MATH2017 Problem Set 3: Integration

Submit on Gradescope by 17:00, Monday 20 March 2023

1. (a) Construct the first Taylor approximant to $f : (-1, 1) \to \mathbb{R}$, $f(x) = \sin^{-1} x$ about $a = \frac{1}{2}$.

   (b) Use this approximant, and Taylor's Theorem, to find upper and lower bounds on $\sin^{-1}(3/4)$.

2. Let $f : [-1, 2] \to \mathbb{R}$, $f(x) = |x|$, and $\mathscr{D} = \{-1, 0, 1/2, 2\}$.

   (a) Draw the graph of f.

   (b) Compute the lower and upper Riemann sums $l_{\mathscr{D}}(f)$, $u_{\mathscr{D}}(f)$.

3. (a) Prove that $\displaystyle\sum_{j=1}^{n} j^3 = \frac{1}{4} n^2 (n+1)^2$.

   (b) Evaluate $\int_0^1 x^3\, dx$ directly from the definition, without using the Fundamental Theorem of the Calculus. *(Hint: use Lemma 5.13 or Theorem 5.16.)*

4. Let $f : (-1, 1) \to \mathbb{R}$, $f(x) = (1 + x^3)^{-3}$. Compute $g' : (0, 1) \to \mathbb{R}$ where

   (a) $\displaystyle g(x) = \int_0^x f.$

   (b) $\displaystyle g(x) = \int_{-x}^x f.$

   (c) $\displaystyle g(x) = \int_0^{f(x)} f.$

5. Let $f : \mathbb{R} \to \mathbb{R}$ be Riemann integrable on every interval $[a, b] \subset \mathbb{R}$ and define $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = \int_0^x f$. Prove that g is continuous. (Warning: you may **not** assume that f is continuous.)