# BPES - Finance and Financial Management

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## Contents

# 1 Lecture 1: Introduction to Finance

"qualitative in changing input on output, how the price would change."

## 1.1 Economics

**what is economics?**

- study of the production, distribution, and consumption of goods and services
- or can be interpreted as there's a lot of stuffs, how do you get stuffs to people
- focused on analysing behaviour and maximising welfare. (positive vs normative)
  - positive analysis (factual): how do things work; normative: how should things work.
- standard reasons for money:
  - store of value: moving value through time.
  - medium of exchange: common goods for exchange
  - unit of account: compare goods, one common denominator for comparison.

**why economics paid attention with finance?**

The financial problem blew up which also affects severely on the real economic, proving there're not unrelated but somehow intertwine, leading to *macro-finance*

## 1.2 Finance

**what is finance?**

- Finance is the study of investments.
- The mean-variance approach (micro-level)
- The CAPM (macro-level)
- Two basic functions:
  - Valuation: objective-independent, how are assets valued? How should they be valued?

### 1.2.1 Primary Market

**primary market** is the financial market where entities such as companies, governments and other institutions obtain funds through the sale of debt and equity-based securities. selling part of the company and in return to share their profit.

### 1.2.2 Secondary Market

**secondary markets** is the financial market where investors buy and sell securities from other investors (stock exchange)

### 1.2.3 Purpose of Financial Markets

**why do we need financial market?**

1. Allows trading to offset and reduce risks (e.g. corn production with other investment, income is no longer unpredictable, as corn price might drop, profits from investing can save some downsides)
2. (Setting Prices) The average aggregated information is reflected in the prices (more people want to sell, price goes down), which is valuable for people outside the market, companies, and policy makers.
3. (Transferring Risks) People expose to risk that doesn't want to be can have a place to sell it

It essential for both group of these people (people with info vs people don't like risk) to make a financial market to function.

### 1.2.4 Important Assumptions

1. Agents are selfish
2. Investors prefer more to less
3. Investors don't like risk investors prefer money now to later
4. No such thing as a free lunch, shouldn't allowed to create when nothing to start with
5. Financial market price to set supply = demand
6. Risk sharing and frictions are central to financial innovation
7. Don't say that a model is unrealistic.

# 2 Lecture 2: Time Value of Money

Time value of money can be understood by expressing as a compensation for lack of liquidity (independent of the riskiness of what im investing) (risk-free), where Assets can be formally defined as (1)**sequence of cashflows**. And it is important to recall that (2)**One dollar today does not equal one dollar tomorrow**

## 2.1 Future Value (FV)

$$\mathbf{FV = PV(1 + RT)}$$

## 2.2 Present Value (PV) or Price

$$\mathbf{PV = \frac{FV}{(1+R)^T}}$$

## 2.3 Yield (R): Rate of return

$$\mathbf{R = \left(\frac{FV}{PV}\right)^{1/T} - 1}$$

## 2.4 Discount Factor

**Discount factors** are how we value future cashflow, my willingness to buy and sell will depend on discount factor, how you convert future cashflow to today

- (Risk-free) Discount factor for a period of t is:

$$\mathbf{\frac{1}{(1+R)^t}}$$

- if my discount factor is **higher** than yours, that means I'm willing to **pay more** (higher PV) today than you are, i.e. willing to buy a bond from you. less risk averse (lower R)
- **lower** the discount factor, lower the patient you are willing to pay, and higher the the need for cash right now. **more risk averse** (higher R)

## 2.5 Perpetuities

How much is an infinite cashflow of C each year worth?

$$PV = \frac{C}{(1+R)} + \frac{C}{(1+r)^2} + \cdots = \frac{C}{R}$$

### 2.5.1 Growing/Infinite Perpetuities

gets fixed amount of cash for a infinite number of periods. How much is an *infinite* cash-flow of C growing at g each year worth?

$$PV = \frac{C}{1+R} + \frac{C(1+g)}{(1+R)^2} + \frac{C(1+g)^2}{(1+R)^3} + \ldots$$
$$PV = \frac{C}{R-g}$$

- for this to be well-defined, we need R > g
- If g = R, that means g is perfectly compensating for my time value of money. i.e. All rhe growth in the payment getting are perfectly compensating me for the amount of time I'm losing, which indicates each cashflow is equally weighted for me.
- Effectively, this stream of infinite money is worth, in terms of how much I'm willing to pay is infinite amount of money. (value it infinitely because I'd be willing to pay infinitely amount), therefore there's NO well-defined price I can arrive to at for this.

## 2.6 Fixed Horizon Annuities

you get fixed amount of cash every period for a finite number of periods How much is a fixed horizon cash-flow of C each year worth?

$$PV = \frac{C}{(1+R)} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+R)^T}$$
$$(1+R)PV = C + \frac{C}{1+R} + \cdots + \frac{C}{(1+R)^{T-1}}$$
$$RPV = C - \frac{C}{(1+R)^T}$$
$$PV = \frac{C}{R} - \frac{C}{R(1+R)^T}$$

T-period Annuity = Infinite Perpetuity - Date-T Perpetuity

# 3 Lecture 3: Portfolio Selection

"The fundamental decision of investing is the allocation of your assets: How much should you own in stock? How much should you own in bonds? How much should you own in cash reserves"

## 3.1 Expected Value

Suppose that a return $R_i$ on asset i is equal to $R_i(s)$ in state s. And that states' probabilities of occurrence are given by p(s), for s = 1, ..., S.

$$E[R_i] = \sum_{s=1}^{S} R_i(s)p(s)$$

## 3.2 Variance

The volatility or risk of $R_i$ is often refer to the **standard deviation** $\sigma_i$ (will be redefined later)

$$\begin{aligned} Var[R_i] = \sigma_i^2 &= E[(R_i(s) - E[R_i])^2] \\ &= \sum_{s=1}^{S} p(s)(R_i(s) - E[R_i])^2 \end{aligned}$$

