Tidal effects in compact binaries

Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation
Galileo Galilei Institute

March 10-15, 2025

Andrea Maselli
Costantino Pacilio

Plan of the course

Why do we care about tidal effects in compact binaries?

fundamental physics

distinguishing BH from stars

matter under extreme conditions

exotic compact objects or GR deviations

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what's the imprint of tidal interactions on GW?

how do we measure it [hands on]

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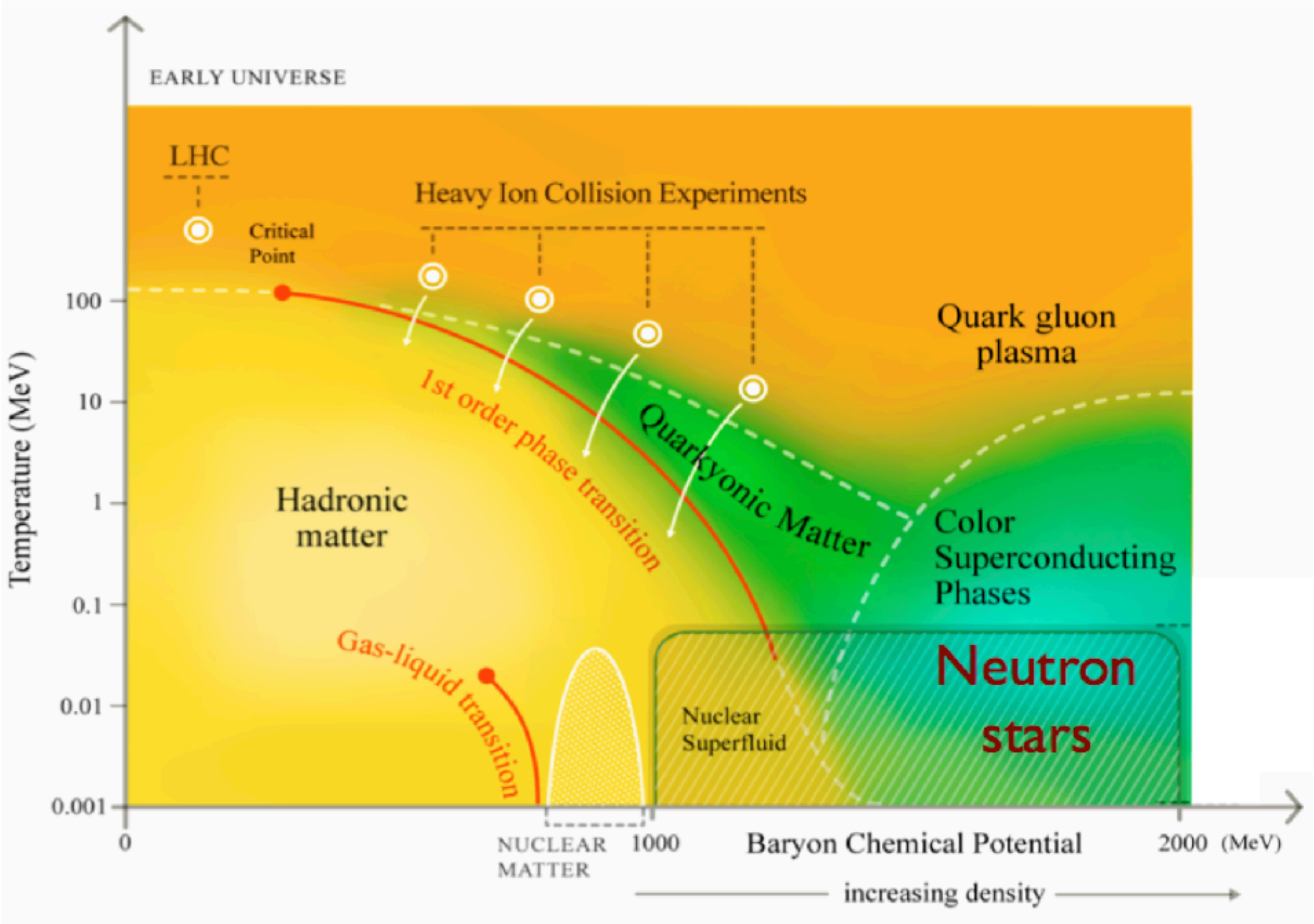
GW ? properties and binary modelling

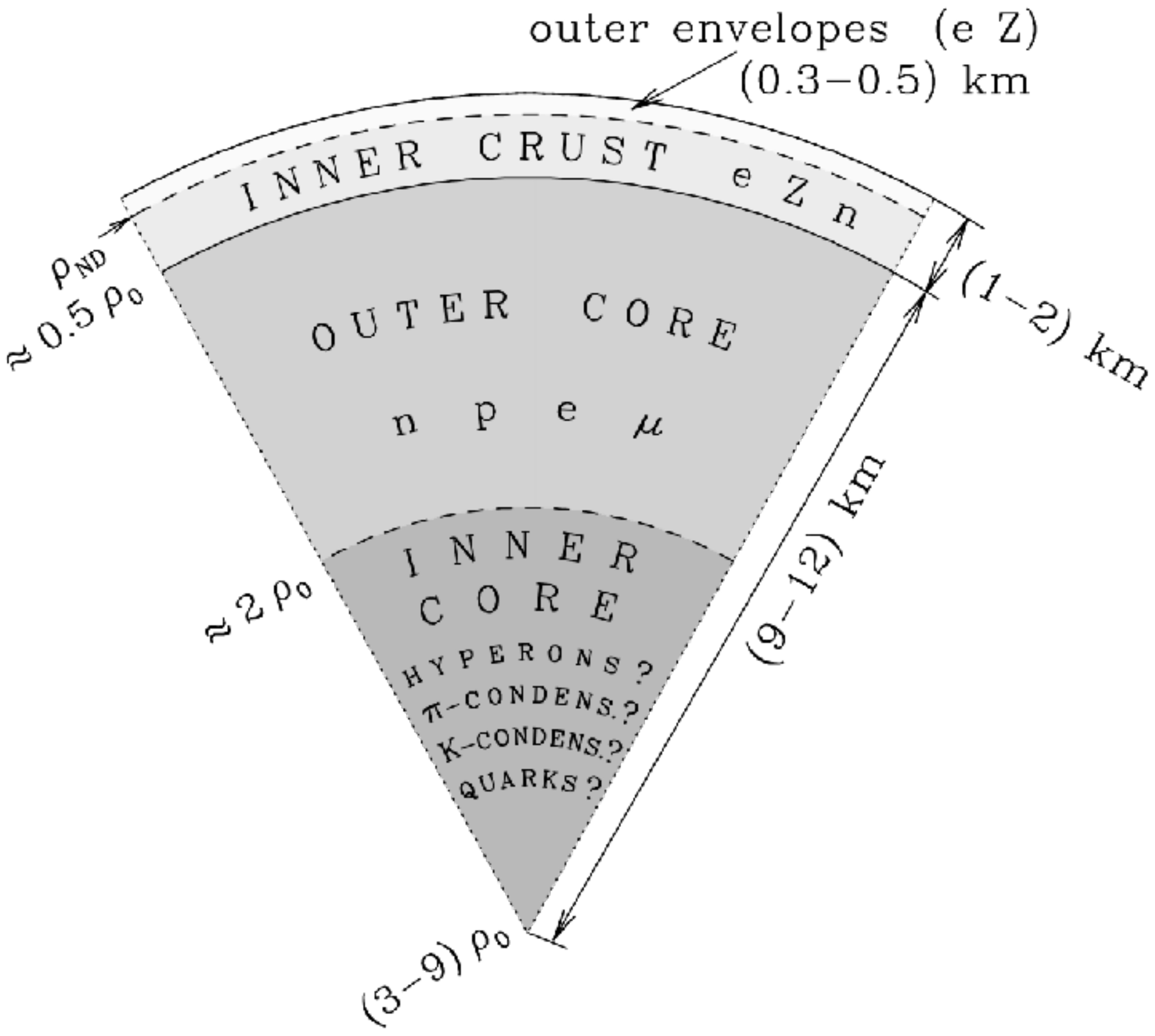
Love numbers of compact objects in GR

State of matter

Magnifying lenses of fundamental forces



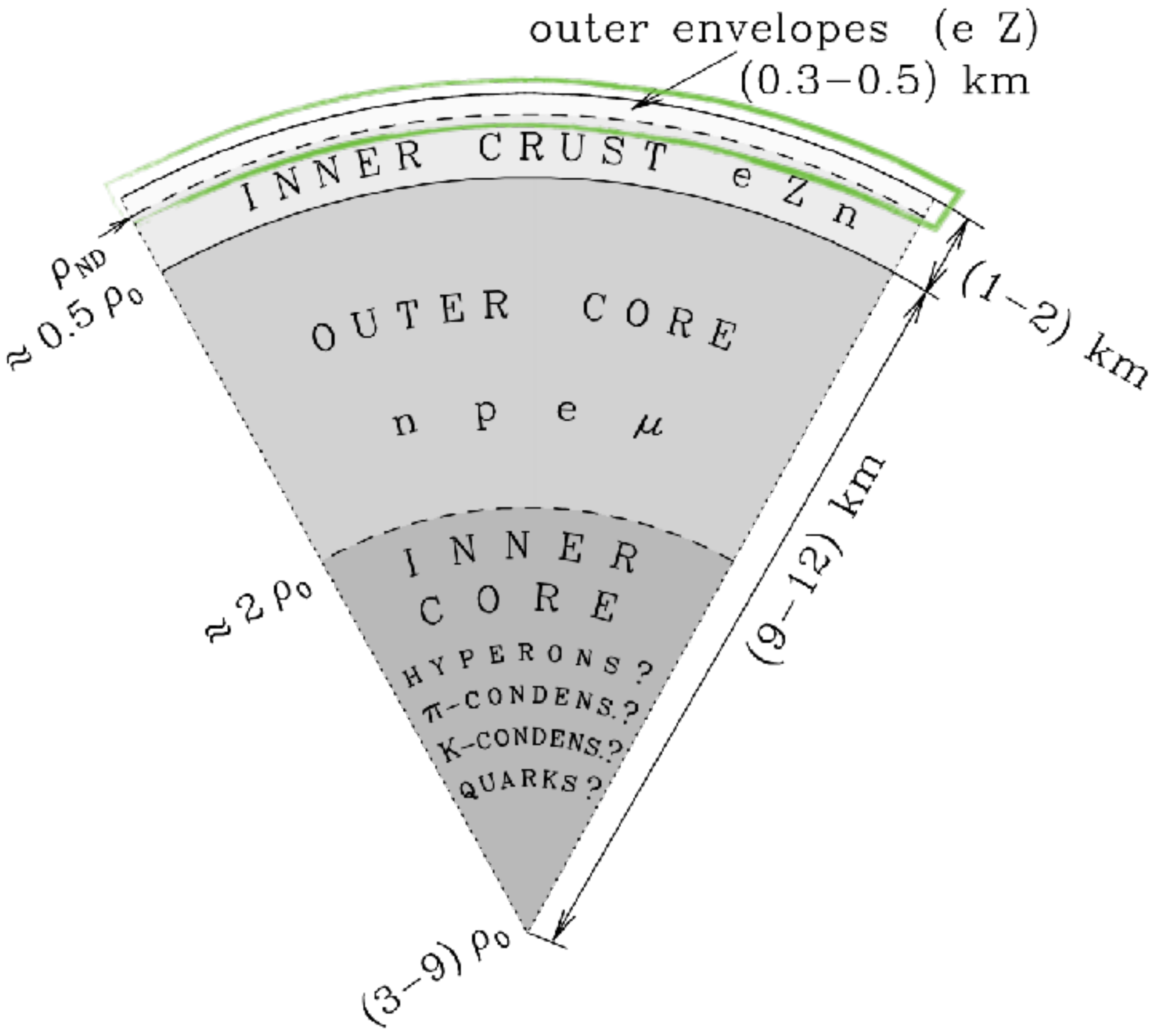
State of matter



$$\rho_0 \simeq 2.67 \times 10^{14} \mathrm{g\ cm^{-3}}$$

J. Lattimer & M. Prakas, Phys. Rep. 621 (2016)

State of matter



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Nuclei lattice within e^- gas

J. Lattimer & M. Prakas, Phys. Rep. 621 (2016)

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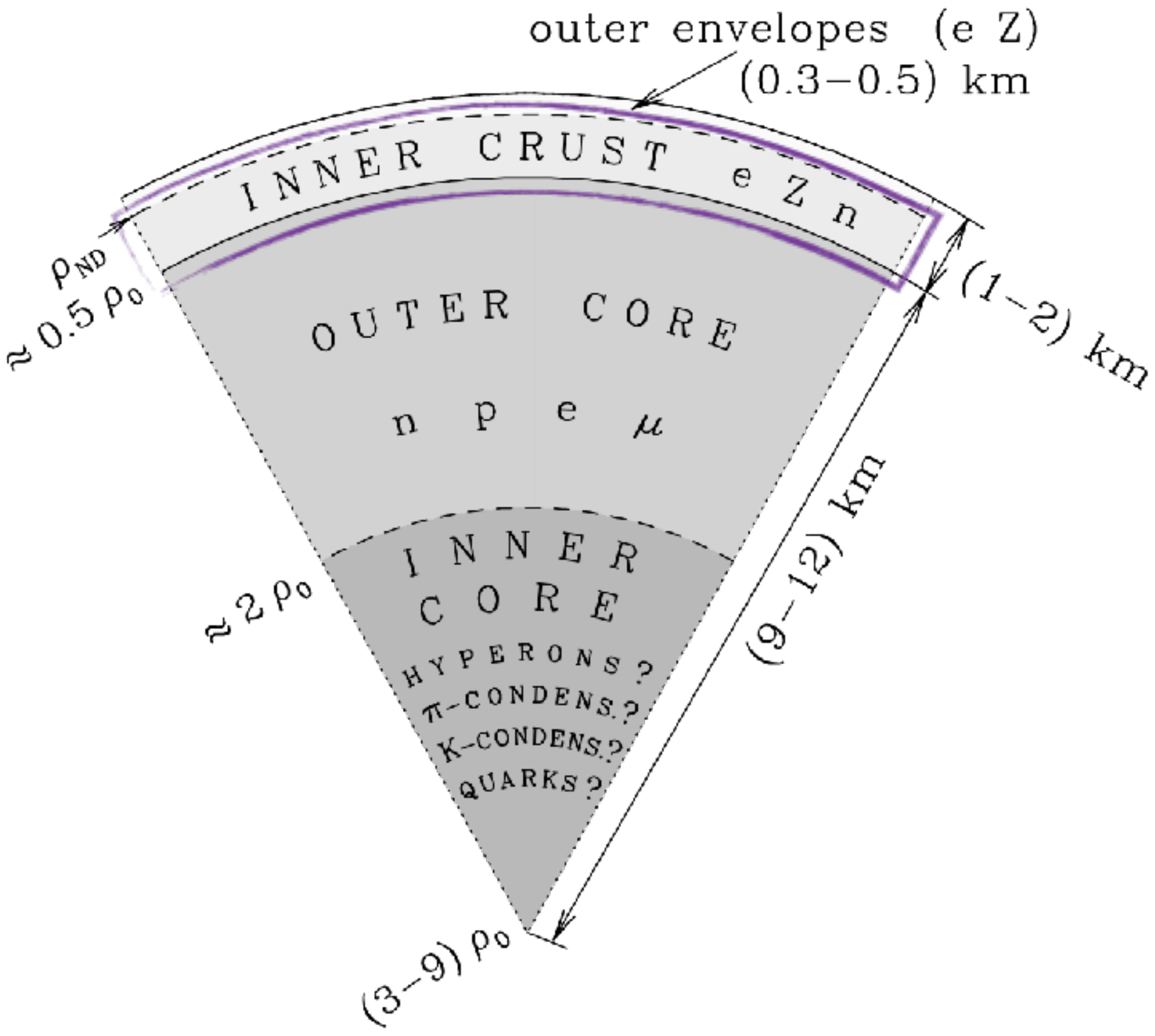

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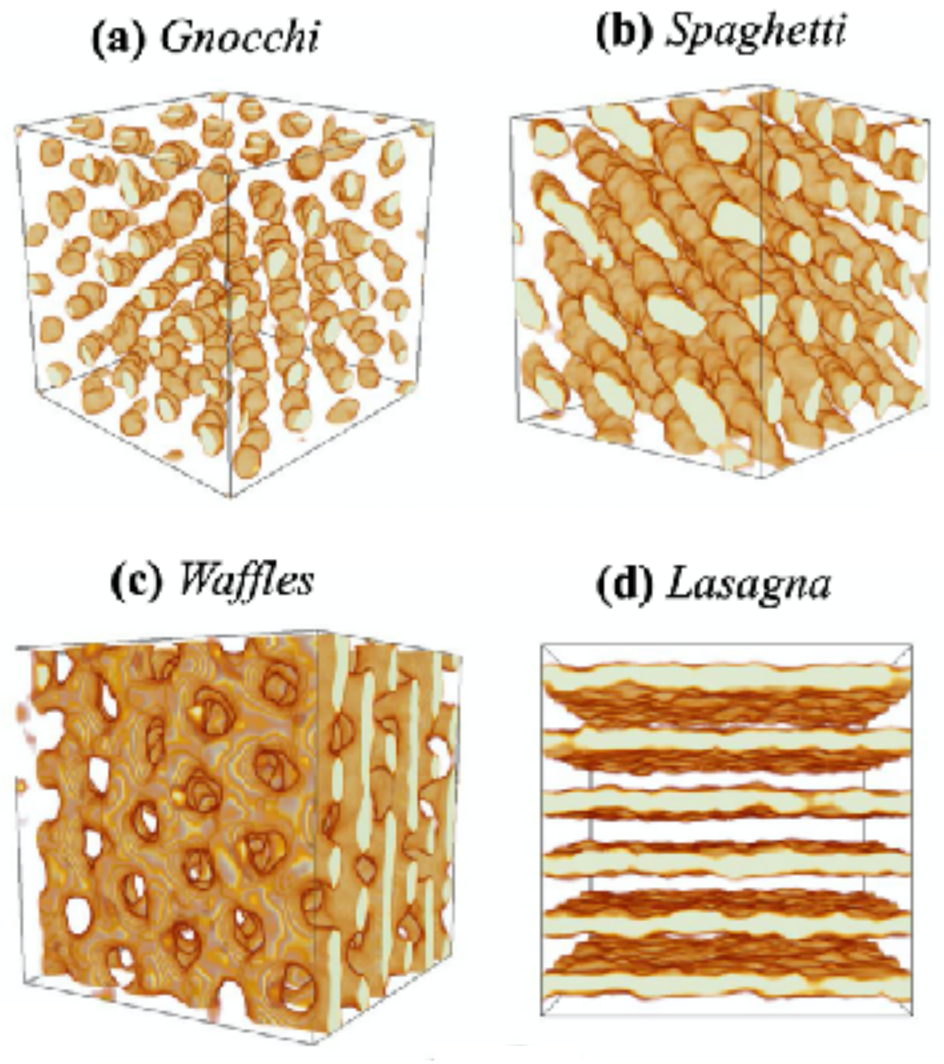
Nuclei lattice within e^- gas

