

Start-up Financing, Entry and Innovation

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Abstract

Venture capital (VC) is the key source of financing for high-growth start-ups, yet VC activity varies markedly across countries and sectors. To study the drivers and implications of these patterns, I develop and estimate a tractable equilibrium model of the VC market. The model features frictional matching between entrepreneurs and VCs, and VCs endogenously stage capital injections over time to limit losses from hidden failure by entrepreneurs. The model maps directly to observed funding histories, enabling estimation and policy counterfactuals. Estimated on US start-ups first funded in 2005–2015, I find that removing financing frictions in the VC market raises the share of high-growth start-ups that achieve acquisitions or IPOs from 30% to 50%. To examine cross-country differences, I estimate the model separately on UK microdata and find that differences in VC activity reflect tighter financing and thinner acquisition markets, not start-up quality. Counterfactuals show gains from retargeting existing government support: reallocating 5% of total VC funding from early to late-stage start-ups increases IPOs by 15%. Finally, the theory implies a bias against long-horizon projects with few acquisition opportunities; a sectoral calibration indicates that, in a frictionless benchmark, the share of VC-backed software and services start-ups would fall from 61% to 53%, offset by gains in science-based sectors.

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1 Introduction

Entrepreneurship is a central driver of economic growth, but realising its benefits relies on the efficient allocation of capital. While most firms secure loans against pledgeable assets or predictable cash flows, high-growth start-ups with limited revenues face difficulty financing operations with debt (Hall and Lerner, 2010). These firms instead seek funding from specialist intermediaries – venture capitalists – who, unlike banks, supply equity and finance operating losses in pursuit of substantial payoffs. Venture capital (VC) has funded many of the largest and most innovative US firms; yet, concerns remain that VC fails to penetrate certain sectors and that limited access to VC for European start-ups constrains innovation.¹

Understanding VC allocation and outcomes is challenging because geographic and sectoral patterns confound the quality of projects with financing conditions. Lower VC activity in Europe and in science-based sectors could reflect more limited opportunities or frictions in allocating capital to entrepreneurs; and the drivers matter for policy design. However, because *ex ante* start-up quality is poorly observed, disentangling supply and demand for capital is challenging. Furthermore, issues in obtaining external financing mean some start-up projects may be terminated prematurely. Empirically, distinguishing poor fundamentals from premature termination is difficult because the counterfactual under continued financing is unobserved.

To address these challenges, I develop and estimate a tractable equilibrium model of the VC market. The model connects tightly to microdata on start-up funding histories, enabling estimation and policy counterfactuals, and separating project quality from financing conditions. The model features frictional matching and staged financing; VCs inject capital in stages to limit losses when entrepreneurs hide failure. However, staging means the start-up must reach profitability or raise more capital before funding runs out. In turn, difficulties raising follow-on funding create financing risk, akin to rollover risk in debt markets, and mean some viable start-ups may fail to secure additional funding and shut down prematurely.² Using the model, I quantify the effects of financing frictions in the VC market on start-up activity in the US, unpack cross-country differences in VC activity, and assess its sectoral concentration.

For the US, removing matching and agency frictions increases the share of start-ups with successful exits (acquisition or IPO) by 20pp (from 30% to 50%). Across countries, I find that differences in VC activity – specifically between the US and UK – reflect tighter financing conditions and thinner acquisition markets, rather than the quality of projects. In the UK, retargeting existing support improves outcomes: reallocating 5% of total VC from early to late-stage rounds raises IPOs by 15%. Finally, the model indicates a bias against long-horizon projects with few acquisition opportunities: without frictions, the share of VC-backed software and services start-ups would fall from 61% to 53%, offset by growth in science-driven sectors.

The model features frictional matching between entrepreneurs and VCs and a principal-

¹Since 1978, approximately 50% of US IPOs were VC-backed, generating over 90% of R&D spending (Gornall and Strebulaev, 2021); 1978 marks the start of the modern VC model, when the capital gains and ERISA “prudent man” reforms were enacted (Gompers, 1994).

²Nanda and Rhodes-Kropf (2017) introduce financing risk to the academic literature. Paul Graham (Y Combinator founder) frames this as being “default alive” vs “default dead”: “do they make it to profitability on the money they have left?” He cautions that founders often assume raising again will be easy, but “that assumption is often false.” See Default Alive or Default Dead?, <https://www.paulgraham.com/aord.html>.

agent problem: entrepreneurs can hide failure *ex post*, creating an external-finance premium. In response, VCs stage capital injections over time so only a small tranche is at risk between reviews. However, because VCs typically rely on future investors for additional rounds, initial commitments are forward looking and VCs commit more funding upfront when follow-on funding is uncertain. This margin was evident in 2008 when Sequoia advised portfolio companies to “raise as much as possible”.³ Incorporating these mechanisms, the model maps directly to microdata on funding histories (timing, size and frequency), which enables estimation and counterfactuals. It also delivers implications for the sectoral allocation of VC investment.

The matching friction captures the difficulty in evaluating projects, which complicates the process of securing investment, and may delay market clearing and create congestion. These problems arise even for ultimately successful ventures: for example, Airbnb’s Brian Chesky described facing a series of rejections when introduced to prominent Silicon Valley investors.⁴

If a VC agrees to invest, the separation of ownership and control exposes them to moral hazard. Entrepreneurs invest in R&D and come to learn more about the project’s viability, but this information is private and can be hidden from VCs, leading to “inefficient continuation” (Gompers, 1995).⁵ This behaviour induces an external-finance premium that depends on: (i) the project’s novelty, which affects the likelihood of negative news; (ii) the amount of capital injected; and (iii) the project’s development horizon, which together determine how much capital is left when negative news arrives. The dependence of the premium on novelty and development horizon implies that financing frictions interact with project characteristics, and the dependence on the size of the capital injection incentivises staged financing.

Staging capital injections over time means that when the entrepreneur hides failure, less capital is wasted. However, because VCs often lack the ability or will to fund a start-up to positive cash flow on their own, follow-on funding rounds require attracting new investors. The information and project screening issues that arose when the firm first sought funding resurface, and additional funding may not be forthcoming even for viable projects.⁶ The tendency of the VC market to suddenly dry up exacerbates this problem (Janeway et al., 2021).

These dynamics create “financing risk”: viable start-ups may not secure follow-on funding (Nanda and Rhodes-Kropf, 2017).⁷ To capture this in the model, start-ups must match with a new VC for each capital injection and, during their search, the lack of funding can force premature closure. Small capital injections mitigate agency costs but increase reliance on future funding, raising exposure to financing risk. Market tightness determines the availability of follow-on funding and so financing risk is endogenous. The optimal capital commitment balances these considerations, prompting small capital injections when agency frictions are severe, and large commitments when the VC forecasts limited future funding availability.

³See <https://articles.sequoiacap.com/rip-good-times>. Implicit is that Sequoia was not offering to provide follow-on funding to unilaterally, instead relying on other investors to meet their financing needs.

⁴See <https://medium.com/@bchesky/7-rejections-7d894cbaa084>. On the investor side, Bessemer Venture Partners, a leading VC firm, maintains a list of companies that it chose not to invest in, including Apple, Intel and Google (see www.bvp.com/anti-portfolio).

⁵Entrepreneurs derive private benefits from operating their own firms (Hurst and Pugsley, 2011) and so are unlikely to want to shut them down upon the arrival of bad news.

⁶For investors, syndication aids diversification by enabling them to spread capital among more firms, and provides external validation to investments (Kerr et al., 2014; Nanda and Rhodes-Kropf, 2018).

⁷Financing risk has parallels to rollover risk in debt markets (Acharya et al., 2011; He and Xiong, 2012).

Three key insights emerge from the theoretical analysis. First, some projects are more difficult to finance than others. This is true for more novel projects, as in Nanda and Rhodes-Kropf (2017), but also for long-horizon projects. For these projects, *ex ante* investment incentives are weaker – so fewer are funded – and, conditional on funding, success is less likely. Second, funding histories encode information about project characteristics and the funding environment. Specifically, moments such as the number of and duration between financing rounds depend on the tightness in the financing market, project novelty and development horizon, and a notion of project quality. Therefore, the model provides a structural interpretation of the data, which facilitates learning these primitives from observables. Third, there is a novel contracting externality. The choice of capital injection by a given start-up and VC affects the meeting rate in the financing market; for instance, because longer-duration funding means the start-up is less likely to seek follow-on funding in the future. Agents in the model do not internalise this externality, so contracts are generally inefficient.

I estimate the model using VC funding round and exit data for all US start-ups that received their first early-stage funding round between 2005 and 2015. To make realistic quantitative statements while retaining the core economics, I make two simple adjustments to the baseline model. First, I include a distinct role for acquisitions, which act as a substitute for VC financing. Most start-up exits occur through acquisitions, making their inclusion essential. Second, I introduce multiple hurdles in a start-up’s journey to full development. This adjustment means the model can capture differences in how early-stage and late-stage start-ups access funding, reflecting the greater information investors have as start-ups progress.

The model captures rich patterns of the data, including the full shape of the distributions of the number of funding rounds, the duration between rounds, exit multiples, the burn rate (i.e. rate of capital utilisation) and time-to-exit dynamics. These features emerge naturally from the model’s structure, rather than from estimating any flexible functional forms. A key insight is that observed differences in funding patterns and outcomes across firms can be explained by shocks and information revealed after initial funding, even for start-ups that appear identical at the outset. This uncertainty highlights a core challenge for VCs: identifying which start-ups in their portfolios will succeed and which will not (Kerr et al., 2014).

Using the estimated model, I analyse the role of project characteristics versus financing in determining start-up outcomes. Simulating the model, I find that roughly 60% of US VC-backed start-ups are either acquired, remain independent and scale, or face technological or commercial challenges that lead to closure. The remainder succumb to issues in accessing capital, implying that they shut down with positive NPV projects or, equivalently, that continuation would have been optimal for a cash-rich entrepreneur. When matching and agency frictions are removed, these firms are funded to completion, which increases the overall share that remain independent and scale – e.g. through an IPO – from 5% to 11% (+6pp) and pre-commercial acquisitions from 25% to 39% (+14pp); as a result, overall successful exits rise from 30% to 50% (+20pp).

I then use the model to study cross-country differences in venture capital activity. The US is not alone in its strong scientific capabilities but is far superior in commercialisation.⁸ Difficulties

⁸ Among the top 1% most cited scientific publications, China accounted for 27.3%, the US 21.7%, the EU 17.8% and the UK 6.1% in 2020 (European Commission, 2024). However, among the 2,000 largest R&D investors in 2023, 681 were US-headquartered, 524 from China, 322 from the EU and 63 from the UK (Nindl et al., 2024).

in translating discovery into commercial application in Europe have led to the promotion of policies to tackle a perceived “funding gap” (Draghi, 2024); however, it is *a priori* unclear whether lower VC activity in Europe reflects more limited investment opportunities or issues in the efficient allocation of risk capital to entrepreneurs. The lack of available metrics of *ex ante* start-up quality compounds the identification challenge. I overcome this problem using the model: both project quality and financing constraints can explain lower VC activity compared to the US, but they leave different footprints in funding histories. Estimating the model then allows me to uncover the root causes of cross-country differences. I focus on a comparison of the US and UK, where London’s status as a global financing centre, its strong university system, and comparable legal environment make the relative lack of VC activity particularly puzzling.⁹

The estimation reveals that UK VC-backed start-ups face tighter financing conditions, compounded by a thinner acquisition market, with no discernible difference in project quality. This justifies ongoing policy initiatives to release institutional capital and government schemes that attempt to plug funding gaps. However, I show that the effectiveness of government intervention depends on where it is targeted. Many programmes have traditionally targeted seed and early-stage finance, reflecting more severe information asymmetries and limited pledgeable collateral at this stage (Wilson, 2015). However, high-growth start-ups typically remain unprofitable for many years and late-stage capital is critical to support these firms to positive cash flow.¹⁰ Furthermore, in the UK, a lower share of VC-backed start-ups access late-stage capital.¹¹ In a counterfactual exercise, I find that a policy that reallocates 5% of total VC funding from the early to late-stage improves outcomes: fewer firms are initially funded, but are more likely to raise late-stage capital and scale independently, rather than be acquired at pre-commercial stages. Pre-commercial acquisitions fall by approximately 9%, but the number of firms that scale projects independently rises by 15%, leading to greater value creation. Reassuringly, recent interventions recognise the importance of supporting late-stage start-ups.¹²

Finally, I study the allocation of venture capital funding across technologies. The theoretical analysis highlights that projects with long development horizons and limited established exit pathways are particularly susceptible to financing frictions. My calibration suggests that the effects could be sizeable and points to rather substantial misallocation of VC funding across sectors. Without financing frictions, the share of VC-backed start-ups in ‘Computer Software and Services’ and ‘Internet Specific’ sectors, both of which are dominated by software and service firms, would fall from 61% to 53%, offset by growth in science-driven sectors.

My results have implications for competition policy on start-up acquisitions. The policy debate weighs potential harms, such as market power and dampened innovation incentives for incumbents, while recognising that acquisitions induce start-up creation (Fons-Rosen et al., 2021). In the calibration, marginal changes to acquisition prospects have limited effect on entry incentives for short-horizon projects, where acquisition prospects are already plentiful. However, the effects are amplified for technologies with long gestation lags and where the acquisition

⁹Evidence from developing-country private equity shows that common-law jurisdictions use US-style separation of cash-flow and control rights (Lerner and Schoar, 2005); the US and UK share this legal origin.

¹⁰Figure B.1 shows that most U.S.-headquartered, VC-backed IPO firms were unprofitable at the time of listing.

¹¹Just 33% of UK start-ups in my sample raise late-stage funding within seven years of their first funding round, compared to 42% in the US.

¹²In 2018, the British Business Bank launched British Patient Capital, a late-stage fund of funds.

market is thin. Practically, increased scrutiny of start-up acquisitions for software and service start-ups is therefore unlikely to have dramatic consequences for start-up creation on the margin, but in science-based sectors, policymakers should be more wary of detrimental effects.

Related literature The paper relates to several areas of existing study. A large literature in entrepreneurial finance studies the control mechanisms VCs use to facilitate funding for projects that would typically be rationed from credit markets.¹³ Screening/selection (Sørensen, 2007; Gompers et al., 2020), active monitoring and expertise (Hellmann and Puri, 2002; Hsu, 2004; Bernstein et al., 2016), and contracts that separate cash flow and control rights (Kaplan and Strömberg, 2003; Ewens et al., 2022) are important determinants of outcomes.

I focus on staged financing and the need for multiple investors to coordinate to fund a firm until it becomes cash-flow positive. Staging is designed as a form of monitoring to curb inefficient continuation (Gompers, 1995; Lerner, 1998; Tian, 2011) and is the primary governance tool used by VCs (Sahlman, 1990; Hall and Lerner, 2010).¹⁴ However, initial investors rarely commit to fund a firm unilaterally and later rounds typically bring in new investors.¹⁵ This exposes firms with little interim cash flow to financing risk (Nanda and Rhodes-Kropf, 2017); when follow-on funding is unavailable, viable projects can be forced to shut down. Relative to this literature, my contribution is to embed these mechanisms in a tractable equilibrium model that can be estimated from round-level data. In the model, VCs worry that entrepreneurs hide failure and therefore stage injections over time; but this exposes the firm to financing risk when it seeks follow-on funding.¹⁶ I use the framework to quantify financing risk in the US, assess funding gaps across markets and consider policy interventions, and study the allocation of funding across technologies, providing insights into issues funding certain sectors (Lerner and Nanda, 2020).

This paper also builds on existing models of the venture capital market, which fall into two main strands: (i) theoretical analyses of the VC market directly; and (ii) macroeconomic models that embed VC into quantitative environments. In line with the former, I focus on developing a theoretical equilibrium model of the VC market; like the latter, I estimate the model and consider policy-relevant counterfactuals.

Among theoretical contributions, Inderst and Müller (2004) is first to model VC as a search-and-matching market and studies how market tightness shapes contracting under double-sided moral hazard. Michelacci and Suarez (2004) and Silveira and Wright (2016) also develop search and matching models¹⁷; Sannino (2024) studies VC value-added in an equilibrium environment; and Jovanovic and Szentes (2013) links VC excess returns to VC scarcity (and estimates them).

¹³Shocks to VC supply matter for innovation and start-up activity (Kortum and Lerner, 2000; Samila and Sorenson, 2011; Chen and Ewens, 2025), indicating that VC is not a mere substitute for traditional finance.

¹⁴Staging can also mitigate hold-up/renegotiation (Neher, 1999) and facilitate learning (Bergemann and Hege, 1998). Importantly, the VC retains abandonment options (see e.g. Cornelli and Yosha 2003; Bergemann and Hege 2005; Dahiya and Ray 2012).

¹⁵See Figure 1; section 2.1 includes a further discussion. From an optimal contracting perspective, Admati and Pfleiderer (1994) show that combining new and existing investors at subsequent funding rounds disciplines insiders' overpricing of new securities to outsiders and mitigates the *ex post* information monopoly the firm faces when raising new capital solely from insiders (as in Sharpe 1990; Rajan 1992).

¹⁶Mayer (2022) studies hidden failure; optimal contracts induce disclosure and can imply premature termination. I abstract from failure payments and study market equilibrium with many investors, but the contract retains the feature that VCs risk premature termination to curb inefficient continuation decisions by the entrepreneur.

¹⁷They study the role of the stock market in recycling capital and the VC fund lifecycle, respectively.

I follow Inderst and Müller (2004) in adopting a search-and-matching framework, but my focus on staged financing differs from existing work. Theoretically, firms compete for capital across “cohorts” and contracting is generally inefficient because neither entrepreneurs nor VCs internalise how contract choices affect (i) the timing of future fundraising and (ii) the availability of funds for other start-ups. Empirically, my focus on staging means the model is readily confronted with data on start-up funding rounds.¹⁸ The result is a theory that draws closely on insights from entrepreneurial finance, can be transparently estimated from funding histories and outcomes, and delivers quantitative measures of market-level distortions. This, in turn, allows me to consider the determinants of the geographic and sectoral concentration of VC funding.

Several papers incorporate VC into quantitative models.¹⁹ Closest to my paper, Greenwood et al. (2022) develop a dynamic contracting view of VC within an endogenous growth model; VCs offer long-term contracts, similar to those studied in Bergemann and Hege (1998) but with the addition of re-evaluation between capital injections. Two differences are worth noting. First, I stress the practical consideration that VCs do not finance a start-up to positive cash flow unilaterally. One key area where this matters is for assessments of the development of the VC market – relevant to both papers’ cross-country analyses – because less-experienced investors often struggle to raise large funds, rendering the full single-investor long-term contract impractical.²⁰ Second, the source of contracting inefficiencies differ; my focus is hidden failure/inefficient continuation, whereas Greenwood et al. (2022) considers diversion of funds. In that sense, the approaches are complementary. However, I show that focussing on inefficient continuation yields a structural interpretation of VC data, which facilitates estimation of project fundamentals and funding market conditions from standard data on funding histories.

Finally, my applications to cross-country differences and sectoral concentration in VC activity relate to the literature on interaction of financial and economic development (King and Levine, 1993; Rajan and Zingales, 1996). A large literature studies the effect of financial frictions on entrepreneurs quantitatively (e.g. Buera et al. 2011; Cavalcanti et al. 2023). A common – albeit not universal (e.g. Greenwood et al. 2010) – feature of these models in the macro-development tradition is that entrepreneurs can self-finance and gradually relax constraints via retained earnings. This differs from my model, where cash flows are delayed and firms rely exclusively on external financing. The distinction matters for the design of policy interventions. In a model in the macro-development tradition calibrated to Europe, Aragonese and Saxena (2025) find welfare gains from reallocating government VC budgets from the late to early-stage. In my UK analysis, I find the opposite, despite estimating that funding gaps are more severe in

¹⁸With severe agency issues, the duration between rounds should shorten and the frequency of injections rise (Gompers, 1995; Hall and Lerner, 2010). However, short duration funding raises financing risk (Nanda and Rhodes-Kropf, 2017); much as short-duration contracts increase rollover risk in debt markets (Flannery, 1986; Diamond, 1991; He and Xiong, 2012; Brunnermeier and Oehmke, 2013). So, when the market tightens, durations lengthen and the frequency falls. I formalise these insights and use them to estimate primitives via the model.

¹⁹Ates (2018) and Akcigit et al. (2022) embed VC in endogenous-growth models where VCs as provide of expertise as well as finance; Ando (2024) considers angels, in addition to VCs and banks; Ando et al. (2025) studies VC and advanced technology adoption; and Opp (2019) develops an endogenous growth model in the macro-finance tradition, focussing on VC’s risk properties and cyclical nature.

²⁰In Europe, a common concern is that limited fund size constrains capital for start-ups, particularly for larger, late-stage investments (Arnold et al., 2024). Because investor experience/quality drives returns (Sørensen, 2007; Ewens and Rhodes-Kropf, 2015) but is imperfectly observed (Gompers, 1996), more experienced investors tend to raise larger funds (Gompers and Lerner, 1998; Kaplan and Schoar, 2005).

the early stage (as in Aragonese and Saxena 2025). In my setting, both entry and early-stage investment hinge on the availability of late-stage capital, which is needed for firms to reach profitability. Taken together, the results are complementary and highlight an important nuance of policy design. When self-financing is most difficult, governments should be especially concerned about late-stage funding gaps; otherwise, relaxing constraints for early-stage start-ups is likely optimal. Relatedly, I show that acquisitions are most important in mitigating financing frictions for long-horizon technologies, where cash flows are most delayed. This has implications for debates on the role of start-up acquisitions in driving, or deterring, innovation (e.g. Kamepalli et al. 2020; Fons-Rosen et al. 2021; Cunningham et al. 2021; Berger et al. 2025).

The remainder of the paper is organised as follows. In section 2, I motivate the modelling environment, specify a baseline tractable model and then introduce two extensions to a quantitative framework. Section 3 estimates the model for the US, section 4 conducts a cross-country comparison, and section 5 analyses venture capital across sectors. Section 6 concludes.

2 Model

2.1 Economic environment and modelling choices

In this section, I describe the key building blocks for the equilibrium model. The modelling environment is motivated by three features of the market. First, difficulty evaluating projects complicates the process of finding willing investors. Second, agency conflicts lead investors to be cautious of committing too much capital to any given project. Third, institutional features of the market create a need for multiple investors to coordinate to fund a given start-up.

In the model, I incorporate the first and third feature by requiring that entrepreneurs meet with new investors for each capital injection in a frictional matching market. I also provide microfoundations for an agency conflict that creates a wedge between the internal and external cost of funds, which leads to staged financing. In so doing, I make assumptions to retain tractability: my objective is to develop a model that admits closed-form solutions, both to elucidate the interaction of matching and agency frictions and to make the mapping to data transparent, so that it is clear which features of the data identify the model’s primitives.

The remainder of this section provides further details on the three features of the market that motivate the modelling environment and discuss how they can be translated into a tractable theory of the venture capital market. Section 2.2 specifies the model.

The economic setting Consider an entrepreneur with a new business idea. The project requires a continuous R&D investment k to learn about its viability, which is revealed at an uncertain future date $T_\kappa \sim \text{Exp}(\kappa)$. The idea is successful with probability p , yielding a payoff π , and otherwise fails, returning zero. The parameters (k, κ, p, π) are the project’s *characteristics*, which summarise the capital intensity, development horizon, and payoff profile of the project.

Suppose that the entrepreneur is wealthy and can self-finance the project. Assuming risk neutrality and discount rate ρ , the project NPV is given by

$$\bar{V}_s = E \left[e^{-\rho T_\kappa} p \pi - \int_0^{T_\kappa} e^{-\rho t} k dt \right] = \frac{\kappa p \pi - k}{\rho + \kappa} \quad (1)$$

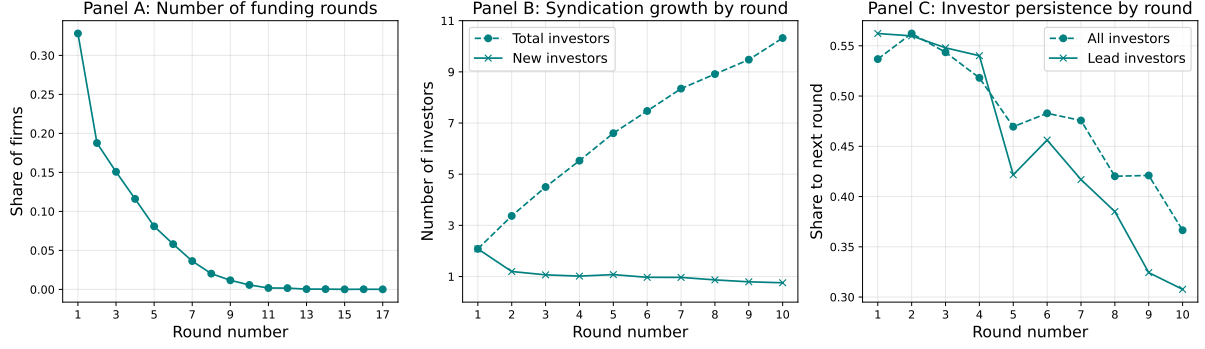


Figure 1: Staged financing and investor syndication

Source: Thomson Reuters, US VC-backed start-ups first funded between 2005–2015, including data to end-2022 (see section B.1). Data is censored at 7 years after initial funding (panel A, to ensure uniform censoring) or 12 years (panels B and C). Panel A shows the distribution of the number of rounds. Panels B and C summarise syndication patterns. Panel B shows the average total (dashed) and new (solid) investors by round; only one undisclosed investor is included (per Nanda and Rhodes-Kropf (2018)). Panel C shows the share of investors in round n that reinvest in round $n + 1$, conditional on subsequent funding; undisclosed investors are excluded.

where expectations are taking over $T_\kappa \sim \text{Exp}(\kappa)$. The entrepreneur should finance the project if \bar{V}_s is positive and abandon otherwise. In practice, the entrepreneur requires external financing. The project derives no intermediate cash flows and cannot be collateralised, so debt is unsuitable and the entrepreneur seeks equity financing from a VC firm.

Obtaining investment from a VC faces two broad challenges: asymmetric information and moral hazard. First, investors may be concerned that the entrepreneur has more information they do, leading to adverse selection. The difficulty evaluating projects exacerbates this issue. Second, the separation of ownership and control induces moral hazard. For instance, the entrepreneur may misuse funds or conceal bad news from the investor.

The asymmetric information problem prolongs the process of finding an investor and funding may conceivably never arrive. Furthermore, should a VC choose to invest, the agency conflicts drive the effect investment cost above k .²¹ To overcome these issues, VCs adopt a broad range of strategies, including intensive screening and active monitoring of their investments.²² Among these strategies, the staging of capital injections over time is often considered the most effective (Sahlman, 1990; Hall and Lerner, 2010).

Staging helps to overcome the problem of “inefficient continuation”, in which the entrepreneur continues to operate the project beyond the date that the VC would rather close it down as see unused capital returned (Gompers, 1995; Lerner, 1998). With a smaller initial capital injection, K , less capital remains when the project is complete (whether successful or not), minimising the VC’s exposure. This lowers the cost of funds to the entrepreneur because the VC requires less compensation *ex ante* for this opportunistic behaviour *ex post*. Staging also provides an opportunity to reassess the project before committing additional funds, acting as an information production tool and creating option value (e.g. Cornelli and Yosha 2003; Dahiya and Ray 2012). Panel A of Figure 1 shows that the typical VC-backed start-up in the US raises multiple funding rounds.

²¹The first problem could also generate this result. I abstract from this margin in the analysis.

²²For a survey, see Da Rin et al. (2013).

In practice, it is common for *new* investors to participate in subsequent funding rounds, while some incumbents do not reinvest (Panels B and C, Figure 1). This reflects various practical considerations, such as diversification incentives and fund-lifecycle dynamics, and the desire for external validation (Kerr et al., 2014) and expertise (Brander et al., 2002).²³ However, for the entrepreneur, the need to seek additional investors for subsequent rounds means that matching issues they faced initially reappear.

In light of these concerns, the initial investor(s) may increase upfront funding to reduce the need for subsequent rounds. At first glance, the notion that initial investor(s) could provide extra funding *ex ante*, but is unwilling to do so *ex post*, at least unilaterally, is puzzling. However, portfolio concerns, VC fund lifecycle dynamics and liquidity issues imply that capital now and capital later are imperfect substitutes. Negative shocks to one portfolio company can restrict the availability of follow-on funding for other firms in a VC’s portfolio (Townsend, 2015). VC fund structure – typically fixed-duration limited partnerships – also mechanically limits the scope for follow-on investments.²⁴ Furthermore, VCs face liquidity risk.²⁵ Anticipating potential liquidity issues, they prefer to make investments early in the fund’s lifecycle (Maurin et al., 2023) and, when liquidity issues arise, they make fewer investments (Li, 2024).²⁶ These liquidity shocks may have a common component and affect aggregate capital commitments, meaning that the need for new investors may arise exactly when securing funding is most difficult.²⁷ Relatedly, Nanda and Rhodes-Kropf (2017) show that the market can dry up precisely *because* multiple investors need to coordinate to fund the firm. Investor beliefs of limited future funding can be self-fulfilling, unless early investors can meet firms’ funding needs unilaterally. This leads to *financing risk*, in which projects are forced to shut down prematurely due to a lack of funding.

In this setting, the financing environment can influence what gets funded and whether projects scale successfully: the project may not be funded at all, or not funded to completion, leading to premature closure. Furthermore, the types of contracts written between entrepreneurs and VCs encode information about the extent of agency frictions and aggregate financing conditions, as they shape the capital injection decisions at any given funding round.

So far, the discussion has centred around a single entrepreneur. How does the setting change when there are many entrepreneurs pursuing projects? If many entrepreneurs are seeking funding contemporaneously, the market may take longer to clear. In that regard, congestion effects

²³The distribution of start-up outcomes is heavily right-skewed (Hall and Woodward, 2010) and the formation of investor syndicates aids diversification across investments (Pitchbook–NVCA Yearbook, 2024). Lerner (1998) also notes that bringing on new investors can help to solve agency conflicts within the VC fund that may lead to inefficient refinancing decisions.

²⁴Haran Rosen et al. (2025) show that start-ups receiving investment later in the fund’s lifecycle are less likely to receive follow-on investment.

²⁵Liquidity risk arises because VC funds make investments by pooling capital *commitments* from their investors (limited partners, LPs) on fund initiation and later draw down these commitments through capital *calls* when investment opportunities arise.

²⁶The sample in Li (2024) includes both buyout and venture capital funds. The penalties for default on capital calls are severe, including forfeiting invested capital (Robinson and Sensoy, 2016), but as Li (2024) points out, the liquidity pressure on LPs may induce VC funds to limit drawdowns for relationship building motives. Therefore, liquidity concerns need not trigger actual default to impact investment decisions at the fund level.

²⁷For instance, in 2008 the appetite among traditional LPs to make new private equity commitments fell markedly. Concurrently, “certain LPs also had issues with funding their current commitments and formally or informally communicated [...] a desire to defer capital calls [...] or to reduce the overall size of their [existing] commitments to the funds.” (Weil, Gotshal & Manges LLP, n.d.)

may be important. Furthermore, the potential to invest in various different projects implies that the outside options of investors may interact with the investment decision. Together, these interactions suggest that a full characterisation of the financing environment requires specifying an equilibrium model of the funding market.

An operational theory of the market The preceding discussion motivates a model of the venture capital market with the following structure.

Entrepreneurs with projects and VCs with capital meet in a frictional search-and-matching market and write incomplete contracts, which leads to the problem of “inefficient continuation”. The matching friction serves several purposes. On the one hand, it is a tractable representation of the difficulties in evaluating projects, which implies that the market can take time to clear. Relatedly, it endogenises the availability of capital based on the relative supply from investors and demand from entrepreneurs. On the other hand, it allows me to capture coordination issues among investors across funding rounds in a simple manner. Specifically, I require that an entrepreneur searches for a new VC each time it raises capital.²⁸ During their search, they may be forced to close down positive NPV projects, and so are subject to financing risk.

Contracting is incomplete, which leads the VC to infuse capital in stages. In the stylised example, the size of the capital injection, K , and the funding horizon, or “runway”, were equivalent; a start-up with initial investment K could conduct R&D for a period $T_\omega = K/k$. In practice, the environment is uncertain and *ex ante* identical firms may exhaust capital at different rates *ex post*. For instance, the investment plan may need to be altered or the project delivers some uncertain intermediate cash-flows (positive or negative). Therefore, I introduce some noise in the relationship between K and T_ω .

Introducing this noise directly into the funding horizon, T_ω , rather than the run-rate, k , and allowing the entrepreneur and VC to contract on the (expected) funding horizon keeps the model tractable. Specifically, suppose that the entrepreneur and VC write a contract that commits the VC to pay the investment cost k until time T_ω , a random variable. Setting $T_\omega \sim \text{Exp}(\omega)$, the funding commitment expires at constant hazard rate ω , and neither the capital stock, nor time since last funding, need to be carried as state variables.²⁹ Furthermore, all state-to-state transitions then have constant hazard rates, implying that the start-up lifecycle is determined by a simple continuous time Markov chain. The parameter ω replaces K as a feature of the contract and I refer to it as the “contract rate”. A given contract rate, ω , implies a certain expected capital commitment, $K(\omega)$, the analogue to K .

The model is closed by specifying an entry condition for start-ups and a matching technology. In equilibrium, the meeting rates in the financing market must be consistent with the entry decisions of entrepreneurs and the contracts signed between entrepreneurs and VCs.

²⁸I make this assumption for tractability. In practice, the existing VC(s) may participate in or lead a follow-on funding round, but empirically VC syndicates grow as the firm continues to raise funding. I abstract from the precise formation of these syndicates in this model and focus on the need for multiple investors to coordinate.

²⁹A subtle point is that the capital injection becomes a flow, rather than a stock.

2.2 The baseline model: specification

This section presents the baseline equilibrium model; section 2.3 presents a characterisation of the equilibrium. Time is continuous and the horizon is infinite. I focus on the steady-state equilibrium, so all transition rates and stocks are constant over time.

2.2.1 Agents and technology

There are two types of agents, entrepreneurs and venture capital firms (VCs). All agents are risk neutral and discount the future at rate ρ . At any instant, an exogenous flow Γ of entrepreneurs have a one-time chance to create a *start-up*, drawing some entry cost $c \sim U[0, \sigma]$. A start-up is created if the entrepreneur finds it profitable to pay the entry cost c ; otherwise, the idea is lost.

Following entry, all start-ups are *ex ante* homogenous. Each start-up's project requires flow capital investment k . Conditional on paying k , the results of the development process are realised at Poisson arrival rate κ and are positive (success) with probability p and negative (failure) with probability $1 - p$. Success results in a payoff π , whereas a failure returns zero. In addition to any pecuniary return associated with the project, entrepreneurs also derive a non-pecuniary flow benefit $x_b > 0$ whenever the project is actively being funded. For technical reasons set out in section 2.2.2, I let $x_b \rightarrow 0$.

Entrepreneurs have no wealth and must seek external financing from VCs. In the baseline, I assume that entrepreneurs have all the bargaining power in contracting; I relax this in section A.2 of the Appendix. There is a measure M of VCs, each of which may finance at most one project at a time. VCs face no capital constraints with respect to a single investment, but cannot provide follow-on funding to an entrepreneur that they previously invested in, should they require it.

Information plays a key role in the model. Firstly, I endow VCs with a technology to screen out projects that have already failed: this implies that a start-up that has previously received funding and realised that their project has failed cannot secure additional funding from a new VC. Without such a technology, failed entrepreneurs would seek new funding to obtain private benefits, but the existence of such a technology implies that doing so derives no value. Then, only firms with projects that have not yet failed seek funding and, conditional on a meeting, the entrepreneur and VC both agree on the project characteristics (k, κ, p, π) . Secondly, once a VC has committed capital to a start-up, the arrival of news about the project is private information of the entrepreneur. This implies that any contract that conditions on the arrival of information about T_κ must be incentive compatible, in the sense that the entrepreneur must find it advantageous to report truthfully.³⁰

³⁰The model can be extended to include costly monitoring. For now, I note only that contracting upon such information would require that it is 'hard', or verifiable. In practice, the highly innovative nature of these projects means that much of the information produced is 'soft', or non-verifiable, and monitoring would be ineffective. The model presented here is the special case in which all information produced by the entrepreneur is soft. Staging then emerges as the only information production tool, because it permits VCs to make continuation/termination decisions on the basis of both hard and soft information.

2.2.2 The financing market

Access to external finance is subject to two frictions: Es and VCs meet in a frictional search market, and contracting is subject to moral hazard of the type described in section 2.1.

Search-and-matching friction Start-ups search for a VC following entry or the exhaustion of previous funding, provided they did not realise a negative result. VCs search for a start-up to invest in if they are not currently committed to another start-up. Search is costly due to discounting, but there are no explicit search costs. I denote by μ_s the measure of start-ups searching for capital and by μ_{vc} the measure of searching VCs. The measures μ_s and μ_{vc} are the aggregate state variables in the model and determine the flow of meetings via the matching function

$$m(\mu_s, \mu_{vc}) = \varepsilon \mu_s^\alpha \mu_{vc}^\beta \quad (2)$$

where $\alpha, \beta > 0$. The Poisson arrival rate of meetings for a given entrepreneur and for a VC are

$$\nu = \frac{1}{\mu_s} m(\mu_s, \mu_{vc}) \quad \text{and} \quad v = \frac{1}{\mu_{vc}} m(\mu_s, \mu_{vc}), \quad (3)$$

respectively. During the search process, start-ups fail at rate λ , which is exogenous. In contrast, VCs are infinitely lived.

Contracting and moral hazard Once an entrepreneur and VC meet, they negotiate an equity contract to fund the start-ups development cost, k . Contracts specify an equity stake ς for the VC and a funding termination rule, which has two components. Firstly, the funding termination rule may condition on the arrival of news about the project at date T_κ . Secondly, the entrepreneur and VC may agree to a random date, $T_\omega \sim \text{Exp}(\omega)$, at which the funding commitment expires.

Consider first the contingent component of the funding termination rule, which plays a role when news arrives prior to the expiration of the funding commitment; that is, when $T_\kappa < T_\omega$ *ex post*. Given the information asymmetry, any incentive compatible contract requires that the entrepreneur find it profitable to report the arrival of news at time T_κ to the VC.

There are two cases to consider, success or failure. Given their private benefits, the entrepreneur never reports failures.³¹ In contrast, the entrepreneur may report successes. Reporting success immediately at date T_κ leads the VC to terminate funding, meaning the entrepreneur forgoes any additional non-pecuniary benefit. However, concealing the news delays the pecuniary payoff by T_ω .³² In other words, the entrepreneur must decide whether the value of the private benefit is sufficient to compensate for the loss in their pecuniary payoff due to discounting, which leads to an incentive compatibility constraint.³³

³¹Concealing failure allows the entrepreneur to continue to reap their private benefits, $x_b > 0$, until funding expires at date T_ω , whereas reporting failure allows the VC to shut down the venture. This logic also applies to any time after the realisation of the result because the environment is stationary.

³²The environment is stationary, so if the entrepreneur finds it profitable to delay reporting at the time the result is obtained, they will never report until the contract expires, which happens at date T_ω .

³³See Remark A.2. The IC depends on features of the project and environment, (π, ρ, x_b) , but also on the share, $1 - \varsigma$, that the entrepreneur maintains in the project. This leads to the implication that the entrepreneur must retain sufficient “skin-in-the-game” to report truthfully.

Proposition A.1 in the Appendix shows that, for any set of model parameters, there always exists an upper bound $\bar{x}_b > 0$ such that for $x_b < \bar{x}_b$, then the entrepreneur always reports positive results. Intuitively, if the non-pecuniary benefit is sufficiently low (in a relative sense), the entrepreneur would never willingly delay the realisation of their pecuniary payoff to continue to enjoy private benefits. Therefore, given $x_b < \bar{x}_b$, the entrepreneur always reports successes to the VC, but never reports failures; this appears to accord with anecdotal accounts of entrepreneur behaviour.³⁴ The assumption $x_b \rightarrow 0$ guarantees that this is the case.³⁵

With the contingent features of the contract determined, it is simple to map out the implications of the choice of non-contingent funding termination, $T_\omega \sim \text{Exp}(\omega)$. Analogous to a larger capital injection, a lower “contract rate” ω allows the entrepreneur to conduct development for a longer duration before seeking additional funding. However, in this incomplete contracting environment, it increases the VC’s exposure to the entrepreneur’s opportunistic behaviour, because it increases both the probability that the entrepreneur will uncover negative news *and* the outstanding capital commitment when that information arrives. Specifically, the expected capital commitment associated with contract (ς, ω) is given by³⁶

$$\begin{aligned}
K(\omega) &= \underbrace{\frac{k}{\rho + \kappa + \omega}}_{\text{Frictionless expected capital cost}} + \underbrace{\frac{\kappa(1-p)}{\kappa + \omega}}_{\text{Pr(negative result)}} \underbrace{\frac{k(\kappa + \omega)}{(\rho + \omega)(\rho + \kappa + \omega)}}_{\text{Remaining commitment}} \\
&= \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k}{\rho + \kappa + \omega}.
\end{aligned} \tag{4}$$

where $K'(\omega) < 0$ and $K''(\omega) > 0$. The expected capital cost is composed of two terms. First, in a complete contracting environment, the entrepreneur and VC could write a contract to terminate funding at the realisation of the result, date $T_\kappa \sim \text{Exp}(\kappa)$. The contract rate would be $\omega = 0$, so that funding would never be withdrawn prematurely, and the VC would expect to inject $k/(\rho + \kappa)$ into the start-up before the result is realised. The first term reflects this benchmark, which is equivalent to the capital cost that would arise if the entrepreneur could invest their own funds.³⁷ However, the agency conflict introduces an additional term: the entrepreneur conceals the arrival of bad news, engaging in inefficient continuation. The second line of equation (4) shows that the capital cost now takes the form of a wedge over the frictionless benchmark due to this agency conflict.