## 3.3 Covariance

The degree of two assets in a portfolio move in the same direction at the same time.

$$Cov(R_i, R_j) = \sum_{s=1}^{S} p(s)(R_i(s) - E[R_i])(R_j(s) - E[R_j])$$

$$Var[wA + vB] = w^2 Var[A] + v^2 Var[B] + 2wvCov[A, B]$$

## 3.4 Correlation

Measure the strength and direction two assets in a portfolio moves at the same time.

$$Corr(R_i, R_j) = \rho_{ij} = \frac{Cov(R_i, R_j)}{\sigma_i \sigma_j}$$

## 3.5 Portfolio Allocation

### 3.5.1 1 risky, 1 risk-free

**Risk-free asset:**
A baseline to compare with risky assets. normally deemed as the US Treasury Bill (not true this year).

$$E[R_f] = R_f, \quad V[R_f] = 0, \quad Cov[R_f] = 0$$

The expected return for this kind of portfolio can be express as,

$$\begin{aligned} E[R_p] &= E[wR_i + (1 - w)R_f] \\ &= \boxed{R_f + w\overbrace{E[\underbrace{R_i - R_f}_{\text{excess return}}]}^{\text{risk premium}}} \end{aligned}$$

and the variance of the portfolio will be

$$\begin{aligned} V[R_p] &= V[wR_i + (1 - w)R_f] = w^2\sigma_i^2 \\ \sigma_p &= |w|\sigma_i \end{aligned}$$

**The Captial allocation line:**
substituting the above, we can re-express expected value as the following, this is also know as the Capital Allocation Line, where the only possible combination of 1 risky and 1 risk-free lies on this line. What the investor *can* get.

$$E[R_p] = R_f + wE[R_i - R_f] = R_f + \underbrace{\frac{E[R_i - R_f]}{\sigma_i}}_{\text{Sharpe ratio}} \sigma_p$$

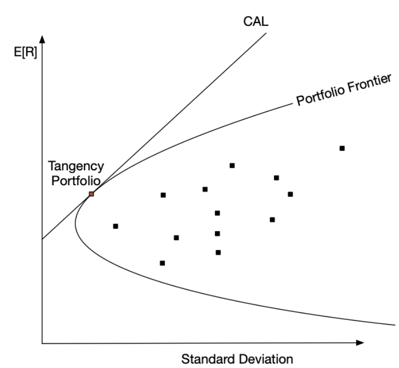
**Indifference Curve:**
Indifference curves expresses sets of portfolios that would make investors equally happy, they must slope upwards, and it also increases value from left to right. Indifference curves represent what investors want. If risk-neutral, Indifference curves becomes straight lines.

**Utility Function:**
uses mean-variance utility to measure how risk tolerant and risk averse investor will value stocks

$$E[U(R_p)] = E(R_p) - 0.5A \cdot V(R_p)$$

$A$ is a measure of risk aversion, the bigger the A is, the more risk averse the investor is.

### 3.5.2 2 risky, 1 risk-free




### 3.5.3 Equal-Weighted Portfolio

Equally weighted portfolio of independent assets. $Cov(R_i, R_j) = 0$, variance of the portfolio then becomes,

$$\sigma_p^2 = \frac{1}{N}\sum_{i=1}^{N}\sigma_i^2 = \frac{1}{N}E[\sigma_i^2]$$

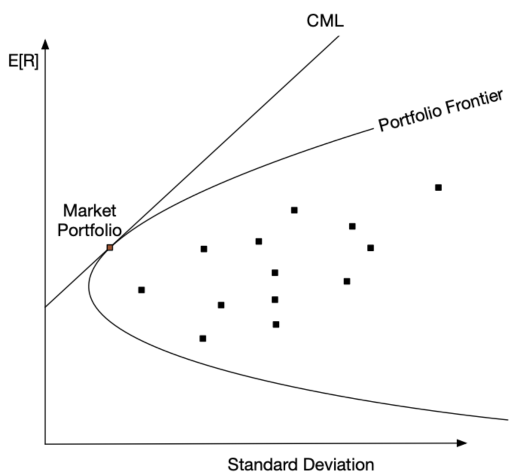
Risk decreases with the number of assets. Standard deviation declines with the number of assets.

# 4 Lecture 4: Capital Asset Pricing Model (CAPM)

Now we need to find a way to quantify risk, especially systemic risk (idiosyncratic risk can be diversified away), of an asset or an asset in our portfolio.

In the CAPM framework, all investors are assumed to have access to the same information and hold homogeneous expectations about future returns, and the market portfolio is considered to be the only efficient portfolio, meaning that it offers the highest expected return for a given level of risk.

Then we can effectively transfer our original Tangency Portfolio into our **Market portfolio**, and our capital allocation line (CAL) into **Capital Market Line**, the line that connects the Market Portfolio to the risk-free rate




Given the assumption, the risk-reward ratio of any asset or portfolio is determined solely by its beta, and the expected return for each unit of systematic risk should be the same for all assets and portfolios.

$$\frac{E[R_i] - R_f}{Cov[R_M, R_i]} = \frac{E[R_M] - R_f}{V[R_M]}$$

$$E[R_i] = (E[R_M] - R_f)\frac{Cov(R_M, R_i)}{V[R_M]} + R_f$$

$$\boxed{E[R_i] = R_f + \beta(E[R_M] - R_f)}$$

$\beta$ is therefore the measure of systemic risk of an asset. This is because according to CAPM in an efficient market, investors are only compensated for bearing systematic risk, which is the risk that cannot be diversified away by holding a diversified portfolio.

## 4.1 Application of CAPM (1) – Excess Return

Analyst compare stocks' returns with their fair expected return from the CAPM

$$\alpha = E[R_i] - [R_f + \beta(E[R_M] - R_f)]$$

A stock's $\alpha$ is the unexpected deviation from the fair return. Stocks with high alpha are under-valued and should be bought.

