Start-up Financing, Entry and Innovation

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Abstract

I develop a tractable equilibrium model of start-up development to examine and quantify the implications of frictions in access to venture capital (VC) funding. The model features endogenous entry, search-and-matching dynamics between start-ups and VCs and endogenous financial contracts subject to agency conflicts. A key feature of the model is that firms may fail due to financing risk—the inability to secure follow-on funding for otherwise viable firms. I estimate the model on US micro data and show that it captures a rich set of empirical moments. My estimates suggest that a substantial share (24%) of US VC-backed start-ups fail due to financing risk and that a significant share (11%) of US VC funding is misallocated to start-ups that should have closed down. To explore the implications of financing risk in other settings, I conduct a case study of the UK. I find that limited funding and acquisition opportunities increase the likelihood of failure by more than 15 percentage points for UK start-ups relative to their US counterparts. Finally, I study how financing risk biases VC funding away from long-term projects with fewer acquisition opportunities, estimating that in its absence, the share of VC-backed start-ups in software and services would fall from 61% to 46%, offset by growth in science-driven sectors.

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1 Introduction

In the United States, venture capital (VC) plays a foundational role in the growth of some of the largest and most innovative companies. Among public companies founded in the past fifty years, VC-backed firms account for the vast majority of market value, R&D spending, and patent stocks by value (Gornall and Strebulaev, 2021). Despite representing a small share of the financial system, VC is a critical gateway for innovation. This central role means that frictions in VC markets can have outsized consequences for the broader economy, with the potential to create bottlenecks for innovation or systematically skew investment towards certain projects.

Existing theories of venture capital markets address matching frictions between entrepreneurs and investors, as well as agency frictions within their relationships. However, these theories often ignore implications of staged-financing, the primary control mechanism adopted by VCs (Sahlman, 1990).[1] Staged financing, in which capital is provided incrementally rather than upfront, mitigates agency frictions by reducing the VC's exposure to potential exploitation by the entrepreneur. Yet, to be effective, staging implies the lack of a firm commitment to follow-on funding, leaving the entrepreneur vulnerable to the state of the financing market when they attempt to raise capital again. As a result, entrepreneurs and investors are concerned about 'financing risk'–the possibility that viable start-ups may fail to secure future funding (Nanda and Rhodes-Kropf, 2017).[2]

I develop a tractable equilibrium model of start-up development in which staged financing and financing risk arise endogenously, and use it to study and quantify the implications of financing frictions for the scale and direction of start-up innovation.[3] Estimating the model for the US, I show that it captures rich features of VC data and provide new estimates of the extent and cost of financing risk. The analysis reveals that 24% of VC-backed start-ups in the US fail due to financing risk and 11% of capital is misallocated to start-ups that should have closed down. I then use the model to consider two distinct issues, which currently lack quantitative analyses. In a case study of the UK, I show that limited funding and acquisition opportunities make UK start-ups more than twice as vulnerable to financing risk than their US counterparts, increasing their failure likelihood by over 15 percentage points. I also explore how financing risk biases innovation towards short-term projects, particularly those lacking established exit pathways. Without financing frictions, the share of VC-backed start-ups in 'Computer Software and Services' and 'Internet Specific' sectors, both of which are dominated by software and service firms, would fall from 61% to 46%, offset by growth in science-driven sectors. These findings provide quantitative support for concerns that VC's narrow sectoral focus may lead it to overlook transformative innovations (Lerner and Nanda, 2020).

Consider an entrepreneur in possession of a risky project but no wealth. To develop the project, the entrepreneur requires external financing from a VC, but finding and contracting with VCs is subject to frictions (Hall and Lerner, 2010). Suppose the entrepreneur successfully secures investment from

[1] One exception is Nanda and Rhodes-Kropf (2017). There is a large literature that motivates the use of staged-financing by VCs; Gompers (1995), Bergemann and Hege (1998) and Neher (1999) are key references.

[2] Nanda and Rhodes-Kropf (2017) introduce the concept of financing risk to the literature and show how it can arise endogenously due to coordination issues between investors across funding rounds. In the discussion of related literature below, I expand on the key insights of their paper and discuss my contribution in light of their work.

[3] Existing theories highlight search and matching frictions in the VC market (Inderst and Müller, 2004; Michelacci and Suarez, 2004) and agency frictions in contracting (see, for example, Jovanovic and Szentes (2013)). In the macroeconomic growth literature, a related paper is Greenwood et al. (2022), who study the growth implications of contracting frictions. However, in their model, all viable start-ups receive funding, and so financing risk is not present.

a VC. Both parties recognise at the outset that the project is high risk and may fail due to unforeseen technological or commercial issues. With the VC's funds, the entrepreneur engages in a process of experimentation to learn about the project's viability, ultimately gaining more information about its prospects than the VC. If the entrepreneur learns negative information, the VC might prefer to terminate the project and recover unused capital. Yet, in practice, the entrepreneur could continue development by concealing the negative information from the VC, leading to the problem of "inefficient continuation" (Gompers, 1995). This leads VCs to be cautious of committing too much capital to a project upfront, instead preferring to infuse capital in tranches, a process known as *staged-financing*.

In the first part of the paper, I introduce a principal-agent problem that reflects these dynamics and show how it raises the cost of investing in a start-up.[4] The key result is that for every \$$1 + x$ invested, the VC expects only \$1 to be used productively by the entrepreneur, creating a *wedge*, x, which raises the cost of capital above the frictionless benchmark. This wedge depends on three factors: the project's riskiness, which determines the likelihood that the entrepreneur learns negative information about the project; the size of the initial capital injection; and the project's development horizon, both of which influence how much capital remains unused at the time any negative information arises. To reduce the cost of capital, the VC and start-up are incentivised to limit the size of the capital injections–engaging in staged-financing.

I embed this principal-agent problem into an equilibrium search and matching framework. This serves two purposes. Firstly, it captures the notion that finding a VC that is willing to invest is challenging due to heterogeneity on both sides of the market.[5] The difficulty in valuing early-stage companies also means that VCs may pass on even the best investments.[6] Consequently, the market takes time to clear and congestion effects may be important (Inderst and Müller, 2004). Secondly, it allows me to capture coordination frictions among VCs across funding rounds tractably. Specifically, I assume that the start-up must search for a new VC each time it raises capital. Most start-ups secure follow-on funding relatively quickly after exhausting their previous funding, but some may struggle to do so for a longer period. During their search, they may fail due to a lack of funding, and so are subject to financing risk.[7] This implies that while start-ups in the model are identical at the outset, their outcomes diverge over time—some succeed, while others fail due to either the inherent shortcomings of

[4] In the environment I consider, the entrepreneur derives private benefits, which leads them to conceal the realisation of negative news from the VC to prevent closure. The relationship between private benefits and inefficient continuation is discussed in Gompers (1995). In their extension to incomplete markets, Nanda and Rhodes-Kropf (2017) make a similar assumption.

[5] Due to their role as providers of "informed capital" (Michelacci and Suarez, 2004), characteristics of the VC and start-up are likely to be important in determining investment returns. In practice, VCs are differentiated by experience (Sørensen, 2007), access to networks (Hochberg et al., 2007), degree of sectoral specialisation (Gompers et al., 2009) and location (Lerner, 1995). Similarly, start-ups vary along several dimensions. The ability to seek external funding can differ across gender and race (Ewens, 2023). Furthermore, in choosing investments, VCs are particularly concerned with the make-up of the founding team, the business model and the market (Gompers et al., 2020). Obtaining investment requires seeking out an investor that is a good fit across several dimensions and an ability to stand out in a crowded market.

[6] An often-cited indication of this is the list of companies that Bessemer Venture Partners, a leading US venture capital firm, chose not to invest in–a list that includes Apple, Intel and Google (see www.bvp.com/anti-portfolio).

[7] Exposure to financing risk depends on the relative demand and supply for capital (market tightness), which is consistent with the view that financing risk is most salient when the supply of VC funding is limited (Janeway et al., 2021). In Nanda and Rhodes-Kropf (2017), coordination issues among investors are modelled using a public signal (a "sunspot"), so that the capital market is either "hot" or "cold" in a way that is devoid of market fundamentals. In reality, these coordination failures are likely a key driver of capital supply, which I do not consider explicitly in this paper. However, by modelling the market using a search and matching framework, I am able to relate the level of capital supply to the extent of financing risk in the market, rather than have financing risk be a binary measure.

their projects or an inability to secure follow-on funding.

The optimal contract in this environment balances the demand for insurance against financing risk—favouring larger capital injections to reduce future reliance on the capital market—against the increased cost of funds that larger injections entail. I analyse the implications of project characteristics and funding market conditions for start-up funding patterns–such as the number of funding rounds and duration between them–and start-up outcomes (success or failure). In the model, a firm that opts to visit the funding market more frequently is more likely to fail due to financing risk. For instance, despite the motivation to match funding duration to project milestones, projects with longer development horizons typically need to visit the capital market more frequently and so are more adversely affected. These results have implications for the effects of frictions on the direction of start-up innovation.[8]

To close the theoretical analysis, I consider market efficiency. Financing risk leads start-ups to secure more capital relative to their annual requirements, thereby increasing the likelihood of inefficient continuation. Inefficient continuation results in capital misallocation: some start-ups continue receiving funding despite their poor prospects, while others with viable projects are actively seeking funding. This misallocation is more severe in less-developed capital markets, where financing risk is a greater concern. Importantly, because misallocation reflects the cost of insuring against financing risk, estimates of its extent provide a metric for measuring the social cost of financing risk for start-ups.[9]

In the second part of the paper, I estimate the model using VC funding round and exit data for all US start-ups that received their first early-stage funding round between 2005 and 2015. The estimation validates the model and quantifies the extent and cost of financing risk for US start-ups. To make realistic quantitative statements while retaining the core economics, I make two simple adjustments to the baseline model. Firstly, I include a distinct role for acquisitions, which act as a substitute for VC financing in the model. Most start-ups exits occur through acquisitions, making their inclusion essential for quantifying financing risk. Secondly, I introduce multiple hurdles in a start-up's journey to full development. This adjustment allows me to capture the differences in how earlier-stage and later-stage start-ups access funding, reflecting the greater information investors have as start-ups progress.[10]

The model is able to capture complex features of the data, including the full shape of the distributions of the number of funding rounds, the duration between rounds, exit multiples, the burn rate (i.e. rate of capital utilisation) and time-to-exit dynamics. These features emerge naturally from the model's structure, rather than from estimating any flexible functional forms. Therefore, my paper contributes to the literature that attempts to explain features of venture capital funding patterns (see, for example, Gompers 1995; Neher 1999; Bergemann et al. 2009; Dahiya and Ray 2012, among others). A key insight is that observed differences in funding patterns and outcomes across firms can be explained by shocks and information revealed after initial funding, even for start-ups that appear identical at the outset.

[8] Nanda and Rhodes-Kropf (2013) show that start-ups funded in "hot" markets are less likely to succeed but, conditional on success, have higher valuations, more patents and more citations to their patents. They argue that this is consistent with more novel innovations facing greater costs of insuring against financing risk, as in (Nanda and Rhodes-Kropf, 2017). The same mechanism is present in my paper. In contrast to their work, I highlight the additional interaction between a project's development horizon and financing risk and provide estimates of the extent to which financing risk biases VC funding away from longer-horizon projects.

[9] Capital misallocation is related to the wedge, x, because this pins down the share of capital squandered by the average entrepreneur. Furthermore, since the agency friction incentivises a contract in which $x \to 0$, whereas insurance against financing risk induces greater capital commitments, leading to $x > 0$, misallocation reflects the costs of insurance against financing risk.

[10] As start-ups progress, their longer history implies that investors have greater information with which to evaluate them. This should reduce the difficulty with which viable start-ups secure funding.

This uncertainty highlights a core challenge for VCs: identifying which start-ups in their portfolios will succeed and which will not (Kerr et al., 2014).

Using the estimated model, I quantify the extent and cost of financing risk in the US VC market. My estimates suggest that 24% of VC-backed start-ups that had their first funding round between 2005-2015 failed due to financing risk, which corresponds to approximately one-third of all start-up failures. Notably, most of these failures occurred at the early stage, providing direct evidence for the conjecture that financing risk disproportionately impacts early-stage start-ups (Nanda and Rhodes-Kropf, 2017). Over the same period, I estimate that 11% of VC funding was misallocated, reflecting the cost borne by VCs and start-ups to insure against this risk. These findings highlight significant inefficiencies and suggest that reducing frictions in the VC market could yield substantial welfare gains.

In the final part of the paper, I apply the model in two distinct contexts to further explore its implications. Despite VC's prominence in the US, other countries have typically been unable to replicate the American experience (Gornall and Strebulaev, 2021). To explore the nature of VC in other regions, I focus on the UK, a country with comparable legal and financial institutions, a historically prominent financial centre, and high-quality universities that often underpin entrepreneurial ecosystems (Nicholas, 2019). Yet, on a per-capita basis, fewer start-ups in the UK access VC funding, and those that do are less likely to achieve a successful exit. Disentangling the drivers of these disparities is challenging because VC activity and start-up outcomes reflect the equilibrium between capital supply and demand. By modelling both start-up outcomes and observable features of VC contracts, the model distinguishes between explanations such as the quality and supply of entrepreneurial projects, funding market conditions, and the availability of exit opportunities. My analysis points to differences in financing conditions and limited acquisition opportunities, rather than project quality, as key drivers of the observed disparities. Together, my estimates suggest that roughly two-thirds of UK start-up failures relate to an inability to secure follow-on funding, rather than technical or commercial challenges. This aligns with the empirical findings of Wilson et al. (2018), who argue that start-ups in the UK face significant difficulties in accessing follow-on funding.

Finally, I investigate how financing risk biases innovation towards projects where uncertainty can be resolved quickly, and away from longer-term, potentially transformative technologies. Venture capital typically funds start-ups in narrowly defined sectors, raising concerns that projects with broad societal benefits—from renewable energy to advanced materials technologies—may remain unfunded (Lerner and Nanda, 2020). While Lerner and Nanda (2020) highlight these challenges as critical issues for the VC market, their review does not provide a framework for quantifying the distortions.

To address this, I introduce ex-ante heterogeneity among start-ups in their development horizon and, motivated by data, the availability of acquisition opportunities, into the model. The model highlights that projects with long development horizons and limited acquisition opportunities are particularly vulnerable to financing risk because they typically require more funding rounds to develop their innovations and have fewer alternative exit routes. My estimates suggest that frictions in VC funding distort the sectoral composition of VC-backed start-ups, particularly favouring software and services businesses at the expense of start-ups pursuing science-based technologies. Specifically, in my sample start-ups in 'Computer Software and Services' and 'Internet Specific' sectors account for 64% of VC-backed start-ups, but I estimate that this would fall to 46% in the absence of frictions in access to capital. This shift would be offset by start-ups in biotech, medical and health applications, and semiconductors. The analysis demonstrates how financing risk can systematically distort the allocation of resources away from sectors that often underpin transformative innovation and highlights the need for targeted policy interventions.

Related literature. This paper is related to several strands of literature. Firstly, it relates to the literature on the financing of innovation. The majority of the literature focusses on the relationship between R&D investments and financing for mature firms, considering factors such as the importance of access to public equity markets (Acharya and Xu, 2017), the role of debt financing (Mann, 2018), and the different roles played by internal and external finance (Brown et al., 2009). A growing literature considers issues more specific to start-ups, where agency frictions are likely to be even more pronounced (Hall and Lerner, 2010).

My paper relates most closely to Nanda and Rhodes-Kropf (2017), who introduce the notion of financing risk to the literature.[11] In their paper, coordination issues among investors across funding rounds are modelled using a public signal (a "sunspot"), and the authors show that negative realisations can prevent positive NPV projects from receiving investment. They also consider the possibility that investors could commit more capital upfront to mitigate the need to coordinate with other investors. Their key result is that, due to inefficient continuation, the value of committing additional capital is lower for riskier projects, which makes them more vulnerable to the state of the financing market—a critical insight demonstrating how financing frictions affect the novelty of start-up innovation.

Like in their paper, I consider an incomplete markets setting and motivate inefficient continuation through private benefits. As in their analysis, the cost of providing additional funding is endogenously higher for riskier, or more novel, projects, meaning that their central insight is preserved in my framework. My paper also differs from and extends their work in several ways. First, my contract microfoundations generate the additional insight that the cost of supplying additional funding depends on the project's horizon—the timing of the signal about project quality. Second, while Nanda and Rhodes-Kropf explore incentives for upfront funding under financing risk, they do not derive the optimal contract in the incomplete markets setting. I address this gap, formalising their insights such as the need for riskier projects to raise more rounds of capital and extending them to characterise start-up outcomes by novelty and development horizon, among other characteristics. Third, I employ a search and matching framework that endogenises the extent of financing risk. Finally, I estimate the model to quantify the extent and cost of financing risk in the U.S. market and conduct several other exercises.

Second, this paper is related to the literature on staged-financing. Neher (1999) shows that staging capital injections can help to overcome a commitment problem in light of the potential to renegotiate contracts. Bergemann and Hege (1998) consider dynamic contracts in which capital is infused in stages. I do not consider dynamic contracts and assume that the VC-entrepreneur relationship only lasts for one funding round (although that funding round may be sufficient to cover all the start-ups future funding needs). I motivate the contract based on the insights of Gompers (1995), who attributes staged financing to agency frictions and tests the predictions of the theory on venture capital data. The friction I introduce leads to the problem of inefficient continuation, which is a key problem that staging is designed to address (Lerner, 1998). Relative to the existing literature, the main contribution

[11] In their model, a start-up conducts several experiments to commercialise its innovation, each lasting one period with a certain probability of success. Investors fund one period at a time and rely on others for future funding, so expectations about future funding availability affect the current investment's value by influencing the project's continuation value.

is to integrate the problem of inefficient continuation–microfounded through agency frictions–into a search and matching model that is able to capture rich features of venture capital funding data.

Third, this paper is related to theoretical models of the venture capital market. I follow Inderst and Müller (2004) and Michelacci and Suarez (2004) in adopting a search and matching model. Inderst and Müller (2004) and Jovanovic and Szentes (2013) also model agency frictions explicitly, but neither model features start-ups that engage in staged-financing and financing risk is not a concern. This means the models are unable to speak to data on the profile of financing rounds, such as the number of funding rounds and duration between them. In contrast, my model estimation explicitly targets these statistics, which I show can be used to identify key parameters. Using these estimates, I quantify the extent of frictions in VC markets and explore several related issues.

Fourth, this paper is related to a series of papers in the macroeconomics literature on venture capital and economic growth. For instance, Opp (2019), Akcigit et al. (2022), Greenwood et al. (2022) and Ando (2024) study the role that venture capital plays in fostering economic growth. Notably, Greenwood et al. (2022), who embed a long-term contract in the spirit of Bergemann and Hege (1998) into an endogenous growth model, also considers a cross-country comparison, focussing on the effect of tax rates and monitoring capabilities in explaining differences between France and the US. Relative to this literature, my paper highlights several factors as crucial to understanding the start-up development process and the close mapping to micro data provides support for my approach. First, I highlight the role that coordination issues between investors across funding rounds play in shaping start-up funding patterns and outcomes. These coordination issues generate financing risk, which does not feature in these models. Second and as a result of these coordination issues, the state of the financing market is an important determinant of optimal contracts in my setting. In fact, the key decision for agents in my model is how to trade-off the heightened agency frictions associated with large capital injections against concerns about future funding availability. This implies that funding market conditions matter for start-up outcomes, but also that start-up funding patterns encode information about the extent of agency frictions and the state of the funding market in which start-ups raise capital. Leveraging these insights, I provide a new perspective on the drivers of cross-country differences in VC activity. Third, my analysis highlights that frictions in VC markets matter for the sectoral and technological composition of investment.

Finally, my paper relates to a the literature on frictions in private-sector funding for R&D. Concerns over short-termism among public companies (see, for example, Terry 2023) leads naturally to considering the capacity of venture capital to fund radical innovations. To explore this, Narain (2024) constructs a measure of the delay from investment to innovation and shows that both VC-backed firms and public firms favour projects with shorter "gestation" periods relative to the distribution of commercial opportunities. Her explanation, based on the common ten-year fund structure adopted by most VCs, is complementary to the explanation based on financing risk explored in this paper. More generally, there are concerns that private-sector investment struggles to fund longer-term projects (Acemoglu, 2011; Budish et al., 2015) and my paper provides specific insights into this issue in the context of venture capital.

The remainder of the paper is organised as follows. In sections 2.1 and 2.2, I introduce the baseline model. Sections 2.3 and 2.4 study properties of this model and section 2.5 extends the model prior to estimation. Section 3 estimates the model for the US, section 4 considers a case study of the UK, and section 5 considers the direction of start-up innovation. Section 6 concludes.

2 Model

The model is in continuous time, the horizon is infinite, and I focus on the steady-state equilibrium. I begin by defining the environment of a single entrepreneur that contracts with a VC to fund their project. In this environment, I introduce an agency friction that affects the ability of the entrepreneur and VC to write a fully state-contingent contract and derive the implied cost of capital. I then embed this setting into an equilibrium search model, which features many start-ups and VCs. Subsequent sections then analyse the properties of the model.

2.1 Technology and financing

Start-up projects and VC contracts. Entrepreneurs (E) are risk neutral and discount the future at rate ρ. Consider a solo E that possesses a risky project that requires a continuous flow of investment, k, to fund its development. Conditional on paying k, the results of the development process are realised at Poisson arrival rate κ and are positive (success) with probability p and negative (failure) with probability $1-p$. Success results in a payoff π, whereas a failure returns zero. The parameters k, κ, p and π capture the project's exogenous characteristics.

The E has no wealth and must seek external funding from a VC to pay the flow cost k. The VC discounts the future at rate ρ and considers the size of the expected discounted capital commitment in negotiations. The E and VC sign an equity contract that determines the duration of funding that the VC provides to the E and an equity stake that the VC receives in return. Specifically, in return for a share, ς, of the value of firm gross of investment costs (i.e. post money), the VC agrees to fund the flow cost k for some period of time. The contract is designed so that the VC's commitment to fund the project expires at a Poisson rate ω and I refer to ω as the contract rate, which is a negotiable feature of the contract. This specification of the contract implies that, absent any future agreement to terminate funding, the VC provides funding to the E until time $T_\omega \sim Exp(\omega)$.[12] The relationship between ω and the expected discounted capital commitment, $K(\omega)$, is mediated through the agency friction, which I describe below.

Once funding is exhausted, the value of the E's project returns to $V_s \geq 0$, which is the value of a project held by an entrepreneur without funding and will be determined endogenously in the next section. The gross pecuniary value of the project with contract (ω, ς), excluding investment costs and denoted by V_d, solves the following HJB equation

$$\rho V_d = \kappa [p \pi - V_d] + \omega [V_s - V_d] \tag{1}$$

Agency friction. I assume that the arrival of the result, success or failure, is observable to the E but

[12] The problem has parallels to an optimal stopping problem–as in Jovanovic and Szentes (2013)–except the funding horizon is stochastic and determined via the hazard rate, ω, rather than being a fixed date. Modelling the contract in this way has a number of advantages. Firstly, *ex ante* homogenous firms with the same capital commitment may exhaust their funding at dates *ex post*, which captures the high degree of uncertainty in this environment. Secondly, when integrating this setup into the search equilibrium, it implies that all state-to-state transitions have constant hazard rates, so that the model features exponential holding times. The lifecycle of a start-up is then determined by a continuous time Markov chain, which facilitates both the theoretical analysis and model simulation.

not to the VC. In addition, the E derives some non-pecuniary benefits of entrepreneurship, $x_e > 0$, whenever the project is being funded. This could reflect social status or access to networks.

In the absence of asymmetric information and private benefits, the optimal state-contingent contract would involve the VC funding the investment cost k until time $T_\kappa \sim Exp(\kappa)$ and no longer. Given that funding could be conditioned to elapse when a result is obtained, this optimal contract can be implemented within the framework considered here by setting $\omega = 0$. In this case, funding is never withdrawn before the realisation of the result.

However, under the stated assumptions, it is not possible to write a contract that conditions the termination of VC funding on the arrival of a result, at date $T_\kappa \sim Exp(\kappa)$, because the E may find it profitable to withhold information about the status of the project. Specifically, suppose that the result has just been realised at time T_κ. If the E reports this information to the VC, funding is withdrawn. By choosing not to divulge this information, the E obtains the non-pecuniary benefit for an additional period $T_\omega \sim Exp(\omega)$, which has positive value since $x_e > 0$. Therefore, if the E learns of a negative result, they would never willingly inform the VC. This logic also applies to any time after the realisation of the result because the environment is stationary. The fact that the E squanders funds following the realisation of a negative news means that they engage in "inefficient continuation" (Gompers, 1995).

In order to determine the cost of a contract (ω, ς) to the VC, it is necessary to determine the E's actions in the case of a success, which is not so straightforward as the case of failure. If the E reports a success to the VC immediately, they forgo any additional non-pecuniary benefit but receive the pecuniary payoff immediately. Conversely, choosing not to report the success allows the E to continue to reap the non-pecuniary benefit, but means they must wait until the contract expires to receive the pecuniary payoff.[13] In other words, the E must decide whether the value of the private benefit is sufficient to compensate for the loss in their pecuniary payoff due to discounting, which leads to an incentive compatibility constraint.[14] Proposition A.1 in the Appendix shows that, for any set of model parameters, there always exists an upper bound $\bar{x}_e > 0$ such that if $x_e < \bar{x}_e$, then the E always reports positive results. Intuitively, if the non-pecuniary benefit is sufficiently low (in a relative sense), the E would never willingly delay the realisation of their payoff in order to continue to obtain the private benefit. Therefore, given $x_e < \bar{x}_e$, the E always reports successes to the VC, but never reports failures.

In what follows, I take the limiting case where $x_e \to 0$, so that the result of inefficient continuation is preserved and the E always reports positive results to the VC. This setting appears to accord with reality.[15] Under the assumption that $x_e \to 0$, the expected capital cost associated with the contract

[13]The environment is stationary, so if the E finds it profitable to delay reporting at the time the result is obtained, they will never report until the contract expires, which happens at rate ω.

[14]See Remark 3 in the Appendix. The IC depends on features of the project and environment, (π, ρ, x_e) but also on the share, $1 - \varsigma$, that the E maintains in the project. This leads to the implication that the E must maintain sufficient skin-in-the-game to report truthfully.

[15]See, for example, the discussion in Chapter 4 of Janeway (2018), "After more than forty years in the game, I have yet to meet the entrepreneur who dallies in delivering word that 'the product works' or 'the sale has closed'. When communication ceases, then the venture capitalist can expect to discover that 'the product needs another rev' or that 'we lost the order'." Janeway refers to this as his Second Law of Venture Capital, "No news is never good news".

(ω, ς) is given by[16]

$$K(\omega) = \underbrace{\frac{k}{\rho+\kappa+\omega}}_{\substack{\text{Frictionless expected}\\ \text{capital cost}}} + \underbrace{\frac{\kappa\,(1-p)}{\kappa+\omega}}_{\text{Pr(negative result)}} \underbrace{\frac{k\,(\kappa+\omega)}{(\rho+\omega)\,(\rho+\kappa+\omega)}}_{\substack{\text{Expected capital cost}\\ \text{from continuation}}} = \left(1+\frac{\kappa\,(1-p)}{\rho+\omega}\right)\frac{k}{\rho+\kappa+\omega}. \quad (2)$$

The expected discounted capital cost, $K(\omega)$, is composed of two terms. In a frictionless world, the E and VC could write a contract to terminate funding at the realisation of the result, date $T_\kappa \sim Exp(\kappa)$. The contract rate would be $\omega = 0$, so that funding is never exhausted prematurely, and the VC would expect to inject $k/(\rho+\kappa)$ into the start-up before the result is realised. The first term reflects this frictionless benchmark and is also the cost of capital that would arise if the E could invest their own funds, because such an E would not engage in inefficient continuation.[17] In practice, there is an additional term that represents the effect of the agency friction on the cost of investment. It is the product of two terms: (i) the probability that a negative result arrives before funding is exhausted; and (ii) the volume of capital wasted by the E following a negative result, appropriately discounted.

As shown in equation (2), the result is that the expected discounted capital cost takes the form of a wedge, $\frac{\kappa(1-p)}{\rho+\omega}$, over the frictionless capital cost. Put differently, this is the wedge between the internal and external cost of capital. The wedge captures the extent of agency friction and depends on three key factors beyond the discount rate. Firstly, a longer funding commitment (a reduction in ω) increases the wedge by raising the likelihood of encountering a negative outcome and, conditional on such an outcome, prolonging the period of inefficient continuation. Secondly, projects with a lower likelihood of success (a lower p) have a higher wedge because they are more likely to enter the state of inefficient continuation for a given contract rate, ω. This aligns with the key insight of Nanda and Rhodes-Kropf (2017): committing more funding becomes costlier for riskier, or more novel, projects. Finally, projects with quicker expected resolution of uncertainty (a higher κ) also face a higher wedge, given the contract rate, as they are more likely to generate an outcome. As I discuss in the coming sections, this provides a motivation to match the duration of funding to the specific milestone that the start-up is attempting to overcome.

2.2 A baseline equilibrium model

In this section, I embed the principal-agent problem of section 2.1 into an equilibrium search model in which the contract (ω, ς) is negotiated optimally. In the model, start-ups make an endogenous entry decision and must seek external funding. Start-ups that have paid the entry cost are *ex ante* homogeneous and are all in possession of a project whose prospects are inherently uncertain. As in Inderst and Müller (2004), the process of finding a VC that is willing to invest is subject to a search and matching friction. Conditional on meeting with a VC, the E and VC determine a contract (ω, ς) to fund the development process. The E then attempts to develop the project until either: (i) a result is obtained (at rate κ), or (ii) funding is exhausted (at rate ω).

If the start-up obtains a positive result, it receives a payoff π, which is split among the E and

[16]See section A.1 for a full derivation.

[17]Specifically, $\frac{k}{\rho+\kappa+\omega}$ is the expected discounted value of the capital injection prior to the realisation of the result or the withdrawal of funding. In the frictionless benchmark, funding would be withdrawn at rate $\omega = 0$, so that $\frac{k}{\rho+\kappa}$ is the expected cost of funding the start-up.

existing investors in proportion to their equity holdings in the firm.[18] In contrast, an E that receives a negative result conceals this information from the VC, engaging in inefficient continuation until funding is eventually exhausted. I will refer to a start-up that has already obtained a negative result but is still receiving development funding as 'unproductive', as opposed to 'productive' start-ups who may still achieve success. The key friction is that, after committing capital to the project, the VC does not know which of these two states the E is in. Finally, if the E is yet to obtain a result and sees its funding exhausted, it must return to the capital market to search for new funding.[19] If successful in securing new funding, the start-up resumes development. However, during its search, it may fail. Therefore, 'financing risk' is present in the model—an otherwise healthy firm may fail to secure follow-on funding. Figure 1 depicts the state-to-state transitions. In the remainder of this section, I outline the model in further detail. The following section then discusses features of the equilibrium, particularly the optimal contract.

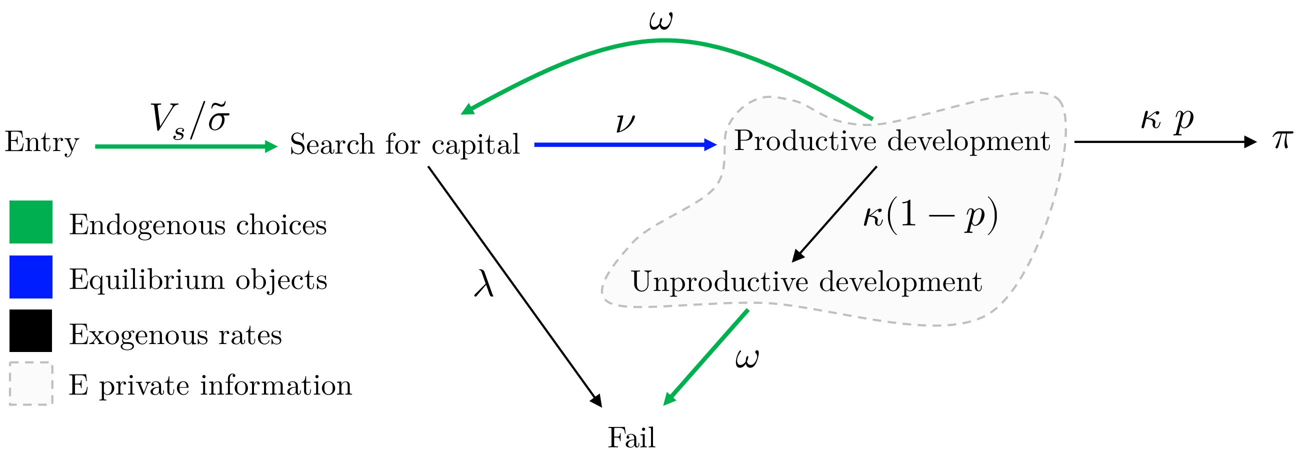

Figure 1: State transitions

Entry reflects an aggregate flow; other arrows represent individual-level transition intensities.