Pasta phases



J. Lattimer & M. Prakas, Phys. Rep. 621 (2016)

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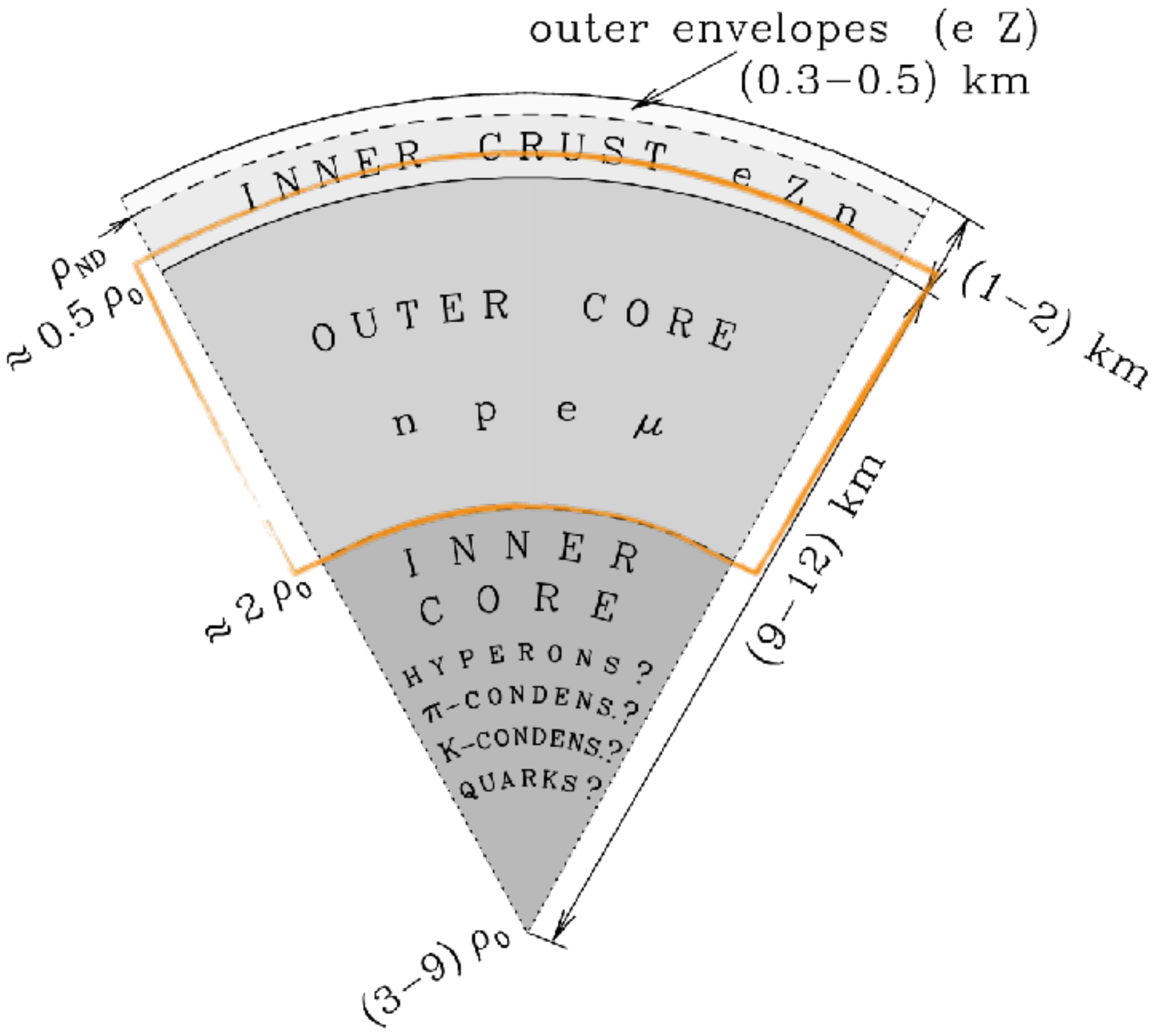

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Nucleonic matter in β^- equilibrium

J. Lattimer & M. Prakas, Phys. Rep. 621 (2016)

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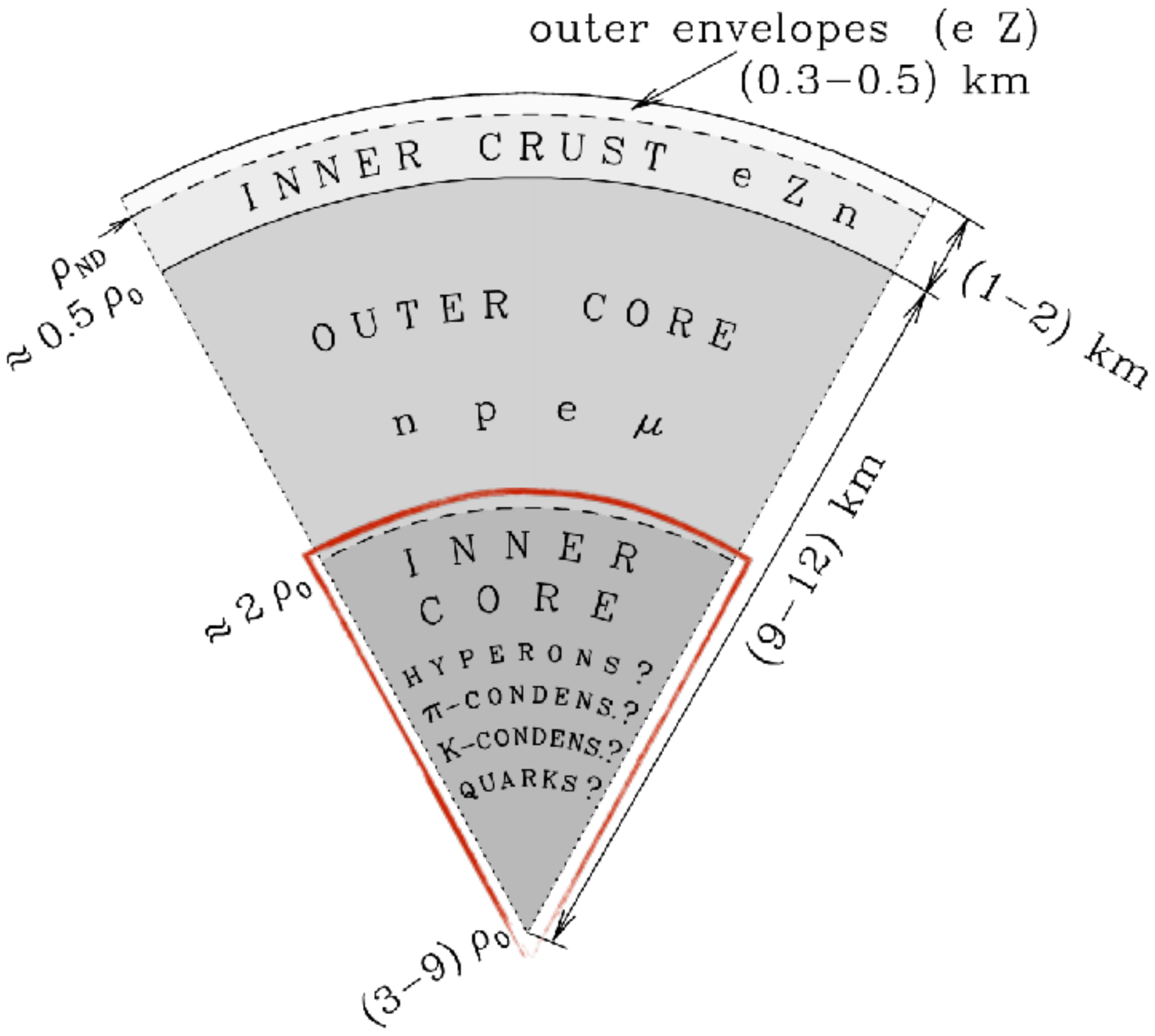

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Nuclei lattice within e^- gas

Pasta phases

Nucleonic matter in β^- equilibrium

- phase transitions
- hyperons

$$\underset{(udd)}{n} + e^- \to \underset{(dds)}{\Sigma^-} + \nu_e$$

- meson condensates
- quark deconfinement

J. Lattimer & M. Prakas, Phys. Rep. 621 (2016)

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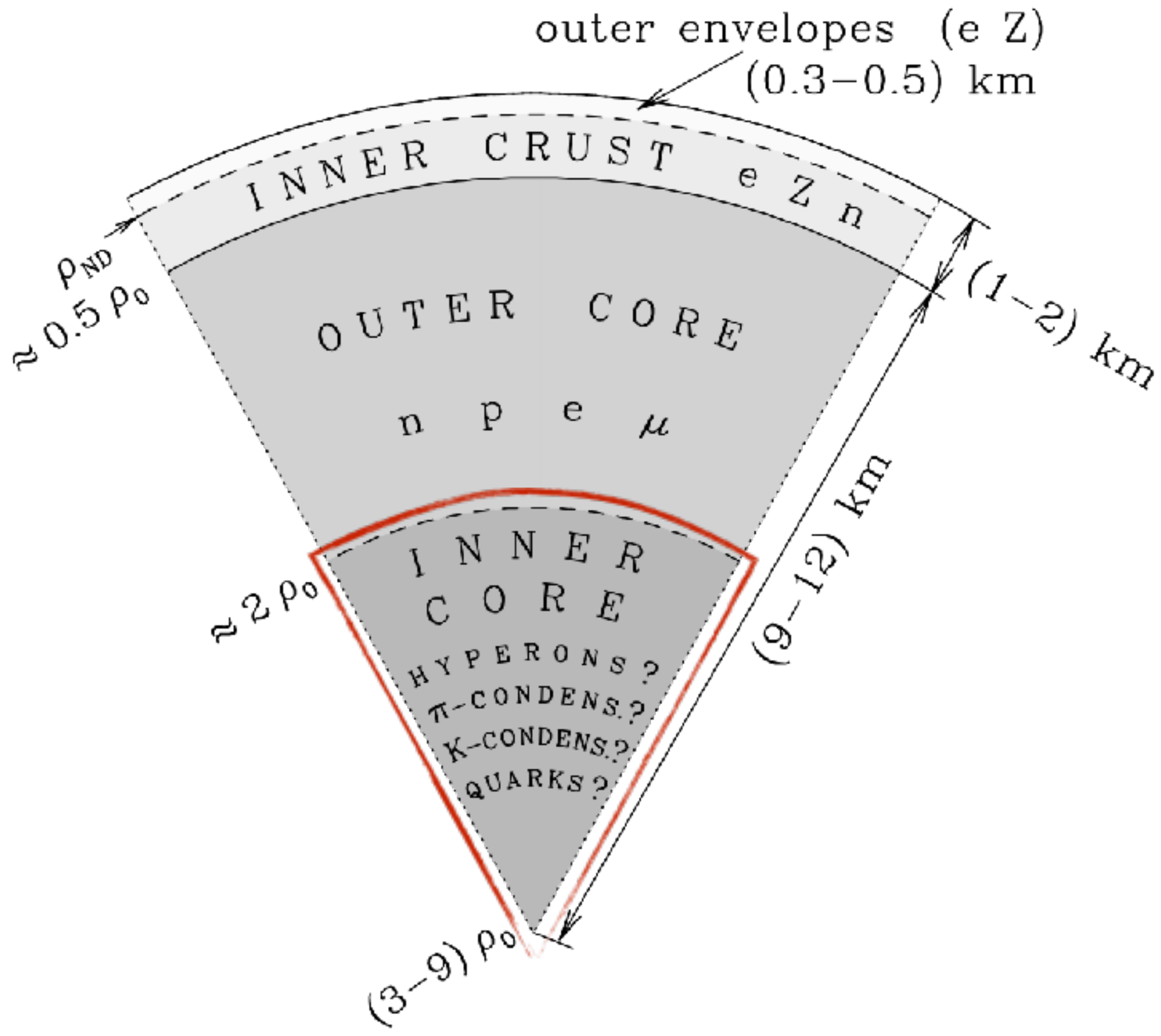

$\rho_0 \simeq 2.67 \times 10^{14}$g cm^{-3}

constraints at supranuclear density

Nuclei lattice within e^- gas

Pasta phases

Nucleonic matter in β^- equilibrium

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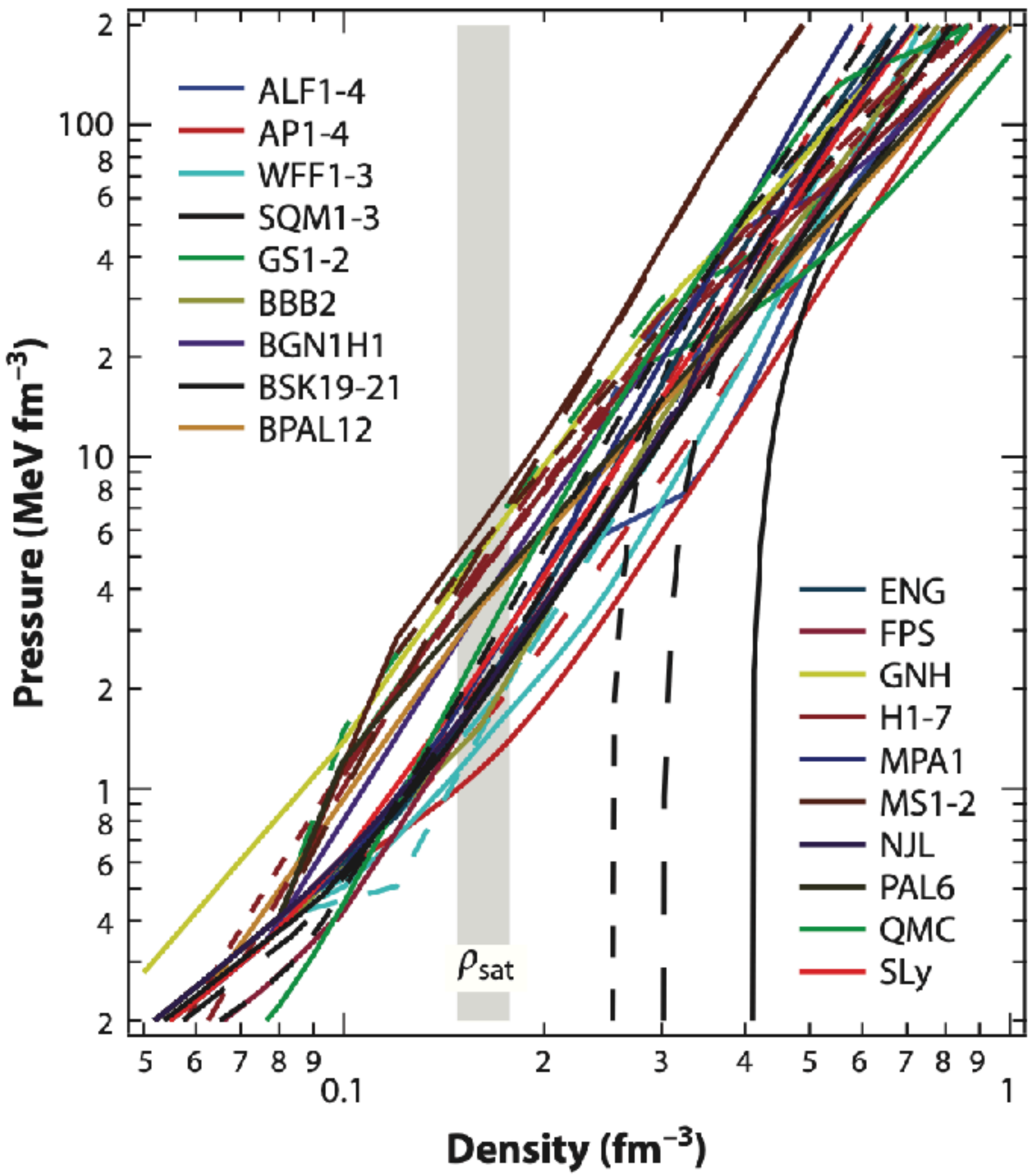
J. Lattimer & M. Prakas, Phys. Rep. 621 (2016)

