Read in the following dictionary:

```
-7.00x_1 - 7.00x_2 + 10.00x_3 + 5.00x_4 - 8.00x_5 + 4.00x_6 + 7.00x_7 - 10.00x_8 + 10.
  x_9
x_{10}
                                  28.0
                                                                         -5.00x_1 - 2.00x_2 - 3.00x_3 + 10.00x_4 + 5.00x_5 + 3.00x_6 - 10.00x_7 - 9.00x_8
                                  35.0
                                                                         -6.00x_1 - 6.00x_2 - 8.00x_3 - 1.00x_4 + 5.00x_5 + 1.00x_6 - 9.00x_7 + 6.00x_8
x_{11}
                              -13.0
                                                                                                                       -3.00x_2 +7.00x_3 +7.00x_4 +5.00x_5 +5.00x_6 -10.00x_7 -5.00x_8
x_{12}
                              -11.0
                                                                         +5.00x_1 - 2.00x_2 + 2.00x_3 + 6.00x_4 - 9.00x_5 + 8.00x_6 + 5.00x_7 - 3.00x_8
x_{13}
                                                                         -4.00x_1 - 4.00x_2 + 4.00x_3 - 5.00x_4 - 4.00x_5 + 3.00x_6 - 3.00x_7 - 3.00x_8
                                     0.0
```

## 0.1 Initialization Phase: Dual Problem Solving

New Objective in primal was changed to:

$$\max \sum_{j=1}^{8} -x_j$$

Primal variable  $x_j$  corresponds to dual variable  $y_j$  for j = 1, ..., 13 Dual Dictionary (with objective changed is):

```
1.0
           +7.00y_9 +5.00y_{10} +6.00y_{11}
                                                            -5.00y_{13}
            +7.00y_9 +2.00y_{10} +6.00y_{11} +3.00y_{12} +2.00y_{13}
y_2
           -10.00y_9 +3.00y_{10} +8.00y_{11} -7.00y_{12} -2.00y_{13}
     1.0
y_3
            -5.00y_9 -10.00y_{10} +1.00y_{11} -7.00y_{12} -6.00y_{13}
     1.0
y_4
           +8.00y_9 -5.00y_{10} -5.00y_{11} -5.00y_{12} +9.00y_{13}
y_5
     1.0
     1.0
            -4.00y_9 -3.00y_{10} -1.00y_{11} -5.00y_{12} -8.00y_{13}
y_6
     1.0
            -7.00y_9 +10.00y_{10} +9.00y_{11} +10.00y_{12} -5.00y_{13}
y_7
     1.0
           +10.00y_9 +9.00y_{10} -6.00y_{11} +5.00y_{12} +3.00y_{13}
     -0
           +7.00y_9 -28.00y_{10} -35.00y_{11} +13.00y_{12} +11.00y_{13}
```

Initialization succeeded in finding final dual dictionary with 4 pivots

```
-5.00y_{13}
                                +7.00y_9 +5.00y_{10} +6.00y_{11}
               1.0
y_1
         1.42857142857
                                +4.86y_9 -2.29y_{10} +6.43y_{11} -0.43y_4 -0.57y_{13}
y_2
     6.93889390391e - 17
                                -5.00y_9 +13.00y_{10} +7.00y_{11} +1.00y_4 +4.00y_{13}
y_3
        0.142857142857
                                -0.71y_9 -1.43y_{10} +0.14y_{11} -0.14y_4 -0.86y_{13}
y_{12}
        0.285714285714
                               +11.57y_9 +2.14y_{10} -5.71y_{11} +0.71y_4 +13.29y_{13}
y_5
                                -0.43y_9 +4.14y_{10} -1.71y_{11} +0.71y_4 -3.71y_{13}
        0.285714285714
y_6
y_7
         2.42857142857
                               -14.14y_9 -4.29y_{10} +10.43y_{11} -1.43y_4 -13.57y_{13}
         1.71428571429
                               +6.43y_9 +1.86y_{10} -5.29y_{11} -0.71y_4 -1.29y_{13}
         1.85714285714
                                -2.29y_9 -46.57y_{10} -33.14y_{11} -1.86y_4 -0.14y_{13}
```