## 4.2 Application of CAPM (2) – Capital Budgeting Decisions

CAPM can also be used to judge whether firms should undertake risky projects. This can be done by calculating the Net Present Value (NPV) or compare the CAPM expected return with internal rate of return (IRR) (the rate of return where NPV = 0). If $E[R_i] > IRR$, then the firm should reject the project.

## 4.3 Index Model

An index model uses an actual stock index to proxy this theoretical market. CAPM makes forward-looking predictions. Index models use historical data. Index model states that

$$R_i - R_f = \alpha_i + \beta_i[R_M - R_f] + \varepsilon_i$$

$\alpha$ is the abnormal return and $\varepsilon$ is the firm specific risk, which is idiosyncratic risk. And the CAPM equation can be rewritten in terms of the index model

$$\boxed{R_i = R_f + \beta_i[R_M - R_f] + \varepsilon_i}$$

The variance percentage of the idiosyncratic risk and systemic risk can therefore be quantify as

$$\begin{aligned} V[R_i] &= V[R_f + \beta_i[R_M - R_f] + \varepsilon_i] \\ \sigma_i^2 &= V[\beta_i(R_M - R_i) + \varepsilon_i] \\ &= \beta_i\sigma_M^2 + \bar{\sigma_i}^2 \end{aligned}$$

$\boxed{\bar{\sigma_i}^2/\sigma_i^2}$ is the variance percentage that is **idiosyncratic** and 1-term will then be the systemic risk

# 5 Lecture 5: Risk Pricing and Arbitrage

Arbitrage is a trading strategy that **(1)** require NO initial investment. **(2)** Has NO negative cash flows at any time. **(3)** Has a positive cash flow at at least one time. Arbitrage also implies and solves mispricing.

Important insight in theoretical finance: (1) True arbitrage cannot exist. (2) Prices would adjust to eliminate it. (3) The no-arbitrage condition.

## 5.1 Bid-Ask Spread

**Bid:** the price at which market maker buys an asset.

**Ask:** the price at which market maker sells an asset.

**Mid:** (Bid+Ask)/2, a way to value holdings.

The Bid-Ask spread is normally set base on liquidity and other reasons, for a market maker, they care about (1) Their risk aversion (2) Internal guidelines on inventory (3) Reputations. (4) **Adverse Selection**. if an investor wished to purchase stocks from a fund and has inside or more research on that specific asset, them the fund might widen their spread to minimise their risk in order to offset this lack of information.

## 5.2 Implication of No Arbitrage

### 5.2.1 The Law of One Price (LOOP)

If two securities have the same payoffs, they have the same price. e.g.

Chase is offering a bond that pays $100 in one year, at $94.34. Merrill Lynch is offering a bond that pays $100 in one year, at $95.23. In this arbitrage example, arbitrageurs tends to borrow/sell Merrill Lynch bond and buy the Chase bond to make profit.However transaction costs make it more difficult to find arbitrages, there isn't necessarily a deterministic price with transaction costs.

### 5.2.2 Replicating Portfolio

If a portfolio has the same payoffs as a security, it must have the same price. e.g.

If a bond three-year bond with a 10% coupon rate is trading for $100. At the same time

- A zero-coupon bond maturing in 1 year costs $98
- A zero-coupon bond maturing in 2 year costs $96
- A zero-coupon bond maturing in 3 year costs $93

From the replicating portfolio, we can refer that $10 in one year is worth $9.8 today; $10 in two year is worth $9.6 today, $110 in three year is worth $102.3 today, the bond should worth $121.7, therefore, buy the bond and short the replicating portfolio if it's priced at $100.

### 5.2.3 Dynamic Hedging Strategy

If a self-financing strategy has the same payoffs as a security, it must have the same price. e.g.

How much should a zero-coupon bond maturing in two years cost if:

- A zero-coupon bond maturing in one year costs $98.
- In one year, a one year zero-coupon bond also costs $98.

Need $100 in two year, so i need $98 in one year (for the second point), therefore, to get only $98 in one year, the bond today will cost 0.98*98= $96.04. If priced at $95: buy the two-year zero. Short 0.98 units of the 1-year zero at $96.04. Make $1.04.

# 6 Lecture 6: Equity Valuation

This section will be discussing on the intrinsic value and market of an equity, particularly on stocks

## 6.1 intrinsic Value

Value $V$ based on a fundamental analysis of a company's current assets and future prospects. It is the discounted value of the cash that can be taken out of a business during its remaining life.

## 6.2 Market Value

Value $P$ based on observed market prices and shares outstanding.

## 6.3 Dividend Discount Model (DDM)

Equity's value is equal to the discounted value of all future dividends.

$$\begin{aligned} V_0 = P_0 &= \frac{E(D_1)}{1+R} + \frac{E(P_1)}{1+R} \\ &= \frac{E(D_1)}{1+R} + \frac{E(\frac{E(D_2)}{1+R} + \frac{E(P_2)}{1+R})}{1+R} \\ &= \frac{E(D_1)}{1+R} + \frac{E(D_2) + E(P_2)}{(1+R)^2} \\ &= \frac{E(D_1)}{1+R} + \frac{E(D_2)}{(1+R)^2} + \frac{E(D_3)}{(1+R)^3} + \cdots \end{aligned}$$

In order to price a equity from its dividends, we first need to find out how to evaluate a company's dividend.

