Workshop 10: questions for week 11

1. (a) Prove that
$$f(x) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)^2} \frac{x^{2n-1}}{1+x^{2n}}$$
converges uniformly on $\mathbb{R}$. *[Hint: Weierstrass M Test!]*

   (b) Compute $\int_0^1 f$. Rigorously justify your answer.

2. Assume that, for all $x \in (0,4)$,
$$\sum_{n=0}^{\infty} a_n (x-2)^n = \frac{1}{x^2}.$$
Find a formula for a_n.

3. Is the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = |x|$ analytic? What about the function $g : \mathbb{R}\backslash\{0\} \to \mathbb{R}$, $g(x) = |x|$?