Primal Dictionary is:

```
2.28571428571
                           -7.00x_1 - 4.86x_2 + 5.00x_3 + 0.71x_{12} - 11.57x_5 + 0.43x_6 + 14.14x_7 - 6.43x_8
x_9
x_{10}
                           -5.00x_1 + 2.29x_2 - 13.00x_3 + 1.43x_{12} - 2.14x_5 - 4.14x_6 + 4.29x_7 - 1.86x_8
       46.5714285714
       33.1428571429
                           -6.00x_1 - 6.43x_2 - 7.00x_3 - 0.14x_{12} + 5.71x_5 + 1.71x_6 - 10.43x_7 + 5.29x_8
x_{11}
       1.85714285714
                                    +0.43x_2 -1.00x_3 +0.14x_{12} -0.71x_5 -0.71x_6 +1.43x_7 +0.71x_8
x_4
      0.142857142857
                           +5.00x_1 + 0.57x_2 - 4.00x_3 + 0.86x_{12} - 13.29x_5 + 3.71x_6 + 13.57x_7 + 1.29x_8
x_{13}
      -1.85714285714
                          -1.00x_1 - 1.43x_2 - 0.00x_3 - 0.14x_{12} - 0.29x_5 - 0.29x_6 - 2.43x_7 - 1.71x_8
```

Primal Dictionary with original objective is:

```
2.28571428571
                           -7.00x_1 - 4.86x_2 + 5.00x_3 + 0.71x_{12} - 11.57x_5 + 0.43x_6 + 14.14x_7 - 6.43x_8
x_9
       46.5714285714
                           -5.00x_1 + 2.29x_2 - 13.00x_3 + 1.43x_{12} - 2.14x_5 - 4.14x_6 + 4.29x_7 - 1.86x_8
x_{10}
x_{11}
       33.1428571429
                           -6.00x_1 - 6.43x_2 - 7.00x_3 - 0.14x_{12} + 5.71x_5 + 1.71x_6 - 10.43x_7 + 5.29x_8
       1.85714285714
                                    +0.43x_2 -1.00x_3 +0.14x_{12} -0.71x_5 -0.71x_6 +1.43x_7 +0.71x_8
x_4
                           +5.00x_1 + 0.57x_2 - 4.00x_3 + 0.86x_{12} - 13.29x_5 + 3.71x_6 + 13.57x_7 + 1.29x_8
      0.142857142857
x_{13}
                           -4.00x_1 - 6.14x_2 + 9.00x_3 - 0.71x_{12} - 0.43x_5 + 6.57x_6 - 10.14x_7 - 6.57x_8
      -9.28571428571
```

## 1 Optimization Phase Simplex

Starting Dictionary is:

```
2.28571428571
                           -7.00x_1 - 4.86x_2 + 5.00x_3 + 0.71x_{12} - 11.57x_5 + 0.43x_6 + 14.14x_7 - 6.43x_8
x_9
       46.5714285714
                           -5.00x_1 + 2.29x_2 - 13.00x_3 + 1.43x_{12} - 2.14x_5 - 4.14x_6 + 4.29x_7 - 1.86x_8
x_{10}
       33.1428571429
                           -6.00x_1 - 6.43x_2 - 7.00x_3 - 0.14x_{12} + 5.71x_5 + 1.71x_6 - 10.43x_7 + 5.29x_8
x_{11}
       1.85714285714
                                    +0.43x_2 -1.00x_3 +0.14x_{12} -0.71x_5 -0.71x_6 +1.43x_7 +0.71x_8
x_4
                           +5.00x_1 + 0.57x_2 - 4.00x_3 + 0.86x_{12} - 13.29x_5 + 3.71x_6 + 13.57x_7 + 1.29x_8
      0.142857142857
x_{13}
      -9.28571428571
                           -4.00x_1 - 6.14x_2 + 9.00x_3 - 0.71x_{12} - 0.43x_5 + 6.57x_6 - 10.14x_7 - 6.57x_8
```

 $x_3$  enters and  $x_{13}$  leaves

```
2.46428571429
                           -0.75x_1 -4.14x_2 -1.25x_{13} +1.79x_{12} -28.18x_5 +5.07x_6 +31.11x_7 -4.82x_8
x_9
       46.1071428571
                           -21.25x_1 + 0.43x_2 + 3.25x_{13} - 1.36x_{12} + 41.04x_5 - 16.21x_6 - 39.82x_7 - 6.04x_8
x_{10}
       32.8928571429
                           -14.75x_1 - 7.43x_2 + 1.75x_{13} - 1.64x_{12} + 28.96x_5 - 4.79x_6 - 34.18x_7 + 3.04x_8
x_{11}
       1.82142857143
                           -1.25x_1 +0.29x_2 +0.25x_{13} -0.07x_{12} +2.61x_5 -1.64x_6 -1.96x_7 +0.39x_8
x_4
      0.0357142857143
                           +1.25x_1 +0.14x_2 -0.25x_{13} +0.21x_{12} -3.32x_5 +0.93x_6 +3.39x_7 +0.32x_8
x_3
      -8.96428571429
                           +7.25x_1 -4.86x_2 -2.25x_{13} +1.21x_{12} -30.32x_5 +14.93x_6 +20.39x_7 -3.68x_8
```