Entry. An exogenous flow Γ of potential start-ups have a one-time chance to enter, drawing entry cost $c \sim U[0,\sigma]$. New entrants begin their life in search of capital, because entrepreneurs have zero wealth. Therefore, a given start-up enters if and only if their entry cost $c \leq V_s$, where V_s is the value of a start-up searching for capital. Given $c \sim U[0,\sigma]$, the probability of drawing an entry cost below the threshold is $Pr(c \leq V_s) = V_s/\sigma$. Each potential entrant faces this same probability, so that the flow of new entrants that enter endogenously is given by $\Gamma\left(V_s/\sigma\right)$. It will not be possible to

[18]With each funding round, the E and any existing investors from previous rounds are diluted.

[19]The assumption that the start-up cannot simply return to its initial funder is clearly a strong one and can be relaxed. In the quantitative model, I allow the meeting rate in the capital market to depend on whether the start-up is and "early-stage" or "late-stage" firm, which allows for matching frictions to become less severe as the start-up progresses. Nevertheless, there are reasons to believe it is a reasonable approximation to reality. Firstly, although a firm will often receive multiple rounds of funding from the same VC, VCs typically syndicate projects within and between rounds, bringing on additional investors as the firm progresses through its development. This serves the role of providing an outsider's perspective on the performance of the firm (Kerr et al., 2014). The fact that new investors are typically sought implies that the matching friction is also present in follow-on rounds. Secondly, there are various reasons why a start-up may have to seek the bulk of its follow-on funding from entirely new investors. This could be because the initial investor is small and cannot fund 'big-ticket' investments, or because the initial funder faces some idiosyncratic shock that leads it to limit further investments. Finally, Nanda and Rhodes-Kropf (2017) show how coordination issues among investors across funding rounds can arise in an equilibrium model, even for start-ups with positive project NPV if fully funded. The assumption aims to capture these ideas while maintaining tractability.

disentangle Γ from σ, so I will substitute in $\tilde{\sigma} = \sigma/\Gamma$ going forwards. The endogenous flow of entry is then $V_s/\tilde{\sigma}$. The parameter $\tilde{\sigma}$ can be interpreted as determining the supply of entrepreneurial projects.

The search process. I assume a fixed measure M of VCs that can fund at most one project at time.[20] Just like start-ups, VCs are risk-neutral and discount the future at rate ρ. The meeting rate in the financing market depends on the measure of start-ups searching for capital, μ_s, and the measure of VCs that are not currently funding a project, μ_{vc}, which I call "unencumbered". I denote by μ_d^p the measure of start-ups in productive development and by μ_d^u the measure of start-ups in unproductive development; these are start-ups that have already obtained a failed result but have not reported this to their VC. Since the total measure of VCs is M and a measure $\mu_d^p + \mu_d^u$ are funding start-ups, the measure of unencumbered VCs is given by $\mu_{vc} = M - \mu_d^p - \mu_d^u$.

The flow of meetings between start-ups in search and unencumbered VCs is determined by a Cobb-Douglas matching function

$$\text{flow of meetings} = m(\mu_s, \mu_{vc}) = m(\mu_s, M - \mu_d^p - \mu_d^u) = \mu_s^\alpha \, (M - \mu_d^p - \mu_d^u)^\beta \tag{3}$$

where $\alpha, \beta > 0$. Therefore, the meeting rate for a start-up in search is given by $\nu = \mu_s^{\alpha-1} \, \mu_{vc}^\beta$.

During the search process, the start-up may fail due to a lack of funding. This occurs at rate λ, which is exogenous, and such a start-up obtains a return of zero. Together with the specification of the matching process and denoting by V^M the value of a meeting with a VC (prior to any agreement), the value of search, V_s, solves the HJB equation

$$(\rho + \lambda) \, V_s = \nu \, [V^M - V_s]. \tag{4}$$

The value of a match to existing shareholders is $V^M = (1 - \varsigma)V_d(\omega)$, where $V_d(\omega)$ solves equation (1) and (ω, ς) are optimal in the sense described below.

The optimal contract. In the baseline model, I abstract from VC bargaining power.[21] Therefore, the optimal contract, (ω, ς), maximises the value of the project to existing shareholders (the entrepreneur and any prior investors) subject to a participation constraint of the new investor.

For a given contract (ω, ς), the VC incurs expected capital costs $K(\omega)$, given by equation (2), and receives a share ς of the (post-money) value of the firm, $\varsigma V_d(\omega)$. Therefore, their participation constraint is given by $\varsigma V_d(\omega) \geq K(\omega)$. The optimal contract (ω, ς) then solves

$$\begin{aligned} V^M = & \sup_{\{\omega \in [0,\infty), \varsigma \in [0,1]\}} \left\{ (1 - \varsigma) \, V_d(\omega) \right\} \\ \text{s.t.} \quad & \varsigma \, V_d(\omega) \geq K(\omega) \end{aligned} \tag{5}$$

where $V_d(\omega)$ is the solution to equation (1), $K(\omega)$ is given by equation (2), and V_s is taken as given in the optimisation. The binding participation constraint pins down ς as a function of ω, which implies that $V^M = V_d(\omega) - K(\omega)$. Therefore, the optimal contract rate, ω, maximises the net (private) value

[20] This has parallels to the "short-run" economy of Inderst and Müller (2004). Jovanovic and Szentes (2013) also assume a fixed supply of VCs.

[21] I consider this extension in section B.2 of the Appendix.

of the firm, inclusive of investment costs. On the one hand, a lower contract rate, ω, typically increases the value of the pie, V_d, but in so doing raises the cost of capital, $K(\omega)$. Crucially, a lower contract rate increases the wedge in the capital cost function, which provides an incentive to set $\omega > 0$.

Equilibrium conditions. At present, the equilibrium objects V_d, V_s, V^M and (ς, ω) are determined for any given meeting rate, ν. To close the model, I need to specify the steady state conditions describing the two state variables, μ_s and μ_{vc}, which pin down the meeting rate, ν, in equilibrium. Going forwards, it will be simpler to consider the three states of the firm: search, μ_s, productive development, μ_d^p and unproductive development, μ_d^u. The measure of unencumbered VCs, μ_{vc}, can then be recovered from the definition $\mu_{vc} = M - \mu_d^p - \mu_p^u$.

In steady-state, the inflows and outflows from each state must be equal to one another. Consider first the measure of firms in productive development. A flow $m(\mu_s, M - \mu_d^p - \mu_p^u) = \nu\, \mu_s$ of firms enter this state from search, having signed contracts with VCs. In steady state, this inflow must equal the outflow, which comes from firms obtaining results, a flow $\kappa\mu_d^p$, and firms who's funding is exhausted, a flow $\omega\mu_d^p$. Whether a results leads to success or to the firm moving to unproductive development is unimportant. Therefore, the steady state condition is

$$\nu\, \mu_s = (\kappa + \omega)\, \mu_d^p \tag{6}$$

Next, consider the measure of firms in unproductive development. Firms enter this state from productive development when they obtain a negative result, a flow $\kappa(1-p)\mu_d^p$. In steady state, this inflow must equate to the outflow from firms who's funding is exhausted, a flow $\omega\mu_d^u$. Therefore, the steady state condition is

$$\kappa(1-p)\mu_d^p = \omega\mu_d^u \tag{7}$$

Finally, consider the measure of firms in search. Firms return to search from productive development when their funding is exhausted, which corresponds to an inflow $\omega\mu_d^p$. I assume that firms in unproductive development whose funding is exhausted would fail to convince another VC to invest in their start-up and therefore find it optimal to close down. This means they do not return to search for more capital. Intuitively, if investors conduct due diligence before funding new ventures, they would learn of the firms weak (here, non-existent) prospects and refuse to offer funding. Realising this, firms would shut down immediately after their current funding is exhausted. In addition to the inflow from productive development, new entrants begin their lifecycle in search of capital, a flow $V_s/\tilde{\sigma}$. In steady state, these inflows must equal outflows, which occur when firms fail during the search process, a flow $\lambda\mu_s$, or meet with VCs and so begin productive development, a flow $\nu\mu_s$. Therefore, the steady state condition is

$$\omega\mu_d^p + \frac{V_s}{\tilde{\sigma}} = (\lambda + \nu)\, \mu_s \tag{8}$$

Equations (6), (7) and (8) represent the steady state conditions. These equations can be written purely in terms of μ_s and μ_{vc}.[22] A steady-state equilibrium is a tuple $(V_d, V_s, V^M, \omega, \varsigma, \mu_s, \mu_{vc})$, where (ω, ς) solve maximisation problem (5), and equations (1), (4) (6), (7) and (8) are satisfied.

[22] From equation (7), $\mu_d^u = \frac{\kappa(1-p)}{\omega}\mu_d^p$, so that $\mu_{vc} = M - \mu_d^p - \mu_d^u = M - \left(1 + \frac{\kappa(1-p)}{\omega}\right)\mu_d^p = M - \left(1 + \frac{\kappa(1-p)}{\omega}\right)\frac{\nu}{\kappa+\omega}\mu_s$, where the last equality follows from equation (6). Furthermore, for μ_d^p in equation (8) from equation (6) yields $\frac{\omega}{\kappa+\omega}\nu\mu_s + \frac{V_s}{\tilde{\sigma}} = (\lambda+\nu)\mu_s$.

2.3 Funding patterns, entry and outcomes

Given a value for the meeting rate in the financing market, ν, the model omits a closed-form solution for value and policy functions. This section analyses properties of the model taking ν as given. The first proposition proves existence of an equilibrium given ν and the resulting corollary derives the optimal contract rate, ω, as a function of model parameters. It also provides conditions under which upfront-financing is maintained in the frictional setting.

Proposition 2.1. *Given ν and for $p\kappa\pi > k$, there is unique solution for $\{V_d, V_s, V^M, (\omega, \varsigma)\}$ in terms of model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, $\omega \in [0, \infty)$.*

Corollary 2.1. *Given ν, the equilibrium contract rate is given by*

$$\omega = \begin{cases} \max\left\{0, -\frac{(\rho+\lambda)(\rho(\kappa p\pi - k) - \kappa k(1-p)) - \sqrt{\kappa k(1-p)(\rho+\lambda)(\kappa p\pi(\kappa(\rho+\lambda+\nu)+\nu\rho) - k(\nu(\rho+\kappa)+\kappa p(\rho+\lambda)))}}{(\rho+\lambda)(\kappa p\pi - k)}\right\} & \text{for } k < \kappa p\pi \\ \infty & \text{else} \end{cases} \tag{9}$$

Furthermore, to sustain upfront-financing in equilibrium ($\omega = 0$), it must be that $p \in (\bar{p}, 1]$ and $\nu \in (0, \bar{\nu}]$ for some $0 < \bar{p} < 1$ and $\bar{\nu} \geq 0$.

Corollary (2.1) is central to the dynamics of the model.[23] Assuming that the project is viable ($k < \kappa p\pi$), the optimal contract may feature *upfront-financing* ($\omega = 0$) or *staged-financing* ($\omega > 0$).[24] When $\omega = 0$, the VC agrees to fund the project until a result is obtained, regardless of its timing. I refer to this as upfront-financing because the start-up obtains a committent from the outset to fund its entire development path and never needs to return to the capital market. Conversely, when $\omega > 0$, there is a positive probability that funding will be exhausted before the project realises a result. If this happens, the firm returns to the funding market to seek more funding in return for a further dilution of the equity stake of existing shareholders. This reflects staged-financing and is an endogenous feature of the model.

Throughout and unless stated otherwise, I make assumptions so that firms engage in staged-financing, which occurs when $\omega \in (0, \infty)$. However, it is worth considering briefly the settings in which upfront-financing arises as the optimal contract. When the meeting rate in the financing market, ν, is sufficiently low *and* the probability of success is sufficiently high, upfront-financing can still be maintained, despite the agency friction. Intuitively, when ν is low, financing risk is extreme, which leads shareholders to increase their demand for insurance against it and so to commit more capital upfront. Providing this insurance is costly, but when agency frictions are sufficiently muted, providing complete insurance in the form of upfront-financing can be optimal. In simple terms, low-risk projects (think real estate investments) can be financed upfront, whereas more novel ventures benefit from a wait-and-see approach.

2.3.1 Staged-financing

I begin by analysing the effect project characteristics and funding market conditions on the optimal contract, before considering their effect on funding patterns, start-up outcomes and entry. I am partic-

[23] The values of $\bar{p}$ and $\bar{\nu}$ are provided in the proof in section A.2 of the Appendix.

[24] When $k \geq \kappa p\pi$, $\omega \to \infty$ is optimal. Note that $\lim_{\omega\to\infty} K(\omega) = 0$, so that the VC provides no funds to the project.

ularly interested in the implications of the frictions for projects with different characteristics, but equal underlying value. For instance, certain projects may be riskier than others or require a long period of investment to uncover uncertainty, but still have the same underlying value because, if successful, they obtain a higher valuation. Therefore, in addition to the simple comparative static exercises that I conduct in this section, I also conduct comparative statics in which I hold the frictionless project value constant.[25] This allows for isolating the effect of frictions on project type, holding fixed the underlying value of the project.

Definition 2.1. A *value-compensating* comparative static with respect to a parameter x is the derivative of a function with respect to x where the payoff, $\pi = \pi(x)$, is adjusted to compensate for the effect of x on the frictionless project value, i.e. $\bar{V}_s = \frac{\kappa p \pi(x) - k}{\rho + \kappa}$ = constant.

The first result considers the effect of project characteristics and funding market conditions on the optimal contract rate.

Proposition 2.2. *The optimal contract rate, ω, is: strictly increasing in ν and k; and strictly decreasing in p, π, λ. For κ, $\exists \bar{\kappa} > k/p\pi$ such that for $\kappa < \bar{\kappa}$, $\omega'(\kappa) < 0$ and otherwise $\omega'(\kappa) \geq 0$. The value-compensating comparative static for p is strictly negative and for κ is strictly positive.*

Corollary 2.2. *The expected duration of productive development,* $1/(\kappa + \omega)$*, is: strictly decreasing in ν and k; strictly increasing in p, π and λ; and ambiguous with respect to κ. The value-compensating comparative static for p is strictly positive and for κ is strictly negative.*

Corollary 2.3. *The expected discounted capital commitment, $K(\omega)$, is: strictly decreasing in ν; strictly increasing in π and λ; and ambiguous with respect to κ, k and p. The value-compensating comparative static for p is ambiguous and for κ is strictly negative.*

Proposition 2.2 is key to the dynamics of the model. Central to the choice of contract is a trade-off between two frictions: the agency friction, which affects financing costs, and the matching friction, which generates financing risk. A lower contract rate, ω, raises the expected duration of productive development, which in turn means that the firm is less likely to need to return to the capital market. Therefore, it acts as insurance against financing risk. However, this insurance is costly: larger capital commitments exacerbate the agency friction and so require the entrepreneur to give up an increasingly large equity stake.

With this trade-off in mind, factors that make searching for capital more costly, either because it takes longer to find a match, a decrease in ν, or because firms are less able to survive without an equity injection, a shorter "runway" and an increase in λ, lead to a lower optimal contract rate–a larger capital committent. Among the various project characteristics, π and k do not directly affect the agency friction, but they do affect project value. For given agency frictions, projects that are more valuable demand more insurance against financing risk because failure due to financing risk implies a larger loss in value, so that the marginal benefit of additional insurance is higher. Therefore, an increase in π, or decrease in k, leads to a lower contract rate, reflecting greater insurance against financing risk.

Finally, characteristics p and κ interact with the value of a project and the agency friction. Holding other parameters constant, frictionless project value is increasing in p and the capital wedge is decreasing

[25] Absent frictions, a start-up meets with a VC the instant it enters and secures funding until the realisation of uncertainty. The post-entry project value is then $\bar{V}_s = \frac{\kappa p \pi - k}{\rho + \kappa}$.

in p. A higher likelihood of success, an increase in p, therefore translates unambiguously into a lower contract rate, ω. This continues to apply when focussing on value-compensating changes in p. Projects that are less likely to succeed but have larger payoffs in the case of success can be considered more radical (Nanda and Rhodes-Kropf, 2017). The implication is that insurance against financing risk is more costly for more radical projects. As I discuss below, this leads them to fail from financing risk at a higher rate.

In contrast, the effect of the development horizon, κ, is ambiguous. On the one hand, an increase in κ increases the frictionless project NPV because the timing of the payoff is brought forwards. This incentivises greater upfront investment. However, as κ continues to rise relative to ω, the agency friction is exacerbated because of maturity mismatch between the realisation of uncertainty and the funding horizon. Intuitively, if uncertainty is expected to be resolved in a matter of months, but funding has been provided for a number of years, then the VC is at risk of exploitation by the entrepreneur. This is an incarnation of the common feature of venture capital financing that capital injections are provided in order for the venture to overcome a particular milestone. Thus, comparing two projects with equal frictionless project value, the one with the shorter development horizon, a higher κ, will opt for a shorter funding duration, a higher ω.

Start-up funding patterns. To build intuition for how these results map into the model, it is informative to consider a number of empirically observable statistics about the firm. I focus on two statistics that are readily observable from data on venture capital funding rounds: the number of rounds and the duration between rounds.[26]

The model's recursive structure permits the precise derivation of distributions for both of these outcomes. While firms are *ex ante* homogeneous, these outcomes are random variables because, *ex post*, firms with identical contracts exhaust their funding and resolve uncertainty at different times. I focus attention on the subset of firms that receive at least one funding round in the model because this has a natural mapping to venture capital data.[27] The following propositions and related corollaries present the key results.

Proposition 2.3. *The distribution of the number of financing rounds, $N_f \sim Geo(q_d)$ where $q_d = \frac{\kappa}{\kappa+\omega} + \frac{\omega}{\kappa+\omega}\frac{\lambda}{\lambda+\nu}$ with support $N_f \in \{1, 2, 3, ...\}$.*

Corollary 2.4. *The expected number of financing rounds, $E[N_f] = 1/q_d$, is: strictly increasing in ν and k; strictly decreasing in p, π, and λ; and ambiguous for κ. The value-compensating comparative static for p is strictly negative and the for κ is ambiguous.*

Proposition 2.4. *The distribution of the duration between successive funding rounds, $T_{br} \sim Hypo(\kappa + \omega, \lambda + \nu)$.*

Corollary 2.5. *The expected duration between successive funding rounds for a given firm, $E[T_{br}] = \frac{1}{\kappa+\omega} + \frac{1}{\lambda+\nu}$, is: strictly decreasing in ν, and k; strictly increasing in p and π; and ambiguous with respect to λ and κ. The value-compensating comparative static for p is strictly positive and for κ is strictly negative.*

[26]The relationship between these features of funding patterns, firm characteristics, and market conditions was first explored in the seminal work of Gompers (1995).

[27]A share $\frac{\lambda}{\nu+\lambda}$ of entrants fail before they meet a VC for the first time and so receive no funding.

Corollaries 2.4 and 2.5 consider how the expected number of funding rounds and the expected duration between rounds responds to project characteristics and funding market conditions.[28,29] When financing risk is a greater concern for start-ups, a higher λ or lower ν, the expected number of funding rounds falls and the duration between them rises, so funding more closely resembles the case of upfront financing. Conversely, when agency frictions are exacerbated, for instance as projects become more novel, the expected number of funding rounds increases and the time between them falls. Figure 2 plots these effects.

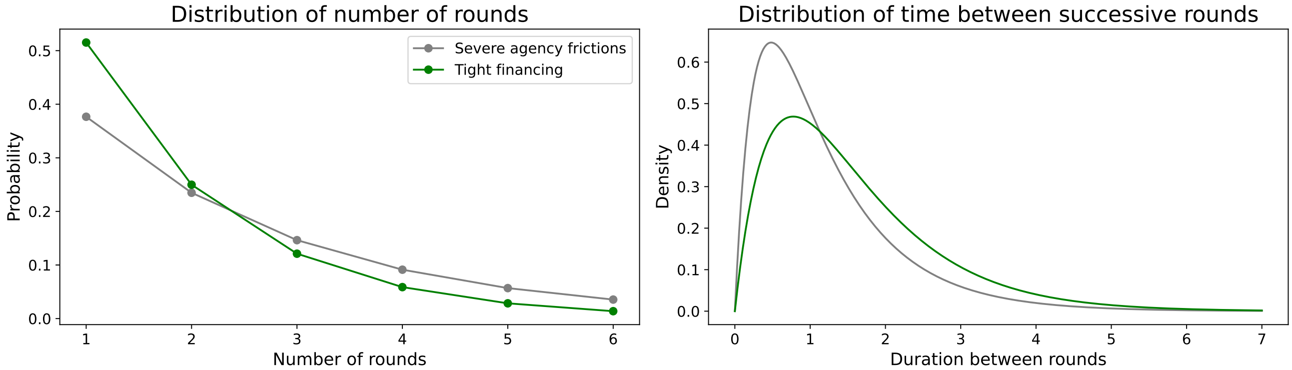

Figure 2: Distribution of the number of and time between funding rounds

The figure depicts the distribution of the number of funding rounds and time between rounds for two hypothetical parameterisations. In the case of “tight financing”, the meeting rate, ν, has been lowered. In the case of “severe agency frictions”, the project is made more novel.

Other parameters are also important for funding patterns. As the project becomes more valuable, for instance because of a higher potential payoff, π, or a lower investment cost, k, the firm receives funding in fewer funding rounds, visiting the capital market less frequently. These comparative statics results will be important in mapping the model to data in the quantitative exercise.

Finally, consider the effect of the development horizon, κ. Holding fixed the frictionless project value, an increase in κ leads to a shorter funding committent, a higher ω, to match the funding duration with the realisation of uncertainty. This naturally results in less time between funding rounds. However, the effect on the number of funding rounds is slightly more nuanced.[30] On the one hand, if a firm needs to fund additional years of development, it will have to visit the market more frequently, holding contracts fixed. However, Proposition 2.2 shows that there is an incentive to match funding duration to the timing of uncertainty realisation. As a result, they raise larger capital injections to support

[28]To build intuition for Proposition 2.3, note that the probability of another funding rounds is memoryless, as it depends only on the current state and not on the number of prior funding rounds. As a result, the distribution of the number of funding rounds follows a Geometric distribution–the only discrete probability distribution with the memoryless property. In practice, this memoryless property is a strong assumption and likely violated in the data. To address this, the quantitative model allows the probability of securing additional funding to vary with the firm's stage of development by reducing matching frictions for “late-stage” firms, mitigating this concern.

[29]To build intuition for Proposition 2.4, consider that between two funding rounds, the start-up conducts development for period $T_d \sim Exp(\kappa + \omega)$ and the searches for duration $T_s \sim Exp(\lambda + \nu)$. Therefore, the time between two successive funding rounds is $T_{br} = T_d + T_s$, the sum of two exponentially distributed random variables. Such a random variable is said to have a hypoexponential distribution, inheriting the rate parameters of the respective exponential distributions from which it derives.

[30]This is counterintuitive, given that generally there is a relationship between the number of funding rounds and the duration between them. However, discussions of the duration between rounds are conditional statements–the firm has to actually meet with a VC for the duration to be meaningful. Conversely, the number of funding rounds is explicitly affected by the extent to which firms fail while searching for capital.

longer periods of experimentation. This adjustment could offset—or even outweigh—the direct effect, potentially reducing the total number of funding rounds a firm conducts. As this discussion elucidates, what matters is *by how much* contracts respond to changes in the development horizon. Specifically, if the semi-elasticity of contracts to funding duration is sufficiently low, $\omega'(\kappa)/\omega < \kappa^{-1}$, then a longer development horizon is associated with more funding rounds, in expectation. In practice, this condition will almost certainly met, and so I will typically refer to projects that have longer development horizons requiring more rounds of funding.[31]

2.3.2 Endogenous start-up outcomes and financing risk

I now consider start-up outcomes. In the frictionless setting, start-up success is an exogenous parameter: a share p of firms are successful in commercialising their innovations. In the frictional setting, this is no longer the case due to financing risk–some otherwise viable firms may fail because they are unable to secure funding. Furthermore, the fact that firms choose optimal contracts implies that their exposure to financing risk, and therefore the likelihood that they are successful, is endogenous.

I denote by p_d and p_s the probability that firm currently in productive development or search, respectively, is successful with its project at some point. The structure of the model allows for writing these probabilities as the solution to the following recursive equations

$$p_d = \frac{\kappa}{\kappa + \omega}\, p + \frac{\omega}{\kappa + \omega}\, p_s, \qquad p_s = \frac{\nu}{\lambda + \nu}\, p_d.$$

Specifically, a firm currently in development can succeed in its current financing round, with probability $\frac{\kappa}{\kappa+\omega}\, p$, or it transitions to search and then is successful with probability p_s. Similarly, a firm currently in search transitions to development with probability $\frac{\nu}{\lambda+\nu}$, in which case its success probability is p_d. This representation allows to solve directly for the success probabilities, which are summarised by the following lemma.

Lemma 1. *The (steady-state) probabilities of success for a firm in productive development, p_d, and in search, p_s, are*

$$p_d = \frac{\kappa\,(\lambda + \nu)}{\omega\,\lambda + \kappa\,(\lambda + \nu)}\, p, \qquad p_s = \frac{\nu}{\lambda + \nu}\, p_d$$

where $p_d, p_s < p$ for $\omega \in (0, \infty)$.

Remark 1. *The (steady-state) probabilities that a firm in productive development, f_d, and in search, f_s, fail due to financing risk are*

$$f_d = \frac{\omega}{\kappa + \omega}\frac{\lambda}{\lambda + \nu} E[N_f], \qquad f_s = \frac{\lambda}{\lambda + \nu} E[N_f]$$

where $E[N_f]$ is the expected number of funding rounds. Note that $p_d = (1 - f_d)\, p$ and $p_s = (1 - f_s)p$.

Before analysing the expressions in more detail, two points are worth noting. Firstly, when $\omega = 0$, $p_d = p$: upfront-financing perfectly insures the start-up against financing risk. Secondly, in general, $p_d < p$ and start-up funding patterns matter for outcomes. All else equal, a higher contract rate implies

[31] It can be shown that if $\pi < \bar{\pi}$ for some $\bar{\pi}$ that depends on model parameters, then $\omega'(\kappa)/\omega < \kappa^{-1}$ is satisfied. In practice, this value of $\bar{\pi}$ is very high, i.e. orders of magnitude higher than the value estimated in the quantitative model, so I will assume that this result holds in most of the discussion.

that firms engage in more funding rounds and consequently are more exposed to financing risk (Remark 1). This point is made explicit in the following lemma.

Lemma 2. *Consider two start-ups $i = a, b$ facing the same funding conditions, ν and λ. If $E[N_f^a] > E[N_f^b]$, then $f_d^a > f_d^b$ and $f_s^a > f_s^b$.*

Lemma 2 establishes a direct link between funding patterns, start-up outcomes, and (endogenous) exposure to financing risk. It demonstrates that firms adopting funding strategies with more funding rounds are more likely to succumb to financing risk. This result is intuitive: such firms visit the funding market more frequently, increasing their vulnerability to fluctuations in market conditions.

Importantly, using the insights of Lemma 2, it becomes clear which start-ups are most adversely affected by financing risk. On the one hand, low quality projects typically demand less insurance against financing risk, visiting the market more frequently, and so are endogenously more exposed.[32] On the other hand, highly novel start-ups, even with fixed frictionless project value, face greater exposure to financing risk because insuring against this risk is more costly for them. This leads them to visit the market more frequently, increasing their exposure. Finally, as a start-up's development horizon lengthens, the preceding discussion shows that it will engage in more funding rounds, and so such projects are more exposed. In the final part of the paper, I use this finding to motivate a study of the extent to which financing risk biases innovation away from longer-horizon, potentially more transformative technologies.

Finally, in Proposition A.3 in the Appendix, I explore the implications of financing conditions, ν and λ, for start-up outcomes. In general, they have ambiguous effects, which reflects two opposing forces. For instance, the direct effect of tighter funding markets is to increase the risk of failure from financing risk. However, as Proposition 2.2 illustrates, a decrease in ν or increase in λ induces a response in optimal contracts: firms raise more capital to insure against financing risk. When markets are particularly tight, i.e. ν is low, this effect can dominate, so that in general the effect is ambiguous.

2.3.3 Endogenous entry

Before moving to consider the full equilibrium model, I analyse the effect of model parameters of the flow rate of entry, $V_s/\tilde{\sigma}$.[33] Studying entry incentives–and particularly how they depend on start-up characteristics and funding patterns–is key to understanding how these frictions affects start-up creation across technologies. Furthermore, although in this model entry is a decision made by the entrepreneur, in practice start-ups typically receive funding prior to engagement with formal VC investors–for instance, by Angel investors. With this in mind, the entry results considered here can also be framed in terms of investment incentives for investors at earlier stages in the development process.

To build intuition, it is useful to consider the case of $\rho \to 0$. In this case, the value of entry can be written simply as

$$V_s = \frac{\nu}{\lambda + \nu}\Big(p_d \times \pi - E[N_f] \times K(\omega)|_{\rho \to 0}\Big) \tag{10}$$

[32] This might seem to contradict the findings of Gompers (1995), that successful firms undergo more financing rounds than unsuccessful ones. However, in the baseline model with *ex ante* homogeneous firms, the number of funding rounds is independent of outcomes (success or failure). Furthermore, in the quantitative model, the firms that are ultimately successful secure, on average, more rounds of funding. The introduction of an intermediate milestone is important for this result.

[33] Note that this ignores the equilibrium effect on ν, which feeds back into the value of entry, V_s.

where p_d is defined in Lemma 1, $E[N_f]$ is the expected number of funding rounds, given in Corollary 2.4 and $K(\omega)$ is the expected discounted capital cost of contract ω, defined in equation (2).

In this limit case, the timing of the payoffs becomes irrelevant. The first term, $\nu/(\lambda + \nu)$, simply reflects the probability of obtaining any funding following entry. More notably, entry increases with the expected payoff, $p_d \times \pi$, which depends on the probability that the start-up is successful, an endogenous object that incorporates the effect of financing risk. Conversely, entry is decreasing in the total volume of capital that the start-up expects to raise, $E[N_f] \times K(\omega)|_{\rho \to 0}$, which depends on the expected number of rounds and the amount of capital raised in each round. This reflects the cost of funding, which adjusts to account for the start-up's incentives to secure insurance against financing risk. The following proposition presents the key results for entry

Proposition 2.5. *The flow rate of entry, $V_s/\tilde{\sigma}$, is: strictly increasing in ν, p, π and κ; and strictly decreasing in k, λ and $\tilde{\sigma}$. The value-compensated comparative static for p is positive and for κ is ambiguous.*

In most cases, the intuition for the sign is relatively intuitive. Entry is higher when: (i) financing is easier to access, an increase in ν or decrease in λ; (ii) projects have higher frictionless NPV, higher (p, π, κ) or lower k; or (iii) when entry costs, $\tilde{\sigma}$ are lower. In section 4, these insights will be useful in thinking about the drivers of lower venture capital activity in the UK.

For the value-compensated effects, it is helpful to refer to equation (10). As projects become more novel, insurance against financing risk becomes more costly, and the probability of success, p_d falls. This suggests that financing risk has particularly damaging effects on entry incentives for highly novel innovations. For projects with longer development horizons, the effect is technically ambiguous, but in general we should expect a negative relationship between project horizon and entry in the frictional environment. In general, these projects are less likely to succeed because they tend to visit the funding market more frequently, as discussed in the previous section. Furthermore, the positive relationship between project horizon and total expected capital raised, $E[N_f] \times K(\omega)|_{\rho \to 0}$, means these projects also anticipate raising more capital. Together, these factors imply that the frictions in this economy disproportionately harm the entry incentives for projects with longer development horizons, even when their frictionless NPV is identical. In section 5, I will extend and quantify these insights by introducing heterogeneity across start-ups in their development horizons.

2.4 Efficiency

In this section, I discuss the efficiency properties of the model. I begin by formalising a notion of capital misallocation, which I argue provides insights into the costs of insuring against financing risk. Capital is misallocated in the economy as a result of inefficient continuation; it would be preferable to transfer capital from a start-up in unproductive development to one in search. However, in light of the frictions present in this economy, a degree capital misallocation is inevitable. Therefore, I then study whether the economy is constrained efficient; that is, taking as given the existence of agency and matching frictions, does the decentralised equilibrium deliver the same allocation as would be chosen by a social planner? I show that social efficiency obtains only in the knife-edge case in which the classical Hosios condition is satisfied (Hosios, 1990).

2.4.1 Capital misallocation

Relative to an economy without frictions, capital is misallocated in the model because start-ups that have received negative news about their prospects–the "unproductive" start-ups–do not inform their investors and so continue to invest in projects with no prospects. The capital would be better invested in a different, productive start-up. Nevertheless, a high degree of capital misallocation is indicative of issues in access to funding. To see this, note that equation (7) allows us to write the share of firms in unproductive development in the baseline model as

$$\frac{\mu_d^u}{\mu_d^u + \mu_d^p} = \frac{\kappa\,(1-p)}{\kappa\,(1-p) + \omega} \tag{11}$$

so that capital misallocation, when understood as the share of firms receiving funding that are unproductive, is decreasing in ω.[34] This leads to the following Corollary (of Proposition 2.2), which characterises factors that affect the extent of capital misallocation in the model.

