AERO70016\97079 Orbital Mechanics coursework

Submission deadline: Friday 18th March 2022 23:00 Europe/London

This coursework is numerical in nature, therefore you will have to use programming languages for scientific computing. Accepted programming languages are MATLAB and Python 3.

If using MATLAB, please make sure not to use any functions outside of those contained in a standard MATLAB installation. If using Python 3, please make sure not to use any functions outside of those in the NumPy and SciPy packages. If the code is not able to run, the corresponding questions will be assigned zero marks. To check whether your code is able to run, it is recommended to use either MATLAB Online or the Imperial RCS Jupyter notebook server.

Question 1.iii requires using a numerical integration algorithm for ordinary differential equations. If using MATLAB, you can employ the `ode45` solver. If using Python 3, you can employ the `scipy.integrate.solve_ivp` function from the SciPy library. In either case, please use relative and absolute tolerances of 10^{-8} or below for the numerical integration.

On Blackboard, please upload in a single .zip file:

- **One or more MATLAB .m files OR Python .py files containing the code used for the calculation of the results and the generation of the plots,**
- **A short technical report (max 3 pages) in PDF format detailing your answers, written using either Microsoft Word or LaTeX.**

Plots in the report must be clearly labelled, and the labels must report units of measure. Any pages past the limit will not be marked.

The coursework has a single question with four parts, provided on the following page.

Please use the physical constants in the table below.

Physical constants

Earth radius, R_E	$6378\ \mathrm{km}$
Earth gravitational parameter, μ_E	$398'600\ \mathrm{km^3 s^{-2}}$
Earth 2nd zonal harmonic coefficient, J_2	0.00108263

Question 1

At time t_0, a spacecraft has the following orbital elements in the Earth-centred inertial frame:

$$a_0 = 6878\ \text{km}, \qquad e_0 = 1.3 \times 10^{-4}, \qquad i_0 = 53°$$
$$\Omega_0 = 290.5°, \qquad \omega_0 = 110°, \qquad \theta_0 = 250°,$$

where θ_0 is the initial true anomaly.

Assume that the trajectory of the spacecraft is affected by the main Earth gravitational acceleration and by the oblateness perturbation exclusively.

i. Transform the initial state vector in osculating orbital elements $\boldsymbol{y}_0 = (a_0, e_0, i_0, \Omega_0, \omega_0, \theta_0)$ to position $\mathbf{r}_0$ and velocity $\mathbf{v}_0$ in the Earth-centred inertial frame. [20%]

ii. Solve the equations of motion in non-dimensionalised coordinates for the perturbed two-body problem numerically using Cowell's method for the interval $[t_0, t_0 + 10T_0]$. Here T_0 is the initial orbital period corresponding to the semi-major axis a_0. Use either the MATLAB `ode45` integrator, setting both absolute and relative tolerances to 10^{-8}. [35%]

iii. Obtain the state vector in osculating orbital elements $\boldsymbol{y}(t) = (a(t), e(t), i(t), \Omega(t), \omega(t), \theta(t))$ for all times $t \in [t_0, t_0 + 10T_0]$. Show the plots of the orbital elements for the entire time interval. Comment on the evolution of the osculating orbital elements in time, and assess whether the evolution of the osculating right ascension of ascending node and of the argument of perigee agree with the respective averaged rates. [25%]

iv. Comment on the relative advantages and disadvantages of using either Encke's method or the Gauss variational equations to numerically integrate this orbit. [20%]