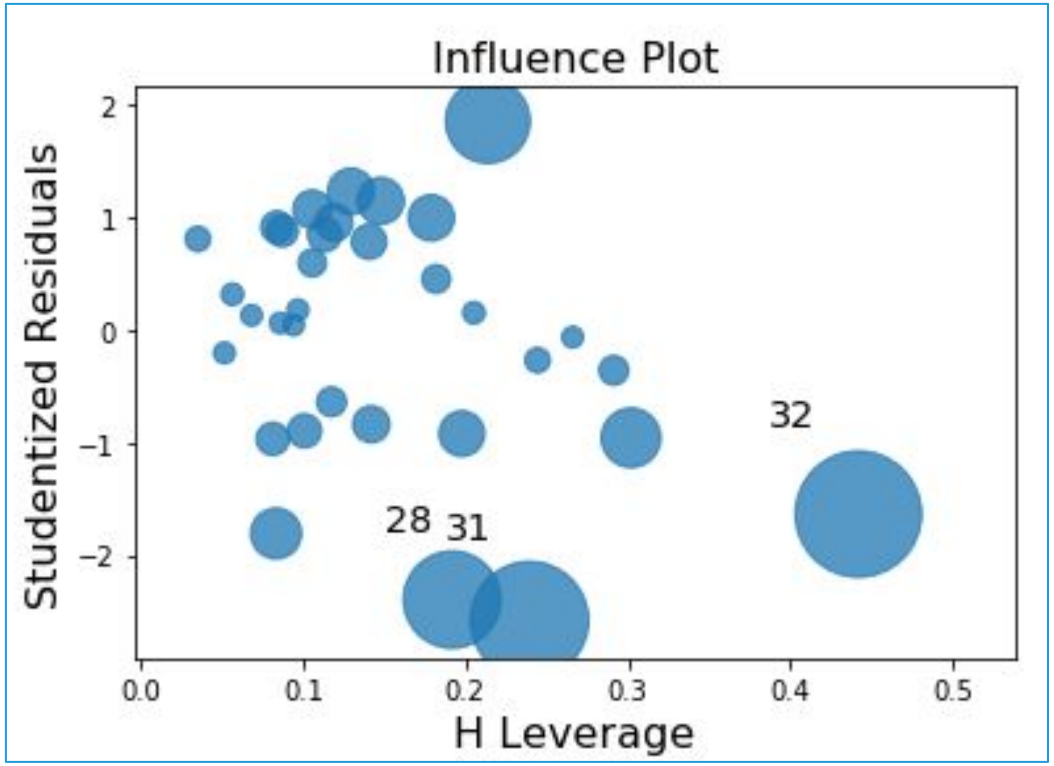
```
#Influence Plot
```

```
from statsmodels.graphics.regressionplots import *
influence_plot(jpimodel, criterion = 'Cooks')
```

- *influence_plot() creates a "bubble" plot of Studentized residuals by hat values, with the areas of the circles representing the observations proportional to criteria specified (in this case Cook's distance).*

Influence Plot in Python

```
# Output
```



Interpretation:
The data points 28,31 and 32 are detected as influential observations.

Quick Recap

Normality Assumption	• Error terms should be normally distributed
Homoscedasticity	• Errors should have constant variance across X values
Residual v/s Predicted Plot	• Ideally, residuals should be randomly distributed
Residual v/s Independent variables Plot	• Ideally,residuals should be randomly distributed
QQ Plot	• Used to check if errors follow Normal distribution
Shapiro Wilk Test	• Test for Normality assessment of errors

Quick Recap

Box Cox Transformation	• Transforming non normal response to normal
Influential Observations in Python	• **`get_influence()`** produces object giving influential observations by different measures • **`Influence_plot()`** creates a “bubble” plot of Studentized residuals by hat values, with the areas of the circles representing the observations proportional to Cook’s distances

Multiple Linear Regression

Cross Validation - I

Contents

Cross Validation in Predictive Modeling

Cross Validation is a

process of evaluating the model on

'Out of Sample' data

- **Model performance measures** such as R-squared or Root Mean Squared Error (RMSE) **tend to be optimistic on 'In sample data'**

- **Model performance on out of sample data** gives more realistic picture of model performance.

