Time Series Analysis

Stationarity of Time Series - I

Contents

What is Stationarity of Time Series ?

Time series process is called **Stationary if statistical properties of the process remain unchanged over time.**

If Y_t is a **stationary** time series where t=1,2,3,...

then,

$E(Y_t) = \mu_t = \mu$ (constant)

$Var(Y_t) = \sigma_t^2 = \sigma^2$ (constant)

$cov(Y_t, Y_{t+s})$ depends only on **s** (lag), and is independent of **t** (time)

Stationary vs. Non-Stationary Time Series

Fig. 1: Stationary Time Series

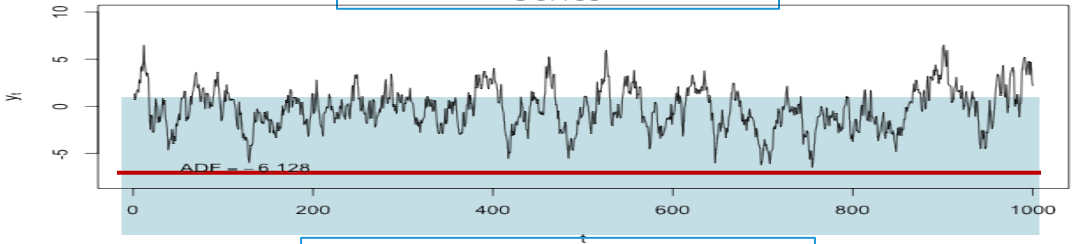
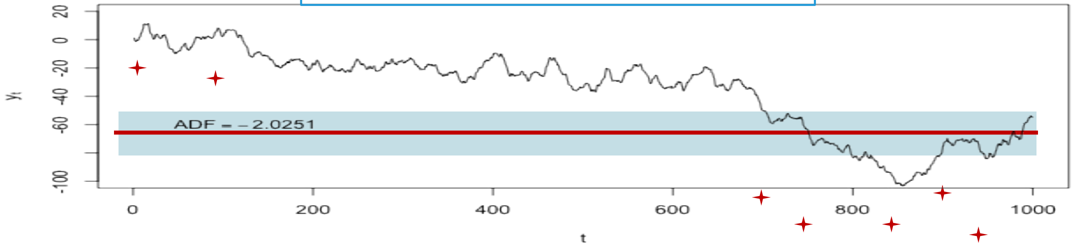


Fig. 2: Non-Stationary Time Series



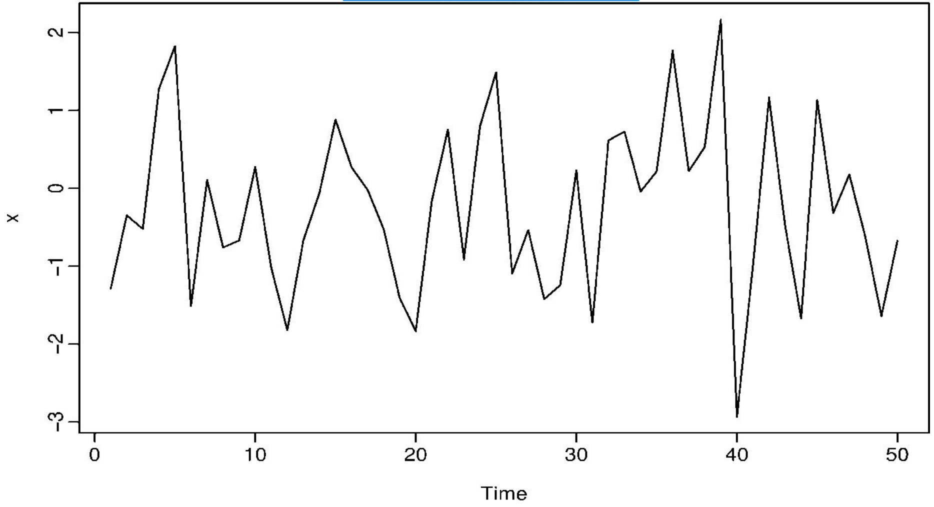
Interpretation :

- A stationary time series has a constant long term mean and variance.
- The first diagram shows a stationary time series whereas the second shows a non-stationary series.

