

Start-up Financing, Entry and Innovation

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Abstract

Venture capital (VC) is the key source of financing for high-growth start-ups, but with few alternatives, limited access can leave viable projects unfunded and constrain innovation. I develop and estimate an equilibrium model of the VC market to quantify these distortions in the US, explain cross-country differences in VC activity, and diagnose VC's sectoral concentration. In the model, entrepreneurs and VCs meet in a frictional matching market and VCs endogenously stage capital injections over time to limit losses from hidden failure by entrepreneurs; however, reliance on follow-on funding exposes the start-up to premature closure if funding does not materialise. The model maps directly to observed funding histories, enabling estimation and policy counterfactuals. For US start-ups first funded in 2005–2015, my estimates suggest that 40% shut down despite having positive continuation value; with continued funding, half would reach an acquisition or IPO. I then estimate the model on UK microdata and find that financing conditions and acquisition opportunities, not project quality, drive US–UK differences; financing conditions account for two-thirds of the entry gap. Because UK start-ups struggle to reach late-stage rounds, retargeting existing support towards late-stage start-ups improves outcomes. Finally, the theory offers an explanation for VC's concentration in software and services: frictions are least severe for short-horizon projects with ample acquisition opportunities. Absent frictions, the share of VC-backed software and services start-ups falls from 61% to 53%, offset by gains in science-based sectors.

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1 Introduction

Entrepreneurship is a central driver of economic growth, but realising its benefits relies on the efficient allocation of capital. While most firms secure loans against pledgeable assets or predictable cash flows, high-growth start-ups with limited revenues face difficulty financing operations with debt (Hall and Lerner, 2010). These firms instead seek funding from specialist intermediaries – venture capitalists – who, unlike banks, supply equity and finance operating losses in pursuit of substantial payoffs. Venture capital (VC) has funded many of the largest and most innovative US firms; yet, concerns remain that VC fails to penetrate certain sectors and that limited access to VC for European start-ups constrains innovation.¹

Understanding the drivers of VC allocation and outcomes is challenging. Lower VC activity in Europe and in science-based sectors may reflect limited opportunities or frictions in allocating capital to entrepreneurs; and the drivers matter for policy design. However, because *ex ante* start-up quality is poorly observed, disentangling supply and demand for capital is difficult. Furthermore, even in the mature US market, financial frictions mean some start-up projects may be terminated prematurely.² Empirically, distinguishing poor fundamentals from premature termination is complicated because the counterfactual under continued financing is unobserved.

Motivated by these challenges, I develop and estimate a tractable equilibrium model of the VC market to study (i) how financing frictions shape start-up outcomes in the US, (ii) why other developed economies have less VC activity, and (iii) whether financing frictions create bottlenecks to innovation in certain sectors. The model yields a structural interpretation of microdata on start-up funding histories, providing a way to tackle the identification challenge and a basis for quantitative counterfactual analyses that unpack start-up outcomes and simulate policy interventions. The model features frictional matching and staged financing: VCs inject capital in stages to limit losses when entrepreneurs hide failure. However, staging means the start-up must reach profitability or raise more capital before funding runs out. In turn, difficulties raising follow-on funding create financing risk, akin to rollover risk in debt markets, and mean some viable start-ups may fail to secure more funding and so shut down prematurely.³

For the US, I estimate that approximately 40% of VC-backed start-ups shut down prematurely, implying that substantial value is left on the table; with continued funding, half of these projects would achieve an acquisition or IPO. Across countries, I find that differences in VC activity – specifically US versus UK – reflect tighter financing conditions and limited acquisition opportunities, not project quality. Simulating a budget-neutral policy that redirects 5% of total UK VC funding (\approx £0.5 – 1Bn/year) from early to late-stage rounds raises IPOs by 15%. Finally, the model indicates that funding is biased away from long-horizon projects with few

¹Since 1978, approximately 50% of US IPOs were VC-backed, generating over 90% of R&D spending (Gornall and Strebulaev, 2021); 1978 marks the start of the modern VC model, when the capital gains and ERISA “prudent man” reforms were enacted (Gompers, 1994).

²Premature termination is common in models of VC contracting (e.g. Bergemann and Hege 1998; Cornelli and Yosha 2003; Mayer 2022). Relatedly, Admati and Pfleiderer (1994) features underinvestment à la Myers (1977). Beyond VC, dynamic long-term contracting can yield inefficient liquidation (e.g. Clementi and Hopenhayn 2006).

³Nanda and Rhodes-Kropf (2017) introduce financing risk to the academic literature. Paul Graham (Y Combinator founder) frames this as being “default alive” vs “default dead”: “do they make it to profitability on the money they have left?” He cautions that founders often assume raising again will be easy, but “that assumption is often false.” See Default Alive or Default Dead?, <https://www.paulgraham.com/aord.html>.

acquisition opportunities; accordingly, counterfactuals imply the share of software and services start-ups is excessive (61% versus 53%), at the expense of science-driven sectors.

The model features frictional matching between entrepreneurs and VCs and a principal-agent problem: entrepreneurs may hide failure *ex post* to reap private benefits, creating an external-finance premium. In response, VCs stage capital injections over time so only a small tranche is at risk between reviews. However, because VCs typically rely on future investors for additional rounds, initial commitments are forward looking and rise when follow-on funding is uncertain. Therefore, funding concerns at the macro level induce front-loading at the micro level, as was evident in 2008 when Sequoia advised portfolio companies to “raise as much as possible”.⁴ Incorporating these mechanisms, the model maps round-level funding histories (timing, size and frequency) to project characteristics and funding conditions, which enables estimation and counterfactuals. The model also delivers implications for the sectoral allocation of VC investment.

The matching friction captures the difficulty evaluating projects, which complicates the process of securing investment, and may delay market clearing and create congestion.⁵ Once a VC agrees to invest, the separation of ownership and control inherent to the relationship exposes them to moral hazard. Entrepreneurs invest in R&D and come to learn more about the project’s viability than the VC, but this information is private and can be hidden from VCs, leading to “inefficient continuation” (Gompers, 1995).⁶ The resulting external finance premium depends on (i) the likelihood that the project delivers a successful outcome; (ii) the development horizon; and (iii) the size of the capital injection. Importantly, because larger capital injections increase the premium, VCs have an incentive to stage capital injections over time.

Staging capital injections means that when entrepreneurs hide failure, less capital is wasted. However, because VCs often lack the ability or will to fund a start-up to positive cash flow on their own, follow-on funding requires attracting new investors. The information and project screening issues that arose when the firm first sought funding resurface and additional funding may not arrive, even for viable projects.⁷ The tendency of the VC market to suddenly dry up exacerbates this problem (Janeway et al., 2021).

These dynamics create “financing risk”: viable start-ups may not secure follow-on funding (Nanda and Rhodes-Kropf, 2017).⁸ Small capital injections mitigate agency costs but increase reliance on future funding, raising exposure to financing risk. Market tightness determines the availability of follow-on funding and so financing risk is endogenous. The optimal capital commitment balances these considerations, prompting small capital injections when agency frictions are severe, and larger injections when the VC forecasts limited future funding availability.

⁴See <https://articles.sequoiacap.com/rip-good-times>. Implicit is that Sequoia was not offering to provide follow-on funding to unilaterally, instead relying on other investors to meet their financing needs.

⁵These problems arise even for successful ventures: for example, Airbnb’s Brian Chesky described facing a series of rejections when introduced to prominent Silicon Valley investors; see <https://medium.com/@bchesky/7-rejections-7d894cbaa084>. On the investor side, Bessemer Venture Partners, a leading VC firm, maintains a list of companies that it chose not to invest in, including Apple, Intel and Google; see www.bvp.com/anti-portfolio.

⁶Entrepreneurs derive private benefits from operating their own firms (Hurst and Pugsley, 2011) and so are unlikely to want to shut them down upon the arrival of bad news.

⁷For investors, syndication aids diversification by enabling them to spread capital among more firms, and provides external validation to investments (Kerr et al., 2014; Nanda and Rhodes-Kropf, 2018).

⁸Financing risk has parallels to rollover risk in debt markets (Acharya et al., 2011; He and Xiong, 2012).

Three key insights emerge from the theoretical analysis. First, some projects are more difficult to finance than others. This is true for more novel projects, characterised by lower success rates and higher payoffs, as in Nanda and Rhodes-Kropf (2017), but it is also true for long-horizon projects. These projects are endogenously funded through more rounds, which increases their exposure to financing risk; *ex ante* investment incentives are then weaker – so fewer are funded – and, conditional on funding, success is less likely. Second, funding histories encode information about project characteristics and the funding environment. Specifically, moments such as the number of and duration between funding rounds depend on financing market tightness, the project’s novelty and development horizon, and a notion of quality. Therefore, the model provides a structural interpretation of the data, which facilitates learning these primitives from observables. Third, there is a novel contracting externality. The choice of capital injection by a given start-up and VC affects the meeting rate in the financing market; for instance, because longer-duration funding reduces the need for follow-on funding in the future. Agents in the model do not internalise this externality, and therefore contracts are generally inefficient.

To bring the model to data and make realistic quantitative statements, I extend it in two ways. First, I include a distinct role for acquisitions, which act as a substitute for VC financing by providing an alternative route to liquidity for pre-commercial projects. Most start-up exits occur through acquisitions, making their inclusion essential. Second, I introduce multiple hurdles in a start-up’s journey to full development. The model can then capture differences in how early-stage and late-stage start-ups access funding, reflecting the greater information investors have as start-ups progress. Using the quantitative model, I study start-up outcomes in the US, unpack cross-country differences in VC activity, and assess its sectoral concentration.

In the US context, I estimate the model on VC funding round and exit data for all start-ups that received their first early-stage funding round between 2005 and 2015. In the baseline quantification, start-ups are *ex ante* homogeneous. Nevertheless, the model captures the rich *ex post* heterogeneity observed in the data, including the full shape of the distributions of the number of funding rounds, the duration between rounds, exit multiples, the burn rate (i.e. rate of capital utilisation) and time-to-exit. This rich *ex post* heterogeneity is explained by shocks and information revealed after initial funding, even among start-ups that appear identical at the outset. Therefore, the analysis provides a structural counterpart to evidence on the difficulty of predicting *ex ante* which start-ups will ultimately be successful (Kerr et al., 2014).

Using the estimated model, I then analyse whether financing frictions lead to premature termination in the US market. Whereas the cause of start-up failures is unobserved empirically, simulating the model allows me to estimate the share of VC-backed start-ups that shut down before uncertainty about their projects is resolved and thus constitute inefficient closures. I find that approximately 60% of US VC-backed start-ups are either acquired, remain independent and scale, or face technological or commercial challenges that lead to closure. The remaining 40% fail due to issues in accessing capital, implying that they shut down with positive NPV projects or, equivalently, that continuation would be optimal for a cash-rich entrepreneur.

When matching and agency frictions are removed, these firms are funded until uncertainty about their potential is resolved and many go on to have successful exits. As a result, the share that scale independently – e.g. through an IPO – rises from 5% to 11% (+6pp) and pre-

commercial acquisitions rise from 25% to 39% (+14pp); together, this implies successful exits rise from 30% to 50% (+20pp). These findings point to substantial losses from frictions in VC markets, but my analysis also highlights an additional margin of inefficiency. To reduce the risk that start-ups shut down prematurely in the future, initial investors extend the funding horizon beyond the level that minimises agency costs. However, this induces inefficient continuation and, therefore, capital is misallocated in equilibrium: premature termination (underinvestment) coexists with inefficient continuation (overinvestment). My estimates suggest that, at any point in time, 12% of start-ups with capital remaining from their previous funding round should have been shut down, whereas 23% of viable start-ups are actively seeking funding for development.

With the US benchmark in hand, I next examine cross-country differences in VC activity. Despite market imperfections, the US VC market has been instrumental in financing many of today’s most R&D-intensive firms (Gornall and Strebulaev, 2021). Conversely, Europe’s strong scientific base has not translated into commercial applications at the same scale, and lower VC activity has prompted policies to address a perceived “funding gap” (e.g. Draghi 2024).⁹ However, whether lower VC activity in Europe reflects fewer opportunities or frictions in allocating capital is *a priori* unclear. The lack of metrics of *ex ante* start-up quality compounds the identification challenge. I address this with the model, which distinguishes quality from financing frictions via their distinct footprints in funding histories. I focus on a US-UK comparison, where London’s status as a global financing centre, its strong universities, and comparable legal environment make the UK’s lower VC intensity particularly puzzling.¹⁰

I find that financing conditions and acquisition opportunities, not project quality, drive the US-UK gap. UK start-ups raise fewer, more widely spaced rounds and are less likely to be acquired, consistent with tighter financing and a thinner acquisition market; conversely, I find no discernible difference in project quality among funded start-ups. Overall, financing conditions are most important, explaining two-thirds of the difference in start-up creation incentives.

These findings justify ongoing policy initiatives to release institutional capital and plug funding gaps, but targeting is important. Many programmes have traditionally focussed on seed and early-stage (Wilson, 2015), but high-growth start-ups often remain unprofitable for years, so late-stage capital is crucial to reach positive cash flow.¹¹ Furthermore, in the UK, fewer start-ups reach late-stage rounds.¹² In a budget-neutral counterfactual, I find gains from reallocating 5% of total VC funding from early to late-stage rounds: fewer firms are initially funded, but are more likely to raise late-stage capital and scale independently, rather than be acquired at pre-commercial stages. Pre-commercial acquisitions fall (−9%) but this is more than offset by an increase (+15%) in projects that are scaled independently (e.g. through an IPO). Reassuringly, recent interventions recognise the importance of supporting late-stage start-ups.¹³

⁹ Among the top 1% most cited scientific publications, China accounted for 27.3%, the US 21.7%, the EU 17.8% and the UK 6.1% in 2020 (European Commission, 2024). However, among the 2,000 largest R&D investors in 2023, 681 were US-headquartered, 524 from China, 322 from the EU and 63 from the UK (Nindl et al., 2024).

¹⁰ Evidence from developing-country private equity shows that high enforcement and common-law jurisdictions use US-style separation of cash-flow and control rights, whereas low enforcement and civil-law countries are more likely to use debt and common stock, relying on majority ownership for control (Lerner and Schoar, 2005). The US and UK share a common-law legal origin.

¹¹ Figure B.1 shows that most U.S.-headquartered, VC-backed IPO firms were unprofitable at the time of listing.

¹² 33% of UK start-ups in my sample (vs. 42% of US) raise late-stage funding within seven years of first funding.

¹³ In 2018, the British Business Bank launched British Patient Capital, a late-stage fund of funds.

Finally, I study the allocation of venture capital funding across technologies. VC typically funds start-ups in narrowly defined sectors, raising concerns that projects with broad societal benefits – from renewable energy to advanced materials technologies – may remain unfunded (Lerner and Nanda, 2020). Although these concerns are longstanding, prior work has not, to my knowledge, attempted to quantify the skew in investment across technologies resulting from frictions in VC markets. My theoretical analysis highlights that projects with long development horizons and limited established exit pathways are particularly susceptible to financing frictions. Specifically, long-horizon projects typically need to raise more rounds of funding, which increases their exposure to the issues in raising additional capital. This problem is exacerbated in cases where acquisition opportunities are limited, because there is no alternative route to liquidity.

A cross-sector calibration for the US suggests these effects are sizeable, indicating substantial sectoral misallocation of funding. Absent frictions, the share of VC-backed start-ups in ‘Computer Software and Services’ and ‘Internet Specific’ sectors, both of which are dominated by software and service firms, falls from 61% to 53%, offset by growth in science-driven sectors.

Beyond the implications for the direction of start-up innovation, these results inform the policy debate on start-up acquisitions.¹⁴ In the calibration, marginal changes to acquisition prospects have limited effect on entry incentives for short-horizon projects, where acquisition prospects are already plentiful. However, the effects are amplified for technologies with long gestation lags and where the acquisition market is thin. Practically, this suggests that increased scrutiny of start-up acquisitions for software and service start-ups is unlikely to have dramatic consequences for start-up creation on the margin, but in science-based sectors, policymakers should be more wary of detrimental effects.

Related literature This paper relates to several areas of existing study. A large literature in entrepreneurial finance studies the control mechanisms VCs use to facilitate funding for projects that would typically be rationed from credit markets.¹⁵ Screening/selection (Sørensen, 2007; Gompers et al., 2020), active monitoring and expertise (Hellmann and Puri, 2002; Hsu, 2004; Bernstein et al., 2016), and contracts that separate cash flow and control rights (Kaplan and Strömberg, 2003; Ewens et al., 2022) are important determinants of outcomes.

I focus on staged financing and the need for multiple investors to coordinate to fund a firm until it becomes cash-flow positive. Staging is designed as a form of monitoring to curb inefficient continuation (Gompers, 1995; Lerner, 1998; Tian, 2011) and is the primary governance tool used by VCs (Sahlman, 1990; Hall and Lerner, 2010).¹⁶ However, initial investors rarely commit to fund a firm unilaterally and later rounds typically bring in new investors.¹⁷ This exposes firms with little interim cash flow to financing risk (Nanda and Rhodes-Kropf, 2017); when follow-on

¹⁴Policy weighs potential harms of acquisitions, such as market power and dampened innovation incentives for incumbents, with the recognition that acquisitions induce start-up creation (e.g. Fons-Rosen et al. 2021).

¹⁵Shocks to VC supply matter for innovation and start-up activity (Kortum and Lerner, 2000; Samila and Sorenson, 2011; Chen and Ewens, 2025), indicating that VC is not a mere substitute for traditional finance.

¹⁶Staging can also mitigate hold-up/renege (Neher, 1999) and facilitate learning (Bergemann and Hege, 1998). Importantly, the VC retains abandonment options (see e.g. Cornelli and Yosha 2003; Bergemann and Hege 2005; Dahiya and Ray 2012).

¹⁷See Figure 1; section 2.1 includes a further discussion. From an optimal contracting perspective, Admati and Pfleiderer (1994) show that combining new and existing investors at subsequent funding rounds disciplines insiders’ overpricing of new securities to outsiders and mitigates the *ex post* information monopoly the firm faces when raising new capital solely from insiders (as in Sharpe 1990; Rajan 1992).

funding is unavailable, viable projects can be forced to shut down. Relative to this literature, my contribution is to embed these mechanisms in a tractable equilibrium model that can be estimated from round-level data. In the model, VCs worry that entrepreneurs hide failure and therefore stage injections over time; but this exposes the firm to financing risk when it seeks follow-on funding.¹⁸ I use the framework to quantify financing risk in the US, assess funding gaps across markets and consider policy interventions, and study the allocation of funding across technologies, providing insights into issues funding certain sectors (Lerner and Nanda, 2020).

This paper also builds on existing models of the venture capital market, which fall into two main strands: (i) theoretical analyses of the VC market directly; and (ii) macroeconomic models that embed VC into quantitative environments. In line with the former, I focus on developing a theoretical equilibrium model of the VC market; like the latter, I estimate the model and consider policy-relevant counterfactuals.

Among theoretical contributions, Inderst and Müller (2004) is first to model VC as a search-and-matching market and studies how market tightness shapes contracting under double-sided moral hazard. Michelacci and Suarez (2004) and Silveira and Wright (2016) also develop search and matching models¹⁹; Sannino (2024) studies VC value-added in an equilibrium environment; and Jovanovic and Szentes (2013) links VC excess returns to VC scarcity (and estimates them). I follow Inderst and Müller (2004) in adopting a search-and-matching framework, but my focus on staged financing differs from existing work. Theoretically, firms compete for capital across “cohorts” and contracting is generally inefficient because neither entrepreneurs nor VCs internalise how contract choices affect (i) the timing of future fundraising and (ii) the availability of funds for other start-ups. Empirically, my focus on staging means the model is readily confronted with data on start-up funding rounds.²⁰ The result is a theory that draws closely on insights from entrepreneurial finance, can be transparently estimated from funding histories and outcomes, and delivers quantitative measures of market-level distortions. This, in turn, allows me to consider the determinants of the geographic and sectoral concentration of VC funding.

Several papers incorporate VC into quantitative models.²¹ Closest to my paper, Greenwood et al. (2022) develop a dynamic contracting view of VC within an endogenous growth model; VCs offer long-term contracts, similar to those studied in Bergemann and Hege (1998) but with the addition of re-evaluation between capital injections. Two differences are worth noting. First, I stress the practical consideration that VCs do not finance a start-up to positive cash flow unilaterally. One key area where this matters is for assessments of the development of the VC market – relevant to both papers’ cross-country analyses – because less-experienced investors

¹⁸Mayer (2022) studies hidden failure; optimal contracts induce disclosure and can imply premature termination. I abstract from failure payments and study market equilibrium with many investors, but the contract retains the feature that VCs risk premature termination to curb inefficient continuation decisions by the entrepreneur.

¹⁹They study the role of the stock market in recycling capital and the VC fund lifecycle, respectively.

²⁰With severe agency issues, the duration between rounds should shorten and the frequency of injections rise (Gompers, 1995; Hall and Lerner, 2010). However, short duration funding raises financing risk (Nanda and Rhodes-Kropf, 2017); much as short-duration contracts increase rollover risk in debt markets (Flannery, 1986; Diamond, 1991; He and Xiong, 2012; Brunnermeier and Oehmke, 2013). So, when the market tightens, durations lengthen and the frequency falls. I formalise these insights and use them to estimate primitives via the model.

²¹Ates (2018) and Akcigit et al. (2022) embed VC in endogenous-growth models where VCs as provide of expertise as well as finance; Ando (2024) considers angels, in addition to VCs and banks; Ando et al. (2025) studies VC and advanced technology adoption; and Opp (2019) develops an endogenous growth model in the macro-finance tradition, focussing on VC’s risk properties and cyclical nature.

often struggle to raise large funds, making single-investor long-term commitments more difficult to sustain in practice.²² Second, the source of contracting inefficiencies differ; my focus is hidden failure/inefficient continuation, whereas Greenwood et al. (2022) considers diversion of funds. In that sense, the approaches are complementary. However, I show that focussing on inefficient continuation yields a structural interpretation of VC data, which facilitates estimation of project fundamentals and funding market conditions from standard data on funding histories.

Finally, my applications to cross-country differences and sectoral concentration in VC activity relate to the literature on interaction between financial and economic development (King and Levine, 1993; Rajan and Zingales, 1998). A large literature studies the effect of financial frictions on entrepreneurs quantitatively (e.g. Buera et al. 2011; Cavalcanti et al. 2023). A common – albeit not universal (e.g. Greenwood et al. 2010) – feature of these models in the macro-development tradition is that entrepreneurs can self-finance and gradually relax constraints via retained earnings. This differs from my model, where cash flows are delayed and firms rely exclusively on external financing. The distinction matters for policy design. In a model in the macro-development tradition calibrated to Europe, Aragonese and Saxena (2025) find aggregate TFP improvements from reallocating government VC budgets from the late to early-stage. By contrast, my UK analysis suggests focus on the late-stage delivers larger gains, even though I estimate tighter early-stage financing conditions. In my setting, entry and early-stage investment are highly sensitive to the availability of late-stage funding, which firms need to reach profitability. Taken together, the results are complementary and highlight an important nuance of policy design. When self-financing is most difficult, governments should be especially concerned about late-stage funding gaps; otherwise, relaxing constraints for early-stage start-ups is likely optimal. Relatedly, I show that acquisitions are most important in mitigating financing frictions for long-horizon technologies, where cash flows are most delayed. This has implications for debates on the role of start-up acquisitions in driving, or deterring, innovation (e.g. Kamepalli et al. 2020; Fons-Rosen et al. 2021; Cunningham et al. 2021; Berger et al. 2025).

The remainder of the paper is organised as follows. In section 2, I motivate the modelling environment, specify a baseline tractable model and then introduce two extensions to a quantitative framework. Section 3 estimates the model for the US, section 4 conducts a cross-country comparison, and section 5 analyses venture capital across sectors. Section 6 concludes.

2 Model

2.1 Economic environment and modelling choices

In this section, I describe the key building blocks for the equilibrium model. The modelling environment is motivated by three features of the market. First, difficulty evaluating projects complicates the process of finding willing investors. Second, agency conflicts lead investors to be cautious of committing too much capital to any given project. Third, institutional features of the market create a need for multiple investors to coordinate to fund a given start-up.

²²In Europe, a common concern is that limited fund size constrains capital for start-ups, particularly for larger, late-stage investments (Arnold et al., 2024). Because investor experience/quality drives returns (Sørensen, 2007; Ewens and Rhodes-Kropf, 2015) but is imperfectly observed (Gompers, 1996), more experienced investors tend to raise larger funds (Gompers and Lerner, 1998; Kaplan and Schoar, 2005).

In the model, I incorporate the first and third feature by requiring that entrepreneurs meet with new investors for each capital injection in a frictional matching market. I also provide microfoundations for an agency conflict that creates a wedge between the internal and external cost of funds, which leads to staged financing. In so doing, I make assumptions to retain tractability: my objective is to develop a model that admits closed-form solutions, both to elucidate the interaction of matching and agency frictions and to make the mapping to data transparent, so that it is clear which features of the data identify the model’s primitives.

The remainder of this section provides further details on the three features of the market that motivate the modelling environment and discuss how they can be translated into a tractable theory of the venture capital market. Section 2.2 specifies the model.

The economic setting Consider an entrepreneur with a new business idea. The project requires a continuous R&D investment k to learn about its viability, which is revealed at an uncertain future date $T_\kappa \sim \text{Exp}(\kappa)$. The idea is successful with probability p , yielding a payoff π , and otherwise fails, returning zero. The parameters (k, κ, p, π) are the project’s *characteristics*, which summarise the capital intensity, development horizon, and payoff profile of the project.

Suppose that the entrepreneur is wealthy and can self-finance the project. Assuming risk neutrality and discount rate ρ , the project NPV is given by

$$\bar{V}_s = E \left[e^{-\rho T_\kappa} p \pi - \int_0^{T_\kappa} e^{-\rho t} k dt \right] = \frac{\kappa p \pi - k}{\rho + \kappa} \quad (1)$$

where expectations are taken over $T_\kappa \sim \text{Exp}(\kappa)$. The entrepreneur should finance the project if \bar{V}_s is positive and abandon otherwise. In practice, the entrepreneur requires external financing. The project derives no intermediate cash flows and cannot be collateralised, so debt is unsuitable and the entrepreneur seeks equity financing from a VC firm.

Obtaining investment from a VC faces two broad challenges: asymmetric information and moral hazard. First, investors may be concerned that the entrepreneur has more information than they do, leading to adverse selection. The difficulty evaluating projects exacerbates this issue. Second, the separation of ownership and control induces moral hazard. For instance, the entrepreneur may misuse funds or conceal bad news from the investor.

The asymmetric information problem prolongs the process of finding an investor and funding may conceivably never arrive. Furthermore, should a VC choose to invest, the agency conflicts drive the effective investment cost above k .²³ To overcome these issues, VCs adopt a broad range of strategies, including intensive screening and active monitoring of their investments.²⁴ Among these strategies, the staging of capital injections over time is often considered the most effective (Sahlman, 1990; Hall and Lerner, 2010).

Staging helps to overcome the problem of “inefficient continuation”, in which the entrepreneur continues to operate the project beyond the date that the VC would rather close it down and see unused capital returned (Gompers, 1995; Lerner, 1998). With a smaller initial capital injection, K , less capital remains when the project is complete (whether successful or not), minimising the VC’s exposure. This lowers the cost of funds to the entrepreneur because the VC requires

²³The first problem could also generate this result. I abstract from this margin in the analysis.

²⁴For a survey, see Da Rin et al. (2013).

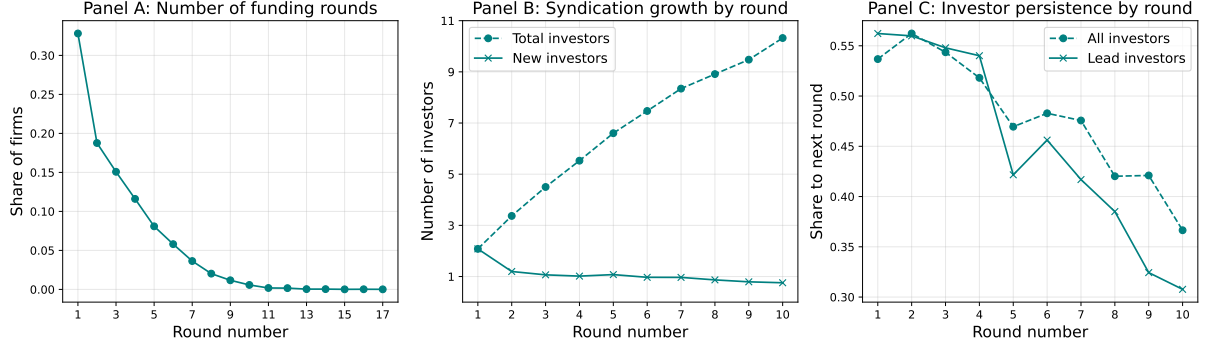


Figure 1: Staged financing and investor syndication

Source: Thomson Reuters, US VC-backed start-ups first funded between 2005–2015, including data to end-2022 (see section B.1). Data is censored at 7 years after initial funding (panel A, to ensure uniform censoring) or 12 years (panels B and C). Panel A shows the distribution of the number of rounds. Panels B and C summarise syndication patterns. Panel B shows the average total (dashed) and new (solid) investors by round; only one undisclosed investor is included (per Nanda and Rhodes-Kropf (2018)). Panel C shows the share of investors in round n that reinvest in round $n + 1$, conditional on subsequent funding; undisclosed investors are excluded.

less compensation *ex ante* for this opportunistic behaviour *ex post*. Staging also provides an opportunity to reassess the project before committing additional funds, acting as an information production tool and creating option value (e.g. Cornelli and Yosha 2003; Dahiya and Ray 2012). Panel A of Figure 1 shows that the typical VC-backed start-up in the US raises multiple funding rounds.

In practice, it is common for *new* investors to participate in subsequent funding rounds, while some incumbents do not reinvest (Panels B and C, Figure 1). This reflects various practical considerations, such as diversification incentives and fund-lifecycle dynamics, and the desire for external validation (Kerr et al., 2014) and expertise (Brander et al., 2002).²⁵ However, for the entrepreneur, the need to seek additional investors for subsequent rounds means that matching issues they faced initially reappear.

In light of these concerns, the initial investor(s) may increase upfront funding to reduce the need for subsequent rounds. At first glance, the notion that initial investor(s) could provide extra funding *ex ante*, but is unwilling to do so *ex post*, at least unilaterally, is puzzling. However, portfolio concerns, VC fund lifecycle dynamics and liquidity issues imply that capital now and capital later are imperfect substitutes. Negative shocks to one portfolio company can restrict the availability of follow-on funding for other firms in a VC’s portfolio (Townsend, 2015). VC fund structure – typically fixed-duration limited partnerships – also mechanically limits the scope for follow-on investments.²⁶ Furthermore, VCs face liquidity risk.²⁷ Anticipating potential liquidity issues, they prefer to make investments early in the fund’s lifecycle (Maurin et al., 2023) and,

²⁵The distribution of start-up outcomes is heavily right-skewed (Hall and Woodward, 2010) and the formation of investor syndicates aids diversification across investments (Pitchbook–NVCA Yearbook, 2024). Lerner (1998) also notes that bringing on new investors can help to solve agency conflicts within the VC fund that may lead to inefficient refinancing decisions.

²⁶Haran Rosen et al. (2025) show that start-ups receiving investment later in the fund’s lifecycle are less likely to receive follow-on investment.

²⁷Liquidity risk arises because VC funds make investments by pooling capital *commitments* from their investors (limited partners, LPs) on fund initiation and later draw down these commitments through capital *calls* when investment opportunities arise.

when liquidity issues arise, they make fewer investments (Li, 2024).²⁸ These liquidity shocks may have a common component and affect aggregate capital commitments, meaning that the need for new investors may arise exactly when securing funding is most difficult.²⁹ Relatedly, Nanda and Rhodes-Kropf (2017) show that the market can dry up precisely *because* multiple investors need to coordinate to fund the firm. Investor beliefs of limited future funding can be self-fulfilling, unless early investors can meet firms’ funding needs unilaterally. This leads to *financing risk*, in which projects are forced to shut down prematurely due to a lack of funding.

In this setting, the financing environment can influence what gets funded and whether projects scale successfully: the project may not be funded at all, or not funded to completion, leading to premature closure. Furthermore, the types of contracts written between entrepreneurs and VCs encode information about the extent of agency frictions and aggregate financing conditions, as they shape the capital injection decisions at any given funding round.

So far, the discussion has centred around a single entrepreneur. How does the setting change when there are many entrepreneurs pursuing projects? If many entrepreneurs are seeking funding contemporaneously, the market may take longer to clear. In that regard, congestion effects may be important. Furthermore, the potential to invest in various different projects implies that the outside options of investors may interact with the investment decision. Together, these interactions suggest that a full characterisation of the financing environment requires specifying an equilibrium model of the funding market.

An operational theory of the market The preceding discussion motivates a model of the venture capital market with the following structure.

Entrepreneurs with projects and VCs with capital meet in a frictional search-and-matching market and write incomplete contracts, which leads to the problem of “inefficient continuation”. The matching friction serves several purposes. On the one hand, it is a tractable representation of the difficulties in evaluating projects, which implies that the market can take time to clear. Relatedly, it endogenises the availability of capital based on the relative supply from investors and demand from entrepreneurs. On the other hand, it allows me to capture coordination issues among investors across funding rounds in a simple manner. Specifically, I require that an entrepreneur searches for a new VC each time it raises capital.³⁰ During their search, they may be forced to close down positive NPV projects, and so are subject to financing risk.

Contracting is incomplete, which leads the VC to infuse capital in stages. In the stylised example, the size of the capital injection, K , and the funding horizon, or “runway”, were equivalent; a start-up with initial investment K could conduct R&D for a period $T_w = K/k$. In practice, the environment is uncertain and *ex ante* identical firms may exhaust capital at

²⁸The sample in Li (2024) includes both buyout and venture capital funds. The penalties for default on capital calls are severe, including forfeiting invested capital (Robinson and Sensoy, 2016), but as Li (2024) points out, the liquidity pressure on LPs may induce VC funds to limit drawdowns for relationship building motives. Therefore, liquidity concerns need not trigger actual default to impact investment decisions at the fund level.

²⁹For instance, in 2008 the appetite among traditional LPs to make new private equity commitments fell markedly. Concurrently, “certain LPs also had issues with funding their current commitments and formally or informally communicated [...] a desire to defer capital calls [...] or to reduce the overall size of their [existing] commitments to the funds.” (Weil, Gotshal & Manges LLP, n.d.)

³⁰I make this assumption for tractability. In practice, the existing VC(s) may participate in or lead a follow-on funding round, but empirically VC syndicates grow as the firm continues to raise funding. I abstract from the precise formation of these syndicates in this model and focus on the need for multiple investors to coordinate.

different rates *ex post*. For instance, the investment plan may need to be altered or the project delivers some uncertain intermediate cash-flows (positive or negative). Therefore, I introduce some noise in the relationship between K and T_ω .

Introducing this noise directly into the funding horizon, T_ω , rather than the run-rate, k , and allowing the entrepreneur and VC to contract on the (expected) funding horizon keeps the model tractable. Specifically, suppose that the entrepreneur and VC write a contract that commits the VC to pay the investment cost k until time T_ω , a random variable. Setting $T_\omega \sim \text{Exp}(\omega)$, the funding commitment expires at constant hazard rate ω , and neither the capital stock, nor time since last funding, need to be carried as state variables.³¹ Furthermore, all state-to-state transitions then have constant hazard rates, implying that the start-up lifecycle is determined by a simple continuous time Markov chain. The parameter ω replaces K as a feature of the contract and I refer to it as the “contract rate”. A given contract rate, ω , implies a certain expected capital commitment, $K(\omega)$, the analogue to K .

The model is closed by specifying an entry condition for start-ups and a matching technology. In equilibrium, the meeting rates in the financing market must be consistent with the entry decisions of entrepreneurs and the contracts signed between entrepreneurs and VCs.

2.2 The baseline model: specification

This section presents the baseline equilibrium model; section 2.3 presents a characterisation of the equilibrium. Time is continuous and the horizon is infinite. I focus on the steady-state equilibrium, so all transition rates and stocks are constant over time.

2.2.1 Agents and technology

There are two types of agents, entrepreneurs and venture capital firms (VCs). All agents are risk neutral and discount the future at rate ρ . At any instant, an exogenous flow Γ of entrepreneurs have a one-time chance to create a *start-up*, drawing some entry cost $c \sim U[0, \sigma]$. A start-up is created if the entrepreneur finds it profitable to pay the entry cost c ; otherwise, the idea is lost.

Following entry, all start-ups are *ex ante* homogeneous. Each start-up’s project requires flow capital investment k . Conditional on paying k , the results of the development process are realised at Poisson arrival rate κ and are positive (success) with probability p and negative (failure) with probability $1 - p$. Success results in a payoff π , whereas a failure returns zero. In addition to any pecuniary return associated with the project, entrepreneurs also derive a non-pecuniary flow benefit $x_b > 0$ whenever the project is actively being funded. For technical reasons set out in section 2.2.2, I let $x_b \rightarrow 0$.

Entrepreneurs have no wealth and must seek external financing from VCs. In the baseline, I assume that entrepreneurs have all the bargaining power in contracting; I relax this in section A.2 of the Appendix. There is a measure M of VCs, each of which may finance at most one project at a time. VCs face no capital constraints with respect to a single investment, but cannot provide follow-on funding to an entrepreneur that they previously invested in, should they require it.

³¹A subtle point is that the capital injection becomes a flow, rather than a stock.

Information plays a key role in the model. Firstly, I endow VCs with a technology to screen out projects that have already failed: this implies that a start-up that has previously received funding and realised that their project has failed cannot secure additional funding from a new VC. Without such a technology, failed entrepreneurs would seek new funding to obtain private benefits, but the existence of such a technology implies that doing so derives no value. Then, only firms with projects that have not yet failed seek funding and, conditional on a meeting, the entrepreneur and VC both agree on the project characteristics (k, κ, p, π) . Secondly, once a VC has committed capital to a start-up, the arrival of news about the project is private information of the entrepreneur. This implies that any contract that conditions on the arrival of information at T_κ must be incentive compatible, in the sense that the entrepreneur must find it advantageous to report truthfully.³²

2.2.2 The financing market

Access to external finance is subject to two frictions: entrepreneurs and VCs meet in a frictional search market, and contracting is subject to moral hazard of the type described in section 2.1.

Search-and-matching friction Start-ups search for a VC following entry or the exhaustion of previous funding, provided they did not realise a negative result. VCs search for a start-up to invest in if they are not currently committed to another start-up. Search is costly due to discounting, but there are no explicit search costs. I denote by μ_s the measure of start-ups searching for capital and by μ_{vc} the measure of searching VCs. The measures μ_s and μ_{vc} are the aggregate state variables in the model and determine the flow of meetings via the matching function

$$m(\mu_s, \mu_{vc}) = \varepsilon \mu_s^\alpha \mu_{vc}^\beta \quad (2)$$

where $\alpha, \beta > 0$. The Poisson arrival rate of meetings for a given entrepreneur and for a VC are

$$\nu = \frac{1}{\mu_s} m(\mu_s, \mu_{vc}) \quad \text{and} \quad v = \frac{1}{\mu_{vc}} m(\mu_s, \mu_{vc}), \quad (3)$$

respectively. During the search process, start-ups fail at rate λ , which is exogenous. In contrast, VCs are infinitely lived.

Contracting and moral hazard Once an entrepreneur and VC meet, they negotiate an equity contract to fund the start-ups development cost, k . Contracts specify an equity stake ς for the VC and a funding termination rule, which has two components. Firstly, the funding termination rule may condition on the arrival of news about the project at date T_κ . Secondly, the entrepreneur and VC may agree to a random date, $T_\omega \sim \text{Exp}(\omega)$, at which the funding commitment expires.

³²The model can be extended to include costly monitoring. For now, I note only that contracting upon such information would require that it is ‘hard’, or verifiable. In practice, the highly innovative nature of these projects means that much of the information produced is ‘soft’, or non-verifiable, and monitoring would be ineffective. The model presented here is the special case in which all information produced by the entrepreneur is soft. Staging then emerges as the only information production tool, because it permits VCs to make continuation/termination decisions on the basis of both hard and soft information.

Consider first the contingent component of the funding termination rule, which matters when news arrives prior to the funding commitment expiring; that is, when $T_\kappa < T_\omega$ *ex post*. Due to the information asymmetry, any contract that conditions on news at time T_κ must be incentive compatible, in the sense that the entrepreneur profits from reporting to the VC.

There are two cases to consider, success or failure. Given their private benefits, the entrepreneur never reports failures.³³ In contrast, the entrepreneur may report successes. Reporting success immediately at date T_κ leads the VC to terminate funding, meaning the entrepreneur forgoes any additional non-pecuniary benefit. However, concealing the news delays the pecuniary payoff by T_ω .³⁴ In other words, the entrepreneur must decide whether the value of the private benefit is sufficient to compensate for the loss in their pecuniary payoff due to discounting, which leads to an incentive compatibility constraint.³⁵

Proposition A.1 in the Appendix shows that, for any set of model parameters, there always exists an upper bound $\bar{x}_b > 0$ such that for $x_b < \bar{x}_b$, then the entrepreneur always reports positive results. Intuitively, if the non-pecuniary benefit is sufficiently low (in a relative sense), the entrepreneur would never willingly delay the realisation of their pecuniary payoff to continue to enjoy private benefits. Therefore, given $x_b < \bar{x}_b$, the entrepreneur always reports successes to the VC, but never reports failures; this appears to accord with anecdotal accounts of entrepreneur behaviour.³⁶ The assumption $x_b \rightarrow 0$ guarantees that this is the case.³⁷

With the contingent features of the contract determined, it is simple to map out the implications of the choice of non-contingent funding termination, $T_\omega \sim \text{Exp}(\omega)$. Analogous to a larger capital injection, a lower “contract rate” ω allows the entrepreneur to conduct development for a longer duration before seeking additional funding. However, in this incomplete contracting environment, it increases the VC’s exposure to the entrepreneur’s opportunistic behaviour, because it increases both the probability that the entrepreneur will uncover negative news *and* the outstanding capital commitment when that information arrives. Specifically, the expected

³³Concealing failure allows the entrepreneur to continue to reap their private benefits, $x_b > 0$, until funding expires at date T_ω , whereas reporting failure allows the VC to shut down the venture. This logic also applies to any time after the realisation of the result because the environment is stationary.

³⁴The environment is stationary, so if the entrepreneur finds it profitable to delay reporting at the time the result is obtained, they will never report until the contract expires, which happens at date T_ω .

³⁵See Remark A.2. The IC depends on features of the project and environment, (π, ρ, x_b) , but also on the share, $1 - \varsigma$, that the entrepreneur maintains in the project. This leads to the implication that the entrepreneur must retain sufficient “skin-in-the-game” to report truthfully.

³⁶See, for example, Chapter 4 of Janeway (2018), “After more than forty years in the game, I have yet to meet the entrepreneur who dallies in delivering word that ‘the product works’ or ‘the sale has closed’. When communication ceases, then the venture capitalist can expect to discover that ‘the product needs another rev’ or that ‘we lost the order.’” Janeway calls this his Second Law of Venture Capital, “No news is never good news”.

³⁷The VC holds a claim that has some resemblance to participating preferred stock, in which the VC receives a liquidation preference and shares in the payoff; such features are common in VC contracts (Kaplan and Strömberg, 2003; Ewens et al., 2022). In the model, when the project has a liquidation value – i.e. when successful – the VC receives a share of the payoff *and* cuts funding, which acts like a claim against the company.

capital commitment associated with contract (ω, ς) is given by³⁸

$$\begin{aligned}
K(\omega) &= \underbrace{\frac{k}{\rho + \kappa + \omega}}_{\text{Frictionless expected capital cost}} + \underbrace{\frac{\kappa(1-p)}{\kappa + \omega}}_{\text{Pr(negative result)}} \underbrace{\frac{k(\kappa + \omega)}{(\rho + \omega)(\rho + \kappa + \omega)}}_{\text{Remaining commitment}} \\
&= \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k}{\rho + \kappa + \omega}.
\end{aligned} \tag{4}$$

where $K'(\omega) < 0$ and $K''(\omega) > 0$. The expected capital cost is composed of two terms. First, in a complete contracting environment, the entrepreneur and VC could write a contract to terminate funding at the realisation of the result, date $T_\kappa \sim \text{Exp}(\kappa)$. The contract rate would be $\omega = 0$, so that funding would never be withdrawn prematurely, and the VC would expect to inject $k/(\rho + \kappa)$ into the start-up before the result is realised. The first term reflects this benchmark, which is equivalent to the capital cost that would arise if the entrepreneur could invest their own funds.³⁹ However, the agency conflict introduces an additional term: the entrepreneur conceals the arrival of bad news, engaging in inefficient continuation. The second line of equation (4) shows that the capital cost now takes the form of a wedge over the frictionless benchmark due to this agency conflict.

