Project 2

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Problem 1)

$u_t - u_{xx} = f(x,t)$

$u(0,t) = 0 \quad , \quad u(1,t) = 0$

$u(x,0) = \sin(\pi x) \quad , \quad f(x,t) = (\pi^2 - 1)e^{-t}\sin(\pi x)$

$\int_0^1 (u_t - u_{xx} - f(x,t))v(x)\,dx = 0 \quad , \quad v(0) = v(1) = 0.$

$\Rightarrow \int_0^1 u_t v\,dx - \int_0^1 u_{xx} v\,dx = \int_0^1 f v\,dx$

$\int_0^1 u_{xx} v\,dx = [u_x \cancel{v}^{\,0}]_0^1 - \int_0^1 u_x v_x\,dx$

$\Rightarrow -\int_0^1 u_{xx} v\,dx = \int_0^1 u_x v_x\,dx$

$\int_0^1 u_t v\,dx + \int_0^1 u_x v_x\,dx = \int_0^1 f(x,t)v(x)\,dx$

$u(0,t) = 0, \; u(1,t) = 0, \; u(x,0) = \sin(\pi x)$

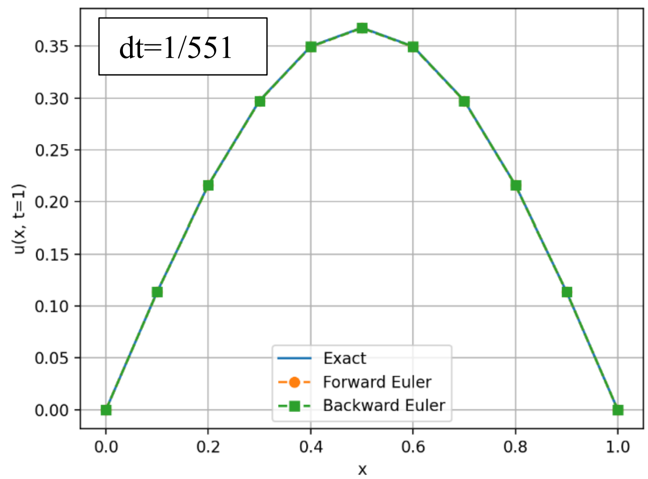
$v(0) = v(1) = 0.$

$$\boxed{\int_0^1 u_t(x,t)v(x)\,dx + \int_0^1 u_x(x,t)v_x(x)\,dx = \int_0^1 f(x,t)v(x)\,dx \;, \quad t > 0.}$$

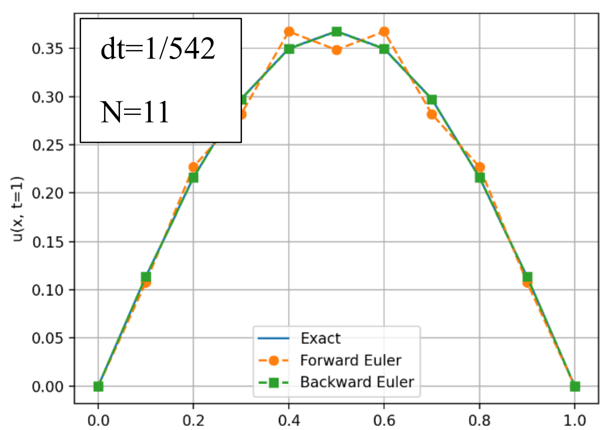
Problem 2)

FE and BE at dt = 1/551:

At this time step, there's no difference between the Forward Euler, the Backward Euler, and the exact curve.



The Forward Euler solution begins to deform at dt = 1/542. At this point, Forward Euler begins to roughly follow the curve but is not exact.



