Workshop 9: questions for week 10

1. Determine whether the following series converge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

2. Let $a_n = 1$ if n is prime and $a_n = 0$ otherwise. Compute the radius of convergence of the power series

$$\sum_{n=2}^{\infty} a_n x^n.$$

3. Let the power series $f(x) = \displaystyle\sum_{n=0}^{\infty} a_n x^n$ and $g(x) = \displaystyle\sum_{n=0}^{\infty} b_n x^n$ have radii of convergence $R_1 > 0$ and $R_2 > 0$ respectively. What can we conclude about R, the radius of convergence of the power series $h(x) = \displaystyle\sum_{n=0}^{\infty} (a_n + b_n) x^n$?