Imperial College London

Orbit Determination and Estimation with Perturbation

AERO70016 Orbital Mechanics Coursework

Academic responsible:	Dr Rob Hewson
	Dr Davide Amato
Department:	Department of Aeronautics
Course:	MEng in Spacecraft Engineering
Module:	AERO70016 Orbital Mechanics
Academic year:	2021/2022
Student:	Chung-Hsun, Chang Chien
CID:	01771697
Date:	16/03/22

Department of Aeronautics
South Kensington Campus
Imperial College London
London SW7 2AZ
U.K.

Contents

0.1 Question (i)

Given the initial state vector of the osculating orbital elements $y_0 = (a_0, e_0, i_0, \Omega_0, \omega_0, \theta_0)$, we can calculate the corresponding initial position and velocity vector from the follow equation,

$$\begin{aligned}\mathbf{r} &= \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}(\cos\theta\mathbf{i}_e + \sin\theta\mathbf{i}_p) \\ \mathbf{v} &= \frac{\mu}{h}[\sin\theta\mathbf{i}_e + (e+\cos\theta)\mathbf{i}_p]\end{aligned} \tag{1}$$

The position and velocity vector calculated from Equation 0.1 requires a transform in reference frame from perifocal frame to Earth Centred Inertial (ECI) reference frame, simply by multiply the $\mathbf{r}$ and $\mathbf{v}$ with a matrix resulting from the unit vector rotation. The resulting initial position and velocity vector is presented below in units of km and km/s.

$$r_0 = \begin{bmatrix} 2.4088 \\ -6.4427 \\ 9.0949e\text{-}16 \end{bmatrix}_{(\times 10^3)} \qquad v_0 = \begin{bmatrix} 4.2908 \\ 1.6052 \\ 6.0795 \end{bmatrix}$$

0.2 Question (ii)

To propagate the trajectory in the required time interval $10T_0$, Cowell's Method was used alongside with MATLAB ode45 integrator, to numerically integrate the orbital equation under gravitational acceleration and oblateness perturbation shown below,

$$\frac{d^2\mathbf{r}}{dt^2} + \mu\frac{\mathbf{r}}{r^3} = \mathbf{a}_d = -\frac{3}{2}\frac{J_2\mu R_\oplus \mathbf{r}}{2r^5}\begin{bmatrix} 1 - \frac{5r_z^2}{r^2} \\ 1 - \frac{5r_z^2}{r^2} \\ 3 - \frac{5r_z^2}{r^2} \end{bmatrix} \tag{2}$$

To improve the accuracy of our solution, the initial state vector was non-dimensionalised by a length unit, DU, and a time unit, TU, respectively to the variables. In order to solve the orbital equation, a function named *EoM.m* was created beforehand to calculate the derivative of our state vector $\dot{\mathbf{x}} = [\dot{\mathbf{r}}\ \dot{\mathbf{v}}]^T$ to pass into the integrator solver. Constraints were also implemented by setting the relative tolerance and absolute tolerance to 1e−8 to reach to a converged solution.

$$r_f = \begin{bmatrix} 2.5348 \\ -6.3583 \\ 0.6759 \end{bmatrix}_{(\times 10^3)} \qquad v_f = \begin{bmatrix} 4.0549 \\ 2.2601 \\ 6.0331 \end{bmatrix}$$

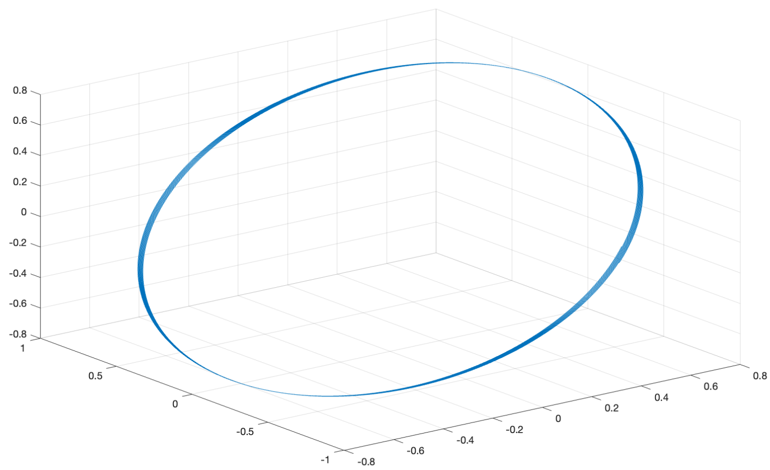

Figure 1: Orbit Trajectory Result

A visualisation of the result orbit trajectory is shown in Figure 1 and with the addition of Earth in Figure 2. The Earth oblateness is not scaled in the following figure, as it serves only the purpose to visualise that Earth is not a perfect sphere as it is has a "bulging" Equator, and slightly "flat" in the poles.

