M368K
Homework #10

Burden and Faires.

Section 11.3 (#2b[1], 9[2]). Section 11.4 (#4a[3]). Section 11.5 (#2[4]).

[1]Explicitly write the central-diff eqns; solve and compare.

[2]Assume $q(x) > 0$ and try using Thm 9.1 instead of Thm 6.31.

[3]Use $N = 3$ instead of the given h. Explicitly write the central-diff eqns; perform one Newton step w/straight guess $y^{(0)}$.

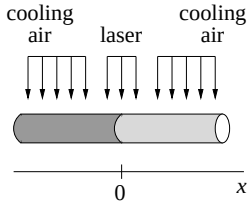
[4]Explicitly write the finite-element eqns; solve and compare at nodes. Note: $\int_0^1 \phi_1 f\, dx = -0.5813$, $\int_0^1 \phi_2 f\, dx = -0.7960$.

Programming mini-project.

A laser is used to weld two electrical conductors of dissimilar material. A model for the temperature profile during the welding process is

$$[-p(x)u']' + q(x)u = f(x), \quad -2 \le x \le 2$$
$$u(-2) = 0, \quad u(2) = 0$$

$$p(x) = \begin{cases} 0.1, & x < 0 \\ 0.2, & x \ge 0 \end{cases} \quad q(x) = \begin{cases} 0 & |x| \le 0.5 \\ \beta, & |x| > 0.5 \end{cases} \quad f(x) = \begin{cases} \gamma & |x| \le 0.4 \\ 0, & |x| > 0.4 \end{cases}$$



where $u(x)$ is the temperature measured with respect to a reference value, $p(x)$ is the thermal conductivity, $q(x)$ is proportional to the cooling air velocity and $f(x)$ is proportional to the laser energy intensity. Here we approximate solutions of this boundary-value problem with discontinuous coefficients using the finite-element method.

(a) Download the C++ files `program10.cpp`, `linearfem.cpp` and `gauss_elim.cpp` from the course webpage. These files implement the finite-element method for linear, two-point boundary-value problems $-[p(x)y']' + q(x)y = f(x)$, $a \le x \le b$, $y(a) = \alpha$ and $y(b) = \beta$; read the files for instructions on how to use.

(b) Consider the case with $\gamma = 50$ and $\beta = 0$. Using a refined grid with $N = 199$ compute and record the temperature at the locations $x = -0.5, 0, 0.5$. Because the materials are different, the maximum temperature does not occur at $x = 0$. Approximately where does it occur and what is its value?

(c) A successful weld requires that the material near the junction reach a relative temperature of 500, while the material away from the junction be kept under 100. Find values of the laser intensity parameter γ and cooling air velocity parameter β to achieve $u \doteq 500$ at $x = 0$, and $u \le 100$ for $|x| \ge 0.5$.

Turn in: modified version of `program10.cpp` used to solve this problem and responses to (b),(c).