A micro-view: the equation of state

Microscopic relation $p = p(\epsilon)$



F. Özel & P. Freire, Ann. Rev. Astr. 5 (2016)

A micro-view: the equation of state

Microscopic relation $p = p(\epsilon)$

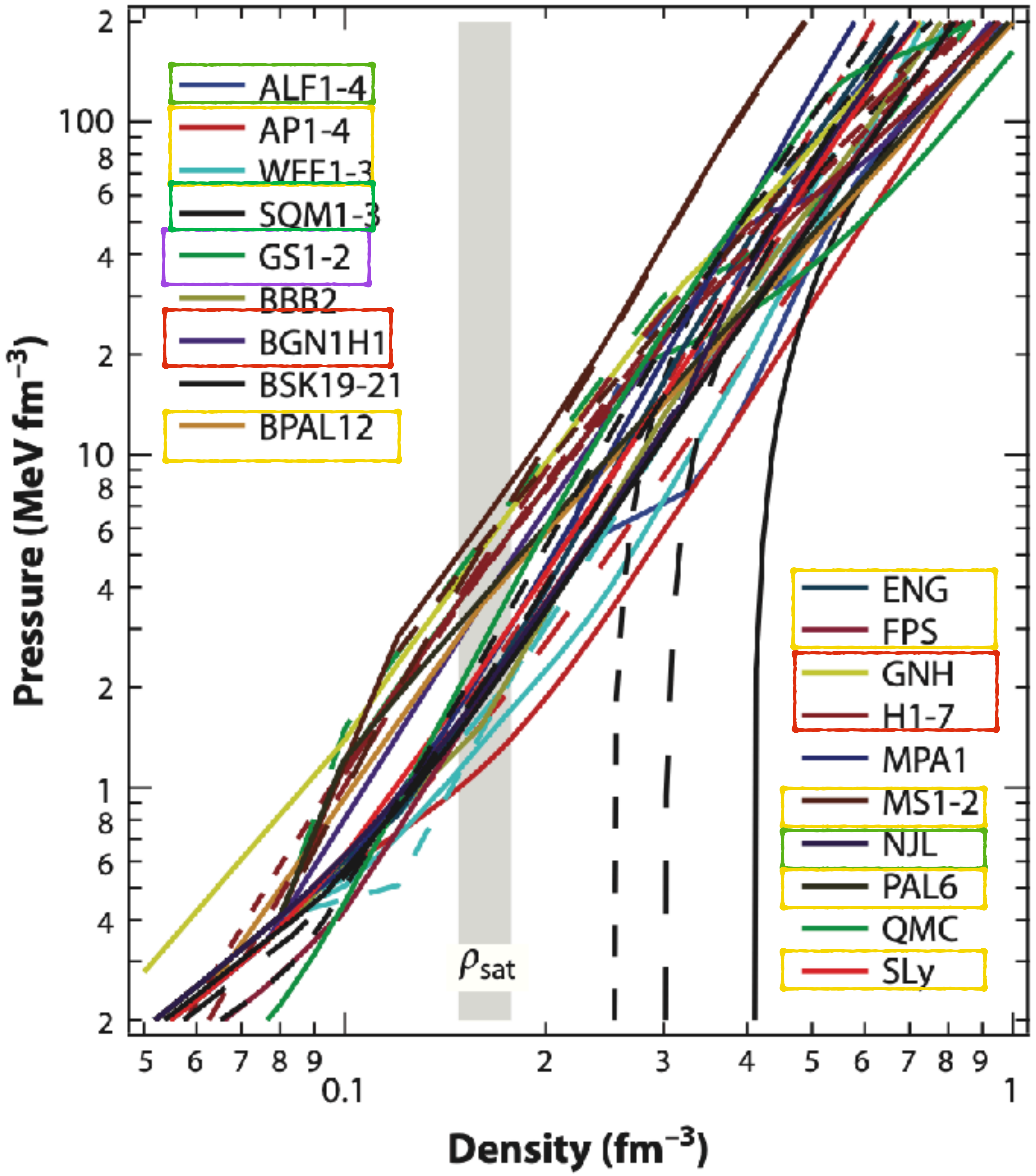
strange stars

nucl matter + quark

hyperons

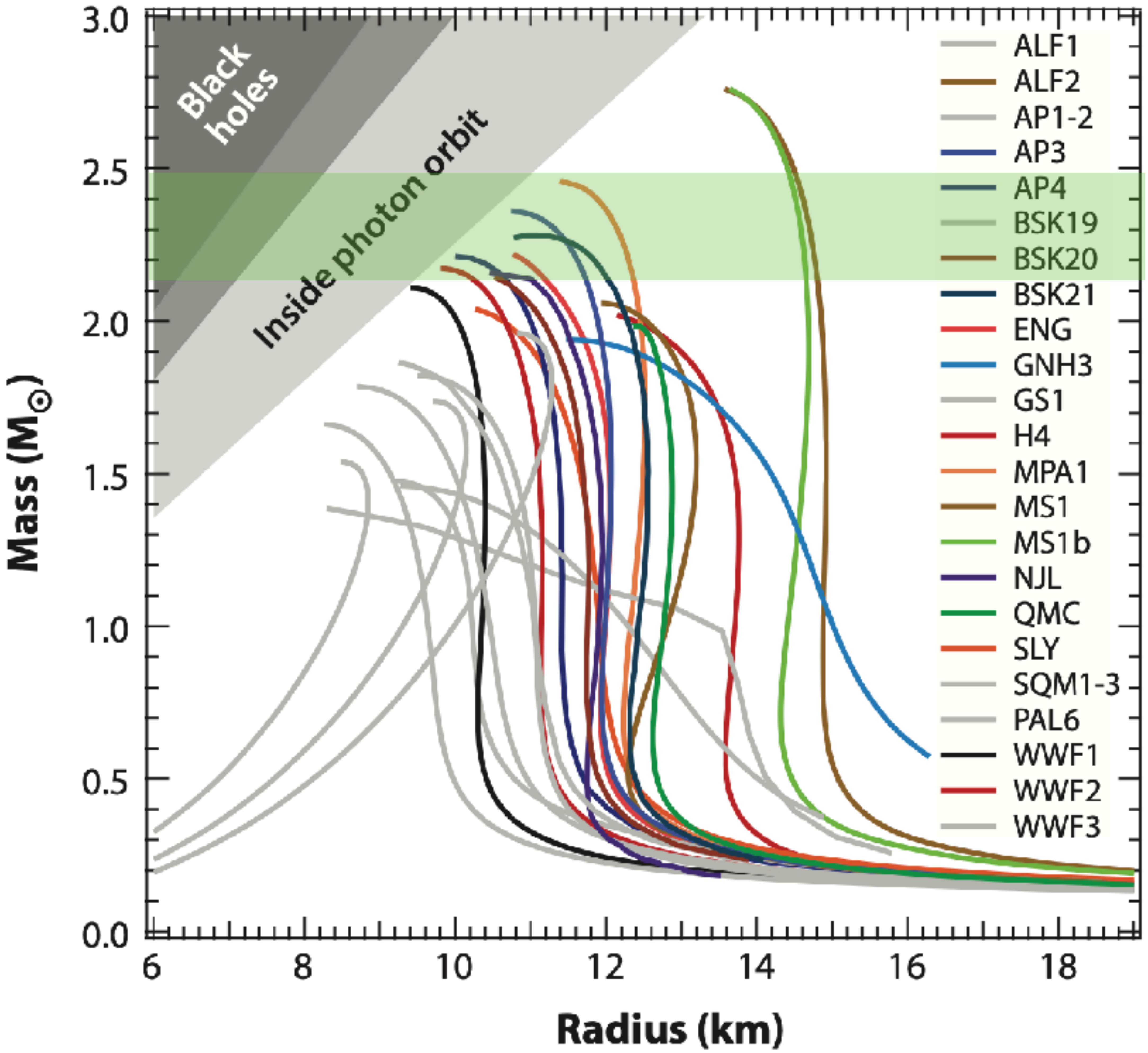
kaons

plain npeμ



F. Özel & P. Freire, Ann. Rev. Astr. 5 (2016)

The equation of state

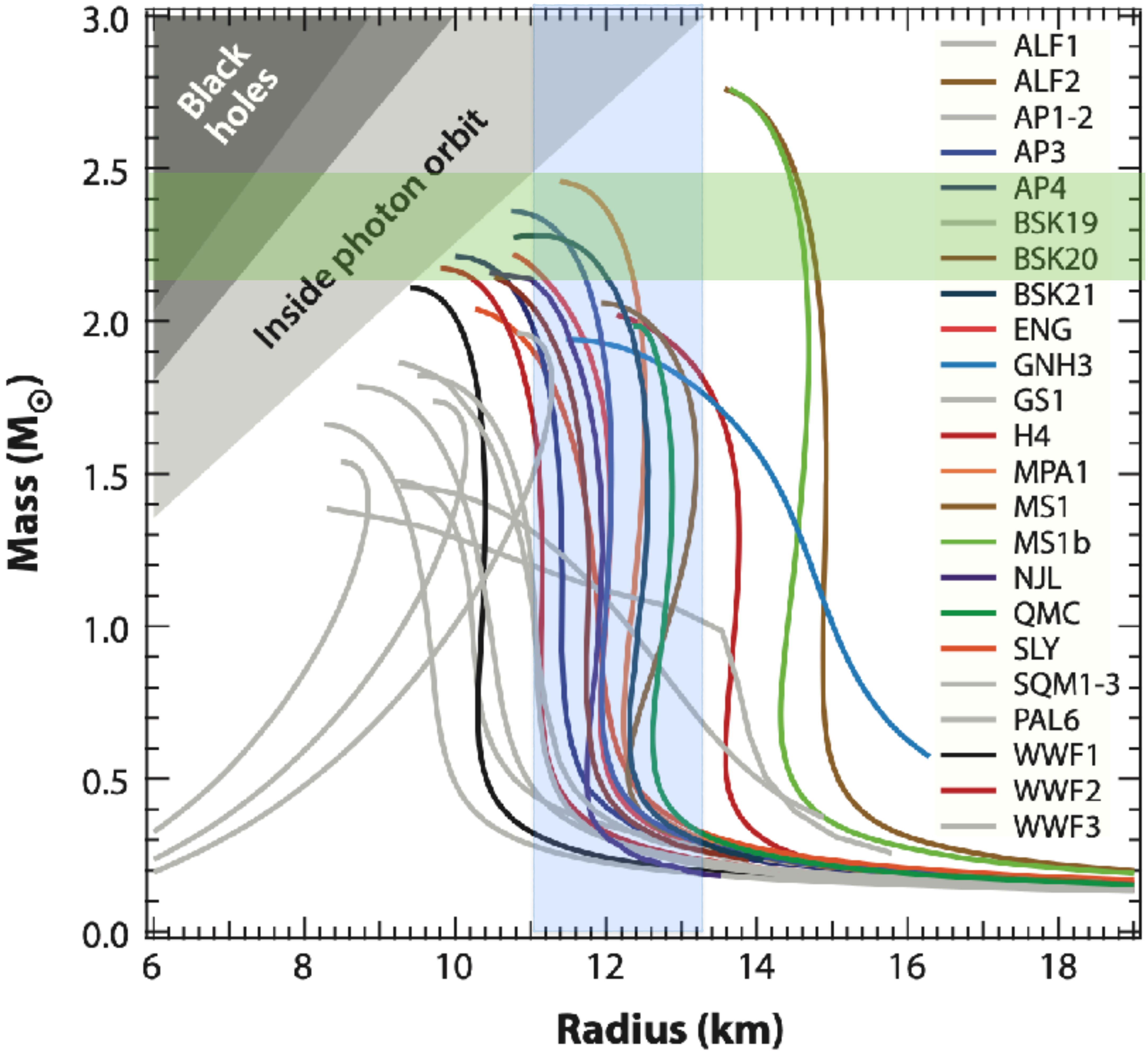


$2.32^{+0.17}_{-0.17} M_\odot$

PSR J0952+0607

F. Özel & P. Freire, Ann. Rev. Astr. 5 (2016)

The equation of state



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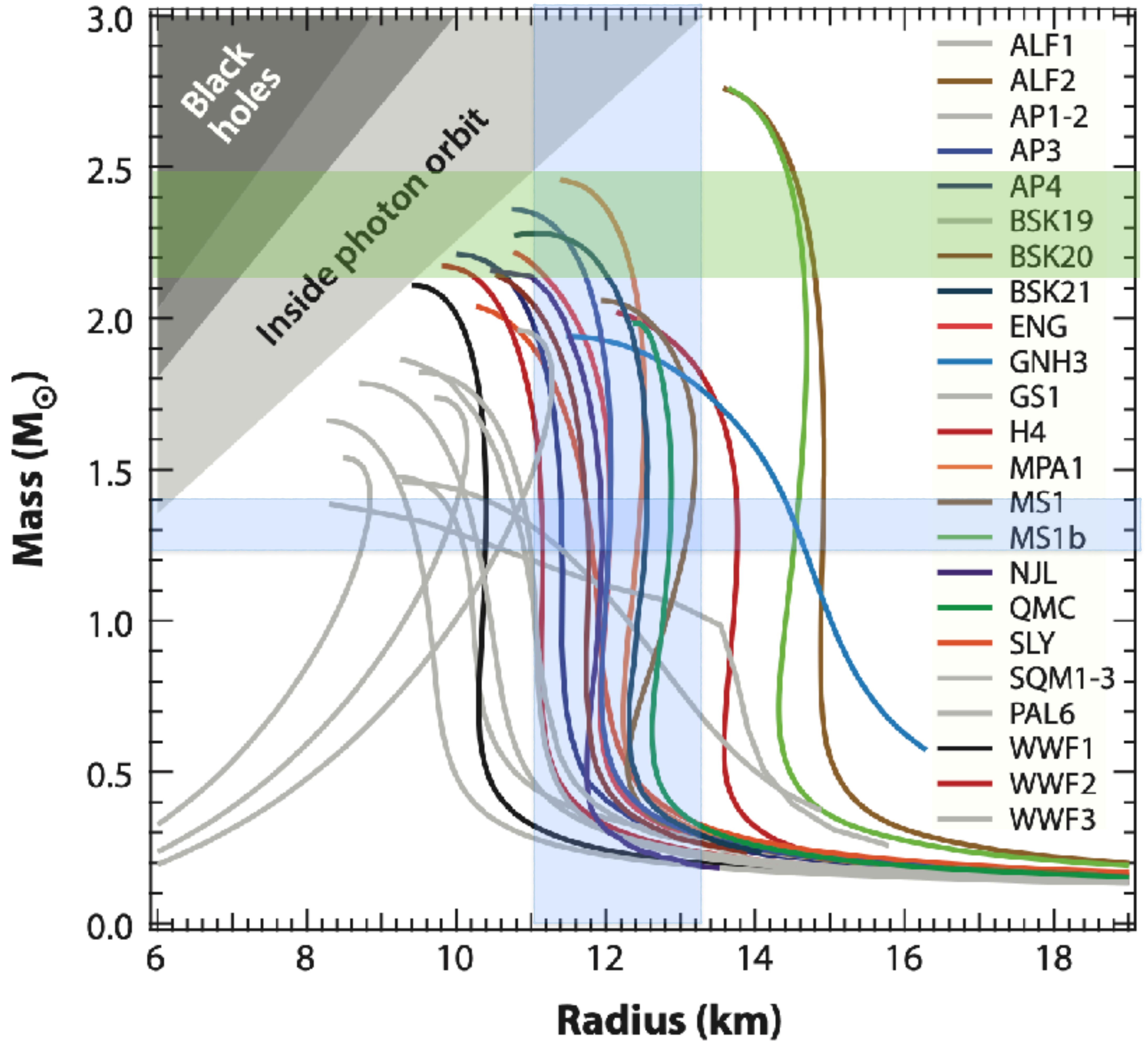

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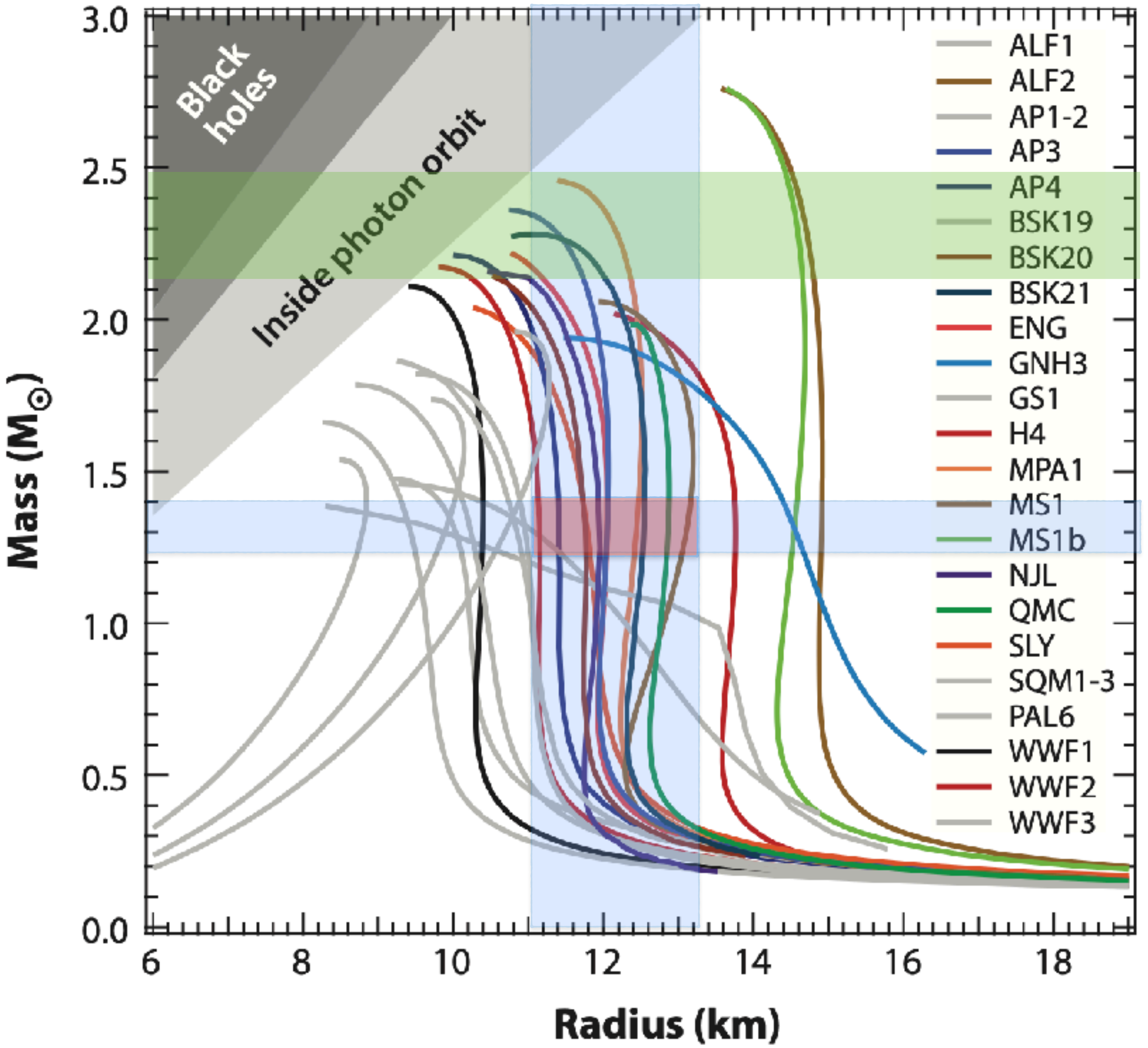

