Workshop 3: questions for week 4

1. Give a direct ε–δ proof that $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2x^2 + x$, is differentiable at 1.

2. (a) What precisely does it mean to say that a function $f : D \to \mathbb{R}$ **attains a maximum** at $c \in D$? Write your answer using quantifiers. What, if any, restrictions must one place on the domain D of f for this definition to make sense?

   (b) Assume that $f : [a, b] \to \mathbb{R}$ is differentiable, and attains a maximum at b. What can you deduce about $f'(b)$? Prove your assertion.

3. Consider the function $f : [0, 1] \to \mathbb{R}$ defined so that $f(0) = 0$, and, for all $x \in (1/(n + 1), 1/n]$, , where n is any positive integer, $f(x) = 1/n$.

   (a) Draw the graph of the function f.

   (b) Is f differentiable at 0? If so, what is $f'(0)$? Prove your assertion.

   (c) What properties does the function f have? (Bounded? Differentiable? Continuous? Surjective? Injective? Monotonic?)