Stationary Time Series - White Noise Process

- **White noise** is the simplest example of stationary time series.
- White Noise time series has **zero mean, constant variance and zero covariance with lagged time series.**

Fig. 3: White Noise



Non Stationary Time Series - Random Walk

- **Random Walk** is the simplest case of Non-stationary time series. It is of the form

$$Y_t = Y_{t-1} + U_t$$
$$t=1,2,3.....$$

- We assume that U_t is a random series with,
  - **Constant mean** μ
  - **Constant variance** σ^2
  - **Serially uncorrelated**

Value of Y at time t is equal to its value at time (t-1) plus a random shock

Why Random Walk is Non Stationary

Let $Y_t = 0$ at time $t = 0$

$$Y_1 = U_1$$

$$Y_2 = Y_1 + U_2 = U_1 + U_2$$

$$Y_3 = Y_2 + U_3 = U_1 + U_2 + U_3$$

So, $Y_t = \sum U_t$ $\qquad t = 1, 2, 3 \ldots$

$Yt = \sum Ut$

$E(Yt) = E(\sum Ut) = t \, . \, \mu$

i.e. Mean is not constant

$Var(Yt) = Var(\sum Ut) = t \, . \, \sigma2$

i.e. Variance is not constant

Therefore, Random Walk is a non-stationary time series

Importance of Stationary Time Series

- **Calibration** (estimation of model parameters using historical data) is an important concept in the **forecasting** of time series values.

- **In the calibration of time series models we need a stationary time series.**

- With a non stationary time series we get into **spurious** regression which badly affects forecasting.

How to Make a Non Stationary Time Series Stationary?

Two Methods for Making Time Series Stationary

Differencing

$$Y_t = Y_{t-1} + U_t \quad ;t=1,2,3.....$$

U_t is a random series with **Constant mean** μ, **Constant variance** σ^2 , and is **serially uncorrelated i.e** (Ut is stationary). Hence, Y_t is differenced:

$$Y_t - Y_{t-1} = \Delta Y = U_t$$

Differencing can be well applied in case of stochastic time series

De-trending

$$Y_t = \beta_1 + \beta_2 t + U_t \quad ;t=1,2,3.....$$

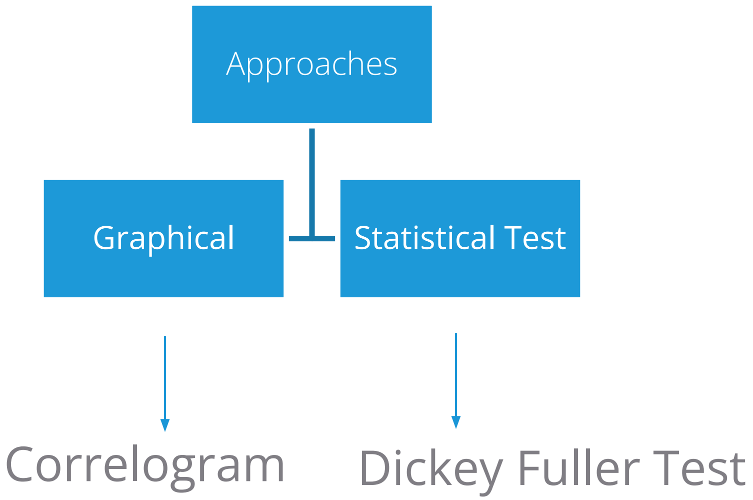
U_t is a stationary with **zero mean** and **constant variance** σ^2 . When Trend element $(\beta_1 + \beta_2 t)$ is subtracted , the result is a stationary process :

$$Y_t - (\beta_1 + \beta_2 t) = U_t$$

De-trending is useful when trend is deterministic

Identifying Stationary Time Series

& Concept of Autocorrelation



Autocorrelation of Lag 1

Data

Year	Sales(m)
Jan	110
Feb	113
Mar	121
Apr	123
May	126
Jun	120
Jul	119
Aug	127
Sep	129
Oct	131
Nov	130
Dec	132

First Order Lag

Year	Yt	Yt-1
Jan	110	
Feb	113	110
Mar	121	113
Apr	123	121
May	126	123
Jun	120	126
Jul	119	120
Aug	127	119
Sep	129	127
Oct	131	129
Nov	130	131
Dec	132	130

