Categorizing (parts of) exam questions

Question type	Marks in 2024	Example from 2019 (or 2023)
State a definition	25	13(a)
Prove a simple claim	31	3(b)
Prove something new	10	9
State a standard theorem	18	4(b)
Prove a standard theorem	5	10(c)
Give an example of...	3	2(b)
Just compute	8	7(b)(i)

State a definition

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- Make sure your quantifiers are in the **right order!**
- Don't waffle. "It's a set that doesn't include its endpoints."

Prove a simple claim

3. (a) Let $D \subseteq \mathbb{R}$, $f : D \to \mathbb{R}$ and $a, L \in \mathbb{R}$. State precisely what is meant by the phrase "*f has limit L at a.*"

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- **Estimate!** I.e. bound complicated quantities by simpler things that you can "control".

Prove something new

9. Let $g : \mathbb{R} \to \mathbb{R}$ be a bounded, differentiable function whose derivative $g' : \mathbb{R} \to \mathbb{R}$ is also bounded. For each $n \in \mathbb{Z}^+$, let $f_n : \mathbb{R} \to \mathbb{R}$ be the function

$$f_n(x) = g(x + 1/n).$$

Prove that the sequence (f_n) converges uniformly, clearly identifying its limit. [6]

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  - Induction?
  - Contradiction? If we assume H is true but C is **false**, can we show this leads to something clearly false?

State a standard theorem

4. (a) Let $D \subseteq \mathbb{R}$, $a \in D$ be a cluster point of D and $f : D \to \mathbb{R}$. State, but do not prove, *Carathéodory's Criterion* for differentiablity.

(b) State and prove the *Chain Rule*.

(c) Assume that $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable, $f(0) = 3$, $f'(0) = 2$, $g(3) = 7$ and $g'(3) = -4$. Compute $(g \circ f)(0)$ and $(g \circ f)'(0)$. [12]

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- Why did I ask this? Is it going to help with a later part of the question?

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10 (c) Let $f : [a,b] \to \mathbb{R}$ be a bounded function. Prove that f is Riemann integrable if and only if there exists a sequence $(\mathcal{D}_n)$ of dissections of $[a,b]$ such that

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- Is there a clever trick?

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4. Break statements down: what do they really mean?
5. Try to figure out the ideas/strategy behind a proof. Direct? Induction? Contradiction? Draw pictures to help you remember. Don't try to memorize verbatim.