

## Workshop 5: questions for week 6

1. Prove that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x + \sin x$  is injective.
2. Use L'Hospital's Rule to compute the limit  $\lim_{x \rightarrow 1} \frac{\sin \pi x}{1 - x^3}$ . Carefully define the objects  $I, a, f, g$  appearing in the statement of the rule (Theorem 4.15).
3. (a) Construct the  $n$ -th Taylor Approximant  $p_n(x)$  for the function  $\ln : (0, \infty) \rightarrow \mathbb{R}$  based at the point  $a = 1$ .  
(b) Use Taylor's Theorem to prove that the Alternating Harmonic Series,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

converges to  $\ln 2$ .