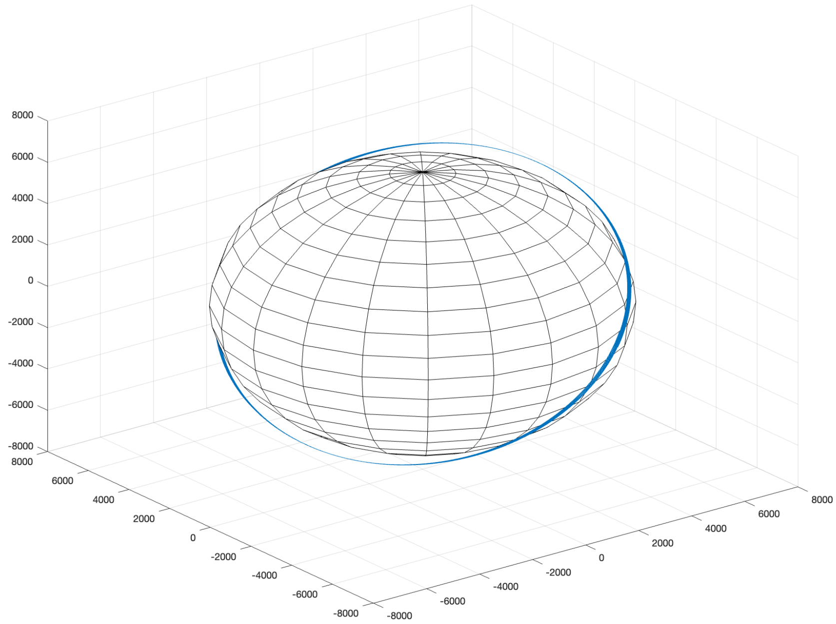

Figure 2: Result Trajectory with Earth Visualisation

0.3 Question (iii)

After acquiring the position and velocity state vector for all time in the time interval, we can then proceeded to calculate the orbital element at each point of time. The plots below represents the propagating osculating orbital elements throughout the time in hrs. We can see that the elements all exhibit an oscillatory behaviour except the Right ascension of ascending node (RAAN), shown in the first subplot in Figure 4.

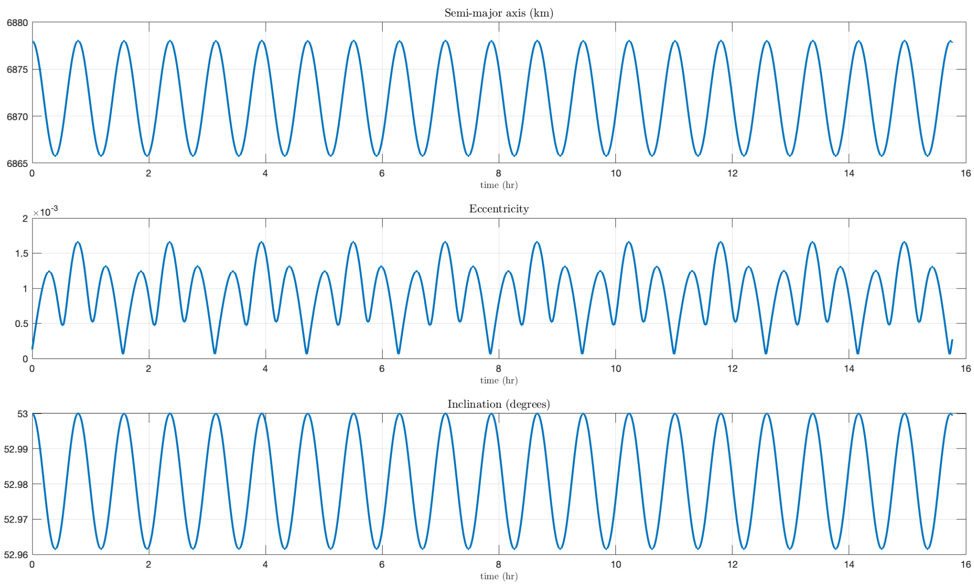

Figure 3: Osculating Orbital Element Through Time Interval

To further investigate the behaviour of the osculating element, we will be focusing primarily on the average rate of RAAN and the argument of perigee, ω, with respect to the average rate obtained using analytical methods. The averaged oblateness perturbation on the orbital element can be simplified and calculated using the averaged Gauss variational equation, as shown in Equation 3.