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From micro to macro

The holy grail of NS astrophysics

- (too) many models describing the NS interior
- how do we identify the correct one?

microscopic Equation of State

$$p = p(\epsilon, \ldots)$$

EM *(pulsar, LMXB..)*
+
Labs

macroscopic observables $(M, R, I, \ldots)$

From micro to macro

The holy grail of NS astrophysics

- (too) many models describing the NS interior
- how do we identify the correct one?

microscopic Equation of State

GWs
from binary NS

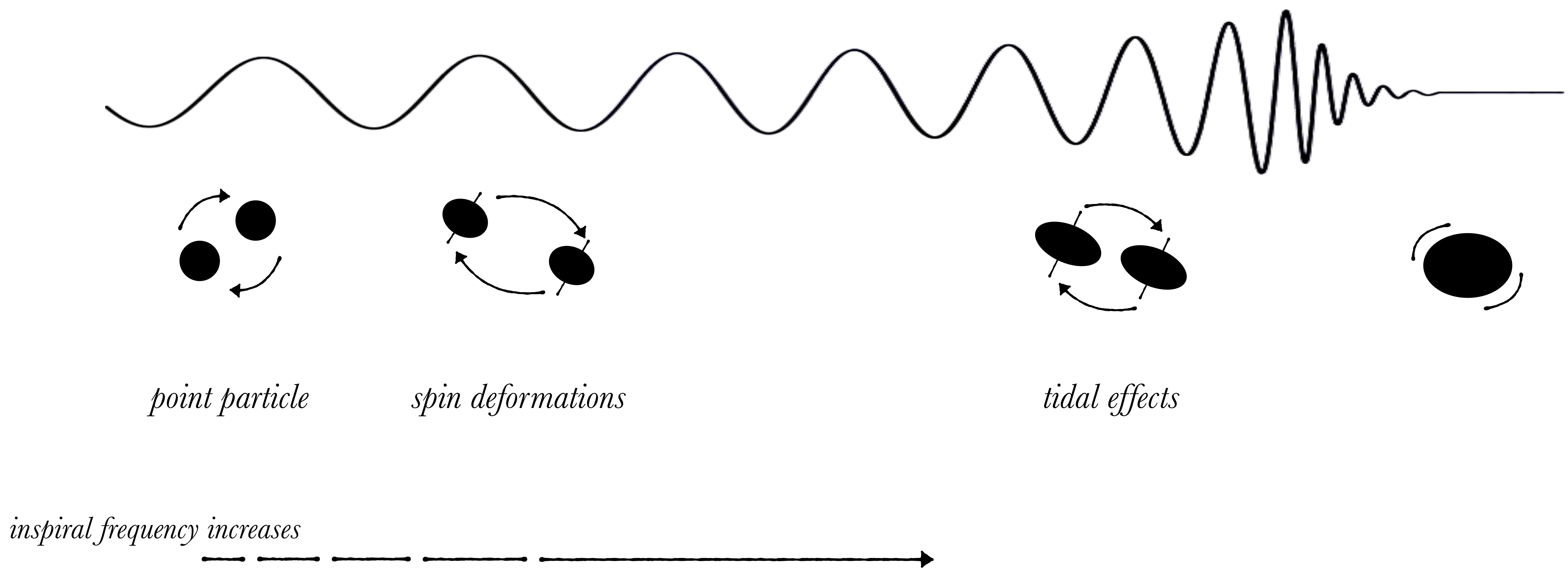
$$p = p(\epsilon, \ldots)$$

EM *(pulsar, LMXB..)*
+
Labs

macroscopic observables $(M, R, I, \ldots)$

Where do we look for matter signatures?

Anatomy of a binary signal



From micro to macro

Tidal interactions leave the footprint of the NS structure on the GW signal

- Deformation properties encoded within the Love numbers

The Yielding of the Earth to Disturbing Forces.

By A. E. H. Love, F.R.S., Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909.)

1. Any estimate of the rigidity of the Earth must be based partly on some observations from which a deformation of the Earth's surface can be inferred, and partly on some hypothesis as to the internal constitution of the Earth. The observations may be concerned with tides of long period, variations of the vertical, variations of latitude, and so on. The hypothesis must relate to the arrangement of the matter as regards density in different parts, and to the state of the parts in respect of solidity, compressibility, and so on. In the simplest hypothesis, the one on which Lord Kelvin's well-known estimate* was based, the Earth is treated as absolutely incompressible and of uniform density and rigidity. This hypothesis was adopted to simplify the problem, not because it is a true one. No matter is absolutely incom-

From micro to macro

Tidal interactions leave the footprint of the NS structure on the GW signal

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star's quadrupole $$Q_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$ *external tidal field*

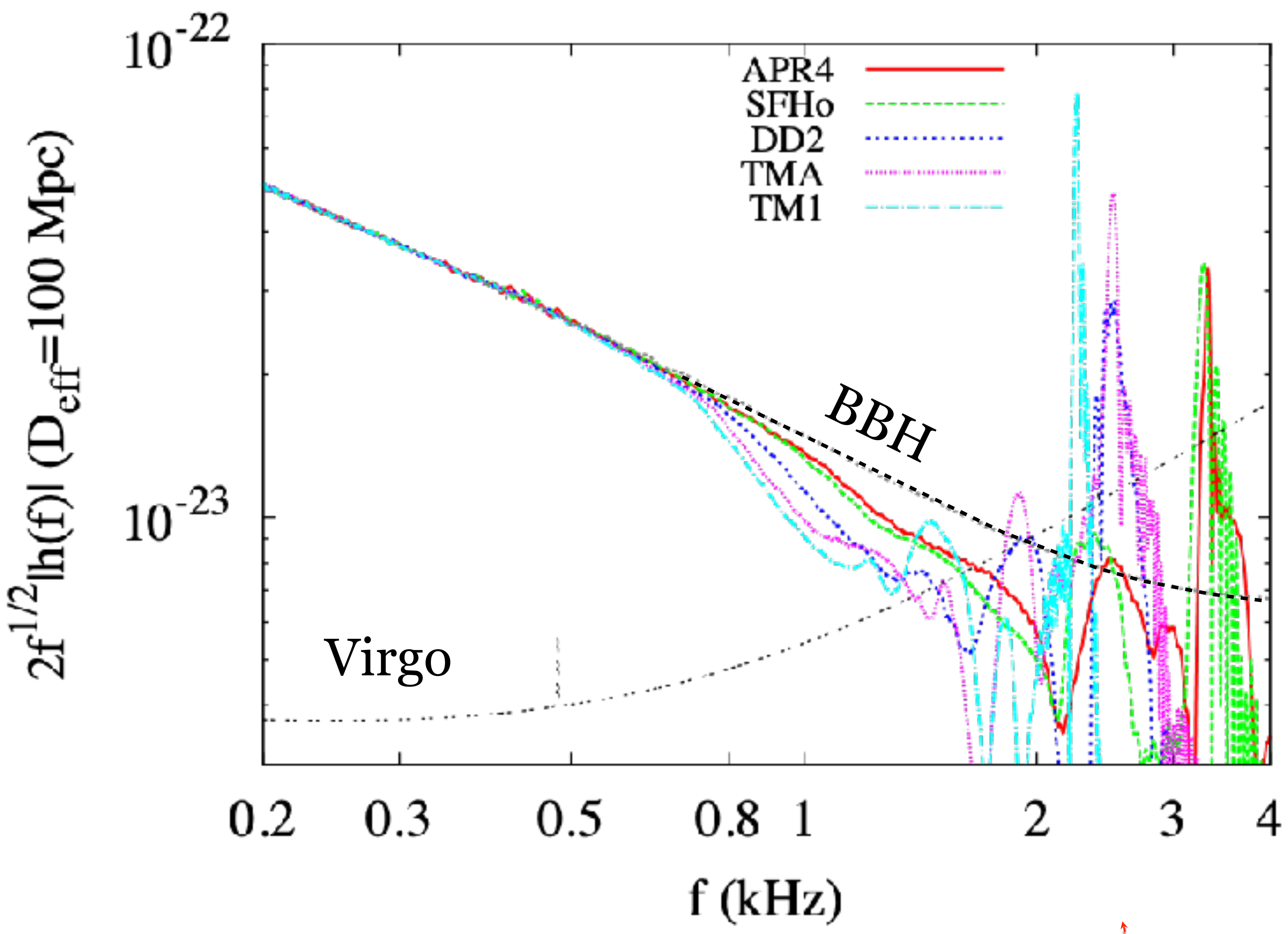
- λ depends on the EoS only, for a given compactness $M_{\rm NS}/R_{\rm NS}$

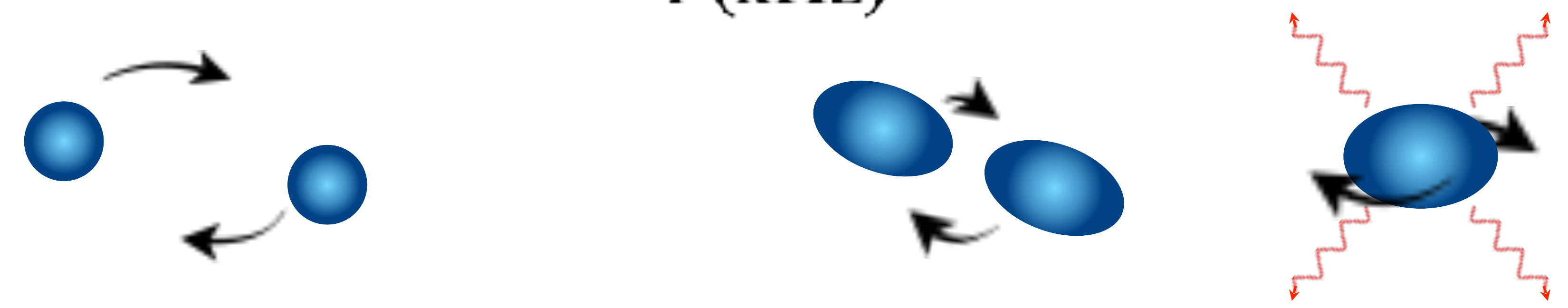
- λ enters within the gravitational waveform

T. Hinderer, The Astroph. J. 677 (2008);
T. Binnington & E. Poisson Phys. Rev. D 80, 084018 (2009)
T. Damour & A. Nagar, Phys. Rev. D 80, 084035 (2009)

How much love?

We classify EoS as stiff v.s. soft





inspiral

late inspiral

EoS	stiffness	R_{NS}	$10^3 \times \lambda$
APR4	very soft	11.09	10
SFHo	soft	11.91	13
DD2	medium soft	13.20	27
TMA	stiff	13.85	37
TM1	very stiff	14.48	45

Soft EoS

- larger densities
- smaller Love numbers

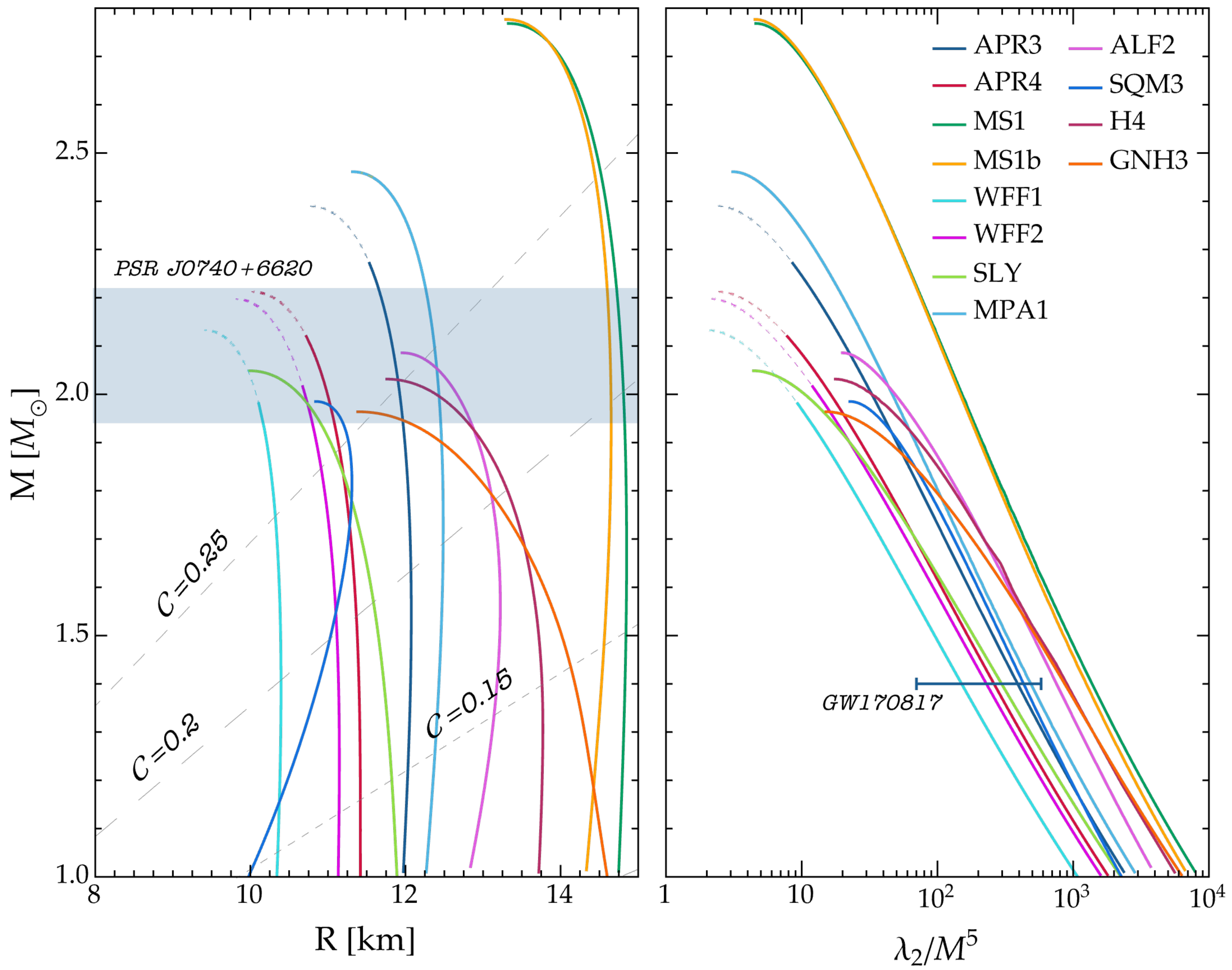
Stiff EoS

- smaller densities
- larger Love numbers

larger effect in the signal

Soft and stiff Love

Build $\lambda - M$ profiles

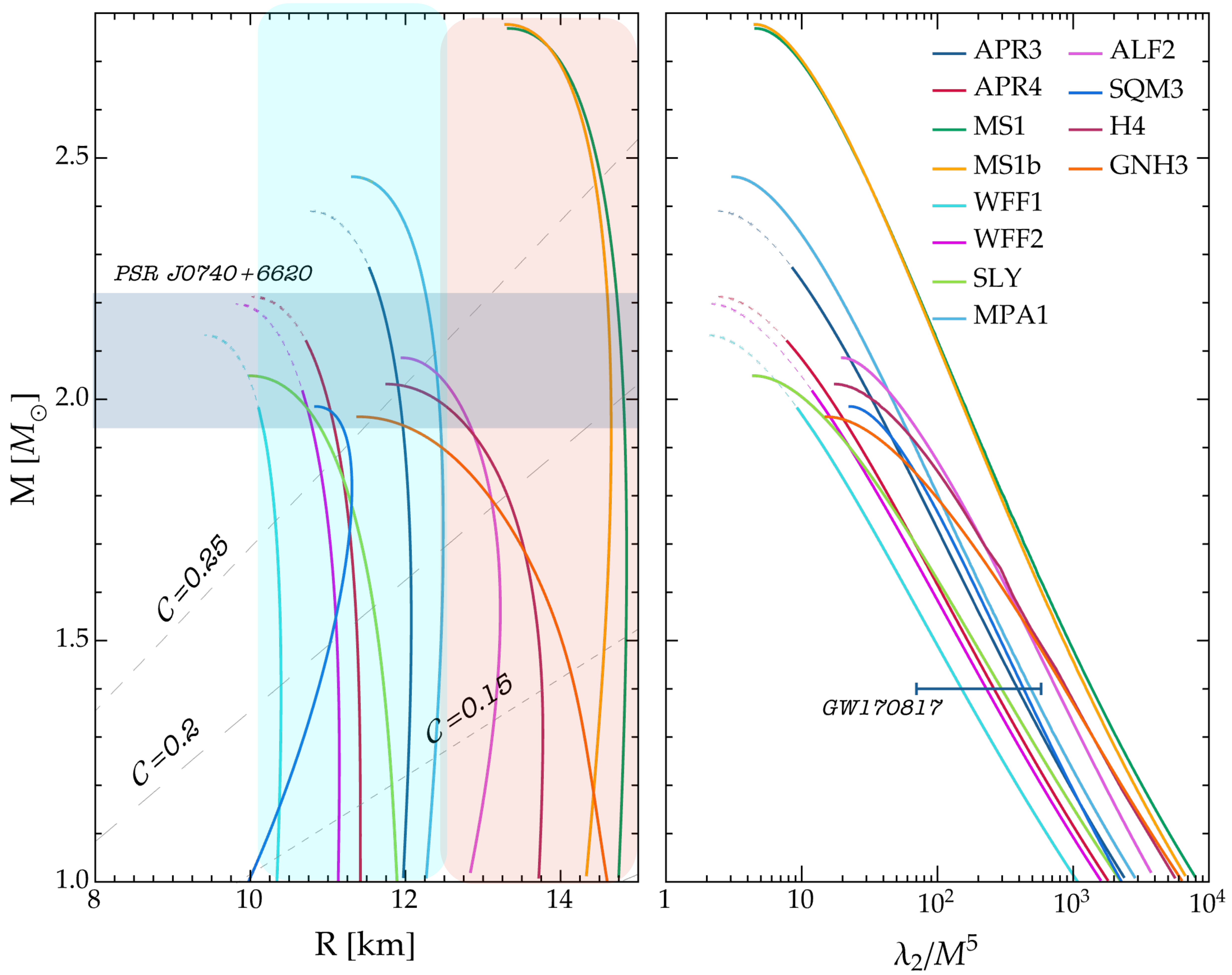


$M_{\rm NS}/R_{\rm NS} \in [0.1 - 0.2]$

$\lambda \sim \mathcal{O}(10^4)$

Soft and stiff Love

Build $\lambda - M$ profiles



soft stiff

$$M_{\rm NS}/R_{\rm NS} \in [0.1 - 0.2]$$

$$\lambda \sim \mathcal{O}(10^4)$$

Where do we look for Love?