## 6.4 Expected Dividend

### 6.4.1 Dividend Payout Ratio

Ratio $d$ is equal to the expected share of earnings that will be paid out as a **dividend**.

$$\boxed{D_t = d_t \times E_t}$$

### 6.4.2 Earnings-Retention Ratio

Ratio $b$ is equal to the expected share of earnings that will be **reinvested**.

$$I_t = b_t \times E_t = (1 - d_t)E_t$$

if the retained earnings fund new projects with return $ri$. the expected return can then expressed as

$$E_{t+1} = E_t + I_t \times ri_{t+1} = E_t + b_t \times E_t \times ri_{t+1}$$

$$\boxed{g_{t+1} = \frac{E_{t+1} - E_t}{E_t} = b_t \times ri_{t+1}}$$

This leads to The **Growth Rate of Earnings**:

## 6.5 DDM Analysis

### 6.5.1 D constant

Suppose that the dividends are **constant**, i.e. $E(D_1) = D_0, E(D_2) = D_0 ...$

$$\begin{aligned} V_0 &= \frac{E(D_1)}{1+R} + \frac{E(D_2)}{(1+R)^2} + \cdots \\ &= \frac{D_0}{1+R} + \frac{D_0}{(1+R)^2} + \cdots = \boxed{\frac{D_0}{R}} \end{aligned}$$

### 6.5.2 D grow at rate g

Suppose now the **dividends grow** at a rate g, i.e. $E(D_1) = (1+g)D_0$, $E(D_2) = (1+g)^2 D_0$

$$\begin{aligned} V_0 &= \frac{D_0(1+g)}{1+R} + \frac{D_0(1+g)^2}{(1+R)^2} + \cdots \\ &= \frac{D_0(1+g)}{R-g} = \boxed{\frac{(1-b)E(E_1)}{R-g}} \end{aligned}$$

This is also called the **Gordon Growth Model**

From the table below, the amount of investment increases from left to right, and price adjust accordingly. If the return on new investment is exactly **equal** to the **CAPM implied return**, the things I'm investing in are compensating me exactly as well as the stock should be compensating me, so the price will remain unchanged

If the new investment return is **lower** than my CAPM implied return, that means I'm investing into something bad that doesn't gives me enough return, therefore, the **stock price must decrease** in order to **increase** the expected return.

Likewise, if my new investment return is **higher** than my CAPM implied return, than the stock **price must increase** in order to **decrease** the expected return.[1]

## 6.6 Two stage valuation

If a company dividends grow at different rates over time, i.e. first stage at rate $g_1$, second stage $g_2$

$$V_0 = \sum_{t=1}^{T} \frac{D_0(1+g_1)^t}{(1+R)^t} + \frac{D_T(1+g_2)}{(R-g_2)(1+R)^T}$$

[1] a stock with a very high price will require a larger dollar change in price to achieve the same percentage return as a stock with a very low price, resulting in a lower percentage return for the higher-priced stock. E.g. Stock A is priced at \$500 per share and Stock B is priced at \$50 per share. If both stocks increase in price by \$50 per share over a period of time, Stock A would have a 10% return (\$50 / \$500), while Stock B would have a 100% return (\$50 / \$50)

## 6.7 Valuation Ratio

### 6.7.1 Price-dividend ratio

$$\frac{P}{D} = \frac{1+g}{R-g}$$

### 6.7.2 Price-earnings ratio

$$\frac{P}{E} = \frac{(1+g)(1-b)}{R-g}$$

| | b = 0 | b = 0.25 | b = 0.5 | b = 0.75 | b = 0.99 |
|---|---|---|---|---|---|
| ri = 15% | 4.24 | 4.11 | 3.86 | 3.27 | 0.39 |
| ri = 21% | 4.24 | 4.65 | 5.74 | 19.42 | ND |
| ri = 27% | 4.24 | 5.35 | 11.18 | ND | ND |
| ri = 33% | 4.24 | 6.30 | 220.89 | ND | ND |

# 7 Lecture 7: Fixed Income Valuation

## 7.1 Bond Pricing

$V_0$ is the current value of the bond (at time 0). $C_t$ is the coupon payment at time $t$. $FV_T$ is the face value at maturity. $r_{0,t}$ is the current interest rate for investment maturing at time T.

$$V_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r_{0,t})^t} + \frac{FV_T}{(1+r_{0,T})^T}$$

## 7.2 Yield To Maturity (YTM)

- Yield to maturity (YTM) is the total return anticipated on a bond if the bond is held until it matures, accounting for the present value of a bond's future coupon payments
- Yield to maturity is the total rate of return that will have been earned by a bond when it makes all interest payments and repays the original principal.
- YTM is essentially a bond's internal rate of return (IRR) if held to maturity.

Given a price $P_0$, the the yield to maturity is the discount rate $y$ that sets the value $V_0$ equal to the price:

$$y \quad s.t. \quad P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{FV_T}{(1+y)^T}$$

## 7.3 Forward Interest Rate

The forward rate is the *inferred* rate from the yield curve. Say we have spot interest rate for investments with a one-year maturity, $\underline{r_{0,1} = 0.26\%}$. and spot interest rate for a two-year maturity, $\underline{r_{0,2} = 0.68\%}$, what would the Forward interest rate for one year investments starting in one year, $f_{1,2}$ be?

$$(1+r_{0,2})^2 = (1+r_{0,1})(1+f_{1,2})$$

This can be express as the **(1)** two year maturity value must equal to the one year maturity + the forward rate value due to arbitrage limitation. we can also be inferred that the **(2)** two year bond interest rate will be an average of the combination of the one year + forward interest rate. Thus, for an upward slopping yield curve, the **forward interest rate must be higher than both the one year and the two year**

## 7.4 Monetary Policy (Central Bank)

Purpose of messing with money supply: (1) Control inflation (2) low and sustainable unemployment.

$\boxed{\textbf{Inflation:}}$

- need to raise interest rate → take money out of circulation(market) → sell lots of government bond → bond price drops → increases interest rate and yield
- people tends to other use of money to get better profits (i.e. save in the bank), thus, bond prices tend to drop to attract.
- When **interest rates rise**, existing bonds paying lower interest rates become less attractive, causing their **price to drop** below their initial par value in the secondary market. (The coupon payments remain unaffected.) **also increases the yield**
- if the expected interest rate between now and one year is going to grow, that means the economy is expected to grow, the central bank is then expected to raise interest rates to make sure it doesn't grow too fast.

$\boxed{\textbf{Recession:}}$

- need to lower interest rate → put money into circulation(market) → buy lots of government bond → bond price increases (supply drops) → decreases interest rate and yield
- if the expected future interest rate between now and one year is going to drop, that means an recession is expected, therefore the central back cuts rate.