³⁴See, for example, Chapter 4 of Janeway (2018), “After more than forty years in the game, I have yet to meet the entrepreneur who dallies in delivering word that ‘the product works’ or ‘the sale has closed’. When communication ceases, then the venture capitalist can expect to discover that ‘the product needs another rev’ or that ‘we lost the order.’” Janeway calls this his Second Law of Venture Capital, “No news is never good news”.

³⁵The VC holds a claim that has some resemblance to participating preferred stock, in which the VC receives a liquidation preference and shares in the payoff; such features are common in VC contracts (Kaplan and Strömberg, 2003; Ewens et al., 2022). In the model, when the project has a liquidation value – i.e. when successful – the VC receives a share of the payoff *and* cuts funding, which acts like a claim against the company.

³⁶See section A.4 for the derivation.

³⁷Specifically, $\frac{k}{\rho + \kappa + \omega}$ is the expected discounted value of the capital injection prior to the realisation of the result or the withdrawal of funding. In the frictionless benchmark, funding would be withdrawn at rate $\omega = 0$, so that $\frac{k}{\rho + \kappa}$ is the expected cost of funding the start-up.

The optimal contract Abstracting from VC bargaining power, the optimal contract, (ω, ς) , simply maximises the value of the project to existing shareholders (the entrepreneur and any prior investors) subject to a participation constraint (PC) of the new investor.³⁸

For a given contract (ω, ς) , the VC incurs expected capital costs $K(\omega)$, given by equation (4), and receives a share ς of the value of the firm gross of their investment, which I denote by $V_d(\omega)$.³⁹ Therefore, the PC is $\varsigma V_d(\omega) \geq K(\omega)$. The optimal contract (ω, ς) then solves

$$\begin{aligned} V^M &= \sup_{\{\omega \in [0, \infty), \varsigma \in [0, 1]\}} \left\{ (1 - \varsigma) V_d(\omega) \right\} \\ \text{s.t.} \quad &\varsigma V_d(\omega) \geq K(\omega) \end{aligned} \quad (5)$$

where $K(\omega)$ is given by equation (4), $V_d(\omega)$ solves the HJB equation

$$\rho V_d = \kappa [p \pi - V_d] + \omega [V_s - V_d] \quad (6)$$

and V_s is the value of a start-up in search, which is taken as given in the optimisation. To see why V_d solves equation (6), recall that news about the project arrives at rate κ and is successful with probability p , in which case the project yields a payoff π . However, at rate ω , the funding commitment terminates, in which case the entrepreneur must search for new funding; its value then becomes V_s . In equilibrium, the value of search solves

$$(\rho + \lambda) V_s = \nu [V^M - V_s]. \quad (7)$$

which follows because a start-up in search fails at exogenous rate λ and meets with a VC at rate ν , in which case it has value V^M . The binding participation constraint pins down ς as a function of ω , which then implies that $V^M = V_d(\omega) - K(\omega)$. Therefore, the optimal contract rate, ω , maximises the (private) value of the firm net of investment costs.

2.2.3 Closing the model

To close the model, I specify the entry decision and the conditions that determine the two state variables, μ_s and μ_{vc} , which pin down the equilibrium meeting rates, ν and v .

Entry Given V_s , the entry decision is a simple cutoff rule simply. A given start-up enters if and only if their entry cost $c \leq V_s$, where V_s is the value of a start-up searching for capital. Given $c \sim U[0, \sigma]$, the probability of drawing an entry cost below the threshold is $Pr(c \leq V_s) = V_s/\sigma$. Each potential entrant faces this same probability, so that the flow of new entrants that enter endogenously is given by $\Gamma(V_s/\sigma)$. It will not be possible to disentangle Γ from σ , so I will substitute in $\tilde{\sigma} = \sigma/\Gamma$ going forwards. The endogenous flow of entry is then $V_s/\tilde{\sigma}$. The parameter $\tilde{\sigma}$ can be interpreted as determining the supply of entrepreneurial projects.

Steady-state conditions Finally, it remains to specify the conditions that determine μ_s and μ_{vc} . Rather than specify a condition for μ_{vc} directly, it will be simpler (and more intuitive) to

³⁸I consider the extension to VC bargaining power in section A.2 of the Appendix. The model remains tractable.

³⁹This is equivalent to the “post-money” value of the firm.

consider how many firms receive funding in equilibrium and to determine μ_{vc} as the residual given the fixed measure M of VCs. In this spirit, I will refer to a start-up that has already obtained a negative result but is still receiving funding as ‘unproductive’, as opposed to ‘productive’ start-ups that have funding and may still achieve success. I denote by μ_d^p the measure of start-ups in productive development and by μ_d^u the measure of start-ups in unproductive development. Since the total measure of VCs is M and a measure $\mu_d^p + \mu_d^u$ are funding start-ups, the measure of searching VCs is given by $\mu_{vc} = M - \mu_d^p - \mu_d^u$.

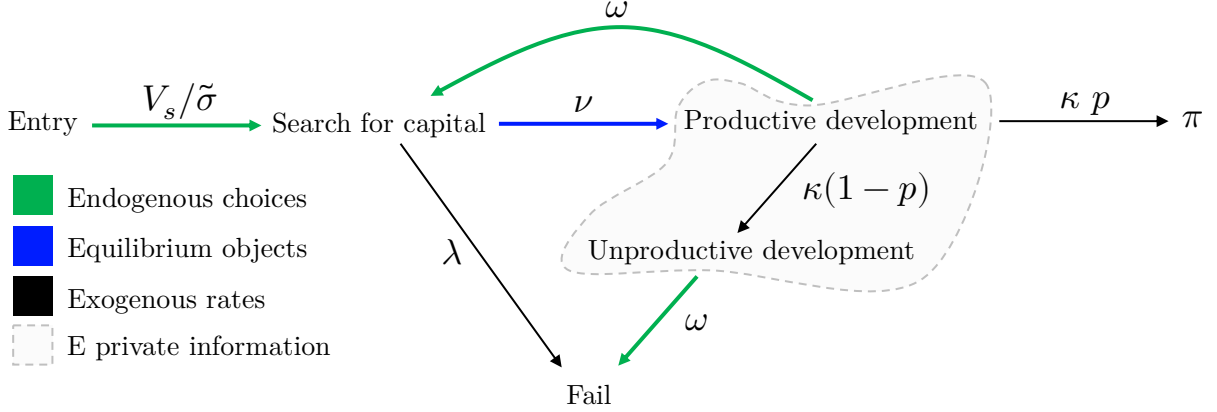


Figure 2: State transitions

Entry reflects an aggregate flow; other arrows represent individual-level transition intensities.

The state-to-state transitions for a single firm are summarised by Figure 2. In steady-state, the inflows and outflows from each state must be equal to one another. Consider first the measure of firms in productive development. A flow $\nu\mu_s$ of start-ups enter this state from search, having signed contracts with VCs, and a flow $(\kappa + \omega)\mu_d^p$ exit this state when realising a result about their project or seeing their funding run out. Next, consider the measure of firms in unproductive development. Firms enter this state from productive development when they obtain a negative result, a flow $\kappa(1-p)\mu_d^p$, and exit this state when their funding runs out, a flow $\omega\mu_d^u$. As Figure 2 illustrates, transitions into this state are unobserved to the VC. Finally, consider the measure of firms in search. Firms return to search from productive development when their funding runs out, an inflow $\omega\mu_d^p$. Since firms in unproductive development would be screened out by subsequent VCs, they find it optimal to close down and so do not return to search. In addition to the inflow from productive development, new entrants begin their lifecycle in search of capital, a flow $V_s/\tilde{\sigma}$. Conversely, start-ups exit search when they fail during the search process or meet with VCs, a flow $(\lambda + \nu)\mu_s$. Therefore, the steady-state conditions governing μ_d^p , μ_d^u and μ_s are given by

$$\mu_d^p : \quad \nu \mu_s = (\kappa + \omega) \mu_d^p \quad (8)$$

$$\mu_d^u : \quad \kappa(1-p)\mu_d^p = \omega\mu_d^u \quad (9)$$

$$\mu_s : \quad \omega\mu_d^p + \frac{V_s}{\tilde{\sigma}} = (\lambda + \nu) \mu_s \quad (10)$$

These equations can be written purely in terms of μ_s and μ_{vc} .⁴⁰ I then define an equilibrium as follows.

Definition 2.1. A steady-state equilibrium is a tuple $(V_d, V_s, V^M, \omega, \varsigma, \mu_s, \mu_{vc})$, where (ω, ς) solve maximisation problem (5) given the meeting rates (ν, v) in equation (3), and equations (6), (7) (8), (9) and (10) are satisfied.

2.3 The baseline model: equilibrium characterisation

In this section, I analyse properties of the equilibrium. First, taking financing conditions in the matching market as given, I solve the contracting problem and consider comparative statics to study: (i) its properties; (ii) the implications for funding histories; (iii) the implications for start-up outcomes and entry. Then, I consider efficiency and discuss the feedback between the agency and matching frictions which endogenously determines financing conditions.

2.3.1 The equilibrium contract

Given a value for the meeting rate in the financing market, ν , the model admits closed-form solutions for value and policy functions.⁴¹ The first proposition proves existence of a unique equilibrium, given ν , and the resulting corollary derives the optimal contract rate, ω , as a function of model parameters.

Proposition 2.1. *Given ν and for $k < \kappa p \pi$, there is unique solution for $\{V_d, V_s, V^M, (\omega, \varsigma)\}$ in terms of model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, $\omega \in [0, \infty)$.*

Corollary 2.1. *Given ν and $k < \kappa p \pi$, the equilibrium contract rate is given by*

$$\omega = \max \left\{ 0, -\rho + \mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} \right\}, \quad \text{where } \mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \nu \times \frac{\rho + \kappa}{\rho + \lambda} \quad (11)$$

and $\omega \rightarrow \infty$ otherwise. \mathcal{C} captures characteristics and \mathcal{F} captures need for and access to follow-on funding.

The optimal contract depends on both characteristics of the project, via \mathcal{C} , and is forward looking through its dependence on \mathcal{F} . Comparative statics are analysed in section 2.3.2. Here, it is worth noting briefly that there are two classes of contract. The contract may feature $\omega = 0$, in which case the VC commits to fund the project to completion with certainty and corresponds to *upfront financing*. Conversely, for $\omega > 0$, the entrepreneur may require additional funding for their project; this is *staged-financing* and is an endogenous feature of the model. In what follows, I will analyse the model under staged financing, which is the empirically relevant case. Upfront financing can arise in the model and requires both that the agency friction is sufficiently muted and the financing market is sufficiently tight. In the general, agency frictions will be severe when financing highly-innovative projects, so staged-financing emerges in equilibrium. In

⁴⁰From equation (9), $\mu_d^u = \frac{\kappa(1-p)}{\omega} \mu_d^p$, so that $\mu_{vc} = M - \mu_d^p - \mu_d^u = M - \left(1 + \frac{\kappa(1-p)}{\omega}\right) \mu_d^p = M - \left(1 + \frac{\kappa(1-p)}{\omega}\right) \frac{\nu}{\kappa + \omega} \mu_s$, where the last equality follows from equation (8). Furthermore, for μ_d^p in equation (10) from equation (8) yields $\frac{\omega}{\kappa + \omega} \nu \mu_s + \frac{V_s}{\sigma} = (\lambda + \nu) \mu_s$.

⁴¹In the generalised Nash bargaining case, it becomes necessary to specify both ν and v .

simple terms, low-risk projects (think real estate investments) can be financed upfront, whereas more novel ventures benefit from a wait-and-see approach.

2.3.2 Comparative statics

In this subsection, I explore features of the model's equilibrium, given ν . I start with simple comparative statics of the contract rate, ω . Then, to build intuition, I discuss the implications of project characteristics and funding conditions for financing patterns; that is, the features of contracts written between Es and VCs. These results will also be relevant for mapping the model to data. Finally, I discuss the implications of financial frictions for start-up outcomes and the entry.

I am particularly interested in understanding the implications of financial frictions for projects with different characteristics, but equal quality. For instance, certain projects may be riskier than others or require a long period of investment to uncover uncertainty, but still have the same underlying value because, if successful, they obtain a higher valuation. To this end, I define a notion of underlying quality based on the NPV of a project to an entrepreneur that can self-finance development, which is given by \bar{V}_s in equation (1) and restated below. Under risk-neutrality, exponential discounting, and perfect capital markets, \bar{V}_s is a sufficient statistic for investment decisions. Therefore, it constitutes a relevant frictionless benchmark. However, when access to capital is subject to frictions, a project's underlying characteristics may be important for investment decisions for reasons beyond their effect on quality, \bar{V}_s . To investigate these effects, I define the notion of a quality-compensating comparative static (QCCS) in which project characteristics are changed while adjusting π to keep quality constant.

Definition 2.2. Consider a project with characteristics (k, κ, p, π) . I define

- (a) Project *quality* as $\bar{V}_s = \frac{\kappa p \pi - k}{\rho + \kappa}$;
- (b) A *quality-compensating comparative static* (QCCS) in parameter x as the derivative with respect to x while adjusting π so that quality, \bar{V}_s , is held constant.

The optimal contract The optimal contract rate, ω , trades off the incentive to reduce agency costs, incentivising higher ω , with insurance against the need for follow-on funding, which incentivises lower ω .

The effects are summarised by two terms, \mathcal{C} and \mathcal{F} ; see equation (11). Firstly, the contract rate is increasing in \mathcal{C} , a composite measure of project *characteristics* that reflects the extent of agency frictions relative to the underlying project value. Holding the contract rate, ω , fixed, the agency friction is more severe for short-horizon or risky projects, which leads $\kappa(1 - p)$ to appear in the numerator of the wedge in $K(\omega)$, shown in equation (4). Intuitively, for a given funding commitment, ω , a short-horizon project, higher κ , is more likely to realise a result and leaves a larger (expected) outstanding capital commitment when news arrives. Furthermore, all else equal, a higher likelihood of project failure, an increase in $(1 - p)$, makes it more likely that the entrepreneur will engage in inefficient continuation when news about the project arrives. Both of these factors increase agency costs. To militate against these issues, the entrepreneur and VC reduce the capital commitment, increasing ω . Doing so reduces the likelihood that

inefficient continuation will occur and the losses in the case that it does (see equation (4)). However, the incentive to reduce the capital commitment also depends on the size of cash flows that the project produces, relative to investment costs, $\frac{\kappa p \pi - k}{k}$. When this is high, the loss in value resulting from delay, or outright failure, during search is larger. Therefore, Es pursuing such projects are more willing to endure higher costs of capital to reduce exposure to future financing conditions.

Secondly, the contract rate is increasing in \mathcal{F} , a composite measure of the firms need and ability to seek *follow-on financing*. The development horizon, κ , affects the *need* for follow-on financing: holding the contract rate, ω , constant, a longer-horizon project, a lower κ , is more likely to require additional financing.⁴² The meeting rate and failure rate in search affect the *ability* to obtain follow-on financing: a lower meeting rate, ν , or higher failure rate, λ , make securing additional financing more difficult. When additional financing becomes more difficult to obtain, the existing investor increases their capital commitment by lowering ω , a form of insurance. These effects are summarised in the following proposition.

Proposition 2.2. *The optimal contract rate, ω , is: strictly increasing in ν and k ; and strictly decreasing in p , π , λ . For κ , $\exists \bar{\kappa} > k/p\pi$ such that for $\kappa < \bar{\kappa}$, $\omega'(\kappa) < 0$ and otherwise $\omega'(\kappa) \geq 0$. The QCCS for p is strictly negative and for κ is strictly positive.*

Most of the comparative statics follow immediately from the preceding discussion. However, the probability that news is positive, p , and the project horizon, κ , interact with the contract choice through multiple channels. A higher likelihood of success, p , raises both project quality and reduces the extent of the agency friction. This implies that both the standard and quality-compensated comparative static are negative, corresponding to longer funding commitments for projects with higher p .

In contrast, the effect of the development horizon, κ , is ambiguous. On the one hand, as κ rises relative to ω , the agency friction is exacerbated because of maturity mismatch between the realisation of uncertainty and the funding horizon. Intuitively, if uncertainty is expected to be resolved in a matter of months, but funding has been provided for a number of years, then the VC is at risk of exploitation by the entrepreneur. Furthermore, holding ω fixed, a rise in κ reduces the need for follow-on funding, which means the entrepreneur and VC require less insurance against financing risk; this generates an additional motive to match the development horizon with the funding horizon. Together, these factors imply that the QCCS is positive. However, κ also affects project quality: an increase in κ increases quality by bringing the payoff forwards and reducing the total investment required. This puts downward pressure on \mathcal{C} , counteracting the quality-compensated effect and dominating for small κ .

Funding histories To build intuition, I now consider how the choice of contract maps into observable features of venture capital funding data. I focus on three readily observable features: the size of each capital injection, the number of funding rounds, and the duration between funding rounds.⁴³ To this end, I let $\bar{K} = K(\omega^*)$ denote the VC's expected capital commitment in

⁴²That is, $Pr(T_\kappa < T_\omega)$ is increasing in κ for fixed ω .

⁴³The relationship between these features of funding patterns, firm characteristics, and market conditions was first explored in the seminal work of Gompers (1995).

the optimal contract; this is the model counterpart to the capital injections observed empirically. For the number of funding rounds, I focus on the subset of firms that receive at least one funding round to facilitate a natural mapping to data.⁴⁴ The following propositions and related corollaries present the key results.

Corollary 2.2. *The capital injection \bar{K} is: strictly decreasing in ν ; strictly increasing in π and λ ; and ambiguous with respect to κ , k and p . The QCCS for p is ambiguous and for κ is strictly negative.*

Proposition 2.3. *The distribution of the number of financing rounds, $N_f \sim \text{Geo}(q_d)$ where $q_d = \frac{\kappa}{\kappa+\omega} + \frac{\omega}{\kappa+\omega} \frac{\lambda}{\lambda+\nu}$ with support $N_f \in \{1, 2, 3, \dots\}$.*

Corollary 2.3. *The expected number of financing rounds, $E[N_f] = 1/q_d$, is: strictly increasing in ν and k ; strictly decreasing in p , π , and λ ; and ambiguous for κ . The QCCS for p is strictly negative and the for κ is ambiguous.*

Proposition 2.4. *The distribution of the duration between successive funding rounds, $T_{br} \sim \text{Hypo}(\kappa + \omega, \lambda + \nu)$.*

Corollary 2.4. *The expected duration between successive funding rounds for a given firm, $E[T_{br}] = \frac{1}{\kappa+\omega} + \frac{1}{\lambda+\nu}$, is: strictly decreasing in ν , and k ; strictly increasing in p and π ; and ambiguous with respect to λ and κ . The QCCS for p is strictly positive and for κ is strictly negative.*

The results for \bar{K} follow immediately from Proposition 2.2; the fact that many are ambiguous reflects the combination of the partial effect of these parameters on the capital cost directly, see equation (4), combined with the effect through optimal contracting, $K'(\omega) < 0$. For instance, holding ω fixed, $K(\omega)$ is increasing in k , but contracts adjust optimally, and the shorter resulting funding horizon counteracts the direct effect, so that the total effect is ambiguous.

The implications for the number and duration of funding rounds provide more insights. These are random variables because, while start-ups are *ex ante* homogeneous, they face different shocks *ex post*.^{45,46} Figure 3 plots the distributions. The key insight is that factors that raise agency costs reduce the duration between funding rounds and lead the firm to conduct more funding rounds. Conversely, factors that increase the exposure to and costs of financing risk lead to a reduction in the number of funding rounds and an increase in the duration between them. This suggests that by observing VC funding data, it is possible to learn about these primitives. I expand on this in discussion of model identification in section 3.1.1.

The project development horizon is a subtle case. Holding quality fixed, an increase in κ leads to a shorter funding committent, a higher ω , to match the funding and development

⁴⁴ A share $\frac{\lambda}{\nu+\lambda}$ of entrants fail before they meet a VC for the first time and so receive no funding.

⁴⁵ To build intuition for Proposition 2.3, note that the probability of another funding rounds is memoryless, as it depends only on the current state and not on the number of prior funding rounds. As a result, the distribution of the number of funding rounds follows a Geometric distribution—the only discrete probability distribution with the memoryless property. In practice, this memoryless property is a strong assumption and likely violated in the data. The quantitative model relaxes this assumption.

⁴⁶ To build intuition for Proposition 2.4, consider that between two funding rounds, the start-up conducts development for period $T_d \sim \text{Exp}(\kappa + \omega)$ and the searches for duration $T_s \sim \text{Exp}(\lambda + \nu)$. Therefore, the time between two successive funding rounds is $T_{br} = T_d + T_s$, the sum of two exponentially distributed random variables. Therefore, T_{br} is hypoexponential, inheriting the rate parameters of the underlying exponential distributions.

horizons. This naturally results in less time between funding rounds. However, the effect on the number of funding rounds is more nuanced.⁴⁷ On the one hand, there is a direct effect: for given contracts, a longer development horizon implies more funding rounds. On the other hand, contracts adjust to match the funding and development horizons. This adjustment could offset, or even outweigh, the direct effect and reduce the total number of funding rounds. Therefore, what matters is how responsive contracts are to the development horizon. In practice, the response is sufficiently muted and the direct effect dominates.⁴⁸ Therefore, I will typically refer to projects that have longer development horizons requiring more funding rounds.

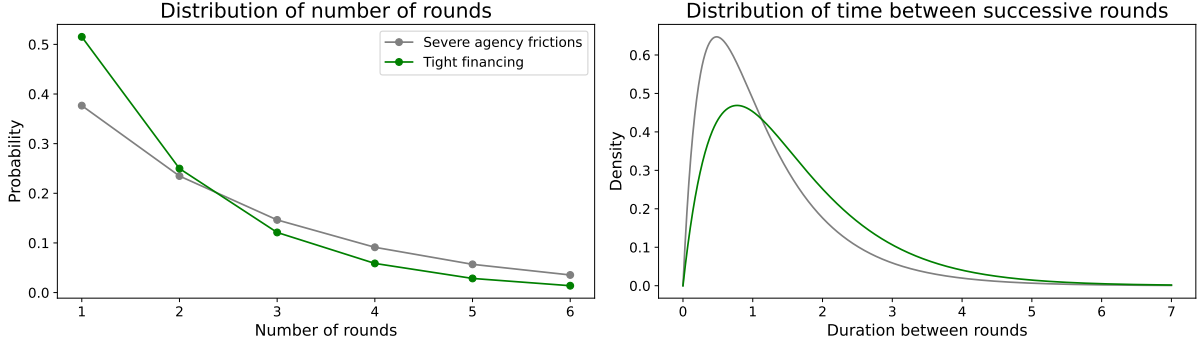


Figure 3: Distribution of the number of and time between funding rounds

The figure depicts the distribution of the number of funding rounds and time between rounds for two hypothetical parameterisations. In the case of “tight financing”, the meeting rate, ν , has been lowered. In the case of “severe agency frictions”, the project is made more novel.

Outcomes With the model apparatus in place, the analysis now turns to the central question: which forces determine whether entrepreneurs ultimately commercialise their innovations, and do all positive-NPV projects reach completion? I proceed in two steps. I start by analysing *outcomes* for start-ups that receive initial funding from VCs. Among these start-ups, the process of securing financing means they fail in search prior to project completion, leaving Es with projects they would continue, could they self-finance. Then, I consider below whether frictions in access to capital affect start-up creation, or *entry*.

Under self-financing, only the exogenous success rate, p , matters for outcomes. However, when the entrepreneur seeks external financing, start-up outcomes are endogenous. I denote by p_d and p_s the probability that firm currently in productive development or search, respectively, is successful with its project at some point. The structure of the model allows for writing these probabilities as the solution to the recursive equations

$$p_d = \frac{\kappa}{\kappa + \omega} p + \frac{\omega}{\kappa + \omega} p_s, \quad p_s = \frac{\nu}{\lambda + \nu} p_d.$$

⁴⁷This is counterintuitive, given that generally there is a relationship between the number of funding rounds and the duration between them. However, discussions of the duration between rounds are conditional statements—the firm has to actually meet with a VC for the duration to be defined. Conversely, the number of funding rounds is explicitly affected by the extent to which firms fail while searching for capital.

⁴⁸Specifically, long-horizon projects require more funding rounds in expectation iff $\frac{\kappa \omega'(\kappa)}{\omega} < 1$; that is, the elasticity of the contract rate, ω , to the completion rate, κ , is less than one. It can be shown that if $\pi < \bar{\pi}$ for some $\bar{\pi}$ that depends on model parameters, then $\omega'(\kappa)/\omega < \kappa^{-1}$ is satisfied. In practice, this value of $\bar{\pi}$ is very high, i.e. orders of magnitude higher than the value estimated in the quantitative model.

Specifically, a firm currently in productive development succeeds in its current financing round with probability $\frac{\kappa}{\kappa+\omega} p$; otherwise, it transitions to search and is successful with probability p_s . Similarly, a firm currently in search transitions to development with probability $\frac{\nu}{\lambda+\nu}$, in which case it succeeds with probability p_d . Solving for (p_d, p_s) leads to the following proposition.

Proposition 2.5. *The (steady-state) probabilities of success for a firm in productive development, p_d , and in search, p_s , are*

$$p_d = (1 - \Delta_p)p, \quad p_s = \frac{\nu}{\lambda + \nu} p_d, \quad \text{where} \quad \Delta_p = \frac{\lambda\omega}{\kappa(\lambda + \nu) + \lambda\omega}$$

with $\Delta_p \in [0, 1)$ and where $p_d, p_s < p$ for $\omega \in (0, \infty)$.

The term Δ_p in Proposition A.4 can be viewed as an *outcome distortion*. Under self-financing, the entrepreneur finances the project until uncertainty is resolved and success occurs with probability p . However, when the entrepreneur seeks external financing, it may not secure sufficient funding to resolve uncertainty and additional funding may not be forthcoming. This lowers the success rate below p . However, this need not be the case: if the entrepreneur and VC find it optimal to commit to *upfront financing*, $\omega = 0$, then $\Delta_p = 0$ and $p_d = p$. This leads to the simple insight that financing matters for start-up outcomes. The following corollary makes the link between financing patterns and start-up outcomes explicit.

Corollary 2.5. *Consider two start-ups $i = a, b$ facing the same funding conditions, ν and λ . If $E[N_f^a] > E[N_f^b]$, then $\Delta_p^a > \Delta_p^b$.*

In words, Corollary 2.5 says that firms adopting funding strategies with more funding rounds face greater outcome distortions. Importantly, it becomes clear which start-ups face the most severe distortions. On the one hand, low quality projects typically demand less insurance against financing risk, visiting the market more frequently, and so are endogenously more exposed to failure in search.⁴⁹ On the other hand, for fixed quality, highly novel start-ups face more severe agency frictions, fund themselves with more rounds, and so see greater outcome distortions. Similarly, longer horizon projects will tend to require more funding rounds and so face greater outcome distortions.

Finally, in Proposition A.4, I explore the implications of financing conditions, ν and λ , for start-up outcomes. In general, they have ambiguous effects, which reflects two opposing forces. The direct effect of tighter funding markets is to increase the risk of failure from financing risk. However, as Proposition 2.2 illustrates, a decrease in ν or increase in λ induces a response in optimal contracts: firms raise more capital to insure against financing risk. When markets are particularly tight, i.e. ν is low, this effect can dominate.

⁴⁹This might seem to contradict the findings of Gompers (1995), that successful firms undergo more financing rounds than unsuccessful ones. However, in the baseline model with *ex ante* homogeneous firms, the number of funding rounds is independent of outcomes (success or failure). Furthermore, in the quantitative model, the firms that are ultimately successful secure, on average, more rounds of funding. The introduction of an intermediate milestone is important for this result.

Entry Turning to entry, recall from section 2.2.3 that the flow rate of entry is given by $V_s/\tilde{\sigma}$; in other words, entry is increasing in the value of search.⁵⁰ Were entrepreneurs able to self-finance, entry would be $\bar{V}_s/\tilde{\sigma}$, where \bar{V}_s is given in Definition 2.2. However, the fact that entrepreneurs must seek external financing may put downward pressure on entry incentives, meaning start-up creation will be suboptimally low. To build intuition, it is useful to consider the case of $\rho \rightarrow 0$, so that the timing of payoffs becomes irrelevant. In that case, the value of entry (search) is simply⁵¹

$$V_s = \frac{\nu}{\lambda + \nu} (1 - \Delta_p) \frac{\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\omega}\right)}{\kappa} \quad (12)$$

This expression has the same form as $\bar{V}_s|_{\rho \rightarrow 0}$ in equation (1), but three differences emerge. First, only a share $\nu/(\lambda + \nu)$ of entrants ever receive funding, reducing the incentive to enter. Second, the outcome distortion attenuates entry incentives. It pre-multiplies the entire expression, reflecting both that $p_d = (1 - \Delta_p)p$ is the relevant success probability, and that the entrepreneur may be forced to shut down before completing the development process, which reduces their total capital investment.⁵² Finally, the agency friction increases the effective flow capital cost by a factor $1 + \kappa(1 - p)/\omega$.

To summarise how these factors influence entry incentives when contracts are determined optimally, it is useful to define an *entry distortion*, Δ_s , analogous to the outcome distortion. The value of search when contracts are set optimally is given by⁵³

$$V_s = (1 - \Delta_s) \bar{V}_s, \quad \text{where} \quad 1 - \Delta_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + 2\mathcal{C} + 2\sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}} \quad (13)$$

where $\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k}$ and $\mathcal{F} = \kappa + \nu \times \frac{\rho + \kappa}{\rho + \lambda}$, as before. Similar to their role in determining the optimal contract rate in equation (11), the composite measures of characteristics, \mathcal{C} , and the financing, \mathcal{F} , are central. The entry distortion, Δ_s , is decreasing in \mathcal{F} , reflecting that a lesser need for follow-on funding, or greater ease of securing it when it becomes necessary, reduces the entry distortion. In contrast, the entry distortion is increasing in \mathcal{C} . On the one hand, this is reassuring because it suggests that high-value projects, for instance with high π , face the least severe distortions. However, this ignores the role of agency costs and, to the extent that a high value of \mathcal{C} reflects these costs, the entry distortion will be significant. The following proposition summarises the key results for entry.

Proposition 2.6. *The value of entry, V_s , is: strictly increasing in ν , p , π and κ ; and strictly decreasing in k , λ . The QCCS for p is strictly positive and for κ is ambiguous.*

In most cases, the intuition for the sign is relatively straightforward. Entry is higher when:

⁵⁰In this simple model, I adopt a unit elasticity of entry with respect to the value of search but this statement clearly holds more generally.

⁵¹Another useful expression is $V_s = \frac{\nu}{\lambda + \nu} (p_d \times \pi - E[N_f] \times K(\omega)|_{\rho \rightarrow 0})$, where p_d is the endogenous success probability for a start-up that has just received funding, $E[N_f]$ is the expected number of funding rounds and $K(\omega)$ is the expected capital commitment per round.

⁵²The outcome distortion, Δ_p , affects total capital expenditure because it affects the duration of investment. In the frictionless case, the expected investment duration is κ^{-1} . In the frictional economy, it is $E[N_f] \times (\kappa + \omega)^{-1} = \kappa^{-1}(1 - \Delta_p)$, where $(\kappa + \omega)^{-1}$ is the average duration in development before returning to search.

⁵³See Appendix A.4.2 for the derivation.

financing is easier to access, an increase in ν or decrease in λ ; or projects are higher quality, higher (p, π, κ) or lower k . For λ and ν , these effects reflect that the entry distortion, Δ_s , falls when financing conditions ease. For project characteristics, (p, π, k) , it reflects the combination of a direct effect on quality, via \bar{V}_s , and on the entry distortion, Δ_s , as higher quality projects face more limited distortions, as do less novel projects. For the development horizon, κ , there are various competing effects. Allowing quality to adjust, entry is higher for short-horizon projects, reflecting both the timing of payoffs and more limited capital requirements associated with a short-horizon project. Holding quality fixed, the effect is technically ambiguous but in practice the effect goes in the same direction as the outcome distortion, Δ_p .⁵⁴ Therefore, because, for given quality, longer-horizon projects are funded with more rounds, in general, they face greater entry distortions.

2.3.3 Efficiency

This section studies efficiency. In the frictionless benchmark, where a wealthy entrepreneur can self-finance the project, the NPV is given by \bar{V}_s in equation (1). The entry rate is then $\bar{V}_s/\bar{\sigma}$ and a share p of projects are successful. When entrepreneurs seek external funding, $V_s < \bar{V}_s$ and $p_d \leq p$; there is less entry and a lower share of projects are successful. However, the model nests the frictionless benchmark in the limit as $\nu \rightarrow \infty$. Indeed, removing the matching friction means $\omega \rightarrow \infty$ is optimal and the firm is financed through an infinite number of instantaneous funding rounds. The value of search then approaches the frictionless value, \bar{V}_s , as summarised by the following lemma.⁵⁵

Lemma 2.1. *As search becomes frictionless, $\nu \rightarrow \infty$, the value of search, V_s , converges to the frictionless value, \bar{V}_s : $\lim_{\nu \rightarrow \infty} V_s = \bar{V}_s$.*

As $\nu \rightarrow \infty$, expected delays between funding rounds vanish. Funding can be split into arbitrarily small, instantaneous rounds, eliminating scope to hide failure and driving the external-finance premium to zero; in other words, staging fully overcomes the agency friction.⁵⁶ The value of search (and thereby entry) therefore coincides with \bar{V}_s .

In practice, $\nu < \infty$ and the economy is inefficient. I begin by formalising a notion of capital misallocation, which provides insights into the costs of this inefficiency. Capital is misallocated in the economy due to inefficient continuation; it would be preferable to transfer capital from a start-up in unproductive development to one in search. However, due to the frictions present in the model, a degree capital misallocation is inevitable. Therefore, I then study whether the decentralised equilibrium is constrained efficient given the agency and matching frictions. I show that social efficiency obtains only in the knife-edge case in which the classical Hosios condition is satisfied (Hosios, 1990).

⁵⁴In the limit where $\rho \rightarrow 0$, Corollary 2.5 can be applied directly to Δ_s .

⁵⁵For finite ν , $\omega \rightarrow \infty$ implies no capital commitment and the total amount of capital raised by the firm over its lifecycle is zero; $\lim_{\omega \rightarrow \infty} E[N_f] \cdot \bar{K} = 0$. However, as $\nu \rightarrow \infty$, $\omega \rightarrow \infty$ more slowly ($\lim_{\nu \rightarrow \infty} \omega(\nu)/\sqrt{\nu} = c \in \mathbb{R}$; see proof to Lemma 2.1). Then, $\lim_{\nu \rightarrow \infty} E[N_f] \cdot \bar{K} = k/\kappa$; the proof to Lemma 2.1 accounts for this, including discounting.

⁵⁶In practice, costs of writing contracts and performing due-diligence between each funding round would overturn this result. I abstract from these concerns to focus on the interplay between agency and matching frictions.

2.3.4 Capital misallocation

Relative to an economy without frictions, capital is misallocated in the model because start-ups that have received negative news about their prospects, the “unproductive” start-ups, do not inform their investors and so continue to invest in projects with no prospects. The capital would be better invested in a different, productive start-up.

A high degree of capital misallocation is indicative of issues in access to funding. Capital misallocation – defined as the share of firms with funding that are unproductive – is given by

$$\frac{\mu_d^u}{\mu_d^u + \mu_d^p} = \frac{\kappa(1-p)}{\kappa(1-p) + \omega} \quad (14)$$

which follows from equation (9).⁵⁷ Capital misallocation is decreasing in ω . This leads to the following Corollary (of Proposition 2.2), which characterises factors that affect the extent of capital misallocation in the model.

Corollary 2.6. *Capital misallocation is: strictly increasing in π and weakly increasing in λ ; strictly decreasing in k and ν ; and ambiguous with respect to κ , p .*

For parameters other than κ and p , factors that push up the contract rate, ω , reduce capital misallocation, whereas factors that lead to a lower contract rate typically increase misallocation. In particular, when financing conditions for start-ups tighten, a lower ν , capital misallocation rises. Intuitively, capital is misallocated in the model because of the interaction of the agency and matching frictions; in an environment with only agency frictions, the optimal contract rate would specify $\omega \rightarrow \infty$ and no capital would be misallocated.⁵⁸ Therefore, capital misallocation reflects insurance against financing risk and my estimates of capital misallocation in section 3.3 are indicative of the costs that firms incur to militate against it.

2.3.5 Welfare analysis

In this section, I provide conditions under which the economy is constrained efficient. In order to do so, I consider the general version of the baseline model that includes VC bargaining power, as set out in section A.2 of the Appendix. The key difference is that I endow the VC with bargaining power $1 - \delta$, where $\delta = 1$ is the case considered in the baseline model. Social efficiency in this setting requires that the flow rate of entry and the contract rate in the decentralised equilibrium coincide with those that would be chosen by a planner maximising total surplus, which is defined as

$$TS = \int_0^\infty e^{-\rho t} \left([\kappa p \pi - k] \mu_{d,t}^p - k \mu_{d,t}^u - \frac{\tilde{\sigma}}{2} \Lambda_t^2 \right) dt \quad (15)$$

where Λ_t denotes the flow rate of entry at time t and $\mu_{d,t}^p$ and $\mu_{d,t}^u$ should be understood to be the total mass of start-ups in productive and unproductive development, respectively, regardless of

⁵⁷In the baseline model where every funding round involves the same amount of capital raised, this is equivalent to the share of capital that is misallocated.

⁵⁸See Lemma 2.1.

their contract rate ω .⁵⁹ The flow value of total surplus is equal to the flow payoff from start-ups in productive development, $\kappa p\pi$, net of the costs of sustaining firms in development, $k(\mu_{d,t}^p + \mu_{d,t}^u)$, and of entry costs $\frac{\bar{\sigma}}{2}\Lambda_t^2$.⁶⁰

Proposition 2.7. *If the matching function exhibits constant returns to scale, $\beta = 1 - \alpha$, and the bargaining weights in the contracting problem are equal to their respective matching function elasticities, $\alpha = \delta$ and $\beta = 1 - \delta$, then the decentralised equilibrium is constrained efficient.*

Proposition 2.7 provides conditions under which the decentralised equilibrium coincides with the solution to the planner’s problem. The conditions are analogous to the Hosios condition typical in the search literature. Intuitively, for a fixed equilibrium contract rate, ω , the standard results from the search literature apply to the entry condition for start-ups: the negative congestion externality implies that entry may be excessive, but a lack of appropriately due to incomplete bargaining power implies that entry might be inefficiently low. These two effects are balanced when the Hosios condition is satisfied.

To provide some intuition on why it follows immediately that the contract rate is socially efficient, it is helpful to conceptualise the contract rate as pinning down the optimal rate of ‘entry’ of productive start-ups and VCs back into the search market: VCs funding start-ups and start-ups in productive development re-enter the search market at rate ω . If the values of start-ups and VCs in search are pinned down optimally by the Hosios condition, then these incentives to ‘re-enter’ the search market must also be optimal. Therefore, the contract chosen in the decentralised equilibrium will ensure the correct flow rates of firms back into the search market and so the decentralised equilibrium is socially efficient. Of course, as pointed out by Inderst and Müller (2004), there is no particularly reason to think that the Hosios condition should be satisfied in the VC market.

2.3.6 Discussion

The model provides several insights. First, the financing environment affects not only the quality of projects that can be funded, but also interacts with project characteristics. This is the key insight of Nanda and Rhodes-Kropf (2017), who show that financing conditions interact with project novelty; Nanda and Rhodes-Kropf (2013) provide empirical support. Relative to their model, I demonstrate this result in an incomplete contracting environment when contracts are negotiated optimally. Furthermore, my model provides the additional insight that a project’s development horizon interacts with the financing environment, which provides an explanation for the tendency of venture capital to fund projects with short gestation periods (Narain, 2024).

Second, the model provides a structural interpretation of funding histories. For instance, in environments where agency frictions are most severe, the model predicts that start-ups should be funded in many smaller funding rounds, visiting the market frequently. Conversely, when the financing market is tight, the funding duration rises among funded firms. Therefore, funding

⁵⁹Out of steady-state, firms may have different contract rates. For instance, denoting the measure of firms at time t in productive development with contract rate ω by $\mu_{d,t}^p(\omega)$, the total mass of firms in productive development at time t is $\mu_{d,t}^p = \int_0^\infty \mu_{d,t}^p(\omega) d\omega$. The proof of Proposition 2.7 takes full account of these dynamics.