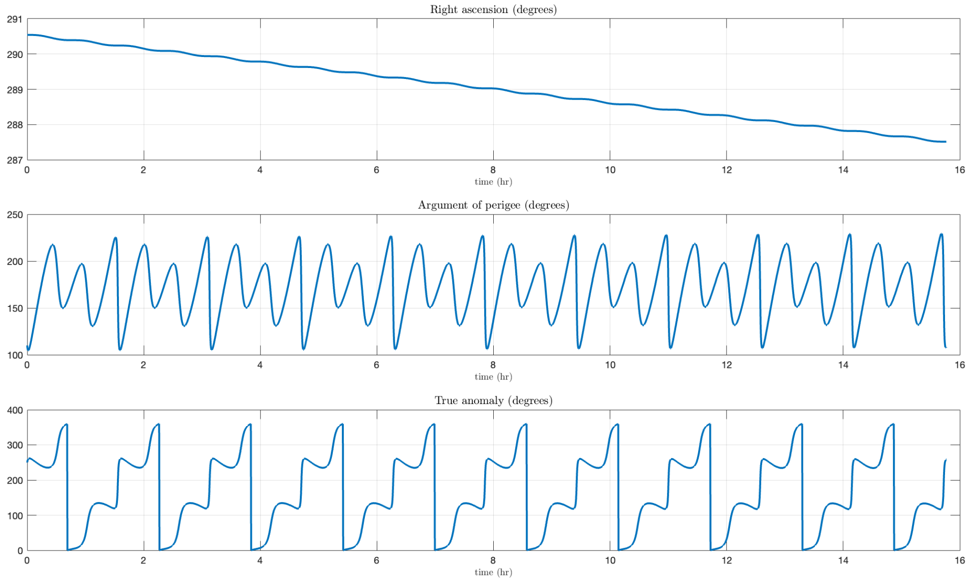

Figure 4: Osculating Orbital Element Through Time Interval (continued)

$$\bar{\dot{\Omega}} = -\frac{3}{2}\frac{nJ_2\mu^2R_\oplus^2}{h^4}\cos i \qquad \bar{\dot{\omega}} = \frac{3}{2}\frac{J_2\mu^2R_\oplus^2}{h^4}\frac{n}{2}(5\cos^2 i - 1) \tag{3}$$

Looking at the rate of change of RAAN in Figure 4, we can clearly see it follows a decreasing trend which calculated to be -0.1926 deg/hr. The averaged rate from the graph coincides with what we got from the analytical formulation, which also suggested a downtrend with similar magnitudes. For the argument of perigee, ω, despite of the oscillatory behaviour from the plots, the trend still indicates a very slight increase overall. The averaged rate from both methods did come in the same magnitude and coincides with each other, but one important thing to take into account is that the graphical averaged rate was taken rather qualitatively and might require more sophisticated calculations to meet up with high accuracy comparison.

	Analytical (deg/hr)	Graph (deg/hr)
$\dot{\Omega}$	-0.1924	-0.1926
$\dot{\omega}$	0.1298	0.1446

Table 1: Average rate of change

0.4 Question (iv)

Encke's method uses the osculating orbit as a reference start point then proceeding to numerically integrate to solve the equation. The advantage of using it is that the perturbations are usually quite small, considering we are not solving it for high accuracy, so the integration can be conducted will less fine time steps. However, the disadvantage of using Encke's methods is we cannot propagate indefinitely without constantly rectifying our deviation from the reference point, meaning the error will start to accumulate.

The Gauss Variational Equations (GVE) serves a convenient way to explicitly express the rate of the element in terms of the disturbing force such as thrust. Such form is also advantageous for non-conservative forces because it is expressed directly from the disturbing acceleration. However, the limitation of the the GVE rises from the equations itself, as it suffers the issue of singularities. The eccentricity must be less than 1 due to the term $\sqrt{1-e^2}$ and $\sin i$, therefore the GVE is more suitable for higher values of eccentricity and inclination.