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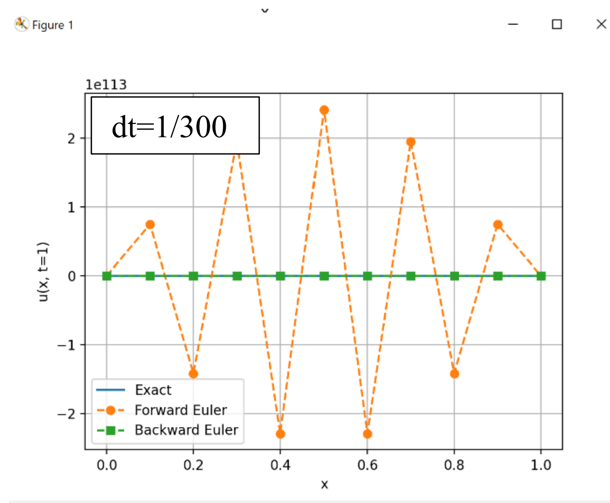
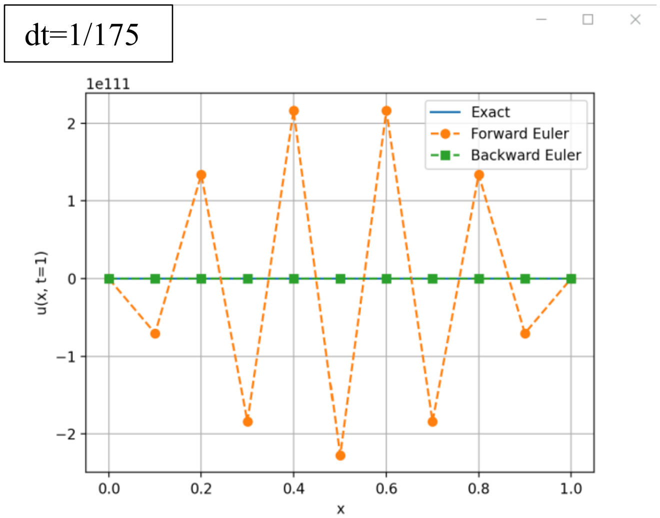
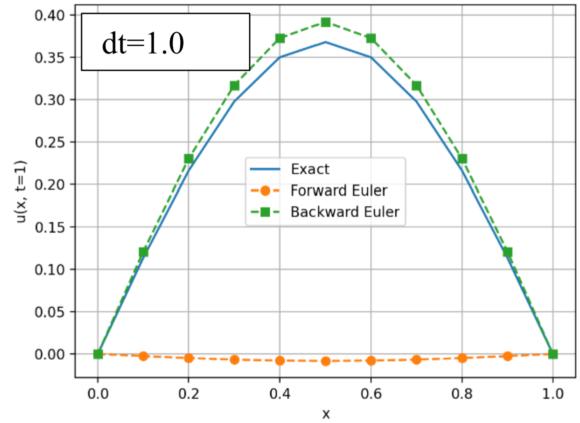
At larger time steps, the Forward Euler approximation explodes, losing all accuracy. This is an example of dt=1/300, the forward euler reaches an estimated value of 1e113. As it increases more, the approximations will become larger and larger, peaking in value at around dt=1/175. At this point, the approximations begin to decrease in value, eventually reaching a value close to zero, which is also an innacurate result.