Tidal effects add linearly to the phase $h(f) = \mathcal{A} e^{i[\psi_{\rm PP}(f) + \psi_{\rm T}(f)]}$

point-particle

$$\psi_{\rm PP} \propto \Bigg[\underset{\uparrow \atop \text{Newt}}{1} - \left(\frac{743}{336} + \frac{11}{4}\eta \right) \underset{\uparrow \atop \text{1 PN}}{\frac{(m\pi f)^{2/3}}{c^2}} + \frac{\dots}{c^3} + \dots + \underset{\uparrow \atop \text{3.5 PN}}{\frac{\dots}{c^7}} \Bigg]$$

tidal

$$\psi_{\rm T} \propto \frac{1}{26} \left[\underbrace{\left(1 + 12\frac{m_2}{m_1}\right)\lambda_1 + \left(1 + 12\frac{m_1}{m_2}\right)\lambda_2}_{\Lambda} \right] \frac{(m\pi f)^{10/3}}{c^{10}} + \frac{\dots}{c^{12}}$$

average tidal deformability

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Newt

1 PN

3.5 PN

tidal

$$\psi_{\rm T} \propto \frac{1}{26}\left[\underbrace{\left(1 + 12\frac{m_2}{m_1}\right)\lambda_1 + \left(1 + 12\frac{m_1}{m_2}\right)\lambda_2}_{\Lambda}\right]\frac{(m\pi f)^{10/3}}{c^{10}} + \frac{\dots}{c^{12}}$$

average tidal deformability

5 PN: small term!

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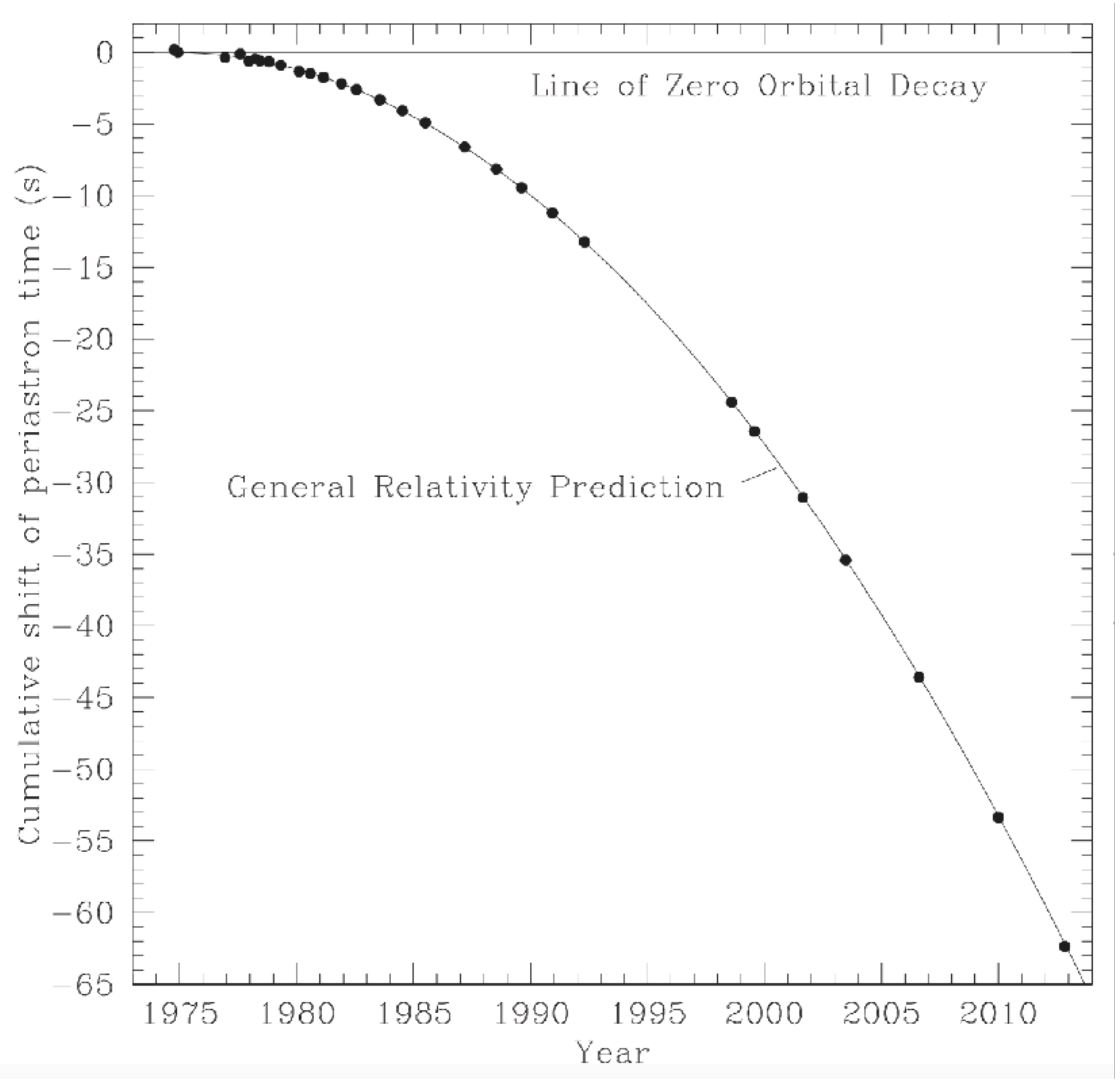
average tidal deformability

$\mathcal{O}(10^4)$

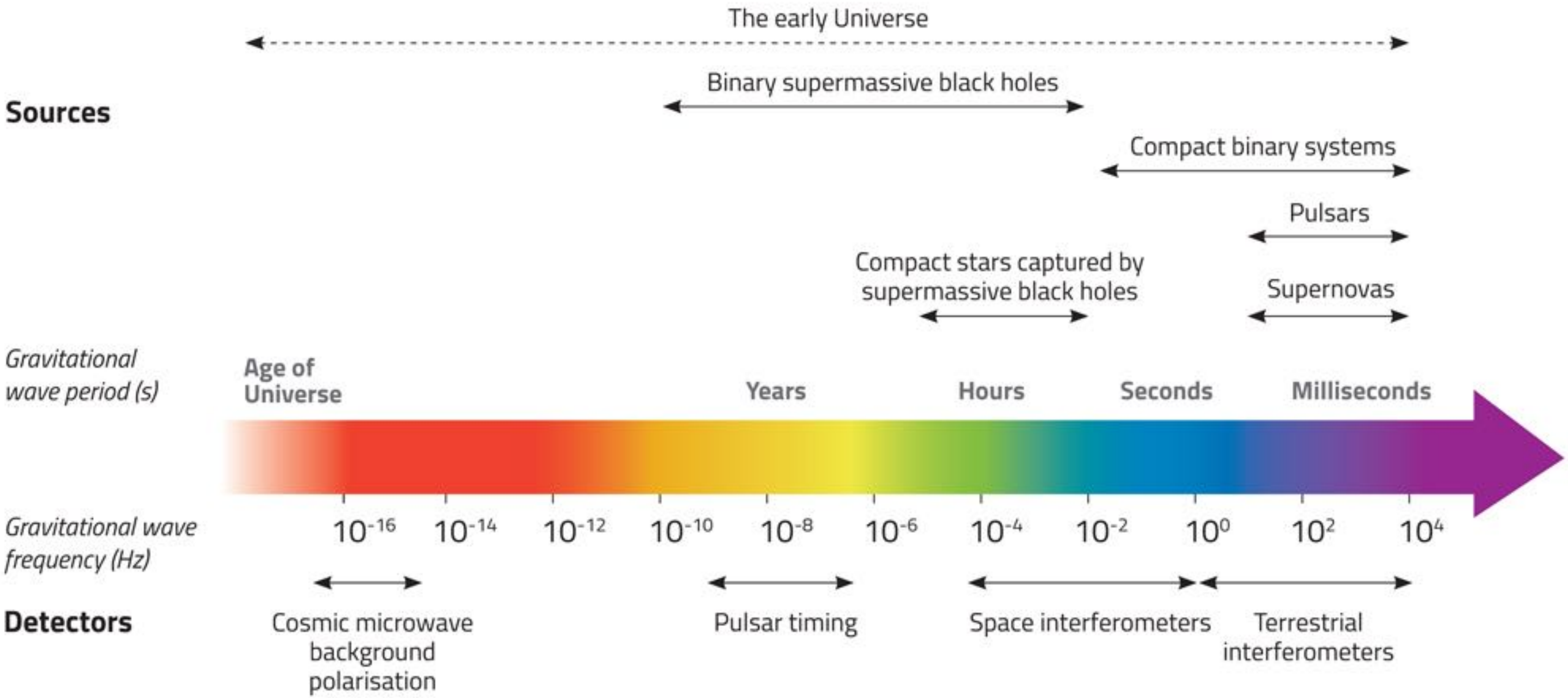
5 PN: small term!

PSR1913+16



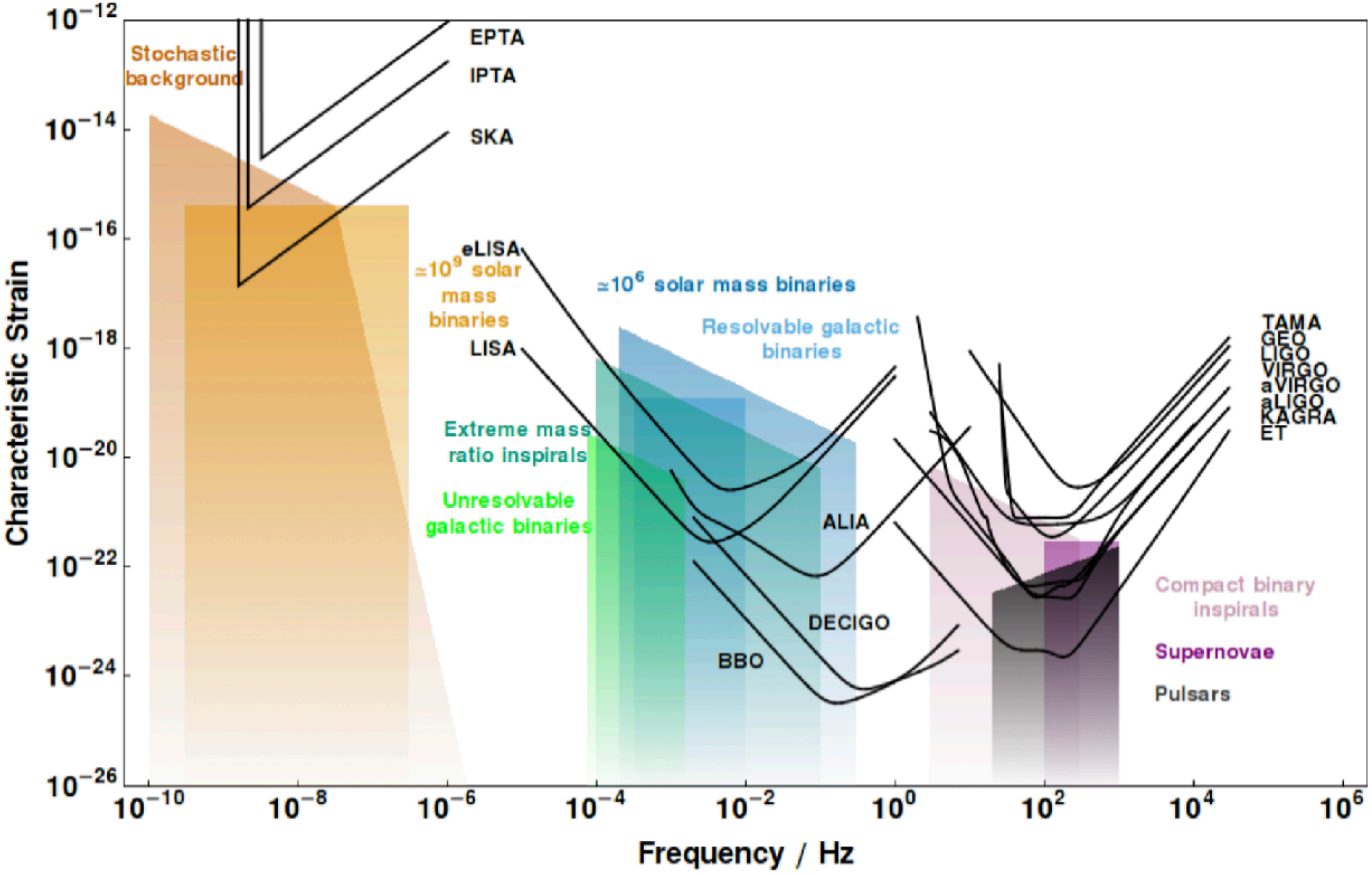
Line of Zero Orbital Decay
General Relativity Prediction
Cumulative shift of periastron time (s)
0
−5
−10
−15
−20
−25
−30
−35
−40
−45
−50
−55
−60
−65
1975
1980
1985
1990
1995
2000
2005
2010
Year

GW frequencies



GW frequencies

C. Moore +, 2014



The Love number

T. Hinderer +, 2010

Love number master equation

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{\Lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$\left(-\frac{6e^{2\Lambda}}{r^2} - 2(\Phi')^2 + 2\Phi'' + \frac{3}{r}\Lambda' + \frac{7}{r}\Phi' - 2\Phi'\Lambda' + \frac{f}{r}(\Phi' + \Lambda')\right) H + \left(\frac{2}{r} + \Phi' - \Lambda'\right) H' + H'' = 0.$$

$$\frac{dH}{dr} = \beta$$

$$\begin{aligned} \frac{d\beta}{dr} = \; & 2\left(1 - 2\frac{m_r}{r}\right)^{-1} H \left\{-2\pi\left[5\epsilon + 9p + f(\epsilon + p)\right]\right. \\ & \left. + \frac{3}{r^2} + 2\left(1 - 2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2} + 4\pi r p\right)^2\right\} \\ & + \frac{2\beta}{r}\left(1 - 2\frac{m_r}{r}\right)^{-1}\left\{-1 + \frac{m_r}{r} + 2\pi r^2(\epsilon - p)\right\} \end{aligned}$$