 $x_1$  enters and  $x_4$  leaves

```
1.37142857143
                                                                                                                          +0.60x_4 -4.31x_2 -1.40x_{13} +1.83x_{12} -29.74x_5 +6.06x_6 +32.29x_7 -5.06x_8
 x_9
                                                                                                                       +17.00x_4 \;\; -4.43x_2 \;\; -1.00x_{13} \; -0.14x_{12} \;\; -3.29x_5 \;\; +11.71x_6 \;\; -6.43x_7 \;\; -12.71x_8 \;\; -2.71x_8 \;\; -2.71x_8
                              15.1428571429
x_{10}
                                                                                                                        +11.80x_4 - 10.80x_2 - 1.20x_{13} - 0.80x_{12} - 1.80x_5 + 14.60x_6 - 11.00x_7 - 1.60x_8
                                                         11.4
x_{11}
                             1.45714285714
                                                                                                                          -0.80x_4 +0.23x_2 +0.20x_{13} -0.06x_{12} +2.09x_5 -1.31x_6 -1.57x_7 +0.31x_8
 x_1
                                                                                                                                                                                                                                                                                +0.14x_{12} -0.71x_5 -0.71x_6 +1.43x_7 +0.71x_8
  x_3
                             1.85714285714
                                                                                                                          -1.00x_4 +0.43x_2
    z
                                                             1.6
                                                                                                                           -5.80x_4 -3.20x_2 -0.80x_{13} + 0.80x_{12} - 15.20x_5 + 5.40x_6 + 9.00x_7 -1.40x_8
```

 $x_6$  enters and  $x_1$  leaves

```
8.08695652174
                         -3.09x_4 - 3.26x_2 - 0.48x_{13} + 1.57x_{12} - 20.13x_5 - 4.61x_1 + 25.04x_7 - 3.61x_8
x_9
x_{10}
      28.1304347826
                         +9.87x_4 -2.39x_2 +0.78x_{13} -0.65x_{12} +15.30x_5 -8.91x_1 -20.43x_7 -9.91x_8
                         +2.91x_4 - 8.26x_2 + 1.02x_{13} - 1.43x_{12} + 21.37x_5 - 11.11x_1 - 28.46x_7 + 1.89x_8
      27.5869565217
x_{11}
      1.10869565217
                         -0.61x_4 + 0.17x_2 + 0.15x_{13} - 0.04x_{12} + 1.59x_5 - 0.76x_1 - 1.20x_7 + 0.24x_8
x_6
       1.0652173913
                         -0.57x_4 + 0.30x_2 - 0.11x_{13} + 0.17x_{12} - 1.85x_5 + 0.54x_1 + 2.28x_7 + 0.54x_8
x_3
                         -9.09x_4 - 2.26x_2 + 0.02x_{13} + 0.57x_{12} - 6.63x_5 - 4.11x_1 + 2.54x_7 - 0.11x_8
      7.58695652174
```

## $x_7$ enters and $x_6$ leaves

```
31.3090909091
                         -15.84x_4 +0.38x_2 +2.71x_{13} +0.65x_{12} +13.11x_5 -20.55x_1 -20.95x_6 +1.40x_8
x_9
x_{10}
      9.18181818182
                         +20.27x_4 -5.36x_2 -1.82x_{13} + 0.09x_{12} - 11.82x_5 +4.09x_1 +17.09x_6 - 14.00x_8
            1.2
                         +17.40x_4 - 12.40x_2 - 2.60x_{13} - 0.40x_{12} - 16.40x_5 + 7.00x_1 + 23.80x_6 - 3.80x_8
x_{11}
      0.927272727273
                         -0.51x_4 +0.15x_2 +0.13x_{13} -0.04x_{12} +1.33x_5 -0.64x_1 -0.84x_6 +0.20x_8
x_7
                         -1.73x_4 +0.64x_2 +0.18x_{13} +0.09x_{12} +1.18x_5 -0.91x_1 -1.91x_6 +1.00x_8
      3.18181818182
x_3
                         -10.38x_4 -1.89x_2 +0.35x_{13} +0.47x_{12} -3.25x_5 -5.73x_1 -2.13x_6 +0.40x_8
      9.94545454545
```