## 7.5 Rational Expectations Theory

**(1)** Long run rates are a geometric average of current and future short rates. **(2)** The expected future interest rate is equal to the forward rate.

$$(1+r_{0,2})^2 = (1+r_{0,1})(1+E[r_{1,2}])$$

Empirically, slope of yield curve is good predictor of GDP growth rate (recession)

## 7.6 Liquidity Preference Theory

**(1)** Long-term lenders require compensation due to lack of liquidity ($\pi_{1,2}$ liquidity premium). **(2)** higher yields for longer maturity bonds, the yield curve will then be upwards slopping independent of the expectations.

$$(1+r_{0,2})^2 = (1+r_{0,1})(1+E[r_{1,2}]+\pi_{1,2})$$

If the yield curve is observed to be upwards sloping, then future spot rates are expected to be higher than current spot rates under Rational Expectations Theory and indeterminate under Liquidity Preference Theory

### 7.7 Segmented Markets Theory

different people are trading different stuffs, thus you can't do much interference on comparing markets. e.g. index tax funds wants to hold the longest safest bonds possible (30 years), bond price increases, yield decreases. This explains persistent differences in rates (20yr vs 30yr). They trade it because they "like" it.

In general, for a downwards sloping yield curve, different interpretation can be inferred from the above theory. From **(1) RET** interest rates expected to fall. From **(2) LPT**, Interest rates expected to fall (by more than RET). From **(3) SMT**, Long maturity investors demand lower returns than short maturity investors.

### 7.8 Duration

Bond prices may be affected severely to interest rate risk, how much will the bond price be affected by changes in interest rate? What happens when the **yield curve shifts up parallel (level).**

$$D = -\frac{dP}{dy}\frac{1+y}{P} = \sum_{t=1}^{T} w_t t \quad \text{where } \; w_t = \frac{\text{CashFlow}_t}{P(1+y)^t}$$

- Duration can measure how long it takes, in years, for an investor to be repaid a bond's price by the bond's total cash flows
- Duration also measures a bond's or fixed income portfolio's price sensitivity to interest rate changes.
- Most often, when interest rates rise, the higher a bond's duration, the more its price will fall.
- Higher Duration → lower coupon rate (longer term to maturity and more volatility) → more vulnerable to interest rate risk
- Higher coupon has the shorter duration.
- e.g. for duration of 5 years, 1% increase in interest rate, bond value decreases by 5%, vice versa.
- Sharpeners and levellers are strategies to **hedge yield curve shape** changing.

# 8 Lecture 8: Options

Contracts that give one party the **(1)** *right* to buy or sell a certain security, target risk better and information. **(2)** Allow us to learn incredibly granular levels of information. **(3)** target risks better and incredibly specific **(4)** Put information in a more targeted way.

## 8.1 Put Option

The right to sell an asset for a certain price at a certain time in the future.

## 8.2 Call Option

The right to buy an asset for a certain price at a certain time in the future.

## 8.3 Long Position

The option buyer or holder pays a premium and receives the right to buy or sell an asset.

## 8.4 Short Position

The option seller or writer receives a premium and has the obligation to deliver or purchase an asset.

## 8.5 Evaluate Options at Expiry

- $S_t$: Spot price, asset trading at time $t$.
- $X$: Strike or exercise price.
- $T$: Strike or expiration date.
- $C_t$: Price of a call option.
- $P_t$: Price of a put option.

| | | Calls | | | Puts | | |
|---|---|---|---|---|---|---|---|
| Strike | Maturity | Bid | Ask | Volume | Bid | Ask | Volume |
| 135 | Feb-23 | 8.95 | 9.15 | 54 | 0.75 | 0.76 | 2,147 |
| 140 | Feb-23 | 5.35 | 5.4 | 849 | 2.1 | 2.12 | 6,134 |
| 145 | Feb-23 | 2.68 | 2.7 | 4,167 | 4.4 | 4.45 | 225 |
| 150 | Feb-23 | 0.99 | 1 | 4,613 | 7.7 | 7.8 | 96 |
| 155 | Feb-23 | 0.29 | 0.3 | 1,332 | 11.9 | 12.15 | 74 |
| 135 | Aug-23 | 18.5 | 18.95 | 11 | 7.65 | 7.95 | 1 |
| 140 | Aug-23 | 15.45 | 15.75 | 2 | 9.5 | 9.8 | - |
| 145 | Aug-23 | 12.665 | 12.9 | - | 11.65 | 11.9 | - |
| 150 | Aug-23 | 10.1 | 10.45 | 1 | 14.1 | 14.1 | 14 |
| 155 | Aug-23 | 8 | 8.2 | - | 16.95 | 17.35 | - |
| 135 | Jan-24 | 23.35 | 23.8 | 10 | 10.1 | 10.45 | 10 |
| 140 | Jan-24 | 20.35 | 20.85 | 31 | 12 | 12.55 | 5 |
| 145 | Jan-24 | 17.5 | 18 | 6 | 14.2 | 14.7 | 7 |
| 150 | Jan-24 | 14.95 | 15.35 | 15 | 16.7 | 17 | 20 |
| 155 | Jan-24 | 12.7 | 13.1 | 1 | 19.2 | 19.85 | 2 |

From the Table above we can infer some information about the stock

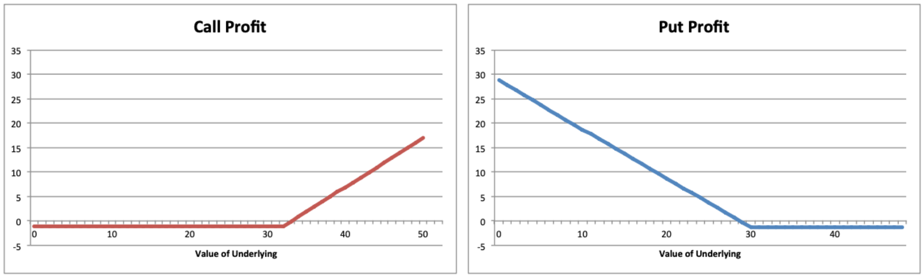
- The price of Call options decreases with increasing strike price, this is because people want to buy the stock at lower price instead of higher price, so I have to pay more for that right than the right to buy it at a more expensive price.
- The opposite is true for put options, where the right to sell at a lower price is less desirable
- For next-day expire option, if I know the price is not gonna change a lot between in one day for not very volatile stocks, and I know the right to buy this asset at say \$135 is \$9 today, it's probably gonna be \$9 tomorrow as well. Therefore, the value of the asset is most likely to be \$9 above \$135, which is around \$144 (or a range).
- options strictly has to be cheaper than the actual asset. It is also cheaper to buy an option if you think the price is going up.