Corollary 2.6. *Capital misallocation is: strictly increasing in π and weakly increasing in λ; strictly decreasing in k and ν; and ambiguous with respect to κ, p.*

For parameters other than κ and p, factors that push up the contract rate, ω, reduce capital misallocation, whereas factors that lead to a lower contract rate typically increase misallocation. In particular, when financing conditions for start-ups tighten, a lower ν, capital misallocation rises. Intuitively, capital is misallocated in the model because of financing risk; in an environment with only agency frictions, the optimal contract rate would specify $\omega \to \infty$ and no capital would be misallocated.[35] Therefore, capital misallocation effectively reflects insurance against financing risk, and the estimates of the extent of capital misallocation that I provide in section 3.3 are indicative of the costs that firms incur to militate against it.

2.4.2 Welfare analysis

In this section, I provide conditions under which the economy is constrained efficient. In order to do so, I consider the general version of the baseline model that includes VC bargaining power, as set out in section B.2 of the Appendix. The key difference is that I endow the VC with bargaining power $1-\delta$, where $\delta = 1$ is the case considered in the baseline model. Social efficiency in this setting requires that the flow rate of entry and the contract rate in the decentralised equilibrium coincide with those that would be chosen by a planner maximising total surplus, which is defined as

$$TS = \int_0^\infty e^{-\rho t}\Big([\kappa p \pi - k]\mu_{d,t}^p - k\mu_{d,t}^u - \frac{\tilde{\sigma}}{2}\Lambda_t^2 \Big)\, dt \tag{12}$$

where Λ_t denotes the flow rate of entry at time t and $\mu_{d,t}^p$ and $\mu_{d,t}^u$ should be understood to be the total mass of start-ups in productive and unproductive development, respectively, regardless of their

[34] In the baseline model where every funding round involves the same amount of capital raised, this is equivalent to the share of capital that is misallocated.

[35] This may appear to conflict with the previous statement that $\omega \to \infty$ implies no funding. In the case where ν is finite, this is indeed true. However, in the limit of no financing risk, $\nu \to \infty$, the start-up meets with a new VC instantaneously, so that they can fund their development with an infinite number of funding rounds, each featuring $\omega \to \infty$.

contract rate ω.[36] The flow value of total surplus is equal to the flow payoff emanating from start-ups in productive development, $\kappa p\pi$, net of the costs of sustaining firms in development, $k(\mu^p_{d,t} + \mu^u_{d,t})$, and of entry costs $\frac{\tilde{\sigma}}{2}\Lambda_t^2$.[37]

Proposition 2.6. *If the matching function exhibits constant returns to scale, $\beta = 1 - \alpha$, and the bargaining weights in the contracting problem are equal to their respective matching function elasticities, $\alpha = \delta$ and $\beta = 1 - \delta$, then the decentralised equilibrium is constrained efficient.*

Proposition 2.6 provides conditions under which the decentralised equilibrium coincides with the solution to the planner's problem. The conditions are analogous to the Hosios condition typical in the search literature. Intuitively, for a fixed equilibrium contract rate, ω, the standard results from the search literature apply to the entry condition for start-ups: the negative congestion externality leads to excessive entry, but a lack of appropriately due to incomplete bargaining power implies that entry might be inefficiently low. These two effects are balanced when the Hosios condition is satisfied. To provide some intuition on why it follows immediately that the contract rate is socially efficient, it is helpful to conceptualise the contract rate as pinning down the optimal rate of 'entry' of productive start-ups and VCs back into the search market: VCs funding start-ups and start-ups in productive development re-enter the search market at rate ω. If the values of start-ups and VCs in search are pinned down optimally by the Hosios condition, then these incentives to 're-enter' the search market must also be optimal. Therefore, the contract chosen in the decentralised equilibrium will ensure the correct flow rates of firms back into the search market and so the decentralised equilibrium is socially efficient. Of course, as pointed out by Inderst and Müller (2004), there is no particularly reason to think that the Hosios condition should be satisfied in the VC market.

2.5 A quantitative equilibrium model

In this section, I discuss two simple extensions of the baseline model which are important for capturing the empirical dynamics. First, I generalise the model to a setting where start-ups must overcome two hurdles to commercialise their innovations.[38] In a practical sense, the introduction of multiple development stages serves three purposes: (i) it allows me to relax the strong assumption that access to capital for firms with viable projects does not improve over their development cycle; (ii) it facilitates a natural mapping to data, where funding rounds are typically considered to be early or late-stage; and (iii) it allows me to capture the time-to-exit dynamics in the data. Regarding (iii), the empirical hazard rate for successful exits is first increasing and then decreasing in the time since first funding (see, for example, Jovanovic and Szentes 2013) and the introduction of two stages allows the model to capture this feature. This is particularly important because I estimate the model on censored data.

[36] Out of steady-state, firms may have different contract rates. For instance, denoting the measure of firms at time t in productive development with contract rate ω by $\mu^p_{d,t}(\omega)$, the total mass of firms in productive development at time t is $\mu^p_{d,t} = \int_0^\infty \mu^p_{d,t}(\omega)\, d\omega$. The proof of Proposition 2.6 takes full account of these dynamics.

[37] Recall that entry is specified as a cutoff rule. If the flow of entry is Λ_t, then the cutoff must be $\hat{c}_t = \tilde{\sigma}\Lambda_t$. The total flow entry costs at time t are calculated as $\int_0^{\hat{c}_t} c/\tilde{\sigma}\, dc = \frac{\tilde{\sigma}}{2}\Lambda_t^2$.

[38] These hurdles–or development stages–may have various interpretations. For instance, Arora et al. (2024) consider a setting in which firms must first overcome technological challenges before considering commercialisation issues. Alternatively, Ewens et al. (2018) frame the multiple development stages as 'experiments' that deliver information about the viability of the project; indeed, the real option to abandon the project after the initial experiment can render a project viable that otherwise would not have been.

Therefore, to limit the influence of the choice of censoring horizon on the results, it is crucial that the model capture the 'time-to-exit' distribution.

Second, I introduce acquisitions as a distinct form of exit for firms in the second (or "late") stage. In practice, firms do not need to fully develop their technologies in order to profit from their efforts; there exists a market in which firms with partially-developed innovations can sell to incumbent firms who may be better placed to commercialise the innovation. Furthermore, I show that acquisitions substitute for financing, formalising the insight of Nanda and Rhodes-Kropf (2016) that a market for ideas can be crucial for overcoming financing risk. Therefore, to properly capture the effect of financing on start-up activity and outcomes, it is important to allow firms to choose between being acquired and self-developing their project. The fact that I permit acquisitions only in the late-stage mirrors the environment of Arora et al. (2024), where a start-up must overcome both *technology risk* and *commercialisation risk* but may sell out to an acquirer before attempting to overcome the second hurdle.

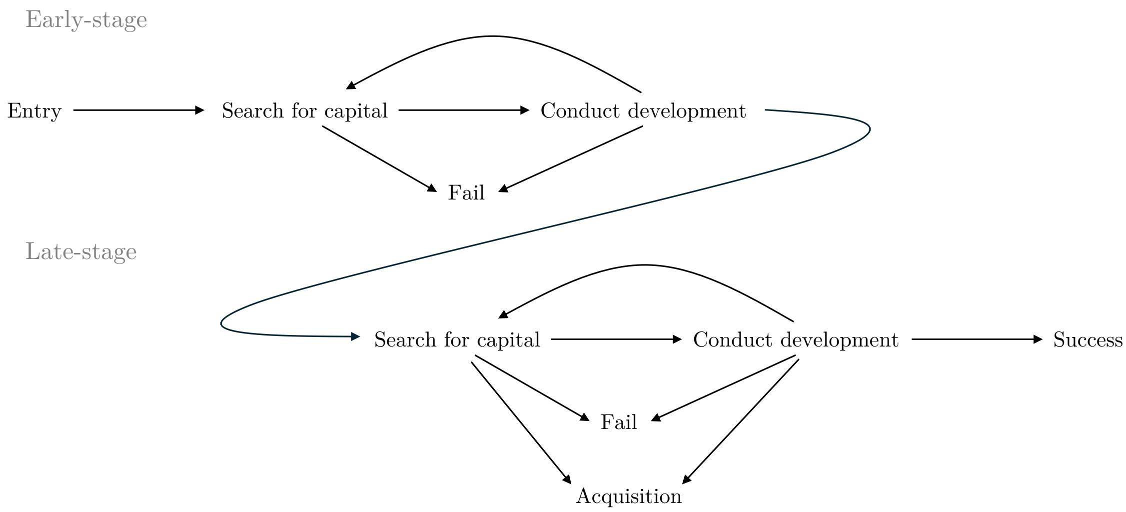

Figure 3: Start-up lifecycle

For ease of exposition, the two states "productive development" and "unproductive development" have been combined into "conduct development".

For clarity, Figure 3 depicts the potential paths that a start-up may take. I now describe the model briefly, focussing on features that differ from the baseline. Before doing so, note that the movement from a single stage to a multi-stage model requires taking a stance on which parameters remain the same across stages and which differ. The choices in this section are identical to those made in the quantification exercise in section 3 and all parameters are shown in Table 1.

The early-stage. Following entry, start-ups seek early-stage funding and have value $V_{s,e}$. During search, they fails at rate λ and meet a VC at rate, ν_e, which is stage-specific. A start-up in the early-stage that invests flow k_e realises a result at rate κ, which is positive with probability p_e/p_l, where p_l is the probability of overcoming late-stage uncertainty, conditional on being in the late-stage and obtaining a result.[39]

[39]This convention means that, in a frictionless world, each start-up in the early-stage would be successful with probability $p_e = (p_e/p_l) \times p_l$. The term p_e/p_l is equivalent to p_s in Ewens et al. (2018).

Conditional on meeting with a VC, the start-up signs a contract with contract rate ω_e, which provides funding for the early-stage only.[40] The contract problem is identical to that laid out in section 2.2, except that the probability p is replaced by p_e/p_l.[41] If successful in overcoming early-stage uncertainty, the start-up progresses to the late-stage and seeks late-stage funding.

The late-stage. A start-up that overcomes early-stage uncertainty begins its search for late-stage capital. During search, it fails at rate λ and finds a VC at rate, ν_l, where, in general, $\nu_l \neq \nu_e$.

Start-ups in the late-stage benefit from acquisition opportunities. I define an acquisition as the sale of a partially-developed project and, therefore, acquisitions may occur whether the firm is in search or development. Specifically, I assume that potential acquirers arrive at rate ϕ, whether the firm is in search or development. This rate is exogenous and the same for all firms.[42] I suppose that the acquirer values the fully-developed project possessed by the start-up at $\epsilon \times \pi$, where $\epsilon \in [0, \infty)$ is a random variable with CDF $F(\cdot)$ and that ϵ is match-specific, common knowledge and *i.i.d.* across matches. However, the acquirer purchases the start-up before it has reached full development so there is residual uncertainty. In the late-stage, a start-ups project would succeed with probability p_l if fully-funded, and I assume that this is the probability with which the acquirer is able to make use of the innovation. Therefore, the acquirer's valuation for the project is $\epsilon\, p_l\, \pi$. The assumptions imply that this is known to the start-up.

I assume that the start-up makes a take-it-or-leave it offer to the acquirer. Given these assumptions, the start-up simply sets the acquisition price at the maximum of its current value in the late-stage, $V_{i,l}$ for $i \in \{s, d\}$, and the valuation of the potential acquirer, $\epsilon\, p_l\, \pi$. If the former exceeds the latter, the potential acquirer will choose not to acquire the start-up. Bringing these insights together, the probability that an acquisition occurs conditional on a meeting is given by

$$\begin{aligned} \text{Development:} \quad & Pr(\epsilon\, p_l\, \pi \geq V_{d,l}) = 1 - F(V_{d,l}/p_l\pi) \\ \text{Search:} \quad & Pr(\epsilon\, p_l\, \pi \geq V_{s,l}) = 1 - F(V_{s,l}/p_l\pi) \end{aligned}$$

which permits defining an effective arrival rate of acquisitions $\hat{\phi}_i = \phi \cdot [1 - F(V_{i,l}/p_l\pi)]$ for $i \in \{s, d\}$. The effective arrival rate of acquisitions modifies the arrival rate of potential acquisitions by the acceptance rules outlined above and so is endogenous. In the quantification, I will impose $\epsilon \sim Exp(1/\xi)$, so that ξ is the average 'synergy'. With this in mind, the HJB equations for late-stage search and development are given by

$$\text{Development:} \quad \rho V_{d,l} = \kappa\big[p_l\pi - V_{d,l}\big] + \omega_l\big[V_{s,l} - V_{d,l}\big] + \hat{\phi}_d \cdot \big[p_l\, \pi E[\epsilon|\epsilon > V_{d,l}/p_l\pi] - V_{d,l}\big] \tag{13}$$

$$\text{Search:} \quad (\rho + \lambda)V_{s,l} = \nu_l\big[V_l^M - V_{s,l}\big] + \hat{\phi}_s \cdot \big[p_l\, \pi E[\epsilon|\epsilon > V_{s,l}/p_l\pi] - V_{s,l}\big] \tag{14}$$

[40] I assume that funding applies only to the start-up's current stage. This aims to capture the fact that VCs tend to specialise in different parts of the development process, so start-ups typically seek funding from larger VCs as they mature.

[41] Specifically, the expected discounted capital cost for the VC is

$$K_e(\omega_e) = \Big(1 + \frac{\kappa(1 - p_e/p_l)}{\rho + \omega_e}\Big)\frac{k_e}{\rho + \kappa + \omega_e}.$$

[42] The model could be generalised in several ways to endogenise ϕ. For instance, in a model with an 'incumbent' sector, ϕ may reflect the intensity with which established firms monitor the start-up sector for new technologies, or it could reflect the stance of antitrust policy.

where V_l^M is the value of a meeting in the late-stage, which is defined analogously to the baseline model. Relative to the baseline model, there is the additional flow value deriving from acquisitions and the conditional expectation obtains because only potential acquirers with sufficiently high synergies result in acquisitions.

An outstanding problem is how to compute the expected capital cost when there are acquisitions. With acquisitions, a firm in productive development experiences an event–a result, acquisition, or funding withdrawal–at rate $\kappa + \omega + \hat{\phi}_d$. I assume that the contract written between the start-up and VC is such that the VC terminates funding when the firm is acquired. For an unproductive firm, I assume that the arrival of a potential acquirer leads the VC to learn that the project has failed. Given these assumptions, a start-up in unproductive development sees its funding withdrawn at rate $\omega + \phi$. The implication is that the expected capital cost associated with contract rate ω_l is[43]

$$K_l(\omega_l) = \left(1 + \frac{\kappa\,(1 - p_l)}{\rho + \omega_l + \phi}\right) \frac{k_l}{\rho + \kappa + \omega_l + \hat{\phi}_d} \tag{15}$$

In effect, this setup implies that the arrival of potential acquirers acts as a monitoring device. Intuitively, if the entrepreneur were to prevent all potential acquirers from conducting due diligence, the venture capital firm might learn that the start-up is of low quality. Similarly, if the VC never learned of any interest from potential acquirers–perhaps because the entrepreneur keeps this information private–they might downgrade their prospects about the firm and have an incentive to monitor. The assumption hopes to capture these ideas in a simple way.

The optimal contract is then found by solving the contract problem, analogous to equation (5), where the value functions are given by equations (13) and (14) and the capital cost by equation (15).

Financing market equilibrium. The equilibrium conditions mirror those for the baseline model, appropriately extended to allow for two stages. They are outlined in section D, along with the full set of equilibrium conditions for this version of the model.

Discussion. With the setup outlined, I highlight a key theoretical insight in the context of acquisitions: start-ups searching for capital accept acquisition offers at a higher rate than those with funding secured. This is formalised in the following proposition.

Proposition 2.7. *Given that the optimal contract rate is finite, $\omega < \infty$, the effective arrival rate of acquisitions is higher in search than in development: $\hat{\phi}_s > \hat{\phi}_d$.*

Proposition 2.7 illustrates that firms are more inclined to accept acquisitions while searching for funding, suggesting that acquisitions substitute for venture capital. The intuition is straightforward:

[43] See Appendix B for a full derivation. To motivate the assumptions imposed here, suppose that the process of an acquisition involves a number of steps. Firstly, the potential acquirer approaches the company and, at this point, the match-specific synergy, $\epsilon \sim F(\epsilon)$ is unknown to both the start-up and potential acquirer. Next, the potential acquirer does some due-diligence in order to determine ϵ. However, during this process of due-diligence, the potential acquirer also learns whether the project has hit a major roadblock and appears unlikely to reach fruition; that is, they learn whether the start-up is productive or unproductive. Finally, having learned this information, the parties negotiate over an acquisition price. I make the simplifying assumption that an unproductive entrepreneur cannot prevent the acquirer from conducting due diligence and learning that its project has failed. The venture capital firm knows that this due diligence process is ongoing and, therefore, also learns this information. Having learned this information, the venture capital firm cuts off funding.

a firm searching for capital has a less favourable outside option in acquisition negotiations, making it more willing to accept a broader range of acquisition offers.

Parameter	Description
ρ	Discount rate
p_e	Early-stage development level
p_l	Late-stage development level
k_e	Early-stage flow investment cost ($Mn USD)
k_l	Late-stage flow investment cost ($Mn USD)
κ	Result arrival rate
π	Payoff in case of success ($Mn USD)
λ	Failure rate in search
ϕ	Acquisition offer arrival rate
ξ	Acquisition price distribution parameters ($\epsilon \sim Exp(1/\xi)$)
ν_e	Early-stage meeting rate
ν_l	Late-stage meeting rate
$\tilde{\sigma}$	Entry cost distribution parameter

Table 1: Parameter definitions

Finally, a word on parameter choices across stages. This choice to permit some parameters to differ across stages, while others remain constant, reflects a balance realism and the ability to identify separately various parameters given the available data. In light of the second motivation, I have assumed that the rate of uncertainty resolution and of failure in search, κ and λ respectively, do not differ across stages. However, I allow $k_e \neq k_e$ and $\nu_e \neq \nu_l$. Gompers (1995) shows that later-stage firms raise larger amounts of capital, which motivates $k_e \neq k_l$ (I anticipate $k_e < k_l$ but do not impose it). Furthermore, to permit the financing risk to depend on the start-up's development stage, I allow $\nu_e \neq \nu_l$. Together with increasing capital requirements as the firm progresses, $k_e < k_l$, permitting differing meeting rates allows the model to capture parsimoniously the idea that VC funds in certain national markets may be incapable of funding a large number of 'big ticket' investments. For ease of reference, the parameters are listed in Table 1.

3 Quantification for the US VC market

In this section, I estimate the model of section 2.5 for the US. Despite it's simple structure, I show that the model is is able to capture a rich set of micro moments related to start-up funding patterns and outcomes. Using the estimated model, I provide estimates of the extent of financing risk and capital misallocation in the US venture capital market. To the best of my knowledge, these estimates are new to the literature.

3.1 Data

My sample includes all firms in the Thomson Reuters venture capital dataset from the US that received their first round of venture capital funding between 2005 and 2015 inclusive. A typical problem in analysing young firms is that insufficient time has passed since their inception to observe their final outcomes. By choosing to include firms receiving their first funding round between 2005 and 2015, I balance the desire for an up-to-date sample with the need to allow sufficient time to elapse since their birth. In order to maintain consistency across the data, I exclude any firm observations that occur more than seven years after their first funding round. This mirrors Ewens and Farre-Mensa (2020), who also consider the status of venture-backed firms seven years after their first funding round.

I apply a number of screens to the data. Financing rounds in the raw data are labelled as one of "seed", "early stage", "expansion", "later stage" or "bridge loan". I disregard bridge loans because their objective is to tie firms over while they raise their next funding round, rather than for conducting major development steps.[44] Next, I label seed and early-stage rounds as "Early" and expansion and later-stage rounds as "Late" and exclude the minority of firms that only have a "Late" funding rounds to maintain consistency with the model.[45] Furthermore, I collapse any two deals for a firm if they are coded with the same date and make a few manual corrections to the data when there are clear errors.[46] Finally, I face two distinct issues of missing data: investment amounts for funding rounds and deal values for some M&A transactions. For investment amounts, I follow Jagannathan et al. (2022) and impute capital injections.[47] For deal values, I complement the dataset with data from Crunchbase where possible. When the deal value remains missing–the majority case–I set it equal to 1.5 times the total capital invested, implying an exit multiple of 1.5X. This approach follows Kerr et al. (2014).

As discussed, a key modification to the baseline model when taking it to the data is to introduce acquisitions as a distinct form of exit from "successes". A success requires that a firm complete its development, whereas an acquisition allows a firm to sell a partially-developed project. In order to map exit outcomes to the model, I define an "acquisition" in the data as an M&A transaction by a non-financial buyer and a "success" as an IPO or an M&A transaction by a financial (including SPAC) buyer. The choice to distinguish between financial and non-financial buyer M&A transactions is motivated by the idea that a non-financial buyer typically has the capabilities to acquire start-ups that have not yet succeeded in commercialising their innovations, whereas a financial buyer is less likely to have such capabilities. This echoes the distinction in the model, in which a "success" requires that the firm reach full development. A summary of the final data is provided in Table 2.

Country	# firms	# rounds	# successes	# acquisitions
US	11,022	33,101	399	2,034
UK	1,066	2,219	26	97

Table 2: Data summary

3.2 Model estimation

I estimate the model in partial equilibrium, treating meeting rates ν_e and ν_l as parameters to estimate. Although less general, estimation of the model in partial equilibrium is sufficient to provide estimates

[44] Despite their labelling, "bridge loans" reported in Thomson Reuters are equity investments.

[45] I provide definitions for the round types that I include in section C.1 of the Appendix. In seed and early-stage rounds, firms are still very much in the ideation and testing stage, whereas firms in the expansion and late stages are attempting to commercialise their projects. The expansion stage is also the point at which institutional investors consider financing start-ups, which offers a degree of market segmentation in the capital market that will map to the model.

[46] For instance, on some occasions Thomson Reuters reports IPOs that were announced by never occurred.

[47] This applies to in approximately 10% of rounds for the US (and slightly more for the UK). The imputation approach is described in section C.2 of the Appendix.

of the extent of financing risk and capital misallocation in the US market and avoids taking a stance on the functional form of the matching function.

Estimation strategy and identification. Table 1 lists the parameters to be estimated. I fix the discount rate $\rho = 0.08$, which is similar to the value used in Jovanovic and Szentes (2013). Towards a strategy for estimating the remaining twelve parameters, note that $V_{s,e}$ is independent of $\tilde{\sigma}$, conditional on ν_e and ν_l. Defining Θ as the set of all parameters to estimate excluding $\tilde{\sigma}$, this implies that $\tilde{\sigma}$ is pinned down uniquely by a free-entry condition, given Θ. In practice, entry is unobserved, but we do observe a closely-related object, the number of first funding rounds per year. In steady state and given Θ, it follows that[48]

$$\tilde{\sigma} = \Big(\frac{\nu_e}{\lambda + \nu_e} \times \frac{1}{\text{flow first-time rounds}}\Big) \times V_{s,e} \tag{16}$$

The estimation strategy then involves two steps. Firstly, the parameters Θ are estimated by minimising the objective function

$$C(\Theta) = \sum_{j \in \Theta} \left(\frac{\tilde{m}_j - m_j(\Theta)}{\frac{1}{2}|\tilde{m}_j| + \frac{1}{2}|m_j(\Theta)|} \right)^2 \tag{17}$$

where $\tilde{m}_j$ is the empirical moment and $m_j(\Theta)$ its model counterpart. Given Θ, $\tilde{\sigma}$ is recovered from equation (16) given data on the number of first funding rounds. For the US, I normalise this to one.[49]

I now discuss the choice of moments to include in $C(\Theta)$. Although the parameters are identified jointly by all moments that enter equation (17), specific moments are particularly informative about certain parameters. The moments are presented with their model and data values in Table 3. Consider first the meeting rates, ν_e and ν_l. In a typical search environment, search is observed and these rates can be read directly from the data.[50] In the setting considered here, search is latent and this creates an identification challenge. Proposition 2.4 provides a route forwards because it enables direct estimation of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ from the data on the duration between funding rounds. In short, the model characterises the distribution of the duration between funding rounds as hypoexponential and the parameters of this distribution can be estimated directly from the data via MLE.[51] I provide more details on this approach in Section C.3 of the Appendix and include these estimates as targetted moments with the aim of providing information on ν_e and ν_l.

Relatedly, Corollary 2.5 states that the duration between rounds is increasing in p; since most fund-

[48] To see this, note that the measure of firms that have paid the entry cost but not yet engaged in a funding round evolves according to

$$\dot{\mu}^0_{s,e,t} = \frac{V_{s,e,t}}{\tilde{\sigma}} - (\lambda + \nu_e)\, \mu^0_{s,e,t}$$

which implies the steady-state condition

$$\tilde{\sigma} = \Big(\frac{1}{\lambda + \nu_e} \times \frac{1}{\mu^0_s}\Big) V_{s,e}$$

where μ^0_s is the steady-state measure of firms that have paid the entry cost but not yet obtained VC funding. μ^0_s is unobserved in the data, but the empirical counterpart to $\nu_e \times \mu^0_s$ is the flow of first funding rounds, which is observed. Substituting in for μ^0_s then yields equation (16).

[49] The choice of normalising constant is irrelevant, given that I am estimating ν_e and ν_l directly. For the UK case-study, I consider the level relative relative to the US (adjusted on a per-capita basis).

[50] For instance, in a frictional labour market setting, those searching for work are tagged as unemployed. Unemployment durations then provide direction information on the meeting rate.

[51] In the late stage, the distribution of the time between funding rounds is $T_{br} \sim Hypo(\kappa + \omega_l + \hat{\phi}_d, \lambda + \nu_l + \hat{\phi}_s)$.

Parameter	Estimate	Moment	Data	Model
p_e	0.17	Mean duration b/w rounds	1.16	1.20
p_l	0.26	Mean # late rounds | progressed to late	2.45	2.46
k_e	2.26	Median burn rate	4.44	4.43
k_l	17.82	Share of capital in late-stage	0.61	0.60
κ	0.30	Share of firms to late-stage	0.42	0.41
π	316.88	Mean # funding rounds	3.00	3.09
λ	0.37	Share receiving funding after year 5	0.20	0.19
ϕ	2.17	Acquisition-to-success ratio	5.10	5.07
ξ	0.20	Share acquisition multiples > 10X	0.08	0.08
ν_e	2.92	Estimate of $\lambda + \nu_e$	3.28	3.29
ν_l	5.20	Estimate of $\lambda + \nu_l + \hat{\phi}_s$	6.00	6.00
σ	4.34	# first funding rounds	1.00	1.00

Table 3: US parameter estimates and targeted moments

The table provides parameter estimates and various targeted moments for US data and from the US estimation. Moments in both cases are computed based on censored data, where observations on firms that occur more than seven years after their first funding round are excluded. The duration between rounds is measured in years, and the burn rate is in units of Mn 2015 USD per year.

ing rounds are early-stage rounds, the mean duration between funding rounds is particularly informative of p_e. To illicit information on p_l, I take the insights of Proposition 2.3 and target the average number of late-stage funding rounds conditional on reaching the late-stage, which I expect to be decreasing in p_l. The conditional, rather than unconditional, average removes the effect of p_l on the probability of reaching the late-stage, which is ambiguous.[52] To pin down k_e, I target the median burn rate, where the burn rate is defined as the rate of cash utilisation between funding rounds.[53] Targeting the median allows for limiting the effect of some incredibly high burn rates in the true data that are not representative and is a statistic more closely related to early-stage rounds, since most rounds are early-stage. For reasonable parameter values, the median burn rate is increasing in k_e.[54] To obtain information on k_l, I target the share of capital that is raised by late-stage firms, which is typically increasing in k_l.

Consider next λ, which I would ideally identify using information on failures. However, such data is generally unreliable as many firms remain "living-dead" (Kerr et al., 2014). Therefore, I target the share of firms that undergo a funding round more than five years after their first. Intuitively, as λ increases, firms that have to search for follow-on funding are more likely to fail from a lack of capital and so not survive to the five-year mark. Furthermore, Proposition 2.3 shows that the expected number of funding rounds is decreasing in λ, so that they are less likely to have a funding round in any given period. For ϕ, I target the ratio of the number of firms who are acquired relative to those that have a success, as I expect a larger set of potential acquirors (higher ϕ) to make acquisitions are more common

[52]As p_l increases, $\hat{p}_e$ falls, so firms that realise a result in the early-stage are less likely to progress. Furthermore, this puts upward pressure on ω_e by Proposition 2.2. However, at the same time $V_{s,l}$ rises, which is equivalent to an increase in π from the perspective of Proposition 2.2, and so puts downward pressure on ω_e. Overall, the effect on ω_e is ambiguous and this translates to an ambiguous effect of p_l on the share of firms that reach late-stage development.

[53]For example, if a firm raises \$10Mn today and raises capital again in two years, then its burn rate is \$5Mn/year. In the model, I compute the burn rate for a given round by dividing the total amount of capital raised, $K_i(\omega_i)$ for $i \in \{s, d\}$, by the duration between successive rounds.

[54]Increasing k_e reduces the time between rounds, pushing up the burn rate. However, Corollary 2.3 shows that $K_e(\omega_e)$ is ambiguous with respect to k. Specifically, $K_e(\omega_e)$ becomes decreasing in k_e as ω_e becomes very large, but for parameter values consistent with the duration between funding rounds, this effect is dominated.

exit strategy. For ξ, which is the mean of the exponentially-distributed 'synergy' distribution, ϵ, I target the tail of the acquisition exit multiple distribution, where the exit multiple is defined as the sale price divided by the total capital invested. Specifically, I target the share of acquisitions that have an exit multiple in excess of 10X, meaning that their sale price was more than ten times larger than the total capital invested.[55] As the average synergy rises, we should expect this share to rise. Next, for π, I target the average number of funding rounds, which Proposition 2.3 suggests is decreasing in π.[56]

Identification of κ is challenging because of the two opposing forces outlined in Proposition 2.2, leading to an ambiguous relationship between κ and ω. The theory, therefore, offers only limited guidance in choosing a relevant target moment. In the estimation exercise, I use the share of firms that reach the late-stage, which I expect to be increasing in κ in general.[57]

Model validation. As Table 3 demonstrates, the model effectively captures the targeted features of the venture capital data. Table 4 highlights several moments that were not directly targeted in the estimation, with four key observations worth noting. First, while the estimation targets the ratio of acquisitions to successes, it does not explicitly target the total number of exits—whether by acquisition or success—yet the model replicates this measure. Second, the model replicates time-to-exit dynamics: the mean time-to-exit is just over 3.67 years in the data and 3.32 years in the model. Furthermore, the model accurately reflects the fact that the mean time-to-success exceeds the mean time-to-acquisition. This is achieved despite the time-to-exit distribution being left fully untargeted. Third, the model performs impressively in capturing the average exit multiple, even though only the tail of the exit multiple distribution for acquisitions was included as a target. Finally, the model correctly reflects the pattern that early-stage rounds tend to have longer durations than late-stage rounds in the censored sample. While the duration of late-stage rounds is somewhat longer in the data than in the model, the overall pattern remains well-replicated.

Figure 4 demonstrates that the model not only replicates basic untargeted moments but also accurately captures the entire distribution of key features of venture capital funding and exit data. First, both the data and the model show a monotonically decreasing PMF for the number of funding rounds. This need not necessarily be the case: if project uncertainty were concentrated in later stages, most start-ups would secure multiple rounds, leading to an initially increasing PMF in the model.[58] Second, the model closely replicates the CDFs of the time of each funding round, $N_f = 1, 2, 3$, etc. and in both the model and data, the mean time to funding round N_f increases with N_f. Empirically, this may not occur if there were substantial ex ante heterogeneity in start-up funding patterns. For example, if firms in some sectors progress quickly through four funding rounds, while firms in the dominant sector only ever require three rounds over a longer period of time, then the mean time to $N_f = 3$ would exceed that to $N_f = 4$ due to selection. Third, the model captures the fact that the burn-rate distribution is heavily right-skewed, although it does fail to capture the full thickness of the right tail of the distribution. Nonetheless, this is interesting because the burn rate in the model is driven predominately by the duration between rounds, since all firms raise the same amount of capital within stage. In reality,

[55] Reporting requirements imply that large acquisition values are typically well-reported, so this is feasible.

[56] An alternative would be to use the average exit multiple. However, in practice moments such as the mean exit multiple are very sensitive to outliers and data on exit values is poor.

[57] See section C.3 for a brief discussion of the argument.

[58] In the baseline model, the number of funding rounds follows a Geometric distribution. However, this is only true *within each stage* in the quantitative model, which can capture more complex dynamics.