Cross validation is important because although a model is built on historical data, ultimately it is to be used on future data. However good the model, if it fails on out of sample data then it defeats the purpose of predictive modeling.

Cross Validation in Predictive Modeling

There are different approaches to cross validation. Five most important of them are:

- Hold-Out Validation
- K-Fold Cross Validation
- Repeated K-Fold Cross Validation
- Leave-One-Out Cross Validation (LOOCV)
- Re-sampling Validation Method (Bootstrap Method)

Case Study – Modeling Motor Insurance Claims

Background

- A car insurance company collects range of information from their customers at the time of buying and claiming insurance. The company wishes to check if any of this information can be used to model and predict claim amount

Objective

- To model motor insurance claim amounts based on vehicle related information collected at the time of registering and claiming insurance

Available Information

- Sample size is 1000
- Independent Variables: Vehicle Information – **Vehicle Age, Engine Capacity, Length and Weight of the Vehicle**
- Dependent Variable: **Claim Amount**

Data Snapshot

Motor_Claims

Independent variables

Dependent variable

Observations

vehage	CC	Length	Weight	claimamt
4	1495	4250	1023	72000
2	1061	3495	875	72000
2	1405	3675	980	50400
7	1298	4090	930	39960
2	1495	4250	1023	106800
1	1086	3565	854	69592.8

Columns	Description	Type	Measurement	Possible values
vehage	Age of the vehicle at the time of claim	integer	Years	positive values
CC	Engine capacity	numeric	cc	positive values
Length	Length of the vehicle	numeric	mm	positive values
Weight	Weight of the vehicle	numeric	kg	positive values
claimamt	Claim amount	numeric	INR	positive values

Data Visualization

```
#Importing the Data
```

```
import pandas as pd
motor=pd.read_csv('Motor_Claims.csv')
```

```
# Install package “seaborn” if not installed previously
# Obtain  scatter plot matrix
```