The optimal contract Abstracting from VC bargaining power, the optimal contract, (ω, ς) , simply maximises the value of the project to existing shareholders (the entrepreneur and any prior investors) subject to a participation constraint (PC) of the new investor.⁴⁰

For a given contract (ω, ς) , the VC incurs expected capital costs $K(\omega)$, given by equation (4), and receives a share ς of the value of the firm gross of their investment, which I denote by $V_d(\omega)$.⁴¹ Therefore, the PC is $\varsigma V_d(\omega) \geq K(\omega)$. The optimal contract (ω, ς) then solves

$$\begin{aligned}
V^M &= \sup_{\{\omega \in [0, \infty), \varsigma \in [0, 1]\}} \left\{ (1 - \varsigma) V_d(\omega) \right\} \\
\text{s.t.} \quad &\varsigma V_d(\omega) \geq K(\omega)
\end{aligned} \tag{5}$$

where $K(\omega)$ is given by equation (4), $V_d(\omega)$ solves the HJB equation

$$\rho V_d = \kappa [p \pi - V_d] + \omega [V_s - V_d] \tag{6}$$

and V_s is the value of a start-up in search, which is taken as given in the optimisation. To see why V_d solves equation (6), recall that news about the project arrives at rate κ and is successful with probability p , in which case the project yields a payoff π . However, at rate ω , the funding commitment terminates, in which case the entrepreneur must search for new funding; its value

³⁸See section A.4 for the derivation.

³⁹Specifically, $\frac{k}{\rho + \kappa + \omega}$ is the expected discounted value of the capital injection prior to the realisation of the result or the withdrawal of funding. In the frictionless benchmark, funding would be withdrawn at rate $\omega = 0$, so that $\frac{k}{\rho + \kappa}$ is the expected cost of funding the start-up.

⁴⁰I consider the extension to VC bargaining power in section A.2 of the Appendix.

⁴¹This is equivalent to the “post-money” value of the firm.

then becomes V_s . In equilibrium, the value of search solves

$$(\rho + \lambda) V_s = \nu [V^M - V_s]. \quad (7)$$

which follows because a start-up in search fails at exogenous rate λ and meets with a VC at rate ν , in which case it has value V^M . The binding participation constraint pins down ς as a function of ω , which then implies that $V^M = V_d(\omega) - K(\omega)$. Therefore, the optimal contract rate, ω , maximises the (private) value of the firm net of investment costs.

2.2.3 Closing the model

To close the model, I specify the entry decision and the conditions that determine the two state variables, μ_s and μ_{vc} , which pin down the equilibrium meeting rates, ν and v .

Entry Given V_s , the entry decision is a simple cutoff rule. A given start-up enters if and only if their entry cost $c \leq V_s$, where V_s is the value of a start-up searching for capital. Given $c \sim U[0, \sigma]$, the probability of drawing an entry cost below the threshold is $Pr(c \leq V_s) = V_s/\sigma$. Each potential entrant faces this same probability, so that the flow of new entrants that enter endogenously is given by $\Gamma(V_s/\sigma)$. It will not be possible to disentangle Γ from σ , so I will substitute in $\tilde{\sigma} = \sigma/\Gamma$ going forwards. The endogenous flow of entry is then $V_s/\tilde{\sigma}$. The parameter $\tilde{\sigma}$ can be interpreted as determining the supply of entrepreneurial projects.

Steady-state conditions Finally, it remains to specify the conditions that determine μ_s and μ_{vc} . Rather than specify a condition for μ_{vc} directly, it will be simpler (and more intuitive) to consider how many firms receive funding in equilibrium and to determine μ_{vc} as the residual given the fixed measure M of VCs. In this spirit, I will refer to a start-up that has already obtained a negative result but is still receiving funding as ‘unproductive’, as opposed to ‘productive’ start-ups that have funding and may still achieve success. I denote by μ_d^p the measure of start-ups in productive development and by μ_d^u the measure of start-ups in unproductive development. Since the total measure of VCs is M and a measure $\mu_d^p + \mu_d^u$ are funding start-ups, the measure of searching VCs is given by $\mu_{vc} = M - \mu_d^p - \mu_d^u$.

The state-to-state transitions for a single firm are summarised by Figure 2. In steady-state, the inflows and outflows from each state must be equal to one another. Consider first the measure of firms in productive development. A flow $\nu\mu_s$ of start-ups enter this state from search, having signed contracts with VCs, and a flow $(\kappa + \omega)\mu_d^p$ exit this state when realising a result about their project or seeing their funding run out. Next, consider the measure of firms in unproductive development. Firms enter this state from productive development when they obtain a negative result, a flow $\kappa(1 - p)\mu_d^p$, and exit this state when their funding runs out, a flow $\omega\mu_d^u$. As Figure 2 illustrates, transitions into this state are unobserved to the VC. Finally, consider the measure of firms in search. Firms return to search from productive development when their funding runs out, an inflow $\omega\mu_d^p$. Since firms in unproductive development would be screened out by subsequent VCs, they find it optimal to close down and so do not return to search. In addition to the inflow from productive development, new entrants begin their lifecycle in search of capital, a flow $V_s/\tilde{\sigma}$. Conversely, start-ups exit search when they fail during the

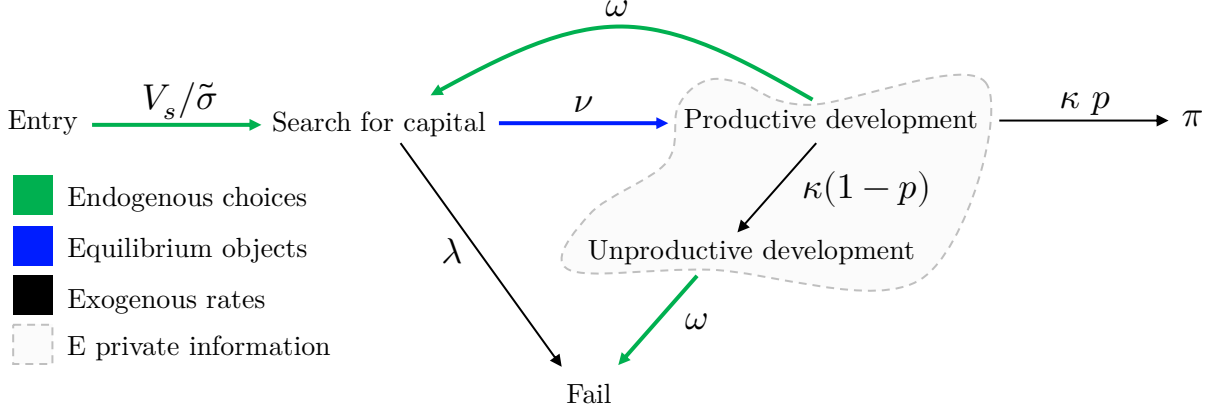


Figure 2: State transitions

Entry reflects an aggregate flow; other arrows represent individual-level transition intensities.

search process or meet with VCs, a flow $(\lambda + \nu)\mu_s$. Therefore, the steady-state conditions governing μ_d^p , μ_d^u and μ_s are given by

$$\mu_d^p : \quad \nu \mu_s = (\kappa + \omega) \mu_d^p \quad (8)$$

$$\mu_d^u : \quad \kappa(1-p)\mu_d^p = \omega \mu_d^u \quad (9)$$

$$\mu_s : \quad \omega \mu_d^p + \frac{V_s}{\tilde{\sigma}} = (\lambda + \nu) \mu_s \quad (10)$$

These equations can be written purely in terms of μ_s and μ_{vc} .⁴² I then define an equilibrium as follows.

Definition 2.1. *A steady-state equilibrium is a tuple $(V_d, V_s, V^M, \omega, \varsigma, \mu_s, \mu_{vc})$, where (ω, ς) solve maximisation problem (5) given the meeting rates (ν, v) in equation (3), and equations (6), (7) (8), (9) and (10) are satisfied.*

2.3 The baseline model: equilibrium characterisation

In this section, I analyse properties of the equilibrium. First, taking financing conditions in the matching market as given, I solve the contracting problem and consider comparative statics. These exercises illustrate: (i) how observable features of VC contracts respond to project characteristics and financing market conditions, thereby encoding information about them, and (ii) how frictions in VC markets shape start-up entry and outcomes. Second, I consider efficiency and discuss the feedback between the agency and matching frictions, which endogenously determines financing conditions.

⁴²From equation (9), $\mu_d^u = \frac{\kappa(1-p)}{\omega} \mu_d^p$, so that $\mu_{vc} = M - \mu_d^p - \mu_d^u = M - \left(1 + \frac{\kappa(1-p)}{\omega}\right) \mu_d^p = M - \left(1 + \frac{\kappa(1-p)}{\omega}\right) \frac{\nu}{\kappa + \omega} \mu_s$, where the last equality follows from equation (8). Furthermore, for μ_d^p in equation (10) from equation (8) yields $\frac{\omega}{\kappa + \omega} \nu \mu_s + \frac{V_s}{\tilde{\sigma}} = (\lambda + \nu) \mu_s$.

2.3.1 The equilibrium contract

Given a value for the meeting rate in the financing market, ν , the model admits closed-form solutions for value and policy functions.⁴³ The first proposition proves existence of a unique equilibrium, given ν , and the resulting corollary derives the optimal contract rate, ω , as a function of model parameters.

Proposition 2.1. *Given ν and for $k < \kappa p \pi$, there is unique solution for $\{V_d, V_s, V^M, (\omega, \varsigma)\}$ in terms of model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, $\omega \in [0, \infty)$.*

Corollary 2.1. *Given ν and $k < \kappa p \pi$, the equilibrium contract rate is given by*

$$\omega = \max \left\{ 0, -\rho + \mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} \right\}, \quad \text{where } \mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \nu \times \frac{\rho + \kappa}{\rho + \lambda} \quad (11)$$

and $\omega \rightarrow \infty$ otherwise. \mathcal{C} captures characteristics and \mathcal{F} captures need for and access to follow-on funding.

The optimal contract depends on both characteristics of the project, via \mathcal{C} , and is forward looking through its dependence on \mathcal{F} . Comparative statics are analysed in section 2.3.2. Here, it is worth noting briefly that there are two classes of contract. The contract may feature $\omega = 0$, in which case the VC commits to fund the project to completion with certainty and corresponds to *upfront financing*. Conversely, for $\omega > 0$, the entrepreneur may require additional funding for their project; this is *staged-financing* and is an endogenous feature of the model. In what follows, I will analyse the model under staged financing, which is the empirically relevant case. Upfront financing can arise in the model and requires both that the agency friction is sufficiently muted and the financing market is sufficiently tight. In general, agency frictions will be severe when financing highly-innovative projects, so staged-financing emerges in equilibrium. In simple terms, low-risk projects (think real estate investments) can be financed upfront, whereas more novel ventures benefit from a wait-and-see approach.

2.3.2 Comparative statics

This section analyses the contract, observable funding histories, and start-up entry and outcomes. In addition to standard comparative statics, which consider how these objects respond to model primitives, I investigate the role of financing frictions across projects with different characteristics conditional on a notion of project quality. This enables analysing the implications of frictions in VC markets for the direction of start-up innovation activity.

To this end, I define quality as the NPV of a project to a wealthy entrepreneur that can self-finance development: \bar{V}_s in equation (1) (restated below). Under risk-neutrality, exponential discounting, and perfect capital markets, \bar{V}_s is a sufficient statistic for investment decisions and therefore constitutes the relevant frictionless benchmark. Conversely, under frictional capital

⁴³Under generalised Nash bargaining case, both ν and v must be specified. VC bargaining power introduces two incentives shaping the optimal contract: (i) VCs profit from each investment but can only invest in one firm at a time, so each deal entails an opportunity cost (cf. Michelacci and Suarez 2004); and (ii) VCs anticipate dilution by future investors who also extract surplus. Effect (i) pushes towards shorter-duration contracts, whereas (ii) induces the opposite – the net effect of VC bargaining power on ω is ambiguous (see Proposition A.2).

markets, entry is proportional to $V_s \leq \bar{V}_s$. To the extent that different projects are equal in \bar{V}_s but not in V_s , the comparative statics reveal how financing frictions create distortions to entry and investment across projects with equal quality, but different characteristics.

To formalise this, I define a quality-compensating comparative static (QCCS) in which project characteristics are varied while the payoff, π , is adjusted to hold quality, \bar{V}_s , fixed. For example, a QCCS with respect to the arrival rate of R&D outcomes, κ , compares a baseline project to a project with a shorter development horizon, higher κ , but lower payoff in case of success, lower π . The compensating decrease in π means I capture the effect of κ coming purely from the timing of the payoff, rather than from changes in the underlying project NPV.

Definition 2.2. Consider a project with characteristics (k, κ, p, π) . I define

- (a) Project quality as $\bar{V}_s = \frac{\kappa p \pi - k}{\rho + \kappa}$;
- (b) A quality-compensating comparative static (QCCS) in parameter x as the derivative with respect to x while adjusting π so that quality, \bar{V}_s , is held constant.

The optimal contract The optimal contract rate, ω , trades off the incentive to reduce agency costs, incentivising higher ω , with insurance against the need for follow-on funding, which incentivises lower ω .

The effects are summarised by two terms, \mathcal{C} and \mathcal{F} ; see equation (11). Firstly, the contract rate is increasing in \mathcal{C} , a composite measure of project *characteristics* that reflects the extent of agency frictions relative to the underlying project value. Holding the contract rate, ω , fixed, the agency friction is more severe for short-horizon or risky projects, which leads $\kappa(1-p)$ to appear in the numerator of the wedge in $K(\omega)$, shown in equation (4). Intuitively, for a given funding commitment, ω , a short-horizon project, higher κ , is more likely to realise a result and leaves a larger (expected) outstanding capital commitment when news arrives. Furthermore, all else equal, a higher likelihood of project failure, an increase in $(1-p)$, makes it more likely that the entrepreneur will engage in inefficient continuation when news about the project arrives. Both of these factors increase agency costs. To militate against these issues, the entrepreneur and VC reduce the capital commitment, increasing ω . Doing so reduces the likelihood that inefficient continuation will occur and the losses in the case that it does (see equation (4)). However, the incentive to reduce the capital commitment also depends on the size of cash flows that the project produces, relative to investment costs, $\frac{\kappa p \pi - k}{k}$. When this is high, the loss in value resulting from delay, or outright failure, during search is larger. Therefore, entrepreneurs pursuing such projects are more willing to endure higher costs of capital to reduce exposure to future financing conditions.

Secondly, the contract rate is increasing in \mathcal{F} , a composite measure of the firms need and ability to seek *follow-on financing*. The development horizon, κ , affects the *need* for follow-on financing: holding the contract rate, ω , constant, a longer-horizon project, a lower κ , is more likely to require additional financing.⁴⁴ The meeting rate and failure rate in search affect the *ability* to obtain follow-on financing: a lower meeting rate, ν , or higher failure rate, λ , make securing additional financing more difficult. When additional financing becomes more difficult

⁴⁴That is, $Pr(T_\kappa < T_\omega)$ is increasing in κ for fixed ω .

to obtain, the existing investor increases their capital commitment by lowering ω , a form of insurance. These effects are summarised in the following proposition.

Proposition 2.2. *The optimal contract rate ω is strictly*

- *increasing in the meeting rate (ν) and the flow investment cost (k);*
- *decreasing in the R&D success probability (p), payoff (π), and failure rate in search (λ).*
- *For the arrival rate of R&D outcomes (κ), there is a threshold, $\bar{\kappa} > k/p\pi$, such that ω is strictly decreasing in κ below this threshold and strictly increasing above it.*

Holding project quality, \bar{V}_s , fixed (QCCS), the contract rate ω is strictly decreasing in the R&D success probability (p) and strictly increasing in arrival rate of R&D outcomes (κ).

Most of the comparative statics follow immediately from the preceding discussion. However, the probability that news is positive, p , and the project horizon, κ , interact with the contract choice through multiple channels. A higher likelihood of success, p , raises both project quality and reduces the extent of the agency friction. This implies that both the standard and quality-compensated comparative static are negative, corresponding to longer funding commitments for projects with higher p .

In contrast, the effect of the development horizon, κ , is ambiguous. On the one hand, as κ rises relative to ω , the agency friction is exacerbated because of maturity mismatch between the realisation of uncertainty and the funding horizon. Intuitively, if uncertainty is expected to be resolved in a matter of months, but funding has been provided for a number of years, then the VC is at risk of exploitation by the entrepreneur. Furthermore, holding ω fixed, a rise in κ reduces the need for follow-on funding, which means the entrepreneur and VC require less insurance against financing risk; this generates an additional motive to match the development horizon with the funding horizon. Together, these factors imply that the QCCS is positive. However, κ also affects project quality: an increase in κ increases quality by bringing the payoff forwards and reducing the total investment required. This puts downward pressure on \mathcal{C} , counteracting the quality-compensated effect and dominating for small κ .

Funding histories To build intuition, I now consider how the choice of contract maps into observable features of venture capital funding data. I focus on three readily observable features: the size of each capital injection, the number of funding rounds, and the duration between funding rounds.⁴⁵ To this end, I let $\bar{K} = K(\omega^*)$ denote the VC's expected capital commitment in the optimal contract; this is the model counterpart to the capital injections observed empirically. For the number of funding rounds, I focus on the subset of firms that receive at least one funding round to facilitate a natural mapping to data.⁴⁶ The following propositions and related corollaries present the key results.

Corollary 2.2. *The capital injection \bar{K} is strictly*

- *increasing in the payoff (π) and failure rate in search (λ);*

⁴⁵The relationship between these features of funding patterns, firm characteristics, and market conditions was first explored in the seminal work of Gompers (1995).

⁴⁶A share $\frac{\lambda}{\nu+\lambda}$ of entrants fail before they meet a VC for the first time and so receive no funding.

- decreasing in the meeting rate (ν);
- and ambiguous with respect to the R&D success probability (p), arrival rate of R&D outcomes (κ), and flow investment cost (k).

Holding project quality, \bar{V}_s , fixed (QCCS), the capital injection \bar{K} is ambiguous with respect to the R&D success probability (p) and strictly decreasing in the arrival rate of R&D outcomes (κ).

Proposition 2.3. *The distribution of the number of funding rounds, $N_f \sim \text{Geometric}(q_d)$ where $q_d = \frac{\kappa}{\kappa+\omega} + \frac{\omega}{\kappa+\omega} \frac{\lambda}{\lambda+\nu}$ with support $N_f \in \{1, 2, 3, \dots\}$.*

Corollary 2.3. *The expected number of funding rounds $E[N_f] = 1/q_d$ is strictly*

- increasing in the meeting rate (ν) and flow investment cost (k);
- decreasing in the R&D success probability (p), payoff (π), and failure rate in search (λ);
- and ambiguous with respect to the arrival rate of R&D outcomes (κ).

Holding project quality, \bar{V}_s , fixed (QCCS), the expected number of funding rounds $E[N_f]$ is strictly decreasing in the R&D success probability (p) and ambiguous with respect to the arrival rate of R&D outcomes (κ).

Proposition 2.4. *The distribution of the duration between successive funding rounds, $T_{br} \sim \text{Hypoexponential}(\kappa + \omega, \lambda + \nu)$.*

Corollary 2.4. *The expected duration between successive funding rounds for a given firm $E[T_{br}] = \frac{1}{\kappa+\omega} + \frac{1}{\lambda+\nu}$ is strictly*

- increasing in the R&D success probability (p) and payoff (π);
- decreasing in the meeting rate (ν) and flow investment cost (k);
- and ambiguous with respect to the failure rate in search (λ) and arrival rate of R&D outcomes (κ).

Holding project quality, \bar{V}_s , fixed (QCCS), the expected duration between funding rounds $E[T_{br}]$ is strictly increasing in the R&D success probability (p) and strictly decreasing in the arrival rate of R&D outcomes (κ).

The results for \bar{K} follow immediately from Proposition 2.2; the fact that many are ambiguous reflects the combination of the partial effect of these parameters on the capital cost directly, see equation (4), combined with the effect through optimal contracting, $K'(\omega) < 0$. For instance, holding ω fixed, $K(\omega)$ is increasing in k , but contracts adjust optimally, and the shorter resulting funding horizon counteracts the direct effect, so that the total effect is ambiguous.

The implications for the number and duration of funding rounds provide more insights. These are random variables because, while start-ups are *ex ante* homogeneous, they face different

shocks *ex post*.^{47,48} Figure 3 plots the distributions. The key insight is that factors that raise agency costs reduce the duration between funding rounds and lead the firm to conduct more funding rounds. Conversely, factors that increase the exposure to and costs of financing risk lead to a reduction in the number of funding rounds and an increase in the duration between them. This suggests that by observing VC funding data, it is possible to learn about these primitives. I expand on this in discussion of model identification in section 3.1.1.

The project development horizon is a subtle case. Holding quality fixed, an increase in κ leads to a shorter funding committent, a higher ω , to match the funding and development horizons. This naturally results in less time between funding rounds. However, the effect on the number of funding rounds is more nuanced.⁴⁹ On the one hand, there is a direct effect: for given contracts, a longer development horizon implies more funding rounds. On the other hand, contracts adjust to match the funding and development horizons. This adjustment could offset, or even outweigh, the direct effect and reduce the total number of funding rounds. Therefore, what matters is how responsive contracts are to the development horizon. In practice, the response is sufficiently muted and the direct effect dominates.⁵⁰ Therefore, I will typically refer to projects that have longer development horizons requiring more funding rounds.

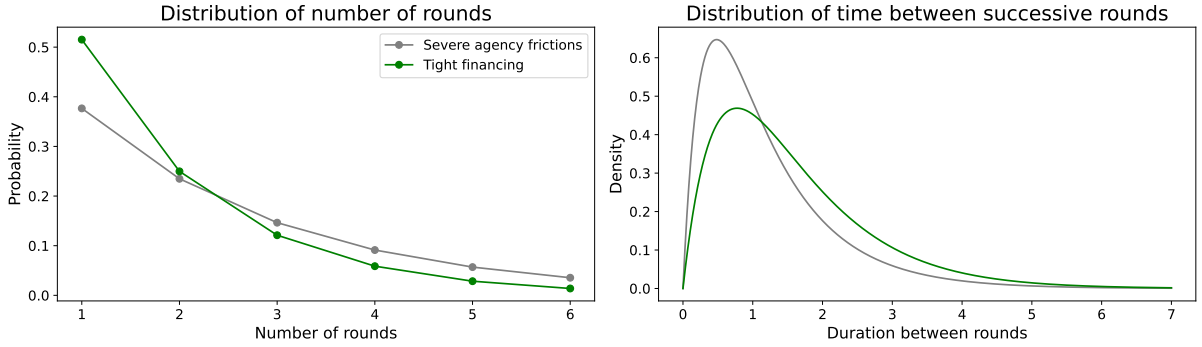


Figure 3: Distribution of the number of and time between funding rounds

The figure depicts the distribution of the number of funding rounds and time between rounds for two hypothetical parameterisations. In the case of “tight financing”, the meeting rate, ν , has been lowered. In the case of “severe agency frictions”, the project is made more novel.

⁴⁷To build intuition for Proposition 2.3, note that the probability of another funding rounds is memoryless, as it depends only on the current state and not on the number of prior funding rounds. As a result, the distribution of the number of funding rounds follows a Geometric distribution—the only discrete probability distribution with the memoryless property. In practice, this memoryless property is a strong assumption and likely violated in the data. The quantitative model relaxes this assumption.

⁴⁸To build intuition for Proposition 2.4, consider that between two funding rounds, the start-up conducts development for period $T_d \sim \text{Exp}(\kappa + \omega)$ and the searches for duration $T_s \sim \text{Exp}(\lambda + \nu)$. Therefore, the time between two successive funding rounds is $T_{br} = T_d + T_s$, the sum of two exponentially distributed random variables. Therefore, T_{br} is hypoexponential, inheriting the rate parameters of the underlying exponential distributions.

⁴⁹This is counterintuitive, given that generally there is a relationship between the number of funding rounds and the duration between them. However, discussions of the duration between rounds are conditional statements—the firm has to actually meet with a VC for the duration to be defined. Conversely, the number of funding rounds is explicitly affected by the extent to which firms fail while searching for capital.

⁵⁰Specifically, long-horizon projects require more funding rounds in expectation iff $\frac{\kappa \omega'(\kappa)}{\omega} < 1$; that is, the elasticity of the contract rate, ω , to the completion rate, κ , is less than one. It can be shown that if $\pi < \bar{\pi}$ for some $\bar{\pi}$ that depends on model parameters, then $\omega'(\kappa)/\omega < \kappa^{-1}$ is satisfied. In practice, this value of $\bar{\pi}$ is very high, i.e. orders of magnitude higher than the value estimated in the quantitative model.

Outcomes With the model apparatus in place, the analysis now turns to the central question: which forces determine whether entrepreneurs ultimately commercialise their innovations, and do all positive-NPV projects reach completion? I proceed in two steps. I start by analysing *outcomes* for start-ups that receive initial funding from VCs. Among these start-ups, the process of securing financing means they fail in search prior to project completion, leaving entrepreneurs with projects they would continue, could they self-finance. Then, I consider below whether frictions in access to capital affect start-up creation, or *entry*.

Under self-financing, only the exogenous success rate, p , matters for outcomes. However, when the entrepreneur seeks external financing, start-up outcomes are endogenous. I denote by p_d and p_s the probabilities that a firm currently in productive development or search, respectively, is successful with its project at some point. The structure of the model allows for writing these probabilities as the solution to the recursive equations

$$p_d = \frac{\kappa}{\kappa + \omega} p + \frac{\omega}{\kappa + \omega} p_s, \quad p_s = \frac{\nu}{\lambda + \nu} p_d.$$

Specifically, a firm currently in productive development succeeds in its current financing round with probability $\frac{\kappa}{\kappa + \omega} p$; otherwise, it transitions to search and is successful with probability p_s . Similarly, a firm currently in search transitions to development with probability $\frac{\nu}{\lambda + \nu}$, in which case it succeeds with probability p_d . Solving for (p_d, p_s) leads to the following proposition.

Proposition 2.5. *The (steady-state) probabilities of success for a firm in productive development, p_d , and in search, p_s , are*

$$p_d = (1 - \Delta_p)p, \quad p_s = \frac{\nu}{\lambda + \nu} p_d, \quad \text{where} \quad \Delta_p = \frac{\lambda\omega}{\kappa(\lambda + \nu) + \lambda\omega}$$

with $\Delta_p \in [0, 1)$ and where $p_d, p_s < p$ for $\omega \in (0, \infty)$.

The term Δ_p in Proposition 2.5 can be viewed as an *outcome distortion*. Under self-financing, the entrepreneur finances the project until uncertainty is resolved and success occurs with probability p . However, when the entrepreneur seeks external financing, it may not secure sufficient funding to resolve uncertainty and additional funding may not be forthcoming. This lowers the success rate below p . However, this need not be the case: if the entrepreneur and VC find it optimal to commit to *upfront financing*, $\omega = 0$, then $\Delta_p = 0$ and $p_d = p$. This leads the simple insight that financing matters for start-up outcomes. The following corollary makes the link between financing patterns and start-up outcomes explicit.

Corollary 2.5. *Consider two start-ups $i = a, b$ facing the same funding conditions, ν and λ . If $E[N_f^a] > E[N_f^b]$, then $\Delta_p^a > \Delta_p^b$.*

In words, Corollary 2.5 says that firms adopting funding strategies with more funding rounds face greater outcome distortions. Importantly, it becomes clear which start-ups face the most severe distortions. On the one hand, low quality projects typically demand less insurance against financing risk, visiting the market more frequently, and so are endogenously more exposed to failure in search.⁵¹ On the other hand, for fixed quality, highly novel start-ups face more severe

⁵¹This might seem to contradict the findings of Gompers (1995), that successful firms undergo more financing

agency frictions, fund themselves with more rounds, and so see greater outcome distortions. Similarly, longer horizon projects will tend to require more funding rounds and so face greater outcome distortions.

Finally, in Proposition A.4, I explore the implications of financing conditions, ν and λ , for start-up outcomes. In general, they have ambiguous effects, which reflects two opposing forces. The direct effect of tighter funding markets is to increase the risk of failure from financing risk. However, as Proposition 2.2 illustrates, a decrease in ν or increase in λ induces a response in optimal contracts: firms raise more capital to insure against financing risk. When markets are particularly tight, i.e. ν is low, this effect can dominate.

Entry Turning to entry, recall from section 2.2.3 that the flow rate of entry is given by $V_s/\tilde{\sigma}$; in other words, entry is increasing in the value of search.⁵² Were entrepreneurs able to self-finance, entry would be $\bar{V}_s/\tilde{\sigma}$, where \bar{V}_s is given in Definition 2.2. However, the fact that entrepreneurs must seek external financing may put downward pressure on entry incentives, meaning start-up creation will be suboptimally low. To build intuition, it is useful to consider the case of $\rho \rightarrow 0$, so that the timing of payoffs becomes irrelevant. In that case, the value of entry (search) is simply⁵³

$$V_s = \frac{\nu}{\lambda + \nu} (1 - \Delta_p) \frac{\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\omega}\right)}{\kappa} \quad (12)$$

This expression has the same form as $\bar{V}_s|_{\rho \rightarrow 0}$ in equation (1), but three differences emerge. First, only a share $\nu/(\lambda + \nu)$ of entrants ever receive funding, reducing the incentive to enter. Second, the outcome distortion attenuates entry incentives. It pre-multiplies the entire expression, reflecting both that $p_d = (1 - \Delta_p)p$ is the relevant success probability, and that the entrepreneur may be forced to shut down before completing the development process, which reduces their total capital investment.⁵⁴ Finally, the agency friction increases the effective flow capital cost by a factor $1 + \kappa(1 - p)/\omega$.

To summarise how these factors influence entry incentives when contracts are determined optimally, it is useful to define an *entry distortion*, Δ_s , analogous to the outcome distortion. The value of search when contracts are set optimally is given by⁵⁵

$$V_s = (1 - \Delta_s) \bar{V}_s, \quad \text{where} \quad 1 - \Delta_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + 2\mathcal{C} + 2\sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}} \quad (13)$$

where $\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k}$ and $\mathcal{F} = \kappa + \nu \times \frac{\rho + \kappa}{\rho + \lambda}$, as before. Similar to their role in determining the

rounds than unsuccessful ones. However, in the baseline model with *ex ante* homogeneous firms, the number of funding rounds is independent of outcomes (success or failure). Furthermore, in the quantitative model, the firms that are ultimately successful secure, on average, more rounds of funding. The introduction of an intermediate milestone is important for this result.

⁵²In this simple model, I adopt a unit elasticity of entry with respect to the value of search but this statement clearly holds more generally.

⁵³Another useful expression is $V_s = \frac{\nu}{\lambda + \nu} (p_d \times \pi - E[N_f] \times K(\omega)|_{\rho \rightarrow 0})$, where p_d is the endogenous success probability for a start-up that has just received funding, $E[N_f]$ is the expected number of funding rounds and $K(\omega)$ is the expected capital commitment per round.

⁵⁴The outcome distortion, Δ_p , affects total capital expenditure because it affects the duration of investment. In the frictionless case, the expected investment duration is κ^{-1} . In the frictional economy, it is $E[N_f] \times (\kappa + \omega)^{-1} = \kappa^{-1}(1 - \Delta_p)$, where $(\kappa + \omega)^{-1}$ is the average duration in development before returning to search.

⁵⁵See Appendix A.4.2 for the derivation.

optimal contract rate in equation (11), the composite measures of characteristics, \mathcal{C} , and the financing, \mathcal{F} , are central. The entry distortion, Δ_s , is decreasing in \mathcal{F} , reflecting that a lesser need for follow-on funding, or greater ease of securing it when it becomes necessary, reduces the entry distortion. In contrast, the entry distortion is increasing in \mathcal{C} . On the one hand, this is reassuring because it suggests that high-value projects, for instance with high π , face the least severe distortions. However, this ignores the role of agency costs and, to the extent that a high value of \mathcal{C} reflects these costs, the entry distortion will be significant. The following proposition summarises the key results for entry.

Proposition 2.6. *The value of entry V_s is strictly*

- *increasing in the meeting rate (ν), R&D success probability (p), payoff (π), and arrival rate of R&D outcomes (κ)*
- *decreasing in the flow investment cost (k) and failure rate in search (λ).*

Holding project quality, \bar{V}_s , fixed (QCCS), the entry value V_s is strictly increasing in the R&D success probability (p) and ambiguous with respect to the arrival rate of R&D outcomes (κ).

In most cases, the intuition for the sign is relatively straightforward. Entry is higher when: financing is easier to access, an increase in ν or decrease in λ ; or projects are higher quality, higher (p, π, κ) or lower k . For λ and ν , these effects reflect that the entry distortion, Δ_s , falls when financing conditions ease. For project characteristics, (p, π, k) , it reflects the combination of a direct effect on quality, via \bar{V}_s , and on the entry distortion, Δ_s , as higher quality projects face more limited distortions, as do less novel projects. For the development horizon, κ , there are various competing effects. Allowing quality to adjust, entry is higher for short-horizon projects, reflecting both the timing of payoffs and more limited capital requirements associated with a short-horizon project. Holding quality fixed, the effect is technically ambiguous but in practice the effect goes in the same direction as the outcome distortion, Δ_p .⁵⁶ Therefore, because, for given quality, longer-horizon projects are funded with more rounds, in general, they face greater entry distortions.

2.3.3 Efficiency

This section studies efficiency. In the frictionless benchmark, where a wealthy entrepreneur can self-finance the project, the NPV is given by \bar{V}_s in equation (1). The entry rate is then $\bar{V}_s/\tilde{\sigma}$ and a share p of projects are successful. When entrepreneurs seek external funding, $V_s < \bar{V}_s$ and $p_d \leq p$; there is less entry and a lower share of projects are successful. However, the model nests the frictionless benchmark in the limit as $\nu \rightarrow \infty$. Indeed, removing the matching friction means $\omega \rightarrow \infty$ is optimal and the firm is financed through an infinite number of instantaneous funding rounds. The value of search then approaches the frictionless value, \bar{V}_s , as summarised by the following lemma.⁵⁷

⁵⁶In the limit where $\rho \rightarrow 0$, Corollary 2.5 can be applied directly to Δ_s .

⁵⁷For finite ν , $\omega \rightarrow \infty$ implies no capital commitment and the total amount of capital raised by the firm over its lifecycle is zero; $\lim_{\omega \rightarrow \infty} E[N_f] \cdot \bar{K} = 0$. However, as $\nu \rightarrow \infty$, $\omega \rightarrow \infty$ more slowly ($\lim_{\nu \rightarrow \infty} \omega(\nu)/\sqrt{\nu} = c \in \mathbb{R}$; see proof to Lemma 2.1). Then, $\lim_{\nu \rightarrow \infty} E[N_f] \cdot \bar{K} = k/\kappa$; the proof to Lemma 2.1 accounts for this, including discounting.

Lemma 2.1. *As search becomes frictionless, $\nu \rightarrow \infty$, the value of search, V_s , converges to the frictionless value, \bar{V}_s : $\lim_{\nu \rightarrow \infty} V_s = \bar{V}_s$.*

As $\nu \rightarrow \infty$, expected delays between funding rounds vanish. Funding can be split into arbitrarily small, instantaneous rounds, eliminating scope to hide failure and driving the external-finance premium to zero; in other words, staging fully overcomes the agency friction.⁵⁸ The value of search (and thereby entry) therefore coincides with \bar{V}_s .

In practice, $\nu < \infty$ and the economy is inefficient. I begin by formalising a notion of capital misallocation, which provides insights into the costs of this inefficiency. Capital is misallocated in the economy due to inefficient continuation; it would be preferable to transfer capital from a start-up in unproductive development to one in search. However, due to the frictions present in the model, a degree capital misallocation is inevitable. Therefore, I then study whether the decentralised equilibrium is constrained efficient given the agency and matching frictions. I show that social efficiency obtains only in the knife-edge case in which the classical Hosios condition is satisfied (Hosios, 1990).

2.3.4 Capital misallocation

Relative to an economy without frictions, capital is misallocated in the model because start-ups that have received negative news about their prospects, the “unproductive” start-ups, do not inform their investors and so continue to invest in projects with no prospects. The capital would be better invested in a different, productive start-up.

A high degree of capital misallocation is indicative of issues in access to funding. Capital misallocation – defined as the share of firms with funding that are unproductive – is given by

$$\frac{\mu_d^u}{\mu_d^u + \mu_d^p} = \frac{\kappa(1-p)}{\kappa(1-p) + \omega} \quad (14)$$

which follows from equation (9).⁵⁹ Capital misallocation is decreasing in ω . This leads to the following Corollary (of Proposition 2.2), which characterises factors that affect the extent of capital misallocation in the model.

Corollary 2.6. *Capital misallocation (equation (14)) is strictly*

- *increasing in the payoff (π) and failure rate in search (λ);*
- *decreasing in the meeting rate (ν) and flow investment cost (k);*
- *and ambiguous with respect to the R&D success probability (p) and arrival rate of R&D outcomes (κ).*

For parameters other than κ and p , factors that push up the contract rate, ω , reduce capital misallocation, whereas factors that lead to a lower contract rate typically increase misallocation. In particular, when financing conditions for start-ups tighten, a lower ν , capital misallocation

⁵⁸In practice, costs of writing contracts and performing due-diligence between each funding round would overturn this result. I abstract from these concerns to focus on the interplay between agency and matching frictions.

⁵⁹In the baseline model where every funding round involves the same amount of capital raised, this is equivalent to the share of capital that is misallocated.

risers. Intuitively, capital is misallocated in the model because of the interaction of the agency and matching frictions; in an environment with only agency frictions, the optimal contract rate would specify $\omega \rightarrow \infty$ and no capital would be misallocated.⁶⁰ Therefore, capital misallocation reflects insurance against financing risk and my estimates of capital misallocation in section 3.3 are indicative of the costs that firms incur to militate against it.

2.3.5 Welfare analysis

In this section, I provide conditions under which the economy is constrained efficient. In order to do so, I consider the general version of the baseline model that includes VC bargaining power, as set out in section A.2 of the Appendix. The key difference is that I endow the VC with bargaining power $1 - \delta$, where $\delta = 1$ is the case considered in the baseline model. Social efficiency in this setting requires that the flow rate of entry and the contract rate in the decentralised equilibrium coincide with those that would be chosen by a planner maximising total surplus, which is defined as

$$TS = \int_0^\infty e^{-\rho t} \left([\kappa p \pi - k] \mu_{d,t}^p - k \mu_{d,t}^u - \frac{\tilde{\sigma}}{2} \Lambda_t^2 \right) dt \quad (15)$$

where Λ_t denotes the flow rate of entry at time t and $\mu_{d,t}^p$ and $\mu_{d,t}^u$ should be understood to be the total mass of start-ups in productive and unproductive development, respectively, regardless of their contract rate ω .⁶¹ The flow value of total surplus is equal to the flow payoff from start-ups in productive development, $\kappa p \pi$, net of the costs of sustaining firms in development, $k(\mu_{d,t}^p + \mu_{d,t}^u)$, and of entry costs $\frac{\tilde{\sigma}}{2} \Lambda_t^2$.⁶²

Proposition 2.7. *If the matching function exhibits constant returns to scale, $\beta = 1 - \alpha$, and the bargaining weights in the contracting problem are equal to their respective matching function elasticities, $\alpha = \delta$ and $\beta = 1 - \delta$, then the decentralised equilibrium is constrained efficient.*

Proposition 2.7 provides conditions under which the decentralised equilibrium coincides with the solution to the planner's problem. The conditions are analogous to the Hosios condition typical in the search literature. Intuitively, for a fixed equilibrium contract rate, ω , the standard results from the search literature apply to the entry condition for start-ups: the negative congestion externality implies that entry may be excessive, but a lack of appropriately due to incomplete bargaining power implies that entry might be inefficiently low. These two effects are balanced when the Hosios condition is satisfied.

To provide some intuition on why it follows immediately that the contract rate is socially efficient, it is helpful to conceptualise the contract rate as pinning down the optimal rate of 'entry' of productive start-ups and VCs back into the search market: VCs funding start-ups and start-ups in productive development re-enter the search market at rate ω . If the values of start-ups and VCs in search are pinned down optimally by the Hosios condition, then these

⁶⁰See Lemma 2.1.

⁶¹Out of steady-state, firms may have different contract rates. For instance, denoting the measure of firms at time t in productive development with contract rate ω by $\mu_{d,t}^p(\omega)$, the total mass of firms in productive development at time t is $\mu_{d,t}^p = \int_0^\infty \mu_{d,t}^p(\omega) d\omega$. The proof of Proposition 2.7 takes full account of these dynamics.

⁶²Recall that entry is specified as a cutoff rule. If the flow of entry is Λ_t , then the cutoff must be $\hat{c}_t = \tilde{\sigma} \Lambda_t$. The total flow entry costs at time t are calculated as $\int_0^{\hat{c}_t} c/\tilde{\sigma} dc = \frac{\tilde{\sigma}}{2} \Lambda_t^2$.

incentives to ‘re-enter’ the search market must also be optimal. Therefore, the contract chosen in the decentralised equilibrium will ensure the correct flow rates of firms back into the search market and so the decentralised equilibrium is socially efficient. Of course, as pointed out by Inderst and Müller (2004), there is no particularly reason to think that the Hosios condition should be satisfied in the VC market.

2.3.6 Discussion

The model provides several insights. First, the financing environment affects not only the quality of projects that can be funded, but also interacts with project characteristics. This is the key insight of Nanda and Rhodes-Kropf (2017), who show that financing conditions interact with project novelty; Nanda and Rhodes-Kropf (2013) provide empirical support. Relative to their model, I demonstrate this result in an incomplete contracting environment when contracts are negotiated optimally. Furthermore, my model provides the additional insight that a project’s development horizon interacts with the financing environment, which provides an explanation for the tendency of venture capital to fund projects with short gestation periods (Narain, 2024).

Second, the model provides a structural interpretation of funding histories. For instance, in environments where agency frictions are most severe, the model predicts that start-ups should be funded in many smaller funding rounds, visiting the market frequently. Conversely, when the financing market is tight, the funding duration rises among funded firms. Therefore, funding histories encode information about the projects characteristics and the financing environment. This provides a route to learning these latent primitives, which I exploit in the estimation.

Finally, the model offers insights into how the venture capital *market* operates, beyond the implications for a *single entrepreneur* attempting to finance their development. In the model, the agency and matching frictions interact in equilibrium. On the one hand, the potential need to secure follow-on funding implies that investment decisions are forward looking and depend on anticipated market tightness. On the other hand, the choice of contracts affects the measure of investors and entrepreneurs in the search market, which feeds back into market tightness. This implies that the choice of contract by one firm has spillovers in the availability of capital to other firms and, to the extent that this is not correctly internalised, the market will be inefficient.

2.4 Quantitative extensions for estimation

This section adapts the baseline model for quantification by relaxing a restrictive assumption and introducing one additional feature.

First, in the baseline model, access to capital for firms with viable projects does not improve over their development cycle. In practice, as more information is produced about the start-up’s potential, the information issues that contribute to the matching friction should ease. However, in the baseline model, there is no intermediate information production and all projects in search are identical. Therefore, I generalise the model to include two hurdles.⁶³ After overcoming the first hurdle, funding conditions may improve, reflecting the better information available to

⁶³These hurdles—or development stages—may have various interpretations. For instance, in Arora et al. (2024), firms must first overcome technological before considering commercialisation challenges. Alternatively, Ewens et al. (2018) frame multiple development stages as ‘experiments’ that deliver information about project viability.

investors about the start-up. In principle, this could be extended to any number of hurdles, but two hurdles facilitates mapping to data, where funding rounds are typically considered to be “early-stage” or “late-stage”. Furthermore, two hurdles is sufficient to capture the qualitative feature of the empirical hazard rate for successful exits, which is first increasing and then decreasing in the time since first funding (Jovanovic and Szentes, 2013). Capturing exit hazards is particularly important because I estimate the model on censored data.