Moment	Data	Model
Share successful	0.04	0.04
Share acquired	0.18	0.20
Share late rounds	0.34	0.33
Mean exit multiple	6.28	5.64
Mean time-to-exit	3.67	3.32
Mean time-to-success	4.43	3.45
Mean time-to-acquisition	3.52	3.29
Mean # early funding rounds	1.98	2.08
Mean # funding rounds | success	4.26	3.99
Mean # funding rounds | acquired	2.62	3.63
Mean # funding rounds | no exit	3.04	2.90
Mean duration b/w rounds | current round early	1.21	1.42
Mean duration b/w rounds | current round late	1.06	0.66

Table 4: US untargeted moments

The table provides a set of untargeted moments for US data and from the US estimation. Moments in both cases are computed based on censored data, where observations on firms that occur more than seven years after their first funding round are excluded.

both the determinants of the burn rate can exhibit substantial variation, in effect offering an additional degree of freedom relative to the model. Finally, the model captures well the shapes of both the exit multiple distribution and the time-to-exit distribution, which is left fully untargeted in the estimation. The ability of the model to replicate such rich patterns in the data provides strong support for its underlying mechanisms and highlights its relevance as a framework for understanding venture capital funding and exit dynamics.

One concern with the estimation approach is that the choice of censoring date, set at seven years, may materially affect the results. Specifically, divergence in the timing of funding rounds and the time-to-exit distribution between the model and data could lead the estimation process to select markedly different parameter values as the censoring threshold is varied. Figure 4 shows that the model and data align well along these margins, as well as in the timing of the final event (funding round or exit) of each firm in the sample, addressing these concerns.[59] A second concern is that attempting to directly estimate $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ directly from data on the duration between funding rounds imposes an unreasonable level of structure on the data. The close mapping between the model and the true distribution of the duration between funding rounds is supportive of the approach taken.[60]

Discussion. I close this section with a brief discussion of the parameter estimates to aid interpretation and as an informal validation check; I focus on the advancement probabilities, p_e and p_l, and the financing market meeting rates, ν_e and ν_l. The estimate $p_e = 0.17$ implies that in the absence of acquisitions and with perfect financing markets, 17% of start-ups would be successful. This is consistent with the high degree of risk associated with innovative projects. The estimate of p_l implies that, conditional on

[59] I have conducted robustness checks regarding this cutoff, and the results remain quantitatively unchanged. While these are not reported here, they will be included in future versions of the paper.

[60] In the baseline model, the distribution of the duration between funding rounds is hypoexponential. In the quantitative model, this is only true within stage, so it is not immediate that the model should capture the shape of this distribution so tightly.

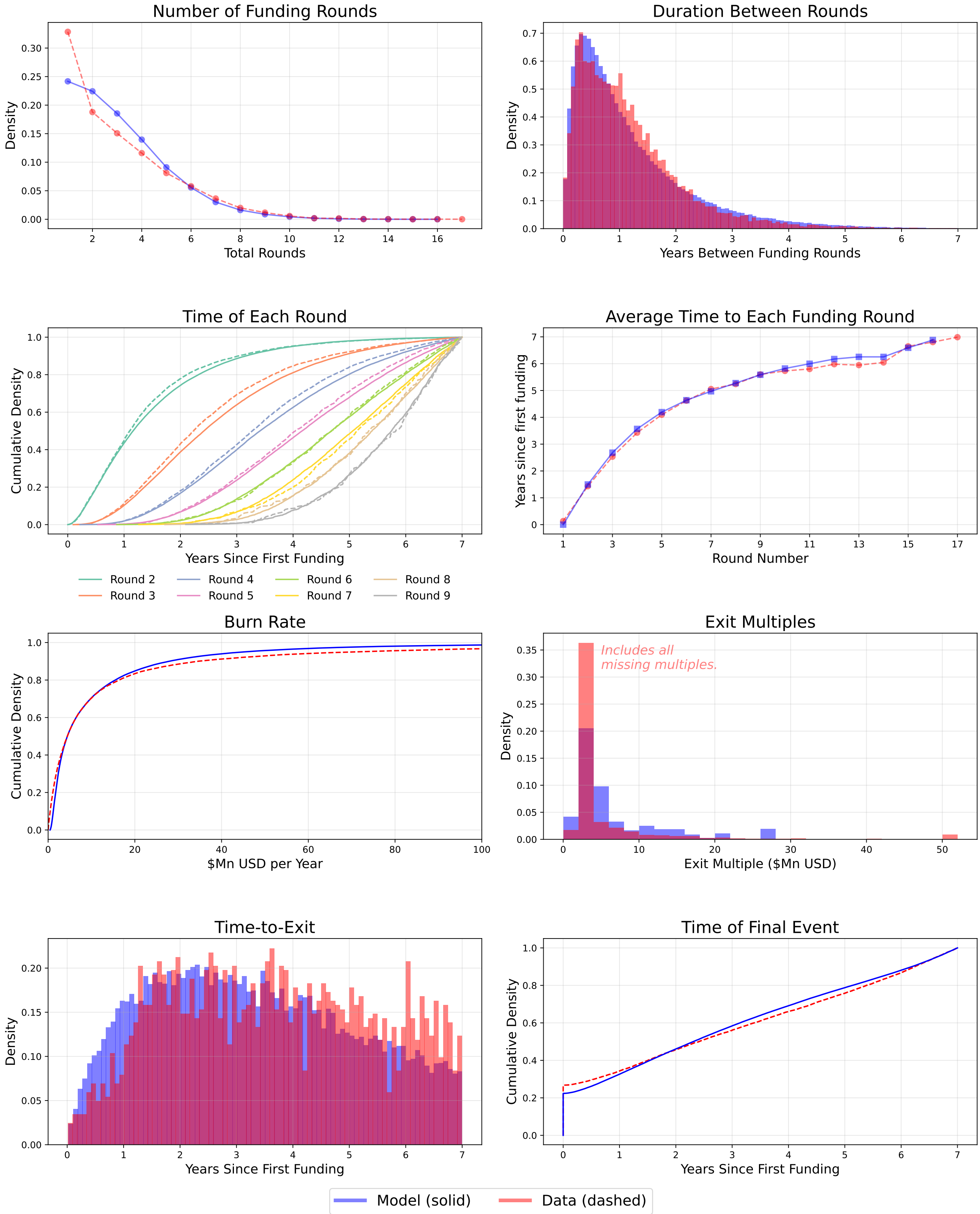

Figure 4: Model validation

The figure depicts a series of outcomes in the data and model for the US. All data is censored seven years after a firm's first funding round. All missing values for exit multiples in the data are set at 1.5X, as in Kerr et al. (2014).

completely early-stage development, firms proceed to the late-stage in 65% (=0.17/0.26) of cases. As a sense check, consider data on the progression rates for clinical drug trials in the United States. Biotech is one of the key industries into which VCs invest and early-stage funding is typically sought in order to complete Phase 1 clinical trials.[61] Although biotech is unlikely to be fully representative of the risk profile of all start-ups, by considering the rates at which drugs typically pass from Phase 1 to Phase 2 and to final approval, we can obtain a model-free counterpart to p_e and p_l that can be used to validate the parameter estimates. Hay et al. (2014) exam the success rates for over 7,300 drug development paths from 835 drug developers, including young biotech and large pharmaceutical companies between 2003 to 2011. They find that the probability of advancing from Phase 1 to final approval is between 10-15%, close to the 17% estimated in the model. Furthermore, the probability of advancing from Phase 1 to Phase 2 is in the range 64-67%, consistent with the early-to-late progression rate of 65% implied by the model.[62]

Finally, turning to the financing market meeting rates, the estimates imply that an early-stage firm would expect to secure funding after an average of 4.1 months and a late-stage firm after an average of 2.3 months.[63] In a guide for start-ups, Pitchbook suggest that "some startups are able to raise the capital they need in three months, while others may take a year", so that these figures seems consistent with the expectations of market participants.[64] Furthermore, the fact that late-stage firms are able to obtain funding more quickly is consistent with the view that there is more information available to investors and so matching frictions and coordination issues are less severe.

3.3 An analysis of the US VC market

In this section, I use the model to analyse the US venture capital market, focussing primarily on estimates of the extent of financing risk and capital misallocation in the market. These metrics give a sense of the cost financial frictions considered in this paper.

Financing risk. Panel A of Table 5 reports the main results related to financing risk. Note that the figures come from uncensored data, so that the share of successes and acquisitions exceed the figures reported in Table 4. In total, model estimates imply that 69% of firms in the sample would experience no successful exit if tracked indefinitely. Among these failures, roughly one third ($\approx 0.24/0.69$) fail due to a lack of funding rather than as a result of realising negative information about their project. Put differently, the model estimates imply that 24% of firms that obtain venture capital funding fail due to an inability to secure follow-on funding when they possess positive NPV projects; that is, they fail due to financing risk. To the best of my knowledge, this constitutes the first estimate of the extent of financing risk in the US venture capital market.

The fact that a significant share of firms fail due to financing risk is somewhat surprising given that financing risk is typically insurable. Entrepreneurs and investors are often concerned with the so-called "second valley of death" (see e.g. Wilson et al. (2018)), which occurs when start-ups transition from the early to late-stage and seek higher volumes of capital in order to commercialise their innovations.

[61] See *Betting on Biotech, Pitchbook, 2020.*

[62] See Figure 1 from Hay et al. (2014).

[63] These are the unconditional wait times, computed simply by inverting the meeting rates. The conditional probabilities would take into account the diverging failure and acquisition rates.

[64] See *PitchBook's guide to VC fundraising for startups, Pitchbook.* Pitchbook is a leading venture capital data provider.

Panel A: Outcomes and financing risk	
Share successful	0.05
Share acquired	0.25
Share failed	0.69
... that obtained a negative result	0.45
... that failed to find funding	0.24
... in the early-stage	0.15
... between the early and late stages	0.03
... in the late-stage	0.06
Panel B: Capital misallocation	
Share unproductive | early-stage	0.18
Share unproductive | late-stage	0.06
Share unproductive	0.16
Share capital misallocated	0.11

Table 5: Financing risk and capital misallocation

Figures are computed from the baseline US calibration and data is uncensored, so that all firms are ultimately successful, acquired, or fail. The share of firms that fail due to a lack of funding "between the early and late stages" refers to firms that have completed early-stage development but are yet to secure their first late-stage funding round. The firms "in the late-stage" are late-stage firms that fail in search but that have already had at least one round of late-stage funding.

In the model, failure at this point is uninsurable because VCs and entrepreneurs in the model are not permitted to sign contracts that span both stages. However, while the estimates suggest that 24% of all firms fail due to financing risk, Table 5 reports that just 3% fail due to this uninsurable financing risk. Instead, the majority of start-ups fail due to financing risk that is insurable; that is, they *could* have written contracts with $\omega_e = 0$ and $\omega_l = 0$ and prevented their failure, but chose not to.

Capital misallocation. Panel B of Table 5 reports the main results related to capital misallocation. Within each stage, capital misallocation is given by equation (11). In the aggregate, misallocation can be computed based on the share of unproductive firms receiving capital or the share of capital allocated to unproductive firms. Both figures are reported in the Table. Overall, 11% of capital is misallocated, whereas 16% of firms are unproductive. The discrepancy results because early-stage firms utilise less capital and but are more likely to be unproductive. Together with the results on financing risk, these estimates suggest that there would be large gains associated with eliminating financing risk.

4 Case study: the UK funding gap

Despite the inefficiencies highlighted in this paper, venture capital plays a pivotal role in driving innovation in the United States. Among US public companies founded in the previous 50 years, firms that received venture capital funding in their infancy account for over 92% of total R&D spending (Gornall and Strebulaev, 2021). However, in many other developed countries, venture capital has not had the same impact. The reasons for this remain somewhat unclear. On the one hand, lower levels of VC activity and weaker performance could be explained by systematic issues in unlocking risk capital, which limits the availability of funds necessary for scaling businesses. On the other hand, it may simply be that there are fewer high-quality investment opportunities, resulting in lower overall activity and fewer successes. The model offers a way to disentangle these competing explanations by tracing their implications to observable features of venture capital funding data.

In this section, I use the model to explore this issue in a case-study of the United Kingdom. Its long history as a financial centre, comparable legal and financial institutions, and strong university infrastructure make it somewhat surprising that the UK has failed to fully harness the benefits of venture capital. Table 6 provides an overview of key differences between the US and UK markets for the period 2005-2015.[65] On a population-adjusted basis, the US venture capital market services roughly twice as many new firms each year compared to the UK. Furthermore, these firms are more likely to have completed a successful exit (including acquisition) within seven years of their first funding round.

	# first funding rounds	Share successful	Share acquired
United States	1.00	0.036	0.185
United Kingdom	0.48	0.024	0.091

Table 6: UK and US start-up activity and outcomes

The table provides an overview of the key aggregate statistics in the United States and United Kingdom. The # first funding rounds refers to the number of firms receiving their early-stage first funding round between 2005-2015, adjusted for differences in population and normalised by the US level. All data is truncated at seven years following the first funding round for the firm. For figures on the actual number of firms, funding rounds and exits, refer to Table 2.

Various explanations have been put forward for these differences. In the European context as a whole, Quas et al. (2022) distinguish between supply-side, demand-side and ecosystem factors, which provides a useful taxonomy.[66] In what follows, I briefly discuss these factors, the extent to which they feature in the model, and their implications for observable features of the data.

Supply-side factors. Supply-side factors refer to the financing conditions facing start-ups seeking VC funding.[67] In the model, financing conditions are captured by the parameters λ, ν_e and ν_l. The meeting rates, ν_e and ν_l, capture the ease with which viable start-ups can raise VC funding, which may differ between the early and late-stages, when firms typically require higher volumes of capital. Furthermore, the failure rate in search, λ, may reflect differences in the availability of alternative funding instruments, such as venture debt, which is used to tie firms over between equity injections (González-Uribe and Mann, 2024). If supply-side factors are first-order, then UK funding patterns should show marked differences with the US along a number of margins. With more limited access to capital, firms that do raise capital should be increasingly concerned about financing risk. This should lead them to raise more capital, relative to their annual capital requirements, completing fewer funding rounds with a longer duration between each round.

Demand-side factors. Demand-side factors refer to the demand for capital by entrepreneurs, which is effected by the quantity and quality of start-ups seeking funding. In the model, the entry cost parameter,

[65] See Table 2 for the number of firms, funding rounds and exits in each country.

[66] For an analysis and comparison of the exit patterns in Europe and the US, see Axelson and Martinovic (2016).

[67] The UK market has had less time to develop than the US market and more experienced investors are typically able to raise more funds from limited partners (Gompers and Lerner, 1999), which suggests that UK VCs may lack the experience to raise substantial amounts of capital. Furthermore, UK pension funds allocate a much lower share of their capital to private equity compared to their international peers, suggesting that VCs themselves may not have access to large pools of capital (see *Comparing the asset allocation of global pension systems*, New Financial, 2024)).

$\tilde{\sigma}$, affects the rate of new project creation. In practice, this may be influenced by the willingness and capability of entrepreneurs to create new firms but also by the aggregate level of innovation in the economy.[68] For entrepreneurs that do create new firms, p_e, p_l and π parameterise the likelihood of success and payoff in case of success, respectively, and so control the quality of these start-ups.

To see the implications of these factors for start-up funding patterns, consider first changes to $\tilde{\sigma}$. In partial-equilibrium, an increase in $\tilde{\sigma}$ reduces the number of firms receiving funding, but has no other effect. However, when the meeting rates, ν_e and ν_l respond in equilibrium, the lower entry rate leads to looser financing conditions for the start-ups that do enter.[69] With looser funding conditions, the implications for funding patterns would be the opposite as those discussed for supply-side factors. Similar arguments apply to project quality. To generate lower entry and worse start-up outcomes, projects in the UK would need to be inherently less likely to succeed, or have lower monetary payoffs. Such projects would demand less insurance against financing risk; they either face more severe agency frictions, or are generally less valuable and so demand less insurance because it is costly. In either case, these forces to lead firms to visit the capital market more frequently.

Ecosystem factors. Ecosystem factors refer to features of the wider environment, including the availability high-skilled labour, the regulatory and tax environment, and market size. Relatedly, the availability of exit opportunities, either by way of IPO or acquisition, impacts the incentive of entrepreneurs to pursue new projects. In the model, acquisition opportunities are captured by the arrival rate of potential acquirors, ϕ, and the acquisition 'synergy' parameters, ξ. Together, these parameters determine the quantity of potential acquirors and the quality of the offers that these acquirors make to start-ups. Furthermore, beyond capturing some demand-side factors, p_e is related to other issues that limit growth potential, such as the availability of high-skilled labour and regulatory factors.[70] Finally, the payoff to successful projects, π, is associated with project quality but also captures factors external to the firm, such as market size, taxation, and the value of exit opportunities, particularly via IPO.

In mapping these factors to funding pattern data, the arguments put forward for demand-side factors apply also to some ecosystem factors, particularly those related to p_e and π. Furthermore, the availability of a market for firms has implications for modes of exit. If firms in the UK find it systematically more difficult to exit via way of acquisition, acquisitions should be a relatively less likely mode of exit.

4.1 Identification exercise

To distinguish between competing explanations for the UK's gap to the US, I analyse differences between funding pattern data in the two countries through the lens of the model. Table 7 reports the targeted moments used in the US quantification alongside their values in the analogous UK sample. Notably, the duration between funding rounds is approximately 14 months (1.16 years) in the US, compared to more than 17 months (1.46 years) in the UK. Furthermore, UK VC-backed start-ups complete fewer funding rounds within the first seven years of funding, with an average of 2.08 compared to 3.00 for US

[68] Risk appetite, availability of management skills, the broader entrepreneurial culture, and the willingness to raise equity financing and so forego control can be important here Quas et al. (2022). Furthermore, Gompers et al. (2005) show that established innovate firms spawn entrepreneurs, who go on to create their own companies and seek VC funding. Such dynamics can generate increasing returns to entrepreneurial activity.

[69] In this thought experiment, the measure of VCs, M, is held constant at its US level as $\tilde{\sigma}$ increases.

[70] Recall that in the baseline model, p, captures the probability of success when there are no financing frictions.

Moment	US Data	UK Data
Mean duration b/w rounds	1.16	1.46
Mean # late rounds | progressed to late	2.45	1.94
Median burn rate	4.44	2.72
Share of capital in late-stage	0.61	0.62
Share of firms to late-stage	0.42	0.33
Mean # funding rounds	3.00	2.08
Share receiving funding after year 5	0.20	0.12
Acquisition-to-success ratio	5.10	3.73
Share acquisition multiples > 10X	0.08	0.11
Estimate of $\lambda + \nu_e$	3.28	2.13
Estimate of $\lambda + \nu_l + \hat{\phi}_s$	6.00	2.99
# first funding rounds	1.00	0.48

Table 7: Targeted moments: cross-country comparison

The table provides comparisons between the UK and US for the set of moments used to estimate the model for the US in section 3.2. All data is censored at seven years.

VC-backed start-ups. Another key difference lies in modes of exit: for every 'success', there are over five acquisitions for US VC-backed start-ups but under four for UK VC-backed start-ups.

The differences in UK and US funding patterns and modes of exit point to issues in the funding market and the availability of acquisition opportunities as the key drivers of gap between the UK and US. Indeed, UK start-ups complete fewer rounds with longer durations between each round, suggesting that insurance against financing risk is a primary concern. Furthermore, the fact that UK start-ups are relatively less likely to exit via acquisition is suggestive of a more limited market for firms. By contrast, if demand-side and ecosystem factors (excluding the acquisition market) were first-order, then UK start-ups would not find it optimal to secure longer funding horizons than their US counterparts. In section E of the Appendix, I investigate these explanations more formally. I show that such explanations are not only inconsistent with the number of and duration between funding rounds, but a swathe of additional moments.

To more formally investigate the hypothesis that issues in the funding market and the availability of acquisition opportunities are central issues for UK start-ups, I conduct the following exercise. I begin with the parameter values estimated for the US and then re-estimate the model parameters related to funding conditions and acquisition opportunities in order to match UK data. I do this in two steps, firstly re-estimating just the parameters related to financing (λ, ν_e, ν_l), and then re-estimating the parameters related to both financing and acquisitions $(\lambda, \nu_e, \nu_l, \phi, \xi)$. To estimate these parameters, I target the UK values of the moments I claim provide identification, as set out in section 3.2. For instance, when targeting ϕ, I target the ratio of acquisitions to successes.

Table 8 reports the parameter estimates from this exercise. In both re-estimations, funding markets are tighter than in the US baseline, reflected by lower values for ν_e and ν_l, and start-ups survive less time in search without capital, a higher λ. When additionally re-estimating acquisition opportunities, UK start-ups have fewer potential acquisition opportunities, a lower ϕ, but slightly higher synergies, a higher ξ.[71] To make the estimated differences in funding markets concrete, it takes approximately twice

[71] One explanation for this is that UK start-ups, realising that they do not have access to sufficient capital to scale-up,

Parameter	λ	ν_e	ν_l	ϕ	ξ
US: baseline	0.37	2.92	5.20	2.17	0.20
UK: financing	0.51	1.61	1.94	2.17	0.20
UK: financing & acquisitions	0.57	1.49	2.64	0.54	0.28

Table 8: Counterfactual exercise: parameter estimates

The table reports the parameter estimates for parameters related to financing (λ, ν_e, ν_l) and acquisitions (ϕ, ξ) in the US baseline and two counterfactual scenarios. In 'UK: financing', λ, ν_e and ν_l are re-estimated to match UK data on the relevant moments, and in 'UK: financing & acquisitions', $\lambda, \nu_e, \nu_l, \phi$ and ξ are re-estimated.

as long to secure funding in the UK counterfactuals relative to the US in both the early and late stages. Figure 5 compares moments in the UK data against moments from the UK counterfactual economies. The set of moments included in the figure are the set of targeted moments from the US estimation and the share of successes and acquisitions.[72] The moments are normalised by the value in the US data, such that values less than 1.0 indicate that the moment is less than the US value, and values greater than 1.0 are higher than in the US data. Finally, circular markers indicate that the moment was untargeted in the re-estimation exercise, and triangles indicate moments that are targeted.[73]

First, consider the left panel of Figure 5, which shows the results of the exercise in which the financing market parameters are re-estimated in isolation. Tighter financing conditions lead to less entry and a lower share of successful firms, consistent with the UK data. Furthermore, tighter funding markets are consistent with features of UK funding patterns, including, but not limited to, the mean duration between funding rounds and the mean number of rounds. Overall, the fit to the UK data is very good, suggesting that financing conditions are a proximate cause of the differences. However, funding conditions alone seem unable to explain acquisition outcomes in the UK. This is unsurprising since the share of acquisitions is unlikely to be highly responsive to the funding market: while firms may be more willing to accept offers as acquisitions substitute for financing, a lack of funding may lead to higher failure rates, reducing overall exits. The right side of 5 re-estimates both funding conditions *and* acquisition opportunities and provides a better fit to the data. The fit is not perfect; in the counterfactual economy, entry is slightly below the true UK level. Nonetheless, the combination of more limited access to funding and acquisition opportunities appears to be consistent not only with lower levels of activity, but also features of UK funding patterns. Together, this suggests that these factors are of first-order importance in explaining the differing levels of activity and start-up outcomes between the UK and US.

To form an idea for the magnitude of the issues, I explore the drivers of UK start-up failure in the counterfactual economy. In section 3, I estimate that among the 69% of start-ups that fail, roughly one-third (35%) of failures are due to financing risk.[74] In the UK counterfactual economy that is hampered by both a weaker financing market and more limited acquisition opportunities (third row of Table 8), roughly 85% of firms fail and roughly two-thirds (63%) of these failures result from an

augment their projects to become more attractive to potential acquirors. However, the estimated difference is small.

[72] Section E.2 in the Appendix reports additional moments, specifically those considered in Table 4.

[73] Specifically, when re-estimating λ, ν_e and ν_l in the left panel, the "share receiving funding after year 5", "estimate of $\lambda + \nu_e$" and "estimate of $\lambda + \nu_l + \hat{\phi}_s$" were targeted. When additionally estimating ϕ and ξ, the "acquisition-to-success ratio" and "share acquisition multiples $>$ 10X" were also included as targets.

[74] Recall from Table 5 that 24% of start-ups in the US are estimated to fail from financing risk.

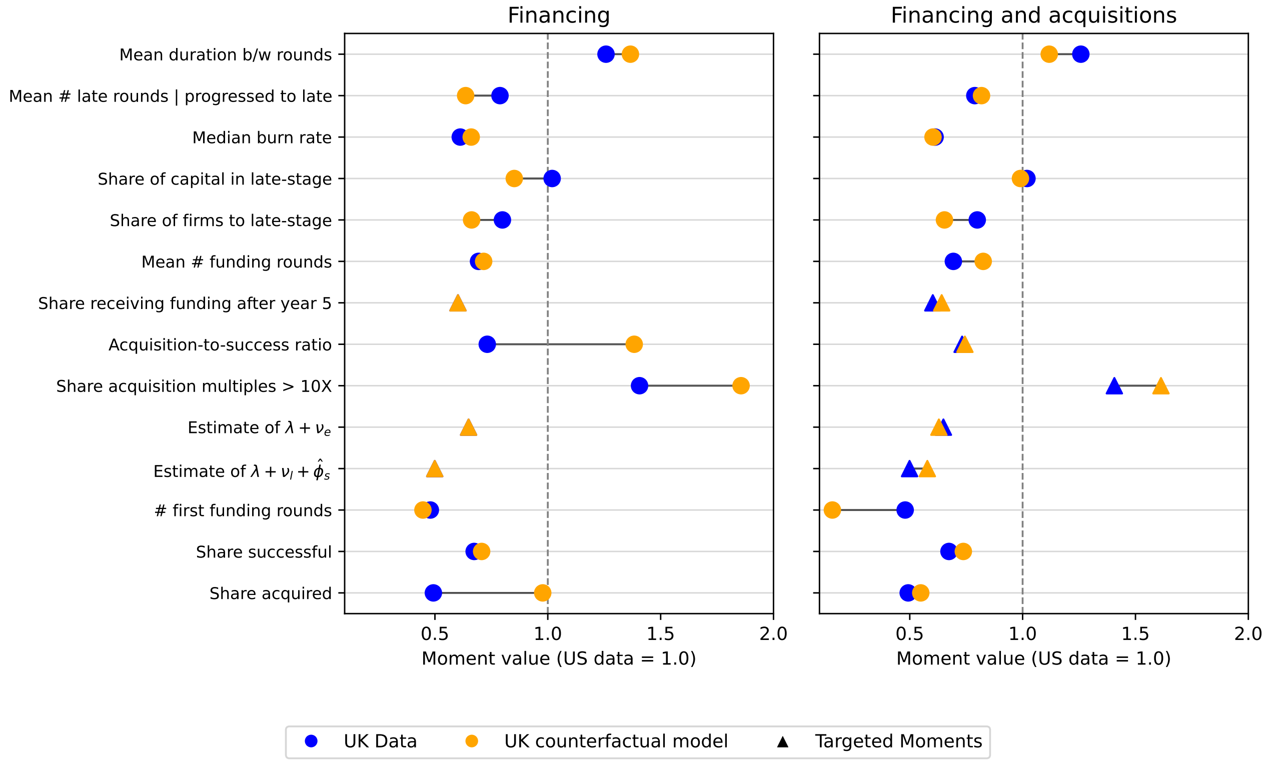

Figure 5: Re-estimation exercise

The figure compares data for the UK with data from the model under various re-estimation exercises. The left panel shows the results when the financing parameters, (λ, ν_e, ν_l), are re-estimated to match data for the UK and the right panel shows the results when the financing parameters and acquisition parameters, $(\lambda, \nu_e, \nu_l, \phi, \xi)$, are re-estimated. Circular markers indicated moments that are untargeted in the re-estimation exercise and triangular markers indicate targeted moments.

inability to secure follow-on funding.[75] This suggest that limited funding and acquisition opportunities increase the likelihood of failure by more than 15 percentage points for UK start-ups relative to their US counterparts.

In practice there are other differences between the UK and US that are not accounted for in this exercise, and the inclusion of such differences may reduce the extent of financing risk estimated here in the UK counterfactual. Nonetheless, the estimation provides strong evidence that UK start-ups face substantial issues in access VC funding. Importantly, the interaction between limited acquisition opportunities and a tight financing market is particularly harmful. In the economy where only financing market conditions are re-estimated (second row of Table 8), 75% of firms fail and roughly one-half (52%) of these failures result from an inability to secure follow-on funding. This highlights the theoretical point made in section 2.5 that acquisitions substitute for financing and help start-ups them to overcome financing risk.

Discussion and external validity. The analysis suggests that UK start-ups require more support in accessing follow-on funding and could benefit from a more vibrant market for firms. Hellmann and Kavadias (2016) also argue that financing conditions are a primary determinant of the issues facing UK VC-backed start-ups.[76] They highlight that UK start-ups secure larger investment rounds from

[75] In line with Table 5, these figure refer to uncensored data.

[76] In a related empirical exercise, Wilson et al. (2018) considers the inability of UK start-ups to secure follow-on funding,

US investors than from domestic ones and that UK VC firms typically raise smaller funds, inhibiting their ability to make later-stage investments.

Relative to their analysis, the use of an equilibrium model with endogenous financing contracts allows me to generate predictions for a broad swathe of observable features of the data and to use these predictions to distinguish between competing explanations. The results of this analysis point strongly towards an explanation that has financing conditions and acquisition opportunities at its centre. Nevertheless, the identification relies heavily on the notion that funding patterns reveal information about the funding conditions facing start-ups when they raise capital. In particular, I have argued that UK start-up funding patterns—such as the frequency of VC funding rounds and the time between raises—are indicative of tight financing conditions and an underdeveloped VC market. To the extent that this is true, there should be similar patterns in the United States when the VC market was in its infancy. Observing such patterns would provide further validity to the approach.

To investigate this, I extend my sample back to the early days of the US VC market. The modern VC firm traces its origins back to the creation of the American Research and Development Corporation in 1946, but the industry remained small for several decades, through the 1970s (Gompers, 1994). In 1979, a clarification to the Employee Retirement Income Security Act (ERISA), which enabled pension funds to increase their exposure to venture capital, led to a large increase in commitments to the venture capital industry (Gompers and Lerner, 1999). I investigate changes in funding patterns in the US market around this date to see whether the influx of capital into the market led to changes in start-up funding patterns in line with the model's predictions.

Figure 6 show the average number of funding rounds, the average duration between rounds, and the number of firms receiving their first early-stage funding round each year from 1970 to 2015. The funding round and duration metrics represent cohort-level averages by year of entry. The sharp increase in funded firms following the 1979 ERISA clarification is evident. Furthermore, firms that received their first funding round in the 1970s tended to complete fewer funding rounds with longer durations between them, relative to their firms that first raised capital after 1980. Through the lens of the model, this is consistent with a tight capital market prior to 1980; in response, firms raised more capital relative to their annual capital requirements, completing fewer rounds with greater intervals between them. These historical patterns in the early US VC industry provide external validation for the identification approach applied to the UK and US today.

5 Financing risk, development horizons and exit opportunities

The quantitative analysis thus far has focussed on a homogenous firm case and, therefore, has had nothing say about the implications of frictions in VC funding for the direction of innovation. Yet, analysis of entry rates and start-up outcomes in the baseline model demonstrated that projects with certain characteristics are more adversely affected by financing risk than others. In this section, I quantify the implications of financing risk for the direction of start-up innovation.

I focus on a specific dimension of heterogeneity, related to the expected project development horizon. Venture capital funding has typically been concentrated in a small number of sectors, specifically those where uncertainty regarding the project's outcome can be resolved quickly (Lerner and Nanda, 2020).

which they refer to as the 'second equity gap' or 'second valley of death'. Their estimates suggest that by 2013, knowledge-intensive firms in the UK faced a funding gap of £1.2 billion at this juncture.

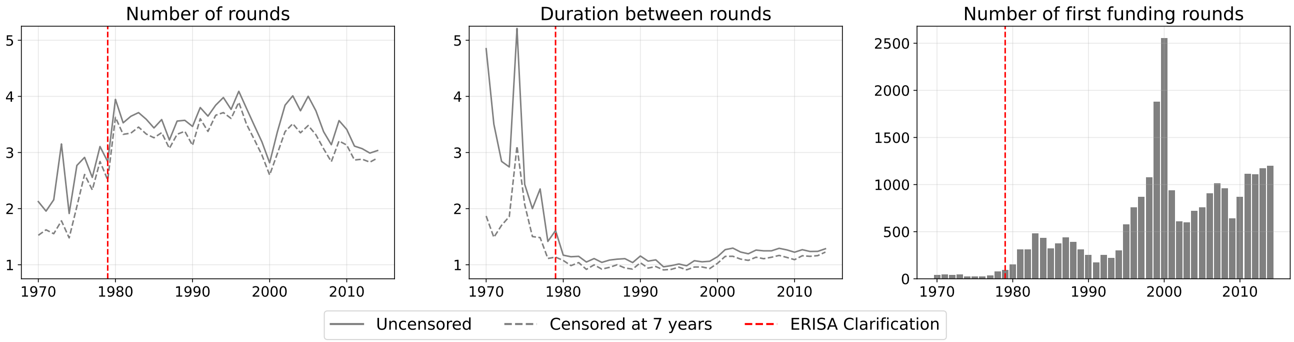

Figure 6: US historical funding patterns

The figure extends the US sample back to 1970 and, for cohorts of firms defined by their year of first funding, reports the average number of funding rounds, the average duration between funding rounds, and the total number of firms. The dashed line shows the results when data is censored at seven years following each firm's first funding round, as in the main estimation exercise. The solid line applies no censoring. The vertical red line refers to the clarification of ERISA in 1979.