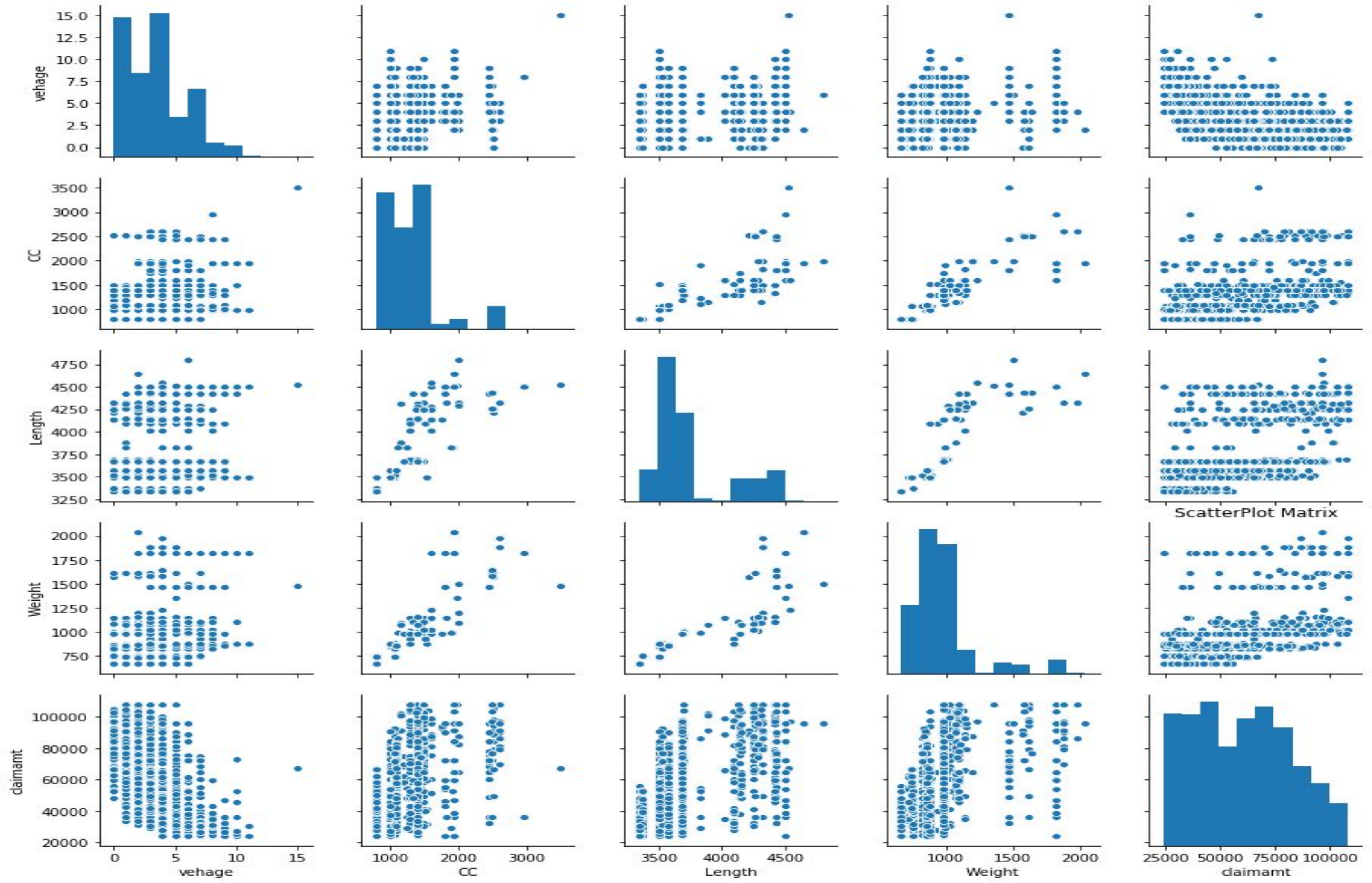
```
import seaborn as sns
import matplotlib.pyplot as plt

sns.pairplot(motor);plt.title('ScatterPlot Matrix')
```

Using the pairplot function in the seaborn library to get a scatter plot of the variables in the data set

Scatter Plot

```
# Output
```



Interpretation :
The scatter plot matrix gives an indication of multicollinearity.

Modeling Using ols Function

```
# Linear regression model
import statsmodels.formula.api as smf
motormodel = smf.ols('claimamt~Length+CC+vehage+Weight', data=motor).fit()
motormodel.summary()
```

```
# Output
                            OLS Regression Results
==============================================================================
Dep. Variable:               claimamt   R-squared:                       0.738
Model:                            OLS   Adj. R-squared:                  0.737
Method:                 Least Squares   F-statistic:                     700.3
Date:                Fri, 25 Oct 2019   Prob (F-statistic):          1.83e-287
Time:                        16:38:15   Log-Likelihood:                -10754.
No. Observations:                1000   AIC:                         2.152e+04
Df Residuals:                     995   BIC:                         2.154e+04
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -5.477e+04   5569.375     -9.833      0.000   -6.57e+04   -4.38e+04
Length        35.4607      1.990     17.824      0.000      31.557      39.365
CC            15.4133      2.114      7.292      0.000      11.265      19.561
vehage     -6637.2134    154.098    -43.071      0.000   -6939.607   -6334.820
Weight       -16.2547      3.678     -4.420      0.000     -23.472      -9.038
==============================================================================
Omnibus:                        7.335   Durbin-Watson:                   2.094
Prob(Omnibus):                  0.026   Jarque-Bera (JB):                9.587
Skew:                          -0.058   Prob(JB):                      0.00828
Kurtosis:                       3.466   Cond. No.                     6.33e+04
==============================================================================
```

Interpretation:
All independent variables in the model are significant.