⁶⁰Recall that entry is specified as a cutoff rule. If the flow of entry is Λ_t , then the cutoff must be $\hat{c}_t = \bar{\sigma}\Lambda_t$. The total flow entry costs at time t are calculated as $\int_0^{\hat{c}_t} c/\bar{\sigma} dc = \frac{\bar{\sigma}}{2}\Lambda_t^2$.

histories encode information about the projects characteristics and the financing environment. This provides a route to learning these latent primitives, which I exploit in the estimation.

Finally, the model offers insights into how the venture capital *market* operates, beyond the implications for a *single entrepreneur* attempting to finance their development. In the model, the agency and matching frictions interact in equilibrium. On the one hand, the potential need to secure follow-on funding implies that investment decisions are forward looking and depend on anticipated market tightness. On the other hand, the choice of contracts affects the measure of investors and entrepreneurs in the search market, which feeds back into market tightness. This implies that the choice of contract by one firm has spillovers in the availability of capital to other firms and, to the extent that this is not correctly internalised, the market will be inefficient.

2.4 Quantitative extensions for estimation

This section adapts the baseline model for quantification by relaxing a restrictive assumption and introducing one additional feature.

First, in the baseline model, access to capital for firms with viable projects does not improve over their development cycle. In practice, as more information is produced about the start-up’s potential, the information issues that contribute to the matching friction should ease. However, in the baseline model, there is no intermediate information production and all projects in search are identical. Therefore, I generalise the model to include two hurdles.⁶¹ After overcoming the first hurdle, funding conditions may improve, reflecting the better information available to investors about the start-up. In principle, this could be extended to any number of hurdles, but two hurdles facilitates mapping to data, where funding rounds are typically considered to be “early-stage” or “late-stage”. Furthermore, two hurdles is sufficient to capture the qualitative feature of the empirical hazard rate for successful exits, which is first increasing and then decreasing in the time since first funding (Jovanovic and Szentes, 2013). Capturing exit hazards is particularly important because I estimate the model on censored data.

Second, I introduce acquisitions as a distinct form of exit for firms in the second (or “late”) stage. In practice, there exists a market in which firms with partially-developed innovations can sell to incumbent firms who may be better placed to commercialise the innovation. Furthermore, I show that acquisitions substitute for financing, formalising the insight of Nanda and Rhodes-Kropf (2016) that a market for ideas can be crucial for overcoming financing risk. Therefore, to properly capture the effect of financing on start-up activity and outcomes, it is important to allow firms to choose between being acquired and scaling independently. I permit acquisitions only in the late-stage, mirroring the environment in Arora et al. (2024), where a start-up must overcome both technology risk and commercialisation risk but may sell out to an acquirer before attempting to overcome the second hurdle.

⁶¹These hurdles—or development stages—may have various interpretations. For instance, in Arora et al. (2024), firms must first overcome technological before considering commercialisation challenges. Alternatively, Ewens et al. (2018) frame multiple development stages as ‘experiments’ that deliver information about project viability.

2.4.1 Model outline

Section A.5 presents the equilibrium conditions; here, I focus on features that differ from the baseline. For clarity, Figure 4 depicts the potential paths that a start-up may take, obscuring the unproductive development states for readability.

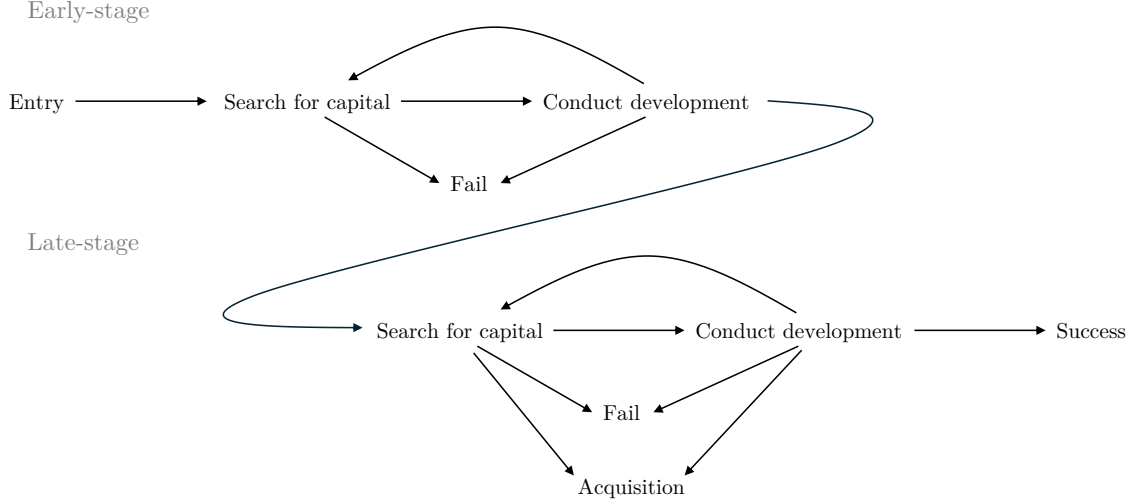


Figure 4: Start-up lifecycle

For ease of exposition, the two states “productive development” and “unproductive development” have been combined into “conduct development”.

The early-stage Following entry, start-ups seek early-stage funding and have value $V_{s,e}$. During search, they fail at rate λ and meet a VC at rate, ν_e , which is stage-specific. A start-up in the early-stage that invests flow k_e realises a result at rate κ , which is positive with probability p_e/p_l , where p_l is the probability of overcoming late-stage uncertainty, conditional on being in the late-stage and obtaining a result.⁶²

Conditional on meeting with a VC, the start-up signs a contract with contract rate ω_e , which provides funding for the early-stage only.⁶³ The contract problem is identical to that laid out in section 2.2, except that the probability p is replaced by p_e/p_l .⁶⁴ If successful in overcoming early-stage uncertainty, the start-up progresses to the late-stage and seeks late-stage funding.

The late-stage A start-up that overcomes early-stage uncertainty begins its search for late-stage capital. During search, it fails at rate λ and finds a VC at rate, ν_l . In general, $\nu_l \neq \nu_e$.

Start-ups in the late-stage may be acquired. I define an acquisition as the sale of a partially-developed project and, therefore, acquisitions may occur whether the firm is in search or development. Specifically, I assume that potential acquirers arrive at rate ϕ , whether the firm is in

⁶²This convention means that, in a frictionless world, each start-up in the early-stage would be successful with probability $p_e = (p_e/p_l) \times p_l$.

⁶³Funding applies only to the current stage. This captures VC specialisation across the start-up lifecycle.

⁶⁴Specifically, the expected discounted capital cost for the VC is

$$K_e(\omega_e) = \left(1 + \frac{\kappa(1 - p_e/p_l)}{\rho + \omega_e}\right) \frac{k_e}{\rho + \kappa + \omega_e}.$$

search or development. This rate is exogenous and the same for all firms. The acquirer values the fully-developed project possessed by the start-up at $\epsilon \times \pi$, where $\epsilon \in [0, \infty)$ is a random variable with CDF $G(\cdot)$ and that ϵ is match-specific, common knowledge and *i.i.d.* across matches. However, the acquirer purchases the start-up before it has reached full development so there is residual uncertainty. In the late-stage, a start-up's project would succeed with probability p_l if fully-funded, so the acquirer's valuation for the project is $\epsilon p_l \pi$. This is known to the start-up.

I assume that the start-up makes a take-it-or-leave-it offer to the acquirer, setting the price at the maximum of its current value in the late-stage, $V_{i,l}$ for $i \in \{s, d\}$, and the valuation of the potential acquirer, $\epsilon p_l \pi$.⁶⁵ If the former exceeds the latter, the potential acquirer will choose not to acquire the start-up. Bringing these insights together, the probability that an acquisition occurs conditional on a meeting is given by

$$\begin{aligned} \text{Development:} \quad & Pr(\epsilon p_l \pi \geq V_{d,l}) = 1 - G(V_{d,l}/p_l \pi) \\ \text{Search:} \quad & Pr(\epsilon p_l \pi \geq V_{s,l}) = 1 - G(V_{s,l}/p_l \pi) \end{aligned}$$

which permits defining an effective arrival rate of acquisitions $\hat{\phi}_i = \phi \cdot [1 - G(V_{i,l}/p_l \pi)]$ for $i \in \{s, d\}$. The effective arrival rate of acquisitions modifies the arrival rate of potential acquisitions by the acceptance rules outlined above and so is endogenous. In the quantification, I will impose $\epsilon \sim \text{Exp}(1/\xi)$, so that ξ is the average 'synergy'. With this in mind, the HJB equations for late-stage search and development are given by

$$\begin{aligned} \text{Development:} \quad & \rho V_{d,l} = \kappa [p_l \pi - V_{d,l}] + \omega_l [V_{s,l} - V_{d,l}] + \hat{\phi}_d \cdot [p_l \pi E[\epsilon | \epsilon > V_{d,l}/p_l \pi] - V_{d,l}] \\ \text{Search:} \quad & (\rho + \lambda) V_{s,l} = \nu_l [V_l^M - V_{s,l}] + \hat{\phi}_s \cdot [p_l \pi E[\epsilon | \epsilon > V_{s,l}/p_l \pi] - V_{s,l}] \end{aligned}$$

where V_l^M is the value of a meeting in the late-stage, which is defined analogously to the baseline model. Relative to the baseline model, there is the additional flow value deriving from acquisitions and the conditional expectation obtains because only potential acquirers with sufficiently high synergies result in acquisitions. Given $\epsilon \sim \text{Exp}(1/\xi)$, the value functions can be written more concisely as

$$\text{Development:} \quad \rho V_{d,l} = \kappa [p_l \pi - V_{d,l}] + \omega_l [V_{s,l} - V_{d,l}] + p_l \pi \xi \phi \exp \left\{ -\frac{V_{d,l}}{p_l \pi \xi} \right\} \quad (16)$$

$$\text{Search:} \quad (\rho + \lambda) V_{s,l} = \nu_l [V_l^M - V_{s,l}] + p_l \pi \xi \phi \exp \left\{ -\frac{V_{s,l}}{p_l \pi \xi} \right\} \quad (17)$$

An outstanding problem is how to compute the expected capital cost when there are acquisitions. With acquisitions, a firm in productive development experiences an event – a result, acquisition, or funding exhaustion – at rate $\kappa + \omega + \hat{\phi}_d$. I assume that the contract written between the start-up and VC is such that the VC terminates funding when the firm is acquired. For an unproductive start-up, the entrepreneur can prevent the acquirer from conducting the

⁶⁵Note that I take an agnostic view on acquisition demand; the model could be generalised in several ways to endogenise ϕ . For instance, ϕ may reflect the intensity of incumbent monitoring of start-up technologies, or the stance of antitrust policy. Without a theory of acquisition demand, little is gained by splitting the surplus according to some Nash bargaining weights. The key point is that acquisitions perform a substitution role (Proposition 2.8), which obtains in the simple case where the start-up makes a take-it-or-leave-it offer.

due-diligence to learn ϵ , because they have control rights. Therefore, the VC is left to honour their commitment, which expires at rate ω_l . The implication is that the expected capital cost associated with contract rate ω_l is

$$K_l(\omega_l) = \left(1 + \frac{\kappa(1-p_l)}{\rho + \omega_l}\right) \frac{k_l}{\rho + \kappa + \omega_l + \hat{\phi}_d} \quad (18)$$

The optimal contract is then found by solving the contract problem, analogous to equation (5), where the value functions are given by equations (16) and (17) and the capital cost by (18).

Financing market equilibrium The equilibrium conditions mirror those for the baseline model, appropriately extended to allow for two stages. They are outlined in section A.5, along with the full set of equilibrium conditions for this version of the model.

2.4.2 Discussion

With the setup outlined, I highlight a key theoretical insight in the context of acquisitions: start-ups searching for capital accept acquisition offers at a higher rate than those with funding secured. This is formalised in the following proposition.

Proposition 2.8. *Given that the optimal contract rate is finite, $\omega < \infty$, the effective arrival rate of acquisitions is higher in search than in development: $\hat{\phi}_s > \hat{\phi}_d$.*

Proposition 2.8 illustrates that firms are more inclined to accept acquisitions while searching for funding, suggesting that acquisitions substitute for venture capital. The intuition is straightforward: a firm searching for capital has a less favourable outside option in acquisition negotiations, making it more willing to accept a broader range of acquisition offers.

Finally, a word on parameter choices across stages. Some parameters differ across stages, while others remain constant, reflecting the need to balance realism and separately identify various parameters with available data. In light of the second motivation, I have assumed that the rate of uncertainty resolution and of failure in search, κ and λ respectively, do not differ across stages. However, I allow $k_e \neq k_l$ and $\nu_e \neq \nu_l$. Gompers (1995) shows that later-stage firms raise larger amounts of capital, which motivates $k_e \neq k_l$ (I anticipate $k_e < k_l$ but do not impose it). Furthermore, to allow access to capital to improve as the start-up develops, reflecting the increased information available about its prospects, I allow $\nu_e \neq \nu_l$.

3 Quantification for the US VC market

In this section, I estimate the model of section 2.4 on US data. First, I discuss how funding histories and start-up outcomes can be used to estimate the model's primitives. Using the estimated model, I then study the determinants of start-up outcomes. The key result is that moving to a frictionless benchmark increases the share of start-ups that achieve successful exits from 30% to 50% (+20pp).

3.1 Identification strategy

This section discusses the identification strategy. To aid intuition, I first discuss how to identify the baseline model of section 2.2, then extend to discuss acquisitions, and finally discuss the full model with acquisitions and two stages. The identification strategy for the full model leverages insights from the simpler environments, but I move from an exactly-identified to over-identified setting and complement the set of moments with parameters from an auxiliary model.⁶⁶

To demonstrate what data is commonly available, Table 1 shows an illustrative dataset. For a given firm, it is typical to observe the dates of each of its funding rounds, an indicator for the ‘type’ of funding round, e.g. early or late, and the amount raised. In certain cases, the firm’s implied valuation is also available but this is not true for the majority of cases and more likely to be available for firms that had a successful exit (Jagannathan et al., 2022). For successful outcomes, data on the valuations is generally available, but missing in a non-negligible share of cases. Conversely, failures are unobserved. For this reason, it is common to assume that firms that have not received funding for a number of years have failed; Kerr et al. (2014) refer to these cases as “living-dead”.⁶⁷ This is unnecessary for my purposes, because the model specifies the full dynamics for funding and outcomes.

Firm ID	Date	Event	Round Type	Amount raised	Valuation
1	28/01/2013	funding round	Early	6.0	-
1	24/09/2013	funding round	Early	6.5	-
1	04/02/2015	funding round	Late	30.0	-
1	17/04/2019	IPO	-	80.5	342.6
2	12/03/2013	funding round	Early	3.0	-
2	19/07/2016	funding round	Late	10.5	-
2	01/01/2017	Acquisition	-	-	89.4
3	06/03/2014	funding round	Early	9.1	-

Table 1: Illustrative dataset

The table shows illustrative data on venture capital funding rounds and exits for three fictitious firms. Amounts raised and valuations should be interpreted in units of million USD.

3.1.1 Identification of the baseline model

The baseline model pools all exits (acquisitions and successes) and ignores the early-stage/late-stage distinction. Taking the discount rate ρ as given, there are seven parameters to estimate: project characteristics, (k, κ, p, π) , financing conditions, (λ, ν) , and the entry cost parameter, $\tilde{\sigma}$. The estimation procedure has two steps. First, $\Theta = (k, \kappa, p, \pi, \lambda, \nu)$ is estimated by minimising the distance between empirical and simulated moments. Second, given Θ , I compute the entry value and recover $\tilde{\sigma}$ from the model condition for the flow rate of first funding rounds.

⁶⁶Estimation targets moments and auxiliary-model parameters; see (Gourieroux et al., 1993) for details on indirect inference.

⁶⁷Most datasets also include information on the venture capital firm(s) making the investments and some include additional information, such as board representation and other contract features, but this information is not required for my purposes.

Not all moments carry separating information about all elements of Θ ; several parameters affect funding-history moments only through the contract rate, ω , which itself depends on two composites $\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k}$ and $\mathcal{F} = \kappa + \nu \times \frac{\rho + \kappa}{\rho + \lambda}$ (see equation (11)). Therefore, it is important to choose moments that also load separately on parameters in Θ .

The frequency and timing of events identify (κ, λ, ν) but only pin down (k, p, π) up to \mathcal{C} . The mean number of funding rounds, duration between rounds, and time-to-exit are given by

$$E[N_f] = \frac{(\kappa + \omega)(\lambda + \nu)}{\kappa(\lambda + \nu) + \lambda\omega} \quad (19)$$

$$E[T_{br}] = \frac{1}{\kappa + \omega} + \frac{1}{\lambda + \nu} \quad (20)$$

$$E[\text{time-to-exit}] = \frac{(\lambda + \nu)^2 + \nu\omega}{(\lambda + \nu)((\kappa + \omega)(\lambda + \nu) - \nu\omega)} \quad (21)$$

which are functions of $(\kappa, \lambda, \nu, \omega)$ alone. Intuitively, when follow-on financing is hard to secure, high λ or low ν , start-ups raise fewer rounds with longer gaps; when agency costs are severe or project quality is low, high \mathcal{C} , the opposite occurs. In addition, time-to-exit is typically longer for longer-horizon projects, low κ .⁶⁸ For a fourth moment, the share of firms that raise capital more than T years after their first funding round (I set $T = 5$) provides separate information on $(\kappa, \lambda, \nu, \omega)$.⁶⁹ Measuring failure times directly would be preferable, but is infeasible in practice.⁷⁰ Intuitively, the share of firms that raise capital later date T is lower when firms struggle to survive without a capital injection, suggesting it is decreasing in λ .

Separately identifying (k, p, π) requires two more moment conditions because the previous moment conditions identify them up to \mathcal{C} . The mean burn rate, defined as the rate of capital utilisation between funding rounds, disciplines k and the mean exit multiple disciplines π .⁷¹ The mean burn rate and mean exit multiple are given by

$$E[\text{burn rate}] = E\left[\frac{\bar{K}}{T_{br}}\right] = \bar{K} \times \left(\frac{(\kappa + \omega)(\lambda + \nu) \ln\left(\frac{\kappa + \omega}{\lambda + \nu}\right)}{(\kappa + \omega) - (\lambda + \nu)} \right) \quad (22)$$

$$E[\text{exit multiple}] = E\left[\frac{\pi}{\bar{K} N_f}\right] = \frac{\pi}{\bar{K}} \frac{q_d}{1 - q_d} \ln(q_d^{-1}) \quad (23)$$

which uses $T_{br} \sim \text{Hypo}(\kappa + \omega, \lambda + \nu)$ and $N_f \sim \text{Geo}(q_d)$, where q_d is defined in Proposition 2.3.

⁶⁸ $E[\text{time-to-exit}]$ may be written, $E[\text{time-to-exit}] = E[N_f](\kappa + \omega)^{-1} + (E[N_f] - 1)(\lambda + \nu)^{-1}$, which adds up the time in development plus time in search (between rounds). Noting $E[N_f](\kappa + \omega)^{-1} = \kappa^{-1}(1 - \Delta_p)$, financing risk brings forward the time-to-exit, because only firms that complete development before funding runs out can be successful. However, the time spent in search delays completion.

⁶⁹ The analytical expression is complex – see section A.4.

⁷⁰ In fact, the time-to-failure is directly informative of p because it affects whether firms fail in search or development, which occur at different times following a previous funding round. Empirical work commonly assumes failure after some period (e.g. two years) following the last observed funding round. However, Panel A of Figure 5 demonstrates this would miss-classify many firms in my sample.

⁷¹ For k , the capital injection itself is non-monotonic (Corollary 2.2), making it less than ideal (see e.g. Strebulaev and Whited 2012). The issue is that, although k directly raises the capital injection, \bar{K} , it causes a reduction in the funding horizon, which may dominate. The burn rate is preferable because it normalises by the time between rounds. For π , using the exit-multiple, rather than the exit valuation directly, is more appropriate when estimating an *ex ante* homogeneous firm model because it better compensates for the implicit pooling of firms that may differ in their levels of both k and π . An alternative is to use a moment that loads on p , not π , such as the share of successes (see Proposition A.4). However, it is important that the model can explain this as an untargted feature of the data.

Finally, to identify $\tilde{\sigma}$, note that Θ determines V_s by equation (13). The entry rate is $V_s/\tilde{\sigma}$, which suggests that $\tilde{\sigma}$ can be backed out given the empirical entry rate. In practice, entry is unobserved, but the model condition determining the flow of first funding rounds can be used instead.⁷² In steady state, it follows that

$$\tilde{\sigma} = \left(\frac{\nu}{\lambda + \nu} \times \frac{1}{\text{flow first-time rounds}} \right) \times V_s \quad (24)$$

which enables recovery of $\tilde{\sigma}$. See section A.4 for the derivation.

3.1.2 Identification of the baseline model with acquisitions

Now consider the single-stage baseline model but allow for acquisitions. This adds two parameters: the arrival rate of offers, ϕ , and the average quality of offers, ξ .

Acquisitions depend jointly on (p, π, ϕ, ξ) , which complicates identification. Specifically, the arrival rate of accepted offers is $\hat{\phi}_i = \phi \exp\{-V_i/p\pi\xi\}$ and the expected acquisition price (conditional on acceptance) is $V_i + p\pi\xi$ for $i \in \{s, d\}$. Both expressions feature p and π multiplicatively with ξ and (p, π, ϕ, ξ) appear multiplicatively in the HJBs (see equations (16) and (17)). As a result, many moments move in the same direction in response to changes in any of (p, π, ϕ, ξ) ; see Figure C.1, which shows the elasticities of targeted moments with respect to (p, π, ϕ, ξ) around their estimated values.

Three moments help to disentangle (p, π, ϕ, ξ) . First, the mode of exit separates (p, π) from (ϕ, ξ) : whereas increases in both (p, π) and (ϕ, ξ) make acquisitions more attractive, (p, π) also affect the success rate directly. Therefore, the acquisition-to-success ratio separates (p, π) and (ϕ, ξ) . Figure C.1 shows the ratio falls with (p, π) , is relatively insensitive to ξ and rises with ϕ . Second, with acquisitions, the overall exit multiple becomes less effective in separating p and π , because acquisition prices are anchored to $p \times \pi$, not π alone. To avoid this confounding, I target the average exit multiple for successes. Finally, separating ϕ from ξ is difficult because both a higher offer arrival rate, ϕ , and higher offer quality, ξ , increase some notion of the arrival rate of “quality-adjusted” offers. Using information on the shape of the distribution of acquisition exit multiples provides a route forward. In particular, the share of high-exit-multiple acquisitions rises with ξ , but is relatively insensitive or decreasing in ϕ ; see Figure C.1. I therefore include the share of acquisition multiples that exceed ten (10X).

3.1.3 Identification of the full model

To estimate the model with two stages and acquisitions, I follow the two-step procedure discussed in section 3.1.1, first estimating the ‘internal’ parameters and then backing out the entry cost distribution parameter, $\tilde{\sigma}$, from the entry condition. Relative to the preceding discussion, I move to an over-identified setting and include two auxiliary-model targets. Furthermore, some parameters are now stage-specific. To identify these parameters, I rely on stage-specific information on funding patterns, which leverages the “tagging” of funding rounds as either early or late-stage in the data. Table C.2 reports the elasticities of targeted moments with respect to all parameters around estimated parameter values.

⁷²Entry and first-rounds differ because a share $\lambda/(\lambda + \nu)$ of entrants do not obtain funding.

In the early-stage, parameters (ν_e, k_e, p_e) are stage specific, whereas (κ, λ) apply across stages. Firms successfully navigate the early-stage with probability p_e/p_l and obtain continuation value $V_{s,l}$; the parallel to separating p from π in the baseline model is to separate p_e/p_l from $V_{s,l}$ here. Rather than use information on the payoff, it is necessary to target p_e/p_l more directly; choosing moments to pin down p_l in the late stage then implies that p_e is identified.⁷³ I include the share of firms that progress to the late-stage and the share of firms with just one funding round to inform p_e/p_l .⁷⁴ Motivated by the discussion in section 3.1.1, I also include the mean number of early-stage rounds and the mean burn rate for early-stage rounds (winsorised at 5% in both tails).⁷⁵ Finally, I introduce an auxiliary model that derives naturally from the model’s structure to provide further information on (λ, ν_e) . Within each stage, the distribution of the time between funding rounds is hypoexponential; for two successive early-stage rounds, $T_{br}^{e \rightarrow e} \sim \text{Hypo}(\kappa + \omega_e, \lambda + \nu_e)$. Estimating this distribution by MLE on the empirical distribution of the duration between early-stage funding rounds solicits direct information on $\lambda + \nu_e$.⁷⁶

My approach to estimating the parameters of the late-stage reflects the discussion in section 3.1.2. I compute the mean number of late-stage rounds conditional on having at least one late-stage round (to remove mechanical effects from the progression share in the unconditional mean), include the 5%-winsorised mean late-stage burn rate, the mean exit multiple for successes, the acquisition-to-success ratio, and the share of acquisition multiples above 10X. I also include an auxiliary-model target, mirroring the approach in the early stage. In the late-stage with acquisitions, the time between rounds is distributed $T_{br}^{l \rightarrow l} \sim \text{Hypo}(\kappa + \omega_l, \lambda + \nu_l + \hat{\phi}_s)$, reflecting that firms in search are acquired at rate $\hat{\phi}_s$. I include the estimate for $\lambda + \nu_l + \hat{\phi}_s$ as a target.

Finally, the parameters (λ, κ) apply across stages. The aforementioned moments are informative of these parameters but, following the discussion in section 3.1.1, I also include the mean time-to-exit and the share of firms with a funding round more than five years after their first.

3.2 Data and estimation

In this section, I present the dataset used to estimate the model and the core estimation results. I then discuss model fit as a validation exercise.

3.2.1 Dataset construction

My sample includes all firms in the Thomson Reuters venture capital dataset from the US that received their first round of venture capital funding between 2005 and 2015 inclusive. A typical problem in analysing young firms is that insufficient time has passed since their inception to observe their final outcomes. In order to maintain consistency across the data, I exclude any firm observations that occur more than seven years after their first funding round. This mirrors

⁷³Data on valuation step-ups when moving from the early to late-stage would offer one route to placing structure on $V_{s,l}$, but data availability is limited.

⁷⁴In the two-stage model, the number of funding rounds (across both stages) is no longer Geometric and p_e/p_l affects the number of funding rounds directly.

⁷⁵Section 3.4 discusses the motivation for winsoring and conducts robustness with respect to the threshold.

⁷⁶Practically, there is an issue because the two parameters of the hypoexponential distribution are interchangeable; I obtain two estimates (h_1, h_2) and must choose which to assign as the counterpart to $\lambda + \nu_e$. I choose to target $\max\{h_1, h_2\}$. See section C.0.1 for a full discussion.

Ewens and Farre-Mensa (2020), who consider the start-up status seven years after first funding. Section 3.4 shows that the estimated parameter values are robust to this choice.

To construct a dataset of the form in Table 1, I apply several screens. Financing rounds in the raw data are labelled as “seed”, “early stage”, “expansion”, “later stage” or “bridge loan”. I disregard bridge loans because their objective is to tie firms over while they raise their next funding round, rather than for conducting major development steps.⁷⁷ Next, I label seed and early-stage rounds as “Early” and expansion and later-stage rounds as “Late” and exclude the minority of firms that only have a “Late” funding rounds to maintain consistency with the model.⁷⁸ Furthermore, I collapse any deals for a firm if their dates coincide and make a few manual corrections to the data when there are clear errors.⁷⁹ I complement the data with market capitalisation data from CRSP for IPOs and M&A price data from Crunchbase, when price data from Thomson Reuters is unavailable. All values are expressed in 2015 USD.

I face two distinct issues of missing data: investment amounts for funding rounds and deal values for some M&A transactions. For investment amounts, I follow Jagannathan et al. (2022) and impute capital injections.⁸⁰ For deal values, I follow Kerr et al. (2014) by setting it equal to 1.5 times the total capital invested, implying an exit multiple of 1.5X.

Finally, I need to map modes of exit in the data to the notion of “success” and “acquisition” in the model. A success requires that a firm complete its development, whereas an acquisition allows a firm to sell a partially-developed project. I define an acquisition in the data as an M&A transaction by a non-financial buyer and a success as an IPO or an M&A transaction by a financial (including SPAC) buyer. The choice to distinguish between financial and non-financial buyer M&A transactions reflects the idea non-financial buyers typically have the capabilities to acquire start-ups at pre-commercial stages, whereas financial buyers do not. This echoes the distinction in the model, in which a “success” requires that the firm reach full development.

The final sample includes 11,030 firms that raised 33,158 funding rounds within seven years of first funding. Among these firms, 400 were successful and 2,034 were acquired within the same period. Section B.1 provides further detail on data construction and Table B.2 includes further summary statistics.

3.2.2 Estimation

The sample I construct is censored at seven years following a firm’s first funding round. To maintain consistency, I censor the simulated data in the same way before computing moments.⁸¹ I fix the discount rate $\rho = 0.08$, which is similar to the value used in Jovanovic and Szentes (2013). A unit of time in the model corresponds to a year. The remaining parameters are estimated in two steps. First, I jointly estimate all parameters except for $\tilde{\sigma}$, denoted by Θ , by

⁷⁷Despite their labelling, “bridge loans” reported in Thomson Reuters are equity investments.

⁷⁸See section B.2 for definitions of each round type. In seed and early-stage rounds, firms are in the ideation/testing stage, whereas firms in the expansion/late stages are attempting commercialisation. The expansion stage also sees institutional investors enter.

⁷⁹For instance, on some occasions Thomson Reuters reports IPOs that were announced by never occurred.

⁸⁰This applies to in approximately 10% of rounds for the US (and slightly more for the UK). The imputation approach is described in section B.1 of the Appendix.

⁸¹Censoring needs to be handled carefully for estimates of $\lambda + \nu_e$ and $\lambda + \nu_i + \hat{\phi}_s$. See section C.0.1.

minimising the criterion function⁸²

$$C(\Theta) = \sum_{j \in \Theta} \left(\frac{\tilde{m}_j - m_j(\Theta)}{\frac{1}{2}|\tilde{m}_j| + \frac{1}{2}|m_j(\Theta)|} \right)^2 \quad (25)$$

where \tilde{m}_j is the empirical moment (or auxiliary parameter) and $m_j(\Theta)$ its model counterpart computed at Θ .⁸³ Second, I recover $\tilde{\sigma}$ from the counterpart to equation (24) for the full model, where I set the flow rate of initial rounds to 1.00 for the US, without loss of generality.

Parameter estimates Table 2 reports the parameter estimates, which should be interpreted as the parameters of the average US VC-backed start-up in my sample.

Parameter		Estimate	Std. Error
Discount rate	ρ	0.08	-
R&D milestone arrival rate	κ	0.26	0.01
Failure rate in search	λ	0.62	0.04
Early-stage development level	p_e	0.23	0.01
Early-stage flow investment cost (\$Mn USD/year)	k_e	4.16	0.18
Early-stage meeting rate	ν_e	2.54	0.13
Late-stage development level	p_l	0.30	0.01
Late-stage flow investment cost (\$Mn USD/year)	k_l	16.15	0.25
Late-stage meeting rate	ν_l	5.20	0.26
Success payoff (\$Mn USD)	π	218.18	15.40
Acquisition offer arrival rate	ϕ	0.65	0.07
Mean acquisition ‘synergy’ ($\varepsilon \sim \text{Exp}(1/\xi)$)	ξ	0.74	0.08
Entry cost distribution ($c \sim U[0, \tilde{\sigma}]$)	$\tilde{\sigma}$	2.68	0.37

Table 2: US parameter estimates

The table reports parameter estimates from the model in section 2.4. Estimation uses a simulated sample of 220,600 firms, $S = 20$ times the size of the empirical sample. $\rho = 0.08$ is fixed externally. Parameters (except $\tilde{\sigma}$) are estimated by minimising (25) (equal weighting of centred percentage deviations; see footnote 83) and I report asymptotic standard errors, where the covariance matrix of moment conditions is computed from 1,000 bootstrap samples, clustered at the firm level. To avoid local minima in the estimation, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points. With the resulting candidate parameter estimates, I run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses. With $S = 20$, simulation noise is miniscule, so I report standard errors that simply reflect sampling variability (applying an adjustment raises standard errors by $< 1\%$). With these estimates, the entry cost parameter, $\tilde{\sigma}$, is recovered from the condition determining the flow rate of first-time funding rounds. To compute its standard error, I take 1,000 samples from the asymptotic distribution of parameter estimates, solve the model, and recover the implied value for $\tilde{\sigma}$. The standard error is the standard deviation of $\tilde{\sigma}$ among this sample. All amounts are in 2015 USD.

The average start-up project requires approximately eight years of funding to fully commercialise ($\approx 2 \times 1/\kappa$), consistent with typical VC timelines. This start-up requires annual investment of $k_e = \$4.16\text{Mn}$ for the first half of this period and $k_l = \$16.15\text{Mn}$ for the second half. If

⁸²To avoid local minima, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points. I then run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses.

⁸³The criterion function sums over squared percentage deviations between the empirical and simulated objects with equal weighting. The choice of equal, rather than optimal, weighting scheme reflects the need to utilise variation in certain moments, specifically coming from exit payoffs, in order to identify the model’s parameters. For instance, the sample features over 30,000 funding rounds, but just 400 successes. The discussion of identification in the baseline model, section 3.1.1, illustrates the importance of exit multiples in differentiating between (p, π) , but an optimal weighting scheme would effectively ignore this variation.

successfully in independently scaling the project, the start-up generates value $\pi = \$218.18\text{Mn}$. However, the start-up may fail due to technological or commercial challenges. Conditional on uncertainty resolution, this happens with probability $1 - p_e/p_l = 23\%$ in the early-stage and $1 - p_l = 70\%$ in the late-stage. In the late-stage, acquisition opportunities arrive at the rate of one every 18 months ($\approx 12/\phi$) and the average offer price is approximately $\$50\text{Mn}$ ($\approx p_l\pi\xi$).

In practice, the start-up’s development may be hindered by access to external finance. In the early-stage, the probability that a viable firm in search fails before securing funding is $\lambda/(\lambda + \nu_e) = 20\%$ and the average duration spent searching is $12/(\lambda + \nu_e) = 3.8$ months. In the late stage, financing conditions are more favourable: the probability that a viable firm in search fails before securing funding falls to 10% and securing funding takes an average of just 2 months.⁸⁴ The improvement in conditions is consistent with greater information as the start-up develops. Finally, the estimates suggest that the average start-up can survive 19 months ($\approx 12/\lambda$) after exhausting its previous capital injection before shutting down.

The figures for the average start-up appear consistent with the type of high-risk innovative firm that VCs seek to invest in. Furthermore, in certain cases, validation can be sought from external data. For instance, biotech is a key sector for venture capitalists and progression rates for clinical drug trials in the United States provides an external measure of the progression probability, p_e/p_l . Biotech start-ups typically seek early-stage funding to fund Phase 1 clinical trials and are often able to exit via IPO before full clinical approval.⁸⁵ Drawing on a sample of over 7,300 drug development paths from 835 drug developers, including young biotech and large pharmaceutical companies between 2003 to 2011, Hay et al. (2014) find that the Phase 1 to Phase 2 progression rate is in the range 64-67% (Hay et al. (2014), Figure 1). With the obvious caveat that biotech start-ups are not fully representative of the “average” start-up, these numbers are broadly in line with the estimated early-to-late progression rate of $p_e/p_l = 77\%$.

The financing market meeting rates, ν_e and ν_l , are important determinants of distortions in the market. To the best of my knowledge, Ewens et al. (2022) provide the only comparable estimates in the literature. They estimate a structural model with heterogeneous entrepreneurs and investors who meet only for first-round funding. Due to heterogeneity, not every “meeting” in their model leads to investment, so the appropriate comparison is between ν_e and their deal frequency.⁸⁶ For entrepreneurs, the average deal frequency is 1.57 per year, ranging from 0.16 to 3.56 across the lowest to highest entrepreneur-quality deciles (Table 6, Ewens et al. (2022)). My estimate of $\nu_e = 2.54$ is within this range. The two models differ markedly in their environments and estimation procedures, so the similarity of estimates is reassuring. The estimates are also in line with reports from market participants.⁸⁷

⁸⁴This is calculated as $\lambda/(\lambda + \nu_l + \hat{\phi}_s) = 0.62/6.15$, where the value for $\lambda + \nu_l + \hat{\phi}_s$ is reported in Table 3. The duration in search before securing funding is $12/(\lambda + \nu_l + \hat{\phi}_s) = 2$ months.

⁸⁵See *Betting on Biotech, Pitchbook, 2020*. Pitchbook is a leading venture capital data provider.

⁸⁶Some matches are not profitable in their model, so do not lead to deals.

⁸⁷Pitchbook suggest that “some startups are able to raise the capital they need in three months, while others may take a year” (*PitchBook’s guide to VC fundraising for startups, Pitchbook*). Furthermore, a founder survey with over 500 respondents by First Round Capital, a leading venture capital firm, shows similar timelines: 25% report securing funding within two months, 45% required between two and four months, and the remainder required more than four months. See Question 12, *State of Startups, First Round Capital, 2018*.

Model fit and validation Panel A of Table 3 reports the targeted moments, which the model captures well.⁸⁸ Panel B shows that the model also captures several key untargeted features of the data. First, although the likelihood of an acquisition relative to a success is targeted, their levels are not, and the model captures the share start-ups with these successful outcomes tightly. Second, the model accurately reflects the fact that the mean time-to-success exceeds the mean time-to-acquisition. However, it is worth noting that successful exits occur slightly too quickly in the model.⁸⁹ Third, the model captures the mean exit multiple, despite only targeting the mean multiple for successes and the right tail of the acquisition multiple distribution.

Panel A: Targeted moments		
Moment	Data	Model
Mean # early rounds	1.98	1.91
Mean # late rounds progressed to late	2.46	2.43
Share one round (%)	32.81	26.47
Mean duration b/w rounds (years)	1.16	1.25
Estimate of $\lambda + \nu_e$	3.30	3.16
Estimate of $\lambda + \nu_l + \hat{\phi}_s$	5.94	6.15
Share funded after year 5 (%)	19.68	18.25
Mean burn rate early round (\$Mn/year)	7.91	7.49
Mean burn rate late round (\$Mn/year)	23.78	24.89
Share to late-stage (%)	41.56	41.86
Mean time-to-exit (years)	3.67	3.33
Acquisition-to-success ratio	5.08	5.09
Mean exit multiple success	9.24	9.06
Share acquisition multiples > 10X (%)	8.01	8.17
# first funding rounds	1.00	1.00
Panel B: Untargeted moments		
Share successful (%)	3.63	3.78
Share acquired (%)	18.44	19.24
Mean time-to-success (years)	4.44	3.40
Mean time-to-acquisition (years)	3.52	3.31
Mean # funding rounds [†]	3.01	2.93
Mean exit multiple	5.79	5.33

Table 3: Targeted and untargeted moments

Panel A shows 15 moments (including two auxiliary parameter estimates) used in the estimation procedure. Empirical moments are computed from the sample of 11,030 firms. Other than for the estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, simulated moments are computed using the parameter values given in Table 2 to simulate a sample of 220,600 ($= 20 \times 11,030$) firms. For $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, the parameter values are taken directly to reduce the computational burden of the estimation procedure. In computing the empirical and simulated moments, data on a given firm is censored seven years after its first funding round. Panel B shows a set of untargeted moments. [†] Note that the mean number of funding rounds is implicitly targeted in the estimation through targeting the mean number of early-stage rounds, the mean number of late-stage rounds conditional on reaching the late-stage, and the share of firms that raise late-stage capital

Figure 5 demonstrates that the model accurately captures the entire distribution of key

⁸⁸For simplicity, I refer to all targeted objects as moments, although two are auxiliary model parameters.

⁸⁹I define time-to-exit as the time to completion in the data. There is often a lag between announcement and completion in the data, so that the model is more closely aligned with the data when using time-to-announcement.

features of VC funding and exit data. Panel A of Figure 5 reports information on the number and timing of funding rounds in the model and data. Three points are particularly noteworthy. First, as shown in Panel A (i), the model replicates the monotonically decreasing probability mass function (PMF) for the number of funding rounds. Unlike in the single-stage model where this is true for all parameterisations, the two-stage model is able to capture more complex dynamics.⁹⁰ For instance, if more project uncertainty were concentrated in the later stages, most start-ups would progress through the early stage and raise multiple funding rounds, leading to an initially increasing PMF. Therefore, that the estimation replicates this feature of the data suggests that the balance of uncertainty is appropriately distributed between the early and late stages. Second, as shown in Panel A (iii), the model closely replicates the average time from the first funding round to each subsequent round. The assumption of *ex ante* homogeneity in the model enforces an upward sloping relationship here: for each firm, round $n+1$ occurs after round n by definition, but when all firms draw from the same distribution, this must also apply to the average (in large samples). The empirical pattern observes this upward sloping relationship, suggesting that unaccounted for *ex ante* heterogeneity is not excessive in the data.⁹¹ Third, the model performs well in replicating funding continuation dynamics, defined as the share of firms that still receive funding more than x years after their first round. This is important because the model is estimated on censored data, and divergence in the timing of funding rounds (and the time-to-exit distribution) could lead the estimation procedure to select markedly different parameter values as the censoring threshold is varied. Section 3.4 discusses this in more detail.