The prevailing 10-year fund structure is often regarded as a key factor and Narain (2024) shows that VCs are more likely to invest in short-term projects as the fund's term limit nears. For longer-horizon projects that do receive VC funding, such as biotechnology and semiconductors, Nanda et al. (2014) suggests that a key driver of capital allocation to these sectors is the availability of an established exit mechanism: the ability of VCs to 'hand over' start-ups at pre-commercial stages to incumbents through acquisitions limits the capital requirements of these longer-horizon projects and their exposure to financing risk. Therefore, in addition to heterogeneity in expected project horizon, I consider that different technologies may also face different acquisition opportunities.

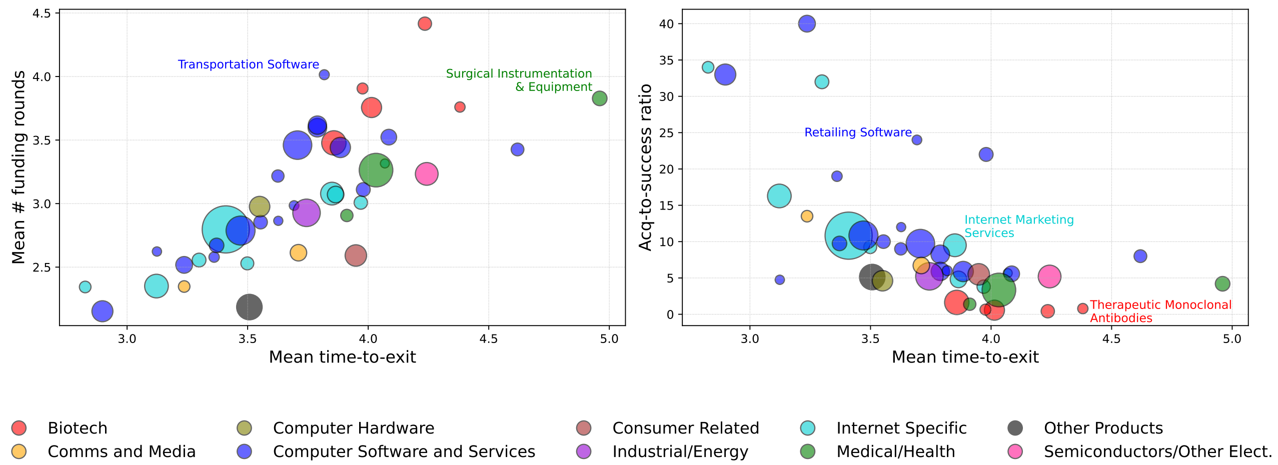

Figure 7: Sectoral heterogeneity

The figure plots the mean number of funding rounds (left) and the acquisition-to-success ratio (right) against the mean time-to-exit across VEIC sectors. The sample period is 2005-2015, consistent with other analysis, and observations are censored seven years following a firm's first funding round. Sectors are included as distinct observations if there are at least 20 exits during the sample period and at least one of each type (acquisition and success). Sectors that do not meet these criteria are grouped at the aggregate sector level and reported together. The membership of each sector to aggregate sectors are indicated through colours, and the size of each point reflects the number of firms in the sector.

Against this backdrop, this section aims to estimate the extent to which financing risk shifts innovation away from longer-term, potentially more transformative technologies, particularly those lacking access to a thriving market for firms. To the best of my knowledge, this is the first attempt in the literature to quantify these effects.

5.1 Data and motivating evidence

The analysis in this section relies on the same dataset as in section 3 but additionally utilises the Venture Economics Industry Classification (VEIC), a granular industry categorisation specifically designed for VC-funded firms. Similar to other industry classification systems such as NAICS, granular VEIC sectors are grouped into aggregate sectors. When relevant, I aggregate to the ten-sector level, which means that each VEIC sector belongs to one of ten aggregate sectors, namely: Biotechnology, Communications and Media, Computer Hardware, Computer Software and Services, Consumer Related, Industrial/Energy, Internet Specific, Medical/Health, Other Products, and Semiconductors/Other Electronics.[77] The Section C.4 of the Appendix provides an overview of the sectoral composition of the sample.

To motivate the analysis, Figure 7 reports two empirical relationships that indicate that firms with longer development horizons face greater exposure to financing risk. Each data point represents a granular VEIC sector, with colours denoting aggregate sector membership. A sector is included if it had at least 20 exits during the sample period and at least one acquisition and one success.[78] Sectors that do not meet these criteria are bundled together and reported at the ten-sector level.[79] The size of each data point is proportional to the number of VC-funded start-ups in each sector. As in all analysis, I censor data seven years after a firm's first funding round, which caps the time-to-exit at seven years.

The figure highlights two findings. First, there is a positive correlation between the average number of funding rounds and the average time-to-exit across sectors. Second, there is a negative correlation between the acquisition-to-success ratio and the average time-to-exit across sectors.

Through the lens of the model, the first finding is consistent with the theoretical prediction that firms with longer development horizons, proxied here by time-to-exit, face greater financing risk. Indeed, the baseline model demonstrated that financing risk is related to the expected number of funding rounds: firms that need to visit the capital market more frequently are more vulnerable to financing risk. Regarding the second finding, recall that the theoretical analysis of acquisitions in section 2.5 showed that acquisitions substitute for financing. Therefore, firms with greater exposure to financing risk should be relatively more willing to accept low-value acquisition offers, predicting a positive correlation between the acquisition-to-success ratio and time-to-exit. However, we observe the opposite at the granular VEIC sector level. This is suggestive evidence that longer-horizon projects face more limited market for firms, underscoring the need to account for this additional margin of heterogeneity—beyond the development horizon—in the analysis.[80]

[77] The highest level of aggregation considers six sectors: Biotechnology, Communications and Media, Computer Related, Medical/Health/Life Science, Non-High-Technology, and Semiconductors/Other Electronics. The key findings of this section remain unchanged under this level of aggregation.

[78] The relationships are not sensitive to this threshold.

[79] For instance, no VEIC sectors within "Semiconductors/ Other Electronics" meet the requirements and so are all included at the aggregate level.

[80] This echoes the discussion in Nanda et al. (2014), who note that start-ups in the clean-energy sector do not have access to a clear exit pathway. They also suggest that a key driver of the allocation of VC funding to longer-horizon sectors, such as biotechnology and semiconductors, is the development of an established exit mechanism via acquisition. Nonetheless, Figure 7 makes it clear that these acquisition opportunities are still more limited (relative to other exit opportunities) than for firms in other sectors.

5.2 Estimates of sectoral misallocation

Motivated by these observations, in the remainder of this section I construct an estimate of the extent to which frictions bias the sectoral allocation of VC funding. Specifically, my objective is to compute an estimate of the sectoral composition of VC funding in a counterfactual world without frictions and to compare it to the true, observed distribution. In what follows, I outline the steps involved in this estimation procedure.

Accounting for heterogeneity in the model. In the first step, I introduce the notion of a 'technology' into the model, which I define as a bundle of start-up characteristics and index by τ. Multiple start-ups may pursue the same technology; for instance, at any one time there may be multiple start-ups attempting to commercialise nuclear fusion technology. However, it may be more difficult to pursue some technologies than others, for example because certain technologies require more specific expertise. To account for this, I assume that for each technology there is a pool of potential entrants, as in the baseline model, but allow the entry cost parameter, $\tilde{\sigma}$ to depend on τ, i.e. $\tilde{\sigma}(\tau)$. Generating estimates of $\tilde{\sigma}(\tau)$ across technologies with different characteristics is key to the second step of the estimation strategy, which I discuss below.

I allow for two dimensions of heterogeneity across technologies: the rate of uncertainty resolution, κ, and the availability of acquisition opportunities, ϕ. However, heterogeneity in (κ, ϕ) affects a project's NPV, even in a frictionless setting and I want to isolate the implications of project characteristics, rather the project quality.[81] To isolate the effect of characteristics, I adjust the project payoff, π, such that all technologies have the same entry value in the frictionless economy.[82] In section F of the Appendix, I provide evidence consistent with the relationship between $\pi(\tau)$ and $(\kappa(\tau), \phi(\tau))$ that this procedure imposes. To summarise, in this section a 'technology' refers to a $(\kappa(\tau), \phi(\tau))$-pair with associated payoff $\pi(\tau)$. Otherwise, all technologies have the same characteristics and face the same funding market conditions. These parameters are set at the values from the US baseline estimation.

Figure 8 plots the ratio of the frictional entry value, $V_{s,e}(\tau)$, to the frictionless entry value, $\bar{V}_{s,e}(\tau)$, across technologies. In general, the ratio is increasing in κ, so that projects for which uncertainty is resolved more quickly are less affected by financial frictions. Furthermore, the ratio is increasing in ϕ, so that projects with greater access to acquisition opportunities are less affected by financing risk, regardless of project horizon. As is clear from the figures, it is firms with limited acquisition opportunities *and* long development horizons that are particularly affected by financing frictions.

Estimating the distribution of technologies. In the second step, I estimate the underlying distribution of start-ups across technologies, which I denote by $\bar{s}(\tau)$. This differs from the distribution of start-ups across technologies that are actually funded, $s(\tau)$, because financial frictions distort entry incentives to differing degrees for each technology τ. Specifically, recall that in the quantitative model, the flow of first funding rounds is proportional to $V_{s,e}/\tilde{\sigma}$, which generalises simply to $V_{s,e}(\tau)/\tilde{\sigma}(\tau)$ at the level of a technology τ, where $V_{s,e}(\tau)$ is the value of early-stage search for a firm with technology τ. Then, the share of projects with technology τ that are funded in frictional economy, $s(\tau)$, and the

[81] For instance, time-to-exit, total capital injection and exit rate all respond endogenously to these parameters.

[82] Specifically, denoting by $\bar{V}_{s,e}(\tau)$ the frictionless NPV for a project with technology τ, I choose $\pi(\tau)$ such that $\bar{V}_{s,e}(\tau) = \bar{V}_{s,e}(\tau')$ for all τ, τ' and set these values equal to the frictionless value for a project with characteristics estimated for the US baseline estimation. The frictionless value is computed under frictionless contracts (i.e. no agency friction, so that the wedge in the cost of capital is removed) and without matching frictions (i.e. $\nu_e \to \infty$ and $\nu_l \to \infty$).

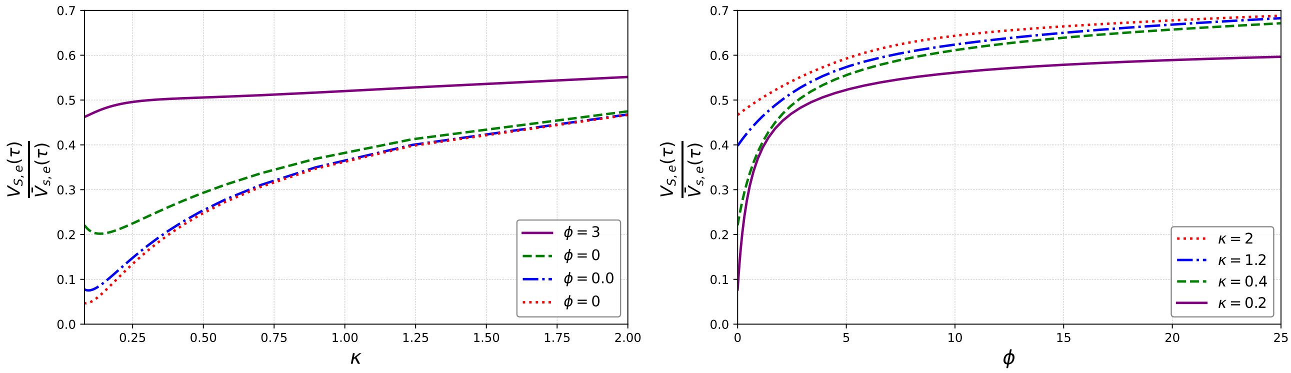

Figure 8: Ratio of frictional to frictionless entry value across technologies

The Figure reports contours of the ratio between the frictional and frictionless project value in the (κ, ϕ) space. Specifically, the left figure holds ϕ fixed at different values as κ is varied, and the right figure does the opposite. In all cases, π is adjusted so as to hold the frictionless project value constant, so that variation is driven solely by the frictional value.

share in the counterfactual, frictionless economy, $\bar{s}(\tau)$, are given by

$$s(\tau) = \frac{V_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \Bigg/ \int \frac{V_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \, d\tau \tag{18}$$

$$\bar{s}(\tau) = \frac{\bar{V}_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \Bigg/ \int \frac{\bar{V}_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \, d\tau = \frac{\tilde{\sigma}(\tau)^{-1}}{\int \tilde{\sigma}(\tau)^{-1} \, d\tau} \tag{19}$$

where the second line in equation (19) follows because $\pi(\tau)$ is chosen such that $\bar{V}_{s,e}(\tau) = \bar{V}_{s,e}(\tau')$.[83]

To estimate $\bar{s}(\tau)$, I require estimates for the distribution of $\tilde{\sigma}(\tau)$, which I recover indirectly from estimates of $s(\tau)$ using equation (18). To estimate the actual distribution of funded start-ups across technologies, $s(\tau)$, I target the joint distribution of the acquisition-to-success ratio and the mean time-to-exit across sectors, as depicted on the right-side of Figure 7. This distribution is informative of the two parameters of interest. Holding fixed ϕ, an increase in κ reduces the development horizon, reducing time-to-exit in equilibrium. Furthermore, as κ increases, start-ups become less exposed to financing risk and therefore less inclined to accept low acquisition offers, lowering the acquisition-to-success ratio. Conversely, holding κ fixed, an increase in ϕ should raise the acquisition-to-success ratio. Since acquisitions tend to occur earlier after first funding, this should also lower time-to-exit. By matching the joint distribution of the acquisition-to-success ratio and the mean time-to-exit across sectors, I allow the data to determine the relative importance of heterogeneity across these two margins.

In order to implement the estimation of $s(\tau)$, I need to introduce some additional notation. I denote the sectors, as depicted in Figure 7, by v (for VEIC) and denote the number of sectors by N_v. Associated with each sector is a weight, $s(v)$, and a point in the (acquisition-to-success, time-to-exit)-space, which I denote by $\mathbf{x}_v$ to simplify notation. Together, $(\mathbf{x}_v, s(v))$ defines the empirical joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space. I then construct a set of technologies, τ, by specifying a grid for κ and ϕ, where the total number of technologies is N_τ.[84] For each technology, I

[83] Note that the flow of first funding rounds in the frictional economy for technology τ is $\frac{V_{s,e}(\tau)}{\tilde{\sigma}(\tau)} \times \frac{\nu_e}{\nu_e+\lambda}$. The financing market rates do not appear in equation (18) because they are identical across technologies, by assumption.

[84] In practice, I experiment with different grid spacings and grid points and choose a sufficiently granular grid such that adding additional grid points has negligible effects on the results.

simulate the model to recover a point in the (acquisition-to-success, time-to-exit)-space, which I refer to as $\mathbf{x}_\tau$. My objective is to choose the shares (or weights), $s(\tau)$, in order to minimise the distance between the empirical joint distribution, $(\mathbf{x}_v, s(v))$, and the model-implied joint distribution, $(\mathbf{x}_\tau, s(\tau))$. In order to compare the distributions, I need to select a measure of distance, of which there are many. I choose to adopt the energy distance metric, which is often used to test for the equality of distributions. Specifically, I select shares $s(\tau)$ to minimize the following distance metric

$$\min_{s(\tau)} \left\{ 2\sum_{v=1}^{N_v}\sum_{\tau=1}^{N_\tau} s(v)s(\tau)||\mathbf{x}_v - \mathbf{x}_\tau|| - \sum_{v=1}^{N_v}\sum_{v'=1}^{N_v} s(v)s(v')||\mathbf{x}_v - \mathbf{x}_{v'}|| - \sum_{\tau=1}^{N_\tau}\sum_{\tau'=1}^{N_\tau} s(\tau)s(\tau')||\mathbf{x}_\tau - \mathbf{x}_{\tau'}|| \right\} \quad (20)$$

subject to $\sum_\tau s(\tau) = 1$ and $s(\tau) \in [0,1]$, where $||\cdot||$ is the Euclidean norm. The energy distance metric minimises the distance between the joint distributions of empirical sectors and model-based technologies, taking account for the location and shape. Figure 9 plots kernel density estimates given the sector weights, $s(v)$, and estimated weights, $s(\tau)$, based on minimisation (20).

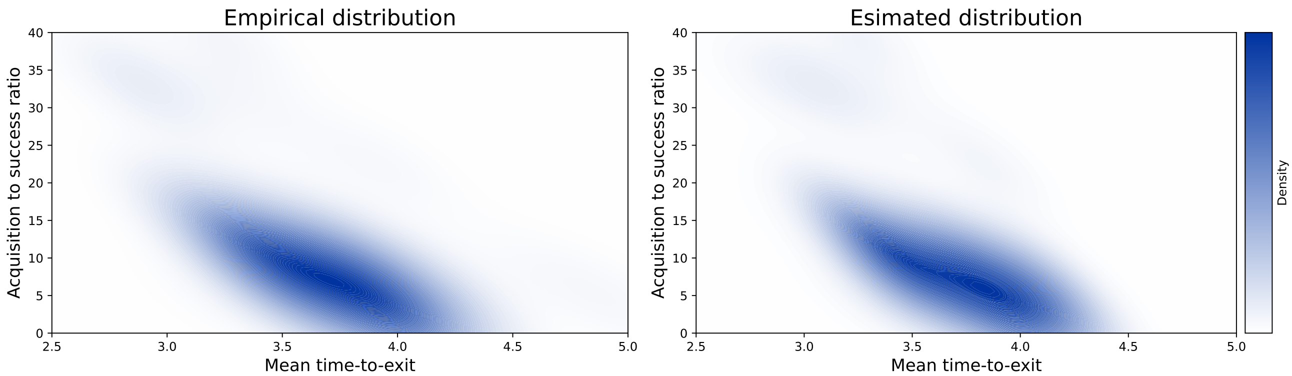

Figure 9: Empirical and model-implied distributions

The left-side figure depicts kernel density estimates for the true joint distribution of sectors in the (acquisition-to-success, time-to-exit)-space, which reflects the data shown in Figure 7. The right-side figure shows the kernel density estimate for the distribution of technologies in the same space, as estimated through minimisation of equation (20).

With the estimated weights, $s(\tau)$, I recover estimates for $\tilde{\sigma}(\tau)$ from equation (18) and compute the adjusted shares, $\bar{s}(\tau)$, via equation (19) that would arise in an economy without frictions. Figure 10 plots the resulting cumulative distributions for the average time-to-exit and acquisition-to-success ratio within *technologies*. The orange (dashed) line is computed using the frictional technology weights, $s(\tau)$, whereas the blue (solid) line recomputes the CDF using the estimates for the frictionless weights, $\bar{s}(\tau)$.[85] The figures show that the removal of financing frictions leads to a relative increase in entry from start-ups pursuing technologies with longer expected time-to-exit and lower acquisition-to-success ratios. In other words, financing risk skews the allocation of VC funding towards projects with shorter time-to-exit and stronger acquisition opportunities, relative to a first-best economy absent frictions.

Sectoral misallocation. In the final step, I map technologies into the ten aggregate sectors shown in Figure 7 to gain an estimate for the magnitude of these effects. Using this mapping and the imputed

[85] In plotting the CDF, I assume that the time-to-exit and acquisition ratios are invariant to frictions for any technology τ. In practice, removing frictions naturally impacts these moments, which are endogenous to financing conditions and optimal acquisition strategies. Nevertheless, depicting the data in this way aids comparison.

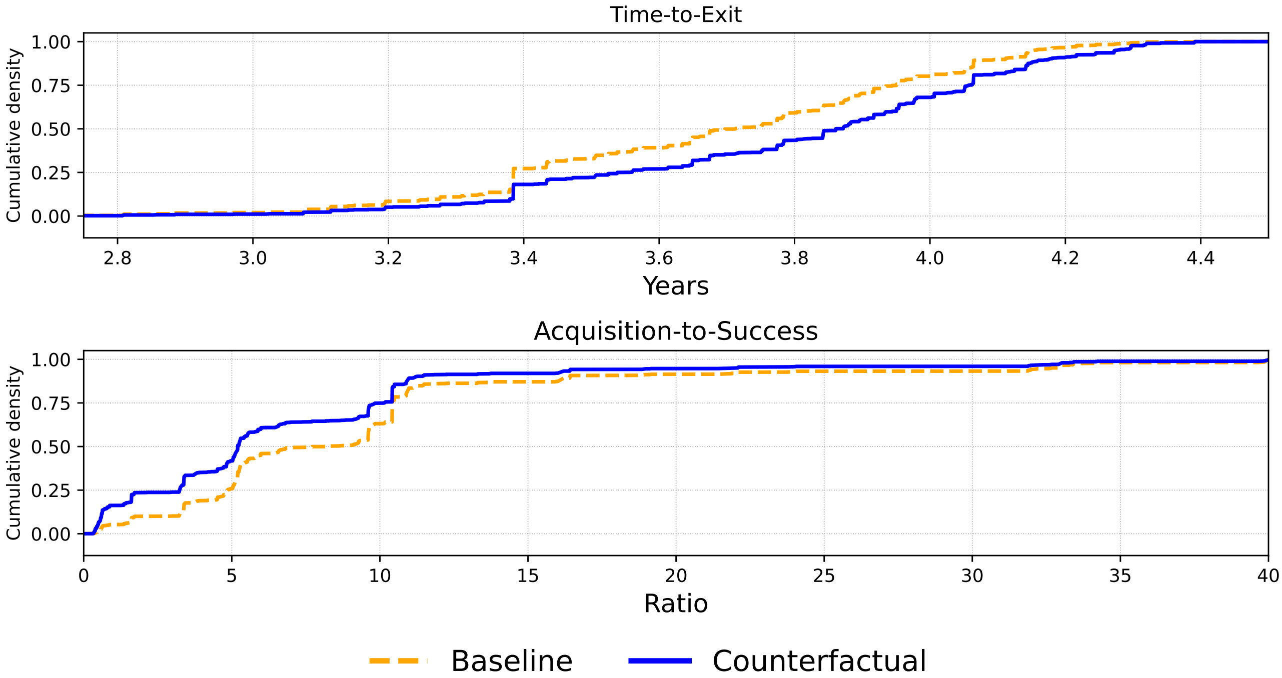

Figure 10: Time-to-exit and outcome ratios: frictional vs. frictionless entry

The figure plots the marginal CDFs for the time-to-exit (top) and acquisition-to-success ratio (bottom) based on the shares frictional shares $s(\tau)$ (orange, dashed) and frictionless shares $\hat{s}(\tau)$ (blue, solid).

shares $\bar{s}(\tau)$, I can recover an estimate of the distortion to the sectoral composition of VC funding induced by the interaction of financing risk, project horizons and acquisition opportunities.

To map from technologies to sectors, I allocate a portion $w(\tau, v)$ of the share of technology τ, $s(\tau)$, to each sector v by matching technologies and sectors in the (acquisition-to-success, time-to-exit)-space. Intuitively, I assign a large weight $w(\tau, v)$ if technology τ and sector v are have similar acquisition-to-success ratios and average time-to-exit. To implement this, I set up a simple optimal transport problem, which I describe in more detail in section F of the Appendix. The weights $w(\tau, v)$ then imply a mapping from technologies to aggregate sectors, since each sector v belongs to one aggregate sector. Finally, using these weights and the frictionless technology shares, $\bar{s}(\tau)$, I can recompute the aggregate sector shares in the counterfactual economy without financing frictions.

Table 9 reports the results of this exercise. My estimates suggest that frictions in VC funding distort the sectoral composition of VC-backed start-ups, particularly favouring start-ups in Computer Software and Services and Internet Specific sectors. The most prominent Internet Specific sectors are service sectors, such as Internet Marketing Services and Finance/Insurance/Real Estate Services. Therefore, these findings suggest that the composition of VC funding is particularly skewed towards software and services. On the other hand, it is start-ups in Biotechnology, Medical/Health and Semiconductors/Other Electronics that are underfunded. For instance, in a frictionless economy, Biotechnology start-ups would comprise roughly 22% of all funded start-ups, compared to just 9% in reality. These findings indicate a substantial misallocation of VC funding across sectors, highlighting that frictions in VC funding affect not only the overall level of investment but also the direction of start-up innovation.

Discussion and validation. A key contribution of this analysis is to quantify the effect of financial

Sector	Observed (%)	Counterfactual (%)	Change
Biotechnology	9.0	21.8	+
Communications and Media	2.6	2.1	−
Computer Hardware	2.7	2.3	−
Computer Software and Services	32.6	25.2	−
Consumer Related	3.0	3.0	−
Industrial/Energy	4.9	4.4	−
Internet Specific	28.1	21.0	−
Medical/Health	9.6	13.0	+
Other Products	4.2	3.4	−
Semiconductors/Other Elect.	3.3	3.8	+

Table 9: Sectoral Misallocation

The table reports the distribution of the number of VC-funded firms in each aggregate sector during the sample period (Observed) against the sector shares in counterfactual economy in which frictions are removed (Counterfactual).

frictions in shaping the direction of start-up activity. By demonstrating the substantial effects, it facilitates the adoption of policies that are specifically designed to address the issues caused by financing risk.[86] A comparative analysis the effect of different policy options is planned for future work.

The interaction of frictions in VC financing with project type also has implications for models that estimate the effect of venture capital financing on economic growth (Greenwood et al., 2022; Akcigit et al., 2022; Ando, 2024). My analysis highlights that frictions in VC markets affect not only the number of projects that are funded and their likelihood of success, but also the composition of VC funding across types of technologies and sectors. This echoes the empirical findings of Nanda and Rhodes-Kropf (2013), who show that start-ups first funded in hot markets are less likely to succeed, but conditional on success, have greater valuations, and more patents of higher quality.

Finally, by way of validation, Figure 11 plots the mean number of funding rounds against the mean time-to-exit and acquisition-to-success ratio for VEIC sectors (red) and model-based technologies (blue). The model replicates the positive association between the number of funding rounds and the average time-to-exit, as well as the negative relationship between the number of funding rounds and the acquisition-to-success ratio. Through the lens of the model, the expected number of funding rounds is related directly to the likelihood that a firm fails to secure follow-on funding at some point during its development process. Therefore, the fact that the model replicates these relationships provides validation of the key message: financing risk drives differences in entry incentives across technologies with different development horizons and acquisition opportunities.

6 Conclusion

In this paper, I study how financial frictions in the VC market impact the scale and direction of start-up innovation. In order to do so, I develop a new model of start-up development featuring

[86] For instance, Narain (2024) shows that grants by the federally-funded SBIR and STTR programmes in the US tend to favour projects with longer lags between investment and innovation, relative to VC-funded projects. In addition, the role of the government as a customer is credited with driving VC financing in Semiconductors and Biotechnology (Janeway et al., 2021) and may be a more appropriate strategy to overcome financing risk by providing a complementary route to liquidity, especially where acquisition opportunities are limited.

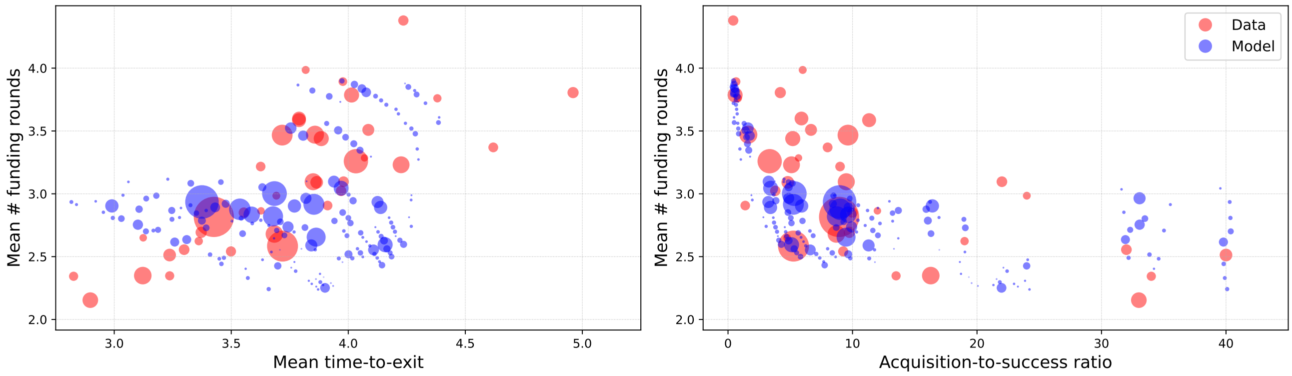

Figure 11: Cross-sectional variation in the model and data

The Figure plots the mean number of funding rounds against the mean time-to-exit and acquisition-to-success ratio for VEIC sectors (red) and model-based technologies (blue). The size of each point reflects its weight, $s(\tau)$ for technologies and $s(v)$ for sectors.

endogenous staged-financing and financing risk. The tight mapping from model to data allows me to assess quantitatively the extent and cost of financing risk for US start-ups, and I apply the model to analyse why VC activity in the UK lags behind the US. I also explore the extent to which financing risk biases VC investment away from certain sectors, which speaks to concerns that venture capital may fail to support some of the most transformative technologies.

One natural avenue for future work is to ask whether policy interventions could dampen the effect of frictions, for instance on the sectoral misallocation of venture capital funding. Comparisons between the efficacy of targeted small business grants–which would aid entry in the model–or government policies to become a customer for certain types of innovations–which would add to potential exit pathways–are feasible within this environment.

Finally, the model's general structure provides a foundation for addressing various applied questions in the literature. For example, considering shocks to the supply of VC funding would allow me to assess the quantitative contribution of financing risk in explaining the findings of Nanda and Rhodes-Kropf (2013), who document that start-ups funded in "hot" markets are less likely to succeed but, conditional on success, have higher valuations, more patents and more citations to their patents. Furthermore, by permitting firms to make endogenous decisions about exit timing, the model offers a framework contribute to the discussion of the drivers of the decline in IPOs over the past two decades.[87] Lastly, the multi-stage model is well-suited to quantitatively explore the role of experimentation costs in shaping the allocation of venture capital funding and to quantify their impact on start-up outcomes.[88]

[87] Gao et al. (2013) and Ewens and Farre-Mensa (2020) consider this question.

[88] The quantitative model has parallels to the environment considered by Ewens et al. (2018). Specifically, the investment cost k_e relates to the cost of experimentation, and the probability p_e/p_l of advancing from the early-to-late stage is relevant to their discussions of "long-shot bets".

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Appendix

A Contract microfoundations

Consider the environment of section 2.1. As discussed in the main text, the E will never report negative results. The contract in the body of the paper makes assumptions such that the E always reports positive results. Specifically, the private benefit is taken to zero in the limit, $x_e \to 0$. The proposition below demonstrates that this is sufficient for the E to always report positive results and never report negative results.

Proposition A.1. *For any set of parameters* $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$*, there exists an* $\bar{x}_e > 0$ *such that if* $x_e < \bar{x}_e$*, then the E reports positive results but does not report negative results.*

Before stating the proof, I give a brief outline of the argument. The proof is formed of three parts. In part 1, I derive the payoffs to the E given a contract (ς, ω) under two cases: (i) the E never reports results and (ii) the E reports only positive results. Then, I show that given (ς, ω), the E always prefers the payoff associated with case (ii). In part 2, I derive a condition under which case (ii) is not incentive compatible; that is, a condition under which a commitment to report positive results is not credible. In part 3, I demonstrate that for any set of parameters, case (ii) can always be made incentive compatible by choosing a suitably small non-pecuniary payoff to the E, x_e. Part 4 concludes the proof.

Proof of Proposition A.1.

Part 1. Without loss of generality, assume $x_e < k$.[89] The E never reports negative results, since $x_e > 0$. Suppose a positive result arrives at time t and call the decision to report a positive result $\chi = 1$, where $\chi = 0$ reflects the choice not to report a positive result. $\chi = 0$ implies that the payoff, π, from success is delayed until time $t + T_\omega$, for $T_\omega \sim Exp(\omega)$, so has time-$t$ expected value $\frac{\omega}{\rho+\omega}\pi$.[90] If $\chi = 0$, the contract ends at date T_ω, regardless of results.

The value of the firm in development, V_d, is determined analogously to equation (1)

$$\rho V_d = \begin{cases} \kappa \left[p\,\pi - V_d\right] + \omega \left[V_s - V_d\right] & \text{if } \chi = 1 \\ \kappa \left[p\,\pi \,\frac{\omega}{\rho+\omega} - V_d\right] + \omega \left[V_s - V_d\right] & \text{if } \chi = 0 \end{cases} \tag{A.1}$$

Denote by $V_d^{np}(\chi)$ the non-pecuniary value of the contract to the E. The value to the E of contract (ς, ω) is $V_E(\omega, \varsigma, \chi) = (1 - \varsigma)\, V_d(\omega, \chi) + V_d^{np}(\omega, \chi)$, where

$$V_d^{np} = \begin{cases} \left(1 + \frac{\kappa\,(1-p)}{\rho+\omega}\right) \frac{x_e}{\rho+\kappa+\omega} & \text{if } \chi = 1 \\ \frac{x_e}{\rho+\omega} & \text{if } \chi = 0 \end{cases} \tag{A.2}$$

where the expression for $V_d^{np}(\omega, \chi)$ is analogous to $K(\omega, \chi = 1)$ in equation (2). Similarly, $K(\omega, \chi = 0) = \frac{k}{\rho+\omega}$ because the contract ends at time T_ω regardless of any potential result. Since the VC has no

[89] This assumption is not restrictive since the proof attempts only to obtain an upper bound on x_e.