Detecting Multicollinearity

```
# Obtaining vif
from patsy import dmatrices
from statsmodels.stats.outliers_influence import
variance_inflation_factor

# Break into left and right hand side; y and X
y, X = dmatrices('claimamt~Length+CC+vehage+Weight', data=motor,
return_type="dataframe")

#Calculating VIF
vif = pd.Series([variance_inflation_factor(X.values, i)for i in
range(X.shape[1])],index=X.columns)
vif
```

```
# Output showing VIF
```

```
Intercept    240.261728
Length         3.396171
CC             5.881428
vehage         1.038357
Weight         6.552811
dtype: float64
```

- *variance_inflation_factor in library statsmodels gives the VIFs of the independent variables in the regression model.*

Interpretation:

- *CC and Weight have VIF >5*

Re- Modeling

```
# New model
motormodel1 = smf.ols('claimamt~Length+CC+vehage', data=motor).fit()
motormodel1.summary()
```

New model after removing weight to remove multicollinearity.

```
# Output of the new model
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:               claimamt   R-squared:                       0.733
Model:                            OLS   Adj. R-squared:                  0.732
Method:                 Least Squares   F-statistic:                     910.3
Date:                Thu, 31 Oct 2019   Prob (F-statistic):          8.79e-285
Time:                        12:48:57   Log-Likelihood:                -10764.
No. Observations:                1000   AIC:                         2.154e+04
Df Residuals:                     996   BIC:                         2.156e+04
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   -4.92e+04   5475.151     -8.985      0.000   -5.99e+04   -3.85e+04
Length        32.0652      1.852     17.312      0.000      28.431      35.700
CC             8.6886      1.481      5.867      0.000       5.783      11.595
vehage     -6638.0765    155.525    -42.682      0.000   -6943.270   -6332.883
==============================================================================
Omnibus:                       10.930   Durbin-Watson:                   2.081
Prob(Omnibus):                  0.004   Jarque-Bera (JB):               15.892
Skew:                          -0.072   Prob(JB):                     0.000354
Kurtosis:                       3.600   Cond. No.                     5.99e+04
==============================================================================
```

Interpretation:
All independent variables in the model are significant.

* **Dropping one independent variable is one of the remedial measures to adjust for multicollinearity (when not many variables are multicollinear). As weight had the maximum VIF value, it is excluded from the model to adjust for multicollinearity.**

VIF of New Model

```
# VIF
```

```
# Break into left and right hand side; y and X
y, X = dmatrices('claimamt~Length+CC+vehage', data=motor,
return_type="dataframe")

#Calculating VIF
vif = pd.Series([variance_inflation_factor(X.values, i)for i in
range(X.shape[1])],index=X.columns)
vif
```

Getting VIFs of the independent variables in the new model

```
# VIFs of variables in the new model
```

```
Intercept     227.959103
Length          2.889718
CC              2.833931
vehage          1.038355
dtype: float64
```

Interpretation:
All VIF s are <5.

RMSE of the Model

```
# RMSE of the model
```

```
motor=motor.assign(res=pd.Series(motormodel1.resid))

from math import sqrt
RMSE = sqrt((motor['res']**2).mean())
RMSE
```

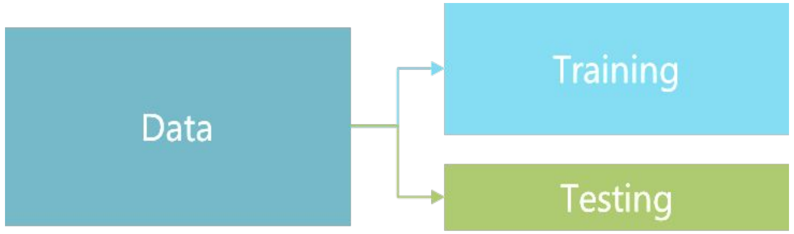
```
# Output
```

```
11444.512861029943
```