Second, I introduce acquisitions as a distinct form of exit for firms in the second (or “late”) stage. In practice, there exists a market in which firms with partially-developed innovations can sell to incumbent firms who may be better placed to commercialise the innovation. Furthermore, I show that acquisitions substitute for financing, formalising the insight of Nanda and Rhodes-Kropf (2016) that a market for ideas can be crucial for overcoming financing risk. Therefore, to properly capture the effect of financing on start-up activity and outcomes, it is important to allow firms to choose between being acquired and scaling independently. I permit acquisitions only in the late-stage, mirroring the environment in Arora et al. (2024), where a start-up must overcome both technology risk and commercialisation risk but may sell out to an acquirer before attempting to overcome the second hurdle.

2.4.1 Model outline

Section A.5 presents the equilibrium conditions; here, I focus on features that differ from the baseline. For clarity, Figure 4 depicts the potential paths that a start-up may take, obscuring the unproductive development states for readability.

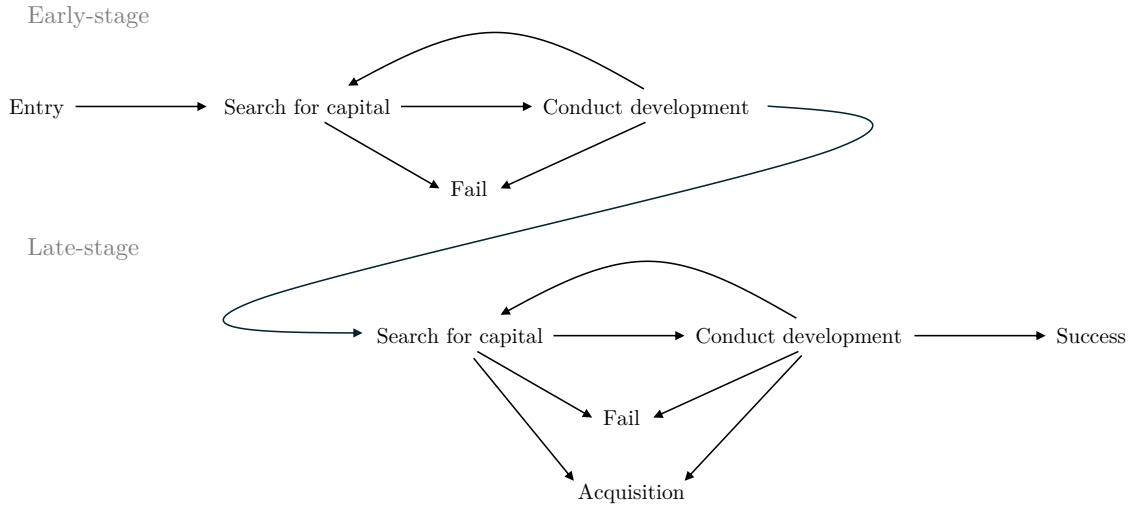


Figure 4: Start-up lifecycle

For ease of exposition, the two states “productive development” and “unproductive development” have been combined into “conduct development”.

The early-stage Following entry, start-ups seek early-stage funding and have value $V_{s,e}$. During search, they fail at rate λ and meet a VC at rate, ν_e , which is stage-specific. A start-up in the early-stage that invests flow k_e realises a result at rate κ , which is positive with probability

p_e/p_l , where p_l is the probability of overcoming late-stage uncertainty, conditional on being in the late-stage and obtaining a result.⁶⁴

Conditional on meeting with a VC, the start-up signs a contract with contract rate ω_e , which provides funding for the early-stage only.⁶⁵ The contract problem is identical to that laid out in section 2.2, except that the probability p is replaced by p_e/p_l .⁶⁶ If successful in overcoming early-stage uncertainty, the start-up progresses to the late-stage and seeks late-stage funding.

The late-stage A start-up that overcomes early-stage uncertainty begins its search for late-stage capital. During search, it fails at rate λ and finds a VC at rate, ν_l . In general, $\nu_l \neq \nu_e$.

Start-ups in the late-stage may be acquired. I define an acquisition as the sale of a partially-developed project and, therefore, acquisitions may occur whether the firm is in search or development. Specifically, I assume that potential acquirers arrive at rate ϕ , whether the firm is in search or development. This rate is exogenous and the same for all firms. The acquirer values the fully-developed project possessed by the start-up at $\epsilon \times \pi$, where $\epsilon \in [0, \infty)$ is a random variable with CDF $G(\cdot)$ and that ϵ is match-specific, common knowledge and *i.i.d.* across matches. However, the acquirer purchases the start-up before it has reached full development so there is residual uncertainty. In the late-stage, a start-up's project would succeed with probability p_l if fully-funded, so the acquirer's valuation for the project is $\epsilon p_l \pi$. This is known to the start-up.

I assume that the start-up makes a take-it-or-leave-it offer to the acquirer, setting the price at the maximum of its current value in the late-stage, $V_{i,l}$ for $i \in \{s, d\}$, and the valuation of the potential acquirer, $\epsilon p_l \pi$.⁶⁷ If the former exceeds the latter, the potential acquirer will choose not to acquire the start-up. Bringing these insights together, the probability that an acquisition occurs conditional on a meeting is given by

$$\begin{aligned} \text{Development:} \quad & Pr(\epsilon p_l \pi \geq V_{d,l}) = 1 - G(V_{d,l}/p_l \pi) \\ \text{Search:} \quad & Pr(\epsilon p_l \pi \geq V_{s,l}) = 1 - G(V_{s,l}/p_l \pi) \end{aligned}$$

which permits defining an effective arrival rate of acquisitions $\hat{\phi}_i = \phi \cdot [1 - G(V_{i,l}/p_l \pi)]$ for $i \in \{s, d\}$. The effective arrival rate of acquisitions modifies the arrival rate of potential acquisitions by the acceptance rules outlined above and so is endogenous. In the quantification, I will impose $\epsilon \sim \text{Exp}(1/\xi)$, so that ξ is the average 'synergy'. With this in mind, the HJB equations for

⁶⁴This convention means that, in a frictionless world, each start-up in the early-stage would be successful with probability $p_e = (p_e/p_l) \times p_l$.

⁶⁵Funding applies only to the current stage. This captures VC specialisation across the start-up lifecycle.

⁶⁶Specifically, the expected discounted capital cost for the VC is

$$K_e(\omega_e) = \left(1 + \frac{\kappa(1 - p_e/p_l)}{\rho + \omega_e}\right) \frac{k_e}{\rho + \kappa + \omega_e}.$$

⁶⁷Note that I take an agnostic view on acquisition demand; the model could be generalised in several ways to endogenise ϕ . For instance, ϕ may reflect the intensity of incumbent monitoring of start-up technologies, or the stance of antitrust policy. Without a theory of acquisition demand, little is gained by splitting the surplus according to some Nash bargaining weights. The key point is that acquisitions perform a substitution role (Proposition 2.8), which obtains in the simple case where the start-up makes a take-it-or-leave-it offer.

late-stage search and development are given by

$$\begin{aligned} \text{Development:} \quad & \rho V_{d,l} = \kappa [p_l \pi - V_{d,l}] + \omega_l [V_{s,l} - V_{d,l}] + \hat{\phi}_d \cdot [p_l \pi E[\epsilon | \epsilon > V_{d,l}/p_l \pi] - V_{d,l}] \\ \text{Search:} \quad & (\rho + \lambda) V_{s,l} = \nu_l [V_l^M - V_{s,l}] + \hat{\phi}_s \cdot [p_l \pi E[\epsilon | \epsilon > V_{s,l}/p_l \pi] - V_{s,l}] \end{aligned}$$

where V_l^M is the value of a meeting in the late-stage, which is defined analogously to the baseline model. Relative to the baseline model, there is the additional flow value deriving from acquisitions and the conditional expectation obtains because only potential acquirers with sufficiently high synergies result in acquisitions. Given $\epsilon \sim \text{Exp}(1/\xi)$, the value functions can be written more concisely as

$$\text{Development:} \quad \rho V_{d,l} = \kappa [p_l \pi - V_{d,l}] + \omega_l [V_{s,l} - V_{d,l}] + p_l \pi \xi \phi \exp \left\{ -\frac{V_{d,l}}{p_l \pi \xi} \right\} \quad (16)$$

$$\text{Search:} \quad (\rho + \lambda) V_{s,l} = \nu_l [V_l^M - V_{s,l}] + p_l \pi \xi \phi \exp \left\{ -\frac{V_{s,l}}{p_l \pi \xi} \right\} \quad (17)$$

An outstanding problem is how to compute the expected capital cost when there are acquisitions. With acquisitions, a firm in productive development experiences an event – a result, acquisition, or funding exhaustion – at rate $\kappa + \omega + \hat{\phi}_d$. I assume that the contract written between the start-up and VC is such that the VC terminates funding when the firm is acquired. For an unproductive start-up, the entrepreneur can prevent the acquirer from conducting the due-diligence to learn ϵ , because they have control rights. Therefore, the VC is left to honour their commitment, which expires at rate ω_l . The implication is that the expected capital cost associated with contract rate ω_l is

$$K_l(\omega_l) = \left(1 + \frac{\kappa(1-p_l)}{\rho + \omega_l} \right) \frac{k_l}{\rho + \kappa + \omega_l + \hat{\phi}_d} \quad (18)$$

The optimal contract is then found by solving the contract problem, analogous to equation (5), where the value functions are given by equations (16) and (17) and the capital cost by (18).

Financing market equilibrium The equilibrium conditions mirror those for the baseline model, appropriately extended to allow for two stages. They are outlined in section A.5, along with the full set of equilibrium conditions for this version of the model.

2.4.2 Discussion

With the setup outlined, I highlight a key theoretical insight in the context of acquisitions: start-ups searching for capital accept acquisition offers at a higher rate than those with funding secured. This is formalised in the following proposition.

Proposition 2.8. *Given that the optimal contract rate is finite, $\omega < \infty$, the effective arrival rate of acquisitions is higher in search than in development: $\hat{\phi}_s > \hat{\phi}_d$.*

Proposition 2.8 illustrates that firms are more inclined to accept acquisitions while searching for funding, suggesting that acquisitions substitute for venture capital. The intuition is

straightforward: a firm searching for capital has a less favourable outside option in acquisition negotiations, making it more willing to accept a broader range of acquisition offers.

Finally, a word on parameter choices across stages. Some parameters differ across stages, while others remain constant, reflecting the need to balance realism and separately identify various parameters with available data. In light of the second motivation, I have assumed that the rate of uncertainty resolution and of failure in search, κ and λ respectively, do not differ across stages. However, I allow $k_e \neq k_l$ and $\nu_e \neq \nu_l$. Gompers (1995) shows that later-stage firms raise larger amounts of capital, which motivates $k_e \neq k_l$ (I anticipate $k_e < k_l$ but do not impose it). Furthermore, to allow access to capital to improve as the start-up develops, reflecting the increased information available about its prospects, I allow $\nu_e \neq \nu_l$.

3 Quantification for the US VC market

The section has three objectives. First, I show how funding histories and exits identify the model’s primitives. The model provides a structural mapping from funding histories (timing, size and frequency) to start-up characteristics and funding conditions, building on insights in entrepreneurial finance (e.g. Gompers 1995; Hall and Lerner 2010; Nanda and Rhodes-Kropf 2017). Because this strategy is novel and underpins the analysis, I develop it in detail.

Second, I validate the model’s ability to replicate key features of the data. A key insight that emerges here is that a model with *ex ante* homogeneous start-ups can capture rich *ex post* heterogeneity in start-up outcomes and funding histories.

Third, I use the estimated model to quantify distortions in the US market. My estimates suggest that 40% of VC-backed start-ups first funded in 2005-2015 shut down prematurely and that, with continued funding, many would have had successful outcomes; absent frictions, successful exits rise from 30% to 50% (+20pp). Furthermore, a substantial share of capital is misallocated to start-ups that should have been closed-down: 12% of capital is in the hands of these start-ups while 23% of viable start-ups are searching for funding at any instant.

3.1 Identification strategy

This section discusses the identification strategy. First, I present a simple identification result, which shows that a set of readily observable moments are sufficient to identify the parameters of the baseline model of section 2.2. I then extend to discuss acquisitions and finally discuss the full model with acquisitions and two stages. The identification strategy for the full model leverages insights from the simpler environments, but I move from an exactly-identified to over-identified setting and complement the set of moments with parameters from an auxiliary model.⁶⁸

Table B.1 in the Appendix shows an illustrative dataset. For each firm, funding rounds and successful exits, whether by IPO or acquisition, are observed. Conversely, failures are typically unobserved. For funding rounds, one observes the date, investment amount, and an indicator for the ‘type’ of round, e.g. early or late. Funding round valuations are often missing (not a random; see Jagannathan et al. 2022). For exits, data on valuations is typically available.⁶⁹

⁶⁸Estimation targets moments and auxiliary-model parameters; see (Gourieroux et al., 1993) for details on indirect inference.

⁶⁹Section 3.2.1 discusses the approach when this is missing.

3.1.1 Identification of the baseline model

This section provides a global identification result for the baseline model. The particular collection of moments I use is sufficient for identification, but not necessary: other just-identified sets would work as well. I focus on this set because it has a natural interpretation and makes the mapping from structural parameters to empirical moments transparent. However, in the quantitative implementation below, I do not include the share of successful firms (included here) as a targeted moment, instead reserving it as an untargeted feature for the model to match.

The baseline model pools all exits (acquisitions and successes) and ignores the early-stage/late-stage distinction. Taking the discount rate ρ as given, there are seven parameters to estimate: project characteristics, (k, κ, p, π) , financing conditions, (λ, ν) , and the entry cost parameter, $\tilde{\sigma}$. Start-up funding histories and outcomes identify $\theta = (k, \kappa, p, \pi, \lambda, \nu)$, and $\tilde{\sigma}$ is then pinned down from the model condition for the flow rate of first funding rounds.

For the first step, identifying θ , consider the following vector of population moments

$$\mathbf{m} = (E[\mathbb{1}\{\text{success}\}], E[N_f], E[T_{br}], E[\text{time-to-exit}], E[\text{burn rate}], E[\text{exit multiple}]) \quad (19)$$

where $E[\mathbb{1}\{\text{success}\}] = p_d$ is the probability of success for funded firms, defined in Proposition 2.5; $E[N_f]$ and $E[T_{br}]$ are, respectively, the expected number of funding rounds and the expected duration between funding rounds, defined in Corollaries 2.3 and 2.4; and $E[\text{time-to-exit}]$, $E[\text{burn rate}]$ and $E[\text{exit multiple}]$ are, respectively, the expected time from first funding to a successful exit, the expected burn rate and the expected exit multiple; see section A.3.4 for closed-form expressions.

Let $\Theta = [0, 1] \times \mathbb{R}_+^5$ denote the parameter space, where $\theta \in \Theta$. Not all values for $\theta \in \Theta$ are reasonable: for $k > \kappa p \pi$, projects are not financed; whereas for some $\theta \in \Theta$, the contract rate $\omega = 0 \implies E[N_f] = 1$, so that staged financing does not arise in equilibrium. Therefore, I define $\Theta^{\text{adm}} \subset \Theta$ as the admissible parameter space, for which the optimal contract rate, ω , is positive and finite. For $\theta \in \Theta^{\text{adm}}$, $E[N_f] > 1$ and all population moments are well-defined.⁷⁰

Definition 3.1. For $\theta \in \Theta^{\text{adm}}$, let

$$\mathbf{m}(\theta) = (E[\mathbb{1}\{\text{success}\}], E[N_f], E[T_{br}], E[T_{\text{exit}}], E[\text{burn rate}], E[\text{exit multiple}]) \in [0, 1] \times \mathbb{R}_+^5$$

denote the vector of population moments implied by the model. Let $\mathcal{M}^{\text{adm}} = \mathbf{m}(\Theta^{\text{adm}})$ denote the corresponding set of admissible moment vectors, and define

$$G : \Theta^{\text{adm}} \rightarrow \mathcal{M}^{\text{adm}}, \quad G(\theta) = \mathbf{m}(\theta).$$

Proposition 3.1. The mapping $G : \Theta^{\text{adm}} \rightarrow \mathcal{M}^{\text{adm}}$ is injective. In particular, for any admissible moment vector $\mathbf{m} \in \mathcal{M}^{\text{adm}}$ there exists a unique $\theta \in \Theta^{\text{adm}}$ such that $G(\theta) = \mathbf{m}$.

Proposition 3.1 provides a route to learning the model's latent structural parameters from the type of data shown in Table B.1. The proof in section A.3.4 proceeds by solving the system

⁷⁰For instance, without multiple funding rounds, the time between funding rounds and the burn rate are not meaningful objects. It becomes infeasible to recover all parameters in this case.

of moment equations for model's structural parameters as functions of \mathbf{m} . The resulting mapping from empirical moments to structural parameters provides insights into how the model's parameters are identified.

The frequency and timing of events discipline the model's hazard structure: the expected time-to-exit, time between rounds, and number of rounds pin down the composite transition rates, $\kappa + \omega$ and $\lambda + \nu$, and the implied cycling intensity between search and productive development, $\nu\omega$. For example, many rounds in quick succession, high $E[N_f]$ and low $E[T_{br}]$, requires at least one of ν or ω to be large. The success probability, expected burn rate, and expected exit multiple then inform (p, k, π) .^{71,72} For example, one can recover the payoff, π , by combining the exit multiple and burn rate; all else constant, a higher exit multiple or burn rate leads to a higher implied exit payoff, π . Finally, optimal contracting condition allows for eliminating ω , meaning that each element of $\theta = (k, \kappa, p, \pi, \lambda, \nu)$ is expressed in terms of \mathbf{m} .

With θ identified, V_s is given by equation (13). The entry rate is $V_s/\tilde{\sigma}$, which suggests that $\tilde{\sigma}$ can be backed out given the empirical entry rate. In practice, entry is unobserved, but the model condition determining the flow of first funding rounds can be used instead.⁷³ In steady state, it follows that

$$\tilde{\sigma} = \left(\frac{\nu}{\lambda + \nu} \times \frac{1}{\text{flow first-time rounds}} \right) \times V_s \quad (20)$$

which enables recovery of $\tilde{\sigma}$. See section A.4 for the derivation.

3.1.2 Identification of the baseline model with acquisitions

Now consider the single-stage baseline model but allow for acquisitions. Doing so adds two parameters: the arrival rate of offers, ϕ , and the average quality of offers, ξ . In this setting, closed forms are no longer available, so I do not provide a global identification proof. Instead, I discuss the moments that are most informative about the acquisition block.

Acquisitions depend jointly on (p, π, ϕ, ξ) : the arrival rate of accepted offers is $\hat{\phi}_i = \phi \exp\{-V_i/p\pi\xi\}$; the expected acquisition price conditional on acceptance is $V_i + p\pi\xi$, for $i \in \{s, d\}$; and (p, π, ϕ, ξ) appear multiplicatively in the HJBs; see equations (16) and (17). As a result, many targeted moments respond similarly to changes in any of (p, π, ϕ, ξ) ; see Figure C.1.

Three moments help to disentangle these parameters. First, exit mode separates (p, π) from (ϕ, ξ) : higher (p, π) increases the success rate directly, while higher (ϕ, ξ) mainly shifts mass toward acquisitions. The acquisition-to-success ratio therefore helps distinguish the two blocks. Second, with acquisitions the overall exit multiple is less useful for separating p from π because

⁷¹The time-to-failure is an alternative that could be used to learn about p , because the probability of success affects whether firms fail in search or development, which occur at different times following a previous funding round. However, failure is poorly observed and whereas empirical work commonly assumes failure after some period (e.g. two years) following the last observed funding round, Panel A of Figure 5 demonstrates this would miss-classify many firms in my sample.

⁷²One could use observed capital injections for k , rather than the burn rate. In practice, the burn rate is cleaner: the capital injection itself is non-monotonic (Corollary 2.2), making it less than ideal (see e.g. Strebulaev and Whited 2012). The issue is that, although k directly raises the capital injection, \bar{K} , it causes a reduction in the funding horizon, which may dominate. The burn rate is preferable because it normalises by the time between rounds. For π , using the exit-multiple, rather than the exit valuation directly, is more appropriate when estimating an *ex ante* homogeneous firm model because it better compensates for the implicit pooling of firms that may differ in their levels of both k and π .

⁷³Entry and first-rounds differ because a share $\lambda/(\lambda + \nu)$ of entrants do not obtain funding.

acquisition prices are anchored to $p \times \pi$, not π alone. I therefore target the mean exit multiple for successes only. Finally, separating ϕ from ξ is difficult because both raise the incidence of “quality-adjusted” offers. Using information on the shape of the distribution of acquisition exit multiples provides a route forward: the share of high-exit-multiple acquisitions rises with ξ , but is relatively insensitive or decreasing in ϕ ; see Figure C.1. I therefore include the share of acquisition multiples that exceed ten (10X).

3.1.3 Identification of the full model

I estimate the two-stage model with acquisitions using the two-step procedure in section 3.1.1: I first estimate the internal parameters and then recover the entry-cost parameter $\tilde{\sigma}$ from the entry condition. Estimation is over-identified, includes two auxiliary duration targets, and exploits early/late round tags to identify stage-specific parameters. Table C.2 reports moment elasticities around the estimates. I leave the share of successful exits untargeted as validation.

In the early stage, (ν_e, k_e, p_e) are stage-specific, while (κ, λ) are shared across stages. Early-stage “success” is progression to the late stage: conditional on completing early-stage R&D, firms progress with probability p_e/p_l and obtain continuation value $V_{s,l}$. Accordingly, the key separation is between p_e/p_l and $V_{s,l}$. Rather than impose additional structure on $V_{s,l}$, I target p_e/p_l directly using progression moments; with p_l pinned down by late-stage moments, these targets discipline p_e .⁷⁴ Specifically, I target (i) the share of firms that reach the late stage and (ii) the share of firms with exactly one round.⁷⁵ I also include the mean number of early-stage rounds and the early-stage mean burn rate (winsorised at 5% in both tails).⁷⁶ To sharpen identification of timing parameters, I add an auxiliary duration target implied by the model. Within stage, inter-round durations are hypoexponential (Proposition 2.4); for two successive early-stage rounds, $T_{br}^{e \rightarrow e} \sim \text{Hypo}(\kappa + \omega_e, \lambda + \nu_e)$. Fitting this distribution by MLE to observed early-stage inter-round durations provides direct information on $\lambda + \nu_e$.⁷⁷

Late-stage targets follow the identification logic in Section 3.1.2. I include the mean number of late-stage rounds among firms that progress to the late-stage, the late-stage mean burn rate (5% winsorised), the mean exit multiple for successes, the acquisition-to-success ratio, and the share of acquisition multiples above 10X. I also include a late-stage auxiliary duration target: with acquisitions, $T_{br}^{l \rightarrow l} \sim \text{Hypo}(\kappa + \omega_l, \lambda + \nu_l + \hat{\phi}_s)$, reflecting acquisitions from search at rate $\hat{\phi}_s$; I therefore target the MLE-based estimate of $\lambda + \nu_l + \hat{\phi}_s$.

Finally, following section 3.1.1, I also target the mean time-to-exit. In addition, I target the share of firms with a funding round more than five years after the first; this helps to ensure that the firm attrition in the model aligns with the data, which is important given that I estimated the model on censored data.

⁷⁴In principle, valuation step-ups at the early-to-late transition could help discipline $V_{s,l}$, but data is limited.

⁷⁵In the two-stage model, the total (i.e. across stages) number of rounds is no longer geometric; it responds directly to p_e/p_l , beyond its indirect effect via ω_e .

⁷⁶Section 3.4 discusses the motivation for winsorising and robustness to the threshold.

⁷⁷This auxiliary target is a convenient way to use within-stage timing information. Because the two hypoexponential rate parameters are unlabelled in the likelihood, I obtain (h_1, h_2) and target $\max\{h_1, h_2\}$ as the counterpart to $\lambda + \nu_e$. See section C.0.1.

3.2 Data and estimation

In this section, I present the dataset used to estimate the model and the core estimation results. I then discuss model fit as a validation exercise.

3.2.1 Dataset construction

My sample includes all firms in the Thomson Reuters venture capital dataset from the US that received their first round of venture capital funding between 2005 and 2015 inclusive. Raw data is censored based on the date of first funding, so to maintain consistency across the data, I exclude any firm observations that occur more than seven years after their first funding round. The approach mirrors Ewens and Farre-Mensa (2020), and section 3.4 shows that the estimated parameter values are robust to this choice.

To construct a dataset of the form in Table B.1, I apply several screens. Financing rounds are labelled as “seed”, “early stage”, “expansion”, or “later stage”.⁷⁸ I denote seed and early-stage rounds as “Early” and expansion and later-stage rounds as “Late”. I also exclude the minority of firms that only have a “Late” funding rounds to maintain consistency with the model.⁷⁹ Furthermore, I collapse any deals for a firm if their dates coincide and make a few manual corrections to the data when there are clear errors.⁸⁰ I complement the data with market capitalisation data from CRSP for IPOs and M&A price data from Crunchbase, when price data from Thomson Reuters is unavailable. All values are in 2015 USD. I face two distinct issues of missing data: investment amounts for funding rounds and deal values for some M&A transactions. I impute missing investment amounts, following Jagannathan et al. (2022), and set missing deal values equal to 1.5 times the total investment, following Kerr et al. (2014).⁸¹

Finally, I need to map modes of exit in the data to the notion of “success” and “acquisition” in the model. In the model, a success is a completed project, whereas acquisitions are of pre-commercial technologies. I define an acquisition in the data as an M&A transaction by a non-financial buyer and a success as an IPO or an M&A transaction by a financial (including SPAC) buyer, reflecting the idea that non-financial buyers typically have the capabilities to acquire start-ups at pre-commercial stages, whereas financial buyers do not.

The final sample includes 11,030 firms and 33,158 funding rounds. Among these firms, 400 were successful and 2,034 were acquired within the same period. Section B.1 provides further details on data construction and Table B.3 includes further summary statistics.

3.2.2 Estimation

To maintain consistency with the data construction procedure, I censor the simulated data in the same way before computing moments.⁸² I fix the discount rate $\rho = 0.08$, which is similar to the value used in Jovanovic and Szentes (2013). A unit of time in the model corresponds

⁷⁸The data also include “bridge loans”, which I disregard bridge loans because they are not designed for conducting major development steps.

⁷⁹See section B.2 for definitions of each round type.

⁸⁰For instance, on some occasions Thomson Reuters reports IPOs that were announced by never occurred.

⁸¹Imputation applies to about 10% of US rounds (and slightly more for the UK); see section B.1.

⁸²Censoring needs to be handled carefully for estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$. See section C.0.1.

to a year. The remaining parameters are estimated in two steps. First, I jointly estimate all parameters except for $\tilde{\sigma}$, denoted by θ , by minimising the criterion function

$$C(\theta) = \sum_{j \in \theta} \left(\frac{\tilde{m}_j - m_j(\theta)}{\frac{1}{2}|\tilde{m}_j| + \frac{1}{2}|m_j(\theta)|} \right)^2 \quad (21)$$

where \tilde{m}_j is the empirical moment (or auxiliary parameter) and $m_j(\theta)$ its model counterpart computed at θ .⁸³ Second, I recover $\tilde{\sigma}$ from the counterpart to equation (20) for the full model, where I set the flow rate of initial rounds to 1.00 for the US, without loss of generality.

Parameter estimates Table 1 reports the parameter estimates and their asymptotic standard errors. Parameter estimates should be interpreted as the values for the average US VC-backed start-up in my sample.

Parameter		Estimate	Std. Error
Discount rate	ρ	0.08	-
R&D milestone arrival rate	κ	0.26	0.01
Failure rate in search	λ	0.62	0.04
Early-stage development level	p_e	0.23	0.01
Early-stage flow investment cost (\$Mn USD/year)	k_e	4.16	0.18
Early-stage meeting rate	ν_e	2.54	0.13
Late-stage development level	p_l	0.30	0.01
Late-stage flow investment cost (\$Mn USD/year)	k_l	16.15	0.25
Late-stage meeting rate	ν_l	5.20	0.26
Success payoff (\$Mn USD)	π	218.18	15.40
Acquisition offer arrival rate	ϕ	0.65	0.07
Mean acquisition ‘synergy’ ($\varepsilon \sim \text{Exp}(1/\xi)$)	ξ	0.74	0.08
Entry cost distribution ($c \sim U[0, \tilde{\sigma}]$)	$\tilde{\sigma}$	2.68	0.37

Table 1: US parameter estimates

The table reports parameter estimates from the model in section 2.4. Estimation uses a simulated sample of 220,600 firms, $S = 20$ times the size of the empirical sample. $\rho = 0.08$ is fixed externally. Parameters (except $\tilde{\sigma}$) are estimated by minimising (21) (equal weighting of centred percentage deviations; see footnote 83) and I report asymptotic standard errors, where the covariance matrix of moment conditions is computed from 1,000 bootstrap samples, clustered at the firm level (see section C.0.1 for details). To avoid local minima in the estimation, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points. With the resulting candidate parameter estimates, I run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses. With $S = 20$, simulation noise is miniscule, so I report standard errors that simply reflect sampling variability (applying an adjustment raises standard errors by $< 1\%$). With these estimates, the entry cost parameter, $\tilde{\sigma}$, is recovered from the condition determining the flow rate of first-time funding rounds. To compute its standard error, I take 1,000 samples from the asymptotic distribution of parameter estimates, solve the model, and recover the implied value for $\tilde{\sigma}$. The standard error is the standard deviation of $\tilde{\sigma}$ among this sample. All amounts are in 2015 USD.

The average start-up project requires approximately eight years of funding to fully commer-

⁸³The criterion function sums over squared percentage deviations between the empirical and simulated objects with equal weighting. The choice of equal, rather than optimal, weighting scheme reflects the need to utilise variation in certain moments, specifically coming from exit payoffs, in order to identify the model’s parameters. For instance, the sample contains over 30,000 funding rounds but only about 400 successes. Because the exit multiple is key for identifying π , equal weighting ensures this low-frequency variation is not dominated by high-frequency moments.

cialise ($\approx 2 \times 1/\kappa$), consistent with typical VC timelines. This start-up requires annual investment of $k_e = \$4.16\text{Mn}$ for the first half of this period and $k_l = \$16.15\text{Mn}$ for the second half. If successfully in independently scaling the project, the start-up generates value $\pi = \$218.18\text{Mn}$. However, the start-up may fail due to technological or commercial challenges. Conditional on uncertainty resolution, this happens with probability $1 - p_e/p_l = 23\%$ in the early-stage and $1 - p_l = 70\%$ in the late-stage. In the late-stage, acquisition opportunities arrive at the rate of one every 18 months ($\approx 12/\phi$) and the average offer price is approximately $\$50\text{Mn}$ ($\approx p_l\pi\xi$).

In practice, the start-up’s development may be hindered by access to external finance. In the early-stage, the probability that a viable firm in search fails before securing funding is $\lambda/(\lambda + \nu_e) = 20\%$ and the average duration spent searching is $12/(\lambda + \nu_e) = 3.8$ months. In the late-stage, financing conditions are more favourable: the probability that a viable firm in search fails before securing funding falls to 10% and securing funding takes an average of just 2 months.⁸⁴ The improvement in conditions is consistent with greater information as the start-up develops. Finally, the estimates suggest that the average start-up can survive 19 months ($\approx 12/\lambda$) after exhausting its previous capital injection before shutting down.

The figures for the average start-up appear consistent with the type of high-risk innovative firm that VCs seek to invest in. Furthermore, in certain cases, validation can be sought from external data. For instance, biotech is a key sector for venture capitalists and progression rates for clinical drug trials in the United States provides an external measure of the progression probability, p_e/p_l . Biotech start-ups typically seek early-stage funding to fund Phase 1 clinical trials and are often able to exit via IPO before full clinical approval.⁸⁵ For a sample including both young biotech and large pharmaceutical companies, Hay et al. (2014) report a Phase 1 to Phase 2 progression rate of between 64-67% (see their Figure 1), which is broadly in line with the early-to-late progression rate of $p_e/p_l = 77\%$ that I estimate, with the obvious caveat that biotech companies are not fully representative of the “average” start-up.

The financing market meeting rates, ν_e and ν_l , are important determinants of distortions in the market. To the best of my knowledge, Ewens et al. (2022) provide the only comparable estimates in the literature. They estimate a structural model with heterogeneous entrepreneurs and investors who meet only for first-round funding. Due to heterogeneity, not every “meeting” in their model leads to investment, so the appropriate comparison is between ν_e and their deal frequency.⁸⁶ For entrepreneurs, the average deal frequency is 1.57 per year, ranging from 0.16 to 3.56 across the lowest to highest entrepreneur-quality deciles (Table 6, Ewens et al. (2022)). My estimate of $\nu_e = 2.54$ is within this range. The two models differ markedly in their environments and estimation procedures, so the similarity of estimates is reassuring. The estimates are also in line with reports from market participants.⁸⁷

⁸⁴This is calculated as $\lambda/(\lambda + \nu_l + \hat{\phi}_s) = 0.62/6.15$, where the value for $\lambda + \nu_l + \hat{\phi}_s$ is reported in Table 2. The duration in search before securing funding is $12/(\lambda + \nu_l + \hat{\phi}_s) = 2$ months.

⁸⁵See *Betting on Biotech, Pitchbook, 2020*. Pitchbook is a leading venture capital data provider.

⁸⁶Some matches are not profitable in their model, so do not lead to deals.

⁸⁷Pitchbook suggest that “some startups are able to raise the capital they need in three months, while others may take a year” (*PitchBook’s guide to VC fundraising for startups, Pitchbook*). Furthermore, a founder survey with over 500 respondents by First Round Capital, a leading venture capital firm, shows similar timelines: 25% report securing funding within two months, 45% required between two and four months, and the remainder required more than four months. See Question 12, *State of Startups, First Round Capital, 2018*.

Model fit and validation Panel A of Table 2 reports the targeted moments, which the model captures well.⁸⁸ Panel B shows that the model also captures several key untargeted features of the data. First, although the likelihood of an acquisition relative to a success is targeted, their levels are not, and the model captures the share start-ups with these successful outcomes tightly. Second, the model accurately reflects the fact that the mean time-to-success exceeds the mean time-to-acquisition. However, it is worth noting that successful exits occur slightly too quickly in the model.⁸⁹ Third, the model captures the mean exit multiple, despite only targeting the mean multiple for successes and the right tail of the acquisition multiple distribution.

Panel A: Targeted moments		
Moment	Data	Model
Mean # early rounds	1.98	1.91
Mean # late rounds progressed to late	2.46	2.43
Share one round (%)	32.81	26.47
Mean duration b/w rounds (years)	1.16	1.25
Estimate of $\lambda + \nu_e$	3.30	3.16
Estimate of $\lambda + \nu_l + \hat{\phi}_s$	5.94	6.15
Share funded after year 5 (%)	19.68	18.25
Mean burn rate early round (\$Mn/year)	7.91	7.49
Mean burn rate late round (\$Mn/year)	23.78	24.89
Share to late-stage (%)	41.56	41.86
Mean time-to-exit (years)	3.67	3.33
Acquisition-to-success ratio	5.08	5.09
Mean exit multiple success	9.24	9.06
Share acquisition multiples > 10X (%)	8.01	8.17
# first funding rounds	1.00	1.00
Panel B: Untargeted moments		
Share successful (%)	3.63	3.78
Share acquired (%)	18.44	19.24
Mean time-to-success (years)	4.44	3.40
Mean time-to-acquisition (years)	3.52	3.31
Mean # funding rounds [†]	3.01	2.93
Mean exit multiple	5.79	5.33

Table 2: Targeted and untargeted moments

Panel A shows 15 moments (including two auxiliary parameter estimates) used in the estimation procedure. Empirical moments are computed from the sample of 11,030 firms. Other than for the estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, simulated moments are computed using the parameter values given in Table 1 to simulate a sample of 220,600 ($= 20 \times 11,030$) firms. For $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, the parameter values are taken directly to reduce the computational burden of the estimation procedure. In computing the empirical and simulated moments, data on a given firm is censored seven years after its first funding round. Panel B shows a set of untargeted moments. [†] Note that the mean number of funding rounds is implicitly targeted in the estimation through targeting the mean number of early-stage rounds, the mean number of late-stage rounds conditional on reaching the late-stage, and the share of firms that raise late-stage capital

Figure 5 demonstrates that the model accurately captures the entire distribution of key

⁸⁸For simplicity, I refer to all targeted objects as moments, although two are auxiliary model parameters.

⁸⁹I define time-to-exit as the time to completion in the data. There is often a lag between announcement and completion in the data, so that the model is more closely aligned with the data when using time-to-announcement.

features of VC funding and exit data. Panel A of Figure 5 reports information on the number and timing of funding rounds in the model and data. Three points are noteworthy. First, Panel A (i) shows that the model replicates the declining distribution of the number of rounds. Unlike in the single-stage model where this is true for all parameterisations, the two-stage model can capture more complex dynamics.⁹⁰ For instance, if most project uncertainty were concentrated in the late-stage, start-ups would typically progress to the late-stage and so raise at least two rounds. Therefore, matching this feature of the data implies that the balance of uncertainty is appropriately distributed between the early and late-stages. Second, Panel A (iii) shows that the model closely replicates the average time from the first funding round to each subsequent round. In the model this profile must be upward sloping because firms are *ex ante* homogeneous.⁹¹ By contrast, unmodelled *ex ante* heterogeneity could induce selection that flattens or even reverses the slope.⁹² The fact that the data preserves the upward slope suggests that such unmodelled *ex ante* heterogeneity is not excessive. Third, the model replicates funding continuation dynamics, defined as the share of firms that receive funding more than T years after their first round. This is important due to censoring in the estimation: divergence in the timing of events between the model and data could lead the estimation procedure to select markedly different parameter values as the censoring threshold is varied. Section 3.4 discusses this in more detail.

Panel B of Figure 5 reports data on capital intensity. In the model, capital injections are constant within stage but there is substantial variation in within-stage capital injections in the data. A key concern is that, by ignoring this heterogeneity, the model omits important dynamics in capital intensity and firm growth. Panel B (i) reports the median and interquartile range for the cumulative amount of capital raised by firms by funding round. Unsurprisingly, the model fails to capture the growing heterogeneity in cumulative capital raised with the number of rounds, but performs much better in replicating the median, capturing the slope well. Relatedly, Panel B (ii) reports the burn rate distribution, pooling across all funding rounds. The model closely captures the skewness in the burn rate distribution, despite variation in the model being driven predominately by heterogeneity in the duration between funding rounds (the denominator), rather than the amount raised (the numerator). Furthermore, differences in the time between rounds in the model reflects *ex post* heterogeneity across firms. The ability to replicate the empirical distribution without large dispersion in round sizes indicates that *ex post* heterogeneity – shocks hitting the firm after it raises capital – is a key driver of dynamics.

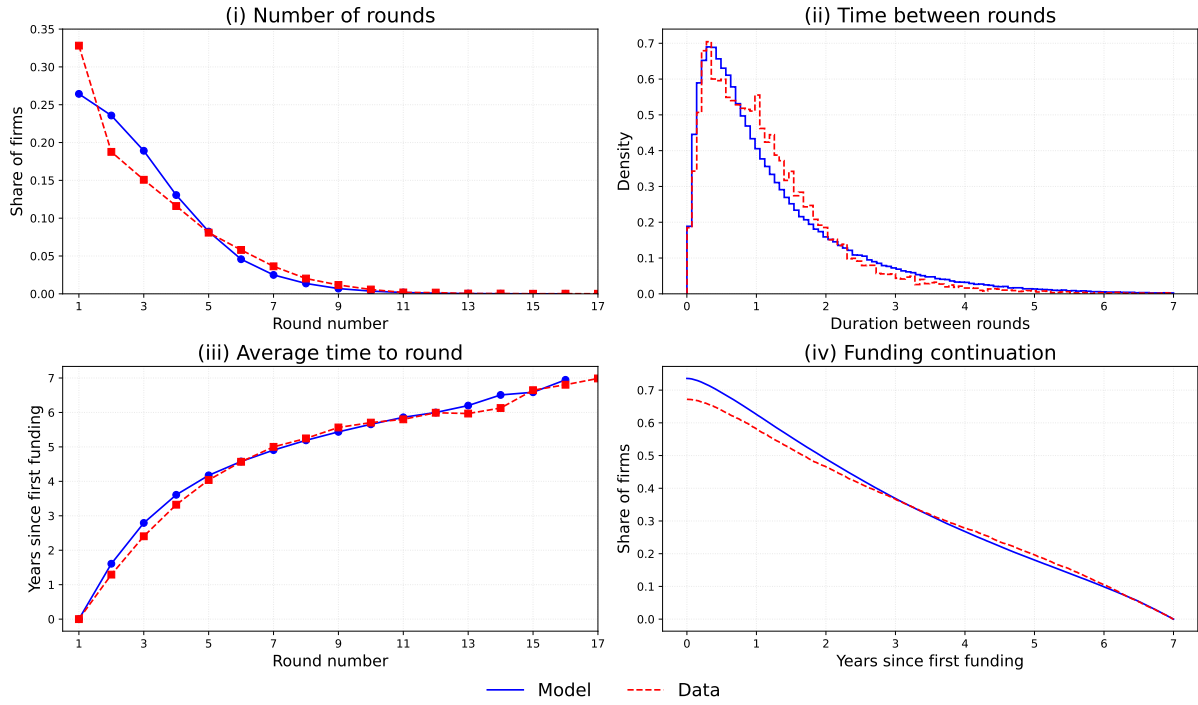
Finally, Panel C of Figure 5 reports information on outcomes. The skewness in start-up outcomes is a key feature of the market (Hall and Woodward, 2010) and Panel C (i) shows that the model does reasonably well in capturing this feature of the data. Furthermore, Panel C (ii) shows that the model captures the full distribution of the timing of exits well. However, it is worth noting that “successes” arrive too early in the model (see Table 2, Panel B), so that this is predominately driven by the model’s ability to capture the timing of acquisitions.

⁹⁰With multiple stages, the distribution of the number of rounds is not Geometric.

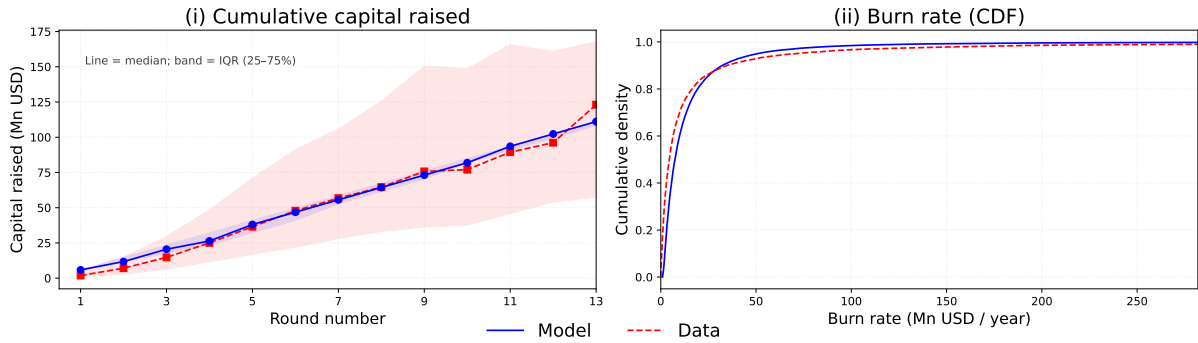
⁹¹Round $n + 1$ occurs after round n for each firm, but all firms draw from the same distribution of round times, so the expected time to round $n + 1$ also exceeds that to round n .

⁹²For example, if firms in some sectors progress quickly through four funding rounds, while firms in the dominant sector only ever require three rounds over a longer period of time, then a selection effect would imply the mean time to $N_f = 3$ exceeds that to $N_f = 4$.