Panel B of Figure 5 reports information on capital intensity. In the model, capital injections are constant within stage: firms either raise \bar{K}_e in an early round or \bar{K}_l in a late round. Conversely, there is substantial variation in within-stage capital injections in the data. A key concern is that, by ignoring this heterogeneity and focussing on a “representative” start-up, the model omits important dynamics in capital intensity and firm growth. Panel B (i) reports the median and interquartile range for the cumulative amount of capital raised by firms by funding round. Unsurprisingly, the model fails to capture the growing heterogeneity in cumulative capital raised with the number of rounds, but performs much better in replicating the median, capturing the slope well. Relatedly, Panel B (ii) reports the burn rate distribution, pooling across all funding rounds. The model closely captures the skewness in the burn rate distribution, despite variation in the model being driven predominately by heterogeneity in the duration between funding rounds (the denominator), rather than the amount raised (the numerator). Furthermore, differences in the time between rounds in the model reflects *ex post* heterogeneity across firms. The ability to replicate the empirical distribution without large dispersion in round sizes indicates that *ex post* heterogeneity – shocks hitting the firm after it raises capital – is a key driver of dynamics.

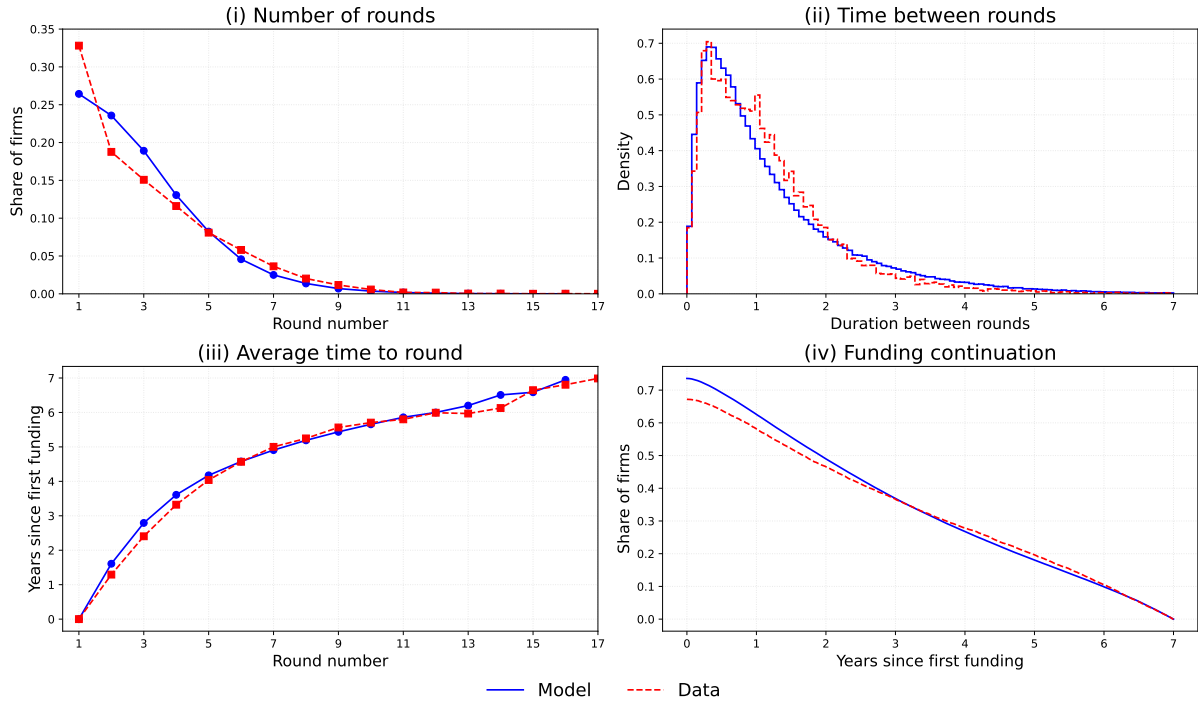
Finally, Panel C of Figure 5 reports information on outcomes. The skewness in start-up outcomes is a key feature of the market (Hall and Woodward, 2010) and Panel C (i) shows that

⁹⁰In the baseline model, the number of funding rounds follows a Geometric distribution. This is not the case when there are multiple stages.

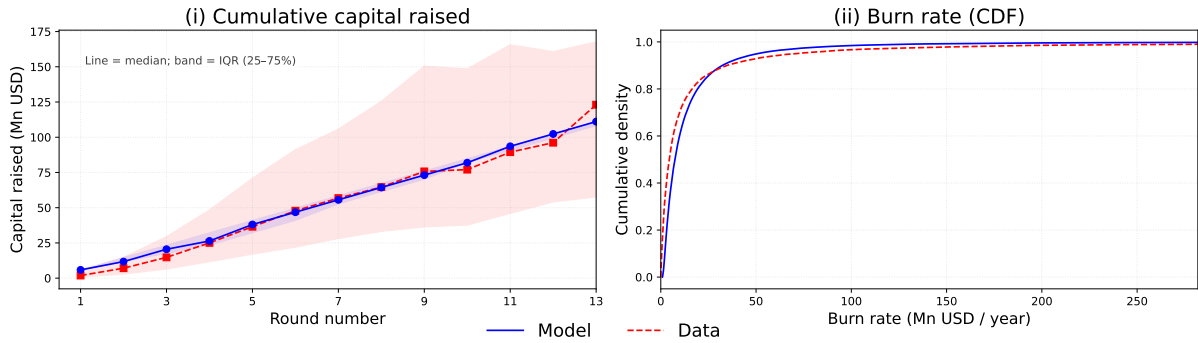
⁹¹For example, if firms in some sectors progress quickly through four funding rounds, while firms in the dominant sector only ever require three rounds over a longer period of time, then a selection effect would imply the mean time to $N_f = 3$ exceeds that to $N_f = 4$.

the model does reasonably well in capturing this feature of the data. Furthermore, Panel C (ii) shows that the model captures the full distribution of the timing of exits well. However, it is worth noting that “successes” arrive too early in the model (see Table 3, Panel B), so that this is predominately driven by the model’s ability to capture the timing of acquisitions.

Panel A: Funding rounds



Panel B: Capital intensity



Panel C: Exits

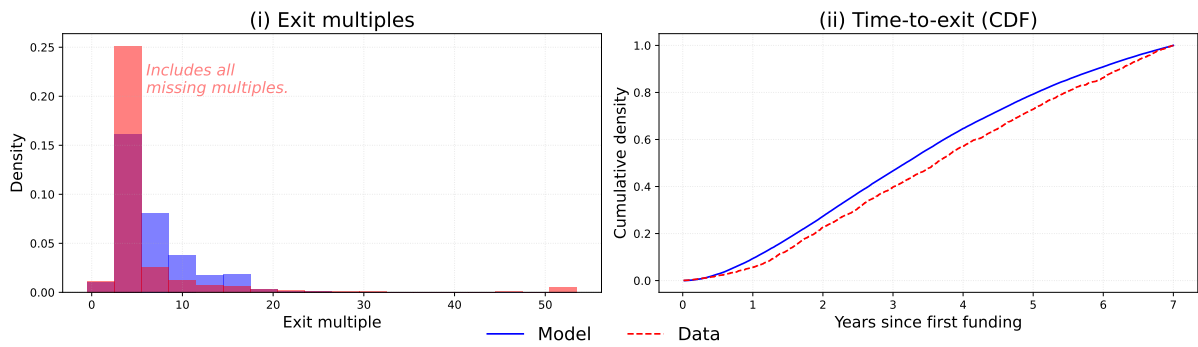


Figure 5: Model validation

The figure shows outcomes in the model and data across funding rounds (panel A), capital intensity (panel B) and exits (panel C). All empirical and simulated data is censored at seven years following a firm's first funding round. "Funding continuation" in Panel A (iv) reports the share of firms that raise capital more than x years after their first funding round. To aid visualisation, cumulative capital raised is shown up to the 13th round; the burn rate CDF is reported up to the 99th percentile; and exit multiples are winsorised at 50X. Missing empirical exit multiples are set to 1.5X, following Kerr et al. (2014).

3.3 Analysis of the US venture capital market

In this section, I show that my estimates imply that frictions in access to venture capital have large effects on entrepreneurial activity. To this end, I quantify the distortions that entrepreneurs face, compared to a frictionless setting in which they can self-finance their projects. The estimates suggest that they are large distortions along both relevant margins: entry and outcomes. Second, I use the model to understand what factors determine whether entrepreneurs can successfully commercialise their innovations. My estimates suggest that a substantial share of entrepreneurial projects are not funded to completion and, therefore, remain potentially viable. Finally, I estimate of capital misallocation in the US venture capital market.

Financing distortions This section examines the effects of financial frictions start-up entry and outcomes. First, I use the model to understand what drives modes of failure for VC-backed start-ups. Then, I compare the estimated economy with financial frictions to a counterfactual economy in which the entrepreneur is wealthy and can self-finance the project.⁹²

Panel A of Table 4 reports a decomposition of start-up outcomes. Column two reports results from censored data used to estimate the model. The estimates suggest that 17.2% of VC-backed start-ups remain active seven years after their first funding round, without having had a successful exit. Column three reports uncensored results, allowing for a full decomposition of start-up failures. Recall that the sample consists of VC-backed start-ups that have at least one funding round. Among these start-ups, approximately 70% fail. Furthermore, the estimates suggest that approximately 40% ($\approx 25.7 + 15.1$) are forced to shut down prematurely, before the resolution of uncertainty. The majority of these failures occur in the early-stage, consistent with more severe matching frictions when little is known about the start-up's prospects.

These estimates are supportive of substantial issues in financing young-innovative firms. However, caution is required in interpreting them. These results *do not imply* that these firms would have been successful, were they able to receive additional funding. Instead, they simply point to premature closure of positive NPV projects. To interpret the losses stemming from these premature closures, it is necessary to consider the counterfactual: what would have happened to these firms, were they to receive more funding.

Panel B of Table 4 reports these measures. In column two of Panel B, I report the success and acquisition distortions, defined analogously to those in section 2.3.2 of the baseline model. Specifically, the success distortion, Δ_p , is defined as wedge between the actual success rate and the success rate under self-financing.⁹³ Analogously, the acquisition distortion, Δ_a , is the wedge between the actual acquisition rate and the acquisition rate under self-financing. Both of these distortions condition on receiving at least one round of funding, mirroring my empirical sample. The probability of success is approximately $\Delta_p = 56\%$ lower when the E has to secure external funding, which corresponds to a 6 percentage point reduction. In other words, if the E could self-finance the share of successful outcomes would rise from 4.9% (see Panel A, column 3) to 11.0%. Acquisition outcomes are also adversely affected: the $\Delta_a = 37\%$ distortion is

⁹²See section A.5.1 for the relevant equations. Equivalently, the frictionless benchmark obtains when $\nu_e \rightarrow \infty$ and $\nu_l \rightarrow \infty$; see Lemma 2.1.

⁹³This is analogous to the “outcome distortion” defined in section 2.3.2. I refer to it here as a “success distortion” because the full model also includes a distinct role for acquisitions.

Panel A: Outcomes and modes of failure	Censored	Uncensored
Share successful (%)	3.8	4.9
Share acquired (%)	19.2	24.5
Share no outcome (%)	17.2	0.0
Share failed (%)	59.7	70.6
... in early-stage development (%)	16.1	18.2
... in early-stage search (%)	22.5	25.7
... in late-stage development (%)	9.0	11.6
... in late-stage search (%)	12.2	15.1
Panel B: Distortions	Agency & Matching	Matching only
Success distortion, Δ_p (%)	55.5	14.9
Acquisition distortion, Δ_a (%)	37.0	5.7
Entry distortion, Δ_s (%)	71.5	31.7
Panel C: Capital misallocation		
Share unproductive early-stage (%)		12.3
Share unproductive late-stage (%)		11.1
Share unproductive (%)		12.0
Share capital misallocated (%)		11.7

Table 4: US market assessment

The table reports start-up outcomes, measures of distortions induced by matching and agency frictions, and capital misallocation for the US estimation. Panel A reports modes of success and failure for firms from data simulated given estimated parameter values. Panel B reports distortions: for an outcome x in the frictional equilibrium, distortions are defined such that $x = (1 - \Delta_x)\bar{x}$, where \bar{x} is the value in the economy without frictions. Distortions relate to the success share, Δ_p , the acquisition share, Δ_a , and the entry value, Δ_s . Column “Agency & Matching” measures the combined effect of both frictions; column “Matching only” removes agency frictions (no hidden failure) while retaining matching frictions. Panel C reports figures on capital misallocation. Within each stage, this is defined as in equation (14). Across stages, the “share unproductive” reports the share of all firms receiving funding that are unproductive, whereas the “share capital misallocated” reports the share of capital allocated to unproductive firms, $(k_e\mu_{d,e}^u + k_l\mu_{d,l}^u)/((k_e(\mu_{d,e}^u + \mu_{d,e}^p) + k_l(\mu_{d,l}^u + \mu_{d,l}^p)))$.

equivalent to a 14 percentage point reduction in the probability of acquisition, from 39% to 25%. The finding of a positive acquisition distortion suggests that the (negative) direct effect of external financing on acquisitions via premature closure dominates the (positive) insurance effect.⁹⁴ Together, the results suggest not only that financial frictions inhibit the ability of entrepreneurs to independently scale their innovations, but they also result in fewer transfers of technologies to other firms operating in the market. Column three of Panel B reports the same distortions in an economy where entrepreneurs can commit not to engage in inefficient continuation, but must still search for funding for each stage.⁹⁵ Abstracting from moral hazard

⁹⁴In principle, the acquisition distortion may be positive or negative, reflecting two opposing forces. On the one hand, when entrepreneurs can self-finance, no viable projects are closed down, increasing the share that are acquired. If this effect dominates, then the acquisition distortion is positive; there are too few acquisitions. On the other hand, Proposition 2.8 shows that acquisitions substitute for financing. Therefore, when there are no issues in raising capital, entrepreneurs are less willing to accept low-value offers. If this effect dominates, then the acquisition distortion is negative; there are too many acquisitions.

⁹⁵In this economy, the E and VC can write a state-contingent contract that terminates funding when news about the project arises, which drives the effective development costs back to k_i for $i \in \{e, l\}$.

in contracting cuts the success distortion $\Delta_p = 56\%$ to 15% and the acquisition distortion $\Delta_a = 37\%$ to 6% . In other words, a substantial share of the lost outcomes are attributable to the interaction between agency and matching frictions.

The success and acquisition distortions allow for understanding the *ex post* effects of financing. To investigate the *ex ante* effects, Panel B also reports the entry distortion, Δ_s , defined, as in section 2.3.2, as the wedge between the actual entry value, $V_{s,e}$, and the entry value under self-financing, $\bar{V}_{s,e}$. The value of the average entrepreneurial project is approximately $\Delta_s = 72\%$ lower when the E cannot self-finance, suggesting that frictions in access to external finance have a large detrimental effects on start-up creation. Converting this into a measure of lost entry requires an estimate of supply curve of entrepreneurial projects, which is beyond the scope of this paper. However, unless the supply of entrepreneurial projects is exceedingly inelastic with respect to project value, the magnitude points to a substantial impact on start-up creation.

Capital misallocation Panel C of Table 4 reports the main results on capital misallocation. Within each stage, capital misallocation is given by equation (14). In the aggregate, misallocation can be computed based on the share of unproductive firms receiving capital or the share of capital allocated to unproductive firms. In practice, both figures are similar: 11.7% of capital is misallocated, whereas 12% of firms are unproductive, reflecting the similar degrees of misallocation across stages. Taken together, these results highlight a key tension in the VC market. On the one hand, roughly 40% of projects are closed down prematurely; if these firms could access additional funding, some would go on to achieve successful outcomes. On the other hand, frictions in allocating financing leads to the opposite problem: a substantial share ($\approx 12\%$) of capital is tied up in firms that have already ceased to create value.

3.4 Robustness

This section describes how the estimates change when I make alternative assumptions in the estimation procedure. First, my baseline sample covers first financings in 2005–2015 with seven-year censoring, so events are observed through end-2022. This window includes the sharp run-up in US VC activity culminating in 2021 and easing in 2022. To check that the estimates are not unduly influenced by this late-sample boom, I re-estimate the model on a 2005–2010 subsample, maintaining seven-year censoring. Table C.3 reports the results. Parameter estimates are qualitatively similar across the two samples and of comparable magnitude. Importantly, estimates of financing conditions, (λ, ν_e, ν_l) , are closely aligned.

Second, my estimation censors events at seven years following a firm’s first funding round. For the baseline (2005-2015), I cannot extend the horizon; however, the 2005-2010 subsample permits a longer window. I therefore re-estimate the model for the 2005-2010 subsample with twelve-year censoring. Table C.3 shows that parameter estimates are very similar across the two strategies. Figure 5 provides a further diagnostic: using the two estimated parameter vectors, I simulate data, apply censoring at $h \in \{3, \dots, 12\}$, and recompute the targeted moments. The simulated moments line up closely across h , including for $h > 7$, which is out of sample for the seven-year censored estimation. This insensitivity to the censoring choice is consistent with the broader model-validation evidence in Figure 5, where the model closely matches the average

time to each funding round, funding continuation (firm attrition), and time-to-exit.

Finally, I winsorise the burn-rate distribution at 5% in both tails for the baseline estimation. Empirically, variation in burn rates reflects both the size of capital injections and the duration between rounds; in the model, within a stage, only the inter-round duration varies across firms. Consequently, extremely large deals values add mass to the right tail of the burn rate distribution that the model does not explicitly capture. I winsorise in the estimation to prevent tail observations – which would primarily identify the *variance* of k_i for $i \in \{e, l\}$ if it were modelled – from having undue influence on average *level* of capital costs across firms. This aligns the estimator with the abstraction in the model, which treats all firms as *ex ante* homogeneous.

Figure 5 shows the model nonetheless produces substantial skewness in burn rates, though it cannot match the full tail thickness driven by heterogeneous deal sizes. Figure C.3 reports estimates under alternative thresholds $w \in \{0, 1, 2.5, 5 \text{ (baseline)}, 7.5, 10\}$. All parameters are stable across w , except the flow capital costs (k_e, k_l) and the payoff from success π . Consistent with the rationale for winsorising, as w falls the empirical mean burn rate rises (more right-tail mass retained), which the model matches via higher (k_e, k_l) . The success payoff, π adjusts to restore fit for the mean success multiple. Figure C.4 confirms that the model tracks the mean burn rate and mean success multiple across specifications. Overall, identification is not sensitive to the winsorisation threshold; only (k_e, k_l) and π adjust mechanically to match higher mean burn rates at lower w .

4 Venture capital across countries

Among the 2,000 firms worldwide that invested the most in R&D in 2023, 681 are headquartered in the US, compared with just 322 in the EU (left panel, Figure 6). Gornall and Strebulaev (2021) argue that the rise of the US venture capital industry catalysed the growth of many young, R&D-intensive firms. In contrast, debates in Europe often invoke the “European Paradox”: strong scientific capabilities coexisting with weaker commercialisation outcomes. Policy responses frequently seek to expand VC investment to close a perceived financing gap for young innovative firms (e.g., Draghi 2024). Consistent with that focus, advanced economies with higher VC intensity have younger firms among their largest R&D investors (right panel, Figure 6). However, compelling evidence that financing frictions – rather than available investment opportunities – are the principal constraint on European start-ups remains limited, which complicates the design of effective interventions. For instance, if less VC activity in Europe reflects weaker investment opportunities, there is less need for capital market intervention.

In this section, I use the model to diagnose the sources of the UK–US disparities and to evaluate policy interventions. The model can separately identify project characteristics from funding conditions, which overcomes the identification challenge posed by a lack of reliable measures of project quality at inception. I focus on the United Kingdom, rather than attempting a Europe-wide assessment, because the evolving nature of European capital market integration creates issues for market definition.⁹⁶ London is a major financial centre, the UK has a strong university system, and its common-law institutions resemble those of the US; yet VC activity

⁹⁶See Ewens and Farre-Mensa (2020) for a discussion of fragmentation in European private equity markets.

and lags the US (right panel, Figure 6).

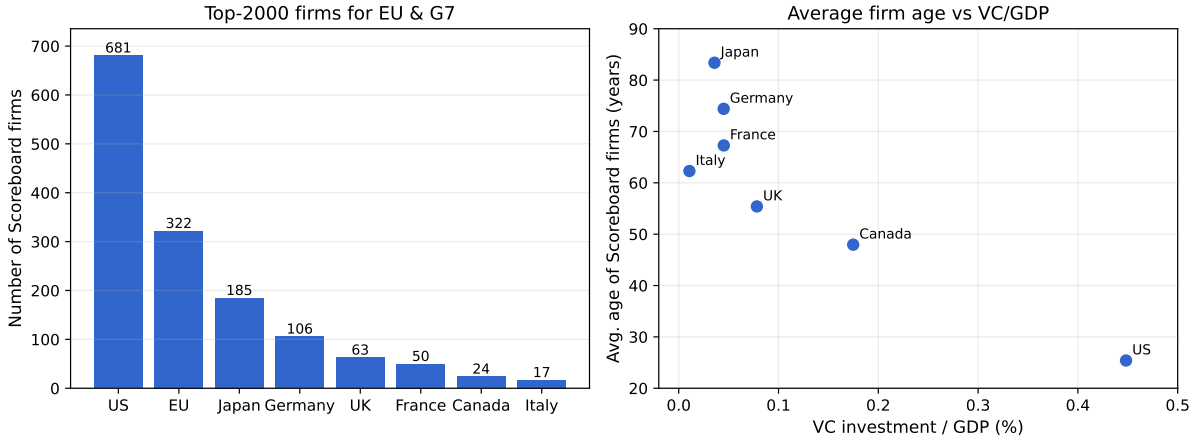


Figure 6: 2024 EU Industrial R&D Investment Scoreboard and VC activity

The figure combines data from the 2024 EU Industrial R&D Investment Scoreboard (Nindl et al., 2024) and venture capital activity data from the OECD. The left panel shows the number of the Scoreboard’s top 2,000 R&D investors in each G7 country and the EU total; the 2024 report provides data for 2023. The right panel reports the average age of these firms against average VC investment as a share of GDP (2010–2022). Age data and firm-country assignments are taken directly from (Nindl et al., 2024). Firms are assigned to the country of their registered office, which may differ from their operational or R&D headquarters.

Estimating the model, I show that access to capital and a thin acquisition market are quantitatively important constraints on UK start-up activity. I then use the model to assess policy interventions. As of end-2015 – the first-round cut-off in my sample – the UK’s development bank, the British Business Bank (BBB), primarily directed its programmes at early-stage venture and angel co-investment, on the premise, per its joint report with European counterparts, that the “equity gap” was most acute at this stage (Fernández Acevedo et al., 2016). In 2018, the BBB introduced British Patient Capital, a scheme targeted to late-stage start-ups.

To study whether government support is better targeted to the early or late-stage, I conduct a counterfactual exercise that reallocates capital stages, while holding total funding fixed. Reallocating 5% of total VC investment from the early to late-stage increases (flow) total surplus by 16-30% between steady-states, depending on the response of start-up creation. The mechanism is compositional: total exits decline marginally, but the composition shifts from acquisitions towards successes; the latter reflect completed projects and, on average, generate higher surplus. These results indicate scope for budget-neutral reallocation to improve outcomes.

The remainder of this section provides the details. Section 4.1 estimates the model for the UK, compares the UK and US markets, and discusses some supplementary evidence consistent with the findings. Section 4.2 conducts the policy exercise.

4.1 Market failure or limited investment opportunities?

Table 5 provides an overview of key differences between the US and UK markets for the period 2005-2015; see Table B.2 for further summary statistics.⁹⁷ On a per-capita basis, the US VC market services twice as many new firms each year as the UK. Furthermore, these firms are

⁹⁷Axelson and Martinovic (2016) analyse the exit patterns in at the European and the US level.

more likely to have completed a successful exit (including acquisition) within seven years of their first funding round.

Country	Firms	Successes	Acquisitions
US	11,030 (1.00)	400 (3.6%)	2,034 (18.4%)
UK	1,066 (0.48)	26 (2.4%)	97 (9.1%)

Table 5: US–UK Comparison

The table provides an overview of key aggregate statistics for the US and UK. Counts shown first; values in parentheses are normalised figures. For ‘Firms’ the value is the population-adjusted ratio to the US (US = 1.00). For ‘Successes’ and ‘Acquisitions’ it is the percentage of firms. All data is censored at 7 years following a firm’s first funding round.

The model captures various explanations for these differences. First, the meeting rates, ν_e and ν_l , capture access to capital, and the model can differentiate between access in early and late-stages. Relatedly, λ captures how long firms can survive without a new equity injection; to the extent that the availability of alternative funding instruments, such as venture debt, differs across countries, estimation of the model will detect it.⁹⁸ Second, estimation of the model allows for uncovering the quality and characteristics of the average start-up that receives VC funding in each market. Finally, the model can capture certain factors external to firms and to the financing environment. In particular, ϕ and ξ parameterise the acquisition market, determining the quantity of potential acquirers and the quality of the offers that these acquirers make to start-ups. The initial success probability, p_e , is also sufficiently reduced from as to capture the availability of high-skilled labour and regulatory factors, while the payoff, π , may reflect market size, taxation and public market valuations.

4.1.1 Estimation

In this section, I provide estimation results for the UK market to uncover factors that drive lagging performance of UK start-ups. I then discuss which features of the data drive the differences in parameter estimates, before providing an interpretation of the results.

Results The parameter estimates in Table 6 point to a tighter financing environment and more limited acquisition opportunities in the UK.⁹⁹ The meeting rates for start-ups in both the early and late-stages, ν_e and ν_l , and the arrival rate of acquisition offers, ϕ , are lower in the UK, and the differences are statistically significant. Conversely, the model cannot differentiate between project characteristics among funded firms in each country.

The differences in access to financing are economically large. In the early-stage, the average time spent in search is $12/(\lambda + \nu_e) = 6.3$ months in the UK, compared to 3.8 months in the US. In the late-stage, the differences are similarly large: 3.9 months compared to just 2 months. Furthermore, UK start-ups more than twice as likely to fail each time they seek funding, compared to comparable US start-ups.¹⁰⁰ The difference in acquisition markets is

⁹⁸Venture debt is often used to tie firms over between equity injections (González-Urbe and Mann, 2024).

⁹⁹The estimation procedure mirrors that for the US in section 3. Section C.1 replicates the model validation exercises for the UK.

¹⁰⁰43% of searches end in failure in the early-stage, and 25% in the late-stage ($=\lambda/(\lambda + \nu_l + \hat{\phi}_s)$); see Table C.4 in the Appendix. These numbers compare to 20% and 10% in the US, respectively.

Parameter	US		UK		$\Delta(\text{US-UK})$
	Estimate	Std. Error	Estimate	Std. Error	
ρ	0.08	—	0.08	—	0.00
κ	0.26	0.01	0.25	0.07	0.01
λ	0.62	0.04	0.81	0.17	-0.19
p_e	0.23	0.01	0.26	0.07	-0.03
k_e	4.16	0.18	3.46	1.02	0.70
ν_e	2.54	0.13	1.09	0.19	1.45***
p_l	0.30	0.01	0.30	0.04	0.00
k_l	16.15	0.25	12.04	2.53	4.11
ν_l	5.20	0.26	2.28	0.41	2.92***
π	218.18	15.40	205.26	66.56	12.92
ϕ	0.65	0.07	0.33	0.07	0.32***
ξ	0.74	0.08	0.81	0.72	-0.07
$\tilde{\sigma}$	2.68	0.37	0.98	2.32	1.70

Table 6: US-UK Parameter Estimates

The table reports parameter estimates from the model in section 2.4 for the US and UK. Estimates for the US are reproduced; see Table 2. The estimation procedure for the UK mirrors the US; I again use a simulated sample of 220,600 firms, although this now corresponds to $S \approx 207$ times the empirical sample size. To avoid local minima in the estimation, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points for each estimation. With the resulting candidate parameter estimates, I run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses. Significance levels for differences in parameter estimates across countries (except $\tilde{\sigma}$) are computed from the asymptotic parameter estimate distributions, assuming independence. For $\tilde{\sigma}$, I take 1,000 samples from the asymptotic distribution of parameter estimates for each country, solve the model, recover the implied value for $\tilde{\sigma}$, and compute the difference $\Delta_{\tilde{\sigma}} = \tilde{\sigma}_{US} - \tilde{\sigma}_{UK}$. The significance level is determined by the share of cases with $\Delta_{\tilde{\sigma}} < 0$. Significance levels for $\Delta(\text{US-UK})$ are: * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

also economically important. While there are no discernible differences in the quality of offers, parameterised by ξ , the arrival rate of offers in the US, ϕ , is approximately twice the UK level and the difference is significant.

Relative to the US, a clear issue in the UK market is difficulty securing follow-on funding. 53% of UK VC-backed do not raise additional funding within seven years of their first round (Table C.4), compared to 33% in the US (Table 3). Relatedly, just 33% of UK start-ups raise late-stage funding, compared to 42% in the US. Consistent with empirical work by Wilson et al. (2018), these issues point to severe issues in securing follow-on funding for UK start-ups. These issues feed into start-up outcomes. UK start-ups are less able to scale independently and are therefore more reliant on pre-commercial acquisitions. In the UK estimation, start-ups accept 66% of acquisition offers in search and 49% in development, compared to 50% and 41% in the US, respectively.¹⁰¹ Tentatively, this suggests that capital may be misallocated between stages: funding fewer projects at the outset but ensuring that follow-on funding is available to those that demonstrate intermediate success may improve outcomes. I consider this counterfactual exercise in section 4.2.

¹⁰¹This is computed as $\hat{\phi}_i/\phi$ for $i \in \{s, d\}$. For the UK, $\hat{\phi}_s = 0.22$ and $\hat{\phi}_d = 0.16$. For the US, $\hat{\phi}_s = 0.33$ and $\hat{\phi}_d = 0.26$. The estimates of ϕ are shown in Table 6.

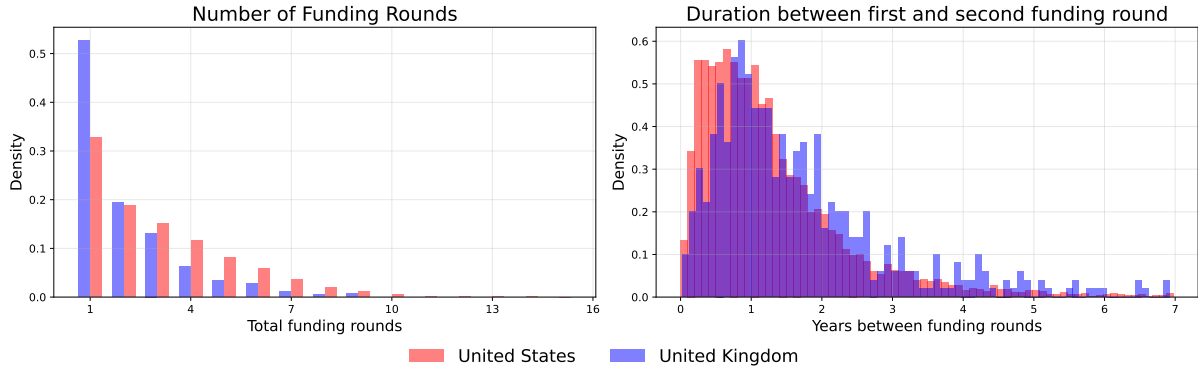


Figure 7: US versus UK funding patterns

The figure shows funding patterns for the UK and US samples. The left plot shows the distribution for the number of funding rounds for US firms (red) and UK firms (blue). The right plot shows the distribution of the duration of time between the first and second funding round for firms in each country. Focussing on the first-to-second round implies that data is uniformly censored; this choice is otherwise immaterial. All data is censored at 7 years following a firm's first funding round.

Identification The model separates financing frictions from other factors because the mechanisms leave distinct footprints in funding histories. In short, if UK underperformance reflected riskier or lower-quality projects, Propositions 2.3 and 2.4 imply more frequent, shorter funding rounds.¹⁰² Conversely, if the funding environment is the constraint, funded start-ups secure longer-duration commitments that insure against returning to the market.

Figure 7 compares funding patterns. On average, UK VC-backed start-ups complete 2.1 rounds within the first seven years of funding, with 17.5 months between rounds; the US averages are 3.0 and 13.9 months, respectively.¹⁰³ Higher project quality in the UK could also explain this pattern, but would be inconsistent with worse start-up outcomes and the lower entry rate. Therefore, the model highlights funding as a key constraint and the estimation cannot distinguish between project characteristics for funded start-ups.¹⁰⁴ The combination of tighter financing conditions and lower acquisition outcomes in the UK – the UK acquisition rate is 9.1%, compared to 18.4% in the US – implies that UK start-ups also face a thinner acquisition market. This follows from Proposition 2.8, which shows acquisitions substitute for financing: tightening finance from the US baseline would raise the acquisition rate. Therefore, the weaker acquisition outcomes in the UK must reflect more limited opportunities.

US projects under UK financing Table 7 reports counterfactual exercises for the US estimation that replace US financing market conditions, specifically ν_e and ν_l , by their estimated values for the UK and compares steady state outcomes. This provides further insights into the extent to which financing conditions shape the volume and outcomes of start-up activity. The table reports the (uncensored) share of successful and acquired firms, conditional on funding, and the entry value, V_s , across baseline and counterfactual economies.

¹⁰²Riskier projects benefit from a wait-and-see-approach, due to increased agency costs, leading to frequent capital injections. Conversely, lower-quality projects face a lower marginal benefit of insurance against financing risk, so optimally lean towards shorter-term funding commitments.

¹⁰³See Table C.4 for the full set of UK moments.

¹⁰⁴My estimation strategy does not identify start-up characteristics for projects that did not receive funding.

Differences in financing conditions explain approximately two-thirds ($\approx (3.3 - 1.6)/(3.3 - 0.8)$) of the gap in entry value between the US and UK; the remainder is explained by acquisition opportunities and other project characteristics. Thicker acquisition markets play a key role in the US, as reflected in the higher acquisition rate in the US counterfactual compared to the UK baseline. Decomposing across stages, rows four and five demonstrate that late-stage funding conditions in the US are particularly important in driving success, relative to acquisitions; in the UK, the similar success rate reflects the more limited acquisition opportunities.¹⁰⁵

	Share successful (%)	Share acquired (%)	Entry value (\$Mn)
UK: baseline	4.4	15.8	0.8
US: baseline	4.9	24.5	3.3
US: ν_e^{UK}, ν_l^{UK}	3.0	19.7	1.6
US: ν_e^{UK}	4.5	22.3	2.2
US: ν_l^{UK}	3.4	22.2	2.5

Table 7: US-UK decomposition

The table reports uncensored outcomes and the entry value across different counterfactual scenarios. Rows one and two show the UK and US baselines. Rows three to five compare different counterfactual scenarios, where the US financing market meeting rates are replaced with their respective UK estimates.

4.1.2 External validity and mechanisms

In this section, I provide supporting evidence consistent with the findings of the estimation exercise, discuss underlying causes, and highlight why some alternative explanations that are not considered within the model are unlikely central to the explanation.

Supporting evidence To address concerns of external validity, I point to two facts. First, the large inflow of capital into the US VC market following the 1979 clarification of the ERISA “prudent man” rule (Gompers and Lerner, 1999) coincided with a shift in start-up funding patterns.¹⁰⁶ Consistent with my identification strategy in the UK-US context, US start-ups funded before 1979 undertook fewer financing rounds and had longer intervals between them, relative to firms funded after 1979 (Figure 8).

Second, venture debt plays a much more significant role in the US market: the share of VC-backed firms using venture debt is approximately double the UK rate (Hellmann and Kavadias, 2016). One role of venture debt is to extend a firm’s runway before securing another equity injection; acting as a “bridge” between rounds (González-Urbe and Mann, 2024). Many start-ups do not have positive cash flows when raising venture debt and lenders expect to be repaid from the proceeds of future equity injections (Davis et al., 2020). Because repayment relies on follow-on financing, lenders effectively bear the firm’s financing risk: when follow-on funding is

¹⁰⁵Note that although the share of successes and acquisitions among funded firms in the US counterfactual with UK early-stage financing exceed that with late-stage financing, the entry value for the US is higher in the latter case. This reflects that not all entrants receive funding.

¹⁰⁶The modern VC firm traces its origins back to the creation of the American Research and Development Corporation in 1946, but the industry remained small for several decades, through the 1970s (Gompers, 1994).

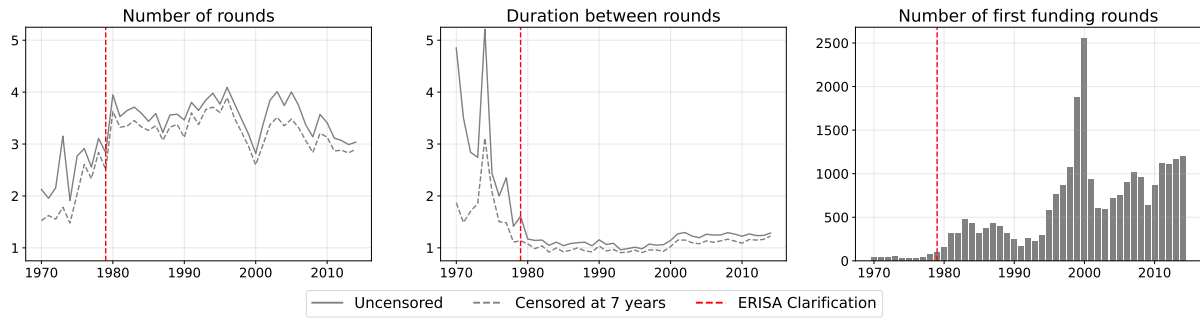


Figure 8: US historical funding patterns

The figure extends the US sample back to 1970 and, for cohorts of firms defined by their year of first funding, reports the average number of funding rounds, the average duration between funding rounds, and the total number of firms. The dashed line shows the results when data is censored at seven years following each firm's first funding round, as in the main estimation exercise. The solid line applies no censoring. The vertical red line refers to the clarification of ERISA in 1979.

harder to secure – as in my estimates – the supply of venture debt should contract. Therefore, the limited availability of venture debt in the UK is consistent with tightness in the VC market.

Underlying causes Outside the model, close inspection of UK and US venture capital markets identifies several differences. For instance, UK VC funds typically manage less capital than US funds, and US VC-backed start-ups raise larger funding rounds, particularly in the late-stage (Hellmann and Kavadias, 2016). My analysis suggests that these features are not symptomatic of weak investment opportunities, but reflect issues in allocating risk capital to start-up projects.¹⁰⁷

Constraints on risk-capital allocation stem from the relationship between venture-capital funds and their limited partners (LPs). Investor experience is a key indicator of the ability to raise large funds (Gompers and Lerner, 1999) and the UK market remained in its infancy for much of my sample. To the extent that investor experience is a constraint, the problem should dissipate as the market matures. However, policy may still be important in attracting talented investors to the UK market. On the other hand, institutional investors need to be willing to allocate funds to VC. UK pension funds have significantly smaller share of capital to private equity compared to international peers, constraining fund scale regardless of investor expertise.¹⁰⁸ Recent UK governments have placed pressure on pension funds to increase private equity allocations, and my analysis suggests that this is warranted.¹⁰⁹

With regards to acquisition markets, a core tension is the relative scarcity of prior successes from the UK VC market. Many of the most frequent acquirers in the US market, such as Amazon, Apple and Google, are themselves products of the US venture capital market. In the same way that these firms create increasing returns through spawning new ventures as spin-outs (Gompers et al., 2005), a similar effect may be present in the acquisition market: past success

¹⁰⁷Some indicators do point to financing issues, rather than demand-side constraints. For example, Hellmann and Kavadias (2016) point to the increasing prevalence of foreign investors in UK deals in later-stages, which suggests that UK VC funds may struggle to raise the large sums required to invest in late-stage ventures.

¹⁰⁸See *Comparing the asset allocation of global pension systems*, New Financial, 2024.

¹⁰⁹For instance, the recent Mansion House Accord has secured a commitment from various pension funds to allocate 5% of their assets under management to UK private equity markets by 2030, releasing around £25Bn.

stories improve the exit opportunities for later innovators, spurring start-up creation.

Alternative explanations I do not model VC quality or skill, which is an important determinant of start-up outcomes (Sørensen, 2007). Broadly, VCs affect outcomes through project evaluation, monitoring, and expertise (Da Rin et al., 2013). In principle, differences in these competencies could explain UK–US differences. In this vein, Greenwood et al. (2022) propose an endogenous growth model with a VC sector and suggest that differences monitoring, project evaluation, and value-add contribute to growth differences between the US and France.

Viewed through my framework, these forces do not appear to be first-order. First, in reduced form, if UK VCs were systematically worse at selecting or steering companies, projects would appear riskier or lower quality in the UK. VCs would then be cautious to commit too much capital, preferring more frequent rounds. Empirically, UK start-ups have fewer rounds than comparable US firms (Figure 7), which contradicts this mechanism. Second, superior monitoring or stronger investor control rights should detect and prevent inefficient continuation more readily, supporting longer-horizon capital commitments. If US VCs held an advantage, US start-ups should obtain longer-duration funding, contrary to the data.¹¹⁰ This suggests that differences in monitoring capabilities or the allocation of control rights are not first-order determinants of US-UK differences.