Interpretation :
RMSE for the model is 1144.51

* RMSE of this model will be used later to measure the performance of the model on cross validation

Hold-Out Validation



In Hold-Out validation method, **available data is split into two non-overlapped parts: 'Training Data' and 'Testing Data'**

- The model is,
  - Developed using training data
  - Evaluated using testing data

Training data should have more sample size. **Typically 70%-80% data is used for model development**

Hold Out Validation in Python

```
# Import train_test_split
# Creation of Datasets for Validation
import numpy as np
from sklearn.model_selection import train_test_split
import statsmodels.formula.api as smf

motor=pd.read_csv('Motor_Claims.csv')

motor_train, motor_test = train_test_split(motor, test_size=0.2,
random_state=0)
```

We use the function train_test_split to split the data 80 -20, with test_size=0.2 specifying that 20 % of data used as test data

```
motor_train.shape
motor_test.shape
```

training and testing data sets for the Motor Insurance Data

```
# Output
(800, 5)
(200, 5)
```

Dimension of training set

Dimension of testing set

Hold Out Validation in Python

```
# RMSE of training data
```

```
motor_model=smf.ols('claimamt~vehage+CC+Length', data =
motor_train).fit()

motor_train=motor_train.assign(res=pd.Series(motor_model.resid))
motor_train.head()

RMSEtrain=pd.Series(np.sqrt((motor_train.res)**2).mean())
RMSEtrain
```

```
# RMSE for testing data
```

```
motor_test=motor_test.assign(pred=pd.Series(motor_model.predict(motor_
test)))
motor_test=motor_test.assign(res=pd.Series(motor_test.claimamt -
motor_test.pred))

RMSEtest=pd.Series(np.sqrt((motor_test.res)**2).mean())
RMSEtest
```

*

For testing data, since we work with predictors, we calculate residuals as observed values minus the predicted values
Note : Since data is randomly taken, output will differ slightly

Hold Out Validation in Python

```
# Output
```

	vehage	CC	Length	Weight	claimamt	res
0	4	1495	4250	1023	72000.0	-3603.862083
2	2	1405	3675	980	50400.0	-17806.818626
3	7	1298	4090	930	39960.0	-8741.177051
4	2	1495	4250	1023	106800.0	18045.949781
6	4	796	3495	740	38400.0	-4537.892032

res column gives the residual for the training set data

11444.512861029943 — *RMSE for the original data with all data points*

11456.386840541307 — *RMSE for the training data*

10846.219628859553 — *RMSE for testing data*

Interpretations :

- *Comparing RMSE of training and testing data shows not much difference between the two and also are in line with the RMSE of the original model. Thus we can say that the model is stable.*

* **There is no rule of thumb for comparison of model performance. The only criterion is that performance measure for training and testing data should not be too diverse**