Panel A: Funding rounds



Panel B: Capital intensity



Panel C: Exits

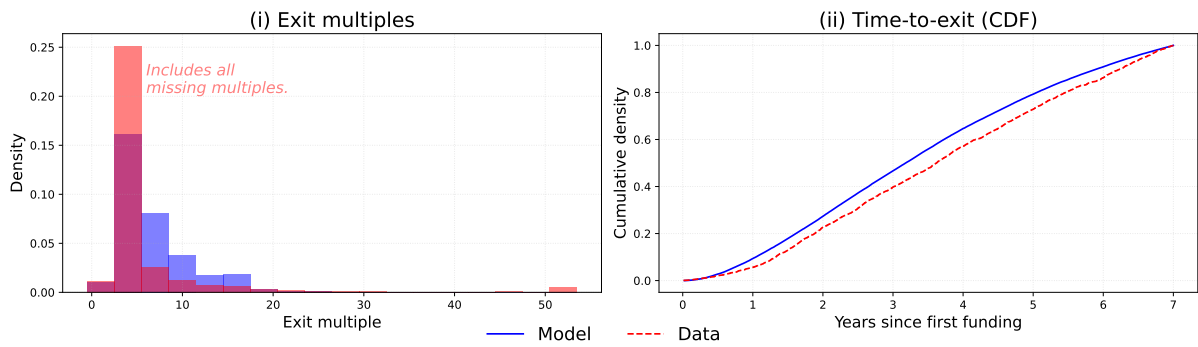


Figure 5: Model validation

The figure shows outcomes in the model and data across funding rounds (panel A), capital intensity (panel B) and exits (panel C). All empirical and simulated data is censored at seven years following a firm's first funding round. "Funding continuation" in Panel A (iv) reports the share of firms that raise capital more than x years after their first funding round. To aid visualisation, cumulative capital raised is shown up to the 13th round; the burn rate CDF is reported up to the 99th percentile; and exit multiples are winsorised at 50X. Missing empirical exit multiples are set to 1.5X, following Kerr et al. (2014).

3.3 Analysis of the US venture capital market

In this section, I show that my estimates imply that frictions in access to venture capital have large effects on entrepreneurial activity. To this end, I quantify the distortions that entrepreneurs face and the frictional economy to a frictionless setting, which is equivalent to the case where wealthy entrepreneurs can finance their own projects.⁹³ The estimates suggest that they are large distortions along both relevant margins: entry and outcomes. Second, I use the model to understand what factors determine whether entrepreneurs can successfully commercialise their innovations. My estimates suggest that a substantial share of entrepreneurial projects are not funded to completion and, therefore, remain potentially viable. Finally, I estimate the extent of capital misallocation in the US venture capital market.

Financing distortions Panel A of Table 3 reports a decomposition of start-up outcomes. Column two reports results from the censored data used to estimate the model. The estimates suggest that 17.2% of VC-backed start-ups remain active seven years after their first funding round, without having had a successful exit. Column three reports uncensored results, permitting a full decomposition of start-up failures. The sample consists of VC-backed start-ups that have at least one funding round and, among these start-ups, approximately 70% fail. However, the estimates suggest that approximately 40% ($\approx 25.7 + 15.1$) are forced to shut down prematurely, before uncertainty is resolved. Most of these failures occur in the early-stage, consistent with more severe matching frictions when little is known about the start-up's prospects.

These estimates suggest there are substantial issues in financing young-innovative firms. However, caution is required in interpreting them. These results *do not imply* that these firms would have been successful, were they able to receive additional funding. Instead, they simply point to premature closure of positive NPV projects. To interpret the losses stemming from these premature closures, I consider the counterfactual: what would have happened to these firms, were they to receive more funding.

Panel B of Table 3 reports these measures. In column two of Panel B, I report the success and acquisition distortions, defined analogously to those in section 2.3.2 of the baseline model. Specifically, the success distortion, Δ_p , is defined as wedge between the actual success rate and the success rate when a wealthy entrepreneur can self-finance the project.⁹⁴ Analogously, the acquisition distortion, Δ_a , is the wedge between the actual acquisition rate and the acquisition rate under self-financing. Both of these distortions condition on receiving at least one round of funding, mirroring my empirical sample. The probability of success is approximately $\Delta_p = 56\%$ lower when the entrepreneur has to secure external funding, which corresponds to a 6 percentage point reduction. In other words, if the entrepreneur were wealthy and could self-finance, the share of successful outcomes would rise from 4.9% (see Panel A, column 3) to 11.0%. Acquisition outcomes are also adversely affected: the $\Delta_a = 37\%$ distortion is equivalent to a 14 percentage point reduction in the probability of acquisition, from 39% to 25%. The finding of a positive acquisition distortion suggests that the (negative) direct effect of external

⁹³See section A.5.1 for the relevant equations. Equivalently, the frictionless benchmark obtains when $\nu_e \rightarrow \infty$ and $\nu_l \rightarrow \infty$; see Lemma 2.1.

⁹⁴This is analogous to the “outcome distortion” defined in section 2.3.2. I refer to it here as a “success distortion” because the full model also includes a distinct role for acquisitions.

Panel A: Outcomes and modes of failure	Censored	Uncensored
Share successful (%)	3.8	4.9
Share acquired (%)	19.2	24.5
Share no outcome (%)	17.2	0.0
Share failed (%)	59.7	70.6
... in early-stage development (%)	16.1	18.2
... in early-stage search (%)	22.5	25.7
... in late-stage development (%)	9.0	11.6
... in late-stage search (%)	12.2	15.1
Panel B: Distortions	Agency & Matching	Matching only
Success distortion, Δ_p (%)	55.5	14.9
Acquisition distortion, Δ_a (%)	37.0	5.7
Entry distortion, Δ_s (%)	71.5	31.7
Panel C: Capital misallocation		
Share unproductive early-stage (%)		12.3
Share unproductive late-stage (%)		11.1
Share unproductive (%)		12.0
Share capital misallocated (%)		11.7
Share viable firms in search (%)		23.0

Table 3: US market assessment

The table reports start-up outcomes, measures of distortions induced by matching and agency frictions, and capital misallocation for the US estimation. Panel A reports modes of success and failure for firms from data simulated given estimated parameter values. Panel B reports distortions: for an outcome x in the frictional equilibrium, distortions are defined such that $x = (1 - \Delta_x)\bar{x}$, where \bar{x} is the value in the economy without frictions. Distortions relate to the success share, Δ_p , the acquisition share, Δ_a , and the entry value, Δ_s . Column “Agency & Matching” measures the combined effect of both frictions; column “Matching only” removes agency frictions (no hidden failure) while retaining matching frictions. Panel C reports figures on capital misallocation. Within each stage, this is defined as in equation (14). Across stages, the “share unproductive” reports the share of all firms receiving funding that are unproductive, whereas the “share capital misallocated” reports the share of capital allocated to unproductive firms, $(k_e\mu_{d,e}^u + k_l\mu_{d,l}^u)/((k_e(\mu_{d,e}^u + \mu_{d,e}^p) + k_l(\mu_{d,l}^u + \mu_{d,l}^p)))$. The share of viable firms in search is computed as $(\mu_{s,e} + \mu_{s,l})/(\mu_{s,e} + \mu_{d,e}^p + \mu_{s,l} + \mu_{d,l}^p)$.

financing on acquisitions via premature closure dominates the (positive) insurance effect.⁹⁵ Together, the results suggest not only that financial frictions inhibit the ability of entrepreneurs to independently scale their innovations, but they also result in fewer transfers of technologies to other firms operating in the market. Column three of Panel B reports the same distortions in an economy where entrepreneurs can commit not to engage in inefficient continuation, but must still search for funding for each stage.⁹⁶ Abstracting from moral hazard in contracting

⁹⁵In principle, the acquisition distortion may be positive or negative, reflecting two opposing forces. On the one hand, when entrepreneurs can self-finance, no viable projects are closed down, increasing the share that are acquired. If this effect dominates, then the acquisition distortion is positive; there are too few acquisitions. On the other hand, Proposition 2.8 shows that acquisitions substitute for financing. Therefore, when there are no issues in raising capital, entrepreneurs are less willing to accept low-value offers. If this effect dominates, then the acquisition distortion is negative; there are too many acquisitions.

⁹⁶In this economy, the entrepreneur and VC can write a state-contingent contract that terminates funding when news about the project arises, which drives the effective development costs back to k_i for $i \in \{e, l\}$.

cuts the success distortion $\Delta_p = 56\%$ to 15% and the acquisition distortion $\Delta_a = 37\%$ to 6% . In other words, a substantial share of the lost outcomes are attributable to the interaction between agency and matching frictions.

The success and acquisition distortions allow for understanding the *ex post* effects of financing. To investigate the *ex ante* effects, Panel B also reports the entry distortion, Δ_s , defined, as in section 2.3.2, as the wedge between the actual entry value, $V_{s,e}$, and the entry value under self-financing, $\bar{V}_{s,e}$. The value of the average entrepreneurial project is approximately $\Delta_s = 72\%$ lower when the entrepreneur relies on venture capital financing, compared to a wealthy entrepreneur that can self-finance. Therefore, frictions in access to external finance have a large detrimental effects on start-up creation. Converting this into a measure of lost entry requires an estimate of supply curve of entrepreneurial projects, which is beyond the scope of this paper. However, unless the supply of entrepreneurial projects is exceedingly inelastic with respect to project value, the magnitude points to a substantial impact on start-up creation.

Capital misallocation In addition to losses from premature terminations, the economy features misallocation: some viable start-ups seek funding, while other start-ups are still having capital from their previous funding round but should have shut down. Panel C of Table 3 reports the main results on capital misallocation. Within each stage, capital misallocation is given by equation (14). In the aggregate, misallocation can be computed based on the share of unproductive firms receiving capital or the share of capital allocated to unproductive firms. In practice, both figures are similar: 11.7% of capital is misallocated, whereas 12% of firms are unproductive, reflecting the similar degrees of misallocation across stages.

Taken together, these results highlight a key tension in the VC market: there coexists premature termination (underinvestment) and inefficient continuation (overinvestment). On the one hand, roughly 40% of projects are closed down prematurely; if these firms could access additional funding, many would go on to achieve successful outcomes. Relatedly, 23% of viable projects are actively searching for capital. On the other hand, a substantial share ($\approx 12\%$) of capital is tied up in firms that have already ceased to create value.

3.4 Robustness

This section describes how the estimates change when I make alternative assumptions in the estimation procedure. First, my baseline sample covers first financings in 2005–2015 with seven-year censoring, so events are observed through end-2022. This window includes the sharp run-up in US VC activity culminating in 2021 and easing in 2022. To check that the estimates are not unduly influenced by this late-sample boom, I re-estimate the model on a 2005–2010 subsample, maintaining seven-year censoring. Table C.3 reports the results. Parameter estimates are qualitatively similar across the two samples and of comparable magnitude. Importantly, estimates of financing conditions, (λ, ν_e, ν_l) , are closely aligned.

Second, my estimation censors events at seven years following a firm’s first funding round. For the baseline (2005-2015), I cannot extend the horizon; however, the 2005-2010 subsample permits a longer window. I therefore re-estimate the model for the 2005-2010 subsample with twelve-year censoring. Table C.3 shows that parameter estimates are very similar across the two

strategies. Figure 5 provides a further diagnostic: using the two estimated parameter vectors, I simulate data, apply censoring at $h \in \{3, \dots, 12\}$, and recompute the targeted moments. The simulated moments line up closely across h , including for $h > 7$, which is out of sample for the seven-year censored estimation. This insensitivity to the censoring choice is consistent with the broader model-validation evidence in Figure 5, where the model closely matches the average time to each funding round, funding continuation (firm attrition), and time-to-exit.

Finally, I winsorise the burn-rate distribution at $w = 5\%$ in both tails in the baseline. Empirically, variation in burn rates reflects both the size of capital injections and the duration between rounds; in the model, within a stage, only the inter-round duration varies across firms. Consequently, extremely large deals values add mass to the right tail of the burn rate distribution that the model does not explicitly capture. I winsorise in the estimation to prevent tail observations – which would primarily identify the *variance* of k_i for $i \in \{e, l\}$ if it were modelled – from having undue influence on average *level* of capital costs across firms. This aligns the estimator with the abstraction in the model, which treats all firms as *ex ante* homogeneous.

Figure 5 shows the model nonetheless produces substantial skewness in burn rates, though it cannot match the full tail thickness driven by heterogeneous deal sizes. Figure C.3 reports estimates under alternative thresholds $w \in \{0, 1, 2.5, 5 \text{ (baseline)}, 7.5, 10\}$. All parameters are stable across w , except the flow capital costs (k_e, k_l) and the payoff from success π . Consistent with the rationale for winsorising, as w falls the empirical mean burn rate rises (more right-tail mass retained), which the model matches via higher (k_e, k_l) . The success payoff, π adjusts to restore fit for the mean success multiple. Figure C.4 confirms that the model tracks the mean burn rate and mean success multiple across specifications. Overall, identification is not sensitive to the winsorisation threshold; only (k_e, k_l) and π adjust mechanically to match higher mean burn rates at lower w .

4 Venture capital across countries: US versus UK

In this section, I diagnose the sources of the UK–US disparities and evaluate policy interventions. I focus on the UK as a case study for Europe: it is Europe’s largest VC market and the closest institutional match to the US, which avoids Europe-wide market-definition issues and provides a conservative setting – if frictions matter here, they likely matter elsewhere.⁹⁷

Despite the focus on closing a perceived financing gap to the US (Draghi, 2024), compelling evidence that financing frictions – rather than available investment opportunities – are the principal constraint on European start-ups remains limited. This complicates policy design: if weaker opportunities are the issue, further capital-market interventions would prove wasteful.

Using the model as an identification device, I find that access to capital and a thin acquisition market are quantitatively important constraints on UK start-up activity. The analysis highlights that start-ups in the UK face particular issues raising follow-on funding. Relatedly, sparse late-stage funding pushes UK start-ups to rely more on pre-commercial acquisitions at the expense of developing their project independently, even though acquisition opportunities are themselves scarce compared with the US.

⁹⁷See Ewens and Farre-Mensa (2020) for a discussion of fragmentation in European private equity.

These findings suggest there may be gains from retargeting existing government support. Over my sample period (to end-2015), the British Business Bank (BBB) primarily supported early-stage venture and angel co-investment.⁹⁸ Using the model as a quantitative device, I run a budget-neutral policy counterfactual that reallocates funding across stages. Reallocating 5% of total VC investment from the early to late-stage increases (flow) total surplus by 16-30% between steady-states, depending on the response of start-up creation. The mechanism is compositional: total exits decline marginally, but the composition shifts from acquisitions towards successes; the latter reflect completed projects and, on average, generate higher surplus. These results indicate scope for budget-neutral reallocation to improve outcomes.

The remainder of this section provides the details. Section 4.1 estimates the model for the UK, compares the UK and US markets, and discusses some supplementary evidence consistent with the findings. Section 4.2 conducts the policy exercise.

4.1 Market failure or limited investment opportunities?

Table 4 provides an overview of key differences between the US and UK markets for the period 2005-2015; see Table B.3 for further summary statistics.⁹⁹ On a per-capita basis, the US VC market services twice as many new firms each year and funded firms are more likely to complete a successful exit (including acquisition) within seven years of their first round.

Country	Firms	Successes	Acquisitions
US	11,030 (1.00)	400 (3.6%)	2,034 (18.4%)
UK	1,066 (0.48)	26 (2.4%)	97 (9.1%)

Table 4: US–UK Comparison

The table provides an overview of key aggregate statistics for the US and UK. Counts shown first; values in parentheses are normalised figures. For ‘Firms’ the value is the population-adjusted ratio to the US (US = 1.00). For ‘Successes’ and ‘Acquisitions’ it is the percentage of firms. All data is censored at 7 years following a firm’s first funding round.

The model captures various explanations for these differences. First, the meeting rates, ν_e and ν_l , capture access to capital, which may differ across stages. Relatedly, λ captures how long firms can survive without a new equity injection; to the extent that the availability of alternative funding instruments, such as venture debt, differs across countries, estimation of the model will detect it.¹⁰⁰ Second, estimation of the model allows for uncovering the quality and characteristics of the average start-up that receives VC funding in each market. Finally, ϕ and ξ parameterise the acquisition market, determining the quantity of potential acquirers and the quality of the offers that these acquirers make to start-ups.

⁹⁸The early-stage focus reflected the view that the “equity gap” was most acute at that stage (Fernández Acevedo et al., 2016). In 2018, the BBB introduced British Patient Capital targeting late-stage start-ups.

⁹⁹Axelson and Martinovic (2016) analyse exit patterns at the European and the US level.

¹⁰⁰Venture debt is not modelled explicitly and I do not observe firm-level venture-debt funding. In aggregate, it is used more in the US than UK (Hellmann and Kavadias, 2016). Venture debt is commonly used to extend runway (González-Urbe and Mann, 2024), with repayment reliant on subsequent equity injections (Davis et al., 2020). Therefore, venture debt providers effectively bear the firm’s financing risk and supply should contract exactly when follow-on funding is hard to secure; i.e. venture debt behaves like a complement to equity, rather than a substitute. If venture debt matters empirically it will be reflected, to a first-order, in the estimated λ .

4.1.1 Estimation

This section estimates the model separately for the UK to unpack cross-country differences in VC activity. I first highlight diagnostics in the UK moments that point to financing and acquisition frictions, then confirm this with a separate UK estimation.

Identification The model separates financing frictions from other factors because each mechanism leaves distinct footprints in funding histories. In short, if UK underperformance reflected riskier or lower-quality projects, Propositions 2.3 and 2.4 imply more frequent, shorter funding rounds.¹⁰¹ Conversely, if the funding environment is the constraint, funded start-ups secure longer-duration commitments to insure against returning to the market.

Figure 6 compares the number of funding rounds and duration between rounds; see Table C.4 for the full set of UK moments. On average, UK VC-backed start-ups complete fewer rounds within the first seven years of funding (2.1 vs 3.0) with longer intervals between rounds (17.5 months vs 13.9 months). Higher project quality in the UK could also explain this pattern, but would be inconsistent with worse start-up outcomes and the lower entry rate.¹⁰²

This suggests that issues in funding, rather than project quality, are a key constraint on UK start-ups. As an external check on this interpretation, when US capital supply increased after the 1979 ERISA “prudent man” clarification – associated with a large inflow of VC funding (Gompers and Lerner, 1999) – cohorts first funded thereafter exhibit more funding rounds and shorter intervals between rounds than earlier cohorts (Figure B.2). Through the lens of the model, this is exactly the footprint of easing financing conditions and aligns with my interpretation of US–UK differences.

Returning to the UK, tighter financing conditions together with worse acquisition outcomes – the UK acquisition rate is 9.1% versus 18.4% in the US – suggest that UK start-ups also face a thinner acquisition market. Intuitively, since acquisitions substitute for financing (Proposition 2.8), tighter financing conditions in the UK should raise the acquisition rate above the US level; observing the opposite points to fewer acquisition opportunities.

Results The parameter estimates in Table 5 confirm this interpretation.¹⁰³ The early and late-stage meeting rates, ν_e and ν_l , and the arrival rate of acquisition offers, ϕ , are lower in the UK and the differences are statistically significant. Conversely, the model cannot differentiate between project characteristics among funded firms in each market.¹⁰⁴

The differences in access to financing are economically large. In the early-stage, the average time spent in search is $12/(\lambda + \nu_e) = 6.3$ months in the UK, compared to 3.8 months in the US. In the late-stage, the differences are similarly large: 3.9 months compared to just 2 months. Furthermore, UK start-ups are more than twice as likely to fail each time they seek

¹⁰¹Riskier projects benefit from a wait-and-see approach, due to increased agency costs, leading to frequent capital injections. Conversely, lower-quality projects face a lower marginal benefit of insurance against financing risk, so optimally lean towards shorter-term funding commitments.

¹⁰²Alternatively, UK start-ups may pursue longer-horizon or less capital-intensive projects. The separate estimation for the UK takes full account of these possibilities.

¹⁰³The estimation procedure mirrors that for the US in section 3. Section C.1 replicates the model validation exercises for the UK.

¹⁰⁴My estimation strategy does not identify start-up characteristics for projects that did not receive funding.

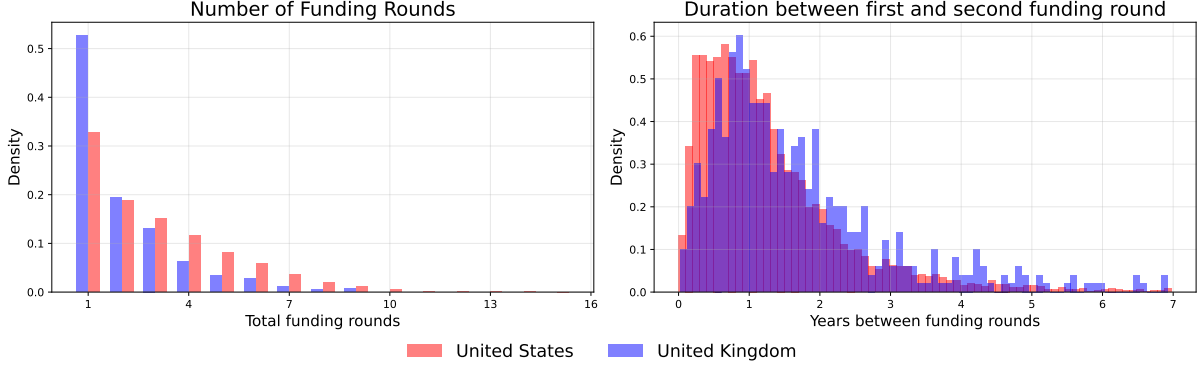


Figure 6: US versus UK funding patterns

The figure shows funding patterns for the UK and US samples. The left plot shows the distribution for the number of funding rounds for US firms (red) and UK firms (blue). The right plot shows the distribution of the duration of time between the first and second funding round for firms in each country. Focussing on the first-to-second round implies that data is uniformly censored; this choice is otherwise immaterial. All data is censored at 7 years following a firm's first funding round.

funding, compared to comparable US start-ups.¹⁰⁵ The difference in acquisition markets is also economically important. While there are no discernible differences in the quality of offers, parameterised by ξ , the arrival rate of offers in the US, ϕ , is approximately twice the UK level and the difference is significant.

Relative to the US, a clear issue in the UK market is difficulty securing follow-on funding. 53% of UK VC-backed firms do not raise additional funding within seven years of their first round (Table C.4), compared to 33% in the US (Table 2). Relatedly, just 33% of UK start-ups raise late-stage funding, compared to 42% in the US. Consistent with empirical work by Wilson et al. (2018), these issues point to severe issues in securing follow-on funding for UK start-ups. These issues feed into start-up outcomes. UK start-ups are less able to scale independently and are therefore more reliant on pre-commercial acquisitions. In the UK estimation, start-ups accept 66% of acquisition offers in search and 49% in development, compared to 50% and 41% in the US, respectively.¹⁰⁶ Tentatively, this suggests that capital may be misallocated between stages: funding fewer projects at the outset but ensuring that follow-on funding is available to those that demonstrate intermediate success may improve outcomes. I consider this counterfactual exercise in section 4.2.

US projects under UK financing To isolate how far financing alone can explain the US-UK gap, Table 6 reports counterfactual exercises for the US estimation that replace US financing market conditions, specifically ν_e and ν_l , by their estimated values for the UK and compares steady state outcomes. This provides further insights into the extent to which financing conditions shape the volume and outcomes of start-up activity. The table reports the (uncensored) share of successful and acquired firms, conditional on funding, and the entry value, V_s , across baseline and counterfactual economies.

¹⁰⁵ 43% of searches end in failure in the early-stage, and 25% in the late-stage ($=\lambda/(\lambda + \nu_l + \hat{\phi}_s)$); see Table C.4 in the Appendix. These numbers compare to 20% and 10% in the US, respectively.

¹⁰⁶ This is computed as $\hat{\phi}_i/\phi$ for $i \in \{s, d\}$. For the UK, $\hat{\phi}_s = 0.22$ and $\hat{\phi}_d = 0.16$. For the US, $\hat{\phi}_s = 0.33$ and $\hat{\phi}_d = 0.26$. The estimates of ϕ are shown in Table 5.

Parameter	US		UK		$\Delta(\text{US-UK})$
	Estimate	Std. Error	Estimate	Std. Error	
ρ	0.08	—	0.08	—	0.00
κ	0.26	0.01	0.25	0.07	0.01
λ	0.62	0.04	0.81	0.17	-0.19
p_e	0.23	0.01	0.26	0.07	-0.03
k_e	4.16	0.18	3.46	1.02	0.70
ν_e	2.54	0.13	1.09	0.19	1.45***
p_l	0.30	0.01	0.30	0.04	0.00
k_l	16.15	0.25	12.04	2.53	4.11
ν_l	5.20	0.26	2.28	0.41	2.92***
π	218.18	15.40	205.26	66.56	12.92
ϕ	0.65	0.07	0.33	0.07	0.32***
ξ	0.74	0.08	0.81	0.72	-0.07
$\tilde{\sigma}$	2.68	0.37	0.98	2.32	1.70

Table 5: US-UK Parameter Estimates

The table reports parameter estimates from the model in section 2.4 for the US and UK. Estimates for the US are reproduced; see Table 1. The estimation procedure for the UK mirrors the US: I again use a simulated sample of 220,600 firms, although this now corresponds to $S \approx 207$ times the empirical sample size. To avoid local minima in the estimation, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points for each estimation. With the resulting candidate parameter estimates, I run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses. Significance levels for differences in parameter estimates across countries (except $\tilde{\sigma}$) are computed from the asymptotic parameter estimate distributions, assuming independence. For $\tilde{\sigma}$, I take 1,000 samples from the asymptotic distribution of parameter estimates for each country, solve the model, recover the implied value for $\tilde{\sigma}$, and compute the difference $\Delta_{\tilde{\sigma}} = \tilde{\sigma}_{US} - \tilde{\sigma}_{UK}$. The significance level is determined by the share of cases with $\Delta_{\tilde{\sigma}} < 0$. Significance levels for $\Delta(\text{US-UK})$ are: * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

Differences in financing conditions explain approximately two-thirds ($\approx (3.3 - 1.6)/(3.3 - 0.8)$) of the gap in entry value between the US and UK; the remainder is explained by acquisition opportunities and other project characteristics. This implies that issues in allocating capital to start-ups is the primary constraint on start-up creation in the UK, relative to the US. Decomposing across stages, rows four and five demonstrate that late-stage funding conditions in the US are particularly important in driving start-ups to scale independently (success) rather than be acquired at pre-commercial stages – the fall in the success share (among funded firms) is more significant when only the late, rather than early-stage, in the US market resembles the UK.¹⁰⁷

4.1.2 Discussion

Outside the model, close inspection of both markets identifies several differences that may account for my estimates. UK VC funds typically manage less capital than US funds, and US VC-backed start-ups raise larger funding rounds, particularly in the late-stage (Hellmann and Kavadias, 2016). Although these features could be symptomatic of weaker investment

¹⁰⁷Note that although the share of successes and acquisitions among funded firms in the US counterfactual with UK early-stage financing exceed that with late-stage financing, the entry value for the US is higher in the latter case. This reflects that not all entrants receive funding.

	Share successful (%)	Share acquired (%)	Entry value (\$Mn)
UK: baseline	4.4	15.8	0.8
US: baseline	4.9	24.5	3.3
US: ν_e^{UK}, ν_l^{UK}	3.0	19.7	1.6
US: ν_e^{UK}	4.5	22.3	2.2
US: ν_l^{UK}	3.4	22.2	2.5

Table 6: US-UK decomposition

The table reports uncensored outcomes and the entry value across different counterfactual scenarios. Rows one and two show the UK and US baselines. Rows three to five compare different counterfactual scenarios, where the US financing market meeting rates are replaced with their respective UK estimates.

opportunities, my analysis suggests they reflect issues in the financing market.¹⁰⁸ Removing constraints on VC fund size may therefore improve outcomes for UK start-ups.

The relationship between VC funds and limited partners (LPs) is important for determining capital supply. Investor experience is a key indicator of the ability to raise large funds (Gompers and Lerner, 1999) and the UK market was relatively young for much of my sample. Here, policy may be important in attracting talented investors to the UK market. However, institutional investors need to be willing to allocate funds to VC and UK pension funds have comparatively small allocations to private equity, constraining fund size regardless of expertise.¹⁰⁹

Finally, there are several alternative explanations for US–UK differences that I do not consider in the model. Specifically, I do not model VC quality or skill, which is an important determinant of start-up outcomes (Sørensen, 2007); VCs shape outcomes through evaluation, monitoring, and their expertise (Da Rin et al., 2013). Greenwood et al. (2022) propose an endogenous growth model with a VC sector and suggest that differences in monitoring, project evaluation, and value-add contribute to growth differences between the US and France. Viewed through my framework, these forces do not appear to be first-order. First, in reduced form, if UK VCs were systematically worse at selecting or steering companies, projects would appear riskier or lower quality in the UK; VCs would then be cautious to commit too much capital, preferring more frequent rounds. Second, superior monitoring or stronger investor control rights should detect and prevent inefficient continuation more readily, supporting longer-horizon capital commitments; if US VCs held an advantage, US start-ups should obtain longer-duration funding.¹¹⁰ Empirically, UK start-ups are funded with fewer rounds at longer intervals, contradicting these mechanisms. Taken together, these alternatives do not appear to be first-order drivers of the UK–US differences in this sample.

¹⁰⁸Some indicators do point to financing issues, rather than demand-side constraints. For example, Hellmann and Kavadas (2016) point to the increasing prevalence of foreign investors in UK deals in later-stages, which suggests that UK VC funds may struggle to raise the large sums required to invest in late-stage ventures.

¹⁰⁹See *Comparing the asset allocation of global pension systems*, New Financial, 2024. Recent UK governments have placed pressure on pension funds to increase private equity allocations: for instance, the recent Mansion House Accord has secured a commitment from various pension funds to allocate 5% of their assets under management to UK private equity markets by 2030, releasing around £25Bn.

¹¹⁰Empirically, Tian (2011) shows that when monitoring costs are high – proxied by geographic distance between investor and firm – the start-up tends to be funded with more rounds, and with shorter durations between them.

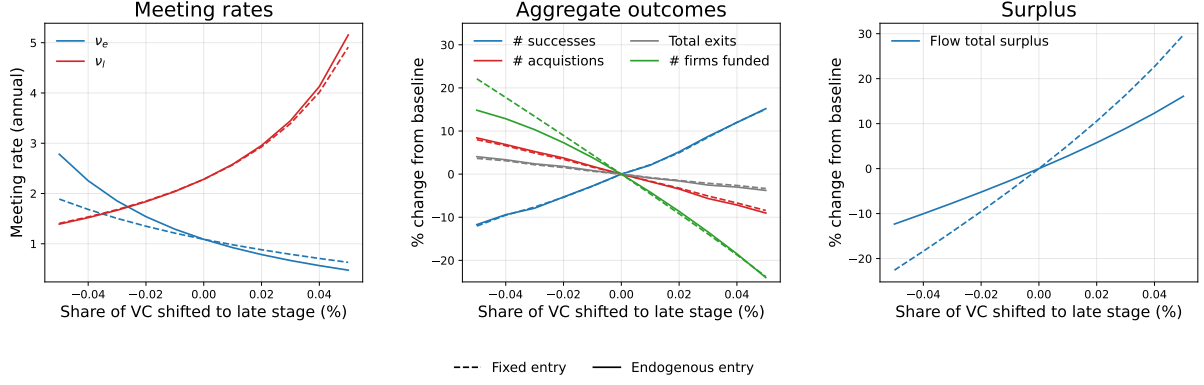


Figure 7: UK policy counterfactual

The figure reports the results of a budget-neutral policy intervention that shifts capital from the early to late-stage. The intervention is implemented by adjusting the meeting rates, (ν_e, ν_l) , while holding total (flow) investment, K^{total} fixed, $K^{\text{total}} = K^e + K^l$ and $K^i = k_i(\mu_{d,i}^p + \mu_{d,i}^u)$ for $i \in \{e, l\}$. The policy intervention is conducted under two assumptions on start-up creation: (i) holding start-up creation constant at its initial value (dashed); (ii) allowing start-up creation to adjust endogenously, according to $V_{s,e}/\bar{\sigma}$ (solid). The left-panel shows the mapping between the meeting rates (y -axis) and the share of capital reallocated from the early to late-stage. The share reallocated is defined as $(K_{\text{new}}^l - K_{\text{init}}^l)/K^{\text{total}}$. The centre-panel shows the change in the number of funded firms (green), the number of successful firms (number funded \times share successful; blue), the number of acquired firms (number funded \times share acquired; red) and the number of total exits (i.e. success + acquisition; gray) as capital is reallocated between stages. The right-panel shows the change in total (flow) surplus. Flow surplus is defined analogously to equation (15), but augmented to account for acquisitions and two stages. Specifically, it is given by $\kappa p_l \pi \times \mu_{d,l}^p + \hat{\phi}_d p_l \pi E[\epsilon | \epsilon > V_{d,l}/p_l \pi] \times \mu_{d,l}^p + \hat{\phi}_s p_l \pi E[\epsilon | \epsilon > V_{s,l}/p_l \pi] \times \mu_{s,l} - K^{\text{total}} - (\bar{\sigma}/2) \times \Lambda^2$, where Λ is the flow rate of start-up creation. In case (i), this is held constant at the initial value of $V_{s,e}/\bar{\sigma}$. In case (ii), it is equals $V_{s,e}/\bar{\sigma}$ as $V_{s,e}$ adjusts endogenously to changes in (ν_e, ν_l) .

4.2 Policy evaluation

In this section, I consider whether there are gains to reallocating capital between stages in the UK market. Existing policy instruments target different stages in the start-up lifecycle, implying that policymakers have access to budget-neutral tools that may improve outcomes.

Policy exercise To understand whether there are potential gains to such a strategy, I conduct the following counterfactual exercise. Starting from the estimated parameter values from Table 5, I reallocate capital between stages by adjusting the meeting rates, (ν_e, ν_l) , while holding constant total (flow) investment in the market.¹¹¹ I do so under two cases: (i) holding start-up creation constant, and (ii) allowing start-up creation to adjust endogenously. Note that in both cases, the total number of firms that receive funding still adjusts because the share of entrants that receives funding depends on ν_e . To aid interpretation, I map from changes in (ν_e, ν_l) to changes in the composition of funding and report results against this basis.

Figure 7 reports the effects across steady states as capital is shifted between stages. In each panel, the x -axis reports the volume of capital reallocated from the early to late-stage, as a share of total investment. The left-panel shows the meeting rates, (ν_e, ν_l) , that are required to generate corresponding capital reallocation; this represents the set of feasible policy interventions.¹¹² The

¹¹¹Total flow investment is defined as the sum of capital investment into start-ups of all stages, $k_e \times (\mu_{d,e}^p + \mu_{d,e}^u) + k_l \times (\mu_{d,l}^p + \mu_{d,l}^u)$. This measure captures both “productive” and “unproductive” start-ups and is analogous to the term that enters equation (15), adapted for the two-stage model.

¹¹²The mapping depends on the response of start-up creation. In practice, reallocating capital to the late-stage increases start-up creation, which raises total early-stage investment for given ν_e . Therefore, when start-up creation is allowed to respond, a larger adjustment in ν_e (and ν_l) is required to deliver a given reallocation.

centre-panel shows the effect on core aggregate statistics. When capital is shifted to the late-stage, the number of firms that are funded falls, but start-ups that are funded are more likely to achieve an exit, whether a success or acquisition. On net, the number of successes rises and the number of acquisitions falls, and this occurs independently of whether start-up creation adjusts.¹¹³ Total exits fall, but the compositional effect implies an increase in value creation (right-panel).¹¹⁴ The compositional effect is welfare-enhancing because successes correspond to fully developed projects, whereas acquired projects are only partially developed. Therefore, when exits tilt towards successes, a greater number of projects lead to commercial applications.

Table 7 considers a particular intervention, in which 5% of total VC investment is shifted from the early to late-stage. This reflects the sort of marginal adjustment that policymakers could induce given existing budgets.¹¹⁵ Furthermore, there is a practical limit to how much capital can be reallocated between stages; without sufficient early-stage funding, there are not enough late-stage firms to absorb an increase in late-stage capital availability.

Reallocating 5% of total capital from the early-stage to late-stage increases the number of successes by 15%, but leads to a fall in pre-commercial acquisitions by 8-9%, depending on the start-up creation adjustment. Although fewer firms receive initial funding (-24%), the share of firms that reach the late-stage rises (+33%), reflecting both that late-stage liquidity raises the marginal benefit of early-stage investment for VCs and that, once overcoming this intermediate milestone, late-stage funding is easier to secure.¹¹⁶ With more favourable late-stage funding conditions, firms are more likely to scale independently, rather than be acquired at pre-commercial stages. Policymakers then face an extensive-intensive margin trade-off: holding budgets fixed, targeting the early-stage means backing many firms, often with insufficient capital to scale, whereas targeting the late-stage means fewer firms are funded, but for those that show early promise, capital is available to scale-up. In the model, targeting the late-stage dominates: total exits fall (-3-4%), but the compositional shift towards successes increases total (flow) surplus by 16-30%, depending on assumptions about the entry response.

Discussion An important assumption in this analysis is that start-ups cannot finance development from retained earnings because they deliver no intermediate cash flows. If start-ups that produce a positive signal in the early-stage could finance late-stage development from retained earnings, then leveraging the extensive margin – providing little funding to many early-stage firms – may instead be optimal. Aragonese and Saxena (2025) consider a related policy exercise in a model calibrated to Europe with collateral constraints in which start-ups can invest

¹¹³Note that the response of the total number of funded firms is relative insensitive to the response of start-up creation. When capital is reallocated to the late-stage and start-up creation responds, a larger decline in ν_e (increase in ν_i) is needed to generate a given reallocation. Then, a smaller share of entrants receive funding, which mostly offsets the effect of start-up creation on the number of firms funded.

¹¹⁴The difference in the response of total surplus when start-up creation is held constant, or is allowed to adjust, is almost entirely accounted for by a change in total entry costs.

¹¹⁵Given recent trends in VC activity, it amounts to between £0.5-1Bn annually; see PitchBook (2025).

¹¹⁶To understand the response of early-stage investors, view the early-stage as an experiment that produces information about the project's viability (Kerr et al., 2014; Ewens et al., 2018). When late-stage capital is scarce, even projects with positive early signals cannot be scaled, which lowers the value of the experiment. Early-stage investors commit less capital to any round, shortening early-stage commitments and leading many funded firms to run out of runway before interim news arrives. Conversely, when late-stage funding is reliable, early investors commit more funding upfront, increasing the likelihood that interim uncertainty is resolved.

Panel A: Policy lever		
Outcome	Fixed Entry	Endogenous Entry
ν_e	0.63	0.48
ν_l	4.91	5.15
Panel B: Outcomes		
Outcome	Fixed Entry	Endogenous Entry
<i>Shares</i>		
Share success (% Δ)	50.94	51.54
Share acquired (% Δ)	20.11	19.68
Share to late (% Δ)	33.25	33.05
<i>Counts</i>		
No. funded firms (% Δ)	-23.76	-23.99
No. successful firms (% Δ)	15.08	15.19
No. acquired firms (% Δ)	-8.43	-9.03
No. firm exits (% Δ)	-3.33	-3.78
<i>Aggregate</i>		
Total flow surplus (% Δ)	29.71	16.09

Table 7: UK budget-neutral reallocation across stages (5% of total investment)

The table reports results from a policy that shifts 5% of total VC investment from the early to late-stage, holding total investment constant. The exercise mirrors that in Figure 7; see notes for details.

out of retained earnings. They show that reallocating interventions towards the early-stage is preferable when frictions are mild, whereas a late-stage focus becomes more attractive as financial market imperfections worsen; in this case, retained earnings are less effective in relaxing constraints. My analysis is complementary and highlights the extreme case in which firms remain unprofitable for many years, which is particularly relevant for the subset of firms access VC financing; since 2000 in the US, the only year in which a majority of VC-backed IPOs were profitable was 2009 (Figure B.1).

5 Venture capital across technologies

Venture capital finances a narrow subset of start-ups, favouring sectors where uncertainty is resolved quickly, particularly software and services (Lerner and Nanda, 2020).¹¹⁷ In long-horizon sectors, uncertainty about future funding availability may depress initial investor returns and lead certain sectors to be underfunded (Nanda et al., 2014). Well-established exit mechanisms provide a route to early liquidity and partially offset this risk (Ghosh and Nanda, 2010).

There is substantial variation in the mode-of and time-to-exit across sectors. Figure 8 reports the acquisition-to-success ratio and average time-to-exit across Venture Economics Industry Classification (VEIC) industries in my sample.¹¹⁸ Sectors with longer average time-to-exit,

¹¹⁷Consistent with this view, Narain (2024) shows that venture capital funding skews towards technologies with short development horizons, whereas government funding tends to support projects with longer timelines.

¹¹⁸The VEIC is a granular industry categorisation specifically designed for VC-funded firms. Given its focus on

which tend to be science-based sectors, are less likely to exit via acquisition, relative to success. Conversely, start-ups in software and service based sectors exit quickly, typically via acquisition. To the extent that these margins of heterogeneity interact with financial frictions, funding may be misallocated away from long-horizon projects without established exit mechanisms.

In this section, I show that the interaction of these features with financial frictions is quantitatively significant and can induce a large reallocation of VC funding away from science-based sectors. In a counterfactual exercise that removes financing frictions, the share of VC-backed start-ups in Biotech, Medical/Health and Semiconductors rises by 8pp, offset by a relative decline in software and service start-ups.

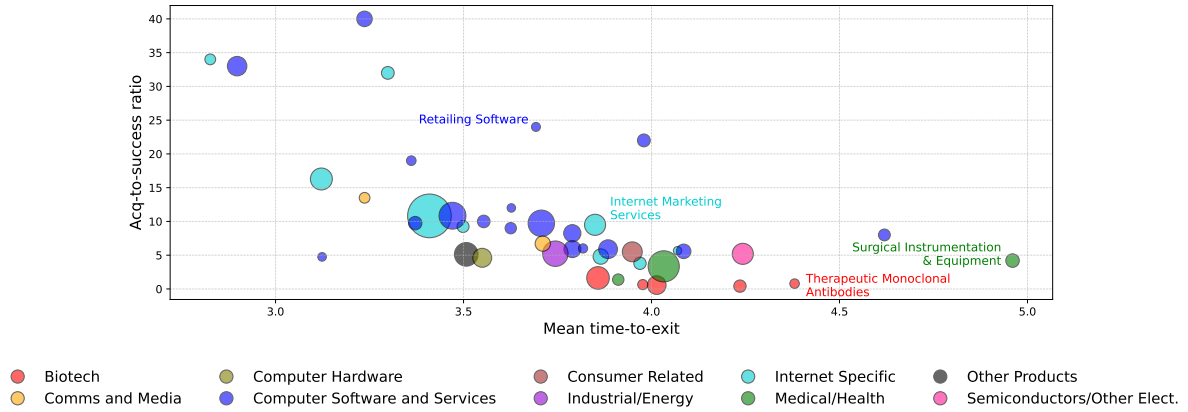


Figure 8: Sectoral heterogeneity

The figure plots the acquisition-to-success ratio against the mean time-to-exit across VEIC sectors. The sample period is 2005-2015, consistent with other analysis, and observations are censored seven years following a firm's first funding round. Sectors are included as distinct observations if there are at least 20 exits during the sample period and at least one of each type (acquisition and success). Sectors that do not meet these criteria are grouped at the aggregate sector level and reported together. The membership of each sector to aggregate sectors are indicated through colours, and the size of each point reflects the number of firms in the sector.

To understand the potential for policy to affect funding misallocation, I assess the effect of start-up acquisitions on start-up creation across sectors. Among the policy levers that affect the market for firms, antitrust policy towards start-up acquisitions is the most natural, but policy that affects market structure among potential acquirers may also be important.¹¹⁹

I find that the incentive effects of acquisitions on start-up creation are strongest in science-based sectors, where funding distortions are most severe. Intuitively, marginal changes in the availability of acquisition opportunities has the strongest effects on start-up creation incentives for start-ups pursuing long-horizon projects in sectors where acquisition opportunities are already limited. Conversely, software start-ups are much less exposed to marginal changes in acquisition opportunities. In these sectors, policymakers may have more leeway to prioritise other objectives – such as limiting entrenchment and incentivising incumbent innovation – without substantial fear of negative side-effects on start-up creation.