 $x_8$  enters and  $x_{11}$  leaves

```
x_9
      31.7511961722
                         -9.43x_4 -4.19x_2 +1.75x_{13} +0.51x_{12} +7.07x_5 -17.97x_1 -12.18x_6 -0.37x_{11}
      4.76076555024
                         -43.83x_4 + 40.32x_2 + 7.76x_{13} + 1.56x_{12} + 48.60x_5 - 21.70x_1 - 70.59x_6 + 3.68x_{11}
x_{10}
      0.315789473684
                         +4.58x_4 -3.26x_2 -0.68x_{13} -0.11x_{12} -4.32x_5 +1.84x_1 +6.26x_6 -0.26x_{11}
x_8
      0.99043062201
                         +0.41x_4 -0.51x_2 -0.01x_{13} -0.06x_{12} +0.46x_5 -0.27x_1 +0.42x_6 -0.05x_{11}
x_7
                         +2.85x_4 -2.63x_2 -0.50x_{13} -0.01x_{12} -3.13x_5 +0.93x_1 +4.35x_6 -0.26x_{11}
       3.4976076555
x_3
                          -8.55x_4 -3.20x_2 +0.07x_{13} +0.43x_{12} -4.98x_5 -4.99x_1 +0.38x_6 -0.11x_{11}
      10.0717703349
```

 $x_6$  enters and  $x_{10}$  leaves

```
30.9299850888
                          -1.86x_4 - 11.14x_2 + 0.41x_{13} + 0.24x_{12} - 1.32x_5 - 14.22x_1 + 0.17x_{10} - 1.00x_{11}
x_9
     0.0674393384845
                          -0.62x_4 +0.57x_2 +0.11x_{13} +0.02x_{12} +0.69x_5 -0.31x_1 -0.01x_{10} +0.05x_{11}
x_6
x_8
     0.738172698929
                          +0.69x_4 +0.31x_2 +0.00x_{13} +0.03x_{12} -0.00x_5 -0.08x_1 -0.09x_{10} +0.06x_{11}
                          +0.15x_4 -0.27x_2 +0.04x_{13} -0.05x_{12} +0.75x_5 -0.40x_1 -0.01x_{10} -0.03x_{11}
      1.01850345669
x_7
                          +0.15x_4 -0.14x_2 -0.02x_{13} +0.08x_{12} -0.14x_5 -0.41x_1 -0.06x_{10} -0.04x_{11}
      3.79124305273
x_3
      10.0972617595
                          -8.78x_4 -2.98x_2 +0.11x_{13} +0.44x_{12} -4.72x_5 -5.11x_1 -0.01x_{10} -0.09x_{11}
z
```

 $x_{12}$  enters and  $x_7$  leaves

```
35.9451476793
                         -1.14x_4 - 12.47x_2 + 0.59x_{13} - 4.92x_7 + 2.38x_5 - 16.17x_1 + 0.14x_{10} - 1.16x_{11}
x_9
      0.535864978903
                         -0.55x_4 + 0.45x_2 + 0.13x_{13} - 0.46x_7 + 1.03x_5 - 0.49x_1 - 0.02x_{10} + 0.04x_{11}
x_6
                         +0.79x_4 +0.13x_2 +0.03x_{13} -0.70x_7 +0.52x_5 -0.36x_1 -0.09x_{10} +0.04x_{11}
      1.44725738397
x_8
                         +3.08x_4 -5.59x_2 +0.75x_{13} -20.75x_7 +15.58x_5 -8.22x_1 -0.12x_{10} -0.64x_{11}
x_{12}
       21.135021097
x_3
      5.52742616034
                         +0.40x_4 -0.60x_2 +0.04x_{13} -1.70x_7 +1.14x_5 -1.08x_1 -0.07x_{10} -0.09x_{11}
z
      19.3755274262
                         -7.43x_4 -5.43x_2 +0.44x_{13} -9.11x_7 +2.12x_5 -8.71x_1 -0.06x_{10} -0.37x_{11}
```

 $x_5$  enters and Unbounded Dictionary!

 $x_5$  enters and Unbounded Dictionary!