A **Call option's value** is:

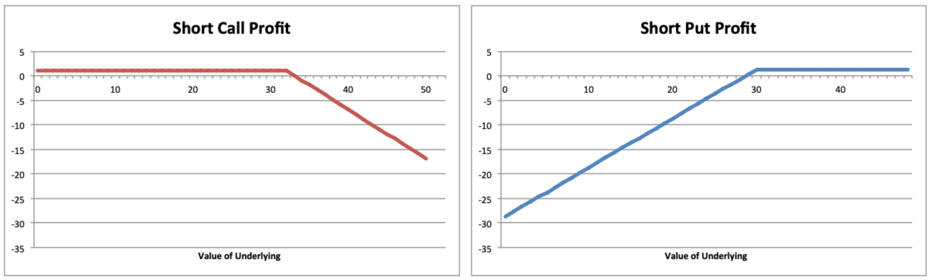
$$C_T = \begin{cases} S_T - X & \text{if } S_T > X \text{ (in the money)} \\ 0 & \text{if } S_T \leq X \text{ (at/out of the money)} \end{cases}$$

A **Put option's value** is:

$$P_T = \begin{cases} 0 & \text{if } S_T \geq X \text{ (at/out of the money)} \\ X - S_T & \text{if } S_T < X \text{ (in the money)} \end{cases}$$




long option

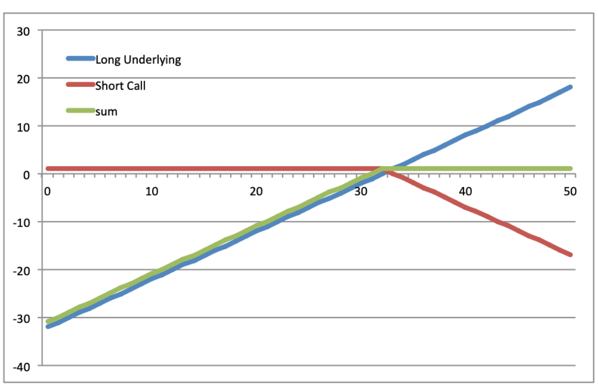



short option

## 8.6 Option Strategies

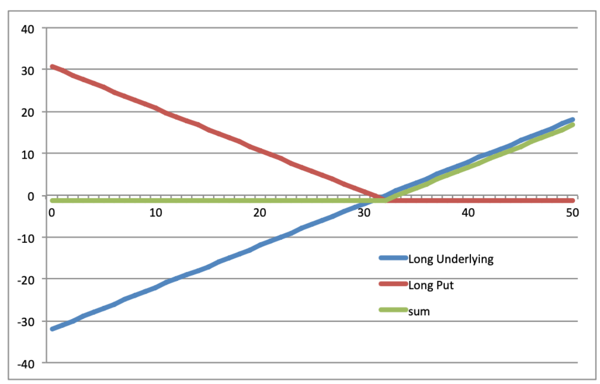
### 8.6.1 Covered Calls

- Buy the stock at 30, and sell the call option.
- if stock price goes up, earn money from stock but lose from selling the call option
- if stock price goes down, lose money from stock and earn a little for not exercising the sell option.
- combination becomes a **short put option**




### 8.6.2 Protected Put

- Buy the underlying, and buy a put option.
- The loss from the assets will be compensated by the profit made from the put option.
- combination becomes a **call option**



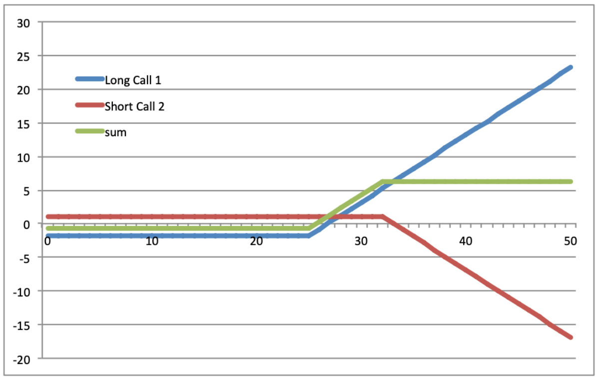

**covered calls** and **protected put** are ways to create the asset I want from assets I have. E.g. if I have the underlying asset and want to create a call option, I'll then buy a put, etc.

### 8.6.3 Bull Spread

- Buy a low-strike call option and sell a high-strike call option.
- given a fair amount of upside for less net losses. why?
- If you expect the asset to rise, but not that much,

- paying for the blue call option will be overpaying, therefore, the green line (sum) will be cheaper than the call option itself.

- popular because it;s a managed cash flow, very low risk asset and fluctuations are small

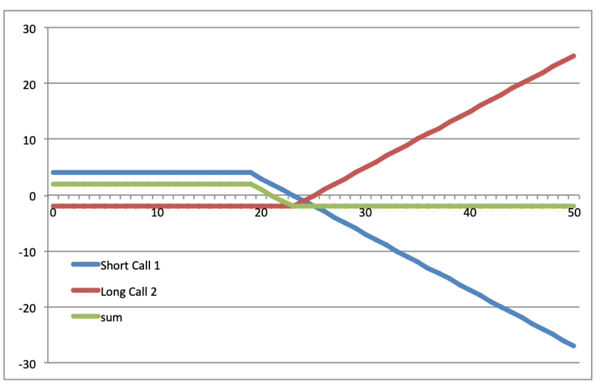



### 8.6.4 Bear Spread

- Buy a high-strike call option and Sell a low-strike call option.

