

MATH2017 Problem Set 1: Limits of functions, differentiability

Submit on Gradescope by 17:00, Friday 17 February 2023

1. Let $D \subseteq \mathbb{R}$. Write down precise mathematical formulations of the following statements, using quantifiers (\forall , \exists):
 - (a) D is bounded above.
 - (b) D has a maximum element.
 - (c) D does not have a maximum element.

2. Prove from first principles (i.e. give a direct ε - N proof) that the following sequence converges:

$$a_n = \frac{3n+1}{2n-1}.$$

3. Show directly from the definition of the Cauchy property that the sequence

$$a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$$

is Cauchy.

4. Give a direct ε - δ proof that

$$\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2} = \frac{1}{2\sqrt{2}}.$$

Interpret this result in terms of differentiability.

5. Give a direct ε - δ proof that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^4$ is differentiable at 1.