# STAC63 Midterm Test

**Due by: Thursday, February 25 at 13:00 EST** via Crowdmark
**Number of questions: 4      Time: 2 hours + 20 minutes      Total points available: 40**

1. (10 points) This question has two independent parts (a) and (b).

   (a) Let $X_1$ and $X_2$ be independent geometric random variables having the same parameter $p$. Guess the value of

   $$\mathbb{P}(X_1 = i | X_1 + X_2 = n).$$

   (b) An unbiased die is successively rolled. Let $X$ and $Y$ denote, respectively, the number of rolls necessary to obtain a six and a five. Find $\mathbb{E}[X]$, $\mathbb{E}[X|Y = 1]$ and $\mathbb{E}[X|Y = 5]$.

2. (10 points) Consider a Markov chain with state space $S = \{1, 2, 3, 4\}$ and transition probabilities

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

(a) Compute $f_{43}$ and $f_{23}$.

(b) Compute $\lim_{n \to \infty} \sum_{n=1}^{\infty} p_{22}^{(n)}$ and determine if state 2 is recurrent or transient.

3. (10 points) Consider a Markov chain with state space $S = \{1, 2, 3\}$ and transition probabilities

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{3}{4} & 0 & \frac{1}{4} \end{pmatrix}.$$

(a) Show that the Markov chain is irreducible and aperiodic.

(b) Compute $\lim_{n \to \infty} p_{12}^{(n)}$.

4. (10 points) Suppose cars can be blue, green or red and that a car will be the same colour with probability $50\%$ and equally likely to be a different colour with probability $25\%$. Moreover, suppose you know the last car that passed by was blue. For example, if the last car was blue, then there is a $50\%$, $25\%$ and $25\%$ chance that the next car will be blue, green or red respectively.

   (a) Write out the transition probability matrix.

   (b) Calculate the expected number of cars until the pattern red-green (a green car passes after a red one) first occurs.