- flipped version of the bull Spread.

- gains and losses are capped, cheaper way to bet on downward price movements.



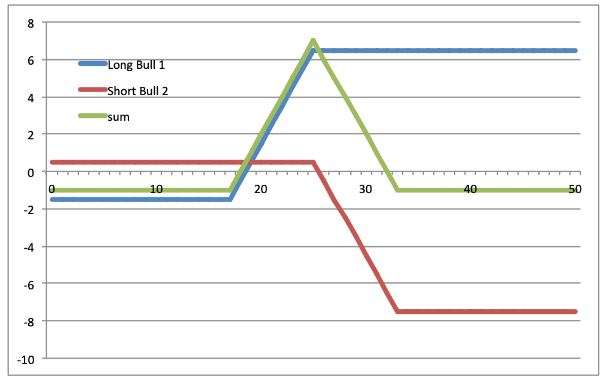

### 8.6.5 Butterfly Spread

- Buy a low strike bull spread and sell a high strike bull spread.

- Call($X_1$)-Call($X_2$) - (Call($X_2$)-Call($X_3$)) = Call($X_1$)-2Call($X_2$)+Call($XX_3$)

- An extremely target bet, therefore, the price of this asset will be very informative about how likely the market thinks the final value of the underlying is gonna stay in that region.

- can use this spread to infer the underlying volatility.



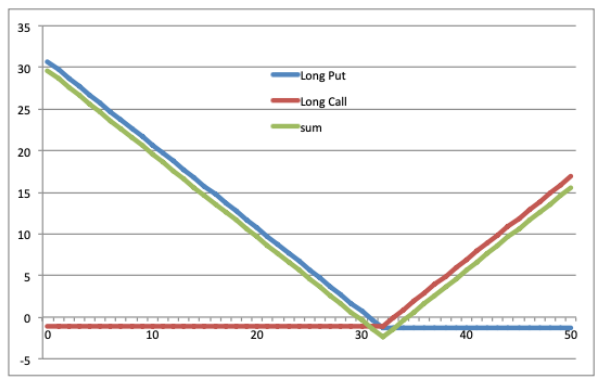

### 8.6.6 Straddles/strangles

- Buy a put option and a call option at the same (different) strikes

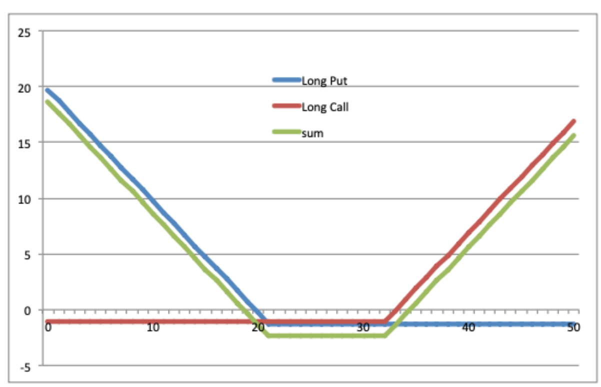
- Bets purely on the volatility of the asset

- for straddle, you make money as long as the price changes.

- for strangle, make money with minimum threshold.



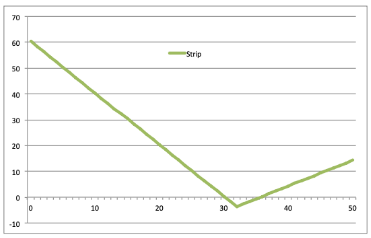

straddle

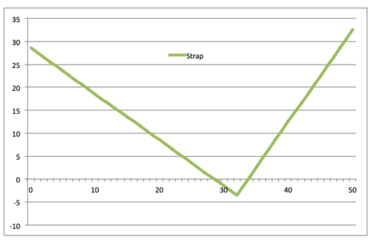



strangle

### 8.6.7 Strips/straps

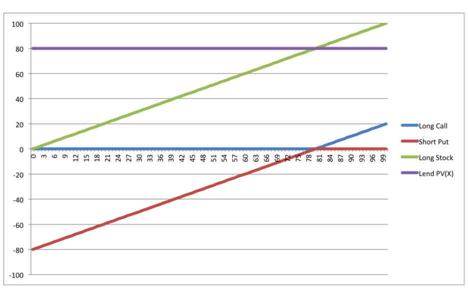
- Strips is a straddle with an additional put
- strips makes money as long as the price moves, but make more if the price moves down
- Strap is a straddle with an addition call
- straps makes money as long as the price moves, but make more if the price moves up




## 8.7 Put-Call Parity

- (1) If I buy a call and sell a put option at the same strike price.
- (2) And I also buy the stock
- (3) Also lend the present value of the strike price.
- Put-Call parity states that the blue line plus the red line plus the purple line should equal to the green line.




$$\boxed{C(K,T) - P(K,T) = S_0 - e^{-rT}K}$$

Where $C$ is the price of a call, $P$ is the price of a put, $T$ is the time to expiry, $K$ is the strike, $S_0$ is the spot price of the underlying, and $r$ is the risk-free rate.

| Action | Today | $S_T$ <X | $S_T$ >X |
|---|---|---|---|
| Buy a call | -C | $S_T$-X | 0 |
| write a put | +P | 0 | $S_T$-X |
| Lend PV(X) at R | -e$^{rT}$X | X | X |
| Net cashflow | $\underbrace{P - C - e^{rT}X}_{S_0}$ | $S_T$ | $S_T$ |

By buying a call, selling a put, and lending the present value of the strike, I ended up with a portfolio $S_T$, which implies that I ended up owning the stock, or owning the cash equivalent of the stock no matter the stock does well or bad. So another way to *own* a stock is to *buy* the stock. Thus, this portfolio perfectly replicates the payoffs of owning this stock.