4.2 Policy evaluation

In this section, I consider whether there are gains to reallocating capital between stages in the UK market. Existing policy instruments target different stages in the start-up lifecycle, implying that policymakers have access to budget-neutral tools that may improve outcomes.

Policy exercise To understand whether there are potential gains to such a strategy, I conduct the following counterfactual exercise. Starting from the estimated parameter values from Table 6, I reallocate capital between stages by adjusting the meeting rates, (ν_e, ν_l) , while holding constant total (flow) investment in the market.¹¹¹ I do so under two cases: (i) holding start-up creation constant, and (ii) allowing start-up creation to adjust endogenously. Note that in both cases, the total number of firms that receive funding still adjusts because the share of entrants that receives funding depends on ν_e . To aid interpretation, I map from changes in (ν_e, ν_l) to changes in the composition of funding and report results against this basis.

Figure 9 reports the effects across steady states as capital is shifted between stages. In each panel, the x -axis reports the volume of capital reallocated from the early to late-stage, as a share of total investment. The left-panel shows the meeting rates, (ν_e, ν_l) , that are required to generate corresponding capital reallocation; this represents the set of feasible policy interventions.¹¹² The

¹¹⁰Empirically, Tian (2011) shows that when monitoring costs are high – proxied by geographic distance between investor and firm – the start-up tends to be funded with more rounds, and with shorter durations between them.

¹¹¹Total flow investment is defined as the sum of capital investment into start-ups of all stages, $k_e \times (\mu_{d,e}^p + \mu_{d,e}^u) + k_l \times (\mu_{d,l}^p + \mu_{d,l}^u)$. This measure captures both “productive” and “unproductive” start-ups and is analogous to the term that enters equation (15), adapted for the two-stage model.

¹¹²The mapping depends on the response of start-up creation. In practice, reallocating capital to the late-stage increases start-up creation, which raises total early-stage investment for given ν_e . Therefore, when start-up creation is allowed to respond, a larger adjustment in ν_e (and ν_l) is required to deliver a given reallocation.

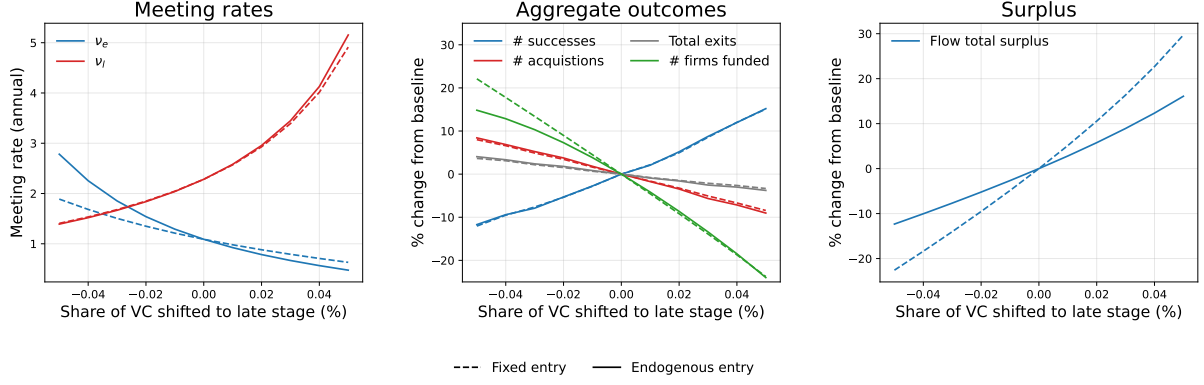


Figure 9: UK policy counterfactual

The figure reports the results of a budget-neutral policy intervention that shifts capital from the early to late-stage. The intervention is implemented by adjusting the meeting rates, (ν_e, ν_l) , while holding total (flow) investment, K^{total} fixed, $K^{\text{total}} = K^e + K^l$ and $K^i = k_i(\mu_{d,i}^p + \mu_{d,i}^u)$ for $i \in \{e, l\}$. The policy intervention is conducted under two assumptions on start-up creation: (i) holding start-up creation constant at its initial value (dashed); (ii) allowing start-up creation to adjust endogenously, according to $V_{s,e}/\bar{\sigma}$ (solid). The left-panel shows the mapping between the meeting rates (y -axis) and the share of capital reallocated from the early to late-stage. The share reallocated is defined as $(K_{\text{new}}^l - K_{\text{init}}^l)/K^{\text{total}}$. The centre-panel shows the change in the number of funded firms (green), the number of successful firms (number funded \times share successful; blue), the number of acquired firms (number funded \times share acquired; red) and the number of total exits (i.e. success + acquisition; gray) as capital is reallocated between stages. The right-panel shows the change in total (flow) surplus. Flow surplus is defined analogously to equation (15), but augmented to account for acquisitions and two stages. Specifically, it is given by $\kappa p_l \pi \times \mu_{d,l}^p + \hat{\phi}_d p_l \pi E[\epsilon | \epsilon > V_{d,l}/p_l \pi] \times \mu_{d,l}^p + \hat{\phi}_s p_l \pi E[\epsilon | \epsilon > V_{s,l}/p_l \pi] \times \mu_{s,l} - K^{\text{total}} - (\bar{\sigma}/2) \times \Lambda^2$, where Λ is the flow rate of start-up creation. In case (i), this is held constant at the initial value of $V_{s,e}/\bar{\sigma}$. In case (ii), it is equal to $V_{s,e}/\bar{\sigma}$ as $V_{s,e}$ adjusts endogenously to changes in (ν_e, ν_l) .

centre-panel shows the effect on core aggregate statistics. When capital is shifted to the late-stage, the number of firms that are funded falls, but start-ups that are funded are more likely to achieve an exit, whether a success or acquisition. On net, the number of successes rises and the number of acquisitions falls, and this occurs independent of whether start-up creation adjusts.¹¹³ Total exits fall, but the compositional effect implies an increase in value creation (right-panel).¹¹⁴ The compositional effect is welfare-enhancing because successes correspond to fully developed projects, whereas acquired projects are only partially developed. Therefore, when exits tilt towards successes, a greater number of projects lead to commercial applications.

Table 8 considers a particular intervention, in which 5% of total VC investment is shifted from the early to late-stage. This reflects the sort of marginal adjustment that policymakers could induce given existing budgets.¹¹⁵ Furthermore, there is a practical limit to how much capital can be reallocated between stages; without sufficient early-stage funding, there are not enough late-stage firms to absorb an increase in late-stage capital availability.

Reallocating 5% of total capital from the early-stage to late-stage is associated with a 15% increase in the number of successes, and a 8-9% decrease in the number of acquisitions, depending on the start-up creation adjustment. On net, total exits fall by 3-4%, but the compositional effect increases total (flow) surplus rises by between 16% and 30%, depending on the extent of

¹¹³Note that the response of the total number of funded firms is relative insensitive to the response of start-up creation. When capital is reallocated to the late-stage and start-up creation responds, a larger decline in ν_e (increase in ν_l) is needed to generate a given reallocation. Then, a smaller share of entrants receive funding, which mostly offsets the effect of start-up creation on the number of firms funded.

¹¹⁴The difference in the response of total surplus when start-up creation is held constant, or is allowed to adjust, is almost entirely accounted for by a change in total entry costs.

¹¹⁵Given recent trends in VC activity, it amounts to between £0.5-1Bn annually; see PitchBook (2025).

the endogenous response of start-up creation.

Panel A: Policy lever		
Outcome	Fixed Entry	Endogenous Entry
ν_e	0.63	0.48
ν_l	4.91	5.15
Panel B: Outcomes		
Outcome	Fixed Entry	Endogenous Entry
<i>Shares</i>		
Share success (% Δ)	50.94	51.54
Share acquired (% Δ)	20.11	19.68
Share to late (% Δ)	33.25	33.05
<i>Counts</i>		
No. funded firms (% Δ)	-23.76	-23.99
No. successful firms (% Δ)	15.08	15.19
No. acquired firms (% Δ)	-8.43	-9.03
No. firm exits (% Δ)	-3.33	-3.78
<i>Aggregate</i>		
Total flow surplus (% Δ)	29.71	16.09

Table 8: UK budget-neutral reallocation across stages (5% of total investment)

The table reports results from a policy that shifts 5% of total VC investment from the early to late-stage, holding total investment constant. The exercise mirrors that in Figure 9; see notes for details.

Discussion When policy targets the late-stage – holding budgets fixed and comparing steady states – fewer start-ups are funded, yet funded firms are more likely to overcome the early-stage milestone because late-stage liquidity raises the marginal benefit of early-stage investment for VCs.¹¹⁶ Therefore, funded firms are more likely to progress to the late-stage (Table 8), where they face more favourable funding conditions. This allows them to scale independently rather rely on acquisitions at pre-commercial stages.

Policymakers then face an extensive-intensive margin trade-off: holding budgets fixed, they must choose between backing many firms, often with insufficient capital to scale, and backing fewer firms while making capital available for them to scale. In the estimated model, the latter effect dominates, so budgets should be reallocated to the late-stage.

An important assumption in this analysis is that start-ups cannot self-finance because they deliver no intermediate cash flows. If start-ups that produce a positive signal in the early-stage could finance late-stage development from cash flows, then leveraging the extensive margin – providing little funding to many early-stage firms – may instead be optimal. Aragoneses and

¹¹⁶To understand the response of early-stage investors, view the early-stage as an experiment that produces information about the project’s viability (Kerr et al., 2014; Ewens et al., 2018). When late-stage capital is scarce, even projects with positive early signals cannot be scaled, which lowers the value of the experiment. Early-stage investors commit less capital to any round, shortening early-stage commitments and leading many funded firms to run out of runway before interim news arrives. Conversely, when late-stage funding is reliable, early investors commit more funding upfront, increasing the likelihood that interim uncertainty is resolved.

Saxena (2025) consider a related policy exercise in a model calibrated to Europe with collateral constraints in which start-ups can invest out of retained earnings. They show that reallocating interventions towards the early-stage is preferable when frictions are mild, whereas a late-stage focus becomes more attractive as financial market imperfections worsen; in this case, retained earnings are less effective in relaxing constraints. My analysis is complementary and highlights the case in which firms remain unprofitable for many years, which is particularly relevant for the subset of firms access VC financing; since 2000 in the US, the only year in which a majority of VC-backed IPOs were profitable was 2009 (Figure B.1).

5 Venture capital across technologies

Venture capital finances a narrow subset of start-ups, favouring sectors where uncertainty is resolved quickly, particularly software and services (Lerner and Nanda, 2020).¹¹⁷ In long-horizon sectors, uncertainty about future funding availability may depress initial investor returns and lead certain sectors to be underfunded (Nanda et al., 2014). Well-established exit mechanisms provide a route to early liquidity and partially offset this risk (Ghosh and Nanda, 2010).

There is substantial variation in the mode-of and time-to-exit across sectors. Figure 10 reports the acquisition-to-success ratio and average time-to-exit across Venture Economics Industry Classification (VEIC) industries in my sample.¹¹⁸ Sectors with longer average time-to-exit, which tend to be science-based sectors, are less likely to exit via acquisition, relative to success. Conversely, start-ups in software and service based sectors exit quickly, typically via acquisition. To the extent that these margins of heterogeneity interact with financial frictions, funding may be misallocated away from long-horizon projects without established exit mechanisms.

In this section, I show that the interaction of these features with financial frictions is quantitatively significant and can induce a large reallocation of VC funding away from science-based sectors. In a counterfactual exercise that removes financing frictions, the share of VC-backed start-ups in Biotech, Medical/Health and Semiconductors rises by 8pp, offset by a relative decline in software and service start-ups.

To understand the potential for policy to affect funding misallocation, I assess the effect of start-up acquisitions on start-up creation across sectors. Among the policy levers that affect the market for firms, antitrust policy towards start-up acquisitions is the most natural, but policy that affects market structure among potential acquirers may also be important.¹¹⁹

I find that the incentive effects of acquisitions on start-up creation are strongest in science-based sectors, where funding distortions are most severe. Intuitively, marginal changes in the availability of acquisition opportunities has the strongest effects on start-up creation incentives for start-ups pursuing long-horizon projects in sectors where acquisition opportunities are already limited. Conversely, software start-ups are much less exposed to marginal changes in ac-

¹¹⁷Consistent with this view, Narain (2024) shows that venture capital funding skews towards technologies with short development horizons, whereas government funding tends to support projects with longer timelines.

¹¹⁸The VEIC is a granular industry categorisation specifically designed for VC-funded firms. Given its focus on VC-funded start-ups, it offers a more granular classification than alternatives, such as NAICS or SIC codes.

¹¹⁹For example, a start-up that appeals to many acquirers can afford to wait for the best match, raising its payoff at exit. In this light, the increase in M&A activity and resulting consolidation in the US defence (Van Reenen et al., 2021) and pharmaceutical sectors (FTC and DOJ, 2022) has the potential to stifle start-up innovation.

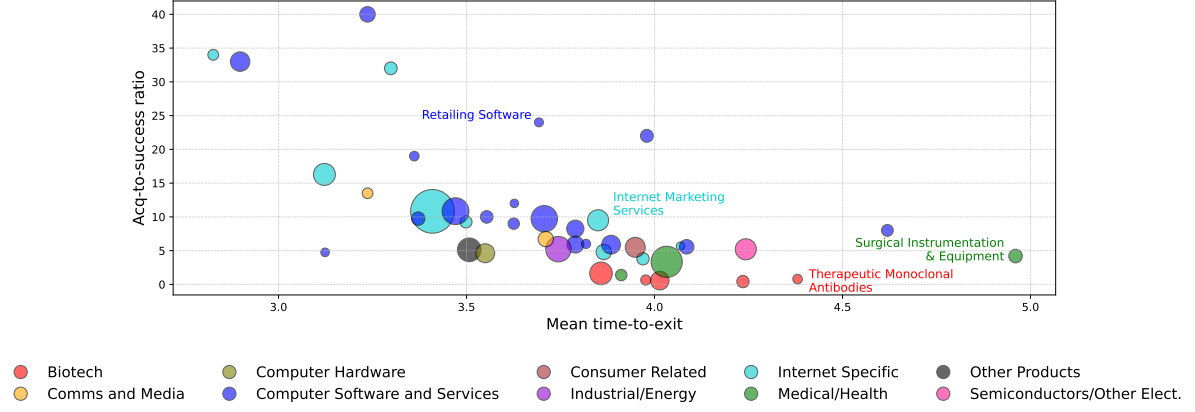


Figure 10: Sectoral heterogeneity

The figure plots the acquisition-to-success ratio against the mean time-to-exit across VEIC sectors. The sample period is 2005-2015, consistent with other analysis, and observations are censored seven years following a firm's first funding round. Sectors are included as distinct observations if there are at least 20 exits during the sample period and at least one of each type (acquisition and success). Sectors that do not meet these criteria are grouped at the aggregate sector level and reported together. The membership of each sector to aggregate sectors are indicated through colours, and the size of each point reflects the number of firms in the sector.

quisition opportunities. In these sectors, policymakers may have more leeway to prioritise other objectives – such as limiting entrenchment and incentivising incumbent innovation – without substantial fear of negative side-effects on start-up creation.

5.1 The allocation of funding

This section constructs the estimates of the skew of VC funding, relative to the benchmark in which entrepreneurs can fund their own projects. The exercise comprises four steps, which I outline briefly in the main text; see section C.2 of the Appendix for full details.

Calibration exercise First, starting from the parameter estimates for the US quantification in section 3, I introduce heterogeneity along two margins: (i) the development horizon, κ ; and (ii) the arrival rate of acquisition offers, ϕ . In practice, introducing heterogeneity along these margins changes investment incentives, independent of how they interact with financial frictions, which is the effect I wish to isolate. Therefore, I adjust the project payoff, π , such that start-ups with different characteristics have the same frictional entry value, $V_{s,e}$, equal to the value in the estimation of section 3. Given this normalisation, any difference in investment incentives in the frictionless counterfactual is due to the interaction of these characteristics with financial frictions, not inherent project quality. In practice, this means that long-horizon projects, low κ , with limited acquisition opportunities, low ϕ , have higher payoffs, π . To validate this normalisation, Figure C.7 shows that the calibration matches the empirical relationship between exit payoffs, time-to-exit and acquisition-to-success ratio across sectors.

Second, I calibrate the degree of heterogeneity by targetting the joint distribution of the average time-to-exit and acquisition-to-success ratio across granular VEIC sectors, as shown in Figure 10. Intuitively, sectors with longer time-to-exit have longer development horizons, whereas sectors with higher acquisition-to-success ratios have better acquisition opportunities.

However, there is an interplay between the two margins of heterogeneity: when acquisition opportunities are plentiful, firms can exit earlier. In addition, for given acquisition opportunities, longer-horizon projects are more likely to exit via acquisition because acquisitions substitute for financing and these firms face more severe financial frictions. By targetting the joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space, I allow the model to determine the relative importance of these channels. To ensure that I capture signal not noise, I consider sectors individually if there were at least 20 exits in my sample period, including at least one acquisition and one success; sectors that do not meet this requirement are aggregated to the ten-sector level. Denoting each ‘technology’ by τ , this calibration exercise produces technology weights, $s(\tau)$. These weights give the baseline shares of firms with each technology.

Third, I recover the frictionless weights, $\bar{s}(\tau)$, by removing frictions and computing the frictionless entry values, $\bar{V}_{s,e}(\tau)$. Specifically, given the estimated weights, $s(\tau)$, and the frictional entry value, $V_{s,e}$, the estimated weights satisfy

$$s(\tau) = \frac{V_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1}}{\int V_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1} d\tau} = \frac{\tilde{\sigma}(\tau)^{-1}}{\int \tilde{\sigma}(\tau)^{-1} d\tau} \quad (26)$$

where $\sigma(\tau)$ is the technology-specific entry parameter and the second equality follows because $\pi(\tau)$ is adjusted such that $V_{s,e}(\tau) = V_{s,e}(\tau') \forall \tau, \tau'$. Given the estimated values $s(\tau)$, equation (26) enables recovery of $\tilde{\sigma}(\tau)^{-1}$ up to a constant factor. Then, the frictionless shares are

$$\bar{s}(\tau) = \frac{\bar{V}_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1}}{\int \bar{V}_{s,e}(\tau)\tilde{\sigma}(\tau)^{-1} d\tau} = \frac{\bar{V}_{s,e}(\tau)s(\tau)}{\int \bar{V}_{s,e}(\tau)s(\tau) d\tau} \quad (27)$$

where the second equality follows from equation (26).¹²⁰

Finally, I map the technologies, τ , to the ten aggregate sectors depicted in Figure 10. Although in practice these sectors may differ on many additional margins, this provides an interpretable metric. Using this mapping and the imputed shares $\bar{s}(\tau)$, I can recover an estimate of the distortion to the sectoral composition of VC funding induced by the interaction of financial frictions, project horizons and acquisition opportunities. To implement the mapping, I set up a simple optimal transport problem, which I run at the granular VEIC sector level, before aggregating to the ten-sector level. In short, the mapping assigns technologies, τ , to sectors based on their proximity in the (acquisition-to-success, time-to-exit)-space.

Results Figure 11 plots the resulting cumulative distributions for the average time-to-exit and acquisition-to-success ratio across project types, comparing the frictional economy (orange, dashed) computed using the true weights, $s(\tau)$, to the frictionless economy (blue, solid) computed using the counterfactual weights, $\bar{s}(\tau)$. Removing financial frictions leads to a relative increase in entry from start-ups with longer expected time-to-exit and lower acquisition-to-success ratios. In other words, financing risk skews the allocation of VC funding towards projects with shorter time-to-exit and stronger acquisition opportunities, relative to a first-best economy.

Figure 12 shows the implications for the sectoral allocation of VC funding. Financial frictions distort the sectoral composition of VC-backed start-ups, particularly favouring start-ups in

¹²⁰Note that the flow of first funding rounds in the frictional economy for technology τ is $\frac{V_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \times \frac{\nu_e}{\nu_e + \lambda}$. The financing market rates do not appear in equation (26) because they are held constant across technologies.

Computer Software and Services and Internet Specific sectors.¹²¹ The share of start-up in these sectors falls from 61% to 53% when financing frictions are removed, with the difference made up by start-ups in Biotechnology, Medical/Health and Semiconductors.¹²²

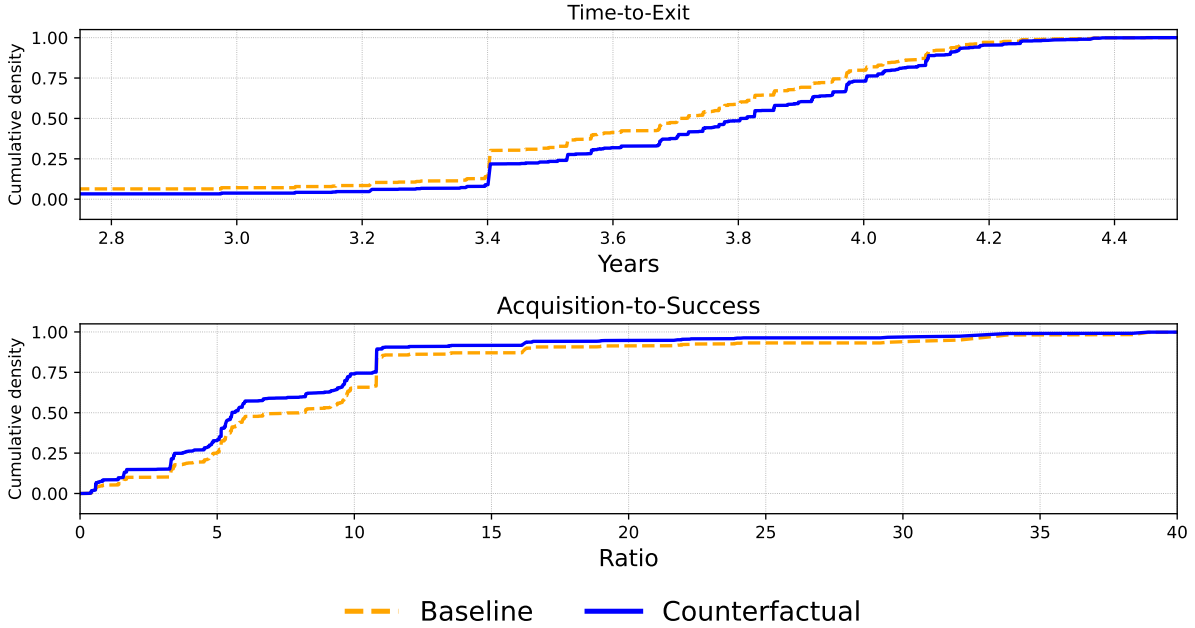


Figure 11: Time-to-exit and outcome ratios: frictional vs. frictionless entry

The figure plots the marginal CDFs for the time-to-exit (top) and acquisition-to-success ratio (bottom) based on the shares frictional shares $s(\tau)$ (orange, dashed) and frictionless shares $\bar{s}(\tau)$ (blue, solid). In plotting the CDFs, I hold the time-to-exit and acquisition-to-success ratio at their values in the baseline economy. These outcomes are endogenous to financial frictions, but depicting the data in this way aids comparison. All data is censored at seven years following a firm's first funding round.

Discussion These findings indicate a substantial misallocation of VC funding across sectors, implying that frictions in VC funding affect not only the overall level of investment but also the direction of start-up innovation. This echoes the empirical findings of Nanda and Rhodes-Kropf (2013), who show that start-ups first funded in hot markets are less likely to succeed, but conditional on success, have greater valuations, and more patents of higher quality. To my knowledge, this is the first quantitative assessment of the interaction between financial frictions and funding allocation across technologies in the VC context.¹²³ Given the importance of young firms in driving economic growth (Decker et al., 2016), the notion that direction of start-up innovation activity may be distorted relative to first-best has implications of the direction of

¹²¹The most prominent Internet Specific sectors are service sectors, such as Internet Marketing Services and Finance/Insurance/Real Estate Services.

¹²²See Table C.5 for the full list of sector reallocation figures.

¹²³A more ambitious quantitative assessment, beyond the scope of this paper, would re-estimate the model at the sector level. At present, (κ, ϕ) are forced to account for all heterogeneity in the acquisition-to-success ratio and time-to-exit across sectors; other primitives that may differ across sectors, such as capital intensity (k_e, k_l) , are held constant at the US baseline. Reversing the conclusions would require science-based sectors to be less risky, muting agency costs, or to face easier financing conditions than software and service start-ups. However, these technologies relate closer to basic science, implying longer development horizons and greater technological uncertainty. In biotech, pre-revenue IPOs can shift some risk to public markets (Ghosh and Nanda, 2010), but high capital intensity simultaneously increases the difficulty of scaling independently.

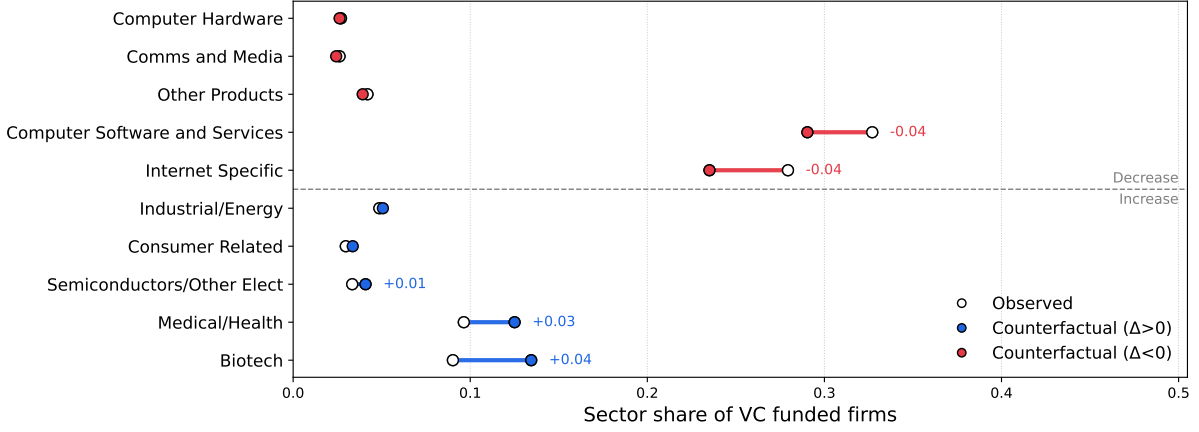


Figure 12: Sector reallocation

The figure shows observed sector weights and counterfactual sector weights under self-financing. Heterogeneity in the (κ, ϕ) -space is first calibrated using cross-sector variation in average time-to-exit and the acquisition-to-success ratio. ‘Technologies’ in the model are then mapped to VEIC sectors based on proximity in the (acquisition-to-success, time-to-exit)-space via optimal transport; see section C.2 for details.

aggregate growth. This is absent from existing models that incorporate venture capital markets into endogenous growth models (Greenwood et al., 2022; Akcigit et al., 2022; Ando, 2024) and provides an important avenue for future research in this area.

5.2 Policy implications and the market for firms

Antitrust authorities are increasingly considering how competition policy shapes innovation, moving beyond the classical static approach. For instance, in Dow–DuPont (2017) the European Commission required divestiture of R&D assets, explicitly invoking an innovation theory of harm.¹²⁴ In the start-up setting, policymakers are concerned about “killer acquisitions” (Cunningham et al., 2021), where established firms buy start-ups to shut down their projects and prevent competition, or that acquisitions in a certain area may reduce entry incentives for follow-on innovations, by creating a “kill zone” (Kamepalli et al., 2020). Albeit lesser-discussed, consolidation among established firms may weaken start-up exit options.¹²⁵ In my model, a similar mechanism is present: when the arrival rate of acquisition offers, ϕ , is low, start-ups have weaker outside options and are subsequently more willing to accept poor quality matches, characterised by lower synergies, ϵ .¹²⁶ The importance of acquisitions is magnified when financial frictions are most severe, as is the case for long-horizon projects.

To illustrate this point, Figure 13 plots the entry-value multiplier, $V_{s,e}(\phi + \Delta\phi)/V_{s,e}(\phi)$, for a small increase in the arrival rate of acquisitions, $\Delta\phi = 0.1$. This increase corresponds to one additional acquisition offer for every ten years spent developing the project. The higher is

¹²⁴See Commission Decision, Case M.7932 – Dow/DuPont.

¹²⁵Science-based start-ups face thinner acquisition markets, which leads them to capture a lower share of the surplus from their innovations (Masclans, 2025). Relatedly, Wang (2018) shows that concentrated acquirer markets depress start-up creation by inventors.

¹²⁶The model could be generalised to allow for the surplus to be split according to some Nash bargaining weights. Given the reduced-form nature of the acquisition market in this model, this additional degree of freedom provides little added value, at the cost of needing to estimate an additional parameter. In practice, the key is that the start-up’s outside option affects the offers it is willing to accept.

the multiplier, the greater the impact of a marginal increase in the arrival rate of acquisitions on the incentives for start-up creation. The left-panel reports the multiplier for projects with different development horizons, κ , and initial acquisition arrival rates, ϕ .¹²⁷ This is greatest for long-horizon projects with limited acquisition opportunities; low κ and ϕ .

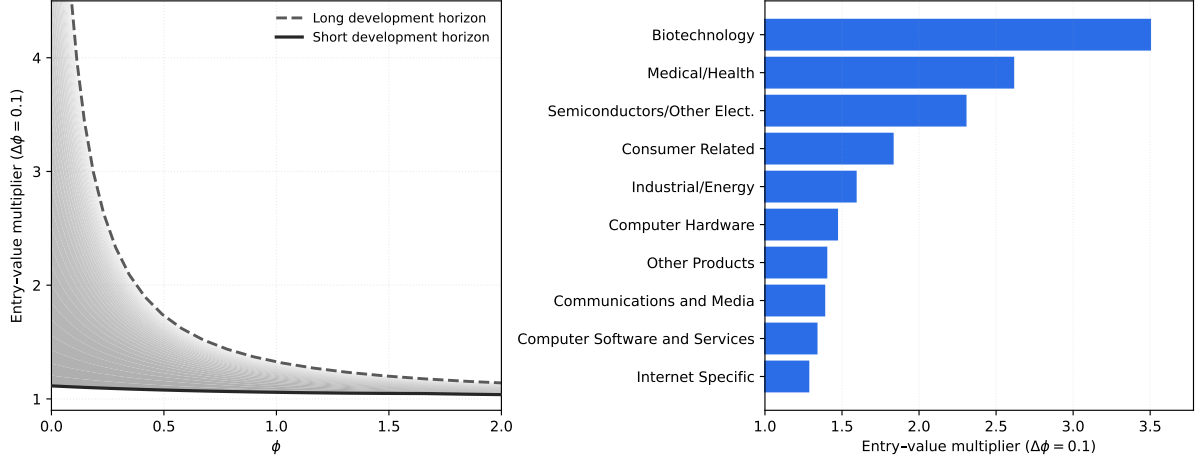


Figure 13: Marginal effect of acquisitions on entry

The figure plots the entry-value multiplier, $V_{s,e}(\phi + \Delta\phi)/V_{s,e}(\phi)$ for a small increase in the arrival rate of acquisitions, $\Delta\phi = 0.1$. The left panel shows how the multiplier varies across given the initial arrival rate of acquisitions, ϕ , for projects with different development horizons (where π is normalised so that all projects have the same frictional entry value, $V_{s,e}(\tau) = V_{s,e}(\tau') \forall \tau, \tau'$). Specifically, the dashed “long development horizon” sets $\kappa = 0.08$, implying an expected development horizon of $2/\kappa = 25$ years if fully funded; the solid “short development horizon” sets $\kappa = 2$, implying an expected development horizon of $2/\kappa = 1$ year if fully funded; between these extremes, darker shading corresponds to shorter development horizons. The right figure plots the weighted average entry-value multiplier across aggregate VEIC sectors. The weighting utilises the estimated weights, $s(\tau)$, and the mapping from technologies, τ , to granular VEIC sectors, as in Figure 12.

The right panel of Figure 13 uses the estimated weights, $s(\tau)$, and mapping from technologies to VEIC sectors to provide an indication of the role that acquisitions play in stimulating start-up creation across sectors. Acquisitions are most important for stimulating start-up creation in sectors where financial frictions create the largest distortions. In Biotech, Medical/Health, and Semiconductors, one additional acquisition offer per ten-years in development more than doubles start-up entry value. Conversely, for firms in Computer Software and Services and Internet Specific sectors, the entry-value multiplier is less than 1.5, reflecting their shorter development horizons and ample existing acquisition opportunities.

This result has implications for competition policy. Marginal changes in the leniency of antitrust policy to start-up acquisitions of software and service-based start-ups has limited effects on start-up creation in those sectors, but effects for science-based sectors are substantial.¹²⁸ Relatedly, increased sensitivity to marginal changes in acquisition opportunities in science-based sectors suggests that consolidation among incumbent firms may distort start-up innovation. In this regard, recent consolidation in the pharmaceutical industry provide cause for concern.

¹²⁷As in all analysis in this section, π is adjusted to hold the frictional entry value constant across technologies; i.e. $V_{s,e}(\tau) = V_{s,e}(\tau') \forall \tau, \tau'$.

¹²⁸The idea that changes to antitrust policy may have heterogeneous effects on start-up creation incentives across sectors is considered by Fons-Rosen et al. (2021). In their model, the difference between the acquisition price and outside option of the start-up to independently scale the innovation is a key calibration target: when this value is sufficiently high, a ban on start-up acquisitions is welfare enhancing.

6 Conclusion

This paper develops a tractable theory of the VC market that facilitates a tight mapping to data. The model incorporates empirical and theoretical insights from entrepreneurial finance into an equilibrium setting. The analysis highlights that financing frictions distort start-up entry and outcomes, particularly for novel, long-horizon projects. Because primitives map to observed funding histories, the framework permits transparent estimation on standard VC data.

My estimates suggest that significant issues remain in the funding of highly innovative ventures, even in the developed US market. However, entrepreneurs in other developed economies face more severe difficulties. In the UK, lower levels of VC activity reflect issues in allocating risk capital to start-ups and thinner acquisition markets, not limited investment opportunities. UK start-ups face particular issues in securing follow-on funding, suggesting that government support should focus existing efforts on late-stage start-ups. Finally, venture capital struggles to penetrate science-based sectors, particularly where a path to early liquidity for initial investors via acquisition opportunities is lacking. Scientific innovations with long development horizons face issues in scaling their projects independently because they typically need to visit the capital market more frequently, increasing their exposure to financing risk. Policymakers can help to mitigate these issues by facilitating – or not inhibiting – technology transfer from start-ups to incumbents, who may be better placed to commercialise the innovations.

This paper is a first step in bringing VC-market insights into a structural framework, and several extensions follow naturally. For example, the model is silent on an explicit role for existing investors in facilitating follow-on funding and existing research suggests that VC networks play an important role (Hochberg et al., 2007). Incorporating this channel would induce different meeting rates for follow-on funding depending on the identify of initial investors, which I abstract from. More generally, start-ups and VCs are homogenous in the baseline model; allowing heterogeneity would permit more intricate sorting and may facilitate revisiting the contribution of VC skill or sorting in the persistence of VC returns (Sørensen, 2007).

An additional limitation of the present analysis is that staging is the only information production tool.¹²⁹ Extensions to monitoring are feasible and act as a substitute for staging (Tian, 2011), permitting the VC to detect hidden failure and enforce closure. The current environment is the special case where information is non-verifiable and staging emerges as the only information production tool. Given the innovative and intangible nature of start-up projects, this is likely a reasonable approximation to reality. However, sectoral differences in “hard” versus “soft” information creates interesting heterogeneity. I conjecture that hard information is more readily available in consumer-orientated start-ups, such as software and services, as revenues arrive earlier in the lifecycle and provide a verifiable measure of intermediate success. This likely amplifies the sectoral misallocation emphasised in this paper. Finally, in this context, VC specialisation may matter, both because specialised VCs face lower monitoring costs and because they design experiments that reveal intermediate success more effectively. These topics offer fruitful areas for future research.

¹²⁹In the VC context, the term “monitoring” is often used to describe efforts on part of the VC to improve start-ups outcomes, rather than effort to detect foul play (see e.g. Kaplan and Strömberg 2001).

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Appendix

A Theory appendix

A.1 The contract

Consider the environment of section 2.2. As discussed in the main text, the entrepreneur (E) will never report negative results. The contract in the body of the paper makes assumptions such that the E always reports positive results. Specifically, the private benefit is taken to zero in the limit, $x_b \rightarrow 0$. The proposition below demonstrates that this is sufficient for the E to always report positive results and never report negative results.

Proposition A.1. *For any set of parameters $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$, there exists an $\bar{x}_b > 0$ such that if $x_b < \bar{x}_b$, then the E reports positive results but does not report negative results.*

Before stating the proof, I give a brief outline of the argument. In part 1, I derive the payoffs to the E given a contract (ς, ω) under two cases: (i) the E never reports results and (ii) the E reports only positive results. I show that given (ς, ω) , the E always prefers the payoff associated with case (ii). In part 2, I derive a condition under which case (ii) is not incentive compatible; that is, a condition under which a commitment to report positive results is not credible. In part 3, I demonstrate that for any set of parameters, case (ii) can always be made incentive compatible by choosing a suitably small non-pecuniary payoff to the E, x_b . Part 4 concludes the proof.

Proof of Proposition A.1.

Part 1. Without loss of generality, assume $x_b < k$.¹³⁰ The E never reports negative results, since $x_b > 0$. Suppose a positive result arrives at time t and call the decision to report a positive result $\chi = 1$, where $\chi = 0$ reflects the choice not to report a positive result. $\chi = 0$ implies that the payoff, π , from success is delayed until time $t + T_\omega$, for $T_\omega \sim \text{Exp}(\omega)$, so has time- t expected value $\frac{\omega}{\rho + \omega} \pi$.¹³¹ If $\chi = 0$, the contract ends at date T_ω , regardless of the result.

The value of the firm in development, V_d , is determined analogously to equation (6)

$$\rho V_d = \begin{cases} \kappa [p \pi - V_d] + \omega [V_s - V_d] & \text{if } \chi = 1 \\ \kappa [p \pi \frac{\omega}{\rho + \omega} - V_d] + \omega [V_s - V_d] & \text{if } \chi = 0 \end{cases} \quad (\text{A.1})$$

Denote by $V_d^{np}(\chi)$ the non-pecuniary value of the contract to the E. The value to the E of contract (ς, ω) is $V_E(\omega, \varsigma, \chi) = (1 - \varsigma) V_d(\omega, \chi) + V_d^{np}(\omega, \chi)$, where

$$V_d^{np} = \begin{cases} \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{x_b}{\rho + \kappa + \omega} & \text{if } \chi = 1 \\ \frac{x_b}{\rho + \omega} & \text{if } \chi = 0 \end{cases} \quad (\text{A.2})$$

¹³⁰This assumption is not restrictive since the proof attempts only to obtain an upper bound on x_b .

¹³¹ $E[e^{-\rho T_\omega} \times \pi] = \frac{\omega}{\rho + \omega} \pi$, where expectations are taken over T_ω .

where the expression for $V_d^{np}(\omega, \chi)$ is analogous to $K(\omega, \chi = 1)$ in equation (4). Similarly, $K(\omega, \chi = 0) = \frac{k}{\rho + \omega}$ because the contract ends at time T_ω regardless of any potential result. Since the VC has no bargaining power, their participation constraint binds, $\varsigma V_d(\omega, \chi) = K(\omega, \chi)$. The value of the contract to the E is then $V_E = (1 - \varsigma)V_d + V_d^{np} = V_d + V_d^{np} - K(\omega)$, or

$$V_E(\omega, \chi) = \begin{cases} \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k - x_b}{(\rho + \kappa + \omega)} & \text{for } \chi = 1 \\ \frac{\kappa p \pi \frac{\omega}{\rho + \omega} + \omega V_s}{\rho + \kappa + \omega} - \frac{k - x_b}{\rho + \omega} & \text{for } \chi = 0 \end{cases} \quad (\text{A.3})$$

This leads to the following remark, which states that the E prefers ex-ante the contract in which they report positive results to the VC. Intuitively, given $x_b < k$, the cost of compensating the VC exceeds the private benefit they attain from continuation, so they would prefer to commit to stop development at time T_κ .

Remark A.1. Given $0 < x_b < k$ and for any contract (ω, ς) , $V_E(\omega, \chi = 1) > V_E(\omega, \chi = 0)$.

Part 2. The benefit to the E of delay, $\chi = 0$, upon observing a positive result is $\frac{x_b}{\rho + \omega}$. The cost to the E is to delay the pecuniary payoff, of which they obtain share $1 - \varsigma$. The delay therefore costs the E $E[(1 - e^{-\rho T_\omega})(1 - \varsigma)\pi] = \frac{\rho}{\rho + \omega}(1 - \varsigma)\pi$, where expectations are taken over T_ω .¹³² This leads to the following incentive compatibility constraint, which states that the E's share of the pecuniary payoff must be sufficiently large for them to optimally report positive results.

Remark A.2. $\chi = 1$ is incentive compatible iff $1 - \varsigma \geq \frac{x_b}{\rho\pi}$.

Part 3. Consider the contract problem when the E commits to report positive results, $\chi = 1$

$$\begin{aligned} & \sup_{\{\omega \in [0, \infty), \varsigma \in [0, 1]\}} \left\{ \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k - x_b}{(\rho + \kappa + \omega)} \right\} \\ \text{s.t.} \quad & 1 - \varsigma \geq \frac{x_b}{\rho\pi}, \quad \varsigma \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} = \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k}{(\rho + \kappa + \omega)} \end{aligned}$$

where V_s is taken as given and the IC constraint from Remark A.2 is imposed. I begin by solving this problem without consideration for the incentive compatibility and feasibility constraint for ς ; thereafter, I show that x_b can be chosen such that they are slack.

