

Temporal aggregation of (growth) timeseries

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1 Introduction

This vignette discusses temporal aggregation of timeseries, i.e. the conversion of a timeseries to a lower frequency. The vignette is an extension of the vignette “Introduction to package regts”, but can be read independently.

Section 2 describes the standard aggregation function **aggregate**. Section 3 discusses aggregation for growth timeseries with several methods, which are described in detail in section 4.

2 Function aggregate

The standard function **aggregate** can be used to convert a timeseries to a lower frequency, for example a quarterly to an annual timeseries, or a monthly to a quarterly timeseries.

An example with a monthly timeseries:

```
> regm <- regts(1:24, start = "2016M1")
> regm
```

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 2016 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2017 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

```
> aggregate(regm, FUN = mean, nfrequency = 4)
```

| | Qtr1 | Qtr2 | Qtr3 | Qtr4 |
|------|------|------|------|------|
| 2016 | 2 | 5 | 8 | 11 |
| 2017 | 14 | 17 | 20 | 23 |

The variables in the timeseries are split into blocks of length **frequency/nfrequency** and **FUN** is applied to each such block. The result returned is a timeseries with frequency **nfrequency** holding the aggregated values.

Another example with a quarterly timeseries:

```
> regq <- regts(1:10, start = "2016q1")
> regq
```

| | Qtr1 | Qtr2 | Qtr3 | Qtr4 |
|------|------|------|------|------|
| 2016 | 1 | 2 | 3 | 4 |
| 2017 | 5 | 6 | 7 | 8 |
| 2018 | 9 | 10 | | |

```
> aggregate(regq, FUN = sum)
```

| | +1 |
|------|-------|
| 2016 | 10 26 |

The quarters in 2016 add up to 10, the quarters in 2017 to 26. Note that the 2 quarters in 2018 are ignored. The **aggregate** function skips the incomplete years at the end.

Aggregate also skips incomplete years at the beginning. For example, the next timeseries starts in the second quarter:

```
> regq2 <- regts(2:10, start = "2016q2")
```

```
> regq2
```

| | Qtr1 | Qtr2 | Qtr3 | Qtr4 |
|------|------|------|------|------|
| 2016 | | 2 | 3 | 4 |
| 2017 | 5 | 6 | 7 | 8 |
| 2018 | 9 | 10 | | |

```
> aggregate(regq2, FUN = sum)
```

```
2017 26
```

The quarters in 2017 add up to 26 as before. But now the information for 2016 is also not complete, and therefore ignored.

Thus for **regts** objects the **aggregate** function skips all incomplete years at the beginning and end of the timeseries. However for standard timeseries (**ts**) the implementation of **aggregate** only skips incomplete years at the end.

The previous example but now with a **ts** timeseries:

```
> tq2 <- ts(2:10, start = c(2016,2), frequency = 4)
> tq2
```

| | Qtr1 | Qtr2 | Qtr3 | Qtr4 |
|------|------|------|------|------|
| 2016 | | 2 | 3 | 4 |
| 2017 | 5 | 6 | 7 | 8 |
| 2018 | 9 | 10 | | |

```
> aggregate(tq2, FUN = sum)
```

```
Time Series:
Start = 2016.25
End = 2017.25
Frequency = 1
[1] 14 30
```

The result is an unconventional timeseries with frequency **year** but a shifted period (the start is 2016.25). The result for 2016.25 is the sum of the observations in period 2016Q2/2017Q1.

In all other cases the **aggregate** function works exactly the same for **regts** and **ts** objects.

3 Aggregation of growth timeseries

As is shown in the next section the function **aggregate** does not yield correct results for timeseries with absolute, relative or percentage changes.

For growth timeseries the function **aggregate_gr** can be used. This function implements several methods and can be employed for both **regts** and **ts** timeseries.

All methods convert input timeseries with high frequency to low frequency outputseries. For different types of input timeseries there are four different type of methods : **dif1**, **dif1s**, **rel** and **pct**.

Methods **dif1** and **dif1s** assume that the input timeseries is a first difference:

$$x_t = z_t - z_{t-1}$$

where **t** is the period (like year or quarter). For method **dif1s** the input series is assumed to be scaled with the output frequency (for example an annualized quarterly series):

$$x_t = n(z_t - z_{t-1})$$

where n is the ratio of the high and the low frequency. These methods calculate a timeseries with first difference in the output frequency.

The `rel` and `pct` methods assume that the input timeseries is a one-period relative or percentage change and calculate the exact relative or percentage change for the output timeseries. The methods are described in detail in the next section.

First a timeseries is created containing differences:

```
> xdif <- diff(regts(c(1,3,4,3,6,2,4,1,3,2), start = "2015q4"))
> xdif
```

| | Qtr1 | Qtr2 | Qtr3 | Qtr4 |
|------|------|------|------|------|
| 2016 | 2 | 1 | -1 | 3 |
| 2017 | -4 | 2 | -3 | 2 |
| 2018 | -1 | | | |

Two examples of aggregation methods are shown, the results are annual timeseries:

```
> aggregate_gr(xdif, method = "dif1", nfrequency = 1)
```

2017 -6

```
> aggregate_gr(xdif, method = "dif1s") # result is also scaled
```

2017 -1.5

4 Description aggregation methods

This section describes the growth methods implemented in function `aggregate_gr`, including the derivation of the corresponding mathematical formulas.

Suppose we are temporally aggregating a timeseries x with n subperiods to a timeseries X with a lower frequency. In other words we want to convert a timeseries x with a high frequency (monthly, quarterly) to a new timeseries with a lower frequency (quarterly, annual).