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- Reduce to a couple system of 1st order ODEs

$$\frac{dH}{dr} = \beta$$

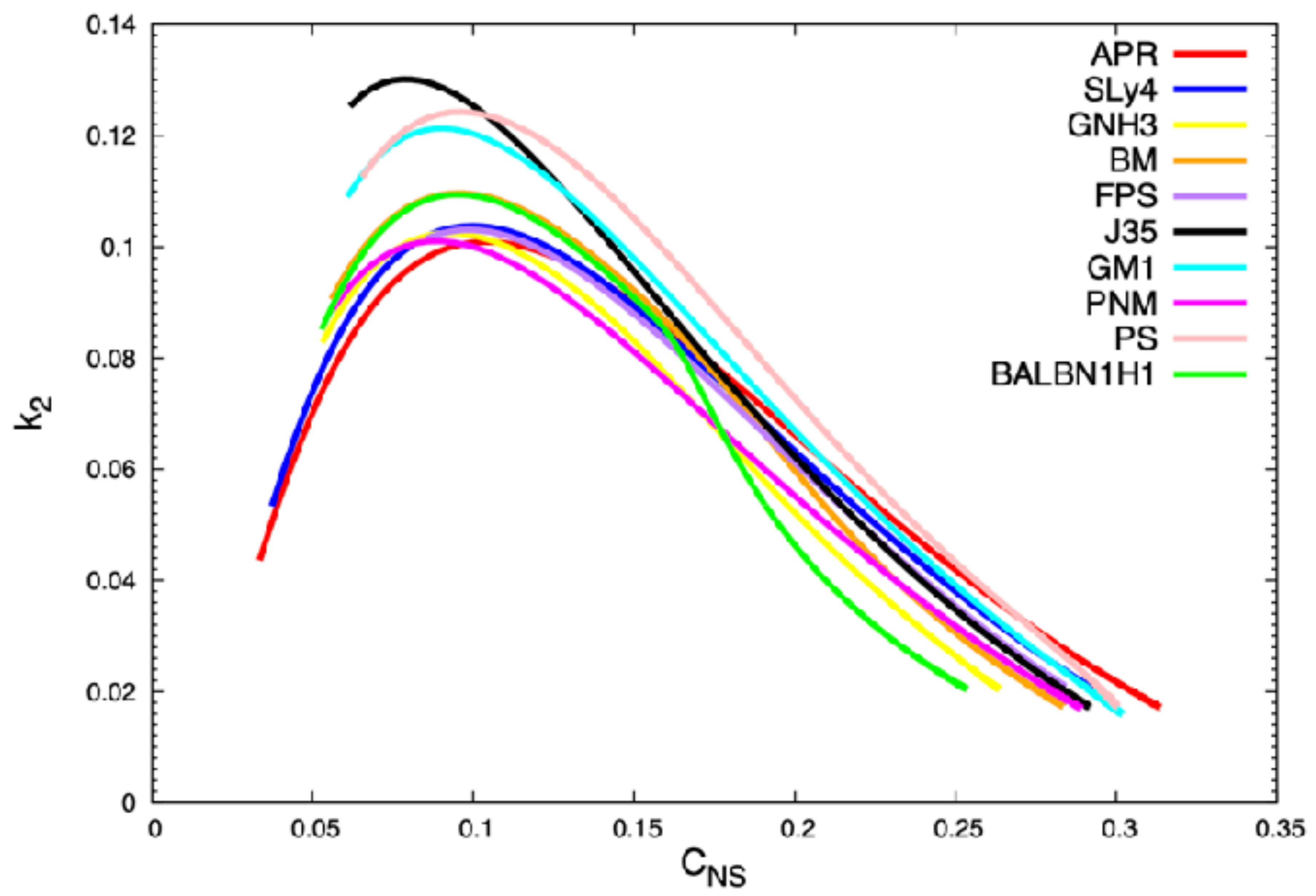
$$\begin{aligned}\frac{d\beta}{dr} = & \; 2\left(1 - 2\frac{m_r}{r}\right)^{-1} H \left\{-2\pi\left[5\epsilon + 9p + f(\epsilon + p)\right]\right. \\ & \left. + \frac{3}{r^2} + 2\left(1 - 2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2} + 4\pi r p\right)^2\right\} \\ & + \frac{2\beta}{r}\left(1 - 2\frac{m_r}{r}\right)^{-1}\left\{-1 + \frac{m_r}{r} + 2\pi r^2(\epsilon - p)\right\}\end{aligned}$$

- Integrate with initial conditions at the center $H(r \to 0) = a_0 r^2$ and $\beta(r \to 0) = 2a_0 r$

The Love number

Love number as a function of the metric perturbation

$$k_2 = \frac{8\mathcal{C}^5}{5}(1-2\mathcal{C})^2[2+2\mathcal{C}(y-1)-y] \times \Big\{2\mathcal{C}(6-3y+3\mathcal{C}(5y-8)) + 4\mathcal{C}^3[13-11y+\mathcal{C}(3y-2)+2\mathcal{C}^2(1+y)] + 3(1-2\mathcal{C})^2[2-y+2\mathcal{C}(y-1)]\log(1-2\mathcal{C})\Big\}^{-1}$$



The Love number

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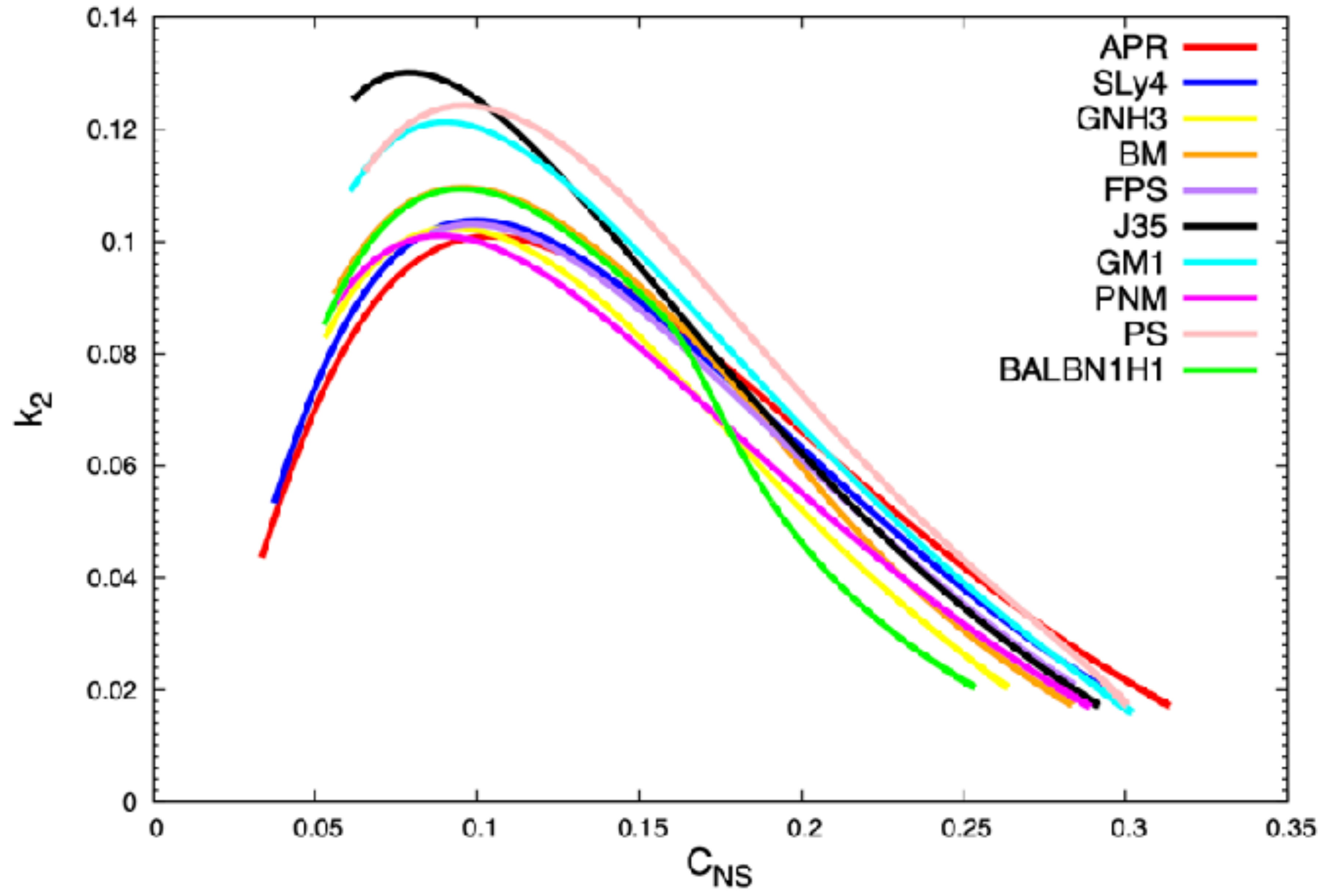
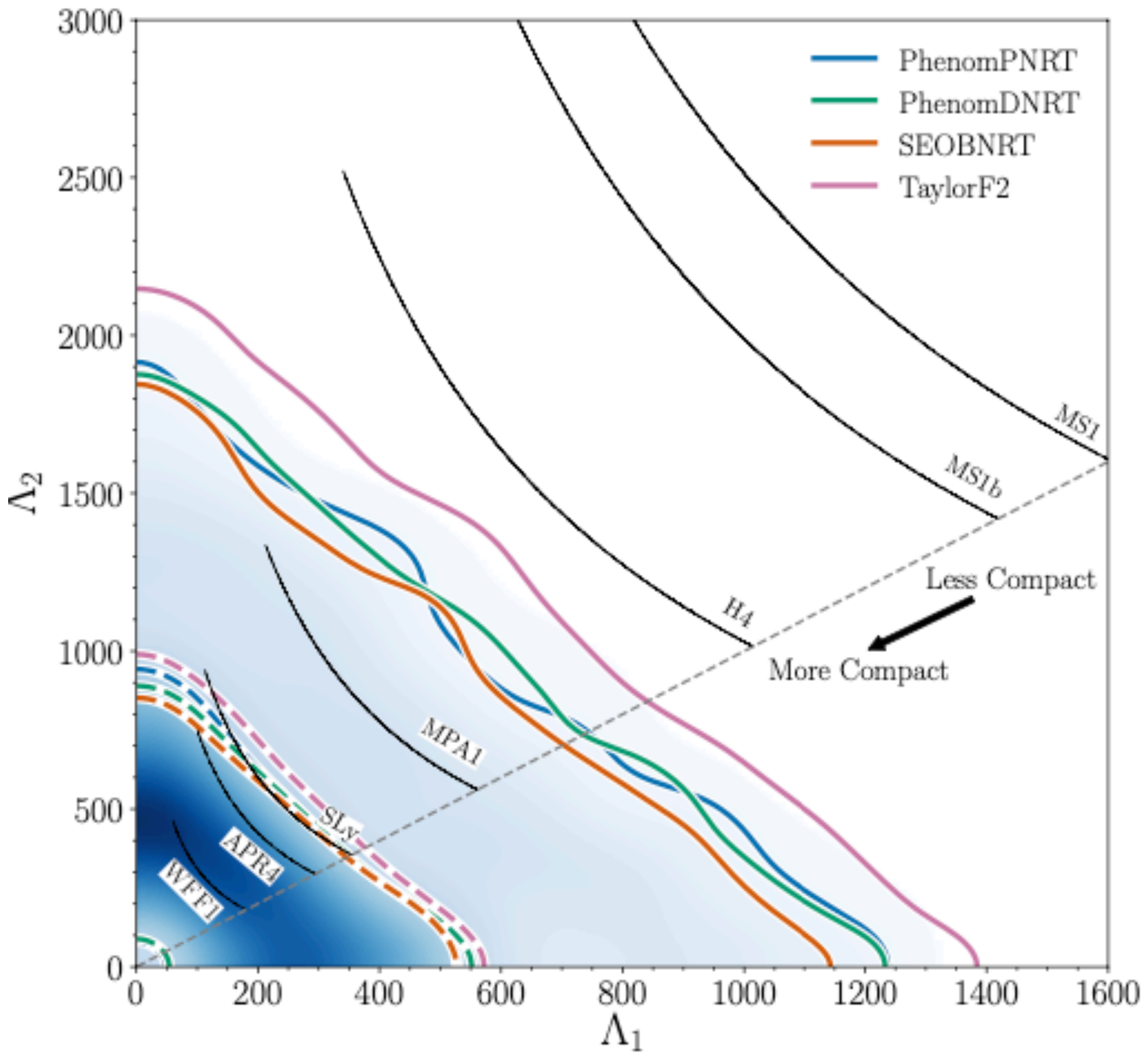
- As the compactness $\mathscr{C} = M/R$ increases the Love number decreases and goes to zero in the $\mathscr{C} \to 1/2$ limit

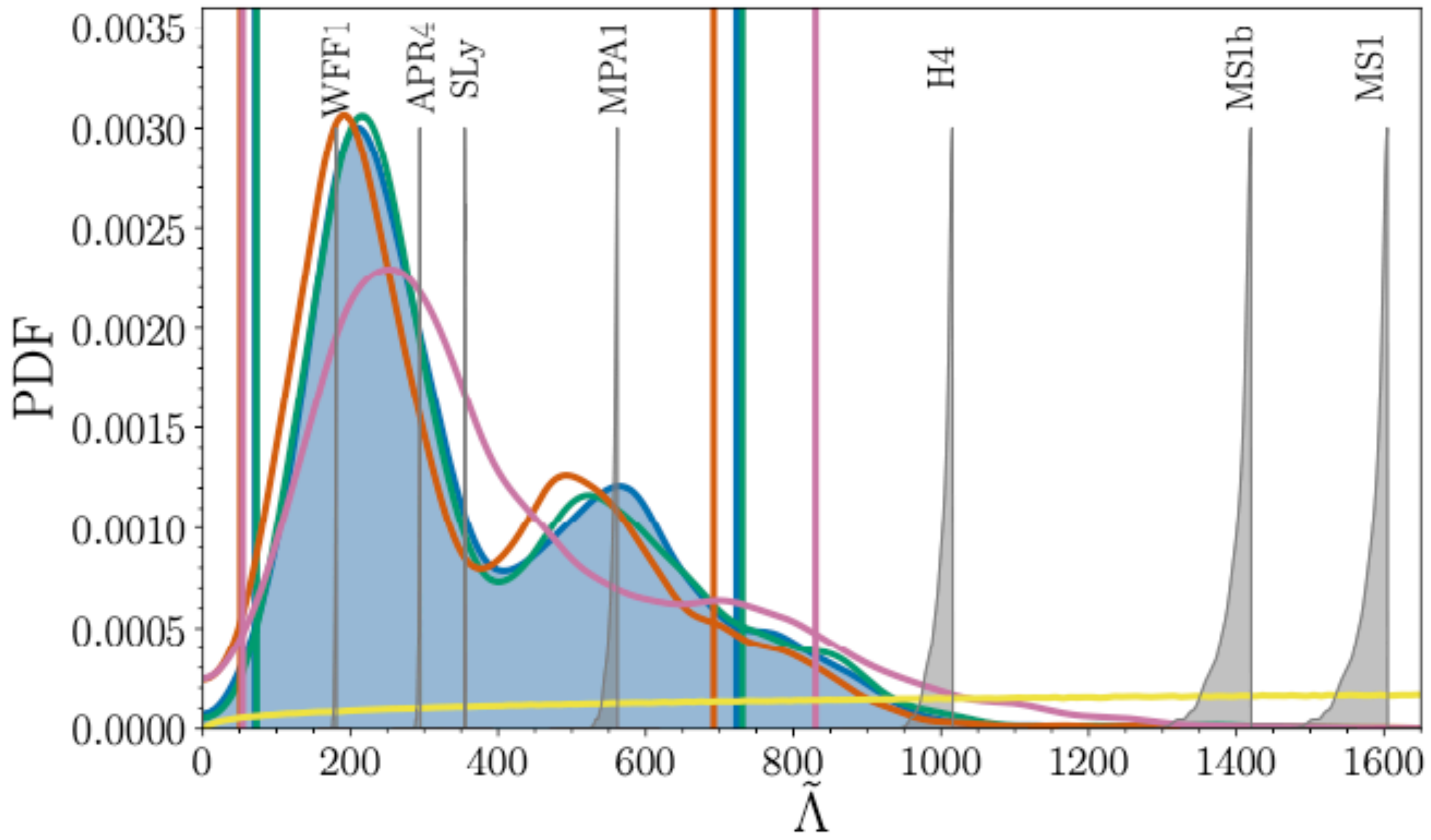


GW170817

The rest is history

$M_1(M_\odot)$	Λ_1	$M_2(M_\odot)$	Λ_2
$1.46^{+0.13}_{-0.09}$	255^{+416}_{-171}	$1.26^{+0.09}_{-0.12}$	661^{+858}_{-375}





LVC, Phys. Rev. Lett. 119-121 (2018)
LVC, Phys. Rev. X 9 (2019)

First BNS detection seems to favour more compact nuclear matter

- ruling out stiff Equations of State

Love for testing vacuum GR

P. Landry & E. Poisson Phys. Rev. D 91, 104018 (2015)
P. Pani + incl. A. M., Phys. Rev. D 92, 024010 (2015)
N. Gürlebeck, Phys. Rev. Lett. 114, 151102 (2015)
A. Le Tiec +, Phys.Rev. D 103 084021 (2021)

Black holes in vacuum General Relativity have <u>zero</u> Love numbers

- New tool for fundamental physics

Compact objects with with <u>non</u>-<u>zero</u> Love numbers

BHs beyond GR

Exotic Compact Objects

BHs in accretion disks/dark matter halo

test of the BH nature
are they all Kerr BH?

probe the astrophysical properties
in which BH evolve

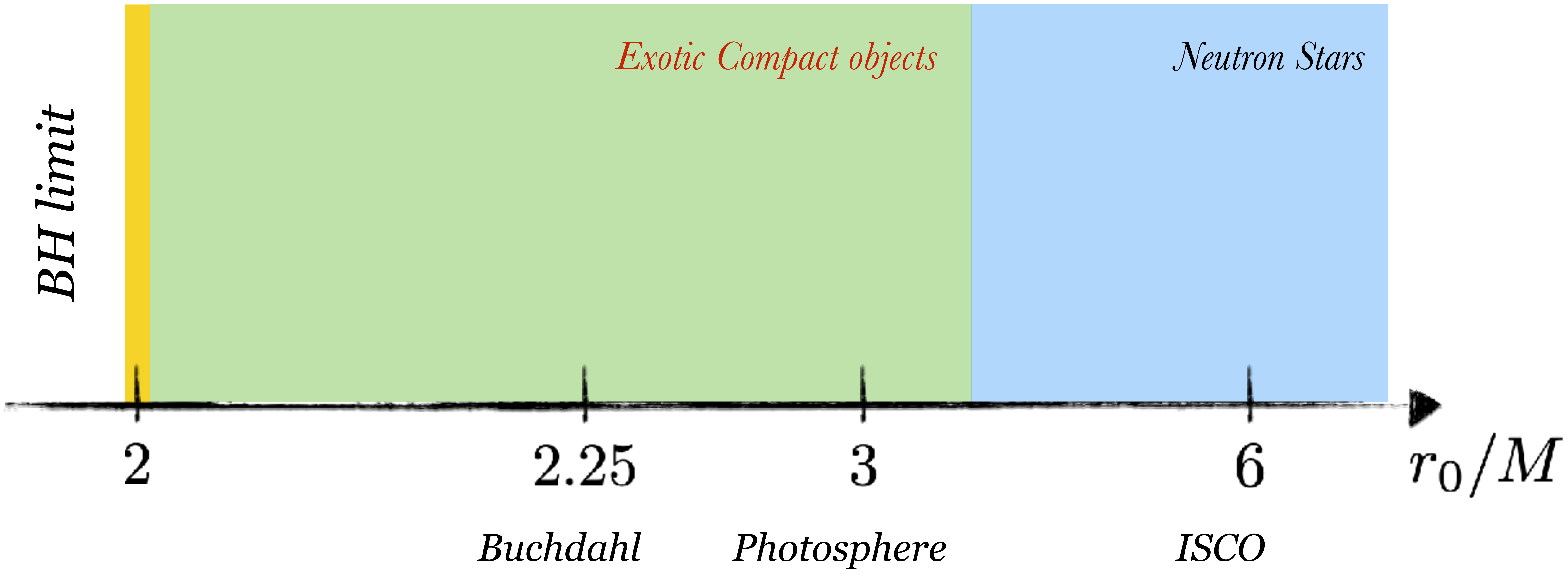
- Possibility to explore a wider range of masses, from stellar to supermassive scales