Quick Recap

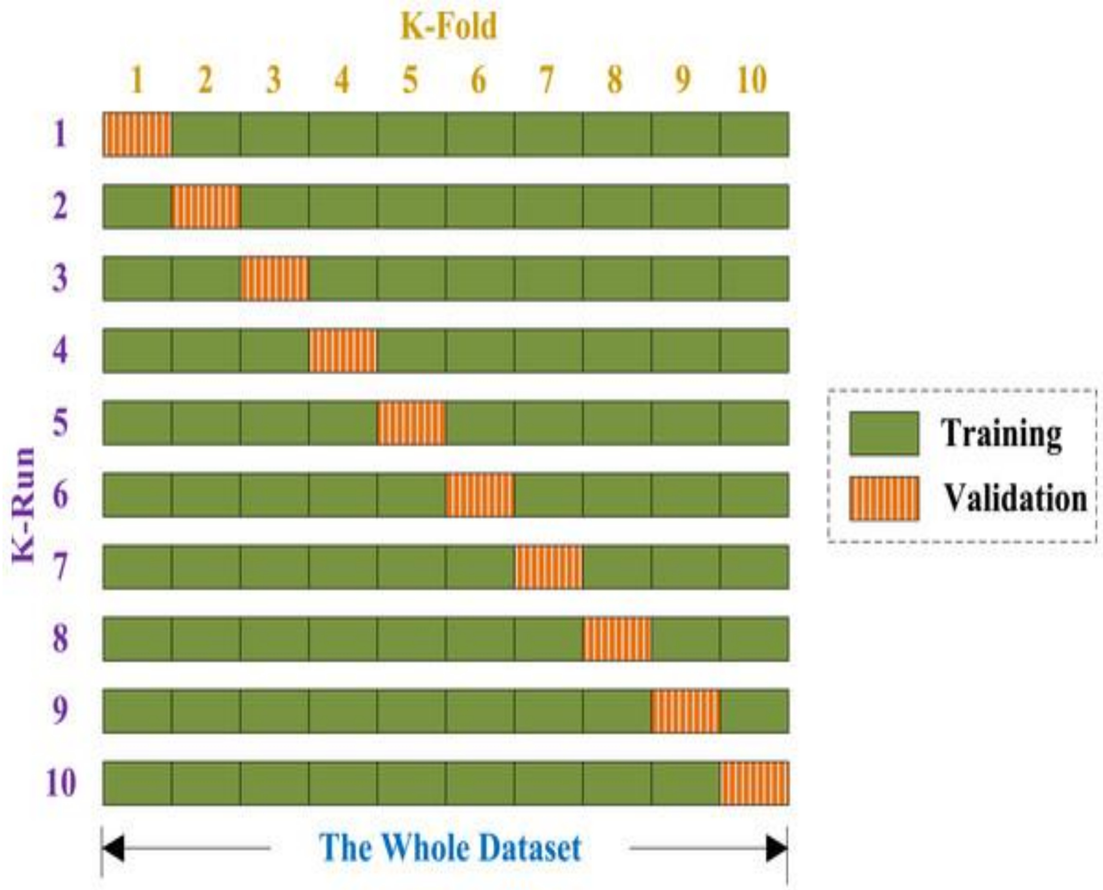
Cross Validation – Meaning and Need	• Process of evaluating the model on 'Out of Sample' data • Important because although a model is built on historical data, ultimately it is to be used on future data
Hold Out Validation	• Data is split into training and testing. Model developed on training and validated on testing

Multiple Linear Regression

Cross Validation - II

Contents

K-Fold Cross Validation



- In k-fold cross-validation the data is first partitioned into k equally (or nearly equally) sized segments or folds
- Then k iterations of training and testing are performed such that each time one fold is kept aside for testing and model is developed using k-1 folds

- Model performance measure is aggregate measure based on above iterations

K-Fold Cross Validation in Python

```
# Import necessary libraries
import pandas as pd
import numpy as np
from sklearn.model_selection import cross_val_score
from sklearn import linear_model

#Splitting data into X variables and Y variable
motor=pd.read_csv('Motor_Claims.csv’)

X=motor.drop(['claimamt'], axis = 1)
y=motor.claimamt

#Regression Model object is lm_reg
lm_reg = linear_model.LinearRegression()
```

K-Fold Cross Validation in Python

```
# Perform K fold cross validation with K=4 and report R squared values
cv_r2_scores_lm = cross_val_score(lm_reg, X, y, cv=4,scoring='r2')
print(cv_r2_scores_lm)
[0.75432031 0.72743804 0.69157553 0.7365273 ]
print("Mean 4-Fold R Squared: {}".format(np.mean(cv_r2_scores_lm)))
Mean 4-Fold R Squared: 0.7274652945788245

cv_rmse_scores= cross_val_score(lm_reg, X, y, cv=4,
scoring='neg_mean_squared_error')
np.sqrt(-(np.mean(cv_rmse_scores)))
11461.73251088066
```

R squared value

RMSE value

- *cross_val_score computes cross validation score*
- *cv=4 sets number of K-folds.*
- *scoring='r2' gives r squared value ; 'neg_mean_squared_error' gives negative mean squared error*

Interpretation :