Let $x_{t,i}$ stand for the value of x in subperiod i of main period t . The index i takes on values in the range $1..n$. We always interpret $x_{t,0}$ to mean $x_{t-1,n}$. The time index t refers to the periods in the time domain of X . Many conversion methods only need the observations subperiod i of main period t . For example the `mean` method calculates X_t as follows

$$X_t = \sum_{i=1}^{i=n} x_{t,i}/n$$

However there are transformations which are not so straightforward. A case in point is when x is a high frequency first difference and must be converted to a low frequency first difference. A simple averaging of n high frequency observations will not give the correct answer. For example, for a quarterly growth series the yearly aggregates also depend on the quarterly observations of the previous year. Therefore we cannot use function `aggregate`, function `aggregate_gr` must be applied. In the next subsections a more detailed description of the available frequency conversion methods for this function is given.

4.1 dif1 method

Define x to be the first difference of z . Thus

$$x_{t,i} = z_{t,i} - z_{t,i-1}$$

We can also define $z_{t,i}$ in terms of x as follows

$$\begin{aligned} z_{t,i} &= z_{t,i-1} + x_{t,i} \\ &= z_{t-1,n} + \sum_{j=1}^i x_{t,j} \end{aligned}$$

Now define Z_t as the level timeseries in time domain t corresponding to z

$$\begin{aligned} Z_t &= \sum_{i=1}^n z_{t,i} \\ &= \sum_{i=1}^n z_{t-1,n} + \sum_{i=1}^n \sum_{j=1}^i x_{t,j} \\ &= \sum_{i=1}^n z_{t-2,n} + \sum_{i=1}^n \sum_{j=1}^n x_{t-1,j} + \sum_{i=1}^n \sum_{j=1}^i x_{t,j} \end{aligned}$$

Then X in any period t can be calculated from

$$X_t = Z_t - Z_{t-1}$$

Since Z_{t-1} can be written as

$$Z_{t-1} = \sum_{i=1}^n z_{t-2,n} + \sum_{i=1}^n \sum_{j=1}^i x_{t-1,j}$$

it is easy to show that X_t does not depend on $z_{t-2,n}$. In any practical algorithm $z_{t-2,n}$ can be set to 0.

A compact equation for X_t can be derived by substituting the equations for Z_t and Z_{t-1} into the definition of X_t

$$X_t = \sum_{i=1}^n \left(\sum_{j=i+1}^n x_{t-1,j} + \sum_{j=1}^i x_{t,j} \right)$$

This expression can be further simplified by changing the order of summation. For the first summation, we can write

$$\sum_{i=1}^n \sum_{j=i+1}^n x_{t-1,j} = \sum_{j=2}^n \sum_{i=1}^{j-1} x_{t-1,j} = \sum_{j=2}^n (j-1)x_{t-1,j}$$

Similarly,

$$\sum_{i=1}^n \sum_{j=1}^i x_{t,j} = \sum_{j=1}^n \sum_{i=j}^n x_{t,j} = \sum_{j=1}^n (n-j+1)x_{t,j}$$

The final equation is given by

$$X_t = \left(\sum_{j=2}^n (j-1)x_{t-1,j} + \sum_{j=1}^n (n-j+1)x_{t,j} \right)$$

4.2 dif1s method

Define x to be the first difference of z scaled to the output frequency. Thus

$$x_{t,i} = n(z_{t,i} - z_{t,i-1})$$

Further derivations are analogous to the case for the `dif1` method described in the previous section.

Now the result for `dif1s` is the result for `dif1` divided by `n`:

$$X_t = \left(\sum_{j=2}^n (j-1)x_{t-1,j} + \sum_{j=1}^n (n-j+1)x_{t,j} \right) / n$$

4.3 rel method

Define x to be the relative change in z . Thus

$$x_{t,i} = (z_{t,i} - z_{t,i-1})/z_{t,i-1}$$

We can also define $z_{t,i}$ in terms of x as follows

$$\begin{aligned} z_{t,i} &= z_{t,i-1}(1 + x_{t,i}) \\ &= z_{t-1,n} \prod_{j=1}^i (1 + x_{t,j}) \\ &= z_{t-2,n} \prod_{j=1}^n (1 + x_{t-1,j}) \prod_{j=1}^i (1 + x_{t,j}) \end{aligned}$$

Now define Z_t as the level timeseries in time domain t corresponding to z

$$\begin{aligned} Z_t &= \sum_{i=1}^n z_{t,i} \\ &= z_{t-1,n} \sum_{i=1}^n \prod_{j=1}^i (1 + x_{t,j}) \\ &= z_{t-2,n} \prod_{j=1}^n (1 + x_{t-1,j}) \sum_{i=1}^n \prod_{j=1}^i (1 + x_{t,j}) \end{aligned}$$

Then X in any period t can be calculated from

$$X_t = Z_t/Z_{t-1} - 1$$

Using the expression for Z_{t-1}

$$Z_{t-1} = z_{t-2,n} \sum_{i=1}^n \prod_{j=1}^i (1 + x_{t-1,j})$$

we obtain

$$X_t = \frac{\prod_{j=2}^n (1 + x_{t-1,j}) \sum_{i=1}^n \prod_{j=1}^i (1 + x_{t,j})}{1 + \sum_{i=2}^n \prod_{j=2}^i (1 + x_{t-1,j})} - 1$$

4.4 pct method

Define x to be the percentage change in z . Thus

$$x_{t,i} = 100(z_{t,i} - z_{t,i-1})/z_{t,i-1}$$

The derivations are analogous to the **rel** method described in the previous subsection.

The result is

$$X_t = 100 \frac{\prod_{j=2}^n (1 + x_{t-1,j}/100) \sum_{i=1}^n \prod_{j=1}^i (1 + x_{t,j}/100)}{1 + \sum_{i=2}^n \prod_{j=2}^i (1 + x_{t-1,j}/100)} - 1$$