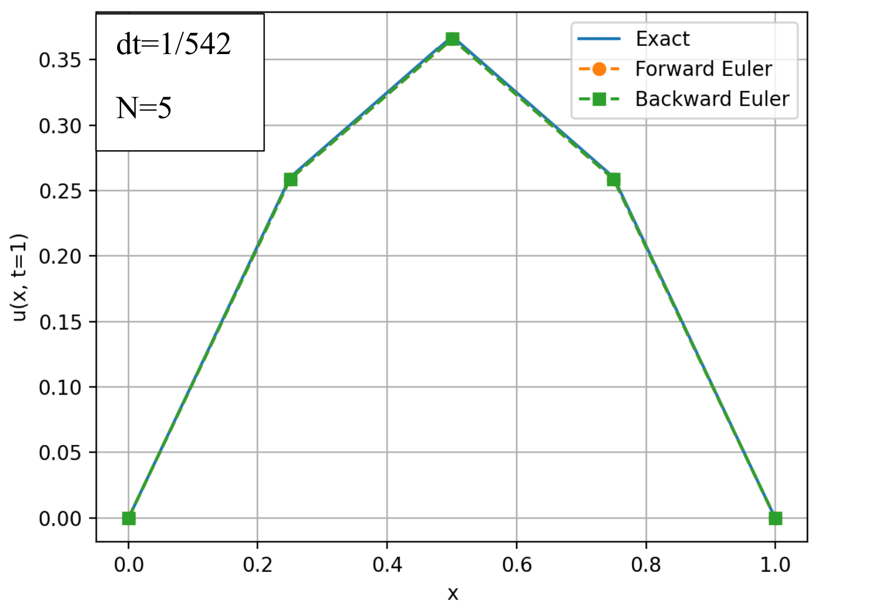






These are example graphs depicting the behavior that was described on the previous page. As you can see, the Forward Euler values decrease, eventually reaching a result very close to zero.

Oddly enough, once our Forward Euler becomes unstable, we can decrease N to regain accuracy. This is seen in the following graph:

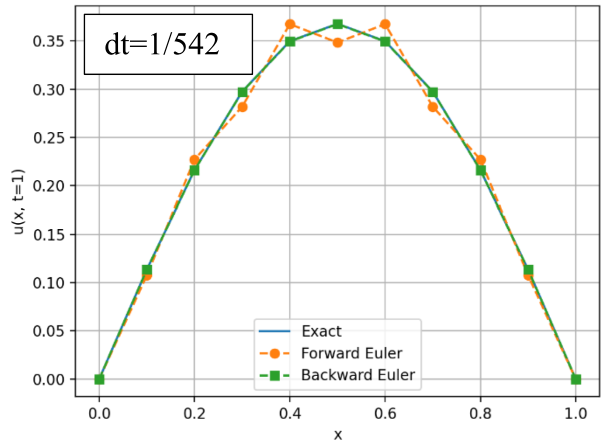


<u>Problem 2 summary:</u>

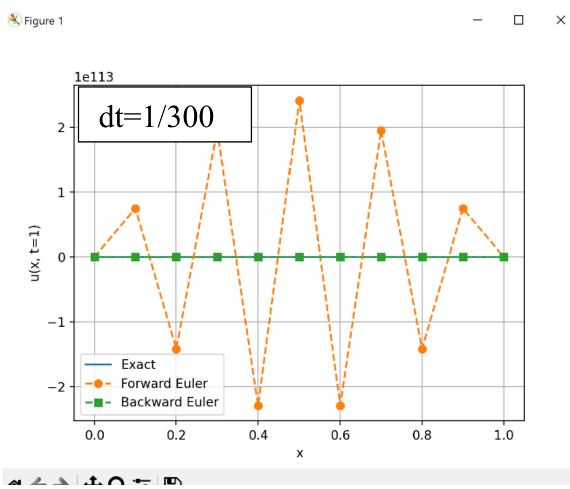
a) At N=11, Forward Euler beings to lose stability at dt=1/542.
b) The solution becomes more accurate as we decrease N.

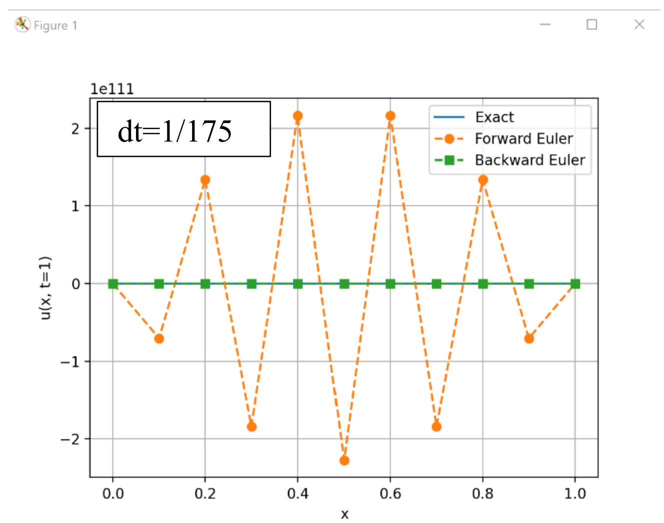
Problem 3)

As we can see, even at dt=1/542, our Backward Euler retains accuracy.