Autocorrelation of Lag 1,
where $\overline{\mathrm{Y}} = \text{Avearge of sales}$

$$\sum_{t=2}^{n} \frac{(Y_t - \overline{Y})(Y_{t-1} - \overline{Y})}{\sum_{t=1}^{n}(Y_t - \overline{Y})^2}$$

- **Autocorrelation** is a correlation between a time series (Y_t) and another time series representing lagged values of the same time series.(Y_{t-k})

Autocorrelation Function & Correlogram

- Plot of the **sample autocorrelation function against lag is called Correlogram.**
- ACF (autocorrelation function) is a general expression for lag k autocorrelation.
- Correlogram is mainly used in checking stationarity of a series.

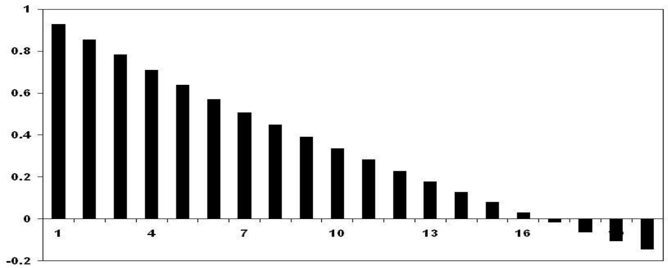
r_K (Sample autocorrelation of lag K) =

$$\sum_{t=k+1}^{n} \frac{(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n} (Y_t - \bar{Y})^2}$$

Correlogram for Checking Stationarity

- For a non-stationary time series showing trend, **slow decay pattern will be observed.**
- If a time series is characterized by seasonal fluctuations, then the **correlogram would also exhibit oscillations** at the same frequency.

Fig. 4: Correlogram



Sample correlogram of *LGDPI*

This figure presents the sample correlogram.

Partial Autocorrelation Function

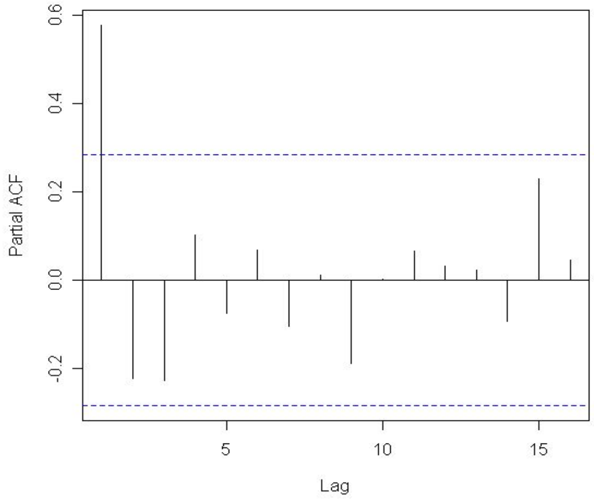
- Partial autocorrelation (PACF) is the

> Autocorrelation between y_t and y_{t-h} after removing any linear dependence on $y_1, y_2, \ldots, y_{t-h+1}$

- PACF gives **partial autocorrelations at various lags.**
- PACF is **mainly used to identify order of moving average present in the process.**

Partial Correlogram

Fig. 5: Partial ACF Plot



- Y-Axis = Partial Autocorrelation Function
- X-Axis = Lag
- Here we see that, after lag 1, the PACF drops dramatically and all PACFs after lag 1 are statistically insignificant.

Quick Recap

What is Stationarity	• Time series is stationary if the statistical properties of the process remain unchanged over time • Time series which has zero mean, constant variance and zero covariance with lagged time series is known as **"White Noise"**
Random Walk	• Is the simplest form of non-stationary time series where $Y_t - Y_{t-1} = U_t$ (U_t is a random error term)
Making a Non-Stationary Time Series Stationary	• Two approaches: **Differencing** (when time series is stochastic) and **De-trending** (when trend is deterministic)
Identifying Stationarity of Time Series	• **Graphical method:** ACF and PACF Correlograms • **Analytical method:** Dickey Fuller Test