[90] $E[e^{-\rho T_\omega} \times \pi] = \frac{\omega}{\rho+\omega}\pi$, where expectations are taken over T_ω.

bargaining power, their participation constraint binds, $\varsigma V_d(\omega,\chi) = K(\omega,\chi)$. The value of the contract to the E is then $V_E = (1-\varsigma)V_d + V_d^{np} = V_d + V_d^{np} - K(\omega)$, or

$$V_E(\omega,\chi) = \begin{cases} \frac{\kappa\, p\, \pi + \omega\, V_s}{\rho+\kappa+\omega} - \left(1 + \frac{\kappa\,(1-p)}{\rho+\omega}\right) \frac{k-x_e}{(\rho+\kappa+\omega)} & \text{for } \chi = 1 \\ \frac{\kappa\, p\, \pi\, \frac{\omega}{\rho+\omega} + \omega\, V_s}{\rho+\kappa+\omega} - \frac{k-x_e}{\rho+\omega} & \text{for } \chi = 0 \end{cases} \tag{A.3}$$

This leads to the following remark, which states that the E prefers ex-ante the contract in which they report positive results to the VC.

Remark 2. *Given* $0 < x_e < k$ *and for any contract* (ω,ς), $V_E(\omega,\chi=1) > V_E(\omega,\chi=0)$.

Part 2. The benefit to the E of delay, $\chi = 0$, upon observing a positive result is $\frac{x_e}{\rho+\omega}$. The cost to the E is to delay the pecuniary payoff, of which they obtain share $1-\varsigma$. The delay therefore costs the E $E[(1-e^{-\rho\, T_\omega})\,(1-\varsigma)\,\pi] = \frac{\rho}{\rho+\omega}\,(1-\varsigma)\,\pi$, where expectations are taken over T_ω. This leads to the following incentive compatibility constraint, which states that the E's share of the pecuniary payoff must be sufficiently large for them to optimally report positive results.

Remark 3. $\chi = 1$ *is incentive compatible iff* $1-\varsigma \geq \frac{x_e}{\rho\pi}$.

Part 3. Consider the contract problem when the E commits to report positive results, $\chi = 1$

$$\sup_{\{\omega\in[0,\infty),\varsigma\in[0,1]\}} \left\{ \frac{\kappa\, p\, \pi + \omega\, V_s}{\rho+\kappa+\omega} - \left(1 + \frac{\kappa\,(1-p)}{\rho+\omega}\right) \frac{k-x_e}{(\rho+\kappa+\omega)} \right\}$$

$$\text{s.t.} \qquad 1-\varsigma \geq \frac{x_e}{\rho\,\pi}, \qquad \varsigma\, \frac{\kappa\, p\, \pi + \omega\, V_s}{\rho+\kappa+\omega} = \left(1 + \frac{\kappa\,(1-p)}{\rho+\omega}\right) \frac{k}{(\rho+\kappa+\omega)}$$

where V_s is taken as given and the IC constraint from Remark 3 is imposed. Begin by considering the problem without consideration for the incentive compatibility and feasibility constraints. Define $\bar{V}_s = \frac{\kappa p\pi - (k-x_e)}{\rho+\kappa}$. For $V_s \geq \bar{V}_s$, the objective function is everywhere increasing in $\omega \in [0,\infty)$ given $x_e < k$, so the solution is $\omega \to \infty$. Restrict attention to $V_s \in [0, \bar{V}_s)$. The first-order condition for an interior optimum is

$$\frac{(k-x_e)(1-p)}{(\rho+\omega)^2} + \frac{(k - x_e - \kappa\pi)\,p + V_s(\rho+\kappa)}{(\rho+\kappa+\omega)^2} = 0$$

which delivers two candidate solutions for ω

$$\omega = \frac{\rho\kappa p\pi - \rho V_s(\rho+\kappa) - (k-x_e)(\kappa(1-p)+\rho) \pm \sqrt{\kappa^2(k-x_e)(1-p)\big(\kappa p\pi - p(k-x_e) - V_s(\rho+\kappa)\big)}}{k - x_e - \kappa p\pi + V_s(\rho+\kappa)}$$

The 'positive' solution is strictly negative on $V_s \in [0,\bar{V}_s)$. Define $\bar{p} = \frac{(k-x_e)(\rho+\kappa)^2}{\kappa(\pi\rho^2 + (k-x_e)(\kappa+2\rho))}$ and $\underline{V_s} = \frac{\kappa p\pi - p(k-x_e)}{\rho+\kappa} - \frac{(k-x_e)(1-p)(\rho+\kappa)}{\rho^2}$. When $p \leq \bar{p}$, the 'negative' solution is positive for $V_s \in [0,\bar{V}_s)$; when $p > \bar{p}$, it is positive for $V_s \in [\underline{V_s}, \bar{V}_s)$. The SOC is satisfied in both cases. Conversely, when $p > \bar{p}$, the solution is negative for $V_s \in [0, \underline{V_s})$. In this case, the optimand is decreasing on $\omega \in [0,\infty)$. The solution is then $\omega = 0$.

It remains to check the feasibility and incentive compatibility constraints. To this end, note the following lemma.

Lemma 3. *If there are parameters such that $\varsigma \in (0,1)$ (i.e. strictly), then $\exists \bar{x}_e$ such that $\forall x_e \in [0, \bar{x}_e)$, $1-\varsigma \geq \frac{x_e}{\rho\pi}$.*

Proof of Lemma 3. The PC gives $\varsigma(\omega) = \frac{k(\kappa(1-p)+\rho+\omega)}{(\rho+\omega)(\kappa p\pi+\omega V s)}$, so that $\varsigma'(\omega) < 0$ on $V_s \in [0, \bar{V}_s)$. Furthermore, from the solution to the contract problem, $\omega'(x_e) \leq 0$, with strict inequality when $\omega > 0$ is optimal. Together, $\varsigma'(x_e) > 0$. Then, suppose the parameters are such that $\varsigma \in (0,1)$ and consider reducing $x_e \to 0$. The E's share must increase, $(1-\varsigma)\uparrow$, from a positive value, whereas $x_e/(\rho\pi) \to 0$. Therefore, there exists a region $[0, \bar{x}_e)$ such that the IC is satisfied, i.e. $1-\varsigma \geq \frac{x_e}{\rho\pi}$.

The remainder of the proof provides sufficient conditions on x_e such that Lemma 3 can be applied. Suppose first that $\omega = 0$ is optimal. Then $\varsigma(0) = \frac{k(\kappa(1-p)+\rho)}{\kappa p\pi\rho}$. Clearly, $\varsigma(0) > 0$ since $p \in (0,1)$. Note that $\varsigma(0) < 1 \iff \kappa p\pi\rho > k(\kappa(1-p)+\rho)$. Furthermore, if $\omega = 0$ is optimal, then (from above) $p > \bar{p} \implies p\kappa\pi\rho > \rho^{-1}\big((k-x_e)(\rho+\kappa)^2 - p\kappa(k-x_e)(\kappa+2\rho)\big)$. Combining these inequalities, a sufficient condition for $\varsigma(0) < 1$ is $\rho^{-1}\big((k-x_e)(\rho+\kappa)^2 - p\kappa(k-x_e)(\kappa+2\rho)\big) > k(\kappa(1-p)+\rho)$. For $x_e < \frac{\kappa(1-p)(\rho+\kappa)}{\rho^2+(1-p)(\kappa^2+2\rho\kappa)}$, this condition is satisfied, so $\varsigma(0) \in (0,1)$. Then, by Lemma 3, it is possible to choose a positive x_e sufficiently small such that the IC is satisfied.

Suppose next that $\omega \in (0,\infty)$ is optimal. Note $\varsigma(\omega) > 0 \ \forall \omega \in (0,\infty)$. It remains to check $\varsigma(\omega) < 1$, which is equivalent to requiring that the pecuniary return to the E is positive, $(1-\varsigma)\, V_d(\omega, \chi=1) = V_d(\omega,\chi=1) - K(\omega,\chi=1) > 0$. Evaluating this expression

$$
\begin{aligned}
V_d(\chi=1) - \Big(1 + \frac{\kappa\,(1-p)}{\rho+\omega}\Big)\, \frac{k}{(\rho+\kappa+\omega)}\Bigg|_{\omega=\omega^*\in(0,\infty)} &= \\
p\pi - \frac{1}{\kappa^2}\Bigg[& \frac{(2k-x_e)\sqrt{\kappa^2(k-x_e)(1-p)\big(p\kappa\pi - p(k-x_e) - V_s(\rho+\kappa)\big)}}{k-x_e} \\
& + \kappa\Bigg(2kp - k + \rho V_s - \frac{(1-p)p(k-x_e)x_e\kappa}{\sqrt{\kappa^2(k-x_e)(1-p)\big(p\kappa\pi - p(k-x_e) - V_s(\rho+\kappa)\big)}}\Bigg)\Bigg]
\end{aligned}
$$

This expression is decreasing in x_e on $x_e > 0$ given $V_s < \frac{\kappa p\pi - p(k-x_e)}{\rho+\kappa}$, which is satisfied whenever $V_s < \bar{V}_s$ since $p \in (0,1)$. Therefore, if $V_d(\omega,\chi=1) - K(\omega,\chi=1) > 0$ at $x_e = 0$, it is positive over some region $x_e \in [0, \bar{x}_e)$ where $\bar{x}_e > 0$. Evaluating $V_d(\omega,\chi=1) - K(\omega,\chi=1) > 0$ at $x_e = 0$ obtains the condition

$$
p\pi - \frac{1}{\kappa}\Big(k(2p-1) + \rho V_s + 2\sqrt{k(1-p)(\kappa p\pi - kp - V_s(\rho+\kappa)}\Big) > 0
$$

which is equivalent to

$$
\kappa p\pi - k(2p-1) - \rho V_s > 2\sqrt{k(1-p)(\kappa p\pi - kp - V_s(\rho+\kappa))}
$$

Note that the left-hand side is positive if $V_s < \frac{\kappa p\pi - k}{\rho} + \frac{2k(1-p)}{\rho}$, which is valid on $V_s \leq \bar{V}_s$ when $x_e = 0$. Therefore, squaring both sides, subtracting the right-hand side from the left-hand side and some algebra obtains

$$
4kV_s(1-p)\kappa + (\kappa p\pi - k - \rho V_s)^2 > 0
$$

which is satisfied given the initial restrictions on parameters (i.e. positivity constraints and $p \in (0,1)$). This implies that the E obtains a positive pecuniary payoff when $x_e = 0$ and, by extension, over some region $x_e \in [0, \bar{x}_e)$. This in turn implies the conditions to apply Lemma 3 are satisfied and there exists a (potentially different) upper bound $\bar{x}_e$ such that $\forall x \in [0, \bar{x}_e)$, the IC is slack.

Suppose finally that $\omega \to \infty$ is optimal. Then $\lim_{\omega\to\infty} \varsigma(\omega) = 0$ and so satisfying the IC requires only that $x_e \leq \rho\pi$, which is easily satisfied.

Part 4. By Remark 2, the E prefers to payoff associated with commitment for any contract (ω, ς). Furthermore, for any contract (ω, ς) and parameters $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$, it is possible to find a strictly positive upper bound on x_e such that the feasibility constraint is slack, i.e. $\varsigma \in (0, 1)$, if x_e is less than this upper bound. Then, by Lemma 3, there is a (potentially different) upper bound on x_e such that the IC constraint is also slack if x_e does not exceed this upper bound. Together, there is an interval $[0, \bar{x}_e)$ such that both the feasibility and incentive compatibility constraints are slack at the optimal contract in which $\chi = 1$. Since the contract with $\chi = 1$ is incentive compatible and for any contract (ω, ς) $\chi = 1$ is preferred ex ante, the optimal contract involves $\chi = 1$ on this interval $x_e \in [0, \bar{x}_e)$. Furthermore, the expected capital cost is given by $K(\omega, \chi = 1)$ and the optimal contract rate is determined as in part 3. ■

A.1 The expected discounted capital commitment

Suppose the contract is signed at date $t = 0$. Until time $T = \min\{T_\kappa, T_\omega\} \sim Exp(\kappa + \omega)$, the E has not obtained a result and funding has not been withdrawn, so the VC pays k. Once time T arrives, there are three potential outcomes: (i) $T = T_\omega$, with probability $\frac{\omega}{\kappa+\omega}$; (ii) $T = T_\kappa$ and the result is a success, with probability $\frac{\kappa p}{\kappa+\omega}$; or (iii) $T = T_\kappa$ and the result is a failure, with probability $\frac{\kappa(1-p)}{\kappa+\omega}$. In cases (i) and (ii), funding ceases. In case (iii), the VC continues to pay k until time $T + T_\omega = T_\kappa + T_\omega$. Therefore, from time $t = 0$ to $t = T$, the flow cost to the E is $e^{-\rho t} k$ from the perspective of time $t = 0$. At time T, the probability that the VC continues to pay k is $\frac{\kappa(1-p)}{\kappa+\omega}$, in which case they pay k from date T to $T + T_\omega$. The expected capital cost is then computed by taking expectations over $T_\omega \sim Exp(\omega)$ and $T \sim Exp(\kappa + \omega)$

$$\begin{aligned} K(\omega) = \int_0^\infty \Bigg(\int_0^T e^{-\rho t} k \, dt \\ + \frac{\kappa(1-p)}{\kappa+\omega} \int_0^\infty \left(\int_T^{T+T_\omega} e^{-\rho s} k \, ds \right) \omega \, e^{-\omega T_\omega} \, dT_\omega \Bigg) (\kappa + \omega) \, e^{-(\kappa+\omega) T} \, dT \\ = \left(1 + \frac{\kappa(1-p)}{\rho + \omega} \right) \frac{k}{\rho + \kappa + \omega} \end{aligned} \tag{A.4}$$

A.2 Existence and uniqueness

In this section, I prove the existence of a unique equilibrium given financing market conditions. I do so in the general case in which the VC has bargaining power; the proof of Proposition 2.1 then follows by setting the VC's bargaining power to zero.

Proposition A.2. *Given ν and υ and for $\kappa p \pi > k$, there is a unique solution for $\{V_d, V_s, U_d^p, U_d^u, U_s, S, \omega, \varsigma\}$ in terms of the model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, $\omega \in [0, \infty)$.*

Proof of Proposition A.2. Given ν, υ, an equilibrium is a solution for $\{V_d, V_s, U_d^p, U_d^u, U_s, S, \omega, \varsigma\}$ to the following set of equations

$$\rho\, V_d = \kappa\,[p\,\pi - V_d] + \omega\,[V_s - V_d] \tag{A.5}$$

$$(\rho + \lambda)V_s = \nu[(1-\varsigma)V_d - V_s] \tag{A.6}$$

$$\rho U_d^p = -k + \kappa[p(\varsigma\,\pi + U_s) + (1-p)U_d^u - U_d^p] + \omega[\varsigma\, V_s + U_s - U_d^p] \tag{A.7}$$

$$\rho U_d^u = -k + \omega[U_s - U_d^u] \tag{A.8}$$

$$\rho U_s = \upsilon[U_d^p - U_s] \tag{A.9}$$

$$S = V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega}\left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \tag{A.10}$$

$$(1-\varsigma)V_d = V_s + \delta S \tag{A.11}$$

$$U_d^p = U_s + (1-\delta)S \tag{A.12}$$

$$\omega = \underset{\omega \in [0,\infty]}{\arg\max}\{S\} \tag{A.13}$$

The first two equations come from the start-up's problem, where I have substituted $V^M = (1-\varsigma)V_d$. The next three equations come from the VC's problem, as set out in section B.2 of the Appendix. The final four equations relate to the optimal contract: they define, respectively, the surplus, the optimality conditions for the equity share in the contract, and the optimal contract rate.[91]

Towards a proof, note that given $(\omega, S, \nu, \upsilon)$, all remaining equilibrium objects are pinned down uniquely. Specifically, combining equations (A.6) and (A.11), $(\rho+\lambda)V_s = \nu\delta S$, so V_s is known given (ν, S). V_d then follows immediately from equation (A.5) given (ω, V_s). The equity stake then follows from equation (A.11). Finally, the VC value functions can then be recovered from equations (A.7), (A.8) and (A.9), given knowledge of $(\omega, \upsilon, \varsigma, V_s)$.

In light of this, to show that the equilibrium, given (ν, υ), is unique, it is sufficient to demonstrate that there is a unique (ω, S) consistent with the equilibrium conditions. To this end, rather than taking the first-order condition as a function of V_s and U_s, as in equation (B.8), and *then* solving the resulting system of equations in (ω, V_s, U_s), it is simpler to first eliminate the search values from the optimisation problem and then solve for the optimal contract. Although the contract is derived taking as given the search values, (V_s, U_s), this approach is valid because, in the background, holding the search values constant is equivalent to preventing agents from internalising the effect of the contract choice, ω, on the equilibrium meeting rats, ν and υ. Towards an expression for S, combining the above expressions

[91] There are more equations than unknowns, but technically one of the equations (A.11) or (A.12) is redundant.

allows to solve for V_s in terms of model parameters and the equilibrium contract rate, ω

$$\begin{aligned}(\rho+\lambda)V_s &= \nu\delta S \\ &= \nu\delta\left[V_d - V_s - \frac{k+\rho U_s}{\rho+\kappa+\omega}\left(1+\frac{\kappa(1-p)}{\rho+\omega}\right)\right] \\ &= \nu\delta\left[\frac{\kappa p\pi+\omega V_s}{\rho+\kappa+\omega} - V_s - \frac{k+\rho U_s}{\rho+\kappa+\omega}\left(1+\frac{\kappa(1-p)}{\rho+\omega}\right)\right] \\ &= \nu\delta\left[\frac{\kappa p\pi+\omega V_s}{\rho+\kappa+\omega} - V_s - \frac{k+\upsilon(1-\delta)\frac{\rho+\lambda}{\nu\delta}V_s}{\rho+\kappa+\omega}\left(1+\frac{\kappa(1-p)}{\rho+\omega}\right)\right]\end{aligned}$$

where the first line follow from combining equations (A.6) and (A.11), the second line uses the definition of S, the third line substitutes for V_d from equation (A.5), and the fourth line uses the relationship between U_s and V_s, which comes from writing these values both in terms of the surplus and then eliminating the surplus from the resulting equations.[92] Then, solving for V_s and using $\nu\delta S = (\rho+\lambda)V_s$ yields an expression for S in terms of model parameters and the contract rate, ω

$$S(\omega) = \frac{(\rho+\lambda)\left[\kappa p\pi - \left(1+\frac{\kappa(1-p)}{\rho+\omega}\right)k\right]}{(1-\delta)\upsilon\,(\rho+\lambda)\left(1+\frac{\kappa(1-p)}{\rho+\omega}\right) - \nu(1-\delta)\,(\rho+\kappa) + (\rho+\kappa)\,(\rho+\lambda+\nu) + (\rho+\lambda)\,\omega} \tag{A.14}$$

The optimal contract rate maximises $S(\omega)$, taking the meeting rates ν and υ as given. Differentiating with respect to ω obtains

$$S'(\omega) = \frac{(\rho+\lambda)(k-\kappa p\pi)(\rho+\omega)^2 + 2\kappa k(1-p)(\rho+\lambda)(\rho+\omega) + k\left(\kappa^2(1-p)(\rho+\lambda+\delta\nu) + \delta\kappa\nu(1-p)\rho\right) + (1-\delta)\kappa^2(1-p)p\pi\upsilon(\rho+\lambda)}{\frac{1}{\rho+\lambda}(\rho+\omega)^2\left((\rho+\kappa)(\delta\nu+\lambda+\rho) + \omega(\rho+\lambda) + (1-\delta)\upsilon(\rho+\lambda)\left(1+\frac{\kappa(1-p)}{\rho+\omega}\right)\right)^2}$$

The denominator of $S'(\omega)$ is strictly positive, so it is sufficient to focus on the numerator. Note firstly that the numerator is a quadratic function of ω. There are two cases to consider. Suppose firstly that $k \geq \kappa p\pi$, in which case $S'(\omega) > 0$ on $\omega \in [0,\infty)$. In this case, the unique solution is $(\omega \to \infty, S = 0)$. Next, suppose that $k < \kappa p\pi$, in which case the numerator is quadratic and concave in ω, and so may omit an interior solution. Setting $S'(\omega) = 0$ and solving for ω obtains

$$\omega = -\frac{(\rho+\lambda)(\rho(\kappa p\pi-k) - \kappa k(1-p)) \pm \sqrt{\kappa(1-p)(\rho+\lambda)(k(\kappa p\pi(\kappa(\rho+\lambda+\delta\nu)+\rho\delta\nu) - k(\delta\nu(\rho+\kappa)+\kappa p(\rho+\lambda))) + (1-\delta)\kappa p\pi\upsilon(\rho+\lambda)(\kappa p\pi-k))}}{(\rho+\lambda)(\kappa p\pi-k)}$$

Among the two solutions, it is simple to see that the 'positive' solution is strictly negative whenever $\kappa p\pi > k$.[93] Therefore, there can be at most one solution on $\omega \in [0,\infty)$. Specifically, since the sign of $S'(\omega)$ is pinned down by a quadratic function in ω that is concave, the greater of the two solutions is a maximum and so is the optimum when it is positive. However, if the greater of the two solution is negative, then the optimum must be at $\omega = 0$, because $S'(\omega)$ is negative on $\omega \in [0,\infty)$. Combining these insights, given ν and υ, is it clear that there is a unique optimum characterised by a finite contract rate when $k < \kappa p\pi$ and so surplus. Given the preceding arguments, the equilibrium objects are uniquely determined. ■

Proof of Corollary 2.1. Again, I consider the general case with VC bargaining power. Solving the

[92] Specifically, $\rho U_s = \upsilon(1-\delta)S$ and $(\rho+\lambda)V_s = \nu\delta S$, so that $\rho U_s = (\rho+\lambda)\frac{1-\delta}{\delta}\frac{\upsilon}{\nu}V_s$

[93] To verify, set the square root term equal to zero and note that the resulting root is negative.

optimisation problem as described, the optimal contract rate is given by

$$\omega = \begin{cases} \max\left\{0, -\frac{(\rho+\lambda)(\rho(\kappa p\pi-k)-\kappa k(1-p))-\sqrt{\kappa(1-p)(\rho+\lambda)(k(\kappa p\pi(\kappa(\rho+\lambda+\delta\nu)+\rho\delta\nu)-k(\delta\nu(\rho+\kappa)+\kappa p(\rho+\lambda)))+(1-\delta)\kappa p\pi\upsilon(\rho+\lambda)(\kappa p\pi-k))}}{(\rho+\lambda)(\kappa p\pi-k)}\right\} & \text{if } k < \kappa p\pi \\ \infty & \text{else} \end{cases} \tag{A.15}$$

To determine when the contract rate is set to zero, corresponding to upfront financing, consider when the max-operator is active. To this end, upfront financing occurs when any of the following conditions are met

$$p \in \left(\frac{k(\rho+\kappa)^2}{\kappa\left(\rho^2\pi + k(\kappa+2\rho)\right)}, 1\right] \quad \textit{and} \quad \begin{cases} \upsilon \geq \frac{1}{p\kappa^2\pi}\Psi & \textit{and} \quad \delta = 0, \quad \textit{or} \\ \nu < \frac{(\rho+\lambda)}{k\kappa\delta(\rho+\kappa)}\Psi & \textit{and} \quad \upsilon \geq f(\nu) \quad \text{and} \quad \delta \in (0,1), \quad \textit{or} \\ \nu \geq \frac{(\rho+\lambda)}{k\kappa(\rho+\kappa)}\Psi & \textit{and} \quad \delta = 1 \end{cases}$$

with

$$\Psi = \frac{1}{1-p}\left(\rho^2(\kappa p\pi - k) - k(1-p)\kappa^2 - 2k(1-p)\rho\kappa\right)$$
$$f(\nu) = \frac{\rho^2(\rho+\lambda)(\kappa p\pi - k) - k\kappa(1-p)\left[\kappa(\rho+\lambda+\delta\nu) + \rho(2\rho+2\lambda+\delta\nu)\right]}{(\rho+\lambda)(1-p)(1-\delta)\kappa^2 p\pi}$$

with $\Psi > 0$ under the restriction on p and $f'(\nu) < 0$. The case given in the main body of the paper simply corresponds to the case of $\delta = 1$. ■

A.3 Additional results

In the main body of the paper, I focus predominately on the effect of start-up characteristics on outcomes. The following proposition provides comparative statics for start-up outcomes with respect to all parameters.[94]

Proposition A.3. *The success probability for firms in productive development, p_d are: strictly increasing in p and π; strictly decreasing in k; and ambiguous with respect to ν, λ, and κ. The success probabilities for firms in search, p_s are: strictly increasing in p and π; strictly decreasing in k; and ambiguous with respect to ν and κ. Furthermore, the value-compensating comparative static of f_d (and f_s) with respect to p is negative and with respect to κ is ambiguous.*

Proof of Proposition A.3. The result for p, π and k follows directly from differentiation of p_d and p_s, given the results from Proposition 2.2. For κ, $\text{sign}\left(\frac{dp_d}{d\kappa}\right) = \text{sign}\left(\frac{dp_s}{d\kappa}\right) = \text{sign}\left(\omega(\kappa) - \kappa\omega'(\kappa)\right)$, which is ambiguous.

Next, consider $\frac{dp_d}{d\nu} = \frac{\kappa p\lambda(\omega-(\lambda+\nu)\omega'(\nu))}{(\kappa(\lambda+\nu)+\lambda\omega)^2} \implies \text{sign}\left(\frac{dp_d}{d\nu}\right) = \text{sign}\left(\omega - (\lambda+\nu)\omega'(\nu)\right)$, which can equivalently be cast as a statement about the semi-elasticity of ω with respect to ν; that is, if $\frac{\omega'(\nu)}{\omega} > \frac{1}{\lambda+\nu}$, then $\frac{dp_d}{d\nu} > 0$. To see that $\frac{dp_d}{d\nu} < 0$ is possible, note that from Corollary 2.1 and Proposition 2.2, there are parameter values such that $\omega = 0$ but $\omega'(\nu) > 0$; specifically, when $p > \bar{p}$ and $\nu = \bar{\nu}$, increasing $\nu > \bar{\nu}$ implies that $\omega > 0$, but $\omega = 0$ for $\nu = \bar{\nu}$. Therefore, in this case $\omega - (\lambda+\nu)\omega'(\nu) = -(\lambda+\nu)\omega'(\nu) < 0$, so $\frac{dp_d}{d\nu} < 0$. To see that $\frac{dp_d}{d\nu} > 0$ is possible, note that $\lim_{\nu\to\infty} p_d = p$ and $p_d \leq p$, so p_d must approach p from below as $\nu \to \infty$. This means there must be some ν for which $\frac{dp_d}{d\nu} > 0$. Finally, to see that $\exists\tilde{\nu}_d \geq 0$

[94] I have not been able to sign $\frac{dp_s}{d\lambda}$. Note that a sufficient condition for $\frac{dp_s}{d\lambda} < 0$ is $\frac{dp_d}{d\lambda} < 0$.

such that if $\nu \geq \tilde{\nu}_d$, $\frac{dp_d}{d\nu} > 0$ and otherwise $\frac{dp_d}{d\nu} < 0$, note that $\frac{d}{d\nu}(\omega - (\lambda+\nu)\omega'(\nu)) = -(\lambda+\nu)\omega''(\nu) > 0$ where the inequality follows since $\omega''(\nu) < 0$, so there at most one root for $\omega - (\lambda+\nu)\omega'(\nu) = 0$. Furthermore, $\frac{d}{d\nu}(\omega - (\lambda+\nu)\omega'(\nu)) > 0$ implies that if $\frac{dp_d}{d\nu} \geq 0$ for some $\nu = \hat{\nu}$, then $\frac{dp_d}{d\nu} > 0 \ \forall \nu > \hat{\nu}$. Therefore, we can define $\tilde{\nu}_d > 0$ s.t. $\omega - (\lambda+\nu)\omega'(\nu) = 0$ or $\tilde{\nu}_d = 0$ if there is no root and the statement holds.

Next, consider $\frac{dp_s}{d\nu} = \frac{\kappa p \lambda(\kappa+\omega-\nu\omega'(\nu))}{(\kappa(\lambda+\nu)+\lambda\omega(\nu))^2} \implies \text{sign}\big(\frac{dp_s}{d\nu}\big) = \text{sign}\big(\kappa + \omega - \nu\omega'(\nu)\big)$. To see that $\frac{dp_s}{d\nu} < 0$ is possible, note that $\frac{d}{d\nu}(\kappa + \omega - \nu\omega'(\nu)) = -\nu\omega''(\nu) > 0$. Therefore, if $\frac{dp_s}{d\nu} < 0$ is to occur, it will occur for low values of ν. Then, take the limit as $\nu \to 0$

$$\lim_{\nu\to 0}(\kappa + \omega - \nu\omega'(\nu)) = \frac{(\rho+\lambda)(k(\kappa p - \rho) + \kappa p \pi(\rho - \kappa)) - \sqrt{\kappa^2 k(1-p)p(\lambda+\rho)^2(\kappa\pi - k)}}{(\rho+\lambda)(k - \kappa p \pi)}$$

where $\omega > 0$ has been assumed. This expression is negative if $p \in (\frac{k\rho^2}{\kappa\pi(\kappa-\rho)^2 - \kappa k(\kappa - 2\rho)}, 1)$. However, the $\omega > 0$ solution may be invalid. From Corollary 2.1, a sufficient condition to take $\nu \to 0$ while maintaining $\omega > 0$ is that $p < \bar{p} = \frac{k(\rho+\kappa)^2}{\kappa(\pi\rho^2 + k(\kappa+2\rho))}$. Both of these conditions on p can be met simultaneously if $\kappa > 2\sqrt{\rho}$, which is admissible. Therefore, $\lim_{\nu\to 0}(\kappa + \omega - \nu\omega'(\nu)) < 0$ in some part of the parameter space and so $\frac{dp_s}{d\nu}$ is feasible. To see that $\frac{dp_s}{d\nu} > 0$ is possible, note that $\lim_{\nu\to\infty} p_s = p$ and $p_s \leq p$, so p_s must approach p from below as $\nu \to \infty$. The remainder of the proof is analogous to that for part (i).

For λ, $\text{sign}\big(\frac{dp_d}{d\lambda}\big) = \text{sign}\big(-\nu\omega - \lambda(\lambda+\nu)\omega'(\lambda)\big)$. To see that this can be be positive, note that there exists a λ such that $\omega = 0$ and $\omega'(\lambda) < 0$; at this point, small reductions in λ imply a movement from upfront financing to staged-financing and so lower the success probability. To see that $\frac{dp_d}{d\lambda} < 0$ is possible, note that $\lim_{\lambda\to 0} p_d = p$ and $p_d \leq p$, so p_d must approach p from below as $\lambda \to 0$, implying there is some region for which $\frac{dp_d}{d\lambda} < 0$. For p_s, clearly $\frac{dp_d}{d\lambda} < 0 \implies \frac{dp_s}{d\lambda} < 0$.

Finally, $\frac{dp_d}{dp} = \frac{\kappa(\lambda+\nu)(\kappa(\lambda+\nu) - \lambda p \omega'(p) + \lambda\omega)}{(\kappa(\lambda+\nu)+\lambda\omega)^2}$ and $\frac{dp_s}{dp} = \frac{\nu}{\lambda+\nu}\frac{dp_d}{dp}$. By Proposition (2.2), $\omega'(p) < 0$, so $\frac{dp_d}{dp} > 0$ and $\frac{dp_s}{dp} > 0$. Furthermore, $\frac{d}{dp}\frac{\kappa\,(\lambda+\nu)}{\omega\,\lambda+\kappa\,(\lambda+\nu)} = -\frac{\kappa\lambda(\lambda+\nu)\omega'(p)}{(\kappa(\lambda+\nu)+\lambda\omega)^2} > 0$ since $\omega'(p) < 0$. The same holds for p, holding $p \times \pi$ constant. ■

A.4 Proofs: partial equilibrium

This section contains the proofs to the partial equilibrium analysis.

Proof of Proposition 2.2. The comparative static for ν, k, p, π and λ is clear from differentiation of equation (9) given parameter restrictions. For κ, a sufficient and necessary condition for $\omega'(\kappa) > 0$ is

$$p\pi(\lambda+\nu+\rho)\kappa^2 - k\left(\rho\nu + 2\kappa(\rho+\lambda+\nu)\left(\sqrt{\frac{(1-p)(\rho+\lambda)}{\rho+\lambda+\nu}} + 1\right)\right) > 0$$

which a quadratic convex function of κ. In order for ω to be finite, it must be that $\kappa > k/(p\pi)$. It is simple to show that the smaller of the two roots to the above expression is strictly less than $k/(p\pi)$, whereas the larger root is strictly greater than $k/(p\pi)$. Therefore, over the admissible region for κ, ω is first decreasing in κ, when κ is smaller than the larger root, and is increasing in κ thereafter.