Define $\bar{V}_s = \frac{\kappa p \pi - (k - x_b)}{\rho + \kappa}$. For $V_s \geq \bar{V}_s$, the objective function is everywhere increasing in $\omega \in [0, \infty)$ given $x_b < k$, so the solution is $\omega \rightarrow \infty$. Next, restrict attention to $V_s \in [0, \bar{V}_s]$. The first-order condition for an interior optimum is

$$\frac{(k - x_b)(1 - p)}{(\rho + \omega)^2} + \frac{(k - x_b - \kappa\pi)p + V_s(\rho + \kappa)}{(\rho + \kappa + \omega)^2} = 0$$

which delivers two candidate solutions for ω

$$\omega = \frac{\rho\kappa p \pi - \rho V_s(\rho + \kappa) - (k - x_b)(\kappa(1 - p) + \rho) \pm \sqrt{\kappa^2(k - x_b)(1 - p)(\kappa p \pi - p(k - x_b) - V_s(\rho + \kappa))}}{k - x_b - \kappa p \pi + V_s(\rho + \kappa)}$$

¹³²Alternatively, one could consider the incentives to delay/report over some finite interval Δt . The environment is stationary, so these two approaches are equivalent.

The ‘positive’ solution is strictly negative on $V_s \in [0, \bar{V}_s)$. Define $\bar{p} = \frac{(k-x_b)(\rho+\kappa)^2}{\kappa(\pi\rho^2+(k-x_b)(\kappa+2\rho))}$ and $\underline{V}_s = \frac{\kappa p\pi - p(k-x_b)}{\rho+\kappa} - \frac{(k-x_b)(1-p)(\rho+\kappa)}{\rho^2}$. When $p \leq \bar{p}$, the ‘negative’ solution is positive for $V_s \in [0, \bar{V}_s)$; when $p > \bar{p}$, it is positive for $V_s \in [\underline{V}_s, \bar{V}_s)$. The SOC is satisfied in both cases. Conversely, when $p > \bar{p}$, the solution is negative for $V_s \in [0, \underline{V}_s)$. In this case, the optimand is decreasing on $\omega \in [0, \infty)$. The solution is then $\omega = 0$.

With candidate solutions in hand, it remains to check the feasibility and incentive compatibility constraints. To this end, note the following lemma.

Lemma A.1. *If there are parameters such that $\varsigma \in (0, 1)$ (i.e. strictly), then $\exists \bar{x}_b$ such that $\forall x_b \in [0, \bar{x}_b)$, $1 - \varsigma \geq \frac{x_b}{\rho\pi}$.*

Proof of Lemma A.1. The PC gives $\varsigma(\omega) = \frac{k(\kappa(1-p)+\rho+\omega)}{(\rho+\omega)(\kappa p\pi+\omega V_s)}$, so that $\varsigma'(\omega) < 0$ on $V_s \in [0, \bar{V}_s)$. Furthermore, from the solution to the contract problem, $\omega'(x_b) \leq 0$, with strict inequality when $\omega > 0$ is optimal. Together, $\varsigma'(x_b) \geq 0$. Then, suppose the parameters are such that $\varsigma \in (0, 1)$ and consider reducing $x_b \rightarrow 0$. The E’s share, $(1 - \varsigma)$, increases weakly from a positive value, whereas $x_b/(\rho\pi) \rightarrow 0$. Therefore, there exists a region $[0, \bar{x}_b)$ such that the IC is satisfied, i.e. $1 - \varsigma \geq \frac{x_b}{\rho\pi}$.

The remainder of the proof provides sufficient conditions on x_b such that Lemma A.1 can be applied. Suppose first that $\omega = 0$ is optimal. Then $\varsigma(0) = \frac{k(\kappa(1-p)+\rho)}{\kappa p\pi\rho}$. Clearly, $\varsigma(0) > 0$ since $p \in (0, 1)$. Note that $\varsigma(0) < 1 \iff \kappa p\pi\rho > k(\kappa(1-p) + \rho)$. Furthermore, if $\omega = 0$ is optimal, then (from above) $p > \bar{p} \implies p\kappa\pi\rho > \rho^{-1}((k-x_b)(\rho+\kappa)^2 - p\kappa(k-x_b)(\kappa+2\rho))$. Combining these inequalities, a sufficient condition for $\varsigma(0) < 1$ is $\rho^{-1}((k-x_b)(\rho+\kappa)^2 - p\kappa(k-x_b)(\kappa+2\rho)) > k(\kappa(1-p) + \rho)$. For $x_b < \frac{\kappa(1-p)(\rho+\kappa)}{\rho^2+(1-p)(\kappa^2+2\rho\kappa)}$, this condition is satisfied, so $\varsigma(0) \in (0, 1)$. Then, by Lemma A.1, it is possible to choose a positive x_b sufficiently small such that the IC is satisfied.

Suppose next that $\omega \in (0, \infty)$ is optimal. Note $\varsigma(\omega) > 0 \forall \omega \in (0, \infty)$. It remains to check $\varsigma(\omega) < 1$, which is equivalent to requiring that the pecuniary return to the E is positive, $(1 - \varsigma) V_d(\omega, \chi = 1) = V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0$. Evaluating this expression

$$\begin{aligned} V_d(\chi = 1) - \left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \frac{k}{(\rho + \kappa + \omega)} \Big|_{\omega = \omega^* \in (0, \infty)} = \\ p\pi - \frac{1}{\kappa^2} \left[\frac{(2k - x_b) \sqrt{\kappa^2(k - x_b)(1-p)(p\kappa\pi - p(k - x_b) - V_s(\rho + \kappa))}}{k - x_b} \right. \\ \left. + \kappa \left(2kp - k + \rho V_s - \frac{(1-p)p(k - x_b)x_b\kappa}{\sqrt{\kappa^2(k - x_b)(1-p)(p\kappa\pi - p(k - x_b) - V_s(\rho + \kappa))}} \right) \right] \end{aligned}$$

This expression is decreasing in x_b on $x_b > 0$ given $V_s < \frac{\kappa p\pi - p(k-x_b)}{\rho+\kappa}$, which is satisfied whenever $V_s < \bar{V}_s$ since $p \in (0, 1)$. Therefore, if $V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0$ at $x_b = 0$, it is positive over some region $x_b \in [0, \bar{x}_b)$ where $\bar{x}_b > 0$. Evaluating $V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0$ at $x_b = 0$ obtains the condition

$$p\pi - \frac{1}{\kappa} \left(k(2p - 1) + \rho V_s + 2\sqrt{k(1-p)(\kappa p\pi - kp - V_s(\rho + \kappa))} \right) > 0$$

which is equivalent to

$$\kappa p\pi - k(2p - 1) - \rho V_s > 2\sqrt{k(1 - p)(\kappa p\pi - kp - V_s(\rho + \kappa))}$$

Note that the left-hand side is positive if $V_s < \frac{\kappa p\pi - k}{\rho} + \frac{2k(1 - p)}{\rho}$, which is valid on $V_s \leq \bar{V}_s$ when $x_b = 0$. Therefore, squaring both sides, subtracting the right-hand side from the left-hand side and some algebra obtains

$$4kV_s(1 - p)\kappa + (\kappa p\pi - k - \rho V_s)^2 > 0$$

which is satisfied given the initial restrictions on parameters (i.e. positivity constraints and $p \in (0, 1)$). This implies that the E obtains a positive pecuniary payoff when $x_b = 0$ and, by extension, over some region $x_b \in [0, \bar{x}_b)$. This in turn implies the conditions to apply Lemma A.1 are satisfied and there exists a (potentially different) upper bound \bar{x}_e such that $\forall x \in [0, \bar{x}_e)$, the IC is slack.

Suppose finally that $\omega \rightarrow \infty$ is optimal. Then $\lim_{\omega \rightarrow \infty} \varsigma(\omega) = 0$ and so satisfying the IC requires only that $x_b \leq \rho\pi$, which is easily satisfied.

Part 4. By Remark A.1, the E prefers to payoff associated with commitment for any contract (ω, ς) . Furthermore, for any contract (ω, ς) and parameters $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$, it is possible to find a strictly positive upper bound on x_b such that the feasibility constraint is slack, i.e. $\varsigma \in (0, 1)$, if x_b is less than this upper bound. Then, by Lemma A.1, there is a (potentially different) upper bound on x_b such that the IC constraint is also slack if x_b does not exceed this upper bound. Together, there is an interval $[0, \bar{x}_b)$ such that both the feasibility and incentive compatibility constraints are slack at the optimal contract in which $\chi = 1$. Since the contract with $\chi = 1$ is incentive compatible and for any contract (ω, ς) $\chi = 1$ is preferred ex ante, the optimal contract involves $\chi = 1$ on this interval $x_b \in [0, \bar{x}_e)$. Furthermore, the expected capital cost is given by $K(\omega, \chi = 1)$ and the optimal contract rate is determined as in part 3. ■

A.2 VC bargaining power

In this section, I generalise the baseline model to allow for VC bargaining power. VC bargaining power influences the choice of contract, (ς, ω) , in two ways. First, current investors face a hold-up problem vis-à-vis future investors because future investors extract surplus in their own negotiations with the start-up. Therefore, VCs see their shareholding in the firm facing greater dilution in future funding rounds relative to the setting where VCs have no bargaining power. The implication is that VCs have an incentive to provide greater funding upfront to reduce the need for the firm to return to the capital market. Second, a VC's outside option in the case that they choose not to invest in a start-up now has positive value. Investing in a given start-up involves forgoing this outside option and, therefore, incurs an opportunity cost that raises the effective flow cost of investing in a start-up above k .¹³³ This second effect has

¹³³A similar effect appears in Michelacci and Suarez (2004): VCs may facilitate an early-stage IPO to “recycle” their capital into new ventures. In my model, the same motive leads to earlier termination by inducing a smaller

the opposite implication to the first, pushing instead in the direction of a higher contract rate. In the remainder of this section, I set up the contracting problem when the VC has bargaining power and consider these effects more formally.

The VC It is now necessary to derive the value of a VC that is searching for investment opportunities or funding a start-up. Consider the value of a VC that is funding a productive start-up, which I denote by U_d^p . The VC incurs capital flow cost k and receives a share ς of the firm. At rate κ , the start-up realises a result, which is a success with probability p and returns the VC their share of the payoff π and the value of search for a VC, U_s , since the VC can now seek out new investment opportunities. However, with probability $1 - p$, the start-up obtains a negative result and so the VC gets the value of funding an unproductive start-up, U_p^u . Finally, at rate ω , the contract expires and the VC obtains their share in the company, a value ςV_s , and returns to search. With this in mind, the HJB for a VC funding a productive start-up is given by

$$\rho U_d^p = -k + \kappa[p(\varsigma \pi + U_s) + (1 - p)U_d^u - U_d^p] + \omega[\varsigma V_s + U_s - U_d^p] \quad (\text{A.4})$$

Consider next the value of funding an unproductive start-up, U_p^u . The VC must continue to pay the flow capital cost k because they are unaware that the start-up is unproductive. In addition, at rate ω , the commitment expires and the VC returns to search. The HJB for a VC funding an unproductive start-up is then given by

$$\rho U_p^u = -k + \omega[U_s - U_p^u] \quad (\text{A.5})$$

Finally, the value of search for a VC depends on the rate at which they meet with start-ups, which I denote by v . At this rate the VC meets with a start-up and obtains the value of funding an productive start-up, U_d^p . The HJB for a VC in search is given by

$$\rho U_s = v[U_d^p - U_s] \quad (\text{A.6})$$

The contract. Upon meeting, the VC and entrepreneur engage in Nash bargaining to determine the contract rate ω and share ς obtained by the VC. The value of the match to the VC is U_d^p and the value of their outside option is U_s . The value of the match to existing shareholders is $(1 - \varsigma)V_d$, because they retain a share $(1 - \varsigma)$ of the gross value of the firm, and the value of their outside option is V_s . If the entrepreneur has bargaining weight δ in the negotiations, then the optimal contract solves

$$\max_{\{\varsigma \in [0,1], \omega_i \in [0,\infty)\}} \left\{ ((1 - \varsigma)V_d - V_s)^\delta (U_d^p - U_s)^{1-\delta} \right\}$$

where V_s and U_s are taken as given in the optimisation. The first-order condition for the equity up-front commitment, after which the start-up must raise additional capital.

stake, ς , yields the following conditions

$$(1 - \varsigma)V_d = V_s + \delta S \quad (\text{A.7})$$

$$U_d^p = U_s + (1 - \delta)S \quad (\text{A.8})$$

where $S = (1 - \varsigma)V_d + U_d^p - V_s - U_s$ is the surplus of the contract. As is standard in these settings, the surplus is split according to the Nash bargaining weights. Solving equations (A.4), (A.5) and (A.6) for U_d^p in terms of U_s allows for writing the surplus as

$$S = V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1 - p)}{\rho + \omega} \right) \quad (\text{A.9})$$

Since the surplus is independent of the share that goes to the VC, the contract rate simply maximises the joint surplus, $\omega = \arg \max_{\omega \in [0, \infty)} \{S\}$. The first-order condition for ω is then

$$V_d'(\omega) = -(k + \rho U_s) \left(\frac{1 - p}{(\rho + \omega)^2} + \frac{p}{(\rho + \kappa + \omega)^2} \right) \quad (\text{A.10})$$

The first-order condition for the optimal contract rate is identical to the case without bargaining power, except that k has been replaced by $k + \rho U_s$. Intuitively, every instant that the VC pays for the investment, they incur capital cost k and *opportunity cost* $\rho \times U_s$, because they cannot engage in other investment opportunities. This induces shorter duration capital commitments. However, in equilibrium, V_s will also adjust due to hold-up vis-a-vis future investors.

As I show in the proof to Proposition A.3, the optimal contract rate when there is bargaining power is

$$\omega = \max \left\{ 0, -\rho + \mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} \right\}, \quad (\text{A.11})$$

where

$$\mathcal{C} = \frac{\kappa(1 - p)k}{\kappa p \pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \delta \nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta) \nu \frac{\kappa p \pi}{k}$$

Clearly, setting $\delta = 1$ recovers equation (11). However, more generally, the bargaining weight has two implications. On the one hand, it affects the collective forward-looking incentives of the current E and VC. Once a VC has invested, it becomes an existing shareholder and has bargaining power δ vis-à-vis future investors. Therefore, δ reflects the size of the potential hold-up problem that the E and current VC face in future investments, which induces dilution. As δ increases, the E and VC worry less about future dilution and so reduce the size of the current capital injection.¹³⁴ Conversely, the bargaining weight affects how the VC prioritises the current investment vis-à-vis other potential investment opportunities. As δ increases, the VCs bargaining power falls in any new investment, which lowers their opportunity cost of funds. This leads them to extend more funding to the current entrepreneur.

¹³⁴Note that from the perspective of the current investment, the need to raise follow-on funding and the associated hold-up acts analogously to financing risk: in the limit where $\delta \rightarrow 0$, current investors would be indifferent between meeting with a new investor and failing due to a lack of follow-on funding.

Comparative statics Here, I provide comparative statics for the optimal contract rate in the model with VC bargaining power.

Proposition A.2. *The optimal contract rate, ω , is: strictly increasing in v , ν and k ; strictly decreasing in p , π , λ ; and ambiguous for κ and δ . The QCCS for p is strictly negative and for κ is strictly positive.*

Comparative statics are preserved for all parameters considered in the $\delta = 1$ case. In addition, the contract rate is increasing in v , the meeting rate for VCs; this reflects the opportunity cost motive. Furthermore, the effect of bargaining power on the optimal contract rate is ambiguous. In fact,

$$\frac{\partial \omega}{\partial \delta} > 0 \iff \theta < \frac{k}{\kappa p \pi} \frac{\rho + \kappa}{\rho + \lambda}$$

Whether increases in the bargaining power of the entrepreneur, δ , increase the contract rate, ω , depends on market tightness, $\theta = \mu_s/\mu_{vc} = v/\nu$. When the market is tight – start-ups are numerous relative to VCs, θ is high – the value of the VC’s outside option is high, which strengthens their motive to keep funding commitments short. In this case, marginal changes to their bargaining power have large effects on their opportunity cost of funds and, therefore, reductions in their bargaining power induce larger capital commitments, $\partial \omega / \partial \delta < 0$. Otherwise, increases in the bargaining power of insiders, δ , allows them to worry less about future dilution, and lead to shorter commitments, $\partial \omega / \partial \delta > 0$.

A.3 Proofs for main results

A.3.1 Existence and uniqueness

In this section, I show that for given meeting rates in the financing market, the value functions and contract are unique. I do so in the general case in which the VC has bargaining power, as described in section A.2. The roof of Proposition 2.1 then follows by setting the VC’s bargaining power to zero.

Proposition A.3. *Given ν and v and for $\kappa p \pi > k$, there is a unique solution for $\{V_d, V_s, U_d^p, U_d^u, U_s, S, \omega, \varsigma\}$ in terms of the model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, $\omega \in [0, \infty)$.*

Proof of Proposition A.3. Given ν, v , an equilibrium is a solution for $\{V_d, V_s, U_d^p, U_d^u, U_s, S, \omega, \varsigma\}$ to the following set of equations

$$\rho V_d = \kappa [p \pi - V_d] + \omega [V_s - V_d] \quad (\text{A.12})$$

$$(\rho + \lambda)V_s = \nu[(1 - \varsigma)V_d - V_s] \quad (\text{A.13})$$

$$\rho U_d^p = -k + \kappa[p(\varsigma \pi + U_s) + (1 - p)U_d^u - U_d^p] + \omega[\varsigma V_s + U_s - U_d^p] \quad (\text{A.14})$$

$$\rho U_d^u = -k + \omega[U_s - U_d^u] \quad (\text{A.15})$$

$$\rho U_s = v[U_d^p - U_s] \quad (\text{A.16})$$

$$S = V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1 - p)}{\rho + \omega} \right) \quad (\text{A.17})$$

$$(1 - \varsigma)V_d = V_s + \delta S \quad (\text{A.18})$$

$$U_d^p = U_s + (1 - \delta)S \quad (\text{A.19})$$

$$\omega = \arg \max_{\omega \in [0, \infty]} \{S\} \quad (\text{A.20})$$

The first two equations come from the start-up's problem, where I have substituted $V^M = (1 - \varsigma)V_d$. The next three equations come from the VC's problem, as set out in section A.2 of the Appendix. The final four equations relate to the optimal contract: they define, respectively, the surplus, the optimality conditions for the equity share in the contract, and the optimal contract rate.¹³⁵

Towards a proof, note that given (ω, S, ν, v) , all remaining equilibrium objects are pinned down uniquely. Specifically, combining equations (A.13) and (A.18), $(\rho + \lambda)V_s = \nu\delta S$, so V_s is known given (ν, S) . V_d then follows immediately from equation (A.12) given (ω, V_s) . The equity stake then follows from equation (A.18). Finally, the VC value functions can then be recovered from equations (A.14), (A.15) and (A.16), given knowledge of $(\omega, v, \varsigma, V_s)$.

In light of this, to show that the equilibrium, given (ν, v) , is unique, it is sufficient to demonstrate that there is a unique (ω, S) consistent with the equilibrium conditions. To this end, rather than taking the first-order condition as a function of V_s and U_s , as in equation (A.10), and *then* solving the resulting system of equations in (ω, V_s, U_s) , it is simpler to first eliminate the search values from the optimisation problem and then solve for the optimal contract. Although the contract is derived taking as given the search values, (V_s, U_s) , this approach is valid because, in the background, holding the search values constant is equivalent to preventing agents from internalising the effect of the contract choice, ω , on the equilibrium meeting rates, ν and v . Towards an expression for S , combining the above expressions allows to solve for V_s in terms of

¹³⁵There are more equations than unknowns, but technically one of the equations (A.18) or (A.19) is redundant.

model parameters and the equilibrium contract rate, ω

$$\begin{aligned}
(\rho + \lambda)V_s &= \nu\delta S \\
&= \nu\delta \left[V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right] \\
&= \nu\delta \left[\frac{\kappa p\pi + \omega V_s}{\rho + \kappa + \omega} - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right] \\
&= \nu\delta \left[\frac{\kappa p\pi + \omega V_s}{\rho + \kappa + \omega} - V_s - \frac{k + v(1-\delta)\frac{\rho+\lambda}{\nu\delta}V_s}{\rho + \kappa + \omega} \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right]
\end{aligned}$$

where the first line follow from combining equations (A.13) and (A.18), the second line uses the definition of S , the third line substitutes for V_d from equation (A.12), and the fourth line uses the relationship between U_s and V_s , which comes from writing these values both in terms of the surplus and then eliminating the surplus from the resulting equations.¹³⁶ Then, solving for V_s and using $\nu\delta S = (\rho + \lambda)V_s$ yields an expression for S in terms of model parameters and the contract rate, ω

$$S(\omega) = \frac{(\rho + \lambda) \left[\kappa p\pi - \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) k \right]}{(1 - \delta)v(\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) - \nu(1 - \delta)(\rho + \kappa) + (\rho + \kappa)(\rho + \lambda + \nu) + (\rho + \lambda)\omega} \quad (\text{A.21})$$

The optimal contract rate maximises $S(\omega)$, taking the meeting rates ν and v as given. Differentiating with respect to ω obtains

$$\begin{aligned}
S'(\omega) &= \frac{1}{\frac{1}{\rho + \lambda}(\rho + \omega)^2 \left((\rho + \kappa)(\delta\nu + \lambda + \rho) + \omega(\rho + \lambda) + (1 - \delta)v(\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \right)^2} \\
&\times \left((\rho + \lambda)(k - \kappa p\pi)(\rho + \omega)^2 + 2\kappa k(1 - p)(\rho + \lambda)(\rho + \omega) \right. \\
&\quad \left. + k(\kappa^2(1 - p)(\rho + \lambda + \delta\nu) + \delta\kappa\nu(1 - p)\rho) + (1 - \delta)\kappa^2(1 - p)p\pi v(\rho + \lambda) \right)
\end{aligned}$$

The denominator of $S'(\omega)$ is strictly positive, so it is sufficient to focus on the numerator. Note firstly that the numerator is a quadratic function of ω . There are two cases to consider. Suppose firstly that $k \geq \kappa p\pi$, in which case $S'(\omega) > 0$ on $\omega \in [0, \infty)$. In this case, the unique solution is $(\omega \rightarrow \infty, S = 0)$. Next, suppose that $k < \kappa p\pi$, in which case the numerator is quadratic and concave in ω , and so may omit an interior solution. Setting $S'(\omega) = 0$ and solving for ω obtains

$$\omega = -\rho + \mathcal{C} \pm \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}$$

where

$$\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p\pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \delta\nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v \frac{\kappa p\pi}{k}$$

Among the two solutions, it is simple to see that the ‘negative’ solution is strictly negative

¹³⁶Specifically, $\rho U_s = v(1 - \delta)S$ and $(\rho + \lambda)V_s = \nu\delta S$, so that $\rho U_s = (\rho + \lambda)^{\frac{1-\delta}{\delta}} \frac{v}{\nu} V_s$

whenever $\kappa p\pi > k$.¹³⁷ Therefore, there can be at most one solution on $\omega \in [0, \infty)$. Specifically, since the sign of $S'(\omega)$ is pinned down by a quadratic function in ω that is concave, the greater of the two solutions is a maximum and so is the optimum when it is positive. However, if the greater of the two solution is negative, then the optimum must be at $\omega = 0$, because $S'(\omega)$ is negative on $\omega \in [0, \infty)$. Combining these insights, given ν and v , it is clear that there is a unique optimum characterised by a finite contract rate when $k < \kappa p\pi$ and so surplus. Given the preceding arguments, the equilibrium objects are uniquely determined. ■

Proof of Corollary 2.1. Again, I consider the general case with VC bargaining power. Solving the optimisation problem as described, the optimal contract rate is given by

$$\omega = \begin{cases} \max \left\{ 0, -\rho + \mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} \right\} & \text{if } k < \kappa p\pi \\ \infty & \text{else} \end{cases} \quad (\text{A.22})$$

where

$$\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p\pi - k} \quad \text{and} \quad \mathcal{F} = \kappa + \delta\nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v \frac{\kappa p\pi}{k}$$

To determine when the contract rate is set to zero, corresponding to upfront financing, consider when the max-operator is active. To this end, upfront financing occurs when any of the following conditions are met

$$p \in \left(\frac{k(\rho + \kappa)^2}{\kappa(\rho^2\pi + k(\kappa + 2\rho))}, 1 \right] \quad \text{and} \quad \begin{cases} v \geq \frac{1}{p\kappa^2\pi}\Psi & \text{and } \delta = 0, \quad \text{or} \\ \nu < \frac{(\rho + \lambda)}{k\kappa\delta(\rho + \kappa)}\Psi & \text{and } v \geq f(\nu) \quad \text{and } \delta \in (0, 1), \quad \text{or} \\ \nu \geq \frac{(\rho + \lambda)}{k\kappa(\rho + \kappa)}\Psi & \text{and } \delta = 1 \end{cases}$$

with

$$\Psi = \frac{1}{1-p}(\rho^2(\kappa p\pi - k) - k(1-p)\kappa^2 - 2k(1-p)\rho\kappa)$$

$$f(\nu) = \frac{\rho^2(\rho + \lambda)(\kappa p\pi - k) - k\kappa(1-p)[\kappa(\rho + \lambda + \delta\nu) + \rho(2\rho + 2\lambda + \delta\nu)]}{(\rho + \lambda)(1-p)(1-\delta)\kappa^2 p\pi}$$

with $\Psi > 0$ under the restriction on p and $f'(\nu) < 0$. The case given in the main body of the paper simply corresponds to the case of $\delta = 1$. ■

A.3.2 Comparative statics

Proof of Propositions 2.2 and A.2 I consider first the standard comparative statics, then the QCCS (see Definition 2.2). Consider the general case of $\delta \in [0, 1]$; Proposition 2.2 considers the special case, $\delta = 1$. Let $k < \kappa p\pi$ and consider the case of a positive contract rate, $\omega > 0$. Then,

$$\omega = -\rho + \mathcal{C}(z) + \sqrt{\mathcal{C}(z)\mathcal{F}(z) + \mathcal{C}(z)^2}$$

¹³⁷ $-\rho + \mathcal{C} - \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2} < -\rho + \mathcal{C} - \sqrt{\mathcal{C}^2} = -\rho$, where the first inequality follows since $\mathcal{C} > 0$ given $k < \kappa p\pi$.

where $\mathcal{C}(z), \mathcal{F}(z)$ makes explicit the dependence on some parameter, z . Differentiating ω with respect to some generic parameter z obtains

$$\frac{\partial \omega}{\partial z} = \left(1 + \frac{2\mathcal{C}(z) + \mathcal{F}(z)}{2\sqrt{\mathcal{C}(z)\mathcal{F}(z) + \mathcal{C}(z)^2}} \right) \mathcal{C}'(z) + \frac{\mathcal{C}(z)}{2\sqrt{\mathcal{C}(z)\mathcal{F}(z) + \mathcal{C}(z)^2}} \mathcal{F}'(z)$$

Now, consider each parameter in turn. For $z = v, \nu, \lambda, \delta$, $\mathcal{C}'(z) = 0 \implies \text{sign}(\frac{\partial \omega}{\partial z}) = \text{sign}(\mathcal{F}'(z))$. Then

- $\mathcal{F}'(v) \geq 0$, with strict equality for $\delta < 1$.
- $\mathcal{F}'(\nu) \geq 0$, with strict equality for $\delta > 0$.
- $\mathcal{F}'(\lambda) \leq 0$, with strict equality for $\delta > 0$.
- $\mathcal{F}'(\delta) = \nu \frac{\rho + \kappa}{\rho + \lambda} - v \frac{\kappa p \pi}{k}$, which has ambiguous sign.

For $z = p, \pi$, $\mathcal{C}'(z) < 0$ (given $k < \kappa p \pi$) but $\mathcal{F}'(z) > 0$. Nevertheless, given $k < \kappa p \pi$, we have $\frac{\partial \omega}{\partial z} < 0$. For example, in the case of $z = \pi$

$$\frac{\partial \omega}{\partial \pi} = -\frac{1}{\kappa p \pi - k} \left(\kappa p \mathcal{C} + \mathcal{C} \frac{\kappa p \left(\kappa + \delta \nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v + 2\mathcal{C} \right)}{2\sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}} \right)$$

which is negative for $k < \kappa p \pi$. Analogous arguments show that $\frac{\partial \omega}{\partial k} > 0$.

For $z = \kappa$, the expression is ambiguous. Indeed, $\mathcal{C}'(\kappa) < 0$, tends to negative infinity as κ approaches its lower bound, $k/(p\pi)$, and tends to zero as $\kappa \rightarrow \infty$. Conversely, \mathcal{F} is increasing and linear in κ . Therefore, for small κ , \mathcal{C} drives the behaviour and for large κ , \mathcal{F} drives the behaviour. More formally, note that $\frac{\partial \omega}{\partial \kappa} > 0$ can be written

$$\mathcal{F}'(\kappa) > -\mathcal{C}'(\kappa) \left(2 + 2\sqrt{1 + \frac{\mathcal{F}(\kappa)}{\mathcal{C}(\kappa)}} + \frac{\mathcal{F}(\kappa)}{\mathcal{C}(\kappa)} \right)$$

The left-hand side is a constant. Given, $k < \kappa p \pi$, the right-hand side is strictly decreasing in κ . Therefore, there can be at most one solution to $\frac{\partial \omega}{\partial \kappa} = 0$. Furthermore, as κ approach $k/(p\pi)$ from above, it is simple to verify that $\frac{\partial \omega}{\partial \kappa} < 0$. Finally, for large κ , note that $\lim_{\kappa \rightarrow \infty} \mathcal{C}(\kappa)$ is a constant and, recalling that $\mathcal{F}(\kappa)$ is linear, it is simple to show that $\lim_{\kappa \rightarrow \infty} \frac{\omega(\kappa)}{\sqrt{\kappa}} = C_0$ for some constant C_0 . Therefore, $\omega(\kappa)$ grows like $\sqrt{\kappa}$, such that $\omega'(\kappa) \rightarrow 0$ from above. Therefore, there exists some threshold $\bar{\kappa}$ such that, for $\kappa < \bar{\kappa}$, $\omega'(\kappa) < 0$ and otherwise $\omega'(\kappa) > 0$.

For the QCCSs, replace $\pi = \frac{1}{p\kappa} [(\rho + \kappa)\bar{V}_s + k]$. Then,

$$\mathcal{C}(z; \bar{V}_s) = \frac{\kappa}{\rho + \kappa} \frac{k(1 - p)}{\bar{V}_s}, \quad \mathcal{F}(z; \bar{V}_s) = \kappa + \delta \nu \times \frac{\rho + \kappa}{\rho + \lambda} + (1 - \delta)v \frac{(\rho + \kappa)\bar{V}_s + k}{k}$$

Then, $\mathcal{C}'(p; \bar{V}_s) < 0$ and $\mathcal{F}'(p; \bar{V}_s) = 0$. It follows immediately that the QCCS for p is strictly negative. Furthermore, $\mathcal{C}'(\kappa; \bar{V}_s) > 0$ and $\mathcal{F}'(\kappa; \bar{V}_s) > 0$, so that the QCCS for κ is strictly positive. ■

Proof of Corollary 2.2 For $x \in \{\nu, \pi, \lambda\}$, $\frac{dK}{dx} = \frac{\partial K}{\partial \omega} \frac{\partial \omega}{\partial x} \implies \text{sign}\left(\frac{dK}{dx}\right) = -\text{sign}\left(\frac{\partial \omega}{\partial x}\right)$ since $\frac{\partial K}{\partial \omega} < 0$. The result follows immediately. For $x \in \{k, p, \kappa\}$, $\frac{dK}{dx} = \frac{\partial K}{\partial x} + \frac{\partial K}{\partial \omega} \frac{\partial \omega}{\partial x}$, where $\frac{\partial K}{\partial \omega} < 0$. For p , $\frac{\partial K}{\partial p} < 0$ and $\frac{\partial \omega}{\partial p} < 0$; for k , $\frac{\partial K}{\partial k} > 0$ and $\frac{\partial \omega}{\partial k} > 0$; and for κ , $\frac{\partial K}{\partial \kappa} < 0$ and the sign of $\frac{\partial \omega}{\partial \kappa}$ is ambiguous. It is not possible to sign these effects in general. The same logic applies for the QCCS for p . However, the QCCS for κ is strictly negative since $\frac{\partial \omega}{\partial \kappa} > 0$ in this case. ■

Proof of Proposition 2.3 Consider a firm in productive development. It may not raise capital again because: (i) it completes R&D in the current financing round, or (ii) its financing is withdrawn and it cannot raise new capital. Together, the probability that a firm in productive development does not raise more capital is given by

$$\begin{aligned} Pr(\text{do not raise again}) &= Pr(\text{result in current round}) + Pr(\text{return to search and fail}) \\ &= \frac{\kappa}{\kappa + \omega} + \frac{\omega}{\kappa + \omega} \frac{\lambda}{\lambda + \nu} \end{aligned}$$

For notational convenience, I denote this probability by q_d . Then

$$Pr(N_f = 1) = q_d, \quad Pr(N_f = 2) = (1 - q_d) q_d, \quad \dots \quad Pr(N_f = n) = (1 - q_d)^{n-1} q_d$$

which is the probability mass function of the Geometric distribution. ■

Proof of Corollary 2.3 Recall $E[N_f] = \left(\frac{\kappa}{\kappa + \omega} + \frac{\omega}{\kappa + \omega} \frac{\lambda}{\lambda + \nu}\right)^{-1}$. Then

- $\frac{d}{d\nu} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(\nu) + \lambda\omega(\nu)(\kappa + \omega(\nu))}{(\kappa(\lambda + \nu) + \lambda\omega(\nu))^2} > 0$ since $\omega'(\nu) > 0$.
- $\frac{d}{d\lambda} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(\lambda) - \nu\omega(\lambda)(\kappa + \omega(\lambda))}{(\kappa(\lambda + \nu) + \lambda\omega(\lambda))^2} < 0$ since $\omega'(\lambda) < 0$.
- For $x \in \{p, \pi, k\}$, including the QCCS for p , $\frac{d}{dx} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(k)}{(\kappa(\lambda + \nu) + \lambda\omega(k))^2} \implies \text{sign}\left(\frac{d}{dx} E[N_f]\right) = \text{sign}(\omega'(x))$. The result then follows from Proposition 2.2.
- $\frac{d}{d\kappa} E[N_f] = \frac{(\lambda + \nu)\nu}{(\kappa(\lambda + \nu) + \lambda\omega(\kappa))^2} (\kappa\omega'(\kappa) - \omega(\kappa))$. This is ambiguous, the QCCS. Note $\frac{d}{d\kappa} E[N_f] < 0$ if $\frac{\partial \omega}{\partial \kappa} \frac{\kappa}{\omega} < 1$; i.e. ω the elasticity of ω with respect to κ is less than one. ■

Proof of Proposition 2.4 Follows immediately from the main text. See footnote 46.

Proof of Corollary 2.4 For all parameters $x \in \{\nu, k, p, \pi, \lambda\}$, $\text{sign}\left(\frac{d}{dz} E[T_{br}]\right) = -\text{sign}(\omega'(x))$. The result then follows from Proposition 2.2. For κ , $\text{sign}\left(\frac{d}{d\kappa} E[T_{br}]\right) = -\text{sign}(1 + \omega'(\kappa))$, which is ambiguous in the general case, but the QCCS is negative, since $\omega'(\kappa) > 0$ in this case. For ν , $\frac{d}{d\nu} E[T_{br}] = -\frac{\omega'(\nu)}{(\kappa + \omega(\nu))^2} - \frac{1}{(\lambda + \nu)^2} < 0$, where the inequality follows from Proposition 2.2, and $\frac{d}{d\lambda} E[T_{br}] = -\frac{\omega'(\lambda)}{(\kappa + \omega(\lambda))^2} - \frac{1}{(\lambda + \nu)^2}$, which is ambiguous since $\omega'(\lambda) < 0$ by Proposition 2.2. ■

Proof of Proposition A.4 Follows immediately from the main text.

Proof of Corollary 2.5. Consider two start-ups $i = a, b$ facing the same funding conditions, ν and λ . Suppose $E[N_f^a] > E[N_f^b]$. Then,

$$\left(\frac{\kappa_a}{\omega_a + \kappa_a} + \frac{\omega_a}{\kappa_a + \omega_a} \frac{\lambda}{\lambda + \nu} \right)^{-1} > \left(\frac{\kappa_b}{\omega_b + \kappa_b} + \frac{\omega_b}{\kappa_b + \omega_b} \frac{\lambda}{\lambda + \nu} \right)^{-1} \implies \frac{\kappa_a}{\omega_a} < \frac{\kappa_b}{\omega_b}$$

The result follows immediately from the expression for $\Delta_p = \frac{\lambda\omega}{\kappa(\lambda+\nu)+\lambda\omega} = \frac{\lambda}{\frac{\kappa}{\omega}(\lambda+\nu)+\lambda}$. ■

Proof of Proposition 2.6. The standard comparative statics follow immediately from differentiating equation (13). For the QCCSs, note that equation (13) can be written

$$V_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\rho + \omega)} \bar{V}_s$$

To compute the QCCS, note first that \bar{V}_s is held constant, so it is sufficient to consider the first term, $\frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\rho + \omega)}$. p appears only via; then, differentiating and using the QCCS for $\omega'(p) < 0$, it follows that the QCCS for entry with respect to p is positive. For κ , we have

$$\frac{\partial}{\partial \kappa} \left(\frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\rho + \omega)} \right) = \frac{\nu}{(\rho + \lambda)(\mathcal{F} + 2(\rho + \omega))^2} \times (\rho + 2[\omega - (\rho + \kappa)\omega'(\kappa)])$$

Therefore, QCCS for κ is positive iff $\rho + 2(\omega - (\rho + \kappa)\omega'(\kappa)) > 0$, but ambiguous in general. For intuition, note that as $\rho \rightarrow 0$, this is equivalent to

$$\frac{\kappa\omega'(\kappa)}{\omega} < 1$$

which is the necessary condition for $\frac{\partial E[N_f]}{\partial \kappa} < 0$; in words, as $\rho \rightarrow 0$, long-horizon projects, $\kappa \downarrow$, face greater distortions when they require more funding rounds. More generally, $\rho + 2(\omega - (\rho + \kappa)\omega'(\kappa)) > 0$ is equivalent to

$$\frac{\kappa\omega'(\kappa)}{\omega} < 1 + \frac{\rho}{\rho + \kappa} \left(\frac{\kappa}{2\omega} - 1 \right)$$

The right-hand side can be less than or exceed one, so it is not possible, in general, to state whether $\rho > 0$ loosens or tightens the elasticity condition under $\rho \rightarrow 0$. ■

Proof of Lemma 2.1 From equation (13), V_s can be expressed as

$$V_s = \frac{\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right)}{(\rho + \kappa) + (\rho + \lambda) \left(\frac{\rho + \kappa + \omega}{\nu} \right)}$$

Under $\kappa p \pi > k$, $\lim_{\nu \rightarrow \infty} \omega(\nu) = \infty \implies \lim_{\nu \rightarrow \infty} \kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho + \omega(\nu)} \right) = \kappa p \pi - k$. Furthermore,

from equation (11)

$$\begin{aligned} \lim_{\nu \rightarrow \infty} \omega(\nu)/\sqrt{\nu} &= \sqrt{\mathcal{C} \times \frac{\rho + \kappa}{\rho + \lambda}} \implies \lim_{\nu \rightarrow \infty} \omega(\nu)/\nu = 0 \\ \implies \lim_{\nu \rightarrow \infty} \left((\rho + \kappa) + (\rho + \lambda) \left(\frac{\rho + \kappa + \omega(\nu)}{\nu} \right) \right) &= \rho + \kappa \end{aligned}$$

where $\mathcal{C} = \frac{\kappa(1-p)k}{\kappa p \pi - k}$. Therefore, $\lim_{\nu \rightarrow \infty} V_s = \frac{\kappa p \pi - k}{\rho + \kappa}$. ■

Proof of Corollary 2.6 The result follows immediately from differentiating the expression and applying the results of Proposition 2.2. ■

Proof of Proposition 2.8 An equivalent statement is that if $\omega < \infty$, then $V_d > V_s$. To see that this must be the case, note that setting $\omega \rightarrow \infty$ implies $V_d = V_s$ and all the value is retained by the existing shareholders. In contrast, a finite contract rate implies that existing shareholders retain value $(1 - \varsigma)V_d|_{\{\omega < \infty\}}$. If this is to be optimal, it must be that $\omega < \infty$ leaves the existing shareholders better off than $\omega \rightarrow \infty$, i.e. $(1 - \varsigma)V_d|_{\{\omega < \infty\}} > V_s \implies V_d|_{\{\omega < \infty\}} > V_s$. Since $\varsigma|_{\{\omega < \infty\}} \in (0, 1)$, this implies $V_d|_{\{\omega < \infty\}} > V_s$. ■

A.3.3 Welfare

In the main body of the paper, only the steady state conditions are considered. Towards solving the planner's problem, it is necessary to consider the behaviour of the system out of steady state. To this end, note that at time t , there may be some firms in productive development with different contract rates $\omega \in [0, \infty)$. The same is true for the unproductive development state. I will use $\mu_{d,t}^p(\omega)$ and $\mu_{d,t}^u(\omega)$ to refer to the measures of firms at time t in productive and unproductive development, respectively, with contract rate ω . The total measure of firms in productive and unproductive development are then given by

$$\mu_{d,t}^p = \int_0^\infty \mu_{d,t}^p(\omega) d\omega, \quad \mu_{d,t}^u = \int_0^\infty \mu_{d,t}^u(\omega) d\omega \quad (\text{A.23})$$

and the measure of VCs not financing start-ups is $\mu_{vc,t} = M - \mu_{d,t}^p - \mu_{d,t}^u$. Firms in search have no contract and so are homogenous—they have measure $\mu_{s,t}$, such that the total flow of matches is $m(\mu_{s,t}, \mu_{vc,t})$.

