Balloon Rise Rate and Bursting Altitude

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This is analysis is provided as a starting point for student experimenters at the Endeavour Balloon Fest STEM event held annually in May or June at Tobin James Cellars.

A meteorological balloon, carrying instruments and transmitting equipment is released, untethered, rises until it bursts, and falls back using a parachute. An onboard radio transmits GPS data so that the payload can be recovered.

The typical balloon for tethered use at Balloon Fest has been model 8236 available from www.ScientificSales.com. This balloon masses 200g, is typically filled to 6.5 ft diameter and has a bursting diameter of 8.0 ft.

The next larger balloon is model 8237. This balloon has a mass of 300g, standard inflation diameter of 8.0 ft and a bursting diameter of 11.0 ft.

At normal ground conditions Helium has a density of 0.179 kg/m^3 and air has a density of 1.225 kg/m^3 for a difference of 1.046 kg/m^3 or 0.02962 kg/ft^3.

The total lift, $L = V(\rho_{air} - \rho_{He})$, and Volume, $V = \pi/6\ D^3$. Substituting yields:

$$\mathbf{L = 0.0155 * D^3} \tag{1}$$

where L is in kg and D is in ft.

Example: a 6 ft diameter spherical He balloon would have 3.35kg of total lift.

The Neck Lift, $\mathbf{N = L - B = P + T + R}$ (2)

Where L is total lift, B is mass of balloon, P is mass of payload, T is mass of tether (if present), & R is residual or net lift. The residual lift is needed in order to provide a reasonable ascent rate or stability against the tether. Tether stability requires at least ½ kg of residual lift or more with wind speeds greater than 5 knots.

The US Standard Atmosphere Model predicts that density changes as:
$\rho = \rho_0\ (1\text{-}h/145442\ ft)^{4.255876}$ For altitudes, h, much less than 145 thousand feet, this is closely approximated (Taylor expansion) by the exponential function:

$$\boldsymbol{\rho/\rho_0 = \exp(-h / 34174\ ft)} \tag{3}$$

If we assume that the helium gas in the balloon maintains equilibrium with the temperature and pressure of the outside air then the proportional change in density will be the same inside as outside. Since $\rho = m / V$ and $V = \pi/6\ D^3$, then:

$\rho/\rho_0 = (D_0/D)^3$ where D_0 is the initial diameter and D is the diameter at H

$D/D_0 = \exp(H / 102500 \text{ ft})$ substituting H_B for H, D_B for D, and D_L for D_0:

Bursting Altitude, $\mathbf{H_B = 102500 \ln(D_B/D_L)}$ (4)
Where D_B is the bursting diameter and D_L is the diameter at launch

The Rate of ascent for a spherical balloon at non-turbulent velocities is:
$\mathbf{V_A = (R\, g / (½\, C_d\, A\, \rho_{air}))^{1/2}}$. In m/s (5)
Where N is net lift in kg, g = 9.80 m/s^2 is gravitational acceleration, C_d = 0.25 is the coefficient of drag for a sphere, $A = \pi r^2$ is the cross-sectional area of the balloon in m^2, and ρ_{air} is the density of air in kg/m^3.

The balloon is presumed to rise at a constant rate because as the cross sectional area gets larger the density decreases. Therefore:
Time to Burst, $\mathbf{T_B = H_B / V_A}$ (6)

An online Balloon Burst Calculator is available at:
http://www.srcf.ucam.org/~cuspaceflight/calc/

Several of the constants need to be reset to match the US Standard Atmosphere model: The air density model is set at 7238.3. It should be 10416. The Air Density is set to 1.2050 and should be 1.2246. The default bursting diameters for various balloon sizes should be reset to manufacturers specification.

Examples:

Using the 200g #8236 balloon with an 8.0 ft bursting diameter, attaching a 3.4 kg payload, and filling it to a Neck Lift of 4.95 kg; the balloon would rise at a rate of 5.33 m/s and in 14 minutes would burst at an altitude of 4530 m (14,800 ft)

Using the 300g #8237, with an 11.0 ft bursting diameter, attaching the same 3.4 kg payload, and filling it to a Neck Lift of 4.97 kg; the balloon would rise at a rate of 5.32 m/s and in 44 minutes would burst at an altitude of 14,200 m (46,600 ft)

8236	8ft	3.4kg	4.95kg	5.33 m/s	14 min	4,530 m	14,800 ft
8236	8ft	1kg	1.95kg	5.57 m/s	41 min	13,600 m	44,800 ft
8237	11ft	3.4kg	4,98kg	5.32 m/s	44 min	14,200 m	46,600 ft
8237	11ft	1kg	1.98kg	5.55 m/s	69 min	22,900 m	75,200 ft