Finally, for the value-compensating comparative statics, I set π such that $\frac{\kappa p \pi - k}{\rho+\kappa} = C$ for some constant $C > 0$. Differentiating the resulting expression with respect to κ and p (separately), substituting back for $C = \frac{\kappa p\pi - k}{\rho+\kappa}$ and checking the sign of the resulting expression obtains the results. ■

Proof of Corollary 2.2. For all parameters $x \in \{\nu, k, p, \pi, \lambda\}$, $\text{sign}\big(\frac{d}{dx}\frac{1}{\kappa+\omega}\big) = -\text{sign}\big(\omega'(x)\big)$. The result therefore follows directly in these cases, including for the value-compensated comparative static for p. For κ, $\text{sign}\big(\frac{d}{d\kappa}\frac{1}{\kappa+\omega}\big) = -\text{sign}\big(1+\omega'(\kappa)\big)$, which is ambiguous in the general case, but the value-compensated comparative static is negative, since $\omega'(\kappa)$ is strictly positive in this case. ■

Proof of Corollary 2.3. For $x \in \{\nu, \pi, \lambda\}$, $\frac{dK}{dx} = \frac{\partial K}{\partial \omega}\frac{\partial \omega}{\partial x} = \implies \text{sign}\big(\frac{dK}{dx}\big) = -\text{sign}\big(\frac{\partial \omega}{\partial x}\big)$ since $\frac{\partial K}{\partial \omega} < 0$. The result follows immediately. For $x \in \{k, p, \kappa\}$, $\frac{dK}{dx} = \frac{\partial K}{\partial x} + \frac{\partial K}{\partial \omega}\frac{\partial \omega}{\partial x}$, where $\frac{\partial K}{\partial \omega} < 0$. For p, $\frac{\partial K}{\partial p} < 0$ and $\frac{\partial \omega}{\partial p} < 0$; for k, $\frac{\partial K}{\partial k} > 0$ and $\frac{\partial \omega}{\partial k} > 0$; and for κ, $\frac{\partial K}{\partial \kappa} < 0$ and the sign of $\frac{\partial \omega}{\partial \kappa}$ is ambiguous. It is not possible to sign these effects in general. The same logic applies for the value-compensated comparative static for p. However, the value compensated comparative static for κ is strictly negative since $\frac{\partial \omega}{\partial \kappa} > 0$ in this case. ■

Proof of Proposition 2.3. Consider a firm in productive development. It may not raise capital again due to either completing its experiment in the current financing round, or because its financing is withdrawn and it cannot raise new capital. Putting this together, the probability that a firm in productive development does not raise more capital is given by

$$\begin{aligned} Pr(\text{do not raise again}) &= Pr(\text{result in current round}) + Pr(\text{return to search and fail}) \\ &= \frac{\kappa}{\kappa+\omega} + \frac{\omega}{\kappa+\omega}\frac{\lambda}{\lambda+\nu} \end{aligned}$$

For notational convenience, I denote this probability by q_d. Then

$$Pr(N_f = 1) = q_d, \quad Pr(N_f = 2) = (1-q_d)\, q_d, \quad \dots \quad Pr(N_f = n) = (1-q_d)^{n-1}\, q_d$$

which is the probability mass function of the Geometric distribution. ■

Proof of Corollary 2.4. Recall $E[N_f] = \big(\frac{\kappa}{\kappa+\omega} + \frac{\omega}{\kappa+\omega}\frac{\lambda}{\lambda+\nu}\big)^{-1}$. Then

- $\frac{d}{d\nu}E[N_f] = \frac{\kappa\nu(\lambda+\nu)\omega'(\nu)+\lambda\omega(\nu)(\kappa+\omega(\nu))}{(\kappa(\lambda+\nu)+\lambda\omega(\nu))^2} > 0$ since $\omega'(\nu) > 0$.
- $\frac{d}{d\lambda}E[N_f] = \frac{\kappa\nu(\lambda+\nu)\omega'(\lambda)-\nu\omega(\lambda)(\kappa+\omega(\lambda))}{(\kappa(\lambda+\nu)+\lambda\omega(\lambda))^2} < 0$ since $\omega'(\lambda) < 0$.
- For $x \in \{p, \pi, k\}$ and $x = p$ in a value-compensating sense, $\frac{d}{dx}E[N_f] = \frac{\kappa\nu(\lambda+\nu)\omega'(k)}{(\kappa(\lambda+\nu)+\lambda\omega(k))^2} \implies \text{sign}\big(\frac{d}{dx}E[N_f]\big) = \text{sign}\big(\omega'(x)\big)$. The result then follows from Proposition 2.2.
- $\frac{d}{d\kappa}E[N_f] = \frac{(\lambda+\nu)(\kappa\nu\omega'(\kappa)-\nu\omega(\kappa))}{(\kappa(\lambda+\nu)+\lambda\omega(\kappa))^2}$. This is ambiguous, including in a value-compensating sense.

Proof of Proposition 2.4. Follows immediately from the main text. Specifically, see footnote 29.

Proof of Corollary 2.5. For all parameters except for ν and λ, the result follows immediately from Corollary 2.2. Then, $\frac{d}{d\nu}E[T_{br}] = -\frac{\omega'(\nu)}{(\kappa+\omega(\nu))^2} - \frac{1}{(\lambda+\nu)^2} < 0$, where the inequality follows from Proposition 2.2, and $\frac{d}{d\lambda}E[T_{br}] = -\frac{\omega'(\lambda)}{(\kappa+\omega(\lambda))^2} - \frac{1}{(\lambda+\nu)^2}$, which is ambiguous since $\omega'(\lambda) < 0$ by Proposition 2.2.

Proof of Lemma 1. Follows directly from solving the system of equations in p_d and p_s.

Proof of Lemma 2. Consider two start-ups $i = a, b$ facing the same funding conditions, ν and λ. Suppose $E[N_f^a] > E[N_f^b]$. Then,

$$\left(\frac{\kappa_a}{\omega_a+\kappa_a} + \frac{\omega_a}{\kappa_a+\omega_a}\frac{\lambda}{\lambda+\nu}\right)^{-1} > \left(\frac{\kappa_b}{\omega_b+\kappa_b} + \frac{\omega_b}{\kappa_b+\omega_b}\frac{\lambda}{\lambda+\nu}\right)^{-1} \implies \frac{\omega_a}{\kappa_a+\omega_a} > \frac{\omega_b}{\kappa_b+\omega_b}$$

The result for f_d follows immediately given the expression in Remark 1. Finally, note that $f_d = \frac{\omega}{\kappa+\omega} f_s \implies f_s = \frac{\lambda}{\lambda+\nu} E[N_f]$. Therefore $f_d^a > f_d^b \implies f_s^a > f_s^b$. ■

Proof of Proposition 2.5. Entry is proportional to the value of search, V_s. Substituting the solution for ω into the value of search, V_s, yields

$$V_s = \frac{\nu^2(\rho+\kappa)(\kappa\pi - k) + \kappa\nu(\rho+\lambda)(\kappa\pi + k(1-2p)) - 2\nu\sqrt{\kappa k(1-p)(\rho+\lambda)(\kappa\pi(\kappa(\rho+\lambda+\nu)+\nu\rho) - k(\nu(\rho+\kappa)+\kappa p(\rho+\lambda)))}}{(\kappa(\rho+\lambda+\nu)+\nu\rho)^2} \tag{A.16}$$

Differentiation of this expression yields the results. ■

A.5 Proofs: welfare analysis

In the main body of the paper, only the steady state conditions are considered. Towards solving the planner's problem, it is necessary to consider the behaviour of the system out of steady state. To this end, note that at time t, there may be some firms in productive development with contract rate ω and others with contract rate ω'. The same is true for the unproductive development state. I will use notation $\mu_{d,t}^p(\omega)$ and $\mu_{d,t}^u(\omega)$ to refer to the measures of firms at time t in productive and unproductive development, respectively, with contract rate ω. The total measure of firms in productive and unproductive development are then given by

$$\mu_{d,t}^p = \int_0^\infty \mu_{d,t}^p(\omega)\, d\omega, \quad \mu_{d,t}^u = \int_0^\infty \mu_{d,t}^u(\omega)\, d\omega \tag{A.17}$$

and the measure of unencumbered VCs is $\mu_{vc,t} = M - \mu_{d,t}^p - \mu_{d,u}^u$. Firm in search have no contract and so are homogenous–they have measure $\mu_{s,t}$, such that the total flow of matches is $m(\mu_{s,t}, \mu_{vc,t})$.

To determine the state transition equations, it is necessary to know which contracts are being signed at each instant. In principle, each firm that signs a contract at time t could choose a different contract rate. In this spirit, I will denote by $\chi_t(\omega)$ the share of contracts signed at time t that have contract rate ω. Naturally, $\chi_t(\omega)$ is subject to the constraints $\chi_t \in [0, 1]$ and $\int_0^\infty \chi_t(\omega)\, d\omega = 1$. The total inflow into state $\mu_{d,t}(\omega)$ from new contracts is then $\chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t})$ and the total outflow of firms from search to development is $\int_0^\infty \chi_t(\omega)m(\mu_{s,t}, \mu_{vc,t})\, d\omega$. With this notation in hand, the state transition equations are given by

$$\dot{\mu}^p_{d,t}(\omega) = \chi_t(\omega)m(\mu_{s,t},\mu_{vc,t}) - (\kappa+\omega)\mu^p_{d,t}(\omega) \tag{A.18}$$

$$\dot{\mu}^u_{d,t}(\omega) = \kappa(1-p)\mu^u_{d,t}(\omega) - \omega\mu^u_{d,t}(\omega) \tag{A.19}$$

$$\dot{\mu}_{s,t} = \int_0^\infty \omega\mu^p_{d,t}(\omega)\,d\omega + \Lambda_t - \lambda\mu_{s,t} - \int_0^\infty \chi_t(\omega)m(\mu_{s,t},\mu_{vc,t})\,d\omega \tag{A.20}$$

where Λ_t is the flow rate of entry at time t, which is equal to $V_{s,t}/\tilde{\sigma}$ in the decentralised equilibrium.

The planner's problem is to choose entry, Λ_t, and the contract rate for each firm at time t in order to maximise total surplus, given by equation (12). In practice, the choice of contract boils down to choosing $\chi_t(\omega)$ for each $\omega \in [0,\infty)$ subject to the aforementioned constraints on $\chi_t(\omega)$. The current value Hamiltonian is given by

$$\begin{aligned} H = \int_0^\infty & \Big([\kappa p\pi - k]\mu^p_{d,t}(\omega) - k\mu^u_{d,t}(\omega) \\ & + \gamma^p_{d,t}(\omega)[\chi_t(\omega)m(\mu_{s,t},\mu_{vc,t}) - (\kappa+\omega)\mu^p_{d,t}(\omega)]\Big)\,d\omega \\ & + \gamma^u_{d,t}(\omega)[\mu^p_{d,t}(\omega)\kappa(1-p) - \omega\mu^u_{d,t}(\omega)] \\ & + \gamma_{s,t}\Big[\int_0^\infty \omega\mu^p_{d,t}(\omega)\,d\omega + \Lambda_t - \lambda\mu_{s,t} - \int_0^\infty \chi_t(\omega)m(\mu_{s,t},\mu_{vc,t})\,d\omega\Big] - \frac{\tilde{\sigma}}{2}\Lambda_t^2 \end{aligned} \tag{A.21}$$

The optimality condition for Λ_t is simply $\frac{\partial H}{\partial \Lambda_t} = \gamma_{s,t} - \tilde{\sigma}\Lambda_t$. For the contracts, the steady-state optimal plan will feature $\chi(\omega) = 1$ for some $\omega \in [0,\infty)$, whereas and $\chi_t(\omega) = 0$ for all $\omega' \neq \omega$. The marginal value of an increase in $\chi_t(\omega)$ is given by

$$\frac{\partial H}{\partial \chi_t(\omega)} = \big[\gamma^p_{d,t}(\omega) - \gamma_{s,t}\big]m(\mu_{s,t},\mu_{vc,t})$$

Imposing the steady-state conditions $\dot{\gamma}_{s,t} = 0$, $\dot{\gamma}^p_{d,t}(\omega) = 0$ and $\dot{\gamma}^u_{d,t}(\omega) = 0$, the shadow-values are given by

$$\begin{aligned} \gamma^p_d(\hat{\omega}) &= \frac{\Big(\rho+\lambda+\frac{\partial m}{\partial \mu_s}\Big)\Big(\kappa p\pi - \Big(1+\frac{\kappa(1-p)}{\rho+\hat{\omega}}\Big)k\Big)}{(\rho+\lambda)\Big(1+\frac{\kappa(1-p)}{\rho+\hat{\omega}}\Big)\frac{\partial m}{\partial \mu_{vc}} + (\rho+\kappa)\frac{\partial m}{\partial \mu_s} + (\rho+\lambda)(\rho+\kappa+\hat{\omega})} \\ &= \frac{(\rho+\lambda+\alpha\,\nu)\Big[\kappa p\pi - \Big(1+\frac{\kappa(1-p)}{\rho+\hat{\omega}}\Big)k\Big]}{\beta\,\upsilon\,(\rho+\lambda)\Big(1+\frac{\kappa(1-p)}{\rho+\hat{\omega}}\Big) - \nu\,(1-\alpha)\,(\rho+\kappa) + (\rho+\kappa)\,(\rho+\lambda+\nu) + (\rho+\lambda)\,\hat{\omega}} \end{aligned} \tag{A.22}$$

and

$$\gamma_s = \frac{\alpha\nu}{\rho+\lambda+\alpha\nu}\gamma^p_d(\hat{\omega}) \tag{A.23}$$

$$\gamma^u_d(\hat{\omega}) = -\frac{k}{\rho+\omega} - \frac{\beta\upsilon(\gamma^p_d(\hat{\omega})-\gamma_s)}{\rho+\hat{\omega}} \tag{A.24}$$

where $\hat{\omega}$ is assumed to be the time-invariant optimal contract rate, i.e. $\chi(\hat{\omega}) = 1$ and $\chi(\omega) = 0$ for all $\omega \neq \hat{\omega}$.[95] The planner sets $\chi(\hat{\omega}) = 1$ to maximise the marginal value of the flow of matches in steady-state, $\gamma_d^p(\hat{\omega}) - \gamma_s$.

Proof of Proposition 2.6. The proof follows immediately from the optimality conditions. First, consider the optimal contract rate. In the decentralised equilibrium, the optimal contract rate ω maximises the surplus, S, which is given by

$$S = \frac{(\rho+\lambda)\left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho+\omega}\right) k\right]}{(1-\delta)\upsilon\,(\rho+\lambda)\left(1 + \frac{\kappa(1-p)}{\rho+\omega}\right) - \nu(1-\delta)\,(\rho+\kappa) + (\rho+\kappa)\,(\rho+\lambda+\nu) + (\rho+\lambda)\,\omega}$$

In the planner's problem, the optimal contract rate maximises $\gamma_d^p(\hat{\omega}) - \gamma_s$, which is given by

$$\gamma_d^p(\hat{\omega}) - \gamma_s = \frac{(\rho+\lambda)\left[\kappa p \pi - \left(1 + \frac{\kappa(1-p)}{\rho+\hat{\omega}}\right) k\right]}{\beta\,\upsilon\,(\rho+\lambda)\left(1 + \frac{\kappa(1-p)}{\rho+\hat{\omega}}\right) - \nu\,(1-\alpha)\,(\rho+\kappa) + (\rho+\kappa)\,(\rho+\lambda+\nu) + (\rho+\lambda)\,\hat{\omega}}$$

Setting $\beta = 1-\delta$ and $\alpha = \delta$ creates an identity, so that the contract decentralised equilibrium equals that chosen by the planner.

Second, consider entry. In the decentralised equilibrium, entry is given by $V_s/\tilde{\sigma}$. In the planner's problem, the optimality condition yields $\Lambda = \gamma_s/\tilde{\sigma}$. Noting that $(\rho+\lambda)V_s = \delta\nu S$ and comparing V_s to γ_s, it is clear, for given optimal contract rate, ω, that entry is efficient if $\beta = 1-\delta$ and $\alpha = \delta$. Finally, since the contract in the decentralised equilibrium coincides with the planner's solution when the same condition is met, entry must also be efficient. ■

Proof of Propositions B.1 and B.2

Both follow directly from differentiation of the solution to ω in section B.2 of the Appendix.

B Extensions

B.1 Acquisitions

This section considers results related to the inclusion of acquisitions. The results are in the context of the one-stage (baseline) model, but apply to the generalised model without modification.

Derivation of expected capital cost. In the setting described in section 2.5, the expected capital

[95]The values for $\gamma_d^p(\omega)$ and $\gamma_d^u(\omega)$ for $\omega \neq \hat{\omega}$ can also be derived, but are not necessary to make the key point.

cost associated with a contract ω when there are acquisitions is given by

$$K_l(\omega_l) = \int_0^\infty \Bigg(\int_0^T e^{-\rho\, t}\, k\, dt + \frac{\kappa\,(1-p)}{\kappa+\omega+\hat{\phi}_d} \int_0^\infty \Big(\int_T^{T+T_{\omega_l}} e^{-\rho\, s}\, k\, ds \Big)\, (\omega+\phi)\, e^{-(\omega_l+\phi)\, T_{\omega_l}}\, dT_{\omega_l} \Bigg) \times (\kappa+\omega+\hat{\phi}_d)\, e^{-(\kappa+\omega+\hat{\phi}_d)\, T}\, dT$$

$$= \left(1 + \frac{\kappa\,(1-p)}{\rho+\omega+\phi} \right) \frac{k}{\rho+\kappa+\omega+\hat{\phi}_d} \tag{B.1}$$

Proof of Proposition 2.7. An equivalent statement is that if $\omega < \infty$, then $V_d > V_s$. To see that this must be the case, note that setting $\omega \to \infty$ implies $V_d = V_s$ and all the value is retained by the existing shareholders. In contrast, a finite contract rate implies that existing shareholders retain value $(1-\varsigma)V_d|_{\{\omega<\infty\}}$. If this is to be optimal, it must be that $\omega < \infty$ leaves the existing shareholders better off than $\omega \to \infty$, i.e. $(1-\varsigma)V_d|_{\{\omega<\infty\}} > V_s \implies V_d|_{\{\omega<\infty\}} > V_s$. Since $\varsigma|_{\{\omega<\infty\}} \in (0,1)$, this implies $V_d|_{\{\omega<\infty\}} > V_s$. ■

B.2 VC bargaining power

I now generalise the baseline model to allow for VC bargaining power. When VCs have bargaining power in contract negotiations with start-ups, they make a positive expected return. This introduces two additional considerations when determining the optimal contract rate. Firstly, current investors face a hold-up problem vis-à-vis future investors because future investors extract surplus in their own negotiations with the start-up. Therefore, VCs see their shareholding in the firm facing greater dilution in future funding rounds relative to the setting where VCs have no bargaining power. The implication is that VCs have an incentive to provide greater funding upfront to reduce the need for the firm to return to the capital market. Secondly, a VC's outside option in the case that they choose not to invest in a start-up now has positive value. Investing in a given start-up involves forgoing this outside option and, therefore, incurs an opportunity cost that raises the effective flow cost of investing in a start-up above k.[96] This second effect has the opposite implication to the first, pushing instead in the direction of a higher contract rate. In the remainder of this section, I set up the optimal contract problem when the VC has bargaining power and consider these effects more formally.

The VC. In order to solve the optimal contract problem, it is now necessary to derive the value of a VC that is searching for investment opportunities or funding a start-up. Consider the value of a VC that is funding a productive start-up, which I denote by U_d^p. The VC incurs capital flow cost k and receives a share ς of the firm. At rate κ, the start-up realises a result, which is a success with probability p and returns the VC their share of the payoff π *and* the value of search for a VC, U_s, since the VC can now seek out new investment opportunities. However, with probability $1-p$, the start-up obtains a negative result and so the VC gets the value of funding an unproductive start-up, U_p^u. Finally, at rate ω, the contract expires and the VC obtains their share in the company, a value ςV_s, and returns

[96] A similar effect is present in Michelacci and Suarez (2004).

to search. With this in mind, the HJB for a VC funding a productive start-up is given by

$$\rho U_d^p = -k + \kappa[p(\varsigma\,\pi + U_s) + (1-p)U_d^u - U_d^p] + \omega[\varsigma\, V_s + U_s - U_d^p] \tag{B.2}$$

Consider next the value of funding an unproductive start-up, U_p^u. The VC must continue to pay the flow capital cost k because they are unaware that the start-up is unproductive. In addition, at rate ω funding is withdrawn and the VC returns to search. The HJB for a VC funding an unproductive start-up is then given by

$$\rho U_d^u = -k + \omega[U_s - U_d^u] \tag{B.3}$$

Finally, the value of search for a VC depends on the rate at which they meet with start-ups, which I denote by υ. At this rate the VC meets with a start-up and obtains the value of funding an productive start-up, U_p^d. The HJB for a VC in search is given by

$$\rho U_s = \upsilon[U_d^p - U_s] \tag{B.4}$$

The contract. Upon meeting, the VC and entrepreneur engage in Nash bargaining to determine the contract rate ω and share ς obtained by the VC. The value of the match to the VC is U_d^p and the value of their outside option is U_s. The value of the match to existing shareholders is $(1-\varsigma)V_d$, because they retain a share $(1-\varsigma)$ of the gross value of the firm, and the value of their outside option is V_s. If the entrepreneur has bargaining weight δ in the negotiations, then the optimal contract solves

$$\max_{\{\varsigma\in[0,1],\omega_i\in[0,\infty)\}} \left\{ \left((1-\varsigma)V_d - V_s\right)^{\delta} \left(U_d^p - U_s\right)^{1-\delta} \right\}$$

where V_s and U_s are taken as given in the optimisation. The first-order condition for the equity stake, ς, yields the following conditions

$$(1-\varsigma)V_d = V_s + \delta S \tag{B.5}$$

$$U_d^p = U_s + (1-\delta)S \tag{B.6}$$

where $S = (1-\varsigma)V_d + U_d^p - V_s - U_s$ is the surplus of the contract. As is standard in these settings, the surplus is split according to the Nash bargaining weights. Solving equations (B.2), (B.3) and (B.4) for U_d^p in terms of U_s allows for writing the surplus as

$$S = V_d - V_s - \frac{k + \rho U_s}{\rho + \kappa + \omega}\left(1 + \frac{\kappa(1-p)}{\rho + \omega}\right) \tag{B.7}$$

Since the surplus is independent of the share that goes to the VC, the contract rate simply maximises the joint surplus, $\omega = \arg\max_{\omega\in[0,\infty)}\{S\}$. The first-order condition for ω is then

$$V_d'(\omega) = -(k + \rho U_s)\left(\frac{1-p}{(\rho+\omega)^2} + \frac{p}{(\rho+\kappa+\omega)^2}\right) \tag{B.8}$$

The first-order condition for the optimal contract rate is identical to the case without bargaining power, except that k has been replaced by $k + \rho U_s$. Intuitively, every instant that the VC pays for the investment, they incur capital cost k and *opportunity cost* $\rho \times U_s$, because they cannot engage in other investment opportunities. In addition to this direct effect, we can expect to see an indirect effect acting through the equilibrium value of V_s, which will be affected because of the future hold-up problem vis-a-vis future investors.

Dynamics. The proof to Proposition A.2 reports the equilibrium contract rate when there is VC bargaining power. To close this section, I formalise the effect of VC bargaining power on the optimal contract rate, ω, in the partial equilibrium model and then discuss whether the results of section 2.2 survive its introduction.

Proposition B.1. *The effect of increasing the bargaining power of existing shareholders, δ, on the contract rate, ω, is ambiguous. Furthermore,*

$$sign\Big(\frac{\partial \omega}{\partial \delta}\Big) = sign\Big((\rho+\kappa)k - (\rho+\lambda)\theta p\kappa\pi\Big)$$

where θ is market tightness.

The fact that the effect is ambiguous reflects the redistribution of returns induced by the change in bargaining power. As the bargaining power of existing shareholders increases, the value of search for a start-up rises and the value of search for a VC falls. This reduces the hold-up problem that current investors face with future investors, but also reduces the opportunity cost of their investment and, ultimately, which of these two effects dominates depends on model parameters.

Finally, I consider whether the core comparative statics results of the baseline model carry over to the model with VC bargaining power. Before doing so, note that it is no longer possible to consider the effect of changes in ν in isolation because ν and υ are jointly determined through the equilibrium conditions and U_s features in the contract problem. Therefore, when considering changes to funding market conditions, I assume that the matching function exhibits constant returns to scale, $m(\mu_s, \mu_{vc}) = \mu_s^{\alpha}\mu_{vc}^{1-\alpha}$, which implies that $\nu = \theta^{\alpha-1}$ and $\upsilon = \theta^{\alpha}$, where $\theta = \mu_s/\mu_{vc}$ is market tightness. I then consider the effect of increasing market tightness, θ, which results in a lower meeting rate for start-ups, a lower ν, and a higher rate for VCs, a higher υ. The next proposition is analogous to Proposition 2.2 in the baseline model with no VC bargaining power.

Proposition B.2. *The optimal contract rate, ω, is: strictly increasing in k; and strictly decreasing in p and π and weakly decreasing in λ; and ambiguous with respect to κ and θ. Furthermore, holding $p \times \pi$ constant, ω is strictly decreasing in p. Finally,*

$$sign\left(\frac{\partial \omega}{\partial \theta}\right) = sign\Big((\rho+\lambda)(1-\delta)\alpha\theta p\kappa\pi - (\rho+\kappa)(1-\alpha)\delta k\Big).$$

The only significant change in the comparative static results is the effect of market tightness. In the baseline model, changes in the meeting rate for VCs is insignificant because they do not face an opportunity cost of investment ($U_s = 0$). Therefore, when the meeting rate for start-ups, ν, falls–equivalently, market tightness increases–the optimal contract rate falls to insure the firm against financing risk. This is clear from Proposition 2.2, which considers the case $\delta = 1$. However, in a setting where the VC has bargaining power, the change in market tightness raises the value of search and so the opportunity cost of investment; this leads the VC to want to set a higher contract rate because they have a greater incentive to avoid funding unproductive start-ups. In the limit as $\delta \to 0$, it is clear from 2.2 that this effect leads the contract rate to be increasing in market tightness.

In practice, we might expect the former effect to dominate. Nanda and Rhodes-Kropf (2017) state that "when venture capital investing dried up in 2009, anecdotal evidence suggests that investors told some firms that they would only invest if the firm took an extra large amount of money relative to their annual capital need to make sure they would not have to come back to the financial markets for an extended period. Thus, while many firms were finding it impossible to raise money others were being asked to take enough for multiple years". These dynamics are consistent with $\frac{\partial \omega}{\partial \theta} < 0$ and so, through the lens of the model, are suggestive of an economy in which existing shareholders extract the majority of the surplus from venture capital investments.

C Data and estimation

C.1 Definitions

Thomson Reuters define funding rounds in the following way.

Seed stage. This stage is a relatively small amount of capital provided to an inventor or entrepreneur to prove a concept. This involves product development and market research as well as building a management team and developing a business plan, if the initial steps are successful. This is a pre-marketing stage.

Early stage. This stage provides financing to companies completing development where products are mostly in testing or pilot production. In some cases, product may have just been made commercially available. Companies may be in the process of organising or they may already be in business for three years or less. Usually such firms will have made market studies, assembled the key management, developed a business plan, and are ready or have already started conducting business.

Expansion stage. This stage involves working capital for the initial expansion of a company that is producing and shipping and has growing accounts receivables and inventories. It may or may not be showing a profit. Some of the uses of capital may include further plant expansion, marketing, working capital, or development of an improved product. More institutional investors are more likely to be included along with initial investors from previous rounds. The venture capitalist's role in this stage evolves from a supportive role to a more strategic role.

Later stage. Capital in this stage is provided for companies that have reached a fairly stable growth rate; that is, not growing as fast as the rates attained in the expansion stages. Again, these companies may or may not be profitable, but are more likely to be than in previous stages of development. Other financial characteristics of these companies include positive cash flow. This also includes companies considering IPO.

C.2 Data processing

Missing raised amounts. For the US, the amount of capital raised is not available for 2,691 of the 34,360 rounds (8%) and a slightly higher share in the UK case. Data on amounts raised is used to compute the burn rate and exit multiples, both of which are used in model calibration. In order to overcome this issue, I follow the literature and impute amounts raised in these cases. The methodology I use follows Jagannathan et al. (2022) closely. Specifically, I estimate the following regression

$$\log(\text{Amount raised}_{i,r}) = \beta_0 + \beta_1 \log(\text{Amount raised}_{i,r-1}) + \beta_2 X_{i,r} + u_{i,r} \tag{C.1}$$

where Amount raised$_{i,r}$ is the amount raised by firm i in round r (in 2015 Mn USD) and X_i is a list of firm-specific controls and fixed effects. I include the investment stage, industry, year-quarter fixed effects, and fixed effect for the number of investors, which I cap at ten investors (i.e. all rounds with 10 or more investors are lumped together). I run this regression separately for the US and UK. In addition, for some rounds I do not observe the previous funding amount and so run the specification with only the controls, $X_{i,r}$.[97] The results are displayed in Table 10. I run this on all deals between 2005 and 2022, which is why the observation counts exceed the sample used in the calibration. Using the results of these regression I impute missing amounts raised using fitted values.

	US		UK	
	Log(Amount$_r$)	Log(Amount$_r$)	Log(Amount$_r$)	Log(Amount$_r$)
Log(Amount$_{r-1}$)	-	0.47***	-	0.55***
Constant	0.43	-0.47	0.49	0.54
Industry FE	Y	Y	Y	Y
Stage FE	Y	Y	Y	Y
# Investors FE	Y	Y	Y	Y
YQ FE	Y	Y	Y	Y
Adj. R^2	0.29	0.50	0.26	0.55
Observations	74,882	43,459	7,133	2,816

Table 10: Imputation model

Notes. The table reports regression results from estimation of equation (C.1). ***, p<0.01.

C.3 Moment selection

In this section, I discuss the choice of moment to identify κ and provide details of how I estimate the values of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, which are then included as targetted moments in the model estimation.

The argument for using the share of firms to obtain late-stage funding to identify κ is as follows. Differentiating p_d from Proposition A.3 with respect to κ, we find that

$$\text{sign}\left(\frac{dp_d}{d\kappa}\right) = \text{sign}\big(\omega - \kappa\omega'(\kappa)\big)$$

For small κ, $\omega'(\kappa) < 0$ so this is always positive. Furthermore, in the limit as $\kappa \to \infty$, $\omega \to \infty$ but $\omega'(\kappa) \to 0$, so again the right-hand side is positive. It is only for intermediate values of κ that

[97] I run this on all the data, regardless of whether previous funding information is available.

$\omega - \kappa\omega'(\kappa) < 0$ is possible and, depending on the parameterisation, it may always be positive. Moving to the full model, p_d is analogous to the share of firms that are successful in navigating the early-stage. Furthermore, κ also enters the late stage and Proposition 2.5 suggests that $V_{s,l}$ is increasing in κ, so firms will write longer contracts to increase their likelihood of navigating the early stage. This intuition is confirmed through exploration of the parameter space, so that the claim that the share of firms reaching the late stage is increasing in κ is fairly robust.

Now consider the procedure to estimate $\lambda+\nu_e$ and $\lambda+\nu_l+\hat{\phi}_s$. At a high level, the idea is to leverage Proposition 2.4, which characterises the model-implied distribution of the duration between funding rounds. Generalising the results from the baseline model, the distribution between any two early-stage funding rounds is $T_{br,e\to e} \sim Hypo(\kappa + \omega_e, \lambda + \nu_e)$ and between any two late-stage funding rounds is $T_{br,l\to l} \sim Hypo(\kappa + \omega_l + \hat{\phi}_d, \lambda + \nu_l + \hat{\phi}_s)$.[98] By estimating this distribution, I can recover estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ in the data.

To see how this is implemented in practice, I need to introduce some notation. I denote by $t^s_{i,n}$ the time of the nth funding round for firm i in state $s \in \{e, l\}$ and the duration between round n and round $n+1$ for firm i in stage s by $\tau^s_{i,n} = t^s_{i,n+1} - t^s_{i,n}$. Firm i completes $N^e_i \in \{1, 2, 3, ...\}$ early-stage rounds and $N^l_i \in \{0, 1, 2, ...\}$ late-stage rounds, but N^e_i and N^l_i are unobserved because the data is truncated.[99]

The objective is to estimate $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ based on the data, $\{\tau^s_{i,n}\}$. However, there are potential concerns of sample selection. The duration between rounds in stage s, $\tau^s_{i,n}$, is only observed for firm i if: (i) firm i has at least $n+1$ funding rounds in stage s, $n+1 \leq N^s_i$; and (ii) both round n and round $n+1$ occur within seven years of the firm's first funding round. Therefore, the duration data that we observe over-samples: (a) durations of firms that have more funding rounds; (b) durations that correspond to funding rounds relatively early in a firm's maturation process; and, relatedly, (c) durations corresponding to low round-number funding rounds (in the sense of n).