To determine the state transition equations, it is necessary to know which contracts are being signed at each instant. In principle, each firm that signs a contract at time t could choose a different contract rate. In this spirit, I will denote by $\chi_t(\omega)$ the share of contracts signed at time t that have contract rate ω . Naturally, $\chi_t(\omega)$ is subject to the constraints $\chi_t \in [0, 1]$ and $\int_0^\infty \chi_t(\omega) d\omega = 1$. The total inflow into state $\mu_{d,t}(\omega)$ from new contracts is then $\chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t})$ and the total outflow of firms from search to development is $\int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) d\omega$. With this notation in hand, the state transition equations are given

by

$$\dot{\mu}_{d,t}^p(\omega) = \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) - (\kappa + \omega)\mu_{d,t}^p(\omega) \quad (\text{A.24})$$

$$\dot{\mu}_{d,t}^u(\omega) = \kappa(1 - p)\mu_{d,t}^p(\omega) - \omega\mu_{d,t}^u(\omega) \quad (\text{A.25})$$

$$\dot{\mu}_{s,t} = \int_0^\infty \omega\mu_{d,t}^p(\omega) d\omega + \Lambda_t - \lambda\mu_{s,t} - \int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) d\omega \quad (\text{A.26})$$

where Λ_t is the flow rate of entry at time t , which is equal to $V_{s,t}/\tilde{\sigma}$ in the decentralised equilibrium.

The planner's problem is to choose entry, Λ_t , and the contract rate for each firm at time t in order to maximise total surplus, given by equation (15). In practice, the choice of contract boils down to choosing $\chi_t(\omega)$ for each $\omega \in [0, \infty)$ subject to the aforementioned constraints on $\chi_t(\omega)$. The current value Hamiltonian is given by

$$\begin{aligned} H = \int_0^\infty & \left([\kappa p \pi - k]\mu_{d,t}^p(\omega) - k\mu_{d,t}^u(\omega) \right. \\ & + \gamma_{d,t}^p(\omega)[\chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) - (\kappa + \omega)\mu_{d,t}^p(\omega)] \\ & + \gamma_{d,t}^u(\omega)[\mu_{d,t}^p(\omega)\kappa(1 - p) - \omega\mu_{d,t}^u(\omega)] \Big) d\omega \\ & + \gamma_{s,t} \left[\int_0^\infty \omega\mu_{d,t}^p(\omega) d\omega + \Lambda_t - \lambda\mu_{s,t} - \int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t}) d\omega \right] - \frac{\tilde{\sigma}}{2}\Lambda_t^2 \end{aligned} \quad (\text{A.27})$$

The optimality condition for Λ_t is simply $\frac{\partial H}{\partial \Lambda_t} = \gamma_{s,t} - \tilde{\sigma}\Lambda_t$. For the contracts, the steady-state optimal plan features $\chi(\omega) = 1$ for some $\omega \in [0, \infty)$, whereas $\chi_t(\omega) = 0$ for all $\omega' \neq \omega$. The marginal value of an increase in $\chi_t(\omega)$ is given by

$$\frac{\partial H}{\partial \chi_t(\omega)} = [\gamma_{d,t}^p(\omega) - \gamma_{s,t}]m(\mu_{s,t}, \mu_{vc,t})$$

Imposing the steady-state conditions $\dot{\gamma}_{s,t} = 0$, $\dot{\gamma}_{d,t}^p(\omega) = 0$ and $\dot{\gamma}_{d,t}^u(\omega) = 0$, the shadow-values are given by

$$\begin{aligned} \gamma_d^p(\hat{\omega}) &= \frac{\left(\rho + \lambda + \frac{\partial m}{\partial \mu_s}\right) \left(\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right) k\right)}{(\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right) \frac{\partial m}{\partial \mu_{vc}} + (\rho + \kappa) \frac{\partial m}{\partial \mu_s} + (\rho + \lambda) (\rho + \kappa + \hat{\omega})} \\ &= \frac{(\rho + \lambda + \alpha \nu) \left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right) k\right]}{\beta \nu (\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}}\right) - \nu (1 - \alpha) (\rho + \kappa) + (\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \hat{\omega}} \end{aligned} \quad (\text{A.28})$$

and

$$\gamma_s = \frac{\alpha \nu}{\rho + \lambda + \alpha \nu} \gamma_d^p(\hat{\omega}) \quad (\text{A.29})$$

$$\gamma_d^u(\hat{\omega}) = -\frac{k}{\rho + \omega} - \frac{\beta \nu (\gamma_d^p(\hat{\omega}) - \gamma_s)}{\rho + \hat{\omega}} \quad (\text{A.30})$$

where $\hat{\omega}$ is assumed to be the time-invariant optimal contract rate, i.e. $\chi(\hat{\omega}) = 1$ and $\chi(\omega) = 0$

for all $\omega \neq \hat{\omega}$.¹³⁸ The planner sets $\chi(\hat{\omega}) = 1$ to maximise the marginal value of the flow of matches in steady-state, $\gamma_d^p(\hat{\omega}) - \gamma_s$.

Proof of Proposition 2.7. The proof follows immediately from the optimality conditions. First, consider the optimal contract rate. In the decentralised equilibrium, the optimal contract rate ω maximises the surplus, S , which is given by

$$S = \frac{(\rho + \lambda) \left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) k \right]}{(1 - \delta) v (\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) - \nu(1 - \delta) (\rho + \kappa) + (\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \omega}$$

In the planner's problem, the optimal contract rate maximises $\gamma_d^p(\hat{\omega}) - \gamma_s$, which is given by

$$\gamma_d^p(\hat{\omega}) - \gamma_s = \frac{(\rho + \lambda) \left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}} \right) k \right]}{\beta v (\rho + \lambda) \left(1 + \frac{\kappa(1-p)}{\rho + \hat{\omega}} \right) - \nu(1 - \alpha) (\rho + \kappa) + (\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \hat{\omega}}$$

Setting $\beta = 1 - \delta$ and $\alpha = \delta$ creates an identity, so that the contract decentralised equilibrium equals that chosen by the planner.

Second, consider entry. In the decentralised equilibrium, entry is given by $V_s/\tilde{\sigma}$. In the planner's problem, the optimality condition yields $\Lambda = \gamma_s/\tilde{\sigma}$. Noting that $(\rho + \lambda)V_s = \delta\nu S$ and comparing V_s to γ_s , it is clear, for given optimal contract rate, ω , that entry is efficient if $\beta = 1 - \delta$ and $\alpha = \delta$. Finally, since the contract in the decentralised equilibrium coincides with the planner's solution when the same condition is met, entry must also be efficient. ■

A.4 Additional results and derivations

A.4.1 Results

Proposition A.4. *The success probability for firms in productive development, p_d is: strictly increasing in p and π ; strictly decreasing in k ; and ambiguous with respect to ν , λ , and κ . The success probabilities for firms in search, p_s is: strictly increasing in p and π ; strictly decreasing in k ; and ambiguous with respect to ν and κ .*

Proof of Proposition A.4. The result for p , π and k follows directly from differentiation of p_d and p_s , given the results from Proposition 2.2. For κ , $\text{sign}(\frac{dp_d}{d\kappa}) = \text{sign}(\frac{dp_s}{d\kappa}) = \text{sign}(\omega(\kappa) - \kappa\omega'(\kappa))$, which is ambiguous.

Next, consider $\frac{dp_d}{d\nu} = \frac{\kappa p \lambda (\omega - (\lambda + \nu)\omega'(\nu))}{(\kappa(\lambda + \nu) + \lambda\omega)^2} \implies \text{sign}(\frac{dp_d}{d\nu}) = \text{sign}(\omega - (\lambda + \nu)\omega'(\nu))$, which can equivalently be cast as a statement about the semi-elasticity of ω with respect to ν ; that is, if $\frac{\omega'(\nu)}{\omega} > \frac{1}{\lambda + \nu}$, then $\frac{dp_d}{d\nu} > 0$. To see that $\frac{dp_d}{d\nu} < 0$ is possible, note that there are parameter values such that $\omega = 0$ but $\omega'(\nu) > 0$ (see the proof to Proposition A.3). In this case $\omega - (\lambda + \nu)\omega'(\nu) = -(\lambda + \nu)\omega'(\nu) < 0$, so $\frac{dp_d}{d\nu} < 0$. To see that $\frac{dp_d}{d\nu} > 0$ is possible, note that $\lim_{\nu \rightarrow \infty} p_d = p$ and $p_d \leq p$, so p_d must approach p from below as $\nu \rightarrow \infty$. This means there must be some ν for which $\frac{dp_d}{d\nu} > 0$. Finally, to see that $\exists \tilde{\nu}_d \geq 0$ such that if $\nu \geq \tilde{\nu}_d$, $\frac{dp_d}{d\nu} > 0$ and otherwise $\frac{dp_d}{d\nu} < 0$, note that $\frac{d}{d\nu}(\omega - (\lambda + \nu)\omega'(\nu)) = -(\lambda + \nu)\omega''(\nu) > 0$ where the inequality follows since $\omega''(\nu) < 0$,

¹³⁸The values for $\gamma_d^p(\omega)$ and $\gamma_d^u(\omega)$ for $\omega \neq \hat{\omega}$ can also be derived, but are not necessary to make the key point.

so there at most one root for $\omega - (\lambda + \nu)\omega'(\nu) = 0$. Furthermore, $\frac{d}{d\nu}(\omega - (\lambda + \nu)\omega'(\nu)) > 0$ implies that if $\frac{dp_d}{d\nu} \geq 0$ for some $\nu = \hat{\nu}$, then $\frac{dp_d}{d\nu} > 0 \forall \nu > \hat{\nu}$. Therefore, we can define $\tilde{\nu}_d > 0$ s.t. $\omega - (\lambda + \nu)\omega'(\nu) = 0$ or $\tilde{\nu}_d = 0$ if there is no root and the statement holds.

Next, consider $\frac{dp_s}{d\nu} = \frac{\kappa p \lambda (\kappa + \omega - \nu \omega'(\nu))}{(\kappa(\lambda + \nu) + \lambda \omega(\nu))^2} \implies \text{sign}(\frac{dp_s}{d\nu}) = \text{sign}(\kappa + \omega - \nu \omega'(\nu))$. To see that $\frac{dp_s}{d\nu} < 0$ is possible, note that $\frac{d}{d\nu}(\kappa + \omega - \nu \omega'(\nu)) = -\nu \omega''(\nu) > 0$. Therefore, if $\frac{dp_s}{d\nu} < 0$ is to occur, it will occur for low values of ν . Then, take the limit as $\nu \rightarrow 0$

$$\lim_{\nu \rightarrow 0} (\kappa + \omega - \nu \omega'(\nu)) = \frac{(\rho + \lambda)(k(\kappa p - \rho) + \kappa p \pi(\rho - \kappa)) - \sqrt{\kappa^2 k(1 - p)p(\lambda + \rho)^2(\kappa \pi - k)}}{(\rho + \lambda)(k - \kappa p \pi)}$$

where $\omega > 0$ has been assumed. This expression is negative if $p \in (\frac{k\rho^2}{\kappa\pi(\kappa - \rho)^2 - \kappa k(\kappa - 2\rho)}, 1)$. However, the $\omega > 0$ solution may be invalid. From Corollary 2.1, a sufficient condition to take $\nu \rightarrow 0$ while maintaining $\omega > 0$ is that $p < \bar{p} = \frac{k(\rho + \kappa)^2}{\kappa(\pi\rho^2 + k(\kappa + 2\rho))}$. Both of these conditions on p can be met simultaneously if $\kappa > 2\sqrt{\rho}$, which is admissible. Therefore, $\lim_{\nu \rightarrow 0} (\kappa + \omega - \nu \omega'(\nu)) < 0$ in some part of the parameter space and so $\frac{dp_s}{d\nu}$ is feasible. To see that $\frac{dp_s}{d\nu} > 0$ is possible, note that $\lim_{\nu \rightarrow \infty} p_s = p$ and $p_s \leq p$, so p_s must approach p from below as $\nu \rightarrow \infty$. The remainder of the proof is analogous to that for part (i).

For λ , $\text{sign}(\frac{dp_d}{d\lambda}) = \text{sign}(-\nu\omega - \lambda(\lambda + \nu)\omega'(\lambda))$. To see that this can be positive, note that there exists a λ such that $\omega = 0$ and $\omega'(\lambda) < 0$; at this point, small reductions in λ imply a movement from upfront financing to staged-financing and so lower the success probability. To see that $\frac{dp_d}{d\lambda} < 0$ is possible, note that $\lim_{\lambda \rightarrow 0} p_d = p$ and $p_d \leq p$, so p_d must approach p from below as $\lambda \rightarrow 0$, implying there is some region for which $\frac{dp_d}{d\lambda} < 0$. For p_s , clearly $\frac{dp_d}{d\lambda} < 0 \implies \frac{dp_s}{d\lambda} < 0$.

Finally, $\frac{dp_d}{dp} = \frac{\kappa(\lambda + \nu)(\kappa(\lambda + \nu) - \lambda p \omega'(p) + \lambda \omega)}{(\kappa(\lambda + \nu) + \lambda \omega)^2}$ and $\frac{dp_s}{dp} = \frac{\nu}{\lambda + \nu} \frac{dp_d}{dp}$. By Proposition (2.2), $\omega'(p) < 0$, so $\frac{dp_d}{dp} > 0$ and $\frac{dp_s}{dp} > 0$. Furthermore, $\frac{d}{dp} \frac{\kappa(\lambda + \nu)}{\omega \lambda + \kappa(\lambda + \nu)} = -\frac{\kappa \lambda (\lambda + \nu) \omega'(p)}{(\kappa(\lambda + \nu) + \lambda \omega)^2} > 0$ since $\omega'(p) < 0$. ■

A.4.2 Derivations

Expected capital cost Suppose the contract is signed at date $t = 0$. Until time $T = \min\{T_\kappa, T_\omega\} \sim \text{Exp}(\kappa + \omega)$, the E has not obtained a result and funding has not been withdrawn, so the VC pays k . Once time T arrives, there are three potential outcomes: (i) funding runs out, $T = T_\omega$, with probability $\frac{\omega}{\kappa + \omega}$; (ii) $T = T_\kappa$ and the result is a success, with probability $\frac{\kappa p}{\kappa + \omega}$; or (iii) $T = T_\kappa$ and the result is a failure, with probability $\frac{\kappa(1-p)}{\kappa + \omega}$. In cases (i) and (ii), funding ceases. In case (iii), the VC continues to pay k until time $T + T_\omega = T_\kappa + T_\omega$. Therefore, from time $t = 0$ to $t = T$, the flow cost to the E is $e^{-\rho t} k$ from the perspective of time $t = 0$. At time T , the probability that the VC continues to pay k is $\frac{\kappa(1-p)}{\kappa + \omega}$, in which case they pay k from date T to $T + T_\omega$. The expected capital cost is then computed by taking expectations over $T_\omega \sim \text{Exp}(\omega)$ and $T \sim \text{Exp}(\kappa + \omega)$

$$\begin{aligned}
K(\omega) &= \int_0^\infty \left(\int_0^T e^{-\rho t} k dt \right. \\
&\quad \left. + \frac{\kappa(1-p)}{\kappa+\omega} \int_0^\infty \left(\int_T^{T+T_\omega} e^{-\rho s} k ds \right) \omega e^{-\omega T_\omega} dT_\omega \right) (\kappa+\omega) e^{-(\kappa+\omega)T} dT \\
&= \left(1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \frac{k}{\rho+\kappa+\omega}
\end{aligned} \tag{A.31}$$

The value of search To obtain equation (13), solve the following system for $\{V_d, V_s\}$

$$\begin{aligned}
\rho V_d &= \kappa[p\pi - k] + \omega[V_s - V_d] \\
(\rho + \lambda)V_s &= \nu[V_d - K(\omega) - V_s]
\end{aligned}$$

which substitutes $V^M = V_d - K(\omega)$. This yields

$$V_s = \frac{\nu \left[\kappa p \pi - k \left(1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \right]}{(\rho + \lambda + \nu)(\rho + \kappa + \omega) - \nu\omega}$$

After some algebra, this is equal to

$$V_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + \frac{\mathcal{C}\mathcal{F} + (\rho+\omega)^2}{\rho+\omega-\mathcal{C}}} \bar{V}_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + \frac{2\mathcal{C}\mathcal{F} + 2\mathcal{C}(\rho+\omega)}{\rho+\omega-\mathcal{C}}} \bar{V}_s = \frac{\mathcal{F} - \kappa}{\mathcal{F} + 2(\mathcal{C} + \sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2})} \bar{V}_s$$

The first value obtains after factoring out \bar{V}_s ; the second after multiplying out $(\rho + \omega)^2$ and simplifying; the third follows by factoring $\sqrt{\mathcal{C}\mathcal{F} + \mathcal{C}^2}$ from $\frac{2\mathcal{C}\mathcal{F} + 2\mathcal{C}(\rho+\omega)}{\rho+\omega-\mathcal{C}}$ and simplifying.

Share receiving funding after year T The probability that a firm receives another funding round after time x is given by

$$\Pr(\text{new funding after year } T) = P_{dd}(T) \left(\frac{\omega}{\kappa+\omega} \frac{\nu}{\lambda+\nu} \right) + P_{ds}(T) \left(\frac{\nu}{\lambda+\nu} \right)$$

where $P_{dd}(T)$ ($P_{ds}(T)$) is the probability of being in state d (s) at time T , given that the firm starts in development (d) at time $t = 0$. Intuitively, given that the firm is in state d at time $t = T$, it raises again with probability $\frac{\omega}{\kappa+\omega} \frac{\nu}{\lambda+\nu}$; and for a firm in state s at time $t = T$, it raises again with probability $\frac{\nu}{\lambda+\nu}$. Then,

$$\begin{pmatrix} P_{dd}(T) & P_{ds}(T) \\ P_{sd}(T) & P_{ss}(T) \end{pmatrix} = e^{Q_T T}, \quad \text{for} \quad Q_T = \begin{pmatrix} -(\kappa+\omega) & \omega \\ \nu & -(\lambda+\nu) \end{pmatrix}$$

where Q_T is the transient block of the Q matrix associated with the CTMC governing state transitions in the model and $e^{Q_T T}$ represents matrix exponentiation. As noted in the main text, the expression for $\Pr(\text{new funding after year } T)$ depends only on $(\kappa, \lambda, \nu, \omega)$ and T , because only these parameters enter Q_T . The closed-form expression can be obtained by solving for

$(P_{dd}(x), P_{ds}(x))$ explicitly.

Flow of first funding rounds To obtain equation (24), note that the measure of firms that have entered but not yet received funding evolves as

$$\dot{\mu}_{s,t}^0 = \frac{V_{s,t}}{\tilde{\sigma}} - (\lambda + \nu_e) \mu_{s,t}^0$$

which implies the steady-state condition

$$\tilde{\sigma} = \left(\frac{1}{\lambda + \nu} \times \frac{1}{\mu_s^0} \right) V_s$$

where μ_s^0 is the steady-state measure of firms that have paid the entry cost but not yet obtained VC funding. μ_s^0 is unobserved in the data, but the empirical counterpart to $\nu_e \times \mu_s^0$ is the flow of first funding rounds, which is observed. Substituting in for μ_s^0 then yields equation (24).

A.5 Quantitative model

A.5.1 Frictionless benchmark

In the frictionless economy without agency and matching frictions, the value functions $\{\bar{V}_{s,e}, \bar{V}_{d,e}, \bar{V}_{s,l}, \bar{V}_{d,l}\}$ solve

$$\rho \bar{V}_{d,l} = -k_l + \kappa [p_l \pi - \bar{V}_{d,l}] + p_l \pi \xi \phi \exp \left\{ -\frac{\bar{V}_{d,l}}{p_l \pi \xi} \right\} \quad (\text{A.32})$$

$$(\rho + \lambda) \bar{V}_{s,l} = p_l \pi \xi \phi \exp \left\{ -\frac{\bar{V}_{s,l}}{p_l \pi \xi} \right\} \quad (\text{A.33})$$

$$\rho \bar{V}_{d,e} = -k_e + \kappa \left[\left(\frac{p_e}{p_L} \right) \max\{\bar{V}_{s,l}, \bar{V}_{d,l}\} - \bar{V}_{d,e} \right] \quad (\text{A.34})$$

$$\bar{V}_{s,e} = \max\{0, \bar{V}_{d,e}\} \quad (\text{A.35})$$

These equations mirror the conditions in the model of section 2.4, but are adjusted for a wealthy entrepreneur that can self-finance the project.¹³⁹ Relative to that benchmark, note that an entrepreneur that enters the late stage may choose not to invest, or decide not to and simply wait for an acquisition offer. Therefore, the relevant continuation value from the early to late-stage is $\max\{\bar{V}_{s,l}, \bar{V}_{d,l}\}$. An entrepreneur that does not invest in the late stage fails at rate λ , and so I retain the subscripts s and d ; although now they refer to no late-stage investment and late-stage investment, respectively. Conversely, in the early-stage the entrepreneur either invests, or the project has zero value.

A.5.2 Equilibrium conditions

In this section, I provide the full set of equilibrium conditions for the model that I outline in section 2.4 and estimate in section 3. I first provide the full set of conditions taking as given the equilibrium in the capital market, and then provide details on the full equilibrium.

¹³⁹ Alternatively, by taking the numerical limit $\nu_e \rightarrow \infty$ and $\nu_l \rightarrow \infty$ and solving the model of section 2.4, one can obtain an approximation to the solution of this system of equations to desired accuracy. See Lemma 2.1.

Taking the meeting rates in the early and late-stage markets, ν_e and ν_l as given, the following conditions characterise the equilibrium

$$\rho V_{d,e}(\omega_e) = \kappa [\hat{p}_e V_{s,l} - V_{d,e}(\omega_e)] + \omega_e [V_{s,e} - V_{d,e}(\omega_e)] \quad (\text{A.36})$$

$$(\rho + \lambda) V_{s,e} = \nu_e [V_e^M - V_{s,e}] \quad (\text{A.37})$$

$$V_e^M = \sup_{\{\omega_e \in [0, \infty), \varsigma_e \in [0, 1]\}} \{(1 - \varsigma_e) V_{d,e}(\omega_e)\} \quad \text{s.t.} \quad \varsigma_e V_{d,e}(\omega_e) \geq K_e(\omega_e), V_{s,e} \text{ given} \quad (\text{A.38})$$

$$K_e(\omega_e) = \left(1 + \frac{\kappa(1 - \hat{p}_e)}{\rho + \omega_e}\right) \frac{k}{\rho + \kappa + \omega_e} \quad (\text{A.39})$$

$$\rho V_{d,l}(\omega_l) = \kappa [p_l \pi - V_{d,l}(\omega_l)] + \omega_l [V_{s,l} - V_{d,l}(\omega_l)] + \hat{\phi}_d [p_l \pi E[\epsilon | \epsilon > V_{d,l}(\omega_l)/(p_l \pi)] - V_{d,l}] \quad (\text{A.40})$$

$$(\rho + \lambda_l) V_{s,l} = \nu_l [V_l^M - V_{s,l}] + \hat{\phi}_s [p_l \pi E[\epsilon | \epsilon > V_{s,l}/(p_l \pi)] - V_{s,l}] \quad (\text{A.41})$$

$$V_l^M = \sup_{\{\omega_l \in [0, \infty), \varsigma_l \in [0, 1]\}} \{(1 - \varsigma_l) V_{d,l}(\omega_l)\} \quad \text{s.t.} \quad \varsigma_l V_{d,l}(\omega_l) \geq K_l(\omega_l), V_{s,l} \text{ given} \quad (\text{A.42})$$

$$K_l(\omega_l) = \left(1 + \frac{\kappa(1 - p_l)}{\rho + \omega_l + \phi}\right) \frac{k}{\rho + \kappa + \omega_l + \hat{\phi}_d} \quad (\text{A.43})$$

where $\hat{p}_e = p_e/p_l$, $\epsilon \sim \text{Exp}(1/\xi)$, $\hat{\phi}_i = \phi[1 - F(V_{i,l}/(p_l \pi))]$ for $i \in \{s, d\}$, and (ω_e, ς_e) and (ω_l, ς_l) are chosen optimally in the sense of equations (A.38) and (A.42), respectively.

The meeting rates, ν_e and ν_l , are determined in equilibrium based on the supply and demand for capital in each stage. I assume that VCs specialise within each stage: a measure M_e serve the early stage and measure M_l serve the late-stage. I denote by $\mu_{s,i}$, $\mu_{d,i}^p$ and $\mu_{d,i}^u$ the measures of firms in search, productive development and unproductive development for stage $i \in \{e, l\}$, respectively. Then, the measure of VCs in each stage that are not funding start-ups is given by $\mu_{vc,i} = M_i - \mu_{d,i}^p - \mu_{d,i}^u$ for $i \in \{e, l\}$ and the flow rate of matches in stage $i = e, l$ is $m_i = m(\mu_{s,i}, \mu_{vc,i})$. Then $\nu_i = \mu_{s,i}^{-1} m(\mu_{s,i}, \mu_{vc,i})$ for $i = e, l$.

Finally, $\mu_{s,i}$, $\mu_{d,i}^p$ and $\mu_{d,i}^u$ for $i = e, l$ are determined in equilibrium by the following steady-state conditions.

$$\text{Early-stage productive development : } \nu_e \mu_{s,e} = (\kappa + \omega_e) \mu_{d,e}^p$$

$$\text{Early-stage unproductive development : } \kappa(1 - \hat{p}_e) \mu_{d,e}^p = \omega_e \mu_{d,e}^u$$

$$\text{Early-stage search : } \omega_e \mu_{d,e}^p + \frac{V_{s,e}}{\tilde{\sigma}} = (\lambda + \nu_e) \mu_{s,e}$$

$$\text{Late-stage productive development : } \nu_l \mu_{s,l} = (\kappa + \omega_l + \hat{\phi}_d) \mu_{d,l}^p$$

$$\text{Late-stage unproductive development : } \kappa(1 - p_l) \mu_{d,l}^p = \omega_l \mu_{d,l}^u$$

$$\text{Late-stage search : } \omega_l \mu_{d,l}^p + \kappa \hat{p}_e \mu_{d,e}^p = (\lambda + \nu_l + \hat{\phi}_s) \mu_{s,l}$$

where $\nu_e = m_e/\mu_{s,e}$ and $\nu_l = m_l/\mu_{s,l}$. Relative to the baseline model, there are two tangible differences. Firstly, the inflow into early-stage search includes new entrants, whereas the inflow into late-stage search includes firms that are successful in overcoming the early-stage uncertainty,

a flow $\kappa_e \hat{p}_e \mu_{d,e}^p$. Secondly, in the late-stage, firms may be acquired from productive development or search.

B Empirical appendix

B.1 Dataset construction

This section discusses dataset construction. My US and UK samples are drawn from Thomson Reuters venture capital dataset; I consider all firms that raised a first “early-stage” funding round between 2005-2015. Before processing the data, I make a small number of manual corrections where there are obvious errors. The majority of errors occur when a funding round coincides with an exit event; I perform manual searches to clarify the true nature of the event and drop the false entry. I also drop duplicated funding rounds.

Among all funding rounds, I keep those tagged as one of “Seed”, “Early Stage”, “Expansion”, “Later Stage”, which reflect increasing levels of progression. If a firm has two deals on the same day, I sum up the respective deal values and record it under the earlier tag. I then label ‘Seed’ “Early Stage” rounds as “early-stage”, and “Expansion” and “Later Stage” rounds as “late-stage”. I drop the minority of firms that only have late-stage funding rounds in the data. I also drop any funding rounds that occur after the firm records an exit.

In some cases, the deal values are missing. This occurs in 2,706 of the 33,158 (8%) US funding rounds, and a slightly higher share of UK rounds. Data on amounts raised is used to compute the burn rate and exit multiples, both of which are used in model estimation. Therefore, I follow existing literature and impute amounts raised in these cases. The methodology I use follows Jagannathan et al. (2022) closely. Specifically, I estimate the following regression

$$\log(\text{Amount raised}_{i,r}) = \beta_0 + \beta_1 \log(\text{Amount raised}_{i,r-1}) + \beta_2 X_{i,r} + u_{i,r} \quad (\text{B.1})$$

where $\text{Amount raised}_{i,r}$ is the amount raised by firm i in round r (in 2015 Mn USD) and X_i is a list of firm-specific controls and fixed effects. I include the investment stage, industry, year-quarter fixed effects, and fixed effect for the number of investors, which I cap at ten investors (i.e. all rounds with 10 or more investors are lumped together). I run this regression separately for the US and UK. In addition, for some rounds I do not observe the previous funding amount and so run the specification with only the controls, $X_{i,r}$.¹⁴⁰ The results are displayed in Table B.1. I run this on all deals between 2005 and 2022, which is why the observation counts exceed the sample used in the estimation (where data is censored). Using the results of these regressions I impute missing amounts raised using fitted values.

Among exit types, I classify IPOs, reverse takeovers, and secondary sales as successes. I also classify “Mergers” as successes when the acquiror is an investor, whereas trade-sales are classified as acquisitions. This categorisation means that SPACs are included as successes. For exit values, I obtain data from CRSP on the market cap of firms six months after IPO for US IPOs and from Bloomberg for UK IPOs.¹⁴¹ Furthermore, a non-negligible share of exit values are missing in the Thomson Reuters data. Where possible, I fill in missing exit values with data

¹⁴⁰I run this on all the data, regardless of whether previous funding information is available.

¹⁴¹Insiders typically cannot cash out until six months after an IPO (Brav and Gompers, 2000).

	United States		United Kingdom	
	Log(Amount _{<i>r</i>})	Log(Amount _{<i>r</i>})	Log(Amount _{<i>r</i>})	Log(Amount _{<i>r</i>})
Log(Amount _{<i>r-1</i>})	-	0.47***	-	0.55***
Constant	0.35	-0.52	0.49	0.54
Industry FE	Y	Y	Y	Y
Stage FE	Y	Y	Y	Y
# Investors FE	Y	Y	Y	Y
YQ FE	Y	Y	Y	Y
Adj. R^2	0.31	0.50	0.26	0.55
Observations	61,004	36,196	7,133	2,816

Table B.1: Imputation model

The table reports regression results from estimation of equation (B.1). ***, $p < 0.01$.

from Crunchbase. When the deal value remains missing – the majority case – I set it equal to 1.5 times total invested capital, implying an exit multiple of 1.5X, following (Kerr et al., 2014).

The final dataset comprises all firms that raised their first early-stage funding round between 2005-2015 inclusive. Table B.2 reports summary statistics for the main sample.

B.2 Definitions

Thomson Reuters define funding rounds in the following way.

Seed stage This stage is a relatively small amount of capital provided to an inventor or entrepreneur to prove a concept. This involves product development and market research as well as building a management team and developing a business plan, if the initial steps are successful. This is a pre-marketing stage.

Early stage This stage provides financing to companies completing development where products are mostly in testing or pilot production. In some cases, product may have just been made commercially available. Companies may be in the process of organising or they may already be in business for three years or less. Usually such firms will have made market studies, assembled the key management, developed a business plan, and are ready or have already started conducting business.

Expansion stage This stage involves working capital for the initial expansion of a company that is producing and shipping and has growing accounts receivables and inventories. It may or may not be showing a profit. Some of the uses of capital may include further plant expansion, marketing, working capital, or development of an improved product. More institutional investors are more likely to be included along with initial investors from previous rounds. The venture capitalist’s role in this stage evolves from a supportive role to a more strategic role.

Later stage Capital in this stage is provided for companies that have reached a fairly stable growth rate; that is, not growing as fast as the rates attained in the expansion stages. Again,

	United States		United Kingdom	
	Count	Share	Count	Share
<i>Sample overview</i>				
Firms	11,030	—	1,066	—
Successes	400	3.6%	26	2.4%
Acquisitions	2,034	18.4%	97	9.1%
Funding rounds — total	33,158	—	2,218	—
Early-stage	21,886	66.0%	1,534	69.2%
Late-stage	11,272	34.0%	684	30.8%
<i>Sectors</i>				
Biotechnology	995	9.0%	97	9.1%
Communications and Media	289	2.6%	43	4.0%
Computer Hardware	298	2.7%	31	2.9%
Computer Software and Services	3,607	32.7%	254	23.8%
Consumer Related	328	3.0%	53	5.0%
Industrial/Energy	537	4.9%	99	9.3%
Internet Specific	3,082	28.0%	246	23.1%
Medical/Health	1,063	9.6%	109	10.2%
Other Products	463	4.2%	85	8.0%
Semiconductors/Other Elect.	368	3.3%	49	4.6%
Total	11,030	100.0%	1,066	100.0%

Table B.2: Sample overview

The table reports the number of firms, funding rounds, and outcomes in the estimation sample, and their sectoral composition for the United States and United Kingdom. Funding rounds and outcomes are censored at seven years following a firm's first funding round. Shares are within-country totals of firms (or rounds where applicable).

these companies may or may not be profitable, but are more likely to be than in previous stages of development. Other financial characteristics of these companies include positive cash flow. This also includes companies considering IPO.

B.3 Additional tables and figures

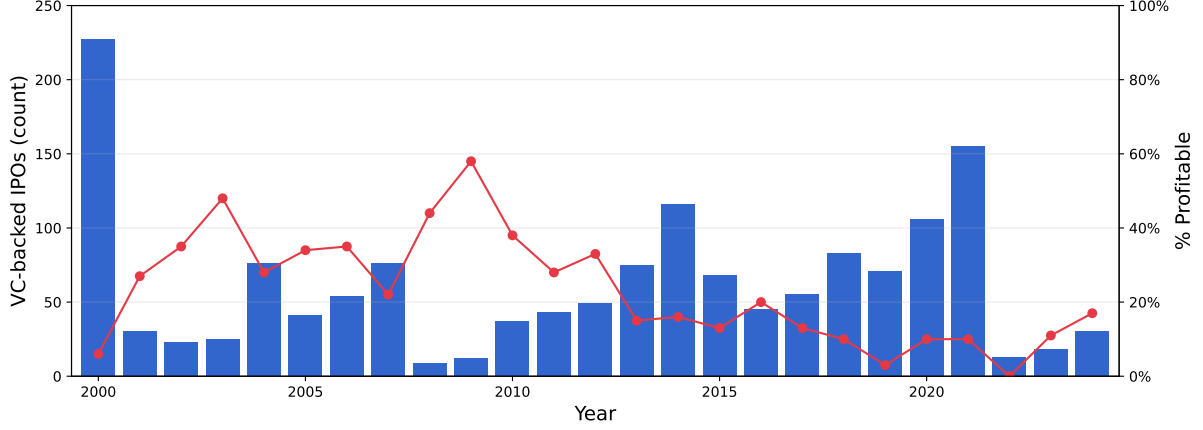


Figure B.1: US-headquartered, VC-backed IPOs

The figure reports the number of VC-backed IPOs of US-headquartered companies from 2000-2024 (bars, left axis), and the share of these companies that were profitable (line, right axis). The data is from Jay Ritter's website (<https://site.warrington.ufl.edu/ritter/>), Table 4d. See source for details.

C Quantitative appendix

C.0.1 Additional details on US estimation

This section provides additional details on the estimation procedure.

Moment computation The estimation procedure involves two steps. First, all parameters except for $\tilde{\sigma}$ are estimated by minimising equation (25). Then, $\tilde{\sigma}$ is recovered from the model condition for the number of first funding rounds. The moments included in equation (25) are reported in Table 3. Before computing moments, I drop observations occurring more than seven years after a firm's first funding round, ensuring uniform censoring.

The complication lies in estimating $\lambda + \nu_e$ and $\lambda + \nu_e + \hat{\phi}_s$ in way consistent with the model's structure. In the model, the within-stage distribution for the time between funding rounds is hypoexponential: for two successive early-stage rounds, $T_{br}^{e \rightarrow e} \sim \text{Hypo}(\kappa + \omega_e, \lambda + \nu_e)$, and for two successive late-stage rounds, $T_{br}^{l \rightarrow l} \sim \text{Hypo}(\kappa + \omega_l, \lambda + \nu_l + \hat{\phi}_s)$. There are two problems. First, the parameters of the hypoexponential distribution are interchangeable, meaning it is unclear which of the estimates reflect $\lambda + \nu_e$ and $\lambda + \nu_e + \hat{\phi}_s$. Second, the data used to estimate the model is censored, meaning these distributions no longer apply.

First, consider the censoring issue. To state the issue more formally, denote by $t_{i,n}^s$ the time of the n th funding round for firm i in state $s \in \{e, l\}$ and the duration between round n and round $n+1$ for firm i in stage s by $\tau_{i,n}^s = t_{i,n+1}^s - t_{i,n}^s$. Firm i completes $N_i^e \in \{1, 2, 3, \dots\}$ early-stage rounds and $N_i^l \in \{0, 1, 2, \dots\}$ late-stage rounds, but N_i^e and N_i^l are unobserved because the data is censored.¹⁴² Without censoring, the duration between funding rounds is drawn from the unconditional distribution with CDF $\Pr(\tau_{i,n}^s \leq \tau)$ for $s \in \{e, l\}$. However, when data is

¹⁴² N_i^e starts at 1, not 0, because the sample is firms that have at least one funding round, which is necessarily an early-stage round.

censored, observations are instead drawn from the distribution

$$\Pr(\tau_{i,n}^s \leq \tau | t_{i,n}^s + \tau_{i,n}^s \leq T)$$

where T is the censoring date, set at seven years after the firms first funding round in the baseline estimation.

In the model, the assumption of *ex ante* homogenous firms implies that the relevant unit of observation within each stage is the funding round, j , not the the firm-round number tuple, (i, n) . Furthermore, the duration to the next round, τ_j^s , is independent of the time of the current round, t_j^s , owing to the model's Markov property. Therefore,

$$\Pr(\tau_{i,n}^s \leq \tau | t_{i,n}^s + \tau_{i,n}^s \leq T) = \Pr(\tau_j^s \leq \tau | \tau_j^s \leq T_j)$$

where $T_j = T - t_j^s$ is the censoring date for a given funding round (e.g. with $T = 7$, the duration to the next round for a round at year $t = 4$ has $T_j = 7 - 4 = 3$).

This means that, under censoring T , funding round j in stage $s \in \{e, l\}$ is drawn from a truncated hypoexponential distribution. The truncation date, T_j is funding round specific and durations are independent across funding rounds in the model. Therefore, the likelihood for the data is given by

$$L^s = \prod_{j=1} \frac{f(\tau_j^s; h_1^s, h_2^s, T_j^s)}{F(T_j^s; h_1^s, h_2^s, T_j^s)}$$

where $f(\cdot)$ and $F(\cdot)$ are, respectively, the PDF and CDF of the hypoexponential distribution conditional on $\tau_j^s \leq T_j^s$ and (h_1^s, h_2^s) are the parameters of the hypoexponential distribution. The estimates are shown in Table C.1.

	UK			US		
	Point estimate	95% CI	N	Point estimate	95% CI	N
h_1^e	1.60	(0.91, 2.29)	468	1.54	(1.47, 1.61)	10,856
h_2^e	2.13	(0.93, 3.32)	468	3.30	(3.02, 3.57)	10,856
h_1^l	0.80	(0.58, 1.02)	331	0.89	(0.84, 0.93)	6,688
h_2^l	2.99	(1.34, 4.65)	331	5.94	(5.05, 6.83)	6,688

Table C.1: Estimates of hypoexponential distribution

The table reports parameter estimates for MLE estimates of hypoexponential distribution parameters, estimated on the duration between funding rounds within stages under round-specific censoring.

The second issue is to assign the estimated parameter values to $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$. I set the estimates for $\lambda + \nu_e = \max\{h_1^e, h_2^e\}$ and $\lambda + \nu_l + \hat{\phi}_s = \max\{h_1^l, h_2^l\}$. Choosing the larger value is conservative: it implies that firms spend relatively less time in search, biasing against finding large frictions. For instance, in the late-stage, setting the estimate for $\lambda + \nu_l + \hat{\phi}_s = 5.94$ implies a search duration of approximately two months, rather than 13 months for $\lambda + \nu_l + \hat{\phi}_s = 0.89$. Furthermore, the value for $\kappa + \omega_l$ should be close to the other parameter value and, in practice, VCs seek to provide funding for one to two years, not several months.

Finally, computing the parameters by MLE is computationally costly. Therefore, for the simulated data, I simply take the values for $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ from the model, rather than estimating them. With $S = 20$ (see below), this is innocuous.

