Workshop 4: questions for week 5

1. Determine whether the following sets are open:

$$[0,1), \quad \mathbb{R}\backslash[0,1), \quad \mathbb{R}\backslash[0,1], \quad \mathbb{R}\backslash\{2^n : n \in \mathbb{Z}\}.$$

2. Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable and satisfy $f(1) = -1$, $f'(1) = 2$, $g(-1) = 1$, $g'(-1) = 7$. Compute:
   (a) $(g \circ f)'(1)$.
   (b) $h'(1)$ where $h(x) = f(f(x)^2)$.

3. (a) Prove that, for all $x, y \in \mathbb{R}$, $\frac{1}{2}(x^2 + y^2) \geq |xy|$.
   (b) Prove that, for all $x, y \in \mathbb{R}$,

$$\left|\ln \frac{4 + x^2}{4 + y^2}\right| \leq \frac{1}{2}|x - y|.$$

   (Hint: use the Mean Value Theorem.)