VC-funded start-ups, it offers a more granular classification than alternatives, such as NAICS or SIC codes.

¹¹⁹For example, a start-up that appeals to many acquirers can afford to wait for the best match, raising its payoff at exit. In this light, the increase in M&A activity and resulting consolidation in the US defence (Van Reenen et al., 2021) and pharmaceutical sectors (FTC and DOJ, 2022) has the potential to stifle start-up innovation.

5.1 The allocation of funding

This section constructs the estimates of the skew of VC funding. In order to do so, I introduce *ex ante* heterogeneity in project types. After calibrating this to heterogeneity across granular sectors, I compare the observed distribution of funding across types to the benchmark in which wealthy entrepreneurs can fund their own projects. The exercise comprises four steps, which I outline briefly in the main text; see section C.2 of the Appendix for full details.

Calibration exercise First, starting from the parameter estimates for the US quantification in section 3, I introduce heterogeneity along two margins: (i) the development horizon, κ ; and (ii) the arrival rate of acquisition offers, ϕ . In practice, introducing heterogeneity along these margins changes investment incentives, independent of how they interact with financial frictions, which is the effect I wish to isolate. Therefore, I adjust the project payoff, π , such that start-ups with different characteristics have the same frictional entry value, $V_{s,e}$, equal to the value in the estimation of section 3. Given this normalisation, any difference in investment incentives in the frictionless counterfactual is due to the interaction of these characteristics with financial frictions, not inherent project quality. In practice, this means that long-horizon projects, low κ , with limited acquisition opportunities, low ϕ , have higher payoffs, π . To validate this normalisation, Figure C.7 shows that the calibration matches the empirical relationship between exit payoffs, time-to-exit and acquisition-to-success ratio across sectors.

Second, I calibrate the degree of heterogeneity by targeting the joint distribution of the average time-to-exit and acquisition-to-success ratio across granular VEIC sectors, as shown in Figure 8. Intuitively, sectors with longer time-to-exit have longer development horizons, whereas sectors with higher acquisition-to-success ratios have better acquisition opportunities. However, there is an interplay between the two margins of heterogeneity: when acquisition opportunities are plentiful, firms can exit earlier. In addition, for given acquisition opportunities, longer-horizon projects are more likely to exit via acquisition because acquisitions substitute for financing and these firms face more severe financial frictions. By targeting the joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space, I allow the model to determine the relative importance of these channels. To ensure that I capture signal not noise, I consider sectors individually if there were at least 20 exits in my sample period, including at least one acquisition and one success; sectors that do not meet this requirement are aggregated to the ten-sector level. Denoting each ‘technology’ by τ , this calibration exercise produces technology weights, $s(\tau)$. These weights give the baseline shares of firms with each technology.

Third, I recover the frictionless weights, $\bar{s}(\tau)$, by removing frictions and computing the frictionless entry values, $\bar{V}_{s,e}(\tau)$. Specifically, given the estimated weights, $s(\tau)$, and the frictional entry value, $V_{s,e}$, the estimated weights satisfy

$$s(\tau) = \frac{V_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1}}{\int V_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1}d\tau} = \frac{\tilde{\sigma}(\tau)^{-1}}{\int \tilde{\sigma}(\tau)^{-1}d\tau} \quad (22)$$

where $\sigma(\tau)$ is the technology-specific entry parameter and the second equality follows because $\pi(\tau)$ is adjusted such that $V_{s,e}(\tau) = V_{s,e}(\tau') \forall \tau, \tau'$. Given the estimated values $s(\tau)$, equation (22) enables recovery of $\tilde{\sigma}(\tau)^{-1}$ up to a constant factor. Then, the frictionless shares are

$$\bar{s}(\tau) = \frac{\bar{V}_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1}}{\int \bar{V}_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1} d\tau} = \frac{\bar{V}_{s,e}(\tau)s(\tau)}{\int \bar{V}_{s,e}(\tau)s(\tau) d\tau} \quad (23)$$

where the second equality follows from equation (22).¹²⁰

Finally, I map the technologies, τ , to the ten aggregate sectors depicted in Figure 8. Although in practice these sectors may differ on many additional margins, this provides an interpretable metric. Using this mapping and the imputed shares $\bar{s}(\tau)$, I can recover an estimate of the distortion to the sectoral composition of VC funding induced by the interaction of financial frictions, project horizons and acquisition opportunities. To implement the mapping, I set up a simple optimal transport problem, which I run at the granular VEIC sector level, before aggregating to the ten-sector level. In short, the mapping assigns technologies, τ , to sectors based on their proximity in the (acquisition-to-success, time-to-exit)-space.

Results Figure 9 plots the resulting cumulative distributions for the average time-to-exit and acquisition-to-success ratio across project types, comparing the frictional economy (orange, dashed) computed using the true weights, $s(\tau)$, to the frictionless economy (blue, solid) computed using the counterfactual weights, $\bar{s}(\tau)$. Removing financial frictions leads to a relative increase in entry from start-ups with longer expected time-to-exit and lower acquisition-to-success ratios. In other words, financing risk skews the allocation of VC funding towards projects with shorter time-to-exit and stronger acquisition opportunities, relative to a first-best economy.

Figure 10 shows the implications for the sectoral allocation of VC funding. Financial frictions distort the sectoral composition of VC-backed start-ups, particularly favouring start-ups in Computer Software and Services and Internet Specific sectors.¹²¹ The share of start-ups in these sectors falls from 61% to 53% when financing frictions are removed, with the difference made up by start-ups in Biotechnology, Medical/Health and Semiconductors.¹²²

Discussion These findings indicate a substantial misallocation of VC funding across sectors, implying that frictions in VC funding affect not only the overall level of investment but also the direction of start-up innovation. This echoes the empirical findings of Nanda and Rhodes-Kropf (2013), who show that start-ups first funded in hot markets are less likely to succeed, but conditional on success, have greater valuations, and more patents of higher quality. To my knowledge, this is the first quantitative assessment of the interaction between financial frictions and funding allocation across technologies in the VC context.¹²³ Given the importance of

¹²⁰Note that the flow of first funding rounds in the frictional economy for technology τ is $\frac{V_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \times \frac{\nu_e}{\nu_e + \lambda}$. The financing market rates do not appear in equation (22) because they are held constant across technologies.

¹²¹The most prominent Internet Specific sectors are service sectors, such as Internet Marketing Services and Finance/Insurance/Real Estate Services.

¹²²See Table C.5 for the full list of sector reallocation figures.

¹²³A more ambitious quantitative assessment, beyond the scope of this paper, would re-estimate the model at the sector level. At present, (κ, ϕ) are forced to account for all heterogeneity in the acquisition-to-success ratio and time-to-exit across sectors; other primitives that may differ across sectors, such as capital intensity (k_e, k_l) , are held constant at the US baseline. Reversing the conclusions would require science-based sectors to be less risky, muting agency costs, or to face easier financing conditions than software and service start-ups. However, these technologies relate closer to basic science, implying longer development horizons and greater technological uncertainty. In biotech, pre-revenue IPOs can shift some risk to public markets (Ghosh and Nanda, 2010), but high capital intensity simultaneously increases the difficulty of scaling independently.

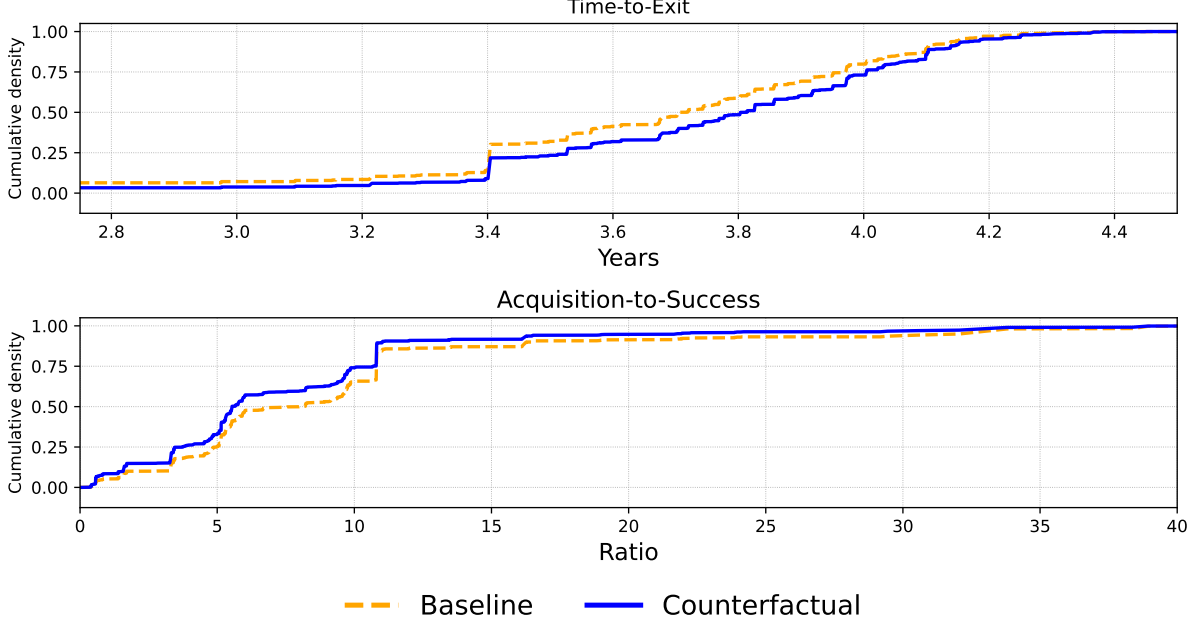


Figure 9: Time-to-exit and outcome ratios: frictional vs. frictionless entry

The figure plots the marginal CDFs for the time-to-exit (top) and acquisition-to-success ratio (bottom) based on the shares frictional shares $s(\tau)$ (orange, dashed) and frictionless shares $\bar{s}(\tau)$ (blue, solid). In plotting the CDFs, I hold the time-to-exit and acquisition-to-success ratio at their values in the baseline economy. These outcomes are endogenous to financial frictions, but depicting the data in this way aids comparison. All data is censored at seven years following a firm’s first funding round.

young firms in driving economic growth (Decker et al., 2016), the notion that direction of start-up innovation activity may be distorted relative to first-best has implications of the direction of aggregate growth. This is absent from existing models that incorporate venture capital markets into endogenous growth models (Greenwood et al., 2022; Akcigit et al., 2022; Ando, 2024) and provides an important avenue for future research in this area.

5.2 Policy implications and the market for firms

Antitrust authorities are increasingly considering how competition policy shapes innovation, moving beyond the classical static approach. For instance, in Dow–DuPont (2017) the European Commission required divestiture of R&D assets, explicitly invoking an innovation theory of harm.¹²⁴ In the start-up setting, policymakers are concerned about “killer acquisitions” (Cunningham et al., 2021), where established firms buy start-ups to shut down their projects and prevent competition, or that acquisitions in a certain area may reduce entry incentives for follow-on innovations, by creating a “kill zone” (Kamepalli et al., 2020). Albeit lesser-discussed, consolidation among established firms may weaken start-up exit options.¹²⁵ In my model, a similar mechanism is present: when the arrival rate of acquisition offers, ϕ , is low, start-ups have weaker outside options and are subsequently more willing to accept poor quality matches, char-

¹²⁴See Commission Decision, Case M.7932 – Dow/DuPont.

¹²⁵Science-based start-ups face thinner acquisition markets, which leads them to capture a lower share of the surplus from their innovations (Masclans, 2025). Relatedly, Wang (2018) shows that concentrated acquirer markets depress start-up creation by inventors.

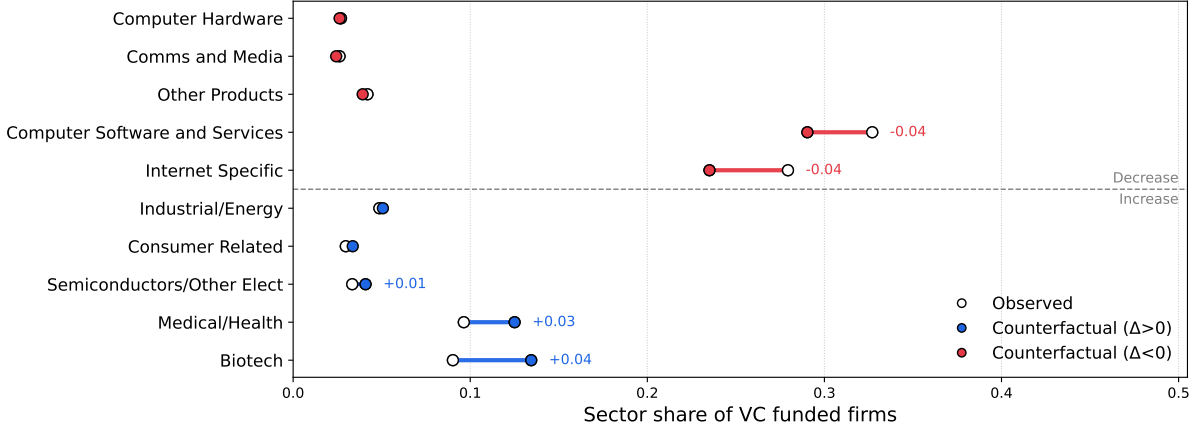


Figure 10: Sector reallocation

The figure shows observed sector weights and counterfactual sector weights under self-financing. Heterogeneity in the (κ, ϕ) -space is first calibrated using cross-sector variation in average time-to-exit and the acquisition-to-success ratio. ‘Technologies’ in the model are then mapped to VEIC sectors based on proximity in the (acquisition-to-success, time-to-exit)-space via optimal transport; see section C.2 for details.

acterised by lower synergies, ϵ .¹²⁶ The importance of acquisitions is magnified when financial frictions are most severe, as is the case for long-horizon projects.

To illustrate this point, Figure 11 plots the entry-value multiplier, $V_{s,e}(\phi + \Delta\phi)/V_{s,e}(\phi)$, for a small increase in the arrival rate of acquisitions, $\Delta\phi = 0.1$. This increase corresponds to one additional acquisition offer for every ten years spent developing the project. The higher is the multiplier, the greater the impact of a marginal increase in the arrival rate of acquisitions on the incentives for start-up creation. The left-panel reports the multiplier for projects with different development horizons, κ , and initial acquisition arrival rates, ϕ .¹²⁷ This is greatest for long-horizon projects with limited acquisition opportunities; low κ and ϕ .

The right panel of Figure 11 uses the estimated weights, $s(\tau)$, and mapping from technologies to VEIC sectors to provide an indication of the role that acquisitions play in stimulating start-up creation across sectors. Acquisitions are most important for stimulating start-up creation in sectors where financial frictions create the largest distortions. In Biotech, Medical/Health, and Semiconductors, one additional acquisition offer per ten-years in development more than doubles start-up entry value. Conversely, for firms in Computer Software and Services and Internet Specific sectors, the entry-value multiplier is less than 1.5, reflecting their shorter development horizons and ample existing acquisition opportunities.

This result has implications for competition policy. Marginal changes in the leniency of antitrust policy to start-up acquisitions of software and service-based start-ups has limited effects on start-up creation in those sectors, but effects for science-based sectors are substantial.¹²⁸ Re-

¹²⁶The model could be generalised to allow for the surplus to be split according to some Nash bargaining weights. Given the reduced-form nature of the acquisition market in this model, this additional degree of freedom provides little added value, at the cost of needing to estimate an additional parameter. In practice, the key is that the start-up’s outside option affects the offers it is willing to accept.

¹²⁷As in all analysis in this section, π is adjusted to hold the frictional entry value constant across technologies; i.e. $V_{s,e}(\tau) = V_{s,e}(\tau') \forall \tau, \tau'$.

¹²⁸The idea that changes to antitrust policy may have heterogeneous effects on start-up creation incentives across sectors is considered by Fons-Rosen et al. (2021). In their model, the difference between the acquisition price and outside option of the start-up to independently scale the innovation is a key calibration target: when

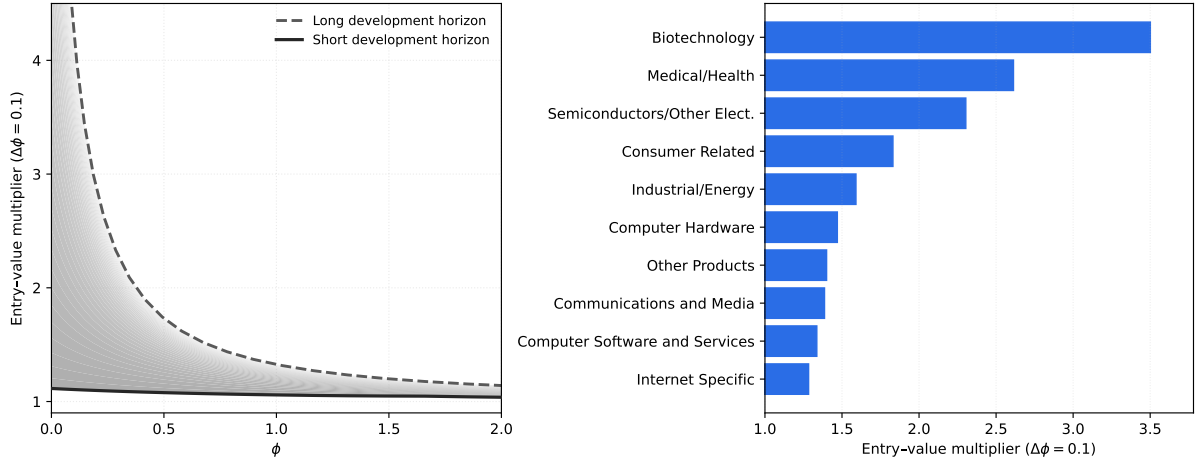


Figure 11: Marginal effect of acquisitions on entry

The figure plots the entry-value multiplier, $V_{s,e}(\phi + \Delta\phi)/V_{s,e}(\phi)$ for a small increase in the arrival rate of acquisitions, $\Delta\phi = 0.1$. The left panel shows how the multiplier varies across given the initial arrival rate of acquisitions, ϕ , for projects with different development horizons (where π is normalised so that all projects have the same frictional entry value, $V_{s,e}(\tau) = V_{s,e}(\tau') \forall \tau, \tau'$). Specifically, the dashed “long development horizon” sets $\kappa = 0.08$, implying an expected development horizon of $2/\kappa = 25$ years if fully funded; the solid “short development horizon” sets $\kappa = 2$, implying an expected development horizon of $2/\kappa = 1$ year if fully funded; between these extremes, darker shading corresponds to shorter development horizons. The right figure plots the weighted average entry-value multiplier across aggregate VEIC sectors. The weighting utilises the estimated weights, $s(\tau)$, and the mapping from technologies, τ , to granular VEIC sectors, as in Figure 10.

latedly, increased sensitivity to marginal changes in acquisition opportunities in science-based sectors suggests that consolidation among incumbent firms may distort start-up innovation. In this regard, recent consolidation in the pharmaceutical industry provide cause for concern.

6 Conclusion

This paper develops a tractable theory of the VC market that facilitates a tight mapping to data. The model incorporates empirical and theoretical insights from entrepreneurial finance into an equilibrium setting. The analysis highlights that financing frictions distort start-up entry and outcomes, particularly for novel, long-horizon projects. Because primitives map to observed funding histories, the framework permits transparent estimation on standard VC data.

My estimates suggest that significant issues remain in the funding of highly innovative ventures, even in the developed US market. However, entrepreneurs in other developed economies face more severe difficulties. In the UK, lower levels of VC activity reflect issues in allocating risk capital to start-ups and thinner acquisition markets, not limited investment opportunities. UK start-ups face particular issues in securing follow-on funding, suggesting that government support should focus existing efforts on late-stage start-ups. Finally, venture capital struggles to penetrate science-based sectors, particularly where a path to early liquidity for initial investors via acquisition opportunities is lacking. Scientific innovations with long development horizons face issues in scaling their projects independently because they typically need to visit the capital market more frequently, increasing their exposure to financing risk. Policymakers

this value is sufficiently high, a ban on start-up acquisitions is welfare enhancing.

can help to mitigate these issues by facilitating – or not inhibiting – technology transfer from start-ups to incumbents, who may be better placed to commercialise the innovations.

This paper is a first step in bringing VC-market insights into a structural framework, and several extensions follow naturally. For example, the model is silent on an explicit role for existing investors in facilitating follow-on funding and existing research suggests that VC networks play an important role (Hochberg et al., 2007). Incorporating this channel would induce different meeting rates for follow-on funding depending on the identity of initial investors, which I abstract from. More generally, start-ups and VCs are homogeneous in the baseline model; allowing heterogeneity would permit more intricate sorting and may facilitate revisiting the contribution of VC skill or sorting in the persistence of VC returns (Sørensen, 2007).

An additional limitation of the present analysis is that staging is the only information production tool.¹²⁹ Extensions to monitoring are feasible and act as a substitute for staging (Tian, 2011), permitting the VC to detect hidden failure and enforce closure. The current environment is the special case where information is non-verifiable and staging emerges as the only information production tool. Given the innovative and intangible nature of start-up projects, this is likely a reasonable approximation to reality. However, sectoral differences in “hard” versus “soft” information creates interesting heterogeneity. I conjecture that hard information is more readily available in consumer-orientated start-ups, such as software and services, as revenues arrive earlier in the lifecycle and provide a verifiable measure of intermediate success. This likely amplifies the sectoral misallocation emphasised in this paper. Finally, in this context, VC specialisation may matter, both because specialised VCs face lower monitoring costs and because they design experiments that reveal intermediate success more effectively. These topics offer fruitful areas for future research.

¹²⁹In the VC context, the term “monitoring” is often used to describe efforts on the part of the VC to improve start-ups’ outcomes, rather than effort to detect foul play (see e.g. Kaplan and Strömberg 2001).

References

- Acevedo, Miguel Fernández, Matt Adey, Claudio Bruno, Gino del Bufalo, Alexandre Gazaniol, Vivien Lo, Georg Metzger, Blanca Navarro Perez, Dan van der Schans, and Baptiste Thornary, “Building Momentum in Venture Capital across Europe: France, Germany, Italy, Spain and the United Kingdom,” December 2016.
- Acharya, Viral V, Douglas Gale, and Tanju Yorulmazer, “Rollover risk and market freezes,” *The Journal of Finance*, 2011, 66 (4), 1177–1209.
- Admati, Anat R and Paul Pfleiderer, “Robust Financial Contracting and the Role of Venture Capitalists,” *The Journal of Finance*, 1994, 49 (2), 371–402.
- Akcigit, Ufuk, Emin Dinlersoz, Jeremy Greenwood, and Veronika Penciakova, “Synergizing ventures,” *Journal of Economic Dynamics and Control*, 2022, 143, 104427.
- Ando, Yoshiki, “Dynamics of High-Growth Young Firms and the Role of Venture Capitalist,” *Working paper*, 2024.
- , Emin Dinlersoz, Jeremy Greenwood, and Ruben Piazzesi, “Technifying Ventures,” Technical Report, Economie d’Avant Garde 2025.
- Aragoneses, Martin and Sagar Saxena, “Industrial Policy via Venture Capital,” *Available at SSRN 5316925*, 2025.
- Arnold, Mr Nathaniel G, Guillaume Claveres, and Jan Frie, *Stepping up venture capital to finance innovation in Europe*, International Monetary Fund, 2024.
- Arora, Ashish, Andrea Fosfuri, and Thomas Rønde, “The missing middle: Value capture in the market for startups,” *Research Policy*, 2024, 53 (3), 104958.
- Ates, Sînâ T, “Beyond cash: venture capital, firm dynamics, and economic growth,” *Unpublished paper. Federal Reserve Board of Governors*, 2018.
- Axelson, Ulf and Milan Martinovic, “European venture capital: myths and facts,” 2016.
- Bergemann, Dirk and Ulrich Hege, “Venture capital financing, moral hazard, and learning,” *Journal of Banking & Finance*, 1998, 22 (6-8), 703–735.
- and —, “The financing of innovation: Learning and stopping,” *RAND Journal of Economics*, 2005, pp. 719–752.
- Berger, Marius, Sara Calligaris, Andrea Greppi, and Dmitri Kirpichev, “Acquisitions and their effect on start-up innovation: Stifling or scaling?,” Technical Report, OECD Publishing 2025.
- Bernstein, Shai, Xavier Giroud, and Richard R Townsend, “The impact of venture capital monitoring,” *The Journal of Finance*, 2016, 71 (4), 1591–1622.

- Brander, James A, Raphael Amit, and Werner Antweiler**, “Venture-capital syndication: Improved venture selection vs. the value-added hypothesis,” *Journal of Economics & Management Strategy*, 2002, 11 (3), 423–452.
- Brav, Alon and Paul A Gompers**, “Insider trading subsequent to initial public offerings: Evidence from expirations of lock-up provisions,” *Available at SSRN 204094*, 2000.
- Brunnermeier, Markus K and Martin Oehmke**, “The maturity rat race,” *The Journal of Finance*, 2013, 68 (2), 483–521.
- Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin**, “Finance and development: A tale of two sectors,” *American economic review*, 2011, 101 (5), 1964–2002.
- Cavalcanti, Tiago, Joseph Paul Kaboski, Bruno Silva Martins, and Cezar Santos**, “Financing costs and development,” Technical Report, IDB Working Paper Series 2023.
- Chen, Jun and Michael Ewens**, “Venture capital and startup agglomeration,” *The Journal of Finance*, 2025.
- Clementi, Gian Luca and Hugo A Hopenhayn**, “A theory of financing constraints and firm dynamics,” *The Quarterly Journal of Economics*, 2006, 121 (1), 229–265.
- Cornelli, Francesca and Oved Yosha**, “Stage financing and the role of convertible securities,” *The Review of Economic Studies*, 2003, 70 (1), 1–32.
- Cunningham, Colleen, Florian Ederer, and Song Ma**, “Killer acquisitions,” *Journal of political economy*, 2021, 129 (3), 649–702.
- Dahiya, Sandeep and Korok Ray**, “Staged investments in entrepreneurial financing,” *Journal of Corporate Finance*, 2012, 18 (5), 1193–1216.
- Davis, Jesse, Adair Morse, and Xinxin Wang**, “The leveraging of silicon valley,” Technical Report, National Bureau of Economic Research 2020.
- Decker, Ryan A, John Haltiwanger, Ron S Jarmin, and Javier Miranda**, “Where has all the skewness gone? The decline in high-growth (young) firms in the US,” *European Economic Review*, 2016, 86, 4–23.
- Diamond, Douglas W**, “Debt maturity structure and liquidity risk,” *the Quarterly Journal of economics*, 1991, 106 (3), 709–737.
- Draghi, Mario**, “The future of European competitiveness,” *Bruxelles: Commission européenne, septembre*, 2024, 2, 5.
- European Commission**, “Science, research and innovation performance of the EU, 2024 – A competitive Europe for a sustainable future,” Technical Report, Directorate-General for Research and Innovation 2024.
- Ewens, Michael, Alexander Gorbenko, and Arthur Korteweg**, “Venture capital contracts,” *Journal of Financial Economics*, 2022, 143 (1), 131–158.

- **and Joan Farre-Mensa**, “The deregulation of the private equity markets and the decline in IPOs,” *The Review of Financial Studies*, 2020, *33* (12), 5463–5509.
 - **and Matthew Rhodes-Kropf**, “Is a VC Partnership Greater than the Sum of its Partners?,” *The Journal of Finance*, 2015, *70* (3), 1081–1113.
 - **, Ramana Nanda, and Matthew Rhodes-Kropf**, “Cost of experimentation and the evolution of venture capital,” *Journal of Financial Economics*, 2018, *128* (3), 422–442.
- Flannery, Mark J**, “Asymmetric information and risky debt maturity choice,” *The journal of finance*, 1986, *41* (1), 19–37.
- Fons-Rosen, Christian, Pau Roldan-Blanco, and Tom Schmitz**, “The effects of startup acquisitions on innovation and economic growth,” *Max Planck Institute for Innovation & Competition Research Paper*, 2021, (23-02).
- FTC and DOJ**, “The Future of Pharmaceuticals: Examining the Analysis of Pharmaceutical Mergers—Workshop Summary,” Technical Report, Federal Trade Commission and U.S. Department of Justice June 2022. Summary of the FTC–DOJ workshop held June 14–15, 2022.
- Ghosh, Shikhar and Ramana Nanda**, “Venture capital investment in the clean energy sector,” *Harvard Business School Entrepreneurial Management Working Paper*, 2010, (11-020).
- Gompers, Paul**, “Optimal investment, monitoring, and the staging of venture capital,” *The journal of finance*, 1995, *50* (5), 1461–1489.
- Gompers, Paul A**, “The rise and fall of venture capital,” *Business and economic history*, 1994, pp. 1–26.
- **, “Grandstanding in the venture capital industry,”** *Journal of Financial economics*, 1996, *42* (1), 133–156.
- **and Josh Lerner**, “What Drives Venture Capital Fundraising?,” *Brookings Papers on Economic Activity*, 1998.
- **, Will Gornall, Steven N Kaplan, and Ilya A Strebulaev**, “How do venture capitalists make decisions?,” *Journal of Financial Economics*, 2020, *135* (1), 169–190.
- Gompers, Paul and Josh Lerner**, “What drives venture capital fundraising?,” 1999.
- González-Uribe, Juanita and William Mann**, “Venture debt as bridge financing,” *Working Paper*, 2024.
- Gornall, Will and Ilya A Strebulaev**, “The economic impact of venture capital: Evidence from public companies,” *Available at SSRN 2681841*, 2021.
- Gourieroux, Christian, Alain Monfort, and Eric Renault**, “Indirect inference,” *Journal of applied econometrics*, 1993, *8* (S1), S85–S118.

- Greenwood, Jeremy, Juan M Sanchez, and Cheng Wang**, “Financing development: The role of information costs,” *American Economic Review*, 2010, *100* (4), 1875–1891.
- , **Pengfei Han, and Juan M Sánchez**, “Financing ventures,” *International Economic Review*, 2022, *63* (3), 1021–1053.
- Hall, Bronwyn H and Josh Lerner**, “The financing of R&D and innovation,” in “Handbook of the Economics of Innovation,” Vol. 1, Elsevier, 2010, pp. 609–639.
- Hall, Robert E and Susan E Woodward**, “The burden of the nondiversifiable risk of entrepreneurship,” *American Economic Review*, 2010, *100* (3), 1163–1194.
- Hay, Michael, David W Thomas, John L Craighead, Celia Economides, and Jesse Rosenthal**, “Clinical development success rates for investigational drugs,” *Nature biotechnology*, 2014, *32* (1), 40–51.
- He, Zhiguo and Wei Xiong**, “Rollover risk and credit risk,” *The Journal of Finance*, 2012, *67* (2), 391–430.
- Hellmann, Thomas and Manju Puri**, “Venture capital and the professionalization of start-up firms: Empirical evidence,” *The journal of finance*, 2002, *57* (1), 169–197.
- and **Stelios Kavadias**, “Scale-up UK: Growing businesses, growing our economy,” *Project report. Barclays*, 2016.
- Hochberg, Yael V, Alexander Ljungqvist, and Yang Lu**, “Whom you know matters: Venture capital networks and investment performance,” *The Journal of Finance*, 2007, *62* (1), 251–301.
- Hosios, Arthur J**, “On the efficiency of matching and related models of search and unemployment,” *The Review of Economic Studies*, 1990, *57* (2), 279–298.
- Hsu, David H**, “What do entrepreneurs pay for venture capital affiliation?,” *The journal of finance*, 2004, *59* (4), 1805–1844.
- Hurst, Erik and Benjamin Wild Pugsley**, “What Do Small Businesses Do?,” *Brookings Papers on Economic Activity*, 2011, *42* (2 (Fall)), 73–142.
- Inderst, Roman and Holger M Müller**, “The effect of capital market characteristics on the value of start-up firms,” *Journal of Financial Economics*, 2004, *72* (2), 319–356.
- Jagannathan, Ravi, Shumiao Ouyang, and Jiaheng Yu**, “Return to Venture Capital in the Aggregate,” *Available at SSRN 4083525*, 2022.
- Janeway, William H**, *Doing capitalism in the innovation economy: Reconfiguring the three-player game between markets, speculators and the state*, Cambridge University Press, 2018.
- , **Ramana Nanda, and Matthew Rhodes-Kropf**, “Venture capital booms and start-up financing,” *Annual Review of Financial Economics*, 2021, *13*, 111–127.

- Jovanovic, Boyan and Bálaazs Szentes**, “On the market for venture capital,” *Journal of Political Economy*, 2013, 121 (3), 493–527.
- Kamepalli, Sai Krishna, Raghuram Rajan, and Luigi Zingales**, “Kill zone,” Technical Report, National Bureau of Economic Research 2020.
- Kaplan, Steven N and Antoinette Schoar**, “Private equity performance: Returns, persistence, and capital flows,” *The journal of finance*, 2005, 60 (4), 1791–1823.
- **and Per Strömberg**, “Venture capitalists as principals: Contracting, screening, and monitoring,” *American Economic Review*, 2001, 91 (2), 426–430.
- **and —**, “Financial contracting theory meets the real world: An empirical analysis of venture capital contracts,” *The review of economic studies*, 2003, 70 (2), 281–315.
- Kerr, William R, Ramana Nanda, and Matthew Rhodes-Kropf**, “Entrepreneurship as experimentation,” *Journal of Economic Perspectives*, 2014, 28 (3), 25–48.
- King, Robert G and Ross Levine**, “Finance and growth: Schumpeter might be right,” *The quarterly journal of economics*, 1993, 108 (3), 717–737.
- Kortum, Samuel and Josh Lerner**, “Assessing the Contribution of Venture Capital to Innovation,” *The RAND Journal of Economics*, 2000, 31 (4), 674–692.
- Lerner, Josh**, “Comment on Bergemann and Hege,” *Journal of Banking & Finance*, 1998, 22 (6-8), 736–740.
- **and Antoinette Schoar**, “Does legal enforcement affect financial transactions? The contractual channel in private equity,” *The Quarterly Journal of Economics*, 2005, 120 (1), 223–246.
- **and Ramana Nanda**, “Venture capital’s role in financing innovation: What we know and how much we still need to learn,” *Journal of Economic Perspectives*, 2020, 34 (3), 237–261.
- Li, Yingxiang**, “Liquidity shocks and private equity investment,” *Available at SSRN 4618348*, 2024.
- Masclans, Roger**, “Science, Startups, and the Problem of Value Capture: Thin Acquisition Markets, Weak Outside Options,” *Working paper*, 2025.
- Maurin, Vincent, David T Robinson, and Per Strömberg**, “A theory of liquidity in private equity,” *Management science*, 2023, 69 (10), 5740–5771.
- Mayer, Simon**, “Financing breakthroughs under failure risk,” *Journal of Financial Economics*, 2022, 144 (3), 807–848.
- Michelacci, Claudio and Javier Suarez**, “Business creation and the stock market,” *The Review of Economic Studies*, 2004, 71 (2), 459–481.

- Myers, Stewart C**, “Determinants of corporate borrowing,” *Journal of financial economics*, 1977, 5 (2), 147–175.
- Nanda, Ramana and Matthew Rhodes-Kropf**, “Investment cycles and startup innovation,” *Journal of financial economics*, 2013, 110 (2), 403–418.
- **and** —, “Financing entrepreneurial experimentation,” *Innovation Policy and the Economy*, 2016, 16 (1), 1–23.
- **and** —, “Financing risk and innovation,” *Management Science*, 2017, 63 (4), 901–918.
- **and** —, “Coordination frictions in venture capital syndicates,” Technical Report, National Bureau of Economic Research 2018.
- , **Ken Younger, and Lee Fleming**, “Innovation and entrepreneurship in renewable energy,” in “The changing frontier: Rethinking science and innovation policy,” University of Chicago Press, 2014, pp. 199–232.
- Narain, Namrata**, “How Patient Is Venture Capital?,” *Working Paper*, 2024.
- Neher, Darwin V**, “Staged financing: an agency perspective,” *The Review of Economic Studies*, 1999, 66 (2), 255–274.
- Nindl, E., L. Napolitano, H. Confraria, F. Rentocchini, P. Fako, J. Gavigan, and A. Tuebke**, “The 2024 EU Industrial R&D Investment Scoreboard,” Technical Report JRC140129, Publications Office of the European Union, Luxembourg 2024.
- Opp, Christian**, “Venture capital and the macroeconomy,” *The Review of Financial Studies*, 2019, 32 (11), 4387–4446.
- PitchBook**, “UK Market Snapshot: Q1 2025,” Technical Report, PitchBook Data, Inc. April 2025.
- Pitchbook–NVCA Yearbook**, “NVCA Yearbook 2024,” 2024.
- Rajan, Raghuram and Luigi Zingales**, “Financial development and growth,” *American economic review*, 1998, 88 (3), 559–586.
- Rajan, Raghuram G**, “Insiders and outsiders: The choice between informed and arm’s-length debt,” *The Journal of finance*, 1992, 47 (4), 1367–1400.
- Reenen, John Van, Sabrina Howell, Jun Wong, and Jason Rathje**, “Opening Up Military Innovation: Causal Effects of ‘Bottom-Up’ Reforms to US Defense Research,” 2021.
- Rin, Marco Da, Thomas Hellmann, and Manju Puri**, “A survey of venture capital research,” in “Handbook of the Economics of Finance,” Vol. 2, Elsevier, 2013, pp. 573–648.
- Robinson, David T and Berk A Sensoy**, “Cyclicality, performance measurement, and cash flow liquidity in private equity,” *Journal of Financial Economics*, 2016, 122 (3), 521–543.

- Rosen, Maya Haran, Alexander Montag, Jonathan Zandberg, and Osnat Zohar**, “The Lifecycle of Venture Capital Funds,” *Working paper*, 2025.
- Sahlman, William A**, “The structure and governance of venture-capital organizations,” *Journal of financial economics*, 1990, *27* (2), 473–521.
- Samila, Sampsa and Olav Sorenson**, “Venture capital, entrepreneurship, and economic growth,” *The Review of Economics and Statistics*, 2011, *93* (1), 338–349.
- Sannino, Francesco**, “The Equilibrium Size and Value-Added of Venture Capital,” *The Journal of Finance*, 2024, *79* (2), 1297–1352.
- Sharpe, Steven A**, “Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships,” *The journal of finance*, 1990, *45* (4), 1069–1087.
- Silveira, Rafael and Randall Wright**, “Venture capital: A model of search and bargaining,” *Review of Economic Dynamics*, 2016, *19*, 232–246.
- Sørensen, Morten**, “How smart is smart money? A two-sided matching model of venture capital,” *The Journal of Finance*, 2007, *62* (6), 2725–2762.
- Strebulaev, Ilya A and Toni M Whited**, “Dynamic models and structural estimation in corporate finance,” *Foundations and Trends® in Finance*, 2012, *6* (1–2), 1–163.
- Tian, Xuan**, “The causes and consequences of venture capital stage financing,” *Journal of Financial Economics*, 2011, *101* (1), 132–159.
- Townsend, Richard R**, “Propagation of financial shocks: The case of venture capital,” *Management Science*, 2015, *61* (11), 2782–2802.
- Wang, Xinxin**, “Catering innovation: Entrepreneurship and the acquisition market,” *Kenan Institute of Private Enterprise Research Paper*, 2018, (18-27).
- Weil, Gotshal & Manges LLP**, “Private Equity Alert: Fear and Greed,” Client newsletter.
- Wilson, K**, “Policy Lessons from Financing Innovative Firms,” Technical Report 24, OECD Publishing, Paris 2015.
- Wilson, Nick, Mike Wright, and Marek Kacer**, “The equity gap and knowledge-based firms,” *Journal of Corporate Finance*, 2018, *50*, 626–649.

Appendix

A Theory appendix

A.1 The contract

Consider the environment of section 2.2. As discussed in the main text, the entrepreneur (E) will never report negative results. The contract in the body of the paper makes assumptions such that the E always reports positive results. Specifically, the private benefit is taken to zero in the limit, $x_b \rightarrow 0$. The proposition below demonstrates that this is sufficient for the E to always report positive results and never report negative results.

Proposition A.1. *For any set of parameters $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$, there exists an $\bar{x}_b > 0$ such that if $x_b < \bar{x}_b$, then the E reports positive results but does not report negative results.*

Before stating the proof, I give a brief outline of the argument. In part 1, I derive the payoffs to the E given a contract (ς, ω) under two cases: (i) the E never reports results and (ii) the E reports only positive results. I show that given (ς, ω) , the E always prefers the payoff associated with case (ii). In part 2, I derive a condition under which case (ii) is not incentive compatible; that is, a condition under which a commitment to report positive results is not credible. In part 3, I demonstrate that for any set of parameters, case (ii) can always be made incentive compatible by choosing a suitably small non-pecuniary payoff to the E, x_b . Part 4 concludes the proof.

Proof of Proposition A.1.

Part 1. Without loss of generality, assume $x_b < k$.¹³⁰ The E never reports negative results, since $x_b > 0$. Suppose a positive result arrives at time t and call the decision to report a positive result $\chi = 1$, where $\chi = 0$ reflects the choice not to report a positive result. $\chi = 0$ implies that the payoff, π , from success is delayed until time $t + T_\omega$, for $T_\omega \sim \text{Exp}(\omega)$, so has time- t expected value $\frac{\omega}{\rho + \omega} \pi$.¹³¹ If $\chi = 0$, the contract ends at date T_ω , regardless of the result.

The value of the firm in development, V_d , is determined analogously to equation (6)

$$\rho V_d = \begin{cases} \kappa [p \pi - V_d] + \omega [V_s - V_d] & \text{if } \chi = 1 \\ \kappa [p \pi \frac{\omega}{\rho + \omega} - V_d] + \omega [V_s - V_d] & \text{if } \chi = 0 \end{cases} \quad (\text{A.1})$$

Denote by $V_d^{np}(\chi)$ the non-pecuniary value of the contract to the E. The value to the E of contract (ς, ω) is $V_E(\omega, \varsigma, \chi) = (1 - \varsigma) V_d(\omega, \chi) + V_d^{np}(\omega, \chi)$, where

$$V_d^{np} = \begin{cases} \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{x_b}{\rho + \kappa + \omega} & \text{if } \chi = 1 \\ \frac{x_b}{\rho + \omega} & \text{if } \chi = 0 \end{cases} \quad (\text{A.2})$$

¹³⁰This assumption is not restrictive since the proof attempts only to obtain an upper bound on x_b .

¹³¹ $E[e^{-\rho T_\omega} \times \pi] = \frac{\omega}{\rho + \omega} \pi$, where expectations are taken over T_ω .

where the expression for $V_d^{np}(\omega, \chi)$ is analogous to $K(\omega, \chi = 1)$ in equation (4). Similarly, $K(\omega, \chi = 0) = \frac{k}{\rho + \omega}$ because the contract ends at time T_ω regardless of any potential result. Since the VC has no bargaining power, their participation constraint binds, $\varsigma V_d(\omega, \chi) = K(\omega, \chi)$. The value of the contract to the E is then $V_E = (1 - \varsigma)V_d + V_d^{np} = V_d + V_d^{np} - K(\omega)$, or

$$V_E(\omega, \chi) = \begin{cases} \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k - x_b}{(\rho + \kappa + \omega)} & \text{for } \chi = 1 \\ \frac{\kappa p \pi \frac{\omega}{\rho + \omega} + \omega V_s}{\rho + \kappa + \omega} - \frac{k - x_b}{\rho + \omega} & \text{for } \chi = 0 \end{cases} \quad (\text{A.3})$$

This leads to the following remark, which states that the E prefers ex-ante the contract in which they report positive results to the VC. Intuitively, given $x_b < k$, the cost of compensating the VC exceeds the private benefit they attain from continuation, so they would prefer to commit to stop development at time T_κ .

Remark A.1. Given $0 < x_b < k$ and for any contract (ω, ς) , $V_E(\omega, \chi = 1) > V_E(\omega, \chi = 0)$.

Part 2. The benefit to the E of delay, $\chi = 0$, upon observing a positive result is $\frac{x_b}{\rho + \omega}$. The cost to the E is to delay the pecuniary payoff, of which they obtain share $1 - \varsigma$. The delay therefore costs the E $E[(1 - e^{-\rho T_\omega})(1 - \varsigma)\pi] = \frac{\rho}{\rho + \omega}(1 - \varsigma)\pi$, where expectations are taken over T_ω .¹³² This leads to the following incentive compatibility constraint, which states that the E's share of the pecuniary payoff must be sufficiently large for them to optimally report positive results.

