# Workshop 8: questions for week 9

1. Let $f : [0, 1] \to \mathbb{R}$, $f(x) = 1/(1 + x)$, and $g : [0, 1] \to \mathbb{R}$, $g(x) = 2x$. Compute

$$(a) \quad \|f\|, \qquad (b) \quad \|g\|, \qquad \text{and} \quad (c) \quad \|f - g\|.$$

   The **sup norm** of a function $\|f\|$ is defined in Definition 7.6.

2. Determine whether each of the following sequences converges pointwise (Definition 7.1). If it converges pointwise determine whether it converges uniformly (Definition 7.8).

   (a) $f_n : [1, \infty) \to \mathbb{R}$, $f_n(x) = nx^{-n}$.

   (b) $f_n : [2, \infty) \to \mathbb{R}$, $f_n(x) = nx^{-n}$.

   (c) $f_n : (1, \infty) \to \mathbb{R}$, $f_n(x) = nx^{-n}$.

3. Construct a sequence of bounded functions $f_n : [0, 1] \to \mathbb{R}$ that converges pointwise to an unbounded function $f : [0, 1] \to \mathbb{R}$.