- *R^2 of the original model is 73.19%*
- *RMSE for the original is model is 11444.51*
- *Comparing the RMSE values, we can say that the model is stable*

Repeated K-Fold Cross Validation

- As the name suggests, repeated k-fold cross validation technique **undertakes cross validation and repeats the process m-number of times**
- This ensures that more robust measure of model performance is generated
- **K-fold** is repeated m times with different randomization in each repetition

For instance,

- Five repeats of 10-fold cross validation will generate 50 total resamples.
- These results are again averaged to produce a single estimate
- This is not the same as 50-fold cross validation

Repeated K-Fold Cross Validation in Python

```
#Creating '5' Folds and '5' repeats
from sklearn.model_selection import RepeatedKFold
rkfold = RepeatedKFold(n_splits=5,n_repeats=5)
```

- *RepeatedKFold() is used to prepare the cross-validation procedure for implementation of repeated k-fold cross-validation.*
- *n_splits = specifies the number of folds.*
- *n_repeats= specifies the number of repeats.*

```
# Finding R squared value & RMSE value

cv_r2_repeated = cross_val_score(lm_reg, X, y, cv=rkfold)
print("Mean 5-Fold R Squared: {}".format(np.mean(cv_r2_repeated)))


cv_rmse_repeated= cross_val_score(lm_reg, X, y, cv=rkfold,
scoring='neg_mean_squared_error')
np.sqrt(-(np.mean(cv_rmse_repeated)))
```

Repeated K-Fold Cross Validation in Python

```
# Output
```

```
Mean 5-Fold R Squared: 0.7325797646609875
```

R squared value

```
11421.269030164527
```

RMSE value

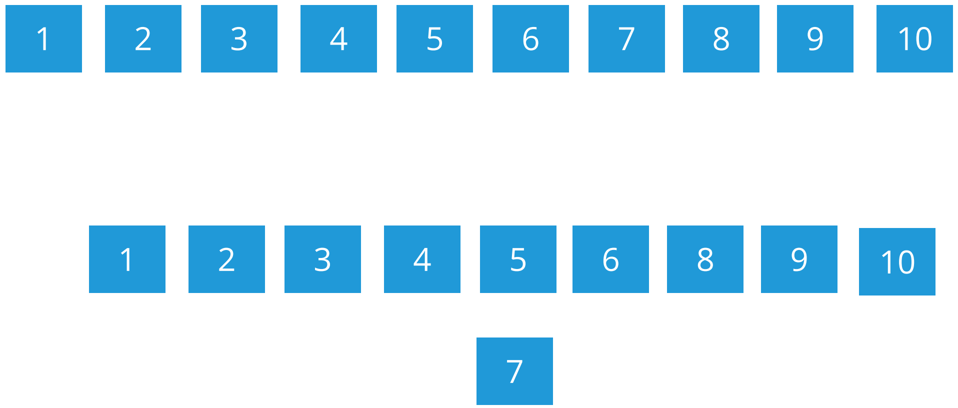
Interpretation :

- R^2 *of the original model is 73.19%*
- *RMSE for the original is 11444.51*
- *RMSE values of the cross validated model indicates stability.*
- *As the observations are selected randomly, output may vary slightly.*

Leave One Out Cross Validation (LOOCV)

- LOOCV is a special case of k-fold cross validation where k equals the sample size (n)
- Each time one observation is kept aside and the model is developed on the remaining data.
- The left out observation is predicted using the model.
- This process is repeated n times
- RMSE is computed based on these predicted residuals

- Sample size is 10 and one observations (say 7) is chosen to be kept aside
- The model is developed on the new sample with n=9 and observation 7 is predicted



Quick Recap

K-Fold Cross Validation	• Data is first partitioned into k equally (or nearly equally) sized segments or folds • Then k iterations of training and testing are performed such that each time one fold is kept aside for testing and model is developed using k-1 folds
Repeated K-Fold Cross Validation	• This is an extension of k-fold method wherein the process is repeated m number of times