V. Cardoso + incl. A. M., Rev. Rev. D 95, 084014 (2017)
A. Maselli +, Phys. Rev. Lett 120, 081101 (2017)
M. Vaglio + incl. A.M., Phys Rev D 108, 023021 (2023)
C. Pacilio + incl. A. M., Phys Rev D 102, 083002 (2020)

Love for testing vacuum GR

Other **Exotic Compact Objects** may be the output of stellar collapse, which form without an event horizon

- Can we distinguish ECOs with no horizon, compact to mimic BHs?



$$M_{\rm BH}/R_{\rm BH} = 0.5 \qquad \textit{v.s.} \qquad M_{\rm ECO}/R_{\rm ECO} = 0.49(99\ldots)$$

[V. Cardoso & P. Pani, 2017]

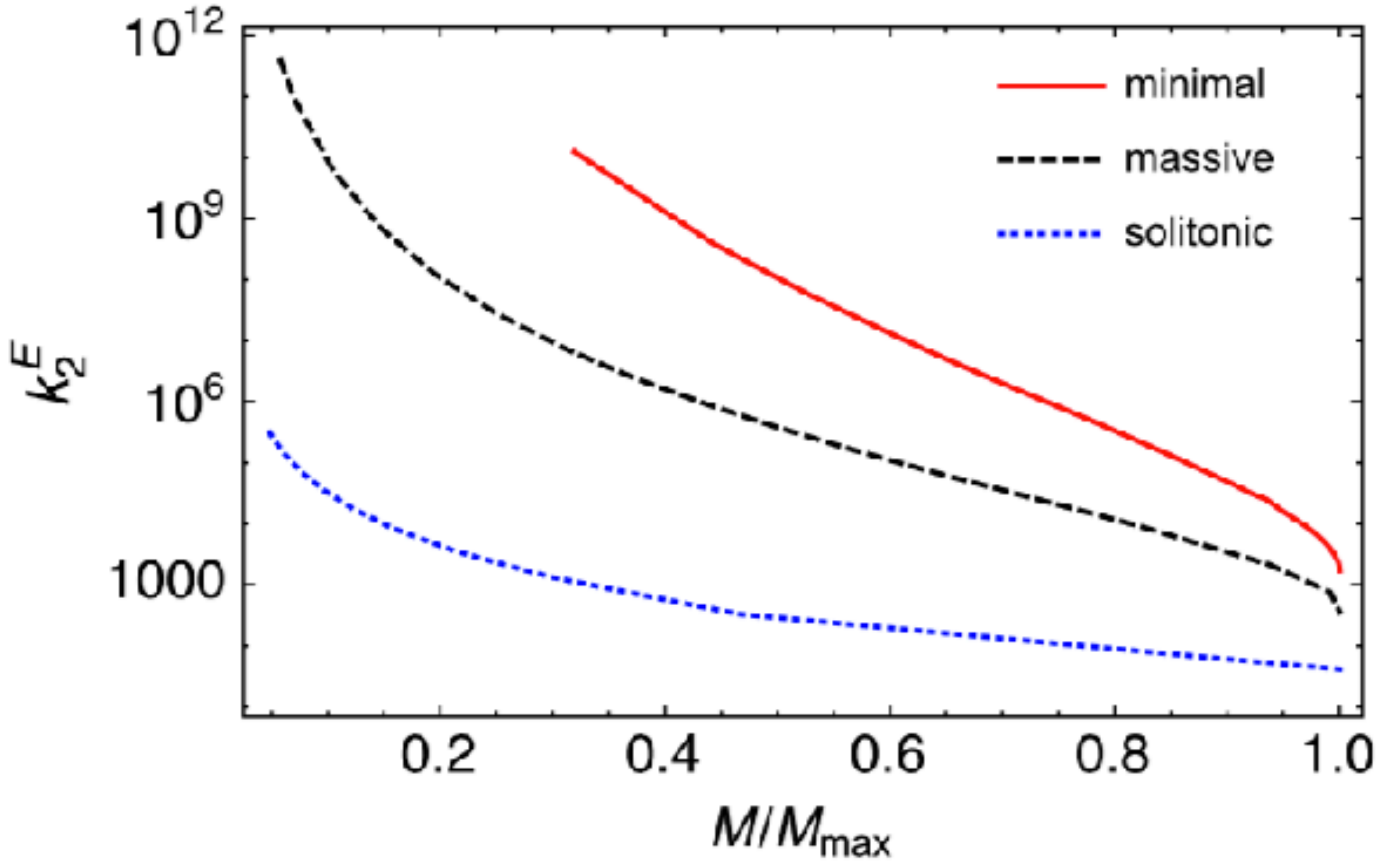
Examples of different types of Love

Scalar field condensates, i.e. Boson Stars

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi} - g^{\alpha\beta} \partial_\alpha \phi^\star \partial_\beta \phi - V(|\phi|^2) \right]$$

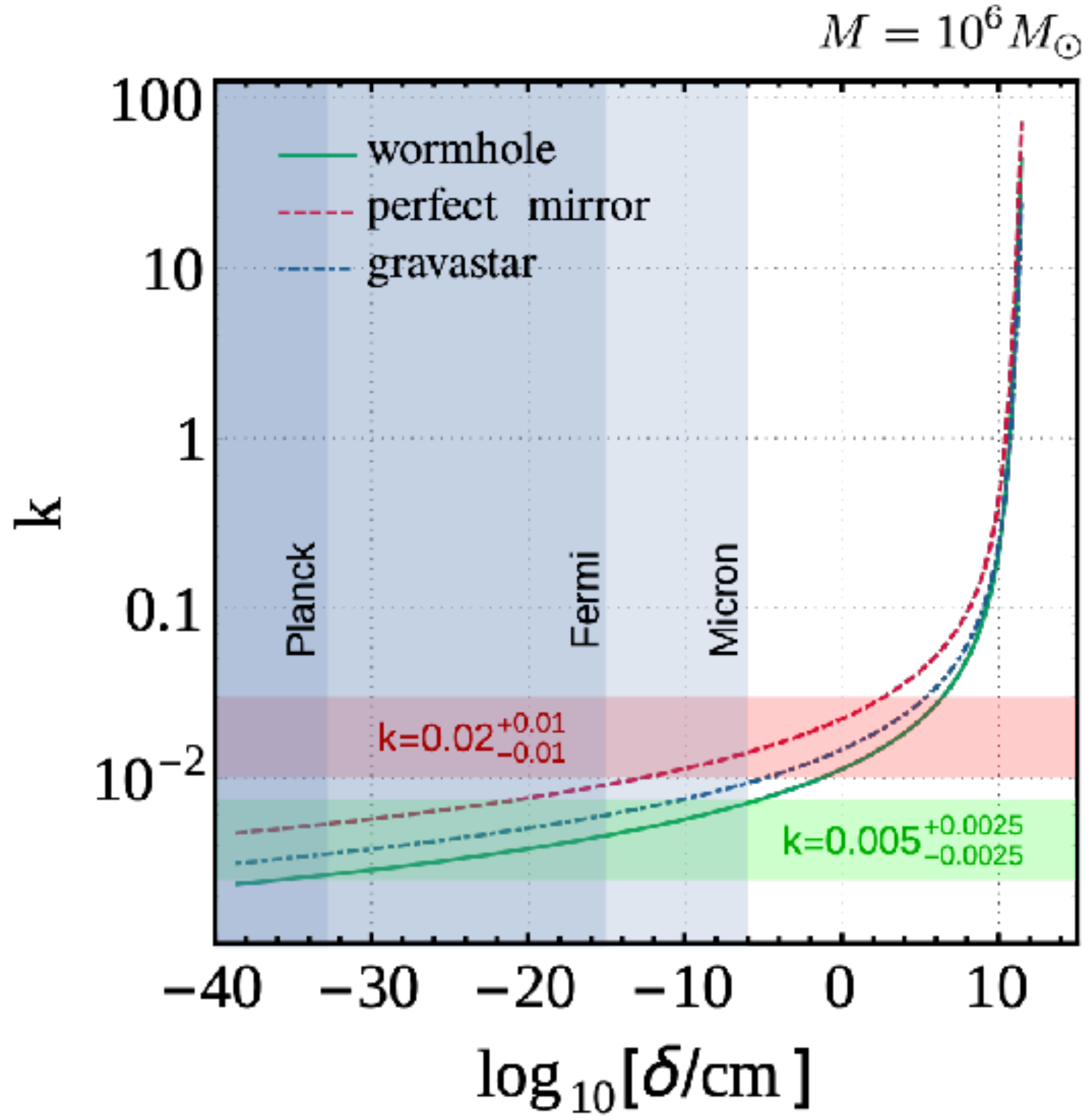
- Behaviour of in qualitative agreement with NSs
- For a compactness $C = M/R \sim 0.2$

$$k_2^{\rm NS} \sim 200 \qquad k_2^{\rm BS} \sim 40$$



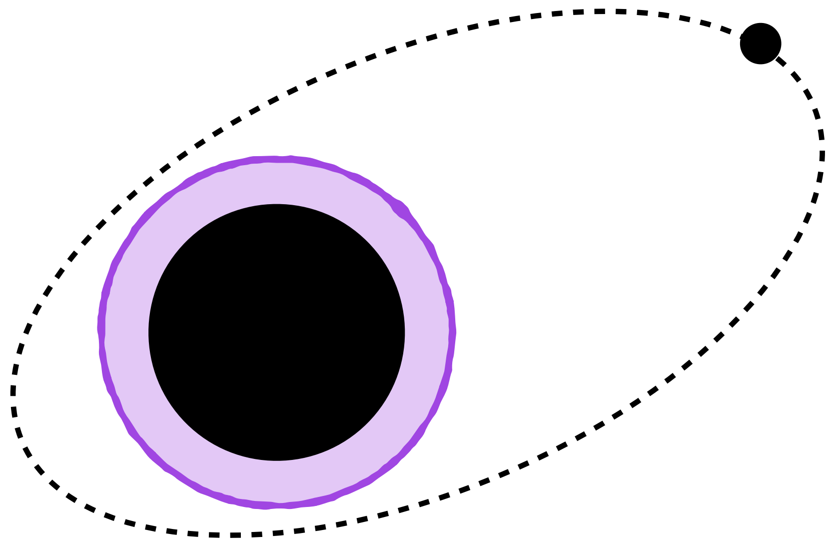
Black hole mimickers, i.e. exotic objects almost as compacts as BHs

$$R = R_{\rm horizon}(1+\delta)$$

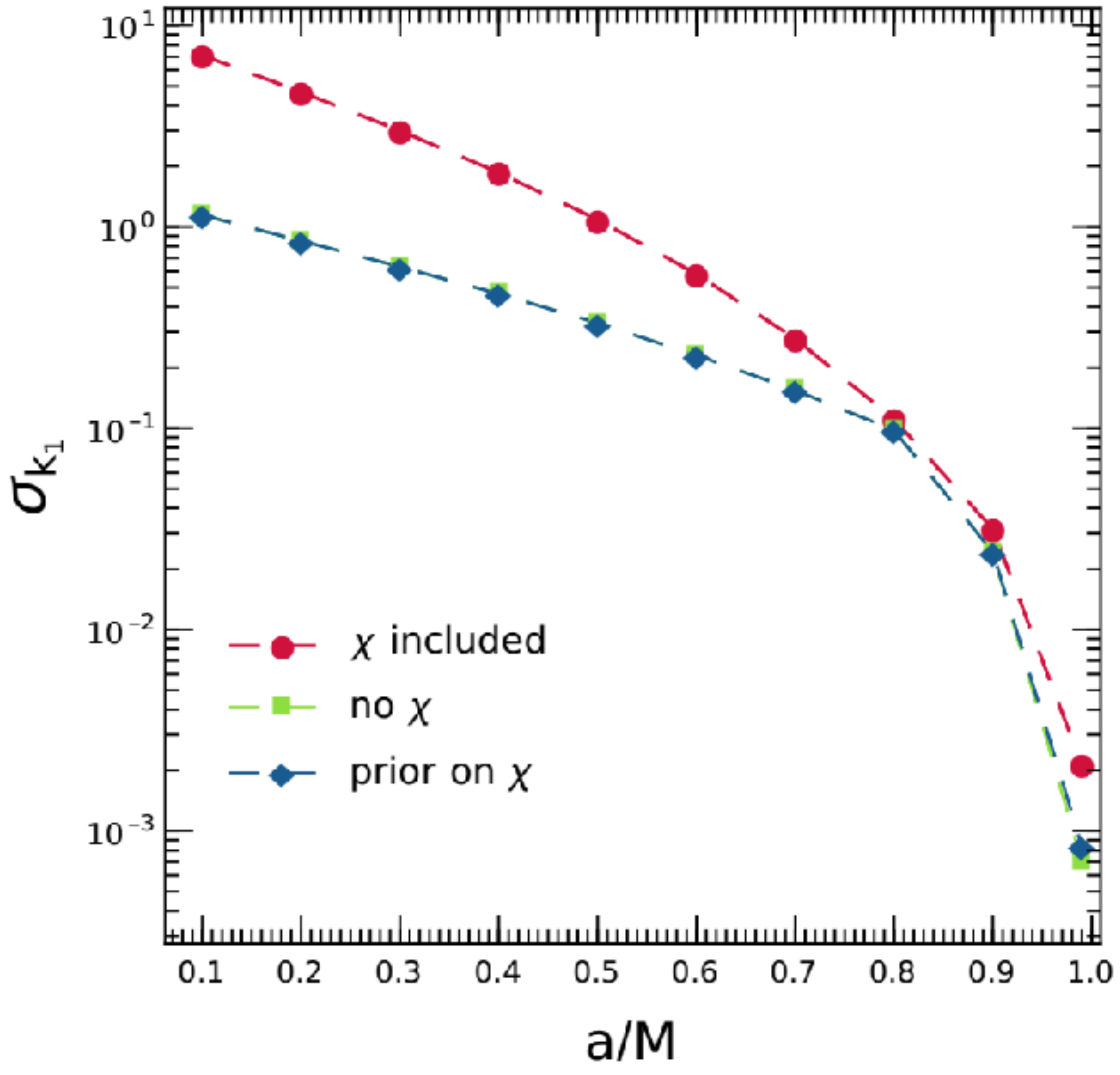


Love numbers and LISA

Measurements of the Love number by Extreme Mass Ratio Inspirals observed by LISA



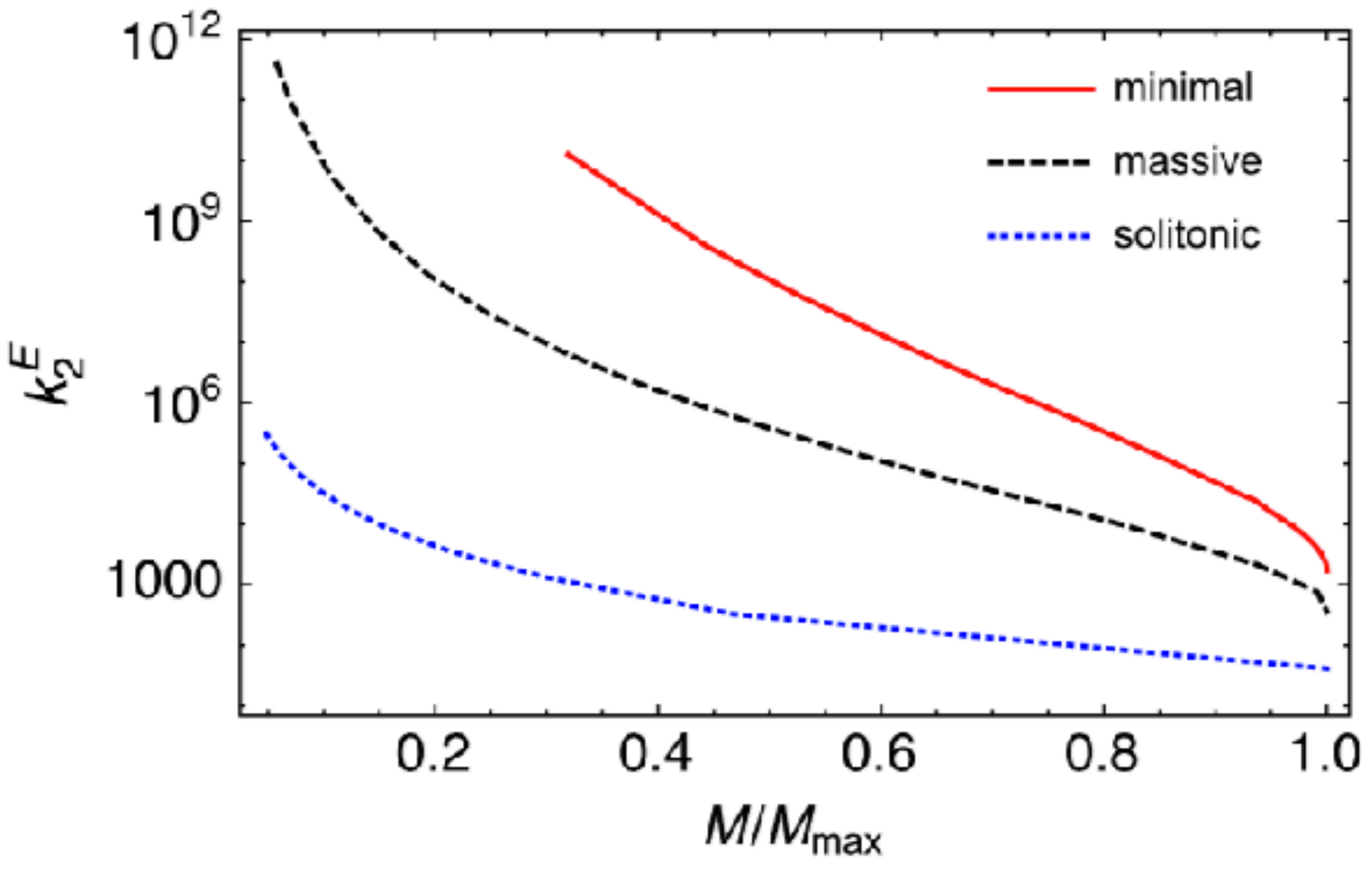
G. Piovano, A. M., P. Pani, PRD 107, 024021 (2023)



