CSC 212: Data Structures and Abstractions
Spring 2018
University of Rhode Island

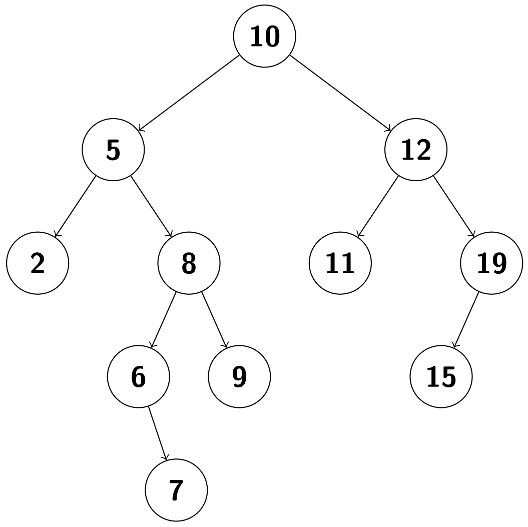
Weekly Problem Set #11 Solutions

Due Thursday 4/19 before class. Please turn in neat, and organized, answers hand-written on standard-sized paper **without any fringe**. At the top of each sheet you hand in, please write your name, and ID. The only library you're allowed to use in your answers is `iostream`.

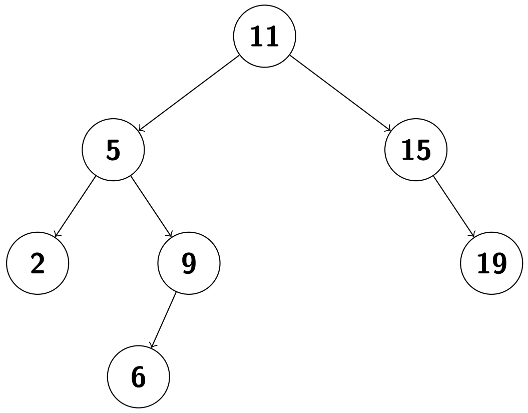
1 Binary Search Trees

1. Draw a binary search tree after the following operations steps:

   (a) Insert: [10, 5, 12, 8, 19, 6, 2, 11, 15, 9, 7]



   (b) Remove: [7, 12, 8, 10]



2. Write a function to delete binary trees. Be sure to remove nodes in the proper order, so that none get orphaned.

```
// Initial call should pass root to clear whole tree.
void clear(BSTNode* node) {
    if (!node) return;

    clear(node->left);
    clear(node->right);
    delete node;
    return;
}
```

3. Assume nodes in a BST contain 4 data members: *data, depth, left, right.* Write a recursive function that, given a pointer to the root of a BST, will update every node's *depth* to it's own depth in the tree.

```
// Initial call should pass 0 for 'depth'.
void update_depth(BSTNode* node, int depth) {
    if (node) {
        node->depth = depth;
        update_depth(node->left, depth+1);
        update_depth(node->right, depth+1);
    }
}
```

4. Briefly explain the difference between in-order, post-order, and pre-order traversals.

   The three terms above signify different methods of traversing a binary tree. From the point of view of the node, in-order signifies visiting the left child, then itself, then the right child; post-order signifies visiting both children before itself; pre-order signifies visiting itself before any children. All three are necessary for different scenarios, as all provide different utility. For example, you cannot delete a node until all children are removed, yet you cannot know the depth of the children without visiting the parent first.

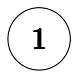
5. Implement a binary search tree with all of the following methods: constructor, destructor, insert, search, remove.

6. Let T be a full k-ary tree, where $k = 2$ (a.k.a. *binary tree*), with n nodes. Let h denote the height of T.

   (a) What is the minimum number of leaves for T? Justify your answer.

$$h + 1$$

   Example when $h = 0$: T, being a *full tree* can have a minimum of 1 leaf.



(b) What is the maximum number of leaves for T? Justify your answer.

$$2^h$$

If we consider the root having height 0, then our maximum number of leaves is when we have a perfect binary tree of height h, thus the maximum number of leaves we can have is:

$$1, 2, 4, 8, \ldots, 2^h$$

(c) What is the minimum number of internal nodes for T? Justify your answer.

$$h$$

Considering the smallest tree, with only one node, we can see that it is possible to have 0 internal nodes.

(d) What is the maximum number of internal nodes for T? Justify your answer.

$$2^h - 1$$

The maximum number of internal nodes can be expressed as the difference between the number of nodes in a tree, and the number of leaves (since nodes can either be leaves, or be internal). The maximum number of internal nodes occurs when a tree is perfect. The number of nodes in a perfect tree is expressed as $2^{h+1} - 1$, the number of leaves in a perfect tree is expressed as 2^h, therefore:

$$(2^{h+1} - 1) - 2^h = 2^h - 1$$

7. Give a $O(n)$ time algorithm for computing the `height` of the tree, where n is the number of nodes.

```
// Initial call should pass depth=-1, this special value
// is returned when the tree is empty.
int height(BSTNode* node, int h) {
    if (node) {
        return max(
            depth(node->left, depth+1),
            depth(node->right), depth+1)
        )
    } else {
        return depth;
    }
}
```

8. Show that the maximum number of nodes in a binary tree of height h is $2^{h+1} - 1$.

$$1 + 2 + 4 + 8 + \ldots + 2^h = 2^{h+1} - 1$$