Remark A.2. $\chi = 1$ is incentive compatible iff $1 - \varsigma \geq \frac{x_b}{\rho \pi}$.

Part 3. Consider the contract problem when the E commits to report positive results, $\chi = 1$

$$\begin{aligned} & \sup_{\{\omega \in [0, \infty), \varsigma \in [0, 1]\}} \left\{ \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k - x_b}{(\rho + \kappa + \omega)} \right\} \\ \text{s.t.} \quad & 1 - \varsigma \geq \frac{x_b}{\rho \pi}, \quad \varsigma \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} = \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k}{(\rho + \kappa + \omega)} \end{aligned}$$

where V_s is taken as given and the IC constraint from Remark A.2 is imposed. I begin by solving this problem without consideration for the incentive compatibility and feasibility constraint for ς ; thereafter, I show that x_b can be chosen such that they are slack.

Define $\bar{V}_s = \frac{\kappa p \pi - (k - x_b)}{\rho + \kappa}$. For $V_s \geq \bar{V}_s$, the objective function is everywhere increasing in $\omega \in [0, \infty)$ given $x_b < k$, so the solution is $\omega \rightarrow \infty$. Next, restrict attention to $V_s \in [0, \bar{V}_s]$. The first-order condition for an interior optimum is

$$\frac{(k - x_b)(1 - p)}{(\rho + \omega)^2} + \frac{(k - x_b - \kappa \pi)p + V_s(\rho + \kappa)}{(\rho + \kappa + \omega)^2} = 0$$

which delivers two candidate solutions for ω

$$\omega = \frac{\rho \kappa p \pi - \rho V_s(\rho + \kappa) - (k - x_b)(\kappa(1 - p) + \rho) \pm \sqrt{\kappa^2(k - x_b)(1 - p)(\kappa p \pi - p(k - x_b) - V_s(\rho + \kappa))}}{k - x_b - \kappa p \pi + V_s(\rho + \kappa)}$$

¹³²Alternatively, one could consider the incentives to delay/report over some finite interval Δt . The environment is stationary, so these two approaches are equivalent.

The ‘positive’ solution is strictly negative on $V_s \in [0, \bar{V}_s)$. Define $\bar{p} = \frac{(k-x_b)(\rho+\kappa)^2}{\kappa(\pi\rho^2+(k-x_b)(\kappa+2\rho))}$ and $\underline{V}_s = \frac{\kappa p\pi - p(k-x_b)}{\rho+\kappa} - \frac{(k-x_b)(1-p)(\rho+\kappa)}{\rho^2}$. When $p \leq \bar{p}$, the ‘negative’ solution is positive for $V_s \in [0, \bar{V}_s)$; when $p > \bar{p}$, it is positive for $V_s \in [\underline{V}_s, \bar{V}_s)$. The SOC is satisfied in both cases. Conversely, when $p > \bar{p}$, the solution is negative for $V_s \in [0, \underline{V}_s)$. In this case, the optimand is decreasing on $\omega \in [0, \infty)$. The solution is then $\omega = 0$.

With candidate solutions in hand, it remains to check the feasibility and incentive compatibility constraints. To this end, note the following lemma.

Lemma A.1. *If there are parameters such that $\varsigma \in (0, 1)$ (i.e. strictly), then $\exists \bar{x}_b$ such that $\forall x_b \in [0, \bar{x}_e)$, $1 - \varsigma \geq \frac{x_b}{\rho\pi}$.*

Proof of Lemma A.1. The PC gives $\varsigma(\omega) = \frac{k(\kappa(1-p)+\rho+\omega)}{(\rho+\omega)(\kappa p\pi+\omega V_s)}$, so that $\varsigma'(\omega) < 0$ on $V_s \in [0, \bar{V}_s)$. Furthermore, from the solution to the contract problem, $\omega'(x_b) \leq 0$, with strict inequality when $\omega > 0$ is optimal. Together, $\varsigma'(x_b) \geq 0$. Then, suppose the parameters are such that $\varsigma \in (0, 1)$ and consider reducing $x_b \rightarrow 0$. The E’s share, $(1 - \varsigma)$, increases weakly from a positive value, whereas $x_b/(\rho\pi) \rightarrow 0$. Therefore, there exists a region $[0, \bar{x}_b)$ such that the IC is satisfied, i.e. $1 - \varsigma \geq \frac{x_b}{\rho\pi}$.

The remainder of the proof provides sufficient conditions on x_b such that Lemma A.1 can be applied. Suppose first that $\omega = 0$ is optimal. Then $\varsigma(0) = \frac{k(\kappa(1-p)+\rho)}{\kappa p\pi\rho}$. Clearly, $\varsigma(0) > 0$ since $p \in (0, 1)$. Note that $\varsigma(0) < 1 \iff \kappa p\pi\rho > k(\kappa(1-p) + \rho)$. Furthermore, if $\omega = 0$ is optimal, then (from above) $p > \bar{p} \implies p\kappa\pi\rho > \rho^{-1}((k-x_b)(\rho+\kappa)^2 - p\kappa(k-x_b)(\kappa+2\rho))$. Combining these inequalities, a sufficient condition for $\varsigma(0) < 1$ is $\rho^{-1}((k-x_b)(\rho+\kappa)^2 - p\kappa(k-x_b)(\kappa+2\rho)) > k(\kappa(1-p) + \rho)$. For $x_b < \frac{\kappa(1-p)(\rho+\kappa)}{\rho^2+(1-p)(\kappa^2+2\rho\kappa)}$, this condition is satisfied, so $\varsigma(0) \in (0, 1)$. Then, by Lemma A.1, it is possible to choose a positive x_b sufficiently small such that the IC is satisfied.

Suppose next that $\omega \in (0, \infty)$ is optimal. Note $\varsigma(\omega) > 0 \forall \omega \in (0, \infty)$. It remains to check $\varsigma(\omega) < 1$, which is equivalent to requiring that the pecuniary return to the E is positive, $(1 - \varsigma) V_d(\omega, \chi = 1) = V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0$. Evaluating this expression

$$\begin{aligned} V_d(\chi = 1) - \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k}{(\rho + \kappa + \omega)} \Big|_{\omega = \omega^* \in (0, \infty)} = \\ p\pi - \frac{1}{\kappa^2} \left[\frac{(2k - x_b) \sqrt{\kappa^2(k - x_b)(1-p)(p\kappa\pi - p(k - x_b) - V_s(\rho + \kappa))}}{k - x_b} \right. \\ \left. + \kappa \left(2kp - k + \rho V_s - \frac{(1-p)p(k - x_b)x_b\kappa}{\sqrt{\kappa^2(k - x_b)(1-p)(p\kappa\pi - p(k - x_b) - V_s(\rho + \kappa))}} \right) \right] \end{aligned}$$

This expression is decreasing in x_b on $x_b > 0$ given $V_s < \frac{\kappa p\pi - p(k-x_b)}{\rho+\kappa}$, which is satisfied whenever $V_s < \bar{V}_s$ since $p \in (0, 1)$. Therefore, if $V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0$ at $x_b = 0$, it is positive over some region $x_b \in [0, \bar{x}_b)$ where $\bar{x}_b > 0$. Evaluating $V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0$ at $x_b = 0$ obtains the condition

$$p\pi - \frac{1}{\kappa} \left(k(2p - 1) + \rho V_s + 2\sqrt{k(1-p)(\kappa p\pi - kp - V_s(\rho + \kappa))} \right) > 0$$

which is equivalent to

$$\kappa p\pi - k(2p - 1) - \rho V_s > 2\sqrt{k(1 - p)(\kappa p\pi - kp - V_s(\rho + \kappa))}$$

Note that the left-hand side is positive if $V_s < \frac{\kappa p\pi - k}{\rho} + \frac{2k(1-p)}{\rho}$, which is valid on $V_s \leq \bar{V}_s$ when $x_b = 0$. Therefore, squaring both sides, subtracting the right-hand side from the left-hand side and some algebra obtains

$$4kV_s(1 - p)\kappa + (\kappa p\pi - k - \rho V_s)^2 > 0$$

which is satisfied given the initial restrictions on parameters (i.e. positivity constraints and $p \in (0, 1)$). This implies that the E obtains a positive pecuniary payoff when $x_b = 0$ and, by extension, over some region $x_b \in [0, \bar{x}_b)$. This in turn implies the conditions to apply Lemma A.1 are satisfied and there exists a (potentially different) upper bound \bar{x}_e such that $\forall x \in [0, \bar{x}_e)$, the IC is slack.

Suppose finally that $\omega \rightarrow \infty$ is optimal. Then $\lim_{\omega \rightarrow \infty} \varsigma(\omega) = 0$ and so satisfying the IC requires only that $x_b \leq \rho\pi$, which is easily satisfied.

Part 4. By Remark A.1, the E prefers to payoff associated with commitment for any contract (ω, ς) . Furthermore, for any contract (ω, ς) and parameters $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$, it is possible to find a strictly positive upper bound on x_b such that the feasibility constraint is slack, i.e. $\varsigma \in (0, 1)$, if x_b is less than this upper bound. Then, by Lemma A.1, there is a (potentially different) upper bound on x_b such that the IC constraint is also slack if x_b does not exceed this upper bound. Together, there is an interval $[0, \bar{x}_b)$ such that both the feasibility and incentive compatibility constraints are slack at the optimal contract in which $\chi = 1$. Since the contract with $\chi = 1$ is incentive compatible and for any contract (ω, ς) $\chi = 1$ is preferred ex ante, the optimal contract involves $\chi = 1$ on this interval $x_b \in [0, \bar{x}_e)$. Furthermore, the expected capital cost is given by $K(\omega, \chi = 1)$ and the optimal contract rate is determined as in part 3. ■

A.2 VC bargaining power

In this section, I generalise the baseline model to allow for VC bargaining power. VC bargaining power influences the choice of contract, (ς, ω) , in two ways. First, current investors face a hold-up problem vis-à-vis future investors because future investors extract surplus in their own negotiations with the start-up. Therefore, VCs see their shareholding in the firm facing greater dilution in future funding rounds relative to the setting where VCs have no bargaining power. The implication is that VCs have an incentive to provide greater funding upfront to reduce the need for the firm to return to the capital market. Second, a VC's outside option in the case that they choose not to invest in a start-up now has positive value. Investing in a given start-up involves forgoing this outside option and, therefore, incurs an opportunity cost that raises the effective flow cost of investing in a start-up above k .¹³³ This second effect has

¹³³A similar effect appears in Michelacci and Suarez (2004): VCs may facilitate an early-stage IPO to “recycle” their capital into new ventures. In my model, the same motive leads to earlier termination by inducing a smaller

the opposite implication to the first, pushing instead in the direction of a higher contract rate. In the remainder of this section, I set up the contracting problem when the VC has bargaining power and consider these effects more formally.

The VC It is now necessary to derive the value of a VC that is searching for investment opportunities or funding a start-up. Consider the value of a VC that is funding a productive start-up, which I denote by U_d^p . The VC incurs capital flow cost k and receives a share ς of the firm. At rate κ , the start-up realises a result, which is a success with probability p and returns the VC their share of the payoff π and the value of search for a VC, U_s , since the VC can now seek out new investment opportunities. However, with probability $1 - p$, the start-up obtains a negative result and so the VC gets the value of funding an unproductive start-up, U_p^u . Finally, at rate ω , the contract expires and the VC obtains their share in the company, a value ςV_s , and returns to search. With this in mind, the HJB for a VC funding a productive start-up is given by

$$\rho U_d^p = -k + \kappa[p(\varsigma \pi + U_s) + (1 - p)U_d^u - U_d^p] + \omega[\varsigma V_s + U_s - U_d^p] \quad (\text{A.4})$$

Consider next the value of funding an unproductive start-up, U_p^u . The VC must continue to pay the flow capital cost k because they are unaware that the start-up is unproductive. In addition, at rate ω , the commitment expires and the VC returns to search. The HJB for a VC funding an unproductive start-up is then given by

$$\rho U_p^u = -k + \omega[U_s - U_p^u] \quad (\text{A.5})$$

Finally, the value of search for a VC depends on the rate at which they meet with start-ups, which I denote by v . At this rate the VC meets with a start-up and obtains the value of funding an productive start-up, U_d^p . The HJB for a VC in search is given by

$$\rho U_s = v[U_d^p - U_s] \quad (\text{A.6})$$

The contract. Upon meeting, the VC and entrepreneur engage in Nash bargaining to determine the contract rate ω and share ς obtained by the VC. The value of the match to the VC is U_d^p and the value of their outside option is U_s . The value of the match to existing shareholders is $(1 - \varsigma)V_d$, because they retain a share $(1 - \varsigma)$ of the gross value of the firm, and the value of their outside option is V_s . If the entrepreneur has bargaining weight δ in the negotiations, then the optimal contract solves

$$\max_{\{\varsigma \in [0,1], \omega_i \in [0,\infty)\}} \left\{ ((1 - \varsigma)V_d - V_s)^\delta (U_d^p - U_s)^{1-\delta} \right\}$$

where V_s and U_s are taken as given in the optimisation. The first-order condition for the equity up-front commitment, after which the start-up must raise additional capital.

stake, ς , yields the following conditions

$$(1 - \varsigma)V_d = V_s + \delta S \quad (\text{A.7})$$

$$U_d^p = U_s + (1 - \delta)S \quad (\text{A.8})$$

where $S = (1 - \varsigma)V_d + U_d^p - V_s - U_s$ is the surplus of the contract. As is standard in these settings, the surplus is split according to the Nash bargaining weights. Solving equations (A.4), (A.5) and (A.6) for U_d^p in terms of U_s allows for writing the surplus as

$$S = V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1 - p)}{\rho + \omega} \right) \quad (\text{A.9})$$

Since the surplus is independent of the share that goes to the VC, the contract rate simply maximises the joint surplus, $\omega = \arg \max_{\omega \in [0, \infty)} \{S\}$. The first-order condition for ω is then

$$V_d'(\omega) = -(k + \rho U_s) \left(\frac{1 - p}{(\rho + \omega)^2} + \frac{p}{(\rho + \kappa + \omega)^2} \right) \quad (\text{A.10})$$

The first-order condition for the optimal contract rate is identical to the case without bargaining power, except that k has been replaced by $k + \rho U_s$. Intuitively, every instant that the VC pays for the investment, they incur capital cost k and *opportunity cost* $\rho \times U_s$, because they cannot engage in other investment opportunities. This induces shorter duration capital commitments. However, in equilibrium, V_s will also adjust due to hold-up vis-a-vis future investors.

As I show in the proof to Proposition A.3, the optimal contract rate when there is bargaining power is

$$\omega = \max \left\{ 0, -\rho + \mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} \right\}, \quad (\text{A.11})$$

where

$$\mathcal{C} = \frac{\kappa(1 - p)k}{\kappa p \pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \delta \nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta) \nu \frac{\kappa p \pi}{k}$$

Clearly, setting $\delta = 1$ recovers equation (11). However, more generally, the bargaining weight has two implications. On the one hand, it affects the collective forward-looking incentives of the current E and VC. Once a VC has invested, it becomes an existing shareholder and has bargaining power δ vis-à-vis future investors. Therefore, δ reflects the size of the potential hold-up problem that the E and current VC face in future investments, which induces dilution. As δ increases, the E and VC worry less about future dilution and so reduce the size of the current capital injection.¹³⁴ Conversely, the bargaining weight affects how the VC prioritises the current investment vis-à-vis other potential investment opportunities. As δ increases, the VCs bargaining power falls in any new investment, which lowers their opportunity cost of funds. This leads them to extend more funding to the current entrepreneur.

¹³⁴Note that from the perspective of the current investment, the need to raise follow-on funding and the associated hold-up acts analogously to financing risk: in the limit where $\delta \rightarrow 0$, current investors would be indifferent between meeting with a new investor and failing due to a lack of follow-on funding.

Comparative statics Here, I provide comparative statics for the optimal contract rate in the model with VC bargaining power.

Proposition A.2. *The optimal contract rate, ω , is: strictly increasing in v , ν and k ; strictly decreasing in p , π , λ ; and ambiguous for κ and δ . The QCCS for p is strictly negative and for κ is strictly positive.*

Comparative statics are preserved for all parameters considered in the $\delta = 1$ case. In addition, the contract rate is increasing in v , the meeting rate for VCs; this reflects the opportunity cost motive. Furthermore, the effect of bargaining power on the optimal contract rate is ambiguous. In fact,

$$\frac{\partial \omega}{\partial \delta} > 0 \iff \theta < \frac{k}{\kappa p \pi} \frac{\rho + \kappa}{\rho + \lambda}$$

Whether increases in the bargaining power of the entrepreneur, δ , increase the contract rate, ω , depends on market tightness, $\theta = \mu_s/\mu_{vc} = v/\nu$. When the market is tight – start-ups are numerous relative to VCs, θ is high – the value of the VC’s outside option is high, which strengthens their motive to keep funding commitments short. In this case, marginal changes to their bargaining power have large effects on their opportunity cost of funds and, therefore, reductions in their bargaining power induce larger capital commitments, $\partial \omega / \partial \delta < 0$. Otherwise, increases in the bargaining power of insiders, δ , allows them to worry less about future dilution, and lead to shorter commitments, $\partial \omega / \partial \delta > 0$.

A.3 Proofs for main results

A.3.1 Existence and uniqueness

In this section, I show that for given meeting rates in the financing market, the value functions and contract are unique. I do so in the general case in which the VC has bargaining power, as described in section A.2. The roof of Proposition 2.1 then follows by setting the VC’s bargaining power to zero.

Proposition A.3. *Given ν and v and for $\kappa p \pi > k$, there is a unique solution for $\{V_d, V_s, U_d^p, U_d^u, U_s, S, \omega, \varsigma\}$ in terms of the model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, $\omega \in [0, \infty)$.*

Proof of Proposition A.3. Given ν, v , an equilibrium is a solution for $\{V_d, V_s, U_d^p, U_d^u, U_s, S, \omega, \varsigma\}$ to the following set of equations

$$\rho V_d = \kappa [p \pi - V_d] + \omega [V_s - V_d] \quad (\text{A.12})$$

$$(\rho + \lambda)V_s = \nu[(1 - \varsigma)V_d - V_s] \quad (\text{A.13})$$

$$\rho U_d^p = -k + \kappa[p(\varsigma \pi + U_s) + (1 - p)U_d^u - U_d^p] + \omega[\varsigma V_s + U_s - U_d^p] \quad (\text{A.14})$$

$$\rho U_d^u = -k + \omega[U_s - U_d^u] \quad (\text{A.15})$$

$$\rho U_s = v[U_d^p - U_s] \quad (\text{A.16})$$

$$S = V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1 - p)}{\rho + \omega} \right) \quad (\text{A.17})$$

$$(1 - \varsigma)V_d = V_s + \delta S \quad (\text{A.18})$$

$$U_d^p = U_s + (1 - \delta)S \quad (\text{A.19})$$

$$\omega = \arg \max_{\omega \in [0, \infty]} \{S\} \quad (\text{A.20})$$

The first two equations come from the start-up's problem, where I have substituted $V^M = (1 - \varsigma)V_d$. The next three equations come from the VC's problem, as set out in section A.2 of the Appendix. The final four equations relate to the optimal contract: they define, respectively, the surplus, the optimality conditions for the equity share in the contract, and the optimal contract rate.¹³⁵

Towards a proof, note that given (ω, S, ν, v) , all remaining equilibrium objects are pinned down uniquely. Specifically, combining equations (A.13) and (A.18), $(\rho + \lambda)V_s = \nu\delta S$, so V_s is known given (ν, S) . V_d then follows immediately from equation (A.12) given (ω, V_s) . The equity stake then follows from equation (A.18). Finally, the VC value functions can then be recovered from equations (A.14), (A.15) and (A.16), given knowledge of $(\omega, v, \varsigma, V_s)$.

In light of this, to show that the equilibrium, given (ν, v) , is unique, it is sufficient to demonstrate that there is a unique (ω, S) consistent with the equilibrium conditions. To this end, rather than taking the first-order condition as a function of V_s and U_s , as in equation (A.10), and *then* solving the resulting system of equations in (ω, V_s, U_s) , it is simpler to first eliminate the search values from the optimisation problem and then solve for the optimal contract. Although the contract is derived taking as given the search values, (V_s, U_s) , this approach is valid because, in the background, holding the search values constant is equivalent to preventing agents from internalising the effect of the contract choice, ω , on the equilibrium meeting rates, ν and v . Towards an expression for S , combining the above expressions allows to solve for V_s in terms of

¹³⁵There are more equations than unknowns, but technically one of the equations (A.18) or (A.19) is redundant.

model parameters and the equilibrium contract rate, ω

$$\begin{aligned}
(\rho + \lambda)V_s &= \nu\delta S \\
&= \nu\delta \left[V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right] \\
&= \nu\delta \left[\frac{\kappa p\pi + \omega V_s}{\rho + \kappa + \omega} - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right] \\
&= \nu\delta \left[\frac{\kappa p\pi + \omega V_s}{\rho + \kappa + \omega} - V_s - \frac{k + v(1-\delta)\frac{\rho+\lambda}{\nu\delta}V_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right]
\end{aligned}$$

where the first line follow from combining equations (A.13) and (A.18), the second line uses the definition of S , the third line substitutes for V_d from equation (A.12), and the fourth line uses the relationship between U_s and V_s , which comes from writing these values both in terms of the surplus and then eliminating the surplus from the resulting equations.¹³⁶ Then, solving for V_s and using $\nu\delta S = (\rho + \lambda)V_s$ yields an expression for S in terms of model parameters and the contract rate, ω

$$S(\omega) = \frac{(\rho + \lambda) \left[\kappa p\pi - \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) k \right]}{(1 - \delta)v(\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) - \nu(1 - \delta)(\rho + \kappa) + (\rho + \kappa)(\rho + \lambda + \nu) + (\rho + \lambda)\omega} \quad (\text{A.21})$$

The optimal contract rate maximises $S(\omega)$, taking the meeting rates ν and v as given. Differentiating with respect to ω obtains

$$\begin{aligned}
S'(\omega) &= \frac{1}{\frac{1}{\rho + \lambda}(\rho + \omega)^2 \left((\rho + \kappa)(\delta\nu + \lambda + \rho) + \omega(\rho + \lambda) + (1 - \delta)v(\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right)^2} \\
&\times \left((\rho + \lambda)(k - \kappa p\pi)(\rho + \omega)^2 + 2\kappa k(1 - p)(\rho + \lambda)(\rho + \omega) \right. \\
&\quad \left. + k(\kappa^2(1 - p)(\rho + \lambda + \delta\nu) + \delta\kappa\nu(1 - p)\rho) + (1 - \delta)\kappa^2(1 - p)p\pi v(\rho + \lambda) \right)
\end{aligned}$$

The denominator of $S'(\omega)$ is strictly positive, so it is sufficient to focus on the numerator. Note firstly that the numerator is a quadratic function of ω . There are two cases to consider. Suppose firstly that $k \geq \kappa p\pi$, in which case $S'(\omega) > 0$ on $\omega \in [0, \infty)$. In this case, the unique solution is $(\omega \rightarrow \infty, S = 0)$. Next, suppose that $k < \kappa p\pi$, in which case the numerator is quadratic and concave in ω , and so may omit an interior solution. Setting $S'(\omega) = 0$ and solving for ω obtains

$$\omega = -\rho + \mathcal{C} \pm \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}$$

where

$$\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p\pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \delta\nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v \frac{\kappa p\pi}{k}$$

Among the two solutions, it is simple to see that the ‘negative’ solution is strictly negative

¹³⁶Specifically, $\rho U_s = v(1 - \delta)S$ and $(\rho + \lambda)V_s = \nu\delta S$, so that $\rho U_s = (\rho + \lambda)\frac{1-\delta}{\delta}\frac{v}{\nu}V_s$

whenever $\kappa p\pi > k$.¹³⁷ Therefore, there can be at most one solution on $\omega \in [0, \infty)$. Specifically, since the sign of $S'(\omega)$ is pinned down by a quadratic function in ω that is concave, the greater of the two solutions is a maximum and so is the optimum when it is positive. However, if the greater of the two solution is negative, then the optimum must be at $\omega = 0$, because $S'(\omega)$ is negative on $\omega \in [0, \infty)$. Combining these insights, given ν and v , it is clear that there is a unique optimum characterised by a finite contract rate when $k < \kappa p\pi$ and so surplus. Given the preceding arguments, the equilibrium objects are uniquely determined. ■

Proof of Corollary 2.1. Again, I consider the general case with VC bargaining power. Solving the optimisation problem as described, the optimal contract rate is given by

$$\omega = \begin{cases} \max \left\{ 0, -\rho + \mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} \right\} & \text{if } k < \kappa p\pi \\ \infty & \text{else} \end{cases} \quad (\text{A.22})$$

where

$$\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p\pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \delta\nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v \frac{\kappa p\pi}{k}$$

To determine when the contract rate is set to zero, corresponding to upfront financing, consider when the max-operator is active. To this end, upfront financing occurs when any of the following conditions are met

$$p \in \left(\frac{k(\rho + \kappa)^2}{\kappa(\rho^2\pi + k(\kappa + 2\rho))}, 1 \right] \quad \text{and} \quad \begin{cases} v \geq \frac{1}{p\kappa^2\pi}\Psi & \text{and } \delta = 0, \quad \text{or} \\ \nu < \frac{(\rho + \lambda)}{k\kappa\delta(\rho + \kappa)}\Psi & \text{and } v \geq f(\nu) \quad \text{and } \delta \in (0, 1), \quad \text{or} \\ \nu \geq \frac{(\rho + \lambda)}{k\kappa(\rho + \kappa)}\Psi & \text{and } \delta = 1 \end{cases}$$

with

$$\Psi = \frac{1}{1-p}(\rho^2(\kappa p\pi - k) - k(1-p)\kappa^2 - 2k(1-p)\rho\kappa)$$

$$f(\nu) = \frac{\rho^2(\rho + \lambda)(\kappa p\pi - k) - k\kappa(1-p)[\kappa(\rho + \lambda + \delta\nu) + \rho(2\rho + 2\lambda + \delta\nu)]}{(\rho + \lambda)(1-p)(1-\delta)\kappa^2 p\pi}$$

with $\Psi > 0$ under the restriction on p and $f'(\nu) < 0$. The case given in the main body of the paper simply corresponds to the case of $\delta = 1$. ■

A.3.2 Comparative statics

Proof of Propositions 2.2 and A.2 I consider first the standard comparative statics, then the QCCS (see Definition 2.2). Consider the general case of $\delta \in [0, 1]$; Proposition 2.2 considers the special case, $\delta = 1$. Let $k < \kappa p\pi$ and consider the case of a positive contract rate, $\omega > 0$. Then,

$$\omega = -\rho + \mathcal{C}(z) + \sqrt{\mathcal{C}(z)\mathcal{F}(z) + \mathcal{C}(z)^2}$$

¹³⁷ $-\rho + \mathcal{C} - \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} < -\rho + \mathcal{C} - \sqrt{\mathcal{C}^2} = -\rho$, where the first inequality follows since $\mathcal{C} > 0$ given $k < \kappa p\pi$.

where $\mathcal{C}(z), \mathcal{F}(z)$ makes explicit the dependence on some parameter, z . Differentiating ω with respect to some generic parameter z obtains

$$\frac{\partial \omega}{\partial z} = \left(1 + \frac{2\mathcal{C}(z) + \mathcal{F}(z)}{2\sqrt{\mathcal{C}(z)\mathcal{F}(z) + \mathcal{C}(z)^2}} \right) \mathcal{C}'(z) + \frac{\mathcal{C}(z)}{2\sqrt{\mathcal{C}(z)\mathcal{F}(z) + \mathcal{C}(z)^2}} \mathcal{F}'(z)$$

Now, consider each parameter in turn. For $z = v, \nu, \lambda, \delta$, $\mathcal{C}'(z) = 0 \implies \text{sign}(\frac{\partial \omega}{\partial z}) = \text{sign}(\mathcal{F}'(z))$. Then

- $\mathcal{F}'(v) \geq 0$, with strict equality for $\delta < 1$.
- $\mathcal{F}'(\nu) \geq 0$, with strict equality for $\delta > 0$.
- $\mathcal{F}'(\lambda) \leq 0$, with strict equality for $\delta > 0$.
- $\mathcal{F}'(\delta) = \nu \frac{\rho + \kappa}{\rho + \lambda} - v \frac{\kappa p \pi}{k}$, which has ambiguous sign.

For $z = p, \pi$, $\mathcal{C}'(z) < 0$ (given $k < \kappa p \pi$) but $\mathcal{F}'(z) > 0$. Nevertheless, given $k < \kappa p \pi$, we have $\frac{\partial \omega}{\partial z} < 0$. For example, in the case of $z = \pi$

$$\frac{\partial \omega}{\partial \pi} = -\frac{1}{\kappa p \pi - k} \left(\kappa p \mathcal{C} + \mathcal{C} \frac{\kappa p \left(\kappa + \delta \nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v + 2\mathcal{C} \right)}{2\sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}} \right)$$

which is negative for $k < \kappa p \pi$. Analogous arguments show that $\frac{\partial \omega}{\partial k} > 0$.

For $z = \kappa$, the expression is ambiguous. Indeed, $\mathcal{C}'(\kappa) < 0$, tends to negative infinity as κ approaches its lower bound, $k/(p\pi)$, and tends to zero as $\kappa \rightarrow \infty$. Conversely, \mathcal{F} is increasing and linear in κ . Therefore, for small κ , \mathcal{C} drives the behaviour and for large κ , \mathcal{F} drives the behaviour. More formally, note that $\frac{\partial \omega}{\partial \kappa} > 0$ can be written

$$\mathcal{F}'(\kappa) > -\mathcal{C}'(\kappa) \left(2 + 2\sqrt{1 + \frac{\mathcal{F}(\kappa)}{\mathcal{C}(\kappa)}} + \frac{\mathcal{F}(\kappa)}{\mathcal{C}(\kappa)} \right)$$

The left-hand side is a constant. Given, $k < \kappa p \pi$, the right-hand side is strictly decreasing in κ . Therefore, there can be at most one solution to $\frac{\partial \omega}{\partial \kappa} = 0$. Furthermore, as κ approach $k/(p\pi)$ from above, it is simple to verify that $\frac{\partial \omega}{\partial \kappa} < 0$. Finally, for large κ , note that $\lim_{\kappa \rightarrow \infty} \mathcal{C}(\kappa)$ is a constant and, recalling that $\mathcal{F}(\kappa)$ is linear, it is simple to show that $\lim_{\kappa \rightarrow \infty} \frac{\omega(\kappa)}{\sqrt{\kappa}} = C_0$ for some constant C_0 . Therefore, $\omega(\kappa)$ grows like $\sqrt{\kappa}$, such that $\omega'(\kappa) \rightarrow 0$ from above. Therefore, there exists some threshold $\bar{\kappa}$ such that, for $\kappa < \bar{\kappa}$, $\omega'(\kappa) < 0$ and otherwise $\omega'(\kappa) > 0$.

For the QCCSs, replace $\pi = \frac{1}{p\kappa} [(\rho + \kappa)\bar{V}_s + k]$. Then,

$$\mathcal{C}(z; \bar{V}_s) = \frac{\kappa}{\rho + \kappa} \frac{k(1 - p)}{\bar{V}_s}, \quad \mathcal{F}(z; \bar{V}_s) = \kappa + \delta \nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v \frac{(\rho + \kappa)\bar{V}_s + k}{k}$$

Then, $\mathcal{C}'(p; \bar{V}_s) < 0$ and $\mathcal{F}'(p; \bar{V}_s) = 0$. It follows immediately that the QCCS for p is strictly negative. Furthermore, $\mathcal{C}'(\kappa; \bar{V}_s) > 0$ and $\mathcal{F}'(\kappa; \bar{V}_s) > 0$, so that the QCCS for κ is strictly positive. ■

Proof of Corollary 2.2 For $x \in \{\nu, \pi, \lambda\}$, $\frac{dK}{dx} = \frac{\partial K}{\partial \omega} \frac{\partial \omega}{\partial x} \implies \text{sign}\left(\frac{dK}{dx}\right) = -\text{sign}\left(\frac{\partial \omega}{\partial x}\right)$ since $\frac{\partial K}{\partial \omega} < 0$. The result follows immediately. For $x \in \{k, p, \kappa\}$, $\frac{dK}{dx} = \frac{\partial K}{\partial x} + \frac{\partial K}{\partial \omega} \frac{\partial \omega}{\partial x}$, where $\frac{\partial K}{\partial \omega} < 0$. For p , $\frac{\partial K}{\partial p} < 0$ and $\frac{\partial \omega}{\partial p} < 0$; for k , $\frac{\partial K}{\partial k} > 0$ and $\frac{\partial \omega}{\partial k} > 0$; and for κ , $\frac{\partial K}{\partial \kappa} < 0$ and the sign of $\frac{\partial \omega}{\partial \kappa}$ is ambiguous. It is not possible to sign these effects in general. The same logic applies for the QCCS for p . However, the QCCS for κ is strictly negative since $\frac{\partial \omega}{\partial \kappa} > 0$ in this case. ■

Proof of Proposition 2.3 Consider a firm in productive development. It may not raise capital again because: (i) it completes R&D in the current financing round, or (ii) its financing is withdrawn and it cannot raise new capital. Together, the probability that a firm in productive development does not raise more capital is given by

$$\begin{aligned} Pr(\text{do not raise again}) &= Pr(\text{result in current round}) + Pr(\text{return to search and fail}) \\ &= \frac{\kappa}{\kappa + \omega} + \frac{\omega}{\kappa + \omega} \frac{\lambda}{\lambda + \nu} \end{aligned}$$

For notational convenience, I denote this probability by q_d . Then

$$Pr(N_f = 1) = q_d, \quad Pr(N_f = 2) = (1 - q_d) q_d, \quad \dots \quad Pr(N_f = n) = (1 - q_d)^{n-1} q_d$$

which is the probability mass function of the Geometric distribution. ■

Proof of Corollary 2.3 Recall $E[N_f] = \left(\frac{\kappa}{\kappa + \omega} + \frac{\omega}{\kappa + \omega} \frac{\lambda}{\lambda + \nu}\right)^{-1}$. Then

- $\frac{d}{d\nu} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(\nu) + \lambda\omega(\nu)(\kappa + \omega(\nu))}{(\kappa(\lambda + \nu) + \lambda\omega(\nu))^2} > 0$ since $\omega'(\nu) > 0$.
- $\frac{d}{d\lambda} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(\lambda) - \nu\omega(\lambda)(\kappa + \omega(\lambda))}{(\kappa(\lambda + \nu) + \lambda\omega(\lambda))^2} < 0$ since $\omega'(\lambda) < 0$.
- For $x \in \{p, \pi, k\}$, including the QCCS for p , $\frac{d}{dx} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(k)}{(\kappa(\lambda + \nu) + \lambda\omega(k))^2} \implies \text{sign}\left(\frac{d}{dx} E[N_f]\right) = \text{sign}(\omega'(x))$. The result then follows from Proposition 2.2.
- $\frac{d}{d\kappa} E[N_f] = \frac{(\lambda + \nu)\nu}{(\kappa(\lambda + \nu) + \lambda\omega(\kappa))^2} (\kappa\omega'(\kappa) - \omega(\kappa))$. This is ambiguous, the QCCS. Note $\frac{d}{d\kappa} E[N_f] < 0$ if $\frac{\partial \omega}{\partial \kappa} \frac{\kappa}{\omega} < 1$; i.e. ω the elasticity of ω with respect to κ is less than one. ■

Proof of Proposition 2.4 Follows immediately from the main text. See footnote 48.

Proof of Corollary 2.4 For all parameters $x \in \{\nu, k, p, \pi, \lambda\}$, $\text{sign}\left(\frac{d}{dz} E[T_{br}]\right) = -\text{sign}(\omega'(x))$. The result then follows from Proposition 2.2. For κ , $\text{sign}\left(\frac{d}{d\kappa} E[T_{br}]\right) = -\text{sign}(1 + \omega'(\kappa))$, which is ambiguous in the general case, but the QCCS is negative, since $\omega'(\kappa) > 0$ in this case. For ν , $\frac{d}{d\nu} E[T_{br}] = -\frac{\omega'(\nu)}{(\kappa + \omega(\nu))^2} - \frac{1}{(\lambda + \nu)^2} < 0$, where the inequality follows from Proposition 2.2, and $\frac{d}{d\lambda} E[T_{br}] = -\frac{\omega'(\lambda)}{(\kappa + \omega(\lambda))^2} - \frac{1}{(\lambda + \nu)^2}$, which is ambiguous since $\omega'(\lambda) < 0$ by Proposition 2.2. ■

Proof of Proposition A.4 Follows immediately from the main text.

Proof of Corollary 2.5. Consider two start-ups $i = a, b$ facing the same funding conditions, ν and λ . Suppose $E[N_f^a] > E[N_f^b]$. Then,

$$\left(\frac{\kappa_a}{\omega_a + \kappa_a} + \frac{\omega_a}{\kappa_a + \omega_a} \frac{\lambda}{\lambda + \nu} \right)^{-1} > \left(\frac{\kappa_b}{\omega_b + \kappa_b} + \frac{\omega_b}{\kappa_b + \omega_b} \frac{\lambda}{\lambda + \nu} \right)^{-1} \implies \frac{\kappa_a}{\omega_a} < \frac{\kappa_b}{\omega_b}$$

The result follows immediately from the expression for $\Delta_p = \frac{\lambda\omega}{\kappa(\lambda+\nu)+\lambda\omega} = \frac{\lambda}{\frac{\kappa}{\omega}(\lambda+\nu)+\lambda}$. ■

Proof of Proposition 2.6. The standard comparative statics follow immediately from differentiating equation (13). For the QCCSs, note that equation (13) can be written

$$V_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\rho + \omega)} \bar{V}_s$$

To compute the QCCS, note first that \bar{V}_s is held constant, so it is sufficient to consider the first term, $\frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\rho + \omega)}$. p appears only via; then, differentiating and using the QCCS for $\omega'(p) < 0$, it follows that the QCCS for entry with respect to p is positive. For κ , we have

$$\frac{\partial}{\partial \kappa} \left(\frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\rho + \omega)} \right) = \frac{\nu}{(\rho + \lambda)(\mathcal{F} + 2(\rho + \omega))^2} \times (\rho + 2[\omega - (\rho + \kappa)\omega'(\kappa)])$$

Therefore, QCCS for κ is positive iff $\rho + 2(\omega - (\rho + \kappa)\omega'(\kappa)) > 0$, but ambiguous in general. For intuition, note that as $\rho \rightarrow 0$, this is equivalent to

$$\frac{\kappa\omega'(\kappa)}{\omega} < 1$$

which is the necessary condition for $\frac{\partial E[N_f]}{\partial \kappa} < 0$; in words, as $\rho \rightarrow 0$, long-horizon projects, $\kappa \downarrow$, face greater distortions when they require more funding rounds. More generally, $\rho + 2(\omega - (\rho + \kappa)\omega'(\kappa)) > 0$ is equivalent to

$$\frac{\kappa\omega'(\kappa)}{\omega} < 1 + \frac{\rho}{\rho + \kappa} \left(\frac{\kappa}{2\omega} - 1 \right)$$

The right-hand side can be less than or exceed one, so it is not possible, in general, to state whether $\rho > 0$ loosens or tightens the elasticity condition under $\rho \rightarrow 0$. ■

Proof of Lemma 2.1 From equation (13), V_s can be expressed as

$$V_s = \frac{\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right)}{(\rho + \kappa) + (\rho + \lambda) \left(\frac{\rho + \kappa + \omega}{\nu} \right)}$$

Under $\kappa p \pi > k$, $\lim_{\nu \rightarrow \infty} \omega(\nu) = \infty \implies \lim_{\nu \rightarrow \infty} \kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho + \omega(\nu)} \right) = \kappa p \pi - k$. Furthermore,

from equation (11)

$$\begin{aligned} \lim_{\nu \rightarrow \infty} \omega(\nu)/\sqrt{\nu} &= \sqrt{\mathcal{C} \times \frac{\rho + \kappa}{\rho + \lambda}} \implies \lim_{\nu \rightarrow \infty} \omega(\nu)/\nu = 0 \\ \implies \lim_{\nu \rightarrow \infty} \left((\rho + \kappa) + (\rho + \lambda) \left(\frac{\rho + \kappa + \omega(\nu)}{\nu} \right) \right) &= \rho + \kappa \end{aligned}$$

where $\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k}$. Therefore, $\lim_{\nu \rightarrow \infty} V_s = \frac{\kappa p \pi - k}{\rho + \kappa}$. ■

Proof of Corollary 2.6 The result follows immediately from differentiating the expression and applying the results of Proposition 2.2. ■

Proof of Proposition 2.8 An equivalent statement is that if $\omega < \infty$, then $V_d > V_s$. To see that this must be the case, note that setting $\omega \rightarrow \infty$ implies $V_d = V_s$ and all the value is retained by the existing shareholders. In contrast, a finite contract rate implies that existing shareholders retain value $(1 - \varsigma)V_d|_{\{\omega < \infty\}}$. If this is to be optimal, it must be that $\omega < \infty$ leaves the existing shareholders better off than $\omega \rightarrow \infty$, i.e. $(1 - \varsigma)V_d|_{\{\omega < \infty\}} > V_s \implies V_d|_{\{\omega < \infty\}} > V_s$. Since $\varsigma|_{\{\omega < \infty\}} \in (0, 1)$, this implies $V_d|_{\{\omega < \infty\}} > V_s$. ■

A.3.3 Welfare

In the main body of the paper, only the steady state conditions are considered. Towards solving the planner's problem, it is necessary to consider the behaviour of the system out of steady state. To this end, note that at time t , there may be some firms in productive development with different contract rates $\omega \in [0, \infty)$. The same is true for the unproductive development state. I will use $\mu_{d,t}^p(\omega)$ and $\mu_{d,t}^u(\omega)$ to refer to the measures of firms at time t in productive and unproductive development, respectively, with contract rate ω . The total measure of firms in productive and unproductive development are then given by

$$\mu_{d,t}^p = \int_0^\infty \mu_{d,t}^p(\omega) d\omega, \quad \mu_{d,t}^u = \int_0^\infty \mu_{d,t}^u(\omega) d\omega \quad (\text{A.23})$$

and the measure of VCs not financing start-ups is $\mu_{vc,t} = M - \mu_{d,t}^p - \mu_{d,t}^u$. Firms in search have no contract and so are homogeneous—they have measure $\mu_{s,t}$, such that the total flow of matches is $m(\mu_{s,t}, \mu_{vc,t})$.

To determine the state transition equations, it is necessary to know which contracts are being signed at each instant. In principle, each firm that signs a contract at time t could choose a different contract rate. In this spirit, I will denote by $\chi_t(\omega)$ the share of contracts signed at time t that have contract rate ω . Naturally, $\chi_t(\omega)$ is subject to the constraints $\chi_t \in [0, 1]$ and $\int_0^\infty \chi_t(\omega) d\omega = 1$. The total inflow into state $\mu_{d,t}(\omega)$ from new contracts is then $\chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t})$ and the total outflow of firms from search to development is $\int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) d\omega$. With this notation in hand, the state transition equations are given

by

$$\dot{\mu}_{d,t}^p(\omega) = \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) - (\kappa + \omega)\mu_{d,t}^p(\omega) \quad (\text{A.24})$$

$$\dot{\mu}_{d,t}^u(\omega) = \kappa(1 - p)\mu_{d,t}^p(\omega) - \omega\mu_{d,t}^u(\omega) \quad (\text{A.25})$$

$$\dot{\mu}_{s,t} = \int_0^\infty \omega\mu_{d,t}^p(\omega) d\omega + \Lambda_t - \lambda\mu_{s,t} - \int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) d\omega \quad (\text{A.26})$$

where Λ_t is the flow rate of entry at time t , which is equal to $V_{s,t}/\tilde{\sigma}$ in the decentralised equilibrium.