- Constraints up to 6 orders of magnitude more stringent than what achievable by current detectors for stellar binaries

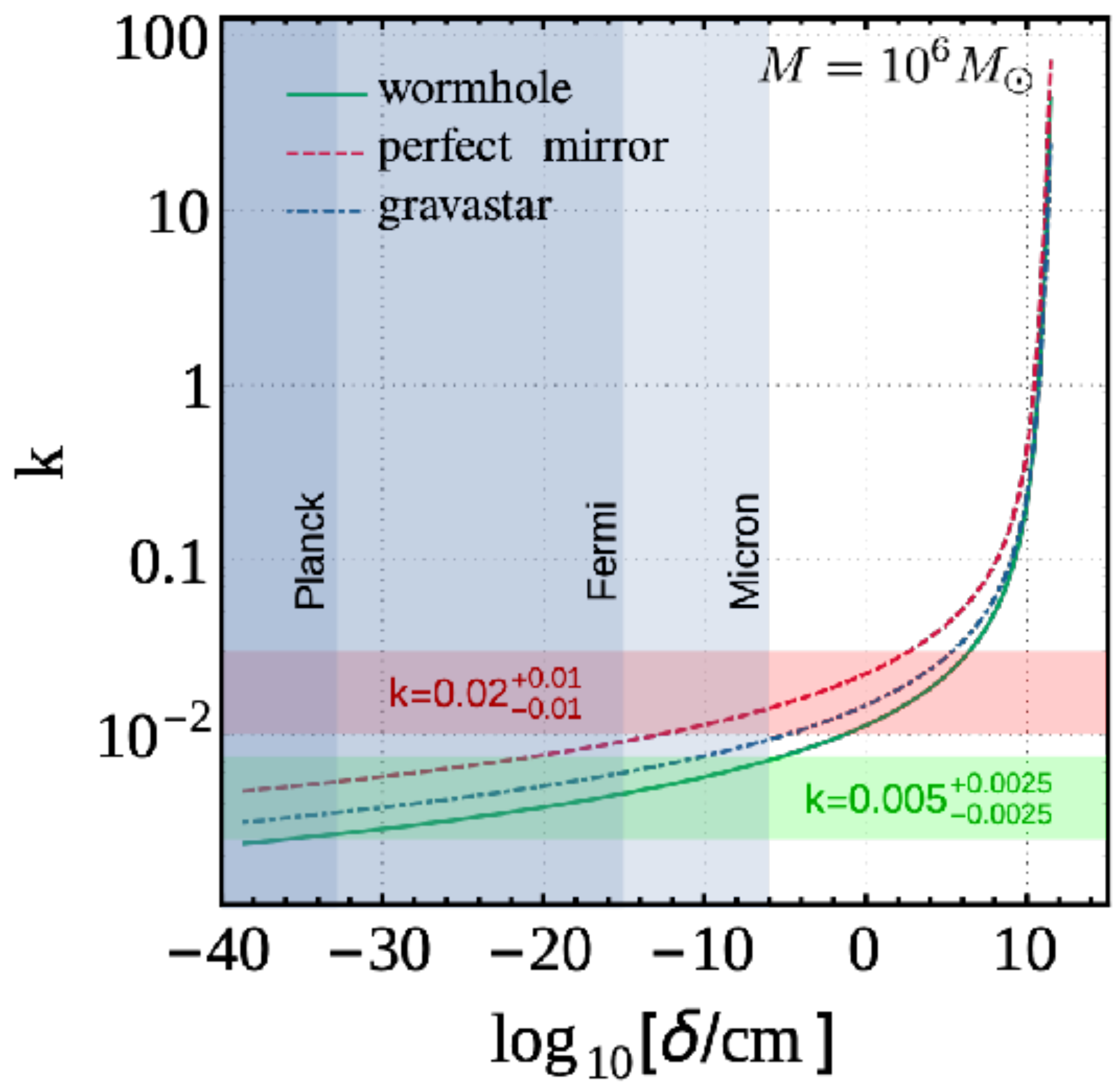
Exotic Love numbers

Boson stars



Black hole mimickers

$R = R_{\rm horizon}(1+\delta)$



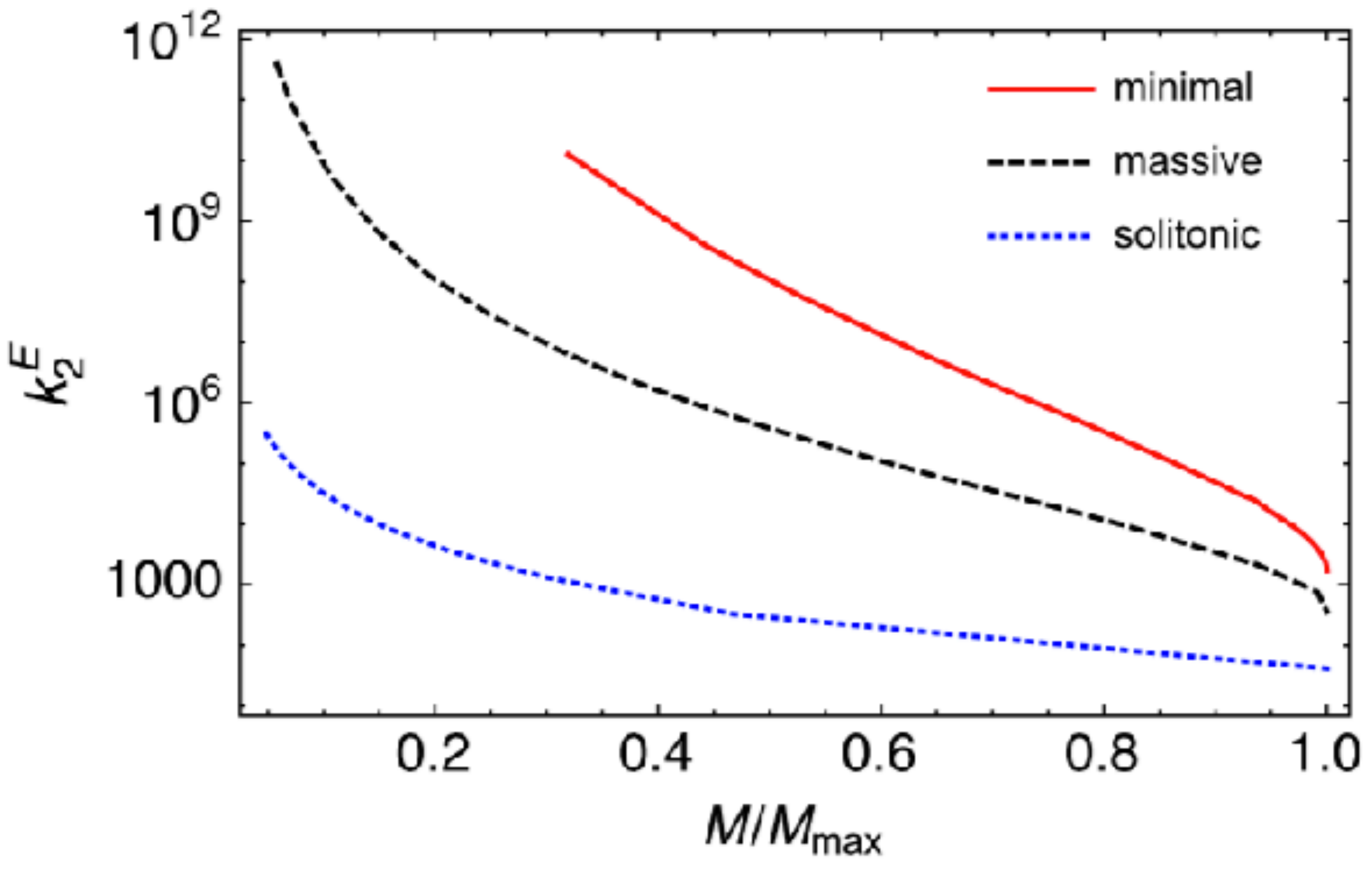
- Behaviour for BS in qualitative agreement with the neutron star case
- For BH mimickers Love numbers show a logarithmic dependence on δ

$\longrightarrow$ Can we measure Love numbers for Compact Objects with masses $M > M_{\rm NS}$

Exotic Love numbers

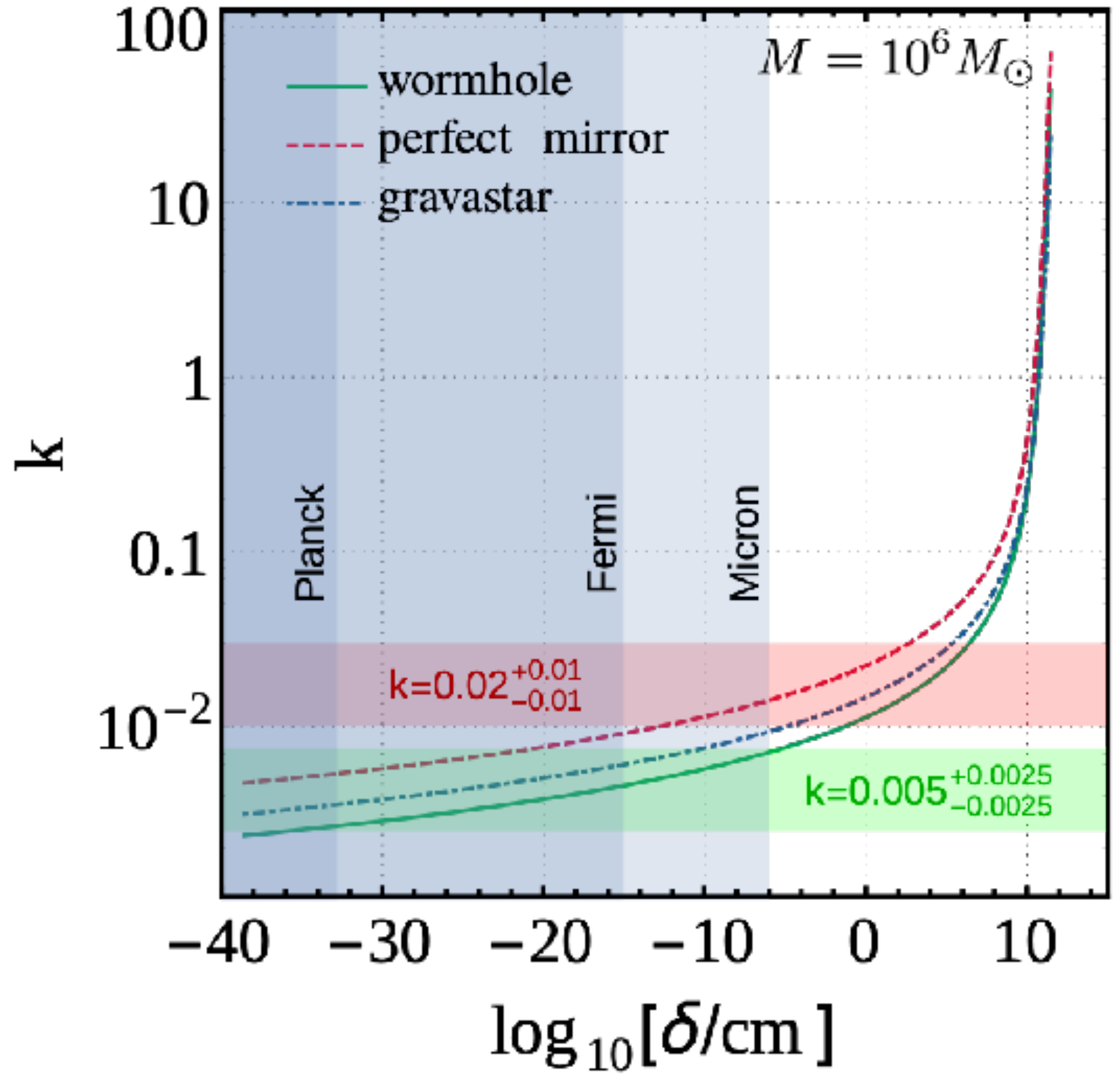
[V. Cardoso + incl. A. M., 2017]

Boson stars



Black hole mimickers

$R = R_{\text{horizon}}(1+\delta)$



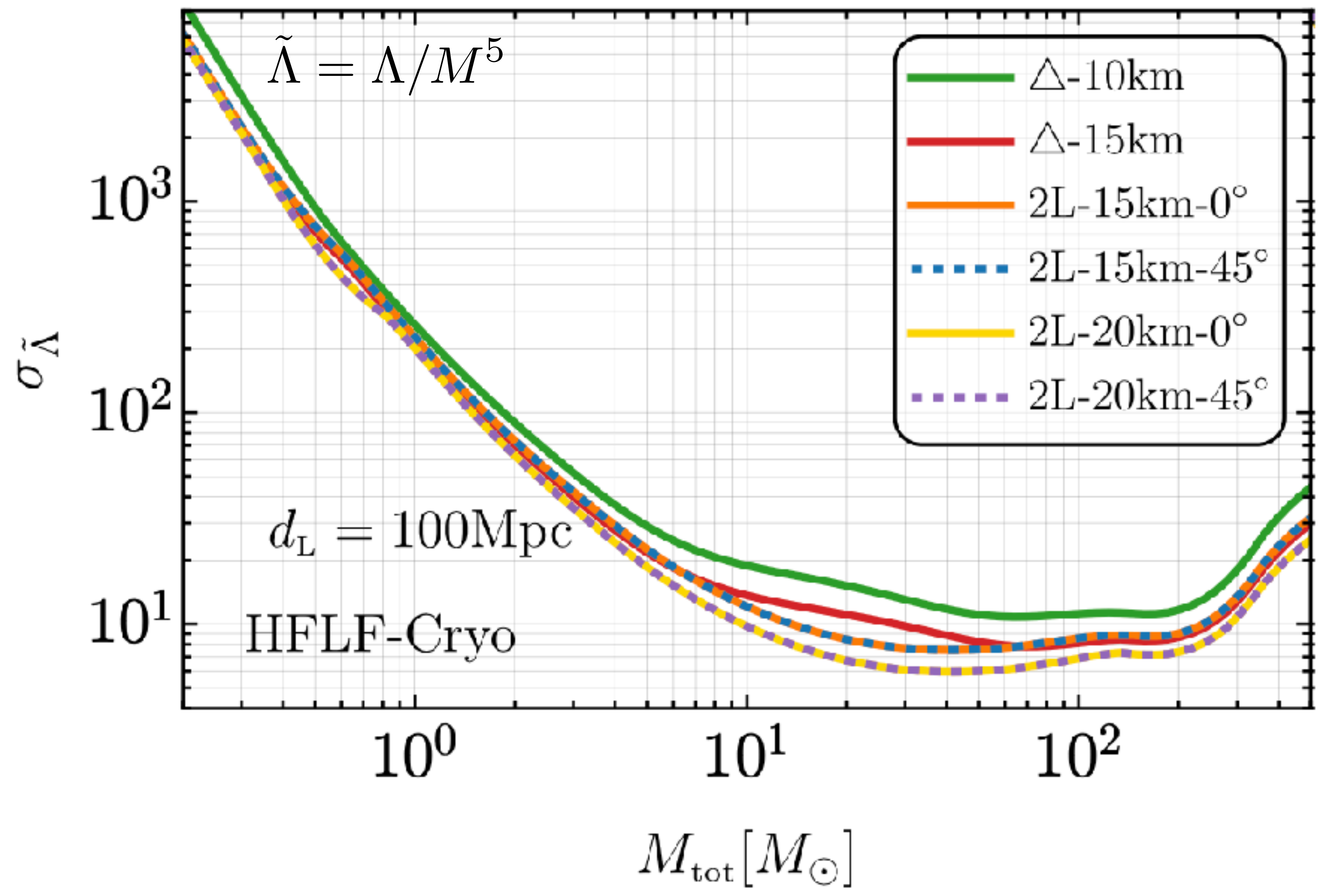
- Behaviour for BS in qualitative agreement with the neutron star case
- For BH mimickers Love numbers show a logarithmic dependence on δ

→ Can we measure Love numbers for Compact Objects with masses $M > M_{\text{NS}}$

Forecasts with ET

[M. Branchesi +, 2023]

Constraints on the tidal deformability and quadrupole deviations from ET observations



$$\xrightarrow[\textit{NS mass}]{\textit{as a reference}} \quad \sigma_{\tilde{\Lambda}} \sim \mathcal{O}(10)$$

$$\sigma_{\tilde{\Lambda}}^{\rm GW170817} \sim \mathcal{O}(100)$$

$$\theta = \{\mathcal{M}, \eta, \chi_1, \chi_2, \Lambda_i, d_L, t_c, \phi_c\}$$

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