Implementation I simulate a sample of 220,600 firms, $S = 20$ times the size of the empirical 2005-2015 sample. Model parameters excluding $\tilde{\sigma}$, Θ , are jointly estimated by minimising the criterion

$$C(\Theta) = g(\Theta)'g(\Theta) \quad (\text{C.1})$$

where the j -th element of $g(\Theta)$ is given by

$$g_j = \frac{\tilde{m}_j - m_j(\Theta)}{\frac{1}{2}|\tilde{m}_j| + \frac{1}{2}|m_j(\Theta)|}$$

and where \tilde{m}_j is the empirical moment and $m_j(\Theta)$ is the model moment, computed under Θ . Standard errors are computed as

$$\widehat{\text{Var}}(\hat{\Theta}) = (G^\top W G)^{-1} G^\top W \Omega W G (G^\top W G)^{-1}$$

where $W = I$ is the identity matrix, Ω is the variance-covariance matrix of $g(\Theta)$, G is the Jacobian of moment conditions, $g(\Theta)$. I compute Ω via bootstrap. I draw $N = 11,030$ firms with replacement from my sample, compute the moments, and compute $g(\Theta)$ at my estimated Θ for that draw. I repeat this $B = 1000$ times. I then compute Ω as

$$\hat{\Omega} = \frac{1}{B-1} (g_b - \bar{g}_b)' (g_b - \bar{g}_b)$$

where g_b is an $B \times J$ matrix of moment conditions at each bootstrap, and \bar{g}_b is a $1 \times J$ vector that averages across bootstrapped samples.

The entry cost parameter, $\tilde{\sigma}$, is estimated from the model condition for the number of first funding rounds. To compute its standard error, I sample parameter Θ from $N(\hat{\Theta}, \text{Var}(\hat{\Theta}))$, $B = 1000$ times, and compute $\tilde{\sigma}$ at each iteration. The standard error is then the standard deviation of the bootstrapped sample.

Parameter	Mean # early rounds	Mean # rounds to late	Mean duration b/w rounds	Estimate of $\lambda + \nu_e$	Estimate of $\lambda + \nu_l + \hat{\phi}_s$	Mean burn rate early round	Mean burn rate late round
Early-stage dev. level, p_e	-1.65	-0.02	0.94	0.00	0.00	1.18	0.02
Late-stage dev. level, p_l	-1.21	-0.75	1.14	0.00	-0.02	1.05	0.83
Early-stage cost, k_e	0.78	-0.01	-0.51	0.00	0.00	0.30	-0.02
Late-stage cost, k_l	0.39	0.65	-0.62	0.00	0.02	-0.32	0.27
R&D arrival rate, κ	-0.75	-0.06	0.04	0.00	-0.01	0.47	0.05
Success payoff, π	-1.19	-0.65	1.07	0.00	-0.02	1.04	0.78
Failure rate search, λ	-0.09	-0.44	0.13	0.20	0.11	-0.03	0.40
Acq. offer rate, ϕ	-0.47	-0.24	0.41	0.00	0.03	0.40	0.10
Mean acq. 'synergy', ξ	-0.82	-0.29	0.67	0.00	-0.00	0.70	0.32
Early-stage rate, ν_e	0.30	0.02	-0.17	0.80	0.00	0.06	0.02
Late-stage rate, ν_l	-0.06	0.35	-0.15	0.00	0.84	0.18	0.17

continued...

Parameter	Mean time-to-exit	Acq-to-success ratio	Mean success multiple	Share one round	Share funded after year 5	Share to late-stage multiple > 10X	Share acq multiple > 10X
Early-stage dev. level, p_e	0.15	-0.31	-0.55	0.02	0.53	1.99	-2.12
Late-stage dev. level, p_l	0.11	-2.24	-1.14	0.70	0.01	0.69	-0.50
Early-stage cost, k_e	-0.02	0.20	-0.17	-0.50	0.02	-0.38	-0.01
Late-stage cost, k_l	-0.07	0.77	0.18	-0.23	-0.06	-0.18	0.54
R&D arrival rate, κ	-0.23	-1.12	0.01	0.40	-0.69	0.71	0.02
Success payoff, π	0.10	-1.19	-0.09	0.69	0.00	0.68	-0.40
Failure rate search, λ	-0.08	-0.02	-0.21	0.42	-0.52	-0.27	-0.39
Acq. offer rate, ϕ	-0.04	0.52	-0.24	0.31	-0.13	0.19	-0.13
Mean acq. 'synergy', ξ	-0.00	-0.30	-0.48	0.49	-0.03	0.41	0.29
Early-stage rate, ν_e	0.03	0.07	-0.02	-0.46	0.31	0.12	0.17
Late-stage rate, ν_l	0.02	-0.17	-0.06	-0.09	0.10	0.18	-0.46

Table C.2: Elasticity of moments with respect to parameters

The table reports the elasticity of each moment (or auxiliary parameter) with respect to each estimated parameter. Starting from the estimated parameter values, each parameter is individually adjusted by $\pm 1\%$, the model is simulated, and moments are recomputed using a centred difference.

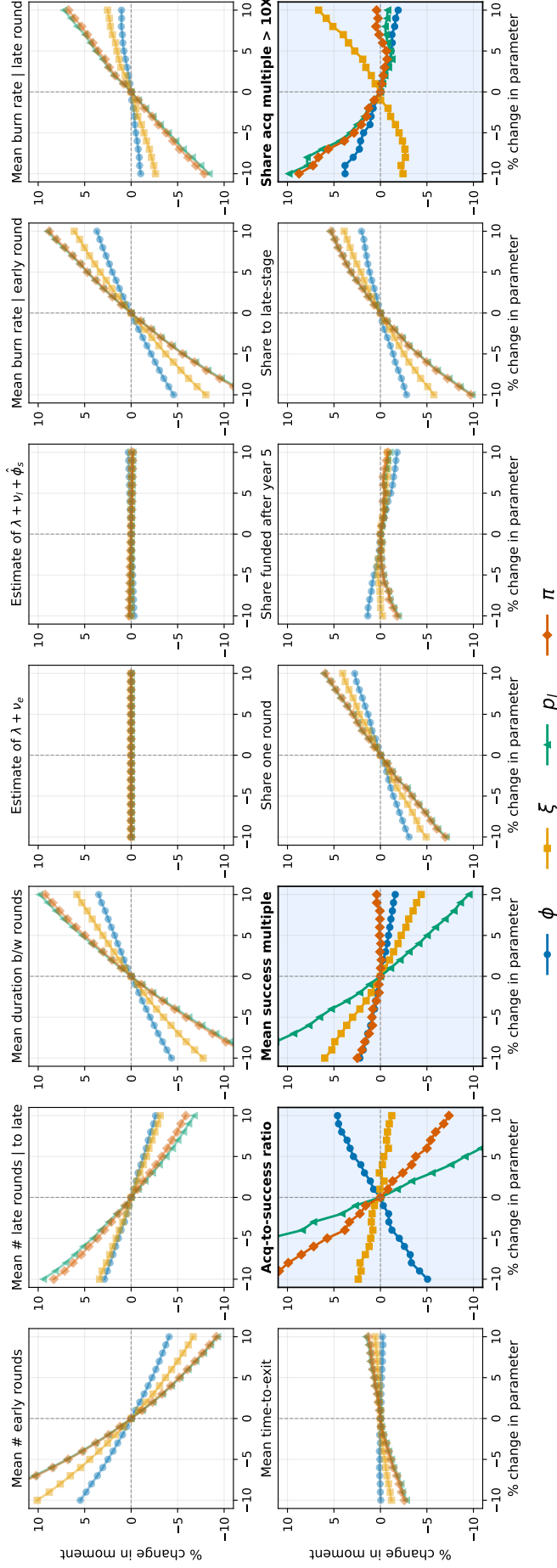


Figure C.1: Comparative statics – moments to parameter changes

Each panel plots the percent change in a moment when varying one parameter by $\pm 10\%$ around the baseline estimate, holding other parameters fixed. Lines correspond to parameters $\{\phi, \xi, \pi, \pi_I\}$. Baselines are normalized to zero; axes report percent changes. For comparability, y-limits are held at $\pm 10\%$ across panels. Shaded panels (acquisition-to-success ratio, mean success multiple, share of acquisitions with multiple $> 10\times$) are those most informative for separating $\{p_I, \pi, \xi, \phi\}$. Moments are computed from simulated data under the estimated baseline, with 1,000,000 simulated firms.

C.0.2 Robustness and extensions

Parameter	US, 2005-2015 (C7)		US, 2005-2010 (C7)		US, 2005-2010 (C12)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
ρ	0.08	—	0.08	—	0.08	—
κ	0.26	0.01	0.25	0.03	0.26	0.02
λ	0.62	0.04	0.71	0.10	0.64	0.15
p_e	0.23	0.01	0.18	0.01	0.18	0.01
k_e	4.16	0.18	3.81	0.35	3.79	0.23
ν_e	2.54	0.13	2.68	0.25	2.75	0.18
p_l	0.30	0.01	0.21	0.01	0.24	0.01
k_l	16.15	0.25	11.44	0.66	11.81	1.06
ν_l	5.20	0.26	5.87	0.63	6.70	0.66
π	218.18	15.40	223.42	32.07	220.95	29.39
ϕ	0.65	0.07	0.46	0.12	0.44	0.10
ξ	0.74	0.08	0.98	0.13	1.01	0.20
$\tilde{\sigma}$	2.68	0.37	1.60	1.00	2.80	1.11

Table C.3: Parameter estimates for the US under alternative sample periods and censoring

The table reports parameter estimates from the model in section 2.4. “US, 2005-2015 (C7)” replicates the US parameter estimates under the baseline sample period and seven-year censoring (see Table 2). “US, 2005-2010 (C7)” and “US, 2005-2010 (C12)” report estimates for the 2005-2010 subsample (firms first funded between 2005-2010), estimated under seven-year and twelve-year censoring, respectively. To maintain consistency, all estimations use a simulated sample of 220,600 firms ($S = 20$ times the size of the 2005-2015 empirical sample). Parameters (except $\tilde{\sigma}$) are estimated by minimising (25) (equal weighting of centred percentage deviations; see footnote 83) and I report asymptotic standard errors, where the covariance matrix of moment conditions is computed from 1,000 bootstrap samples, clustered at the firm level. With $S \geq 20$, simulation noise is minuscule, so I report standard errors that simply reflect sampling variability. To avoid local minima in the estimation, I initialise a simulated annealing algorithm with 200 (randomly selected) starting points for each estimation. With the resulting candidate parameter estimates, I run a local minimisation (Nelder-Mead) using the results from the first step as initial guesses. With these estimates, the entry cost parameter, $\tilde{\sigma}$, is recovered from the condition determining the flow rate of first-time funding rounds. To compute its standard error, I take 1,000 samples from the asymptotic distribution of parameter estimates, solve the model, and recover the implied value for $\tilde{\sigma}$. The standard error is the standard deviation of $\tilde{\sigma}$ among this sample. All amounts are in 2015 USD.

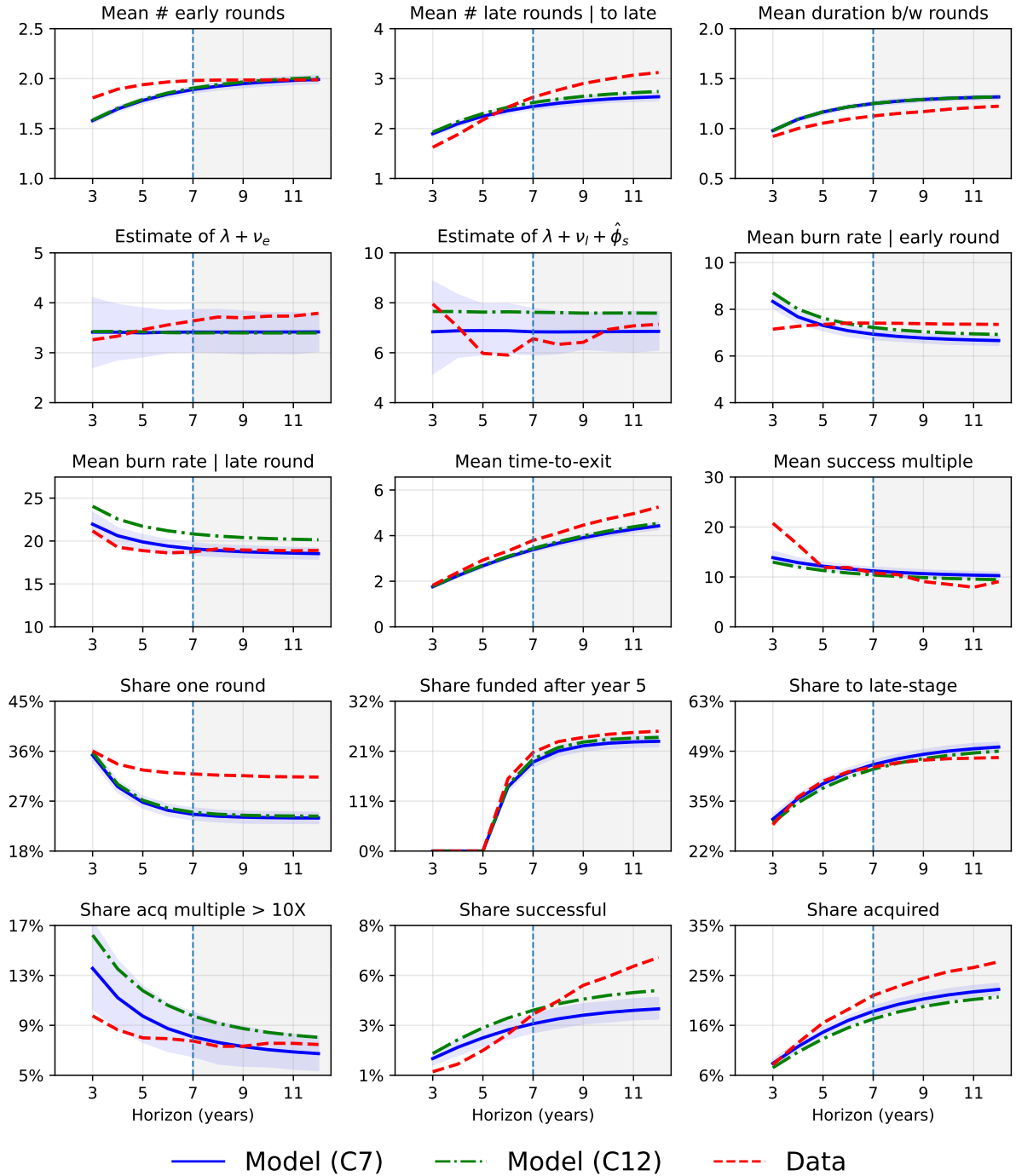


Figure C.2: Out-of-horizon censoring diagnostics (2005-2010 sample)

The figure plots moments computed on data censored at horizon h (x-axis) using the 2005–2010 sample (red, “Data”). Targeted moments and the share of acquisitions and successes are shown; the targeted acquisition-to-success ratio – the ratio of these two measure – is redundant and so excluded. Blue (“Model (C7)”) and green (“Model (C12)”) show model-implied moments when parameters are fixed at the estimates from the 7-year and 12-year censoring specifications, respectively; see Table C.3. For the empirical series, the data *are* re-censored at each h and moments recomputed. For the model series, I simulate 200 samples of the same size as the 2005–2010 data ($N = 5137$) and compute moments after censoring at each h . The blue band is a pointwise 95% simulation interval (2.5–97.5 percent quantiles) and reflects finite-sample variability under the model; parameter uncertainty is not included. The vertical line marks $h = 7$; the shaded region indicates out of sample outcomes with respect to the C7 specification.

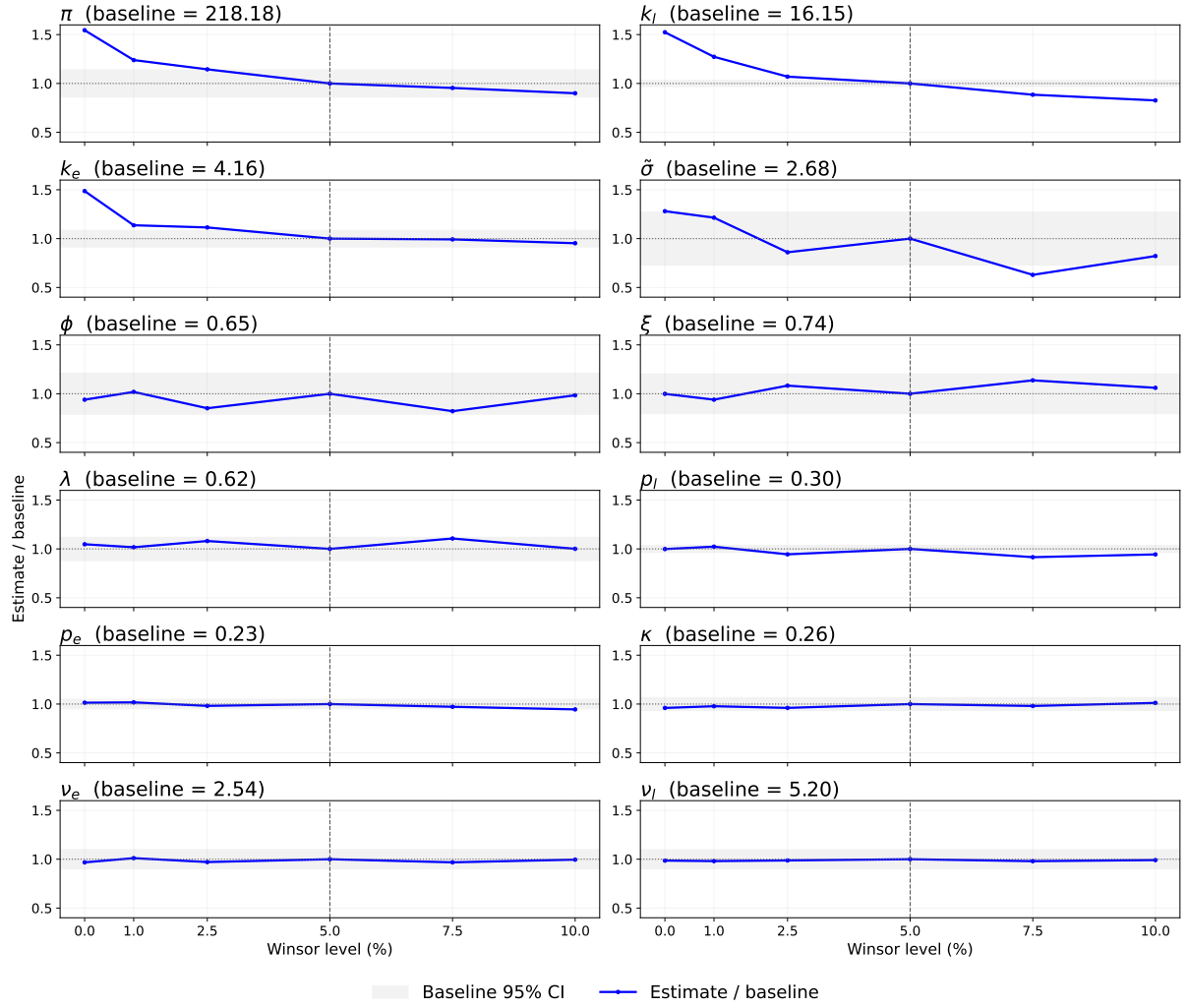


Figure C.3: Parameter estimates under different burn rate winsor thresholds

The figure plots parameter estimates normalised to 1.00 at $w = 5\%$ (baseline), under winsorising thresholds $w = 0\%, 1\%, 2.5\%, 5\%, 7.5\%, 10\%$. Gray bands show 95% confidence intervals for baseline parameter estimates; standard errors are reported in Table 2. Parameters are ordered by maximum percentage deviation from baseline. In each estimation, the mean burn rate is computed after winsorising the burn rate distribution (within stage); the same procedure is applied when computing burn rates from simulated data used in the estimation.

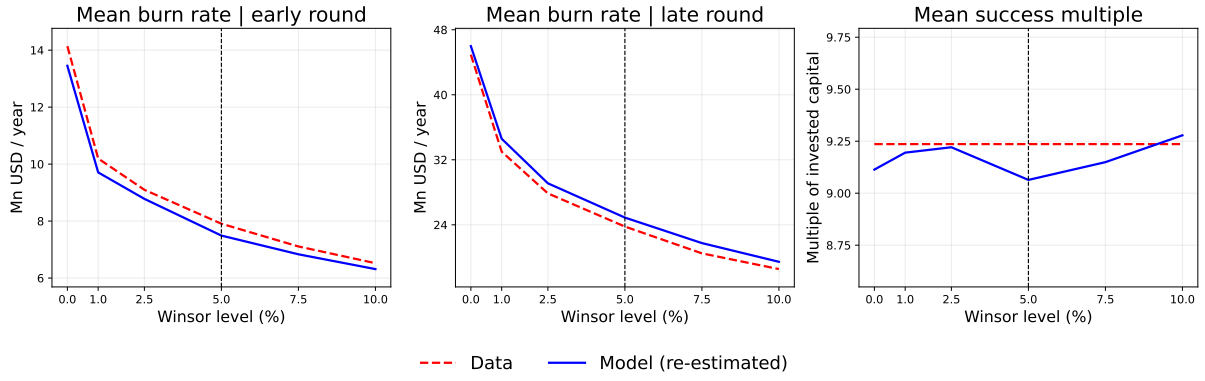


Figure C.4: Model fit under under different burn rate winsor thresholds

The figure reports empirical and model moments under different burn rate winsor thresholds. The model fits well across thresholds; the focus on mean burn rates and the success multiple reflects that movements in k_e , k_l , and π (particularly relevant to these moments) are the largest among all parameters when the winsor threshold is varied.

C.1 Additional details on UK estimation

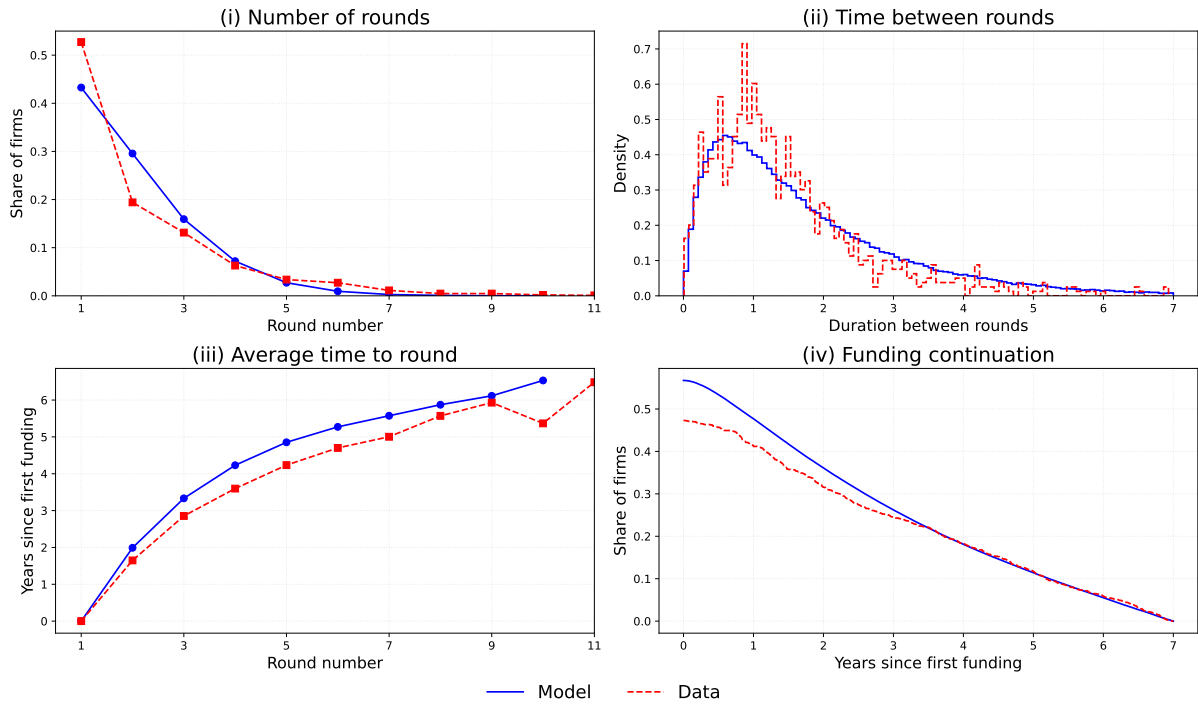
Table C.4 reports targeted and untargeted moments for the UK estimation; the UK parallel to Table 3. Figure C.5 shows the fit of the model to the data across a wider range of outcomes; the UK parallel to Figure 5.

Panel A: Targeted moments		
Moment	Data	Model
Mean # early rounds	1.44	1.44
Mean # late rounds progressed to late	1.94	1.69
Share one round (%)	52.72	43.15
Mean duration b/w rounds (years)	1.46	1.72
Estimate of $\lambda + \nu_e$	2.13	1.90
Estimate of $\lambda + \nu_l + \hat{\phi}_s$	2.99	3.31
Share receiving funding after year 5 (%)	11.73	11.52
Mean burn rate early round (\$Mn/year)	5.80	5.39
Mean burn rate late round (\$Mn/year)	18.79	19.43
Share to late-stage (%)	33.11	34.03
Mean time-to-exit (years)	3.82	3.32
Acquisition-to-success ratio	3.73	3.68
Mean exit multiple success	8.88	9.01
Share acquisition multiples > 10X (%)	11.34	11.54
# first funding rounds	0.48	0.48
Panel B: Untargeted moments		
Share successful (%)	2.44	3.46
Share acquired (%)	9.10	12.73
Mean time-to-success (years)	3.96	3.48
Mean time-to-acquisition (years)	3.78	3.27
Mean exit multiple	5.37	6.10

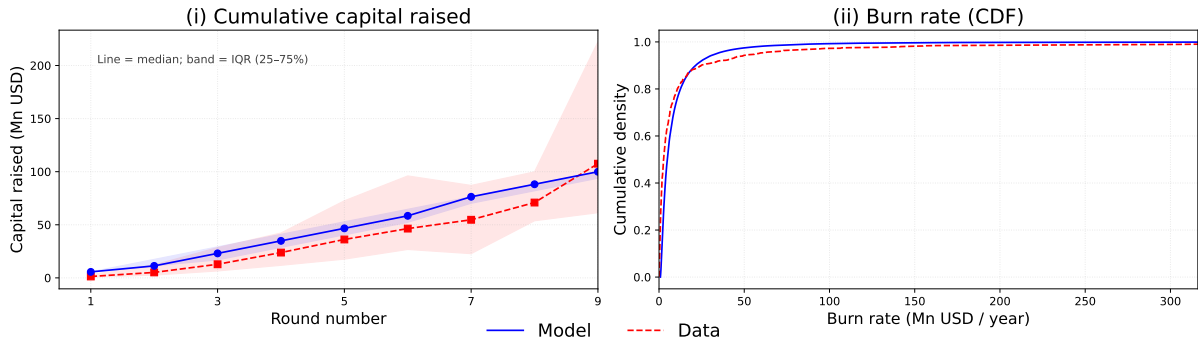
Table C.4: Targeted and untargeted moments

Panel A shows 15 moments (including two auxiliary parameter estimates) used in the estimation procedure. Empirical moments are computed from the sample of 1,066 firms. Other than for the estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, simulated moments are computed using the parameter values given in Table 2 to simulate a sample of 220,600 firms; preserving the same simulated sample size as for the US estimation, for consistency. Note that For $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, the parameter values are taken directly to reduce the computational burden of the estimation procedure. In computing the empirical and simulated moments, data on a given firm is censored seven years after its first funding round. Panel B shows a set of untargeted moments.

Panel A: Funding rounds



Panel B: Capital intensity



Panel C: Exits

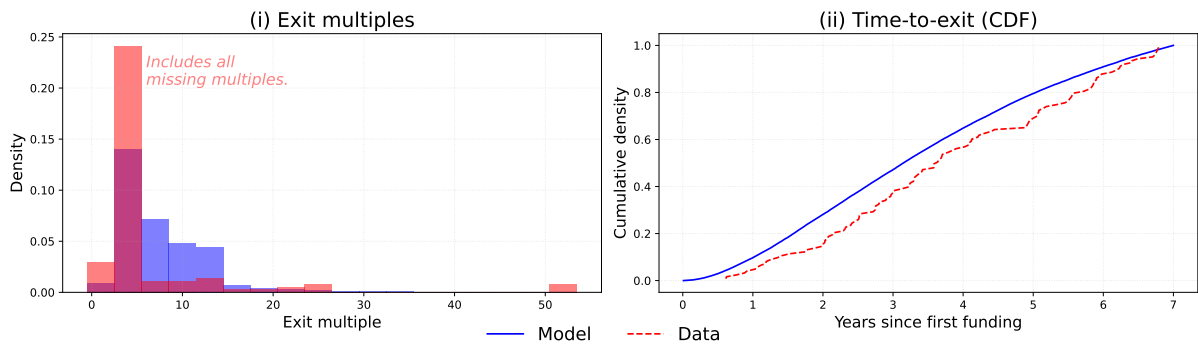


Figure C.5: UK Model validation

The figure shows outcomes in the model and data across funding rounds (panel A), capital intensity (panel B) and exits (panel C). All empirical and simulated data is censored at seven years following a firm's first funding round. "Funding continuation" in Panel A (iv) reports the share of firms that raise capital more than x years after their first funding round. To aid visualisation, cumulative capital raised is shown up to the 9th round; the burn rate CDF is reported up to the 99th percentile; and exit multiples are winsorised at 50X. Missing empirical exit multiples are set to 1.5X, following Kerr et al. (2014).

C.2 Additional details on sector calibration

C.2.1 Calibration strategy

This section provides details of the calibration exercise in section 5.

Accounting for heterogeneity in the model I define a technology, indexed by τ , as a bundle of start-up characteristics. There is a pool of potential entrants for each technology, so that multiple start-ups may pursue the same technology, τ . However, to account for the fact that some technologies may be more difficult to pursue than others, I allow the entry cost, $\tilde{\sigma}$, to be technology specific, i.e. $\tilde{\sigma}(\tau)$.

In the calibration exercise, I allow for two margins of heterogeneity: the rate project development horizon, parameterised by κ , and the availability of acquisition opportunities, parameterised by ϕ . Given my focus on the interaction of project characteristics with financial frictions, I also adjust $\pi(\tau)$ such that all projects have the same entry value, $V_{s,e}$, equal to the value that emerges from estimation of the homogenous firm model in section 3. In practice, this means that for each technology τ with characteristics $(\kappa(\tau), \phi(\tau))$, I solve a fixed point for $\pi(\tau)$ such that $V_{s,e}(\tau) = V_{s,e}$. To summarise, in this section a ‘technology’ refers to a $(\kappa(\tau), \phi(\tau))$ -pair with associated payoff $\pi(\tau)$. Otherwise, all technologies have the same characteristics and face the same funding market conditions. These parameters are set at the values from the US baseline estimation.

Estimating the distribution of technologies I estimate the underlying distribution of start-ups across technologies, which I denote by $s(\tau)$, by targetting cross-sector variation in the data. In order to implement the estimation of $s(\tau)$, I need to introduce some additional notation. I denote the sectors, as depicted in Figure 10, by v (for VEIC) and denote the number of sectors by N_v . Associated with each sector is a weight, $s(v)$, and a point in the (acquisition-to-success, time-to-exit)-space, which I denote by \mathbf{x}_v , to simplify notation. Together, $(\mathbf{x}_v, s(v))$ defines the empirical joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space. I then construct a set of technologies, τ , by specifying a grid for κ and ϕ , where the total number of technologies is N_τ .¹⁴³ For each technology, I simulate the model to recover a point in the (acquisition-to-success, time-to-exit)-space, which I refer to as \mathbf{x}_τ . My objective is to choose the shares (or weights), $s(\tau)$, to minimise the distance between the empirical joint distribution, $(\mathbf{x}_v, s(v))$, and the model-implied joint distribution, $(\mathbf{x}_\tau, s(\tau))$. In order to compare the distributions, I need to select a measure of distance, of which there are many. I choose to adopt the energy distance metric, which is often used to test for the equality of distributions. Specifically, I select shares $s(\tau)$ to minimize the following distance metric

$$\min_{s(\tau)} \left\{ 2 \sum_{v=1}^{N_v} \sum_{\tau=1}^{N_\tau} s(v)s(\tau) \|\mathbf{x}_v - \mathbf{x}_\tau\| - \sum_{v=1}^{N_v} \sum_{v'=1}^{N_v} s(v)s(v') \|\mathbf{x}_v - \mathbf{x}_{v'}\| - \sum_{\tau=1}^{N_\tau} \sum_{\tau'=1}^{N_\tau} s(\tau)s(\tau') \|\mathbf{x}_\tau - \mathbf{x}_{\tau'}\| \right\} \quad (\text{C.2})$$

subject to $\sum_\tau s(\tau) = 1$ and $s(\tau) \in [0, 1]$, where $\|\cdot\|$ is the Euclidean norm. The energy distance metric minimises the distance between the joint distributions of empirical sectors and model-based technologies, taking account for the location and shape. Figure C.6 plots kernel density

¹⁴³In practice, I experiment with different grid spacings and grid points and choose a sufficiently granular grid such that adding additional grid points has negligible effects on the results.

estimates given the sector weights, $s(v)$, and estimated weights, $s(\tau)$, based on minimisation (C.2).

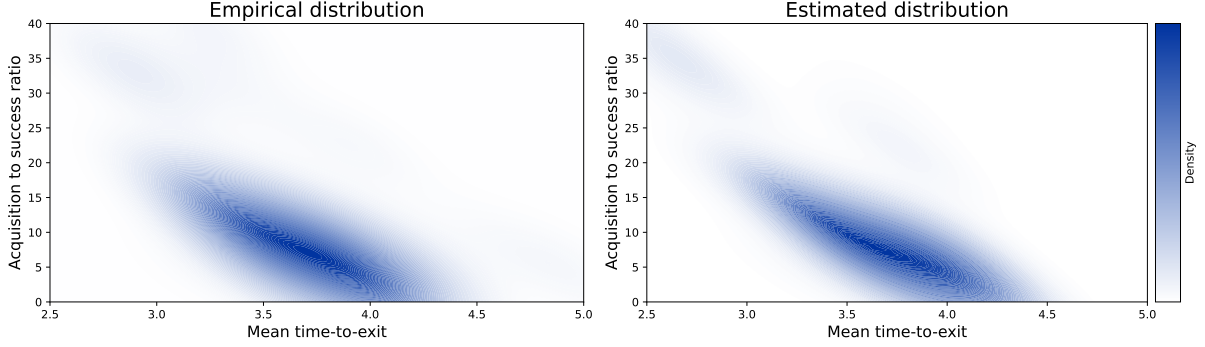


Figure C.6: Empirical and model-implied distributions

The left-side figure depicts kernel density estimates for the true joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space, which reflects the data shown in Figure 10. The right-side figure shows the kernel density estimate for the distribution of technologies in the same space, as estimated through minimisation of equation (C.2).

Recovering the counterfactual distribution of technologies Given the estimated technology shares, $s(\tau)$, it is simple to recover the frictionless shares, which I label $\bar{s}(\tau)$. In particular, the estimated entry shares satisfy equation (26) and the counterfactual entry shares satisfy equation (27). Given the estimates, $s(\tau)$, the technology-specific entry cost parameters, $\tilde{\sigma}(\tau)$, can be recovered up to a constant from (26). Then, it is simple to recover the counterfactual shares from equation (27). Doing so requires the frictionless entry value, $\bar{V}_{s,e}(\tau)$; that is, the entry value under self-financing. For a given technology τ , the frictionless entry value, $\bar{V}_{s,e}(\tau)$, is the solution to the following system of equations given in section A.5.1. Solving this system of equations for $\bar{V}_{s,e}(\tau)$ and substituting into equation (27) yields the counterfactual shares, $\bar{s}(\tau)$.

Mapping from technologies to sectors Finally, I map technologies into the ten aggregate sectors shown in Figure 10. Specifically, I allocate a portion $w(\tau, v)$ of the share of technology τ , $s(\tau)$, to each sector v by matching technologies and sectors in the (acquisition-to-success, time-to-exit)-space; the weights $w(\tau, v)$ then imply a mapping from technologies to aggregate sectors, since each sector v belongs to one aggregate sector. Using these weights and the frictionless technology shares, $\bar{s}(\tau)$, the final step of the counterfactual recomputes the aggregate sector shares in the counterfactual economy without financing frictions.

To implement the assignment between technologies and sectors, I set up a simple optimal transport problem as follows. I have two datasets: (i) an empirical dataset of sectors, $\{\mathbf{x}_v, s(v)\}_{v=1}^{N_v}$, and (ii) a model dataset of technologies $\{\mathbf{x}_\tau, s(\tau)\}_{\tau=1}^{N_\tau}$. The objective is create a mapping from technologies in the model dataset to VEIC sectors in the empirical dataset and, intuitively, I want to map technology τ to sector v if they are close in the (acquisition-to-success, time-to-success)-space, which is denoted by \mathbf{x} . The optimal transport problem selects $w(\tau, v)$

to solve the following problem

$$\min_{\{w(\tau,v)\}} \left\{ \sum_{\tau=1}^{N_\tau} \sum_{v=1}^{N_v} s(\tau) w(\tau, v) c(\tau, v) \right\}$$

subject to

$$\text{Supply constraints: } \sum_{v=1}^{N_v} w(\tau, v) = 1, \quad \forall \tau$$

$$\text{Demand constraints: } \sum_{\tau=1}^{N_\tau} s(\tau) w(\tau, v) = s(v), \quad \forall v$$

$$\text{Non-negativity constraints: } w(\tau, v) \geq 0, \quad \forall \tau, v$$

where $c(\tau, v)$ is the transportation cost from technology τ to sector v , which I specify as the Euclidean distance in the (acquisition-to-success, time-to-success)-space, $c(\tau, v) = \|\mathbf{x}_\tau - \mathbf{x}_v\|_2$.

Briefly, a total mass $s(\tau)w(\tau, v)$ is transported from technology τ to sector v , which has cost $c(\tau, v)$ per unit. The total transportation costs is sums over all transportations. The supply constraint states that all mass from technology τ to sector v is transported somewhere. I define $w(\tau, v)$ as the weight on share of mass from τ that is associated with sector v , so these sum to one. The demand constraint ensures that all sectors are completely accounted for. The optimal transport plan minimises the total transport cost subject to these ‘summing-up’ constraints.

C.2.2 Additional tables and figures for the sector calibration

Sector	Observed	Counterfactual	Δ
Biotech	0.09	0.13	+0.044
Medical/Health	0.10	0.13	+0.029
Semiconductors/Other Elect	0.03	0.04	+0.008
Consumer Related	0.03	0.03	+0.004
Industrial/Energy	0.05	0.05	+0.002
Internet Specific	0.28	0.24	-0.044
Computer Software and Services	0.33	0.29	-0.037
Other Products	0.04	0.04	-0.003
Comms and Media	0.03	0.02	-0.002
Computer Hardware	0.03	0.03	-0.001

Table C.5: Sector reallocation

The table shows observed sector weights and counterfactual sector weights under self-financing. Counterfactual shares are based on the calibration exercise in section 5.1. Heterogeneity in the (κ, ϕ) -space is first calibrated using cross-sector variation in average time-to-exit and the acquisition-to-success ratio. ‘Technologies’ in the model are then mapped to VEIC sectors based on proximity in the (acquisition-to-success, time-to-exit)-space via optimal transport.

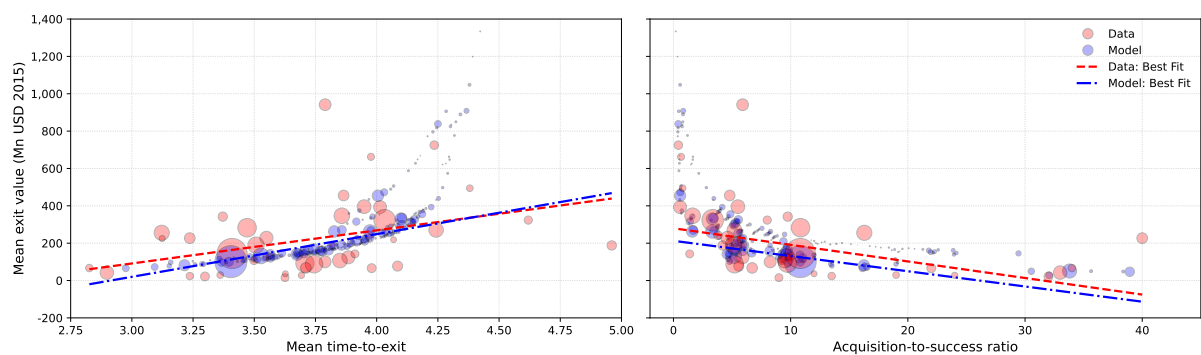


Figure C.7: Exit values

The figure plots the mean exit value against the mean time-to-exit and acquisition-to-success ratio for VEIC sectors (red) and model-based technologies (blue). Missing empirical exit values are populated to yield a 1.5X exit multiple, following Kerr et al. (2014). The size of each point is proportional to the sector/technology's share; lines are of best fit are weighted accordingly.