## 8.8 Option Pricing

### 8.8.1 Risk-Neutral Probability Distribution

Say there is a fair coin ($P(H) = P(T) = 0.5$) option where I offer you $2 if it lands on head and $0 if it lands on tails, how much will you be willing to pay for this option. From an expected future value (EFV) sense, the expected value will be $0.5 \cdot \$2 + 0.5 \cdot \$0 = \$1$, if we account for time discount ($R_f = 3\%$) then the expected discount value (EDV) will be $\$1 \cdot e^{-rT} = \$0.97$. However, because the market tend to be risk averse, the price will tend to what is called the certainty equivalent[2]. Assume that the certainty equivalent is $0.85, then from EFV, the discount rate is now 16.3% ($Pe^{r}T = FV$), which there is a 13.3% risk premium for compensation. The **Risk Neutral Probability** are then defined as the probability that equates the expected discount value to the market price.

$$\$0.85 = [\$2 \cdot p + \$0 \cdot (1-p)]e^{-r_f}$$
$$p = \frac{0.85}{2}e^{r_f} = 43.8\%$$

The term *Risk Neutral* is defined as if we go to an imaginary world where the probability calculated above are accurate for the coin, and then calculate the expected discount value, and under the assumption that we are risk neutral[3], the EDV calculated ($0.85) will thus be a fair market price to pay.

[2]The certainty equivalent cash flow is the risk-free cash flow that an investor or manager considers equal to a different expected cash flow which is higher, but also riskier. It is closely related to the concept of **risk premium** or the amount of additional return an investor requires to choose a risky investment over a safer investment

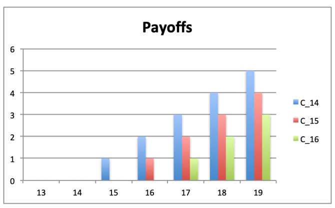
[3]Under the assumption of risk neutrality, investors do not require a higher expected return to compensate them for taking on more risk, and they do not penalize investments with higher risk. Instead, they only care about the expected return of the investment, which is the weighted average of the possible outcomes of the investment, where each outcome is weighted by its probability.

Good states of the world will be undervalued (43.8%), bad states will be overvalued (56.2%). This is how risk aversion shows up in prices. The Risk-Neutral Distribution tells us about the physical probability (P) but also with risk aversion, and it's hard to separate these two.

### 8.8.2 How to get Q distribution

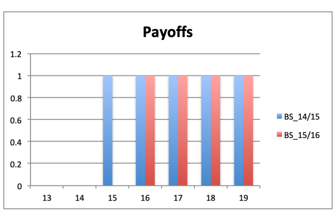
Consider three call options with the same maturity but different strike price:

- Call C(14) @ <u>2.21</u>
- Call C(15) @ <u>1.56</u>
- Call C(16) @ <u>1.05</u>



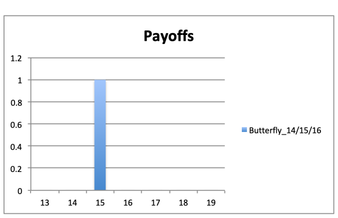

And then we construct a bull spread with these call options (Bull spread: buy low strike call, sell high strike call)

- Bull(14,15) @ (2.21-1.56 = <u>0.65</u>)
- Bull(15,16) @ (1.56-1.05 = <u>0.51</u>)




And finally, if we construct a butterfly spread (Buy low strike bull, sell high strike bull).

- Butterfly(14,15,16) @ (0.65-0.51 = <u>0.14</u>)




This particular butterfly spread price Arrow-Debreu Securities, where if one particular state of the world happens you get \$1 and \$0 if any other state of the world happens.

what does the price **<u>0.14</u>** tells us? 0.14 is the **risk-weighted probability** that includes the risk aversion of people, also reflects the aggregate beliefs. From a mathematical sense, we can also express the above as (per dollar price):

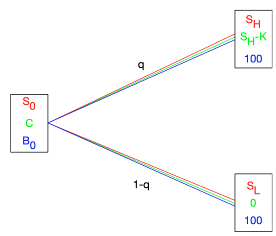
$$\text{Bull spread} = \frac{C(X+\varepsilon) - C(X)}{X+\varepsilon - X} = \boxed{C'(X)}$$

$$\text{Butterfly} = \frac{\text{Bull}(X+2, X+1) - \text{Bull}(X+1, X)}{X+1-X} = \boxed{C''(X)}$$

This shows that we can use options prices to extract risk-neutral probabilities. We can use this probability distribution to account for the volatility of the asset (option), by calculating the standard deviation of the resulting distribution by extracting forward-looking expectation from prices. Because options are stackable, you can create extremely targeted bets, which give you extremely specific pieces of information. This probability includes risk aversion.

### 8.8.3 Option Pricing

I have a stock ($S_0 = 100$), A bond ($B_0 = 100$) with $R_f = 2\%$, and I want to replicate a call option $C$ with strike price $K$.




If I try to replicate the payoffs of the call option by buying $\Delta$ amount of stock $S_0$ and $X$ unit of bond $B$. Assume that for high scenarios, the stock $S_H = 110$, and at low times, the stock $S_L = 90$, whereas the bond pays 100 no matter what. Therefore, the call at maturity for high is \$10, and \$0 at low.

$$\Delta 110 + X100 = 10$$
$$\Delta 90 + X100 = 0$$
$$\therefore \quad \Delta = 0.5, \quad X = -0.45$$

If i can replicate the payoff of the call option, the price of the replicating portfolio must equal to the price of the call option.

$$\Delta \cdot S_0 + X \cdot B_0 = 0.5 \cdot 100 - 0.45 \cdot \frac{100}{1.02} = 5.88$$

If the stock price change by 1 dollar, the value of the call option change by $\Delta$ dollars, sensitivity of the option price to the underlying asset, which is the beta. $X$ tells you the sensitivity of the call option price effectively to the risk-free rate or to the bond price.