M368K
Homework #6

Burden and Faires.

Section 9.3 (#4a[1], 14b[2]). Section 9.4 (#2a). Section 9.5 (#2b[3]).

[1] Use $q = 6$ and $x^{(0)} = (1, 2, 1)^T$.

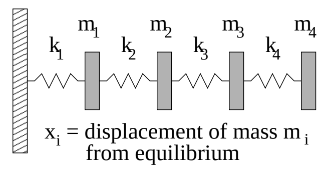
[2] Assume the results of #8b are $\lambda_1 = 5.6658$ and $v_1 = (0.0621, 0.2897, 1.1254, 1.0000)^T$. Find the deflated matrix, and use the general power code from the previous homework to find the requested eigenvalue.

[3] For full credit, use the matrix version of the QR method with zero shifts as described in class.

Programming mini-project.

A model for the vibrational motion of a simple mechanical structure consisting of masses and springs is $M\ddot{x} + Kx = 0$, or equivalently $\ddot{x} + Ax = 0$, where M and K are as shown and $A = M^{-1}K$. The general solution of this equation takes the form $x(t) = \sum_k [\alpha_k \cos(\sqrt{\lambda_k}\, t) + \beta_k \sin(\sqrt{\lambda_k}\, t)]v_k$, where (λ_k, v_k) are the eigenpairs of A and (α_k, β_k) are arbitrary coefficients. Thus $\sqrt{\lambda_k}$ represent the possible vibrational frequencies of the structure; the lowest is called its fundamental frequency.

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix}$$



Here we use the QR method to determine all the eigenpairs of A in the case when $\{m_1, m_2, m_3, m_4\} = \{1, 3, 2, 5\}$ [kg] and $\{k_1, k_2, k_3, k_4\} = \{10, 20, 15, 15\}$ [N/m] and determine the fundamental frequency.

(a) Download the C++ files `program6.cpp` and `qr.cpp` from the course webpage. Complete the file `qr.cpp` so that it implements the matrix version of the QR method with zero shifts.

(b) Use the QR method to find an approximation to all the eigenpairs of $A = M^{-1}K$ using a tolerance of $||R^{(k)}||_\infty \le 10^{-5}$, where $R^{(k)}$ is the off-diagonal portion of $A^{(k)}$. Report the original matrix A, all its approximate eigenvalues $\lambda_1, \ldots, \lambda_4$, ordered from largest to smallest in magnitude, and the lowest eigenpair (λ_4, v_4). What is the fundamental frequency $\omega = \sqrt{\lambda}$ [radian/sec] of the structure?

Turn in: completed version of `qr.cpp` and response to (b).