To be more concrete, the objective is to estimate $Pr(\tau^s_j \leq \boldsymbol{\tau})$ for $s \in \{e, l\}$, where j indexes funding rounds, but the data that we observe is drawn from the following distribution

$$Pr(\tau^s_{i,n} \leq \boldsymbol{\tau} | n+1 \leq N^s_i \text{ and } t^s_{i,n+1} = t^s_{i,n} + \tau^s_{i,n} \leq T)$$

where T is the truncation date, set at 7 years following the firm's first funding round. To the extent that these two distributions differ in a way we cannot control for, estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ based on the data, $\{\tau^s_{i,n}\}$, will be biased.

The structure of the model provides a route forwards. To this end, note that we can write

$$\begin{aligned} Pr(\tau^s_{i,n} \leq \boldsymbol{\tau} | n+1 \leq N^s_i \text{ and } t^s_{i,n+1} = t^s_{i,n} + \tau^s_{i,n} \leq T) &= Pr(\tau^s_{i,n} \leq \boldsymbol{\tau} | \tau^s_{i,n} \leq T - t^s_{i,n}) \\ &= Pr(\tau^s_j \leq \boldsymbol{\tau} | \tau^s_j \leq T - t^s_j) \\ &= Pr(\tau^s_j \leq \boldsymbol{\tau} | \tau^s_j \leq T^s_j) \end{aligned}$$

The first line uses the fact that, in the model, the probability of a subsequent funding round within the current stage is independent of the current round number, n. Specifically, the number of funding rounds within a given stage has a geometric distribution, which is memoryless. Therefore, conditional

[98] It is also possible to characterise the distribution of the time between an early and late-stage funding round as $T_{br,e\to l} \sim Hypo(\kappa + \omega_e, \lambda + \nu_l + \hat{\phi}_s)$, but I will not use this result in the estimation.

[99] N^e_i starts at 1, not 0, because the sample is firms that have at least one funding round, which is necessarily an early-stage round.

on observing the nth funding round, the probability that the firm has an additional funding round in the same stage is independent of n and, therefore, there is no need to condition on $n+1 \leq N_i^s$. The second line makes use of the observation that we do not need to treat the unit of observation as a firm-round tuple, but rather can treat funding rounds themselves, indexed by j, as the unit of observation. This follows because, conditional on the stage, the time to the next funding round is independent of the round number and of the firm, since all firms sign the same contract. This follows from the Markov property of the model and also implies that we need not be concerned with oversampling firms with more funding rounds in any given stage, under the assumptions of the model. A final concern is that the time t_j^s of the current funding round might affect the time to the next round. However, conditional on the funding stage, this is not a concern because of the Markov property. Therefore, in the third line I define the truncation time $T_j^s = T - t_j^s$ for a given funding round.

Given the model implied distribution for the duration, $\tau_j^s \sim Hypo(h_1^s, h_2^s)$, the parameters h_1^s and h_2^s can be estimated via maximum likelihood. The likelihood for the data is given by

$$L^s = \prod_{j=1} \frac{f(\tau_j^s; h_1^s, h_2^s, T_j^s)}{F(T_j^s; h_1^s, h_2^s, T_j^s)}$$

where $f(\cdot)$ and $F(\cdot)$ are, respectively, the PDF and CDF of the hypoexponential distribution conditional on $\tau_j^s \leq T_j^s$. The estimates are displayed in Table 11.

	UK			US		
	Point estimate	95% CI	N	Point estimate	95% CI	N
h_1^e	1.60	(0.91, 2.29)	468	1.54	(1.48, 1.61)	10,833
h_2^e	2.13	(0.93, 3.32)	468	3.28	(3.00, 3.55)	10,833
h_1^l	0.80	(0.58, 1.02)	331	0.88	(0.84, 0.93)	6,665
h_2^l	2.99	(1.34, 4.65)	331	6.00	(5.10, 6.90)	6,665

Table 11: Estimates of hypoexponential distribution

A problem remains, because the parameters of the hypoexponential distribution are interchangeable, so that there is no strict guidance on whether to pick h_1^s or h_2^s (the parameters are ranked based on size in Table 11). In the moment matching exercise, I target h_2^s, the larger of the two parameter estimates, for three reasons. Firstly, suppose instead I were to target h_1^s. Then, the meeting rates ν_s would typically be lower, implying that it would be more difficult for a firm to access follow-on funding and by Proposition 2.2, this would imply a longer contract; a low value for ω_s. But if the model is to provide a good fit to the data, then the (untargeted) value of $\kappa + \omega_e$ and $\kappa + \omega_l + \hat{\phi}_d$ should be close to h_2^e and h_2^l, respectively, which is relatively high. Therefore, such a procedure is unlikely to provide a good fit to the data because the value of ω_s would be inconsistent with h_1^s. Secondly, consider that targetting $h_1^l = \lambda + \nu_l + \hat{\phi}_s$ in the late-stage would imply that the time in late-stage search, $T_{s,l} \sim Exp(h_1^l)$, which means that, for the US, the average time spent in search is 1.11 years (=1/0.90), or roughly 13 months, whereas the average amount of time spent in development would be 0.17 years (=1/5.96), or just over two months. A similar calculation can be made for the early stage, as well as for the UK. Beyond noting that this would be a highly dysfunctional market, it also contradicts conventional wisdom that venture capital firms provide start-ups with enough capital for one to two years of development. Finally and relatedly, the choice to target h_2^s to identify ν_s is a conservative approach: it implies that start-ups spend only a relatively short period of time in search and therefore limits the quantitative impact of the search friction.

C.4 Sectoral composition

Table 12 reports the number of firms and exits in the main sample from each aggregate sector used in the analysis of section 5.

	Firms		Exits	
Aggregate VEIC sector	Number	Share	Number	Share
Biotechnology	995	0.090	270	0.111
Communications and Media	289	0.026	83	0.034
Computer Hardware	298	0.027	73	0.030
Computer Software and Services	3607	0.327	893	0.367
Consumer Related	328	0.030	39	0.016
Industrial/Energy	537	0.049	56	0.023
Internet Specific	3082	0.279	739	0.304
Medical/Health	1063	0.096	170	0.070
Other Products	463	0.042	49	0.020
Semiconductors/Other Elect.	368	0.033	62	0.025

Table 12: Sector-level information

D General model

In this section, I provide the full set of equilibrium conditions for the model that I outline in section 2.5 and estimate in section 3. I first provide the full set of conditions taking as given the equilibrium in the capital market, and then provide details on the full equilibrium.

Taking the meeting rates in the early and late-stage markets, ν_e and ν_l as given, the following conditions characterise the equilibrium

$$\rho V_{d,e}(\omega_e) = \kappa\left[\hat{p}_e V_{s,l} - V_{d,e}(\omega_e)\right] + \omega_e\left[V_{s,e} - V_{d,e}(\omega_e)\right] \tag{D.1}$$

$$(\rho+\lambda)V_{s,e} = \nu_e\left[V_e^M - V_{s,e}\right] \tag{D.2}$$

$$V_e^M = \sup_{\{\omega_e\in[0,\infty),\varsigma_e\in[0,1]\}} \left\{(1-\varsigma_e)\,V_{d,e}(\omega_e)\right\} \quad \text{s.t.} \quad \varsigma_e\,V_{d,e}(\omega_e) \geq K_e(\omega_e),\ V_{s,e} \text{ given} \tag{D.3}$$

$$K_e(\omega_e) = \left(1 + \frac{\kappa\,(1-\hat{p}_e)}{\rho+\omega_e}\right)\frac{k}{\rho+\kappa+\omega_e} \tag{D.4}$$

$$\rho V_{d,l}(\omega_l) = \kappa\left[p_l\pi - V_{d,l}(\omega_l)\right] + \omega_l\left[V_{s,l} - V_{d,l}(\omega_l)\right] + \hat{\phi}_d\left[p_l\pi E[\epsilon|\epsilon > V_{d,l}(\omega_l)/(p_l\pi)] - V_{d,l}\right] \tag{D.5}$$

$$(\rho+\lambda_l)V_{s,l} = \nu_l\left[V_l^M - V_{s,l}\right] + \hat{\phi}_s\left[p_l\pi E[\epsilon|\epsilon > V_{s,l}/(p_l\pi)] - V_{s,l}\right] \tag{D.6}$$

$$V_l^M = \sup_{\{\omega_l\in[0,\infty),\varsigma_l\in[0,1]\}} \left\{(1-\varsigma_l)\,V_{d,l}(\omega_l)\right\} \quad \text{s.t.} \quad \varsigma_l\,V_{d,l}(\omega_l) \geq K_l(\omega_l),\ V_{s,l} \text{ given} \tag{D.7}$$

$$K_l(\omega_l) = \left(1 + \frac{\kappa\,(1-p_l)}{\rho+\omega_l+\phi}\right)\frac{k}{\rho+\kappa+\omega_l+\hat{\phi}_d} \tag{D.8}$$

where $\hat{p}_e = p_e/p_l$, $\epsilon \sim Exp(1/\xi)$, $\hat{\phi}_i = \phi\,[1 - F(V_{i,l}/(p_l\pi))]$ for $i \in \{s, d\}$, and (ω_e, ς_e) and (ω_l, ς_l) are chosen optimally in the sense of equations (D.3) and (D.7), respectively.

The meeting rates, ν_e and ν_l, are determined in equilibrium based on the supply and demand for capital in each stage. I assume a homogeneous set of investors that can fund both early and late-stage start-ups. I denote by $\mu_{s,i}$, $\mu^p_{d,i}$ and $\mu^u_{d,i}$ for the measures of firms in search, productive development and unproductive development for stage $i \in \{e, l\}$, respectively. Then, the measure of unencumbered VCs is given by $\mu_{vc} = M - (\mu^p_{d,e} + \mu^u_{d,e}) - (\mu^p_{d,l} + \mu^u_{d,l})$. To allow for matching efficiencies to differ between stages, I specify the flow of early-stage matches, m_e, and late-stage matches, m_l, as

$$m_e = \frac{\mu_{s,e}}{\mu_{s,e} + \zeta\mu_{s,l}} m(\cdot), \qquad m_l = \frac{\zeta\mu_{s,l}}{\mu_{s,e} + \zeta\mu_{s,l}} m(\cdot).$$

It then follows that $\zeta = \nu_l/\nu_e$ controls the degree of bias towards late-stage start-ups.

Finally, $\mu_{s,i}$, $\mu^p_{d,i}$ and $\mu^u_{d,i}$ for $i \in \{e, l\}$ are determined in equilibrium by the following steady-state conditions.

$$\begin{aligned}
\text{Early-stage productive development}: &\quad \nu_e\mu_{s,e} = (\kappa + \omega_e)\mu^p_{d,e} \\
\text{Early-stage unproductive development}: &\quad \kappa(1 - \hat{p}_e)\mu^p_{d,e} = \omega_e\mu^u_{d,e} \\
\text{Early-stage search}: &\quad \omega_e\mu^p_{d,e} + \frac{V_{s,e}}{\tilde{\sigma}} = (\lambda + \nu_e)\mu_{s,e} \\
\text{Late-stage productive development}: &\quad \nu_l\mu_{s,l} = (\kappa + \omega_l + \hat{\phi}_d)\mu^p_{d,l} \\
\text{Late-stage unproductive development}: &\quad \kappa(1 - p_l)\mu^p_{d,l} = \omega_l\mu^u_{d,l} \\
\text{Late-stage search}: &\quad \omega_l\mu^p_{d,l} + \kappa\hat{p}_e\mu^p_{d,e} = (\lambda + \nu_l + \hat{\phi}_s)\mu_{s,l}
\end{aligned}$$

where $\nu_e = m_e/\mu_{s,e}$ and $\nu_l = m_l/\mu_{s,l}$. Relative to the baseline model, there are two tangible differences. Firstly, the inflow into early-stage search includes new entrants, whereas the inflow into late-stage search includes firms that are successful in overcoming the early-stage uncertainty, a flow $\kappa_e\hat{p}_e\mu^p_{d,e}$. Secondly, in the late-stage, firms may be acquired from productive development or search.

E Case study: the UK funding gap

This section contains two main parts. Firstly, I report a broader set of moments for the counterfactual economies considered in the main text. Secondly, I provide further details on the implications of the other candidate explanations related to the supply of entrepreneurial projects and project quality.

E.1 Financing and acquisitions

Figure 12 is analogous to Figure 5 in the main text, except that it reports the remaining moments listed in Table 4. Note that these moments are untargeted in both the initial US estimation *and* in the re-estimation exercises. The fit remains relatively good across both re-estimation exercises, providing further validation of the identified channels.

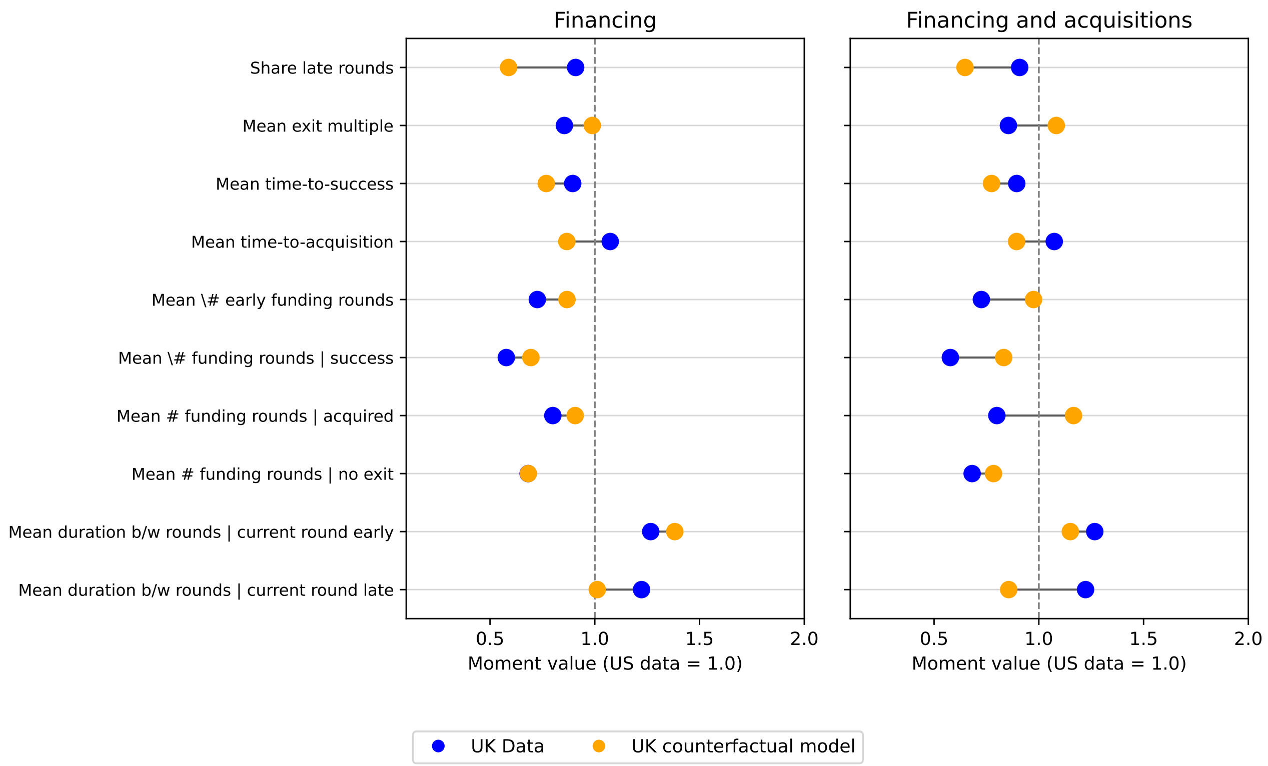

Figure 12: Re-estimation exercise - untargeted moments

The figure compares data for the UK with data from the model under various re-estimation exercises. The moments correspond to the moments listed in Table 4, excluding the share of successes and of acquisitions, which are reported in Figure 5. The left panel shows the results when the financing parameters, (λ, ν_e, ν_l), are re-estimated to match data for the UK and the right panel shows the results when the financing parameters and acquisition parameters, $(\lambda, \nu_e, \nu_l, \phi, \xi)$, are re-estimated.

E.2 Supply of entrepreneurial projects and project quality

In the main body of the paper, I argue that explanations for the disparities between the US and UK in terms of VC activity and start-up outcomes that rely heavily on 'demand-side' factors are inconsistent with the data on start-ups funding patterns, at least as seen through the lens of the model. In this section, I formalise these arguments.

To this end, I conduct two exercises. Firstly, to provide evidence against the argument that there is a more limited *supply of entrepreneurial projects* in the UK, I re-estimate the entry cost parameter, $\tilde{\sigma}$, in order to match the number of first funding rounds (entry rate) in the UK. I do so accounting for the equilibrium effects on the meeting rate and outline this process in more detail below. Secondly, to provide evidence against the argument that UK start-ups are of lower *quality*, or are for structural reasons less likely to succeed, I ask what values of the parameters p_e, p_l and π would be required to match the observed entry rate, success rate and acquisition rate for UK start-ups, again accounting for equilibrium effects. In both cases, there exist values of the parameters that generate an almost perfect match to their respective targets, but in both cases these parameters imply funding patterns at odds with the true UK data, which suggests that these explanations are not the main drivers of the UK's lagging performance.

In order to account for equilibrium effects, it becomes necessary to take a stance on the parameters of the matching function. I follow the approach set out in section D of the Appendix and assume a constant bias towards late-stage start-ups, which for the US gives $\zeta = 1.78$. I then assume that the matching function exhibits constant returns to scale with elasticity $\alpha = 0.5$. This allows me to back out an implied value for the measure of VCs, $M = 16.29$ for the US. Then, for any new set of parameters, $(\tilde{\sigma})$ or (p_e, p_l, π), I solve for the equilibrium meeting rates, ν_e and ν_l, assuming that $\zeta = 1.78$, $\alpha = 0.5$ and $M = 16.29$. These meeting rates will differ from the values estimated for the US because both entry and contracts adjust to changes in the environment and imply changes to equilibrium market tightness.

Panel A of Table 13 provides the parameter estimates. As anticipated, if the issue is the supply of entrepreneurial projects, then $\tilde{\sigma}$ must be higher to reduce entry and, as a result, the firms that do enter face looser financing conditions. On the other hand, in order to generate lower entry, success rates and acquisition rates, the model requires that UK projects are inherently less likely to succeed, lower p_e, with slightly higher payoffs in the case of success, higher π, and little movement in p_l. Such projects are reminiscent of what Ewens et al. (2018) refer to as "long-shot bets", in that they are less likely to overcome the initial hurdle but, conditional on intermediate success, have similar project value. As I argue in the main body of the paper, both cases are inconsistent with the data on UK funding patterns. Figure 13 is analogous to Figure 5 in the main body of the paper. In the left panel, when only the entry cost parameter is re-estimated, all movements in moments are driven by the looser financing conditions facing start-ups that do enter and the counterfactual economy. The outcome is funding patterns that are at odds with the UK data. In the right panel, which considers project quality, the combination of projects that resemble long-shot bets and the looser funding conditions that result from lower entry imply that start-ups raise less capital at each funding round and visit the funding market more frequently. Again, across the board this has implications that are inconsistent with the data on UK funding patterns.

Panel A: Equilibrium model						
Parameter	$\tilde{\sigma}$	p_e	p_l	π	ν_e	ν_l
US: baseline	4.34	0.17	0.26	316.88	2.92	5.20
UK: supply of projects	13.33	0.17	0.26	316.88	7.97	14.20
UK: project quality	4.34	0.08	0.24	375.52	10.41	18.55
Panel B: Partial equilibrium model						
UK: project quality	4.34	0.10	0.26	358.53	2.92	5.20

Table 13: Counterfactual exercise: parameter estimates and equilibrium meeting rates

Panel A reports the parameter estimates for parameters related to entry $(\tilde{\sigma})$ and quality (p_e, p_l, π) in the US baseline and two counterfactual scenarios. In 'UK: supply of projects', $\tilde{\sigma}$ is re-estimated to match UK data on the number of first funding rounds, and in 'UK: project quality', p_e, p_l and π re-estimated to match UK data on the number of first funding rounds, the share of successes and the share of acquisitions. In both cases, meeting rates in the financing market respond, and the final two columns report their values in the new equilibrium. Panel B reports the estimates when the same exercise is conducted for the quality parameters, but the meeting rates are held constant at their US levels in the estimation.

One concern with this approach is that it is sensitive to the choice of matching function elasticities. Another is that it may not be optimal for as many VCs to participate in the UK market if the start-

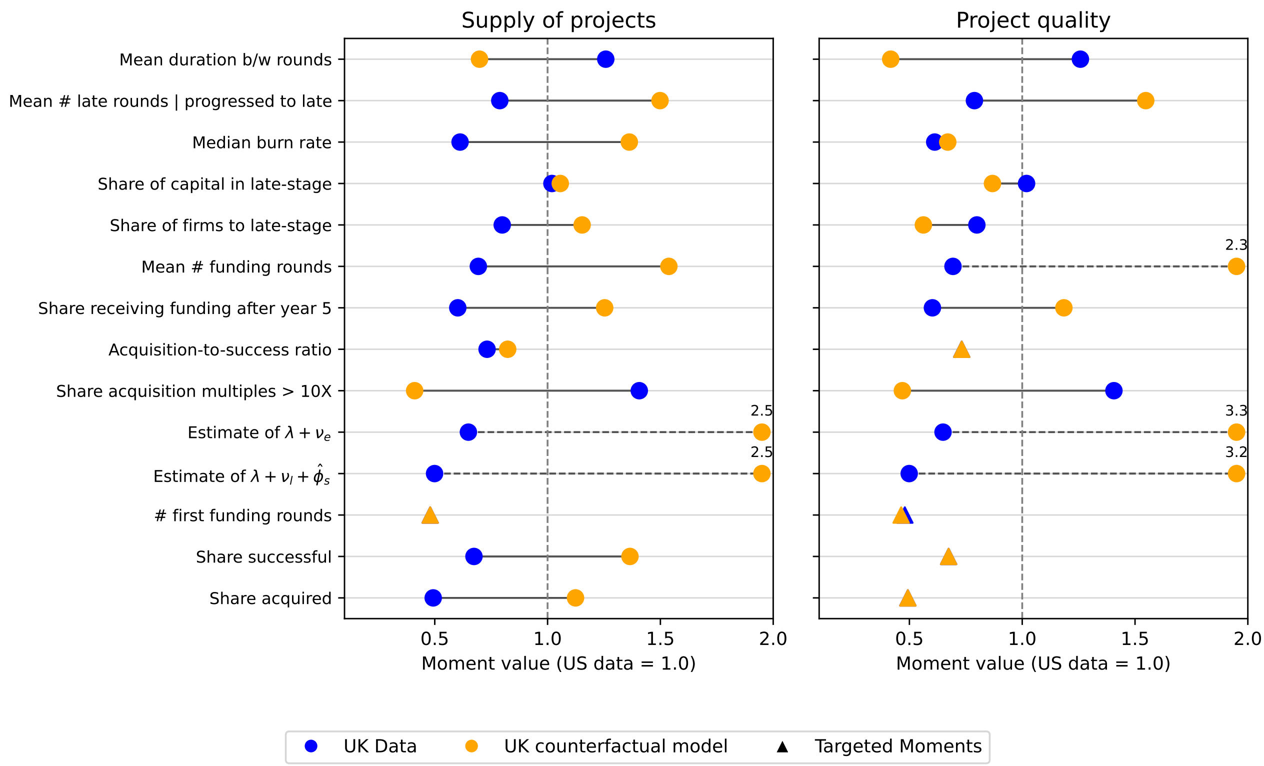

Figure 13: Re-estimation exercise

The figure compares data for the UK with data from the model under various re-estimation exercises. The left panel shows the results when the entry cost parameter, $(\tilde{\sigma})$, is re-estimated to match data for the UK and the right panel shows the results when the project quality parameters, (p_e, p_l, π), are re-estimated. Circular markers indicated moments that are untargeted in the re-estimation exercise and triangular markers indicate targeted moments. In the left panel, the markers for 'Acquisition-to-success ratio' is indicated as targeted, because it is implicitly given the targeting of the share of successes and acquisitions. The x-axis is scaled to aid comparison with Figure 5; where data would not fit within these axis limits, a dashed line joins the data points and the number above the marker indicates the true value.

ups are of lesser quality. Collectively, a reasonable criticism is that the results observed in Figure 13 may reflect the adjustment in equilibrium meetings rates as entry falls, and this adjustment may be counterfactually large and drive the key results. To eschew these concerns, I repeat the exercise for 'project quality' but hold the meeting rates, ν_e and ν_l at their US levels.[100] Such an approach eliminates any influence of adjustments to market tightness on start-up trajectories. Panel B of Table 13 reports the estimated parameter values, where p_e, p_l and π are estimated to match the entry rate, and the share of start-ups that are successful and are acquired. Reassuringly, the parameter estimates are very close to the values estimated for the equilibrium model, as shown in Panel A. Finally, 14 reports the key moments. Relative to the case where the meeting rates adjust, funding patterns are more in line with the data for the UK, but the key differences still remain. Therefore, although adjustments in the financing market do amplify the effect of project type on funding patterns, they are not driving the results. Put differently, even in the extreme case of no funding market adjustment, project quality remains inconsistent with funding pattern data across a number of margins.

[100]For the supply of entrepreneurial projects, removing any equilibrium effects imply that funding patterns are the same as in the US estimation.

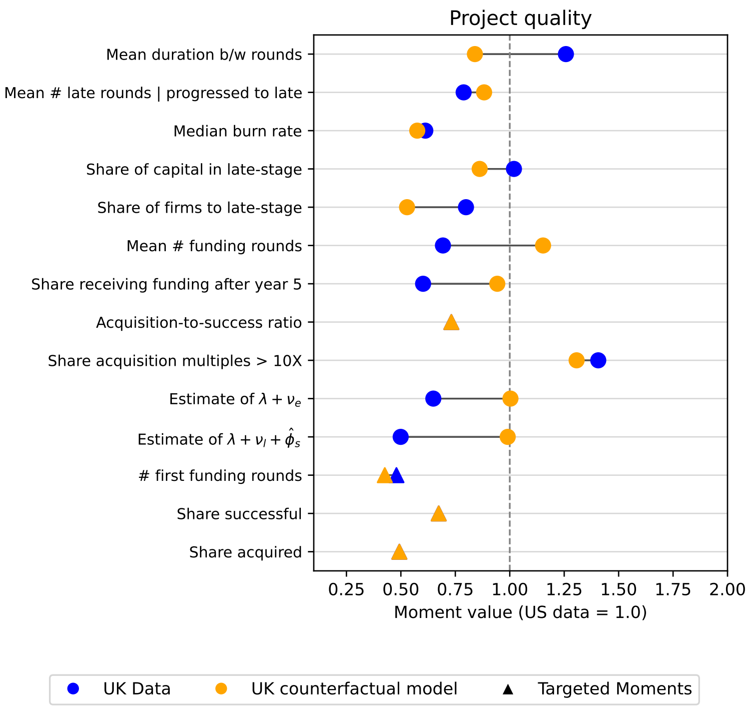

Figure 14: Re-estimation exercise: partial equilibrium

The figure compares data for the UK with data from the model when the project quality parameters, (p_e, p_l, π), are re-estimated. Relative to the right panel of Figure 13, the meeting rates, ν_e and ν_l, are held at their US levels.

F Sectoral misallocation of VC funding

F.1 Project values

In the analysis of section 5, I impose a relationship between project development horizons, κ, access to acquisition opportunities, ϕ, and the payoff to projects that are successful, π. I do this to ensure that frictionless project values are constant across all technologies that I consider, thereby focusing on the effects of project characteristics rather than quality in driving the allocation of funding. In practice, this implies that I impose a negative relationship between κ and π, because shorter-horizon projects face smaller distortions and so require less compensation through the payoff, π. Similarly, it implies a negative relationship between ϕ and π across technologies.

In this section, I provide evidence supporting this approach. For each sector used in the analysis in section 5, Figure 15 plots in red the mean exit value against the average time-to-exit and the acquisition-to-success ratio. For start-ups whose exit values are unreported in the dataset, the exit value is set at 1.5 times their total investment, following Kerr et al. (2014). As is visible from the figure, there is a positive relationship between the mean exit value and time-to-exit and a negative relationship between the mean exit value and the acquisition-to success-ratio across sectors.

This evidence is consistent with the correlation structure imposed in the estimation exercise between (κ, ϕ) and π across technologies, τ. To further confirm this intuition, Figure 15 also plots these relationships for the technologies used in the estimation exercise in blue, demonstrating that the relationships observed in the data are replicated in the model.

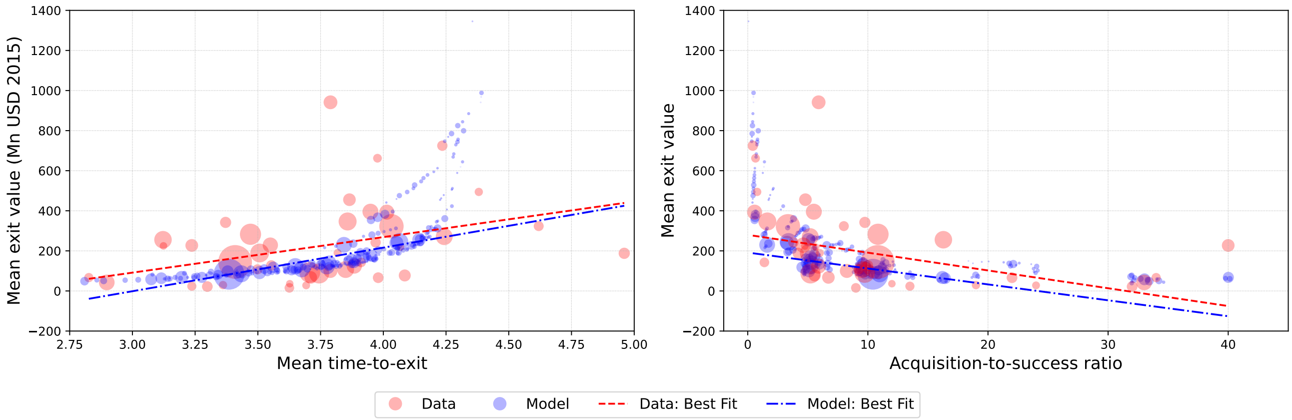

Figure 15: Exit values

The Figure plots the mean exit value against the mean time-to-exit and acquisition-to-success ratio for VEIC sectors (red) and model-based technologies (blue). The size of each point reflects its weight, $s(\tau)$ for technologies and $s(v)$ for sectors. The lines of best fit are weighted by the shares $s(\tau)$ and $s(v)$, respectively.

F.2 Optimal transport

In this section, I specify the optimal transport problem used to map 'technologies' to sectors in the sectoral misallocation exercise of section 5.

Continuing to use the notation introduced in the main text, I have two datasets: (i) an empirical dataset, $\{\mathbf{x}_v, s(v)\}_{v=1}^{N_v}$, and (ii) a model dataset $\{\mathbf{x}_\tau, s(\tau)\}_{\tau=1}^{N_\tau}$. The objective is create a mapping from 'technologies' in the model dataset to VEIC sectors and, intuitively, I want to map technology τ to sector v if they are close in the (acquisition-to-success, time-to-success)-space, which is denoted by $\mathbf{x}$.

To construct this mapping, I assign a portion $w(\tau, v)$ of the share of technology τ, $s(\tau)$, to each sector v. A natural approach is to frame this as a simple optimal transport problem, which attempts to minimise the total transportation cost in the (acquisition-to-success, time-to-success)-space. The problem then becomes one of choosing values of $w(\tau, v)$ to solve the following problem

$$\min_{\{w(\tau,v)\}} \left\{ \sum_{\tau=1}^{N_\tau} \sum_{v=1}^{N_v} s(\tau) w(\tau, v) c(\tau, v) \right\}$$

subject to

$$\begin{aligned}
\text{Supply constraints:} \quad & \sum_{v=1}^{N_v} w(\tau, v) = 1, \quad \forall \tau \\
\text{Demand constraints:} \quad & \sum_{\tau=1}^{N_\tau} s(\tau) w(\tau, v) = s(v), \quad \forall v \\
\text{Non-negativity constraints:} \quad & w(\tau, v) \geq 0, \quad \forall \tau, v
\end{aligned}$$

where $c(\tau, v)$ is the transportation cost from technology τ to sector v, which I specify as the Euclidean distance in the (acquisition-to-success, time-to-success)-space, $c(\tau, v) = ||\mathbf{x}_\tau - \mathbf{x}_v||_2$.

Briefly, a total mass $s(\tau)w(\tau, v)$ is transported from technology τ to sector v, which has cost $c(\tau, v)$ per unit. The total transportation costs is sums over all transportations. The supply constraint states that all mass from technology τ to sector v is transported somewhere. I define $w(\tau, v)$ as the weight on share of mass from τ that is associated with sector v, so these sum to one. The demand constraint ensures that all sectors are completely accounted for. The optimal transport plan minimises the total transport cost subject to these 'summing-up' constraints.