The planner's problem is to choose entry, Λ_t , and the contract rate for each firm at time t in order to maximise total surplus, given by equation (15). In practice, the choice of contract boils down to choosing $\chi_t(\omega)$ for each $\omega \in [0, \infty)$ subject to the aforementioned constraints on $\chi_t(\omega)$. The current value Hamiltonian is given by

$$\begin{aligned} H = \int_0^\infty & \left([\kappa p\pi - k]\mu_{d,t}^p(\omega) - k\mu_{d,t}^u(\omega) \right. \\ & + \gamma_{d,t}^p(\omega)[\chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) - (\kappa + \omega)\mu_{d,t}^p(\omega)] \\ & + \gamma_{d,t}^u(\omega)[\mu_{d,t}^p(\omega)\kappa(1 - p) - \omega\mu_{d,t}^u(\omega)] \Big) d\omega \\ & + \gamma_{s,t} \left[\int_0^\infty \omega\mu_{d,t}^p(\omega) d\omega + \Lambda_t - \lambda\mu_{s,t} - \int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) d\omega \right] - \frac{\tilde{\sigma}}{2}\Lambda_t^2 \end{aligned} \quad (\text{A.27})$$

The optimality condition for Λ_t is simply $\frac{\partial H}{\partial \Lambda_t} = \gamma_{s,t} - \tilde{\sigma}\Lambda_t$. For the contracts, the steady-state optimal plan features $\chi(\omega) = 1$ for some $\omega \in [0, \infty)$, whereas $\chi_t(\omega) = 0$ for all $\omega' \neq \omega$. The marginal value of an increase in $\chi_t(\omega)$ is given by

$$\frac{\partial H}{\partial \chi_t(\omega)} = [\gamma_{d,t}^p(\omega) - \gamma_{s,t}]m(\mu_{s,t}, \mu_{vc,t})$$

Imposing the steady-state conditions $\dot{\gamma}_{s,t} = 0$, $\dot{\gamma}_{d,t}^p(\omega) = 0$ and $\dot{\gamma}_{d,t}^u(\omega) = 0$, the shadow-values are given by

$$\begin{aligned} \gamma_d^p(\hat{\omega}) &= \frac{\left(\rho + \lambda + \frac{\partial m}{\partial \mu_s}\right)\left(\kappa p\pi - \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right)k\right)}{(\rho + \lambda)\left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right)\frac{\partial m}{\partial \mu_{vc}} + (\rho + \kappa)\frac{\partial m}{\partial \mu_s} + (\rho + \lambda)(\rho + \kappa + \hat{\omega})} \\ &= \frac{(\rho + \lambda + \alpha\nu)\left[\kappa p\pi - \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right)k\right]}{\beta\nu(\rho + \lambda)\left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right) - \nu(1 - \alpha)(\rho + \kappa) + (\rho + \kappa)(\rho + \lambda + \nu) + (\rho + \lambda)\hat{\omega}} \end{aligned} \quad (\text{A.28})$$

and

$$\gamma_s = \frac{\alpha\nu}{\rho + \lambda + \alpha\nu}\gamma_d^p(\hat{\omega}) \quad (\text{A.29})$$

$$\gamma_d^u(\hat{\omega}) = -\frac{k}{\rho + \omega} - \frac{\beta\nu(\gamma_d^p(\hat{\omega}) - \gamma_s)}{\rho + \hat{\omega}} \quad (\text{A.30})$$

where $\hat{\omega}$ is assumed to be the time-invariant optimal contract rate, i.e. $\chi(\hat{\omega}) = 1$ and $\chi(\omega) = 0$

for all $\omega \neq \hat{\omega}$.¹³⁸ The planner sets $\chi(\hat{\omega}) = 1$ to maximise the marginal value of the flow of matches in steady-state, $\gamma_d^p(\hat{\omega}) - \gamma_s$.

Proof of Proposition 2.7. The proof follows immediately from the optimality conditions. First, consider the optimal contract rate. In the decentralised equilibrium, the optimal contract rate ω maximises the surplus, S , which is given by

$$S = \frac{(\rho + \lambda) \left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) k \right]}{(1 - \delta) v (\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) - \nu(1 - \delta) (\rho + \kappa) + (\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \omega}$$

In the planner's problem, the optimal contract rate maximises $\gamma_d^p(\hat{\omega}) - \gamma_s$, which is given by

$$\gamma_d^p(\hat{\omega}) - \gamma_s = \frac{(\rho + \lambda) \left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}} \right) k \right]}{\beta v (\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}} \right) - \nu(1 - \alpha) (\rho + \kappa) + (\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \hat{\omega}}$$

Setting $\beta = 1 - \delta$ and $\alpha = \delta$ creates an identity, so that the contract decentralised equilibrium equals that chosen by the planner.

Second, consider entry. In the decentralised equilibrium, entry is given by $V_s/\tilde{\sigma}$. In the planner's problem, the optimality condition yields $\Lambda = \gamma_s/\tilde{\sigma}$. Noting that $(\rho + \lambda)V_s = \delta\nu S$ and comparing V_s to γ_s , it is clear, for given optimal contract rate, ω , that entry is efficient if $\beta = 1 - \delta$ and $\alpha = \delta$. Finally, since the contract in the decentralised equilibrium coincides with the planner's solution when the same condition is met, entry must also be efficient. ■

A.3.4 Identification

This section provides the proof to Proposition 3.1. Before stating the proof, note that the $E[\text{time-to-exit}]$, $E[\text{burn rate}]$ and $E[\text{exit multiple}]$ – the expected time from first funding to a successful exit, the expected burn rate and the expected exit multiple, respectively – are given by¹³⁹

$$E[\text{time-to-exit}] = \frac{(\lambda + \nu)^2 + \nu\omega}{(\lambda + \nu)(\kappa(\lambda + \nu) + \lambda\omega)} \quad (\text{A.31})$$

$$E[\text{burn rate}] = E\left[\frac{\bar{K}}{T_{br}}\right] = \bar{K} \times \left(\frac{(\kappa + \omega)(\lambda + \nu) \ln\left(\frac{\kappa + \omega}{\lambda + \nu}\right)}{(\kappa + \omega) - (\lambda + \nu)} \right) \quad (\text{A.32})$$

$$E[\text{exit multiple}] = E\left[\frac{\pi}{\bar{K} N_f}\right] = \frac{\pi}{\bar{K}} \frac{E[N_f]^{-1}}{1 - E[N_f]^{-1}} \ln(E[N_f]) \quad (\text{A.33})$$

which uses $T_{br} \sim \text{Hypo}(\kappa + \omega, \lambda + \nu)$ and $N_f \sim \text{Geo}(q_d)$, and where \bar{K} is the equilibrium expected capital commitment, defined in Corollary 2.2.

¹³⁸The values for $\gamma_d^p(\omega)$ and $\gamma_d^u(\omega)$ for $\omega \neq \hat{\omega}$ can also be derived, but are not necessary to make the key point.

¹³⁹ $E[\text{time-to-exit}]$ may also be written, $E[\text{time-to-exit}] = E[N_f](\kappa + \omega)^{-1} + (E[N_f] - 1)(\lambda + \nu)^{-1}$, which adds up the time in development plus time in search (i.e. between rounds). Noting $E[N_f](\kappa + \omega)^{-1} = \kappa^{-1}(1 - \Delta_p)$, financing risk brings forward the time-to-exit, because only firms that complete development before funding runs out can be successful. However, the time spent in search delays completion.

Proof of Proposition 3.1. Recall equation (19)

$$\mathbf{m} = (E[\mathbb{1}\{\text{success}\}], E[N_f], E[T_{br}], E[\text{time-to-exit}], E[\text{burn rate}], E[\text{exit multiple}])$$

I refer to element i of \mathbf{m} by m_i . The proof consists of two steps. First, I show how to recover expressions for $\alpha_0 = \kappa + \omega$, $\alpha_1 = \lambda + \nu$, $\alpha_2 = \nu\omega$, $\alpha_3 = \kappa p$, $\alpha_4 = \pi$, and $\alpha_5 = k/(\rho + \omega)$ as functions of observable moments, \mathbf{m} . Then, I use the optimality condition for the contract to eliminate ω ; since this equation is quadratic in ω , it suffices to show that there is at most one positive solution for ω .

Fix some $\mathbf{m} \in \mathcal{M}^{adm}$. Then, using the model-implied expressions for m_1, \dots, m_6 , it is simple to show

$$\begin{aligned}\alpha_0 &= \kappa + \omega = (m_4 - m_3(m_2 - 1))^{-1} \\ \alpha_1 &= \lambda + \nu = (m_2 m_3 - m_4)^{-1} \\ \alpha_2 &= \nu\omega = \alpha_0 \alpha_1 (1 - m_2^{-1}) \\ \alpha_3 &= \kappa p = \alpha_0 m_1 m_2^{-1}\end{aligned}$$

For $\alpha_4 = \pi$, note from equation (A.32)

$$\bar{K} = m_5 \left(\frac{\alpha_0 \alpha_1 \ln(\frac{\alpha_0}{\alpha_1})}{\alpha_0 - \alpha_1} \right)^{-1}$$

then from equation (A.33)

$$\alpha_4 = \pi = \frac{m_5 m_6}{\frac{\alpha_0 \alpha_1 \ln(\frac{\alpha_0}{\alpha_1})}{\alpha_0 - \alpha_1} \frac{m_2^{-1}}{1 - m_2^{-1}} \log(m_2)}$$

Finally, for $\alpha_5 = \frac{k}{\rho + \omega}$, write

$$\bar{K} = \frac{k}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) = \frac{k}{\rho + \omega} \left(1 - \frac{\kappa p}{\rho + \kappa + \omega} \right)$$

then, using the expression for \bar{K}

$$\alpha_5 = \frac{k}{\rho + \omega} = m_5 \left(\frac{\alpha_0 \alpha_1 \ln(\frac{\alpha_0}{\alpha_1})}{\alpha_0 - \alpha_1} \right)^{-1} \left(1 - \frac{\alpha_3}{\rho + \alpha_0} \right)^{-1}$$

Then, the second step eliminates ω from the above relationships. To do so, I use the first-order condition and solve directly for ω in terms of moments, \mathbf{m} . The first order condition for ω can be written

$$\frac{k}{\rho + \omega} \kappa(1-p) = (\rho + \lambda)(\rho + \omega) \frac{V_s(\omega, \theta)}{\nu}$$

where $V_s(\omega, \theta)$ is the value of search, and

$$\begin{aligned} \frac{V_s(\omega, \theta)}{\nu} &= \frac{\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho+\omega}\right)}{(\rho + \lambda + \nu)(\rho + \kappa + \omega) - \nu\omega} = \frac{\kappa p \pi - (\rho + \kappa + \omega)\bar{K}}{(\rho + \lambda + \nu)(\rho + \kappa + \omega) - \nu\omega} \\ &= \frac{\alpha_3 \alpha_4 - (\rho + \alpha_0)m_5 \left(\frac{\alpha_0 \alpha_1 \ln\left(\frac{\alpha_0}{\alpha_1}\right)}{\alpha_0 - \alpha_1}\right)^{-1}}{(\rho + \alpha_1)(\rho + \alpha_0) - \alpha_2} > 0 \end{aligned}$$

which depends only on observables, and where the inequality follows since $\theta \in \Theta^{adm} \implies k < \kappa p \pi$. Denoting $V_s(\omega, \theta)/\nu = \tilde{V}_s(\mathbf{m})$, using $\alpha_5 = k/(\rho + \omega)$ and $\alpha_3 = \kappa p$, and solving for ω , the first-order condition implies

$$\omega = -\rho + \frac{\kappa - \alpha_3}{\rho + \lambda} \frac{\alpha_5}{\tilde{V}_s(\mathbf{m})}$$

Then, to solve for ω , substitute in $\kappa = \alpha_0 - \omega$ and $\lambda = \alpha_1 - \nu = \alpha_1 - \alpha_2/\omega$

$$\omega = -\rho + \frac{\alpha_0 - \omega - \alpha_3}{\rho + \alpha_1 - \alpha_2/\omega} \frac{\alpha_5}{\tilde{V}_s(\mathbf{m})} = -\rho + \frac{(\alpha_0 - \alpha_3 - \omega)\omega}{(\rho + \alpha_1)\omega - \alpha_2} \frac{\alpha_5}{\tilde{V}_s(\mathbf{m})}$$

Rearranging as a quadratic in ω yields

$$[\rho + \alpha_1 + \alpha_5/\tilde{V}_s(\mathbf{m})]\omega^2 + [\rho(\rho + \alpha_1) - \alpha_2 - (\alpha_0 - \alpha_3)\alpha_5/\tilde{V}_s(\mathbf{m})]\omega - \rho\alpha_2 = 0$$

Finally, since $\rho > 0, \alpha_1, \alpha_2, \alpha_5 > 0$ and $\tilde{V}_s(M) > 0$, the coefficient on ω^2 is positive while the constant term is negative; thus the two roots have opposite signs, and exactly one satisfies $\omega > 0$. Then, given $\alpha_0 = \kappa + \omega$, $\alpha_1 = \lambda + \nu$, $\alpha_2 = \nu\omega$, $\alpha_3 = \kappa p$, $\alpha_4 = \pi$, and $\alpha_5 = k/(\rho + \omega)$ and $\omega = \omega(\mathbf{m})$, it follows immediately that there is a unique mapping from empirical moments to model parameters. ■

A.4 Additional results and derivations

A.4.1 Results

Proposition A.4. *The success probability for firms in productive development, p_d is: strictly increasing in p and π ; strictly decreasing in k ; and ambiguous with respect to ν , λ , and κ . The success probabilities for firms in search, p_s is: strictly increasing in p and π ; strictly decreasing in k ; and ambiguous with respect to ν and κ .*

Proof of Proposition A.4. The result for p , π and k follows directly from differentiation of p_d and p_s , given the results from Proposition 2.2. For κ , $\text{sign}\left(\frac{dp_d}{d\kappa}\right) = \text{sign}\left(\frac{dp_s}{d\kappa}\right) = \text{sign}(\omega(\kappa) - \kappa\omega'(\kappa))$, which is ambiguous.

Next, consider $\frac{dp_d}{d\nu} = \frac{\kappa p \lambda (\omega - (\lambda + \nu)\omega'(\nu))}{(\kappa(\lambda + \nu) + \lambda\omega)^2} \implies \text{sign}\left(\frac{dp_d}{d\nu}\right) = \text{sign}(\omega - (\lambda + \nu)\omega'(\nu))$, which can equivalently be cast as a statement about the semi-elasticity of ω with respect to ν ; that is, if $\frac{\omega'(\nu)}{\omega} > \frac{1}{\lambda + \nu}$, then $\frac{dp_d}{d\nu} > 0$. To see that $\frac{dp_d}{d\nu} < 0$ is possible, note that there are parameter values such that $\omega = 0$ but $\omega'(\nu) > 0$ (see the proof to Proposition A.3). In this case $\omega - (\lambda + \nu)\omega'(\nu) = -(\lambda + \nu)\omega'(\nu) < 0$, so $\frac{dp_d}{d\nu} < 0$. To see that $\frac{dp_d}{d\nu} > 0$ is possible, note that $\lim_{\nu \rightarrow \infty} p_d = p$ and

$p_d \leq p$, so p_d must approach p from below as $\nu \rightarrow \infty$. This means there must be some ν for which $\frac{dp_d}{d\nu} > 0$. Finally, to see that $\exists \tilde{\nu}_d \geq 0$ such that if $\nu \geq \tilde{\nu}_d$, $\frac{dp_d}{d\nu} > 0$ and otherwise $\frac{dp_d}{d\nu} < 0$, note that $\frac{d}{d\nu}(\omega - (\lambda + \nu)\omega'(\nu)) = -(\lambda + \nu)\omega''(\nu) > 0$ where the inequality follows since $\omega''(\nu) < 0$, so there at most one root for $\omega - (\lambda + \nu)\omega'(\nu) = 0$. Furthermore, $\frac{d}{d\nu}(\omega - (\lambda + \nu)\omega'(\nu)) > 0$ implies that if $\frac{dp_d}{d\nu} \geq 0$ for some $\nu = \hat{\nu}$, then $\frac{dp_d}{d\nu} > 0 \forall \nu > \hat{\nu}$. Therefore, we can define $\tilde{\nu}_d > 0$ s.t. $\omega - (\lambda + \nu)\omega'(\nu) = 0$ or $\tilde{\nu}_d = 0$ if there is no root and the statement holds.

Next, consider $\frac{dp_s}{d\nu} = \frac{\kappa p \lambda (\kappa + \omega - \nu \omega'(\nu))}{(\kappa(\lambda + \nu) + \lambda \omega(\nu))^2} \implies \text{sign}(\frac{dp_s}{d\nu}) = \text{sign}(\kappa + \omega - \nu \omega'(\nu))$. To see that $\frac{dp_s}{d\nu} < 0$ is possible, note that $\frac{d}{d\nu}(\kappa + \omega - \nu \omega'(\nu)) = -\nu \omega''(\nu) > 0$. Therefore, if $\frac{dp_s}{d\nu} < 0$ is to occur, it will occur for low values of ν . Then, take the limit as $\nu \rightarrow 0$

$$\lim_{\nu \rightarrow 0}(\kappa + \omega - \nu \omega'(\nu)) = \frac{(\rho + \lambda)(k(\kappa p - \rho) + \kappa p \pi(\rho - \kappa)) - \sqrt{\kappa^2 k(1 - p)p(\lambda + \rho)^2(\kappa \pi - k)}}{(\rho + \lambda)(k - \kappa p \pi)}$$

where $\omega > 0$ has been assumed. This expression is negative if $p \in (\frac{k\rho^2}{\kappa\pi(\kappa - \rho)^2 - \kappa k(\kappa - 2\rho)}, 1)$. However, the $\omega > 0$ solution may be invalid. From Corollary 2.1, a sufficient condition to take $\nu \rightarrow 0$ while maintaining $\omega > 0$ is that $p < \bar{p} = \frac{k(\rho + \kappa)^2}{\kappa(\pi\rho^2 + k(\kappa + 2\rho))}$. Both of these conditions on p can be met simultaneously if $\kappa > 2\sqrt{\rho}$, which is admissible. Therefore, $\lim_{\nu \rightarrow 0}(\kappa + \omega - \nu \omega'(\nu)) < 0$ in some part of the parameter space and so $\frac{dp_s}{d\nu}$ is feasible. To see that $\frac{dp_s}{d\nu} > 0$ is possible, note that $\lim_{\nu \rightarrow \infty} p_s = p$ and $p_s \leq p$, so p_s must approach p from below as $\nu \rightarrow \infty$. The remainder of the proof is analogous to that for part (i).

For λ , $\text{sign}(\frac{dp_d}{d\lambda}) = \text{sign}(-\nu\omega - \lambda(\lambda + \nu)\omega'(\lambda))$. To see that this can be positive, note that there exists a λ such that $\omega = 0$ and $\omega'(\lambda) < 0$; at this point, small reductions in λ imply a movement from upfront financing to staged-financing and so lower the success probability. To see that $\frac{dp_d}{d\lambda} < 0$ is possible, note that $\lim_{\lambda \rightarrow 0} p_d = p$ and $p_d \leq p$, so p_d must approach p from below as $\lambda \rightarrow 0$, implying there is some region for which $\frac{dp_d}{d\lambda} < 0$. For p_s , clearly $\frac{dp_d}{d\lambda} < 0 \implies \frac{dp_s}{d\lambda} < 0$.

Finally, $\frac{dp_d}{dp} = \frac{\kappa(\lambda + \nu)(\kappa(\lambda + \nu) - \lambda p \omega'(p) + \lambda \omega)}{(\kappa(\lambda + \nu) + \lambda \omega)^2}$ and $\frac{dp_s}{dp} = \frac{\nu}{\lambda + \nu} \frac{dp_d}{dp}$. By Proposition (2.2), $\omega'(p) < 0$, so $\frac{dp_d}{dp} > 0$ and $\frac{dp_s}{dp} > 0$. Furthermore, $\frac{d}{dp} \frac{\kappa(\lambda + \nu)}{\omega \lambda + \kappa(\lambda + \nu)} = -\frac{\kappa \lambda (\lambda + \nu) \omega'(p)}{(\kappa(\lambda + \nu) + \lambda \omega)^2} > 0$ since $\omega'(p) < 0$. ■

A.4.2 Derivations

Expected capital cost Suppose the contract is signed at date $t = 0$. Until time $T = \min\{T_\kappa, T_\omega\} \sim \text{Exp}(\kappa + \omega)$, the E has not obtained a result and funding has not been withdrawn, so the VC pays k . Once time T arrives, there are three potential outcomes: (i) funding runs out, $T = T_\omega$, with probability $\frac{\omega}{\kappa + \omega}$; (ii) $T = T_\kappa$ and the result is a success, with probability $\frac{\kappa p}{\kappa + \omega}$; or (iii) $T = T_\kappa$ and the result is a failure, with probability $\frac{\kappa(1-p)}{\kappa + \omega}$. In cases (i) and (ii), funding ceases. In case (iii), the VC continues to pay k until time $T + T_\omega = T_\kappa + T_\omega$. Therefore, from time $t = 0$ to $t = T$, the flow cost to the E is $e^{-\rho t} k$ from the perspective of time $t = 0$. At time T , the probability that the VC continues to pay k is $\frac{\kappa(1-p)}{\kappa + \omega}$, in which case they pay k from date T to $T + T_\omega$. The expected capital cost is then computed by taking expectations over $T_\omega \sim \text{Exp}(\omega)$ and $T \sim \text{Exp}(\kappa + \omega)$

$$\begin{aligned}
K(\omega) &= \int_0^\infty \left(\int_0^T e^{-\rho t} k dt \right. \\
&\quad \left. + \frac{\kappa(1-p)}{\kappa+\omega} \int_0^\infty \left(\int_T^{T+T_\omega} e^{-\rho s} k ds \right) \omega e^{-\omega T_\omega} dT_\omega \right) (\kappa+\omega) e^{-(\kappa+\omega)T} dT \\
&= \left(1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \frac{k}{\rho+\kappa+\omega}
\end{aligned} \tag{A.34}$$

The value of search To obtain equation (13), solve the following system for $\{V_d, V_s\}$

$$\begin{aligned}
\rho V_d &= \kappa[p\pi - k] + \omega[V_s - V_d] \\
(\rho + \lambda)V_s &= \nu[V_d - K(\omega) - V_s]
\end{aligned}$$

which substitutes $V^M = V_d - K(\omega)$. This yields

$$V_s = \frac{\nu \left[\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \right]}{(\rho + \lambda + \nu)(\rho + \kappa + \omega) - \nu\omega}$$

After some algebra, this is equal to

$$V_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + \frac{\mathcal{C}\mathcal{F} + (\rho+\omega)^2}{\rho+\omega-\mathcal{C}}} \bar{V}_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + \frac{2\mathcal{C}\mathcal{F} + 2\mathcal{C}(\rho+\omega)}{\rho+\omega-\mathcal{C}}} \bar{V}_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2})} \bar{V}_s$$

The first value obtains after factoring out \bar{V}_s ; the second after multiplying out $(\rho + \omega)^2$ and simplifying; the third follows by factoring $\sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}$ from $\frac{2\mathcal{C}\mathcal{F} + 2\mathcal{C}(\rho+\omega)}{\rho+\omega-\mathcal{C}}$ and simplifying.

Flow of first funding rounds To obtain equation (20), note that the measure of firms that have entered but not yet received funding evolves as

$$\dot{\mu}_{s,t}^0 = \frac{V_{s,t}}{\tilde{\sigma}} - (\lambda + \nu_e) \mu_{s,t}^0$$

which implies the steady-state condition

$$\tilde{\sigma} = \left(\frac{1}{\lambda + \nu} \times \frac{1}{\mu_s^0} \right) V_s$$

where μ_s^0 is the steady-state measure of firms that have paid the entry cost but not yet obtained VC funding. μ_s^0 is unobserved in the data, but the empirical counterpart to $\nu_e \times \mu_s^0$ is the flow of first funding rounds, which is observed. Substituting in for μ_s^0 then yields equation (20).

A.5 Quantitative model

A.5.1 Frictionless benchmark

In the frictionless economy without agency and matching frictions, the value functions $\{\bar{V}_{s,e}, \bar{V}_{d,e}, \bar{V}_{s,l}, \bar{V}_{d,l}\}$ solve

$$\rho \bar{V}_{d,l} = -k_l + \kappa [p_l \pi - \bar{V}_{d,l}] + p_l \pi \xi \phi \exp \left\{ -\frac{\bar{V}_{d,l}}{p_l \pi \xi} \right\} \quad (\text{A.35})$$

$$(\rho + \lambda) \bar{V}_{s,l} = p_l \pi \xi \phi \exp \left\{ -\frac{\bar{V}_{s,l}}{p_l \pi \xi} \right\} \quad (\text{A.36})$$

$$\rho \bar{V}_{d,e} = -k_e + \kappa \left[\left(\frac{p_e}{p_L} \right) \max\{\bar{V}_{s,l}, \bar{V}_{d,l}\} - \bar{V}_{d,e} \right] \quad (\text{A.37})$$

$$\bar{V}_{s,e} = \max\{0, \bar{V}_{d,e}\} \quad (\text{A.38})$$

These equations mirror the conditions in the model of section 2.4, but are adjusted for a wealthy entrepreneur that can self-finance the project.¹⁴⁰ Relative to that benchmark, note that an entrepreneur that enters the late-stage may choose not to invest, or decide not to and simply wait for an acquisition offer. Therefore, the relevant continuation value from the early to late-stage is $\max\{\bar{V}_{s,l}, \bar{V}_{d,l}\}$. An entrepreneur that does not invest in the late-stage fails at rate λ , and so I retain the subscripts s and d ; although now they refer to no late-stage investment and late-stage investment, respectively. Conversely, in the early-stage the entrepreneur either invests, or the project has zero value.

A.5.2 Equilibrium conditions

In this section, I provide the full set of equilibrium conditions for the model that I outline in section 2.4 and estimate in section 3. I first provide the full set of conditions taking as given the equilibrium in the capital market, and then provide details on the full equilibrium.

Taking the meeting rates in the early and late-stage markets, ν_e and ν_l as given, the following

¹⁴⁰Alternatively, by taking the numerical limit $\nu_e \rightarrow \infty$ and $\nu_l \rightarrow \infty$ and solving the model of section 2.4, one can obtain an approximation to the solution of this system of equations to desired accuracy. See Lemma 2.1.

conditions characterise the equilibrium

$$\rho V_{d,e}(\omega_e) = \kappa [\hat{p}_e V_{s,l} - V_{d,e}(\omega_e)] + \omega_e [V_{s,e} - V_{d,e}(\omega_e)] \quad (\text{A.39})$$

$$(\rho + \lambda) V_{s,e} = \nu_e [V_e^M - V_{s,e}] \quad (\text{A.40})$$

$$V_e^M = \sup_{\{\omega_e \in [0, \infty), \varsigma_e \in [0, 1]\}} \{(1 - \varsigma_e) V_{d,e}(\omega_e)\} \quad \text{s.t.} \quad \varsigma_e V_{d,e}(\omega_e) \geq K_e(\omega_e), V_{s,e} \text{ given} \quad (\text{A.41})$$

$$K_e(\omega_e) = \left(1 + \frac{\kappa(1 - \hat{p}_e)}{\rho + \omega_e}\right) \frac{k}{\rho + \kappa + \omega_e} \quad (\text{A.42})$$

$$\rho V_{d,l}(\omega_l) = \kappa [p_l \pi - V_{d,l}(\omega_l)] + \omega_l [V_{s,l} - V_{d,l}(\omega_l)] + \hat{\phi}_d [p_l \pi E[\epsilon | \epsilon > V_{d,l}(\omega_l)/(p_l \pi)] - V_{d,l}] \quad (\text{A.43})$$

$$(\rho + \lambda_l) V_{s,l} = \nu_l [V_l^M - V_{s,l}] + \hat{\phi}_s [p_l \pi E[\epsilon | \epsilon > V_{s,l}/(p_l \pi)] - V_{s,l}] \quad (\text{A.44})$$

$$V_l^M = \sup_{\{\omega_l \in [0, \infty), \varsigma_l \in [0, 1]\}} \{(1 - \varsigma_l) V_{d,l}(\omega_l)\} \quad \text{s.t.} \quad \varsigma_l V_{d,l}(\omega_l) \geq K_l(\omega_l), V_{s,l} \text{ given} \quad (\text{A.45})$$

$$K_l(\omega_l) = \left(1 + \frac{\kappa(1 - p_l)}{\rho + \omega_l + \hat{\phi}}\right) \frac{k}{\rho + \kappa + \omega_l + \hat{\phi}_d} \quad (\text{A.46})$$

where $\hat{p}_e = p_e/p_l$, $\epsilon \sim \text{Exp}(1/\xi)$, $\hat{\phi}_i = \phi [1 - F(V_{i,l}/(p_l \pi))]$ for $i \in \{s, d\}$, and (ω_e, ς_e) and (ω_l, ς_l) are chosen optimally in the sense of equations (A.41) and (A.45), respectively.

The meeting rates, ν_e and ν_l , are determined in equilibrium based on the supply and demand for capital in each stage. I assume that VCs specialise within each stage: a measure M_e serve the early stage and measure M_l serve the late-stage. I denote by $\mu_{s,i}$, $\mu_{d,i}^p$ and $\mu_{d,i}^u$ the measures of firms in search, productive development and unproductive development for stage $i \in \{e, l\}$, respectively. Then, the measure of VCs in each stage that are not funding start-ups is given by $\mu_{vc,i} = M_i - \mu_{d,i}^p - \mu_{d,i}^u$ for $i \in \{e, l\}$ and the flow rate of matches in stage $i = e, l$ is $m_i = m(\mu_{s,i}, \mu_{vc,i})$. Then $\nu_i = \mu_{s,i}^{-1} m(\mu_{s,i}, \mu_{vc,i})$ for $i = e, l$.

Finally, $\mu_{s,i}$, $\mu_{d,i}^p$ and $\mu_{d,i}^u$ for $i = e, l$ are determined in equilibrium by the following steady-state conditions.

$$\text{Early-stage productive development : } \nu_e \mu_{s,e} = (\kappa + \omega_e) \mu_{d,e}^p$$

$$\text{Early-stage unproductive development : } \kappa(1 - \hat{p}_e) \mu_{d,e}^p = \omega_e \mu_{d,e}^u$$

$$\text{Early-stage search : } \omega_e \mu_{d,e}^p + \frac{V_{s,e}}{\tilde{\sigma}} = (\lambda + \nu_e) \mu_{s,e}$$

$$\text{Late-stage productive development : } \nu_l \mu_{s,l} = (\kappa + \omega_l + \hat{\phi}_d) \mu_{d,l}^p$$

$$\text{Late-stage unproductive development : } \kappa(1 - p_l) \mu_{d,l}^p = \omega_l \mu_{d,l}^u$$

$$\text{Late-stage search : } \omega_l \mu_{d,l}^p + \kappa \hat{p}_e \mu_{d,e}^p = (\lambda + \nu_l + \hat{\phi}_s) \mu_{s,l}$$

where $\nu_e = m_e/\mu_{s,e}$ and $\nu_l = m_l/\mu_{s,l}$. Relative to the baseline model, there are two tangible differences. Firstly, the inflow into early-stage search includes new entrants, whereas the inflow into late-stage search includes firms that are successful in overcoming the early-stage uncertainty, a flow $\kappa \hat{p}_e \mu_{d,e}^p$. Secondly, in the late-stage, firms may be acquired from productive development

or search.

B Empirical appendix

B.1 Dataset construction

This section discusses dataset construction. Table B.1 shows an illustrative dataset. To create a dataset of this form, my US and UK samples are drawn from Thomson Reuters venture capital dataset; I consider all firms that raised a first “early-stage” funding round between 2005-2015. Before processing the data, I make a small number of manual corrections where there are obvious errors. The majority of errors occur when a funding round coincides with an exit event; I perform manual searches to clarify the true nature of the event and drop the false entry. I also drop duplicated funding rounds.

Firm ID	Date	Event	Round Type	Amount raised	Valuation
1	28/01/2013	funding round	Early	6.0	-
1	24/09/2013	funding round	Early	6.5	-
1	04/02/2015	funding round	Late	30.0	-
1	17/04/2019	IPO	-	80.5	342.6
2	12/03/2013	funding round	Early	3.0	-
2	19/07/2016	funding round	Late	10.5	-
2	01/01/2017	Acquisition	-	-	89.4
3	06/03/2014	funding round	Early	9.1	-

Table B.1: Illustrative dataset

The table shows illustrative data on venture capital funding rounds and exits for three fictitious firms. Amounts raised and valuations should be interpreted in units of million USD.

Among all funding rounds, I keep those tagged as one of “Seed”, “Early Stage”, “Expansion”, “Later Stage”, which reflect increasing levels of progression. If a firm has two deals on the same day, I sum up the respective deal values and record it under the earlier tag. I then label ‘Seed’ “Early Stage” rounds as “early-stage”, and “Expansion” and “Later Stage” rounds as “late-stage”. I drop the minority of firms that only have late-stage funding rounds in the data. I also drop any funding rounds that occur after the firm records an exit.

In some cases, the deal values are missing. This occurs in 2,706 of the 33,158 (8%) US funding rounds, and a slightly higher share of UK rounds. Data on amounts raised is used to compute the burn rate and exit multiples, both of which are used in model estimation. Therefore, I follow existing literature and impute amounts raised in these cases. The methodology I use follows Jagannathan et al. (2022) closely. Specifically, I estimate the following regression

$$\log(\text{Amount raised}_{i,r}) = \beta_0 + \beta_1 \log(\text{Amount raised}_{i,r-1}) + \beta_2 X_{i,r} + u_{i,r} \quad (\text{B.1})$$

where $\text{Amount raised}_{i,r}$ is the amount raised by firm i in round r (in 2015 Mn USD) and X_i is a list of firm-specific controls and fixed effects. I include the investment stage, industry, year-quarter fixed effects, and fixed effect for the number of investors, which I cap at ten investors

(i.e. all rounds with 10 or more investors are lumped together). I run this regression separately for the US and UK. In addition, for some rounds I do not observe the previous funding amount and so run the specification with only the controls, $X_{i,r}$.¹⁴¹ The results are displayed in Table B.2. I run this on all deals between 2005 and 2022, which is why the observation counts exceed the sample used in the estimation (where data is censored). Using the results of these regressions I impute missing amounts raised using fitted values.

	United States		United Kingdom	
	Log(Amount _{<i>r</i>})	Log(Amount _{<i>r</i>})	Log(Amount _{<i>r</i>})	Log(Amount _{<i>r</i>})
Log(Amount _{<i>r-1</i>})	-	0.47***	-	0.55***
Constant	0.35	-0.52	0.49	0.54
Industry FE	Y	Y	Y	Y
Stage FE	Y	Y	Y	Y
# Investors FE	Y	Y	Y	Y
YQ FE	Y	Y	Y	Y
Adj. R^2	0.31	0.50	0.26	0.55
Observations	61,004	36,196	7,133	2,816

Table B.2: Imputation model

The table reports regression results from estimation of equation (B.1). ***, $p < 0.01$.

Among exit types, I classify IPOs, reverse takeovers, and secondary sales as successes. I also classify “Mergers” as successes when the acquiror is an investor, whereas trade-sales are classified as acquisitions. This categorisation means that SPACs are included as successes. For exit values, I obtain data from CRSP on the market cap of firms six months after IPO for US IPOs and from Bloomberg for UK IPOs.¹⁴² Furthermore, a non-negligible share of exit values are missing in the Thomson Reuters data. Where possible, I fill in missing exit values with data from Crunchbase. When the deal value remains missing – the majority case – I set it equal to 1.5 times total invested capital, implying an exit multiple of 1.5X, following (Kerr et al., 2014).

The final dataset comprises all firms that raised their first early-stage funding round between 2005-2015 inclusive. Table B.3 reports summary statistics for the main sample.

B.2 Definitions

Thomson Reuters define funding rounds in the following way.

Seed stage This stage is a relatively small amount of capital provided to an inventor or entrepreneur to prove a concept. This involves product development and market research as well as building a management team and developing a business plan, if the initial steps are successful. This is a pre-marketing stage.

Early stage This stage provides financing to companies completing development where products are mostly in testing or pilot production. In some cases, product may have just been made

¹⁴¹I run this on all the data, regardless of whether previous funding information is available.

¹⁴²Insiders typically cannot cash out until six months after an IPO (Brav and Gompers, 2000).

	United States		United Kingdom	
	Count	Share	Count	Share
<i>Sample overview</i>				
Firms	11,030	—	1,066	—
Successes	400	3.6%	26	2.4%
Acquisitions	2,034	18.4%	97	9.1%
Funding rounds — total	33,158	—	2,218	—
Early-stage	21,886	66.0%	1,534	69.2%
Late-stage	11,272	34.0%	684	30.8%
<i>Sectors</i>				
Biotechnology	995	9.0%	97	9.1%
Communications and Media	289	2.6%	43	4.0%
Computer Hardware	298	2.7%	31	2.9%
Computer Software and Services	3,607	32.7%	254	23.8%
Consumer Related	328	3.0%	53	5.0%
Industrial/Energy	537	4.9%	99	9.3%
Internet Specific	3,082	28.0%	246	23.1%
Medical/Health	1,063	9.6%	109	10.2%
Other Products	463	4.2%	85	8.0%
Semiconductors/Other Elect.	368	3.3%	49	4.6%
Total	11,030	100.0%	1,066	100.0%

Table B.3: Sample overview

The table reports the number of firms, funding rounds, and outcomes in the estimation sample, and their sectoral composition for the United States and United Kingdom. Funding rounds and outcomes are censored at seven years following a firm's first funding round. Shares are within-country totals of firms (or rounds where applicable).

commercially available. Companies may be in the process of organising or they may already be in business for three years or less. Usually such firms will have made market studies, assembled the key management, developed a business plan, and are ready or have already started conducting business.

Expansion stage This stage involves working capital for the initial expansion of a company that is producing and shipping and has growing accounts receivables and inventories. It may or may not be showing a profit. Some of the uses of capital may include further plant expansion, marketing, working capital, or development of an improved product. More institutional investors are more likely to be included along with initial investors from previous rounds. The venture capitalist's role in this stage evolves from a supportive role to a more strategic role.

Later stage Capital in this stage is provided for companies that have reached a fairly stable growth rate; that is, not growing as fast as the rates attained in the expansion stages. Again, these companies may or may not be profitable, but are more likely to be than in previous stages of development. Other financial characteristics of these companies include positive cash flow. This also includes companies considering IPO.

B.3 Additional tables and figures

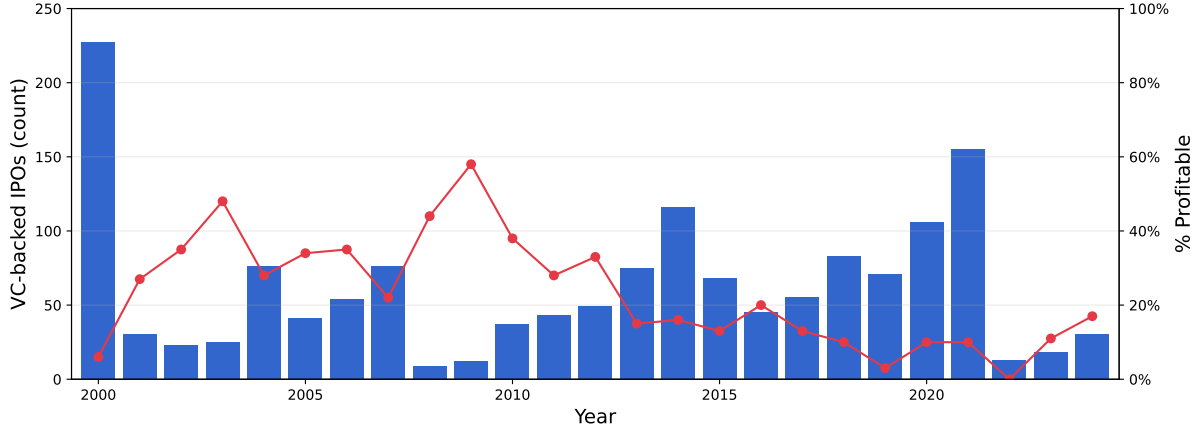


Figure B.1: US-headquartered, VC-backed IPOs

The figure reports the number of VC-backed IPOs of US-headquartered companies from 2000-2024 (bars, left axis), and the share of these companies that were profitable (line, right axis). The data is from Jay Ritter's website (<https://site.warrington.ufl.edu/ritter/>), Table 4d. See source for details.

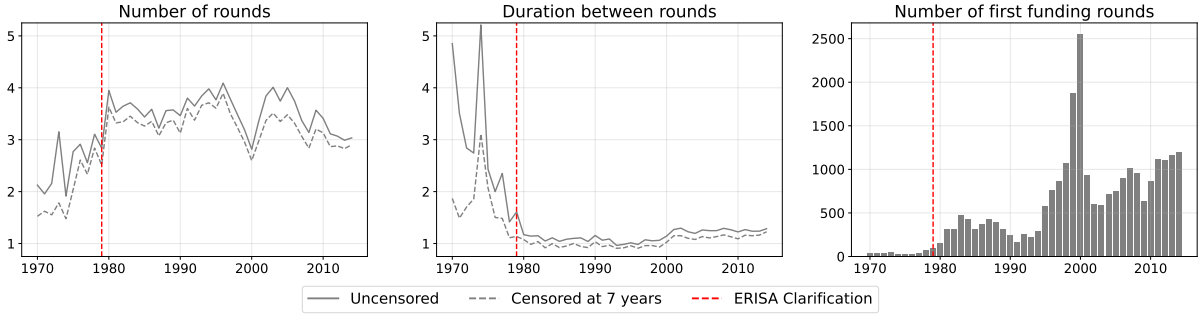


Figure B.2: US historical funding patterns

The figure extends the US sample back to 1970 and, for cohorts of firms defined by their year of first funding, reports the average number of funding rounds, the average duration between funding rounds, and the total number of firms. The dashed line shows the results when data is censored at seven years following each firm's first funding round, as in the main estimation exercise. The solid line applies no censoring. The vertical red line refers to the clarification of ERISA in 1979.

C Quantitative appendix

C.0.1 Additional details on US estimation

This section provides additional details on the estimation procedure.

Moment computation The estimation procedure involves two steps. First, all parameters except for $\tilde{\sigma}$ are estimated by minimising equation (21). Then, $\tilde{\sigma}$ is recovered from the model condition for the number of first funding rounds. The moments included in equation (21) are reported in Table 2. Before computing moments, I drop observations occurring more than seven years after a firm's first funding round, ensuring uniform censoring.

The complication lies in estimating $\lambda + \nu_e$ and $\lambda + \nu_e + \hat{\phi}_s$ in way consistent with the model's structure. In the model, the within-stage distribution for the time between funding rounds is hypoexponential: for two successive early-stage rounds, $T_{br}^{e \rightarrow e} \sim \text{Hypo}(\kappa + \omega_e, \lambda + \nu_e)$, and for two successive late-stage rounds, $T_{br}^{l \rightarrow l} \sim \text{Hypo}(\kappa + \omega_l, \lambda + \nu_l + \hat{\phi}_s)$. There are two problems. First, the parameters of the hypoexponential distribution are interchangeable, meaning it is unclear which of the estimates reflect $\lambda + \nu_e$ and $\lambda + \nu_e + \hat{\phi}_s$. Second, the data used to estimate the model is censored, meaning these distributions no longer apply.

First, consider the censoring issue. To state the issue more formally, denote by $t_{i,n}^s$ the time of the n th funding round for firm i in state $s \in \{e, l\}$ and the duration between round n and round $n+1$ for firm i in stage s by $\tau_{i,n}^s = t_{i,n+1}^s - t_{i,n}^s$. Firm i completes $N_i^e \in \{1, 2, 3, \dots\}$ early-stage rounds and $N_i^l \in \{0, 1, 2, \dots\}$ late-stage rounds, but N_i^e and N_i^l are unobserved because the data is censored.¹⁴³ Without censoring, the duration between funding rounds is drawn from the unconditional distribution with CDF $\Pr(\tau_{i,n}^s \leq \tau)$ for $s \in \{e, l\}$. However, when data is censored, observations are instead drawn from the distribution

$$\Pr(\tau_{i,n}^s \leq \tau | t_{i,n}^s + \tau_{i,n}^s \leq T)$$

where T is the censoring date, set at seven years after the firm's first funding round in the baseline estimation.