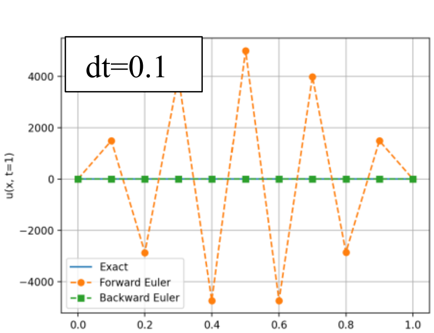


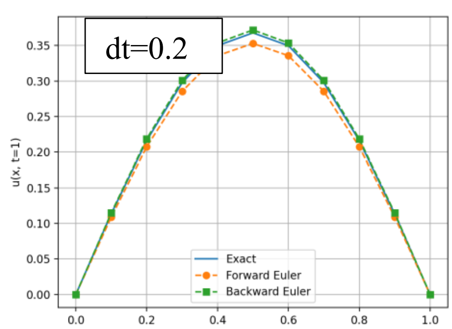
As dt increases further, it retains accuracy, even at the values that our Forward Euler is explosive at.



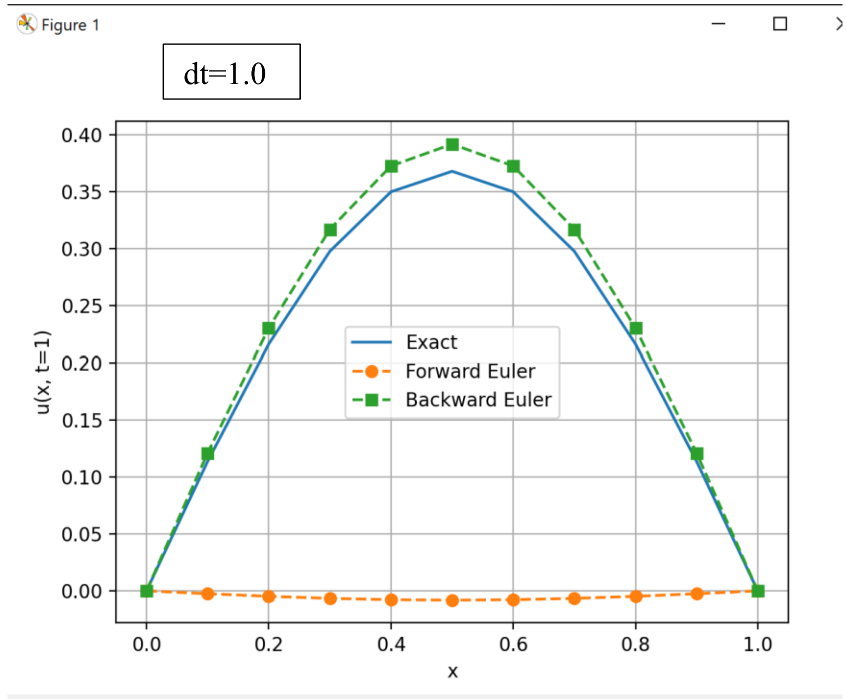


Using larger time steps, we see that the Backward Euler approximation retains accuracy; however, at around dt=0.2, our Backward Euler begins to lose accuracy, though it still retains the same shape as our exact curve.





As we can see, at t=1.0, our Backward Euler still retains the general shape of our exact curve but provides a slight over approximation.



Problem 3 summary:

a) At the time steps that our Forward Euler loses stability at, our Backward Euler retains stability.
b) Our backward Euler does lose some accuracy once our time step becomes larger than our spatial step.

The reason why our Backward Euler becomes less accurate once our time step becomes larger than our spatial step is because it is unconditionally stable, not unconditionally accurate. At larger time steps, Backward Euler becomes overly diffusive, and therefore, less accurate.