In the model, the assumption of *ex ante* homogeneous firms implies that the relevant unit of observation within each stage is the funding round, j , not the firm-round number tuple, (i, n) . Furthermore, the duration to the next round, τ_j^s , is independent of the time of the current round, t_j^s , owing to the model's Markov property. Therefore,

$$\Pr(\tau_{i,n}^s \leq \tau | t_{i,n}^s + \tau_{i,n}^s \leq T) = \Pr(\tau_j^s \leq \tau | \tau_j^s \leq T_j)$$

where $T_j = T - t_j^s$ is the censoring date for a given funding round (e.g. with $T = 7$, the duration to the next round for a round at year $t = 4$ has $T_j = 7 - 4 = 3$).

This means that, under censoring T , funding round j in stage $s \in \{e, l\}$ is drawn from a truncated hypoexponential distribution. The truncation date, T_j is funding round specific and durations are independent across funding rounds in the model. Therefore, the likelihood for the data is given by

$$L^s = \prod_{j=1} \frac{f(\tau_j^s; h_1^s, h_2^s, T_j^s)}{F(T_j^s; h_1^s, h_2^s, T_j^s)}$$

where $f(\cdot)$ and $F(\cdot)$ are, respectively, the PDF and CDF of the hypoexponential distribution conditional on $\tau_j^s \leq T_j^s$ and (h_1^s, h_2^s) are the parameters of the hypoexponential distribution. The estimates are shown in Table C.1.

The second issue is to assign the estimated parameter values to $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$. I set the estimates for $\lambda + \nu_e = \max\{h_1^e, h_2^e\}$ and $\lambda + \nu_l + \hat{\phi}_s = \max\{h_1^l, h_2^l\}$. Choosing the larger value is conservative: it implies that firms spend relatively less time in search, biasing against finding

¹⁴³ N_i^e starts at 1, not 0, because the sample is firms that have at least one funding round, which is necessarily an early-stage round.

	UK			US		
	Point estimate	95% CI	N	Point estimate	95% CI	N
h_1^e	1.60	(0.91, 2.29)	468	1.54	(1.47, 1.61)	10,856
h_2^e	2.13	(0.93, 3.32)	468	3.30	(3.02, 3.57)	10,856
h_1^l	0.80	(0.58, 1.02)	331	0.89	(0.84, 0.93)	6,688
h_2^l	2.99	(1.34, 4.65)	331	5.94	(5.05, 6.83)	6,688

Table C.1: Estimates of hypoexponential distribution

The table reports parameter estimates for MLE estimates of hypoexponential distribution parameters, estimated on the duration between funding rounds within stages under round-specific censoring.

large frictions. For instance, in the late-stage, setting the estimate for $\lambda + \nu_l + \hat{\phi}_s = 5.94$ implies a search duration of approximately two months, rather than 13 months for $\lambda + \nu_l + \hat{\phi}_s = 0.89$. Furthermore, the value for $\kappa + \omega_l$ should be close to the other parameter value and, in practice, VCs seek to provide funding for one to two years, not several months.

Finally, computing the parameters by MLE is computationally costly. Therefore, for the simulated data, I simply take the values for $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ from the model, rather than estimating them. With $S = 20$ (see below), this is innocuous.

Implementation I simulate a sample of 220,600 firms, $S = 20$ times the size of the empirical 2005-2015 sample. Model parameters excluding $\tilde{\sigma}$, Θ , are jointly estimated by minimising the criterion

$$C(\Theta) = g(\Theta)'g(\Theta) \quad (\text{C.1})$$

where the j -th element of $g(\Theta)$ is given by

$$g_j = \frac{\tilde{m}_j - m_j(\Theta)}{\frac{1}{2}|\tilde{m}_j| + \frac{1}{2}|m_j(\Theta)|}$$

and where \tilde{m}_j is the empirical moment and $m_j(\Theta)$ is the model moment, computed under Θ . Standard errors are computed as

$$\widehat{\text{Var}}(\hat{\Theta}) = (G^\top W G)^{-1} G^\top W \Omega W G (G^\top W G)^{-1}$$

where $W = I$ is the identity matrix, Ω is the variance-covariance matrix of $g(\Theta)$, G is the Jacobian of moment conditions, $g(\Theta)$. I compute Ω via bootstrap. I draw $N = 11,030$ firms with replacement from my sample, compute the moments, and compute $g(\Theta)$ at my estimated Θ for that draw. I repeat this $B = 1000$ times. I then compute Ω as

$$\hat{\Omega} = \frac{1}{B-1} (g_b - \bar{g}_b)' (g_b - \bar{g}_b)$$

where g_b is an $B \times J$ matrix of moment conditions at each bootstrap, and \bar{g}_b is a $1 \times J$ vector that averages across bootstrapped samples.

The entry cost parameter, $\tilde{\sigma}$, is estimated from the model condition for the number of first funding rounds. To compute its standard error, I sample parameter Θ from $N(\hat{\Theta}, \text{Var}(\hat{\Theta}))$,

$B = 1000$ times, and compute $\tilde{\sigma}$ at each iteration. The standard error is then the standard deviation of the bootstrapped sample.

C.0.2 Robustness and extensions

Parameter	US, 2005-2015 (C7)		US, 2005-2010 (C7)		US, 2005-2010 (C12)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
ρ	0.08	—	0.08	—	0.08	—
κ	0.26	0.01	0.25	0.03	0.26	0.02
λ	0.62	0.04	0.71	0.10	0.64	0.15
p_e	0.23	0.01	0.18	0.01	0.18	0.01
k_e	4.16	0.18	3.81	0.35	3.79	0.23
ν_e	2.54	0.13	2.68	0.25	2.75	0.18
p_l	0.30	0.01	0.21	0.01	0.24	0.01
k_l	16.15	0.25	11.44	0.66	11.81	1.06
ν_l	5.20	0.26	5.87	0.63	6.70	0.66
π	218.18	15.40	223.42	32.07	220.95	29.39
ϕ	0.65	0.07	0.46	0.12	0.44	0.10
ξ	0.74	0.08	0.98	0.13	1.01	0.20
$\tilde{\sigma}$	2.68	0.37	1.60	1.00	2.80	1.11

Table C.3: Parameter estimates for the US under alternative sample periods and censoring

The table reports parameter estimates from the model in section 2.4. “US, 2005-2015 (C7)” replicates the US parameter estimates under the baseline sample period and seven-year censoring (see Table 1). “US, 2005-2010 (C7)” and “US, 2005-2010 (C12)” report estimates for the 2005-2010 subsample (firms first funded between 2005-2010), estimated under seven-year and twelve-year censoring, respectively. To maintain consistency, all estimations use a simulated sample of 220,600 firms ($S = 20$ times the size of the 2005-2015 empirical sample). Parameters (except $\tilde{\sigma}$) are estimated by minimising (21) (equal weighting of centred percentage deviations; see footnote 83) and I report asymptotic standard errors, where the covariance matrix of moment conditions is computed from 1,000 bootstrap samples, clustered at the firm level. With $S \geq 20$, simulation noise is minuscule, so I report standard errors that simply reflect sampling variability. To avoid local minima in the estimation, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points for each estimation. With the resulting candidate parameter estimates, I run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses. With these estimates, the entry cost parameter, $\tilde{\sigma}$, is recovered from the condition determining the flow rate of first-time funding rounds. To compute its standard error, I take 1,000 samples from the asymptotic distribution of parameter estimates, solve the model, and recover the implied value for $\tilde{\sigma}$. The standard error is the standard deviation of $\tilde{\sigma}$ among this sample. All amounts are in 2015 USD.

Parameter	Mean # early rounds	Mean # rounds to late	Mean duration b/w rounds	Estimate of $\lambda + \nu_e$	Estimate of $\lambda + \nu_l + \hat{\phi}_s$	Mean burn rate early round	Mean burn rate late round
Early-stage dev. level, p_e	-1.65	-0.02	0.94	0.00	0.00	1.18	0.02
Late-stage dev. level, p_l	-1.21	-0.75	1.14	0.00	-0.02	1.05	0.83
Early-stage cost, k_e	0.78	-0.01	-0.51	0.00	0.00	0.30	-0.02
Late-stage cost, k_l	0.39	0.65	-0.62	0.00	0.02	-0.32	0.27
R&D arrival rate, κ	-0.75	-0.06	0.04	0.00	-0.01	0.47	0.05
Success payoff, π	-1.19	-0.65	1.07	0.00	-0.02	1.04	0.78
Failure rate search, λ	-0.09	-0.44	0.13	0.20	0.11	-0.03	0.40
Acq. offer rate, ϕ	-0.47	-0.24	0.41	0.00	0.03	0.40	0.10
Mean acq. 'synergy', ξ	-0.82	-0.29	0.67	0.00	-0.00	0.70	0.32
Early-stage rate, ν_e	0.30	0.02	-0.17	0.80	0.00	0.06	0.02
Late-stage rate, ν_l	-0.06	0.35	-0.15	0.00	0.84	0.18	0.17

continued...

Parameter	Mean time-to-exit	Acq-to-success ratio	Mean success multiple	Share one round	Share funded after year 5	Share to late-stage multiple > 10X	Share acq
Early-stage dev. level, p_e	0.15	-0.31	-0.55	0.02	0.53	1.99	-2.12
Late-stage dev. level, p_l	0.11	-2.24	-1.14	0.70	0.01	0.69	-0.50
Early-stage cost, k_e	-0.02	0.20	-0.17	-0.50	0.02	-0.38	-0.01
Late-stage cost, k_l	-0.07	0.77	0.18	-0.23	-0.06	-0.18	0.54
R&D arrival rate, κ	-0.23	-1.12	0.01	0.40	-0.69	0.71	0.02
Success payoff, π	0.10	-1.19	-0.09	0.69	0.00	0.68	-0.40
Failure rate search, λ	-0.08	-0.02	-0.21	0.42	-0.52	-0.27	-0.39
Acq. offer rate, ϕ	-0.04	0.52	-0.24	0.31	-0.13	0.19	-0.13
Mean acq. 'synergy', ξ	-0.00	-0.30	-0.48	0.49	-0.03	0.41	0.29
Early-stage rate, ν_e	0.03	0.07	-0.02	-0.46	0.31	0.12	0.17
Late-stage rate, ν_l	0.02	-0.17	-0.06	-0.09	0.10	0.18	-0.46

Table C.2: Elasticity of moments with respect to parameters

The table reports the elasticity of each moment (or auxiliary parameter) with respect to each estimated parameter. Starting from the estimated parameter values, each parameter is individually adjusted by $\pm 1\%$, the model is simulated, and moments are recomputed using a centred difference.

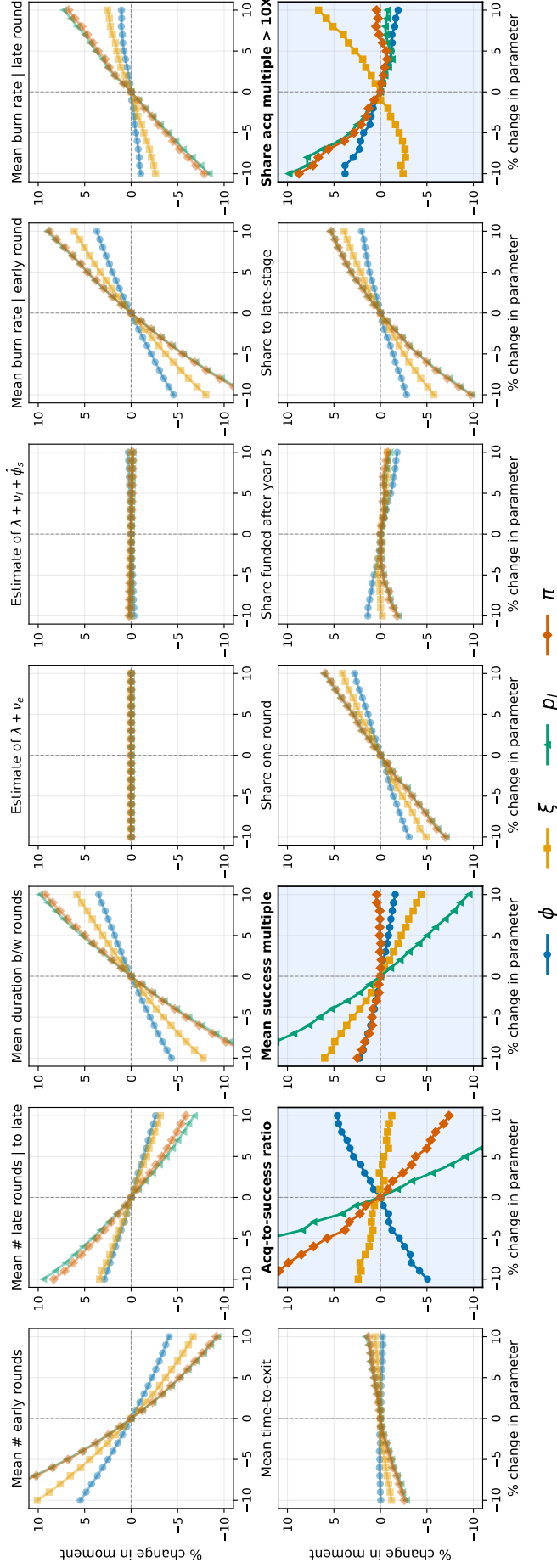


Figure C.1: Comparative statics – moments to parameter changes

Each panel plots the percent change in a moment when varying one parameter by $\pm 10\%$ around the baseline estimate, holding other parameters fixed. Lines correspond to parameters $\{\phi, \xi, p_I, \pi\}$. Baselines are normalized to zero; axes report percent changes. For comparability, y-limits are held at $\pm 10\%$ across panels. Shaded panels (acquisition-to-success ratio, mean success multiple, share of acquisitions with multiple $> 10\times$) are those most informative for separating $\{p_I, \pi, \xi, \phi\}$. Moments are computed from simulated data under the estimated baseline, with 1,000,000 simulated firms.

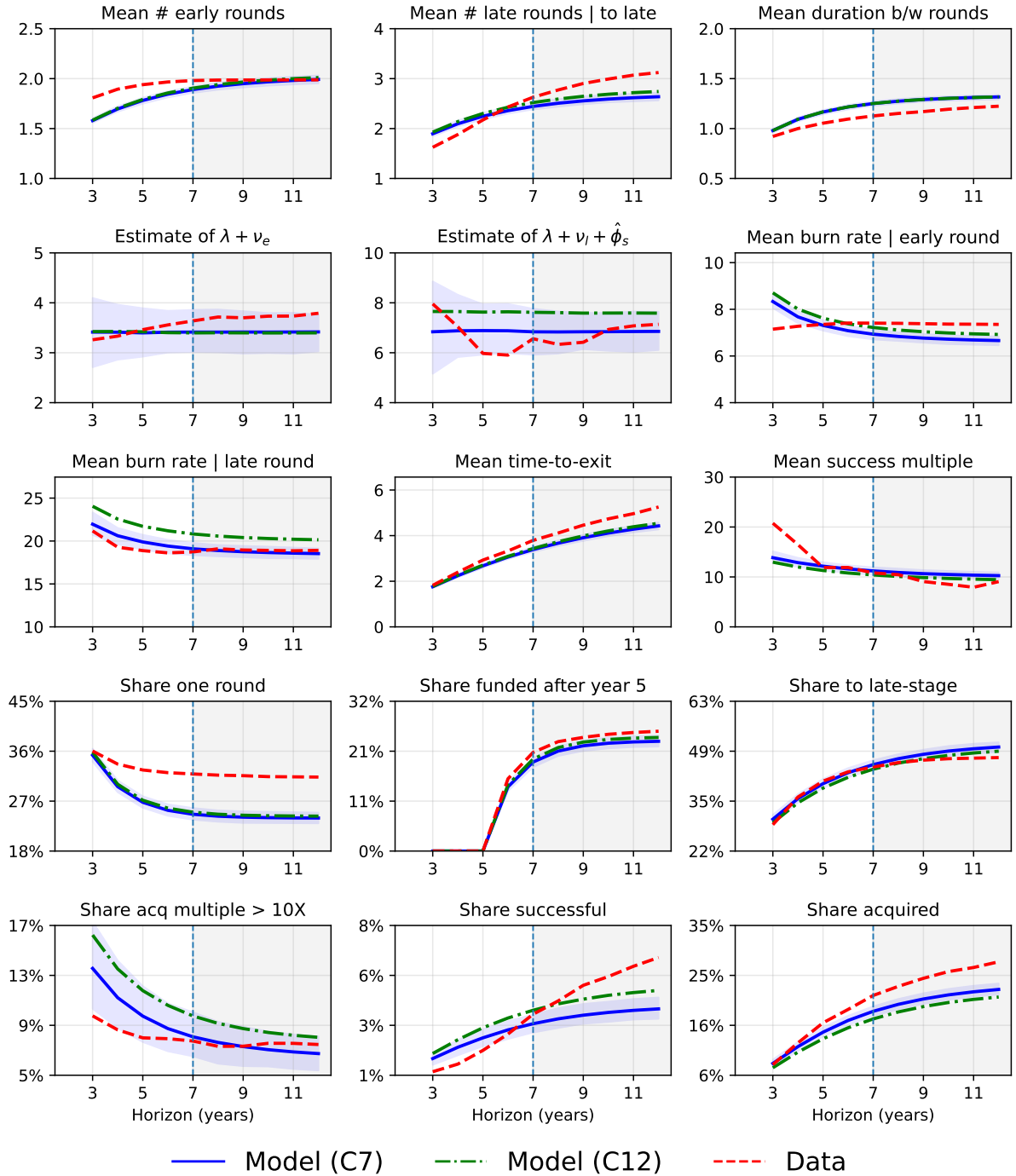


Figure C.2: Out-of-horizon censoring diagnostics (2005-2010 sample)

The figure plots moments computed on data censored at horizon h (x-axis) using the 2005–2010 sample (red, “Data”). Targeted moments and the share of acquisitions and successes are shown; the targeted acquisition-to-success ratio – the ratio of these two measure – is redundant and so excluded. Blue (“Model (C7)”) and green (“Model (C12)”) show model-implied moments when parameters are fixed at the estimates from the 7-year and 12-year censoring specifications, respectively; see Table C.3. For the empirical series, the data *are* re-censored at each h and moments recomputed. For the model series, I simulate 200 samples of the same size as the 2005–2010 data ($N = 5137$) and compute moments after censoring at each h . The blue band is a pointwise 95% simulation interval (2.5–97.5 percent quantiles) and reflects finite-sample variability under the model; parameter uncertainty is not included. The vertical line marks $h = 7$; the shaded region indicates out of sample outcomes with respect to the C7 specification.

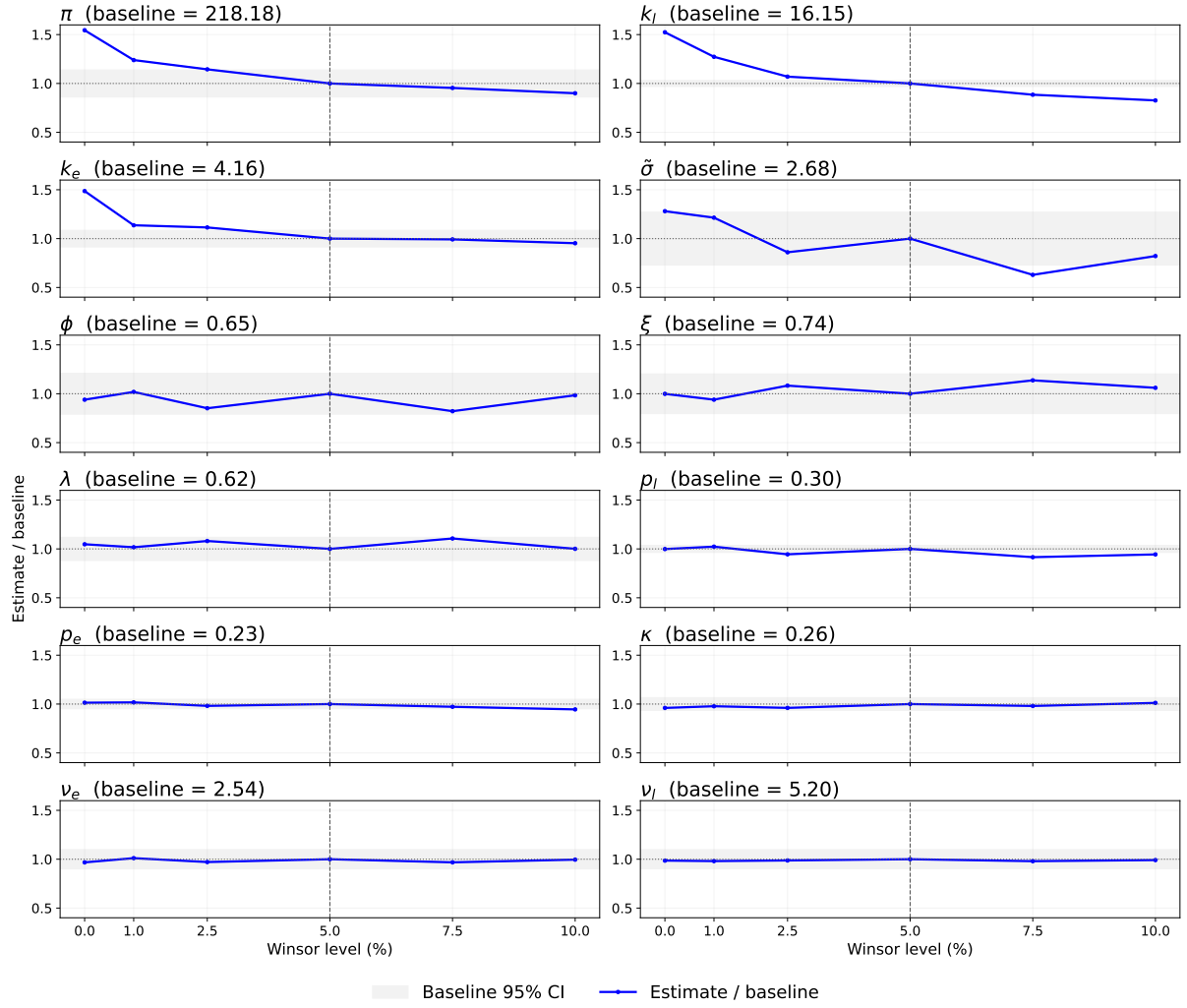


Figure C.3: Parameter estimates under different burn rate winsor thresholds

The figure plots parameter estimates normalised to 1.00 at $w = 5\%$ (baseline), under winsorising thresholds $w = 0\%, 1\%, 2.5\%, 5\%, 7.5\%, 10\%$. Gray bands show 95% confidence intervals for baseline parameter estimates; standard errors are reported in Table 1. Parameters are ordered by maximum percentage deviation from baseline. In each estimation, the mean burn rate is computed after winsorising the burn rate distribution (within stage); the same procedure is applied when computing burn rates from simulated data used in the estimation.

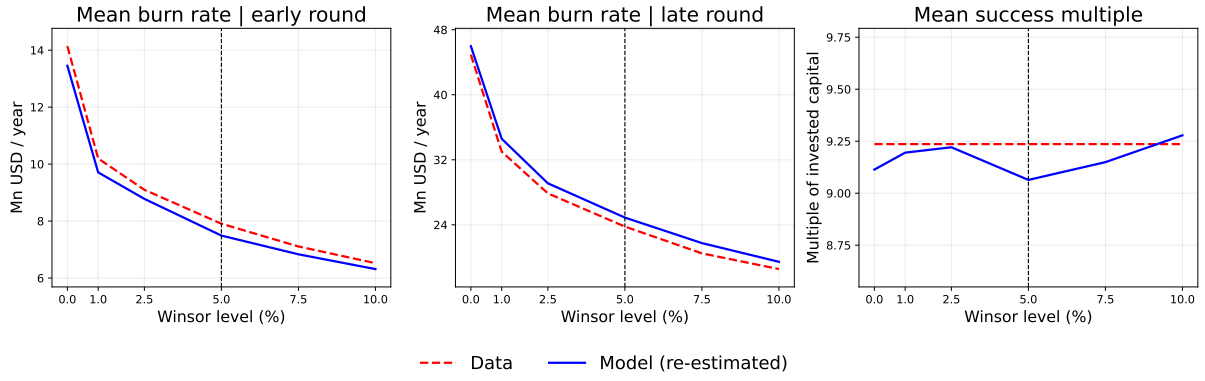


Figure C.4: Model fit under different burn rate winsor thresholds

The figure reports empirical and model moments under different burn rate winsor thresholds. The model fits well across thresholds; the focus on mean burn rates and the success multiple reflects that movements in k_e , k_l , and π (particularly relevant to these moments) are the largest among all parameters when the winsor threshold is varied.

C.1 Additional details on UK estimation

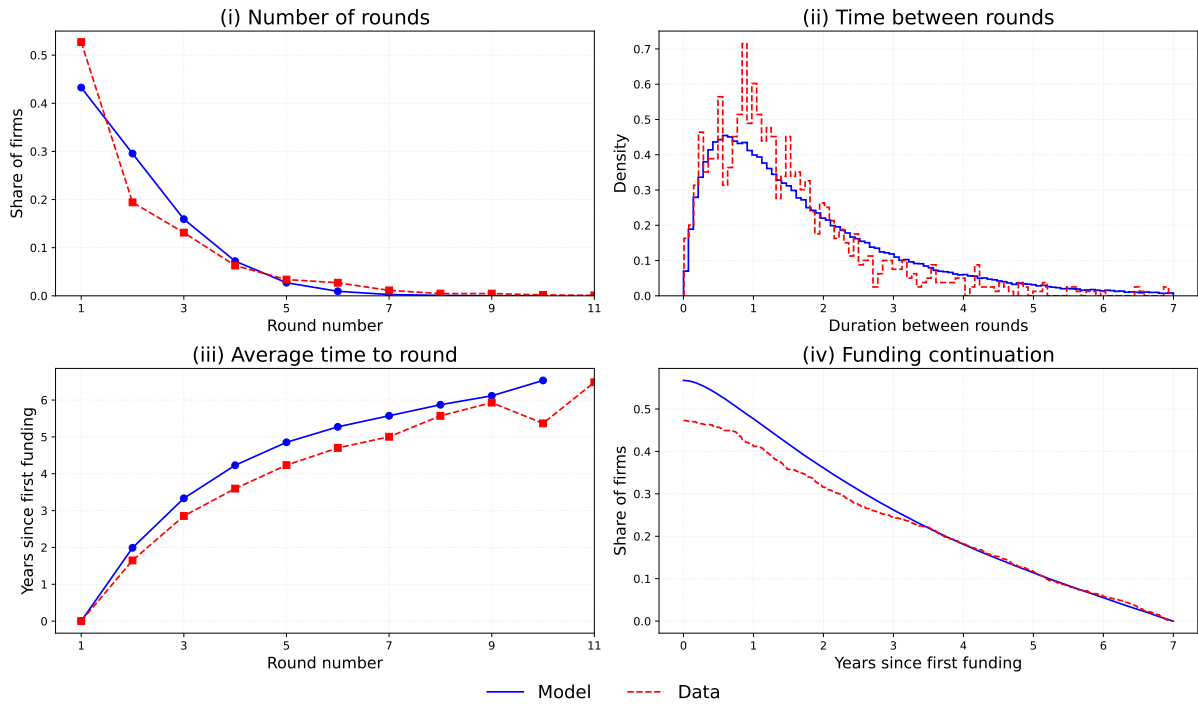
Table C.4 reports targeted and untargeted moments for the UK estimation; the UK parallel to Table 2. Figure C.5 shows the fit of the model to the data across a wider range of outcomes; the UK parallel to Figure 5.

Panel A: Targeted moments		
Moment	Data	Model
Mean # early rounds	1.44	1.44
Mean # late rounds progressed to late	1.94	1.69
Share one round (%)	52.72	43.15
Mean duration b/w rounds (years)	1.46	1.72
Estimate of $\lambda + \nu_e$	2.13	1.90
Estimate of $\lambda + \nu_l + \hat{\phi}_s$	2.99	3.31
Share receiving funding after year 5 (%)	11.73	11.52
Mean burn rate early round (\$Mn/year)	5.80	5.39
Mean burn rate late round (\$Mn/year)	18.79	19.43
Share to late-stage (%)	33.11	34.03
Mean time-to-exit (years)	3.82	3.32
Acquisition-to-success ratio	3.73	3.68
Mean exit multiple success	8.88	9.01
Share acquisition multiples > 10X (%)	11.34	11.54
# first funding rounds	0.48	0.48
Panel B: Untargeted moments		
Share successful (%)	2.44	3.46
Share acquired (%)	9.10	12.73
Mean time-to-success (years)	3.96	3.48
Mean time-to-acquisition (years)	3.78	3.27
Mean exit multiple	5.37	6.10

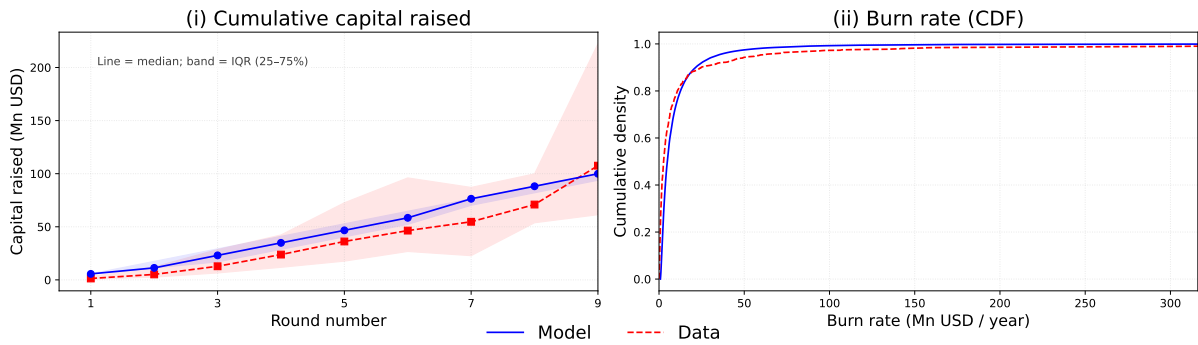
Table C.4: Targeted and untargeted moments

Panel A shows 15 moments (including two auxiliary parameter estimates) used in the estimation procedure. Empirical moments are computed from the sample of 1,066 firms. Other than for the estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, simulated moments are computed using the parameter values given in Table 1 to simulate a sample of 220,600 firms; preserving the same simulated sample size as for the US estimation, for consistency. Note that For $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, the parameter values are taken directly to reduce the computational burden of the estimation procedure. In computing the empirical and simulated moments, data on a given firm is censored seven years after its first funding round. Panel B shows a set of untargeted moments.

Panel A: Funding rounds



Panel B: Capital intensity



Panel C: Exits

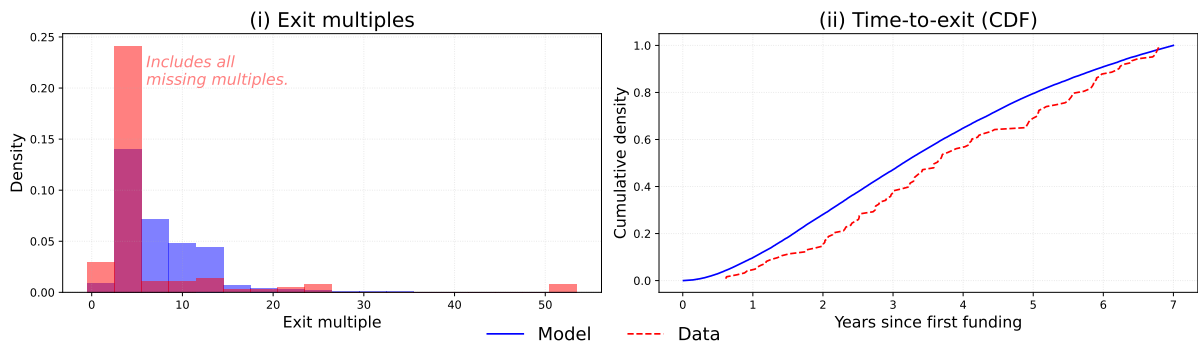


Figure C.5: UK Model validation

The figure shows outcomes in the model and data across funding rounds (panel A), capital intensity (panel B) and exits (panel C). All empirical and simulated data is censored at seven years following a firm's first funding round. "Funding continuation" in Panel A (iv) reports the share of firms that raise capital more than x years after their first funding round. To aid visualisation, cumulative capital raised is shown up to the 9th round; the burn rate CDF is reported up to the 99th percentile; and exit multiples are winsorised at 50X. Missing empirical exit multiples are set to 1.5X, following Kerr et al. (2014).

C.2 Additional details on sector calibration

C.2.1 Calibration strategy

This section provides details of the calibration exercise in section 5.

Accounting for heterogeneity in the model I define a technology, indexed by τ , as a bundle of start-up characteristics. There is a pool of potential entrants for each technology, so that multiple start-ups may pursue the same technology, τ . However, to account for the fact that some technologies may be more difficult to pursue than others, I allow the entry cost, $\tilde{\sigma}$, to be technology specific, i.e. $\tilde{\sigma}(\tau)$.

In the calibration exercise, I allow for two margins of heterogeneity: the rate project development horizon, parameterised by κ , and the availability of acquisition opportunities, parameterised by ϕ . Given my focus on the interaction of project characteristics with financial frictions, I also adjust $\pi(\tau)$ such that all projects have the same entry value, $V_{s,e}$, equal to the value that emerges from estimation of the homogeneous firm model in section 3. In practice, this means that for each technology τ with characteristics $(\kappa(\tau), \phi(\tau))$, I solve a fixed point for $\pi(\tau)$ such that $V_{s,e}(\tau) = V_{s,e}$. To summarise, in this section a ‘technology’ refers to a $(\kappa(\tau), \phi(\tau))$ -pair with associated payoff $\pi(\tau)$. Otherwise, all technologies have the same characteristics and face the same funding market conditions. These parameters are set at the values from the US baseline estimation.

Estimating the distribution of technologies I estimate the underlying distribution of start-ups across technologies, which I denote by $s(\tau)$, by targeting cross-sector variation in the data. In order to implement the estimation of $s(\tau)$, I need to introduce some additional notation. I denote the sectors, as depicted in Figure 8, by v (for VEIC) and denote the number of sectors by N_v . Associated with each sector is a weight, $s(v)$, and a point in the (acquisition-to-success, time-to-exit)-space, which I denote by \mathbf{x}_v , to simplify notation. Together, $(\mathbf{x}_v, s(v))$ defines the empirical joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space. I then construct a set of technologies, τ , by specifying a grid for κ and ϕ , where the total number of technologies is N_τ .¹⁴⁴ For each technology, I simulate the model to recover a point in the (acquisition-to-success, time-to-exit)-space, which I refer to as \mathbf{x}_τ . My objective is to choose the shares (or weights), $s(\tau)$, to minimise the distance between the empirical joint distribution, $(\mathbf{x}_v, s(v))$, and the model-implied joint distribution, $(\mathbf{x}_\tau, s(\tau))$. In order to compare the distributions, I need to select a measure of distance, of which there are many. I choose to adopt the energy distance metric, which is often used to test for the equality of distributions. Specifically, I select shares $s(\tau)$ to minimize the following distance metric

$$\min_{s(\tau)} \left\{ 2 \sum_{v=1}^{N_v} \sum_{\tau=1}^{N_\tau} s(v)s(\tau) \|\mathbf{x}_v - \mathbf{x}_\tau\| - \sum_{v=1}^{N_v} \sum_{v'=1}^{N_v} s(v)s(v') \|\mathbf{x}_v - \mathbf{x}_{v'}\| - \sum_{\tau=1}^{N_\tau} \sum_{\tau'=1}^{N_\tau} s(\tau)s(\tau') \|\mathbf{x}_\tau - \mathbf{x}_{\tau'}\| \right\} \quad (\text{C.2})$$

subject to $\sum_\tau s(\tau) = 1$ and $s(\tau) \in [0, 1]$, where $\|\cdot\|$ is the Euclidean norm. The energy distance metric minimises the distance between the joint distributions of empirical sectors and model-based technologies, taking account for the location and shape. Figure C.6 plots kernel density

¹⁴⁴In practice, I experiment with different grid spacings and grid points and choose a sufficiently granular grid such that adding additional grid points has negligible effects on the results.

estimates given the sector weights, $s(v)$, and estimated weights, $s(\tau)$, based on minimisation (C.2).

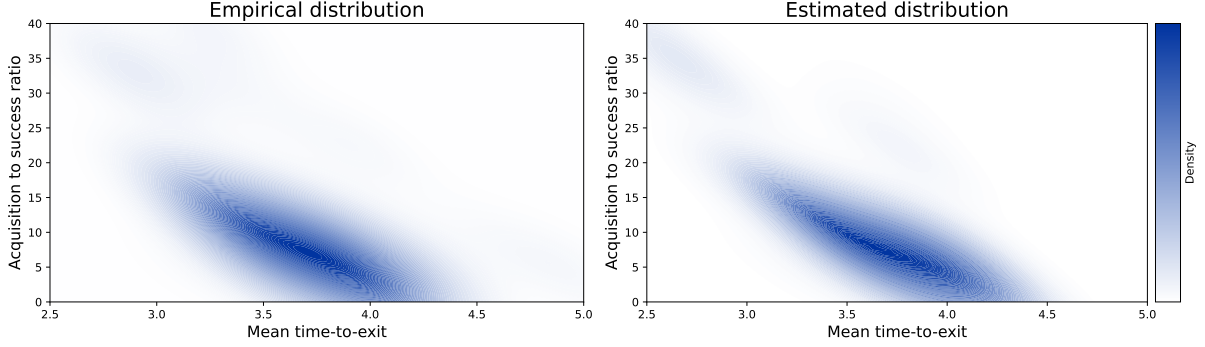


Figure C.6: Empirical and model-implied distributions

The left-side figure depicts kernel density estimates for the true joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space, which reflects the data shown in Figure 8. The right-side figure shows the kernel density estimate for the distribution of technologies in the same space, as estimated through minimisation of equation (C.2).

Recovering the counterfactual distribution of technologies Given the estimated technology shares, $s(\tau)$, it is simple to recover the frictionless shares, which I label $\bar{s}(\tau)$. In particular, the estimated entry shares satisfy equation (22) and the counterfactual entry shares satisfy equation (23). Given the estimates, $s(\tau)$, the technology-specific entry cost parameters, $\tilde{\sigma}(\tau)$, can be recovered up to a constant from (22). Then, it is simple to recover the counterfactual shares from equation (23). Doing so requires the frictionless entry value, $\bar{V}_{s,e}(\tau)$; that is, the entry value under self-financing. For a given technology τ , the frictionless entry value, $\bar{V}_{s,e}(\tau)$, is the solution to the following system of equations given in section A.5.1. Solving this system of equations for $\bar{V}_{s,e}(\tau)$ and substituting into equation (23) yields the counterfactual shares, $\bar{s}(\tau)$.

Mapping from technologies to sectors Finally, I map technologies into the ten aggregate sectors shown in Figure 8. Specifically, I allocate a portion $w(\tau, v)$ of the share of technology τ , $s(\tau)$, to each sector v by matching technologies and sectors in the (acquisition-to-success, time-to-exit)-space; the weights $w(\tau, v)$ then imply a mapping from technologies to aggregate sectors, since each sector v belongs to one aggregate sector. Using these weights and the frictionless technology shares, $\bar{s}(\tau)$, the final step of the counterfactual recomputes the aggregate sector shares in the counterfactual economy without financing frictions.

To implement the assignment between technologies and sectors, I set up a simple optimal transport problem as follows. I have two datasets: (i) an empirical dataset of sectors, $\{\mathbf{x}_v, s(v)\}_{v=1}^{N_v}$, and (ii) a model dataset of technologies $\{\mathbf{x}_\tau, s(\tau)\}_{\tau=1}^{N_\tau}$. The objective is create a mapping from technologies in the model dataset to VEIC sectors in the empirical dataset and, intuitively, I want to map technology τ to sector v if they are close in the (acquisition-to-success, time-to-success)-space, which is denoted by \mathbf{x} . The optimal transport problem selects $w(\tau, v)$

to solve the following problem

$$\min_{\{w(\tau, v)\}} \left\{ \sum_{\tau=1}^{N_\tau} \sum_{v=1}^{N_v} s(\tau) w(\tau, v) c(\tau, v) \right\}$$

subject to

$$\text{Supply constraints: } \sum_{v=1}^{N_v} w(\tau, v) = 1, \quad \forall \tau$$

$$\text{Demand constraints: } \sum_{\tau=1}^{N_\tau} s(\tau) w(\tau, v) = s(v), \quad \forall v$$

$$\text{Non-negativity constraints: } w(\tau, v) \geq 0, \quad \forall \tau, v$$

where $c(\tau, v)$ is the transportation cost from technology τ to sector v , which I specify as the Euclidean distance in the (acquisition-to-success, time-to-success)-space, $c(\tau, v) = \|\mathbf{x}_\tau - \mathbf{x}_v\|_2$.

Briefly, a total mass $s(\tau)w(\tau, v)$ is transported from technology τ to sector v , which has cost $c(\tau, v)$ per unit. The total transportation costs is sums over all transportations. The supply constraint states that all mass from technology τ to sector v is transported somewhere. I define $w(\tau, v)$ as the weight on share of mass from τ that is associated with sector v , so these sum to one. The demand constraint ensures that all sectors are completely accounted for. The optimal transport plan minimises the total transport cost subject to these ‘summing-up’ constraints.

C.2.2 Additional tables and figures for the sector calibration

Sector	Observed	Counterfactual	Δ
Biotech	0.09	0.13	+0.044
Medical/Health	0.10	0.13	+0.029
Semiconductors/Other Elect	0.03	0.04	+0.008
Consumer Related	0.03	0.03	+0.004
Industrial/Energy	0.05	0.05	+0.002
Internet Specific	0.28	0.24	-0.044
Computer Software and Services	0.33	0.29	-0.037
Other Products	0.04	0.04	-0.003
Comms and Media	0.03	0.02	-0.002
Computer Hardware	0.03	0.03	-0.001

Table C.5: Sector reallocation

The table shows observed sector weights and counterfactual sector weights under self-financing. Counterfactual shares are based on the calibration exercise in section 5.1. Heterogeneity in the (κ, ϕ) -space is first calibrated using cross-sector variation in average time-to-exit and the acquisition-to-success ratio. ‘Technologies’ in the model are then mapped to VEIC sectors based on proximity in the (acquisition-to-success, time-to-exit)-space via optimal transport.

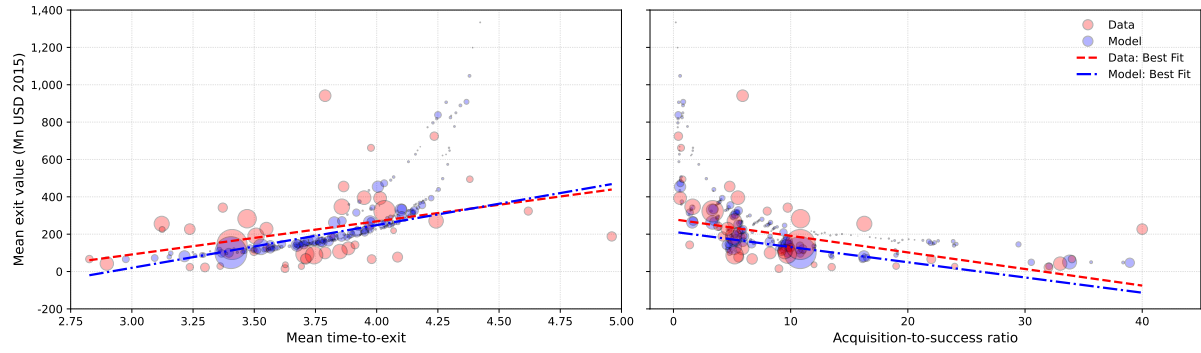


Figure C.7: Exit values

The figure plots the mean exit value against the mean time-to-exit and acquisition-to-success ratio for VEIC sectors (red) and model-based technologies (blue). Missing empirical exit values are populated to yield a 1.5X exit multiple, following Kerr et al. (2014). The size of each point is proportional to the sector/technology's share; lines are of best fit are weighted accordingly.