Introduction to package regts

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1 Introduction

When you start looking in R for timeseries you easily find several classes and packages, class ts from the stats package and zoo being mentioned most.

The function ts creates a regular timeseries object with class ts. This can be a univariate or multivariate object. It has a start, an end, a frequency and you can print, plot, extend and update it. But class ts has no easy way to deal with fixed periods.

Regts is a package defined for regular timeseries objects built on class ts, but with features for easier selecting periods. The regts package defines a class regts that is an extension to the ts class. Because of this the regts class benefits of all the functionality of ts. The period handling is much easier than in the ts class, particularly for monthly, quarterly and annual timeseries.

Package xts also facilitates period selection, but concentrates more on daily (or weekly) based data.

Package regts provides some more extensions to package ts:

- functions for easily reading and writing timeseries to Excel and csv
- the use of labels, a description of the timeseries
- an easy way to convert a data frame to a timeseries object
- a function to calculate differences between multiple timeseries
- and furthermore a special set of aggregation functions for timeseries representing growth rates

To use this package regts load it with:

```
> library(regts)
```

2 Creation regts

2.1 Single timeseries

A single regts timeseries is created with the function regts. An example for a quarterly timeseries:

```
> tq <- regts(1:10, start = "2016Q3")
```

The end period of the timeseries is based on the length of the input data. The period can also be a month, like "2016M3", or a year:

```
> ty <- regts(c(.11, .29, .18, .24), start = "2017")
+1 +2 +3</pre>
```

```
+1 +2 +3
2017 0.11 0.29 0.18 0.24
```

Note that annual timeseries are printed differently than quarterly timeseries.

The class of tq and ty is regts, and because of the inheritance, also ts.

```
> class(tq)
```

```
[1] "regts" "ts"
```

If the input data is longer than you require you can make a selection with the end argument:

```
> ts_data <- regts(1:10, start = "2016Q1", end = "2016Q4")
```

If the input data is shorter than the period determined by the start and the end argument, the data is repeated:

As you can see the length of the period doesn't have to be a multiple of the length of the input data.

Class regts recognizes special formats for quarterly (2016Q1) and monthly (2016M1) timeseries. To create timeseries with a different frequency, specify the frequency argument:

```
> ts_data <- regts(1:10, start = "2016-1", end = "2017-1", frequency = 2)
```

Instead of arguments start and end you can also specify argument period:

```
> ts_data <- regts(1:10, period = "2016Q1/2016Q4")
```

Input for this period may be a character string as shown above, but also a variable of type period_range. This type will be explained in section 10.1.

2.2 Multivariate timeseries

2017

1

Sometimes it is more convenient to combine several timeseries with the same period in a single object: a multivariate timeseries. A multivariate timeseries is just a matrix with two or more columns. multivariate regts is such a matrix with some extra attributes.

Argument names can be used to set the column names. If omitted then the column names of the input matrix are used. In the next example the input matrix has no column names:

```
> rts1 <- regts(matrix(1:9, ncol = 3), start = "2016Q1")
> rts1
```

```
[,1] [,2] [,3]
2016Q1 1 4 7
2016Q2 2 5 8
2016Q3 3 6 9
```

A multivariate timeseries inherits the classes mts (multivariate ts), ts and matrix:

```
> class(rts)
```

```
[1] "regts" "mts" "ts" "matrix"
```

Timeseries names are preferably valid R names (only include a-z, A-Z, _, and 0-9 and start with a letter) and well chosen. In general they will be brief. If more documentation is needed for a timeseries than just this (short) name, optionally labels can be added (see also section labels).

2.2.1 Column selection and creation

Column selection for regts is the same as for matrices.

a1_ts is now a univariate timeseries, ac_ts a multivariate subset of rts.

For the regts class we can also create new columns:

This is not possible for classes ts and matrix.

2.2.2 Selecting columns with a regular expression

The regts package contains a function select_columns to select columns of an R object with column names. Besides regts this can be a matrix or a data frame. The columns with names matching a given regular expression are selected. A few examples:

The syntax of regular expression patterns is described in the R Documentation.

2.3 Matrices with one column

If a regts is created from a matrix with only one column, the timeseries is single (or univariate). So there are two types of single timeseries: one that is based on a vector and one that is based on a one-column matrix:

They have identical output:

```
> ts
```

[,1] 2016Q1 1 2016Q2 2 2016Q3 3 2016Q4 4 2017Q1 5 2017Q2 6 2017Q3 7

and identical classes:

```
> class(ts)
```

2017Q4

```
[1] "regts" "ts"
> class(ts_1col)
```

```
[1] "regts" "ts"
```

but different underlying data.

Another difference between the 'vector' version and the 'matrix-1-column' version is that the latter can have a column name:

```
> ts_1col <- regts(matrix(1:8, ncol = 1), start = "2016Q1", names = "a")
```

The column name is not printed with the output, but can be shown with the View or the colnames function:

```
> colnames(ts_1col)
```

```
[1] "a"
```

Single 'vector' timeseries never have a column name.

A single 'matrix-1-column' timeseries is also created when a single column is selected from a multivariate timeseries with the argument <code>drop = FALSE</code>. In this case the column name is preserved. If this argument is omitted (or <code>drop = TRUE</code> is used) a single 'vector' timeseries is created. A possible column name disapppears in this case. An example:

```
> rts <- regts(matrix(1:8, ncol = 2), start = "2016Q1", names = c("a", "b"))
>
> rts[ ,1, drop = FALSE] # result is a one-dimensional matrix with column name "a"
> rts[ ,2] # result is a vector, column name "b" is lost
```

3 Period selection

The package regts is specially designed for the selection of periods. Where for a ts timeseries you have to use the window function:

with a regts you can select periods with the selection operator []. Period selection is a form of subsetting.

```
> ts <- regts(1:8, start = "2016Q1", end = "2017Q4")
> ts["2016Q4/2017Q2"]
```

Besides a range you can also select one period:

```
> ts["2017Q1"]
```

or a range with an open beginning or end:

```
> ts["/2017Q2"]
> ts["2017Q1/"]
```

or a range with a lower frequency:

```
> ts["2017"]
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2017 5 6 7 8
```

If a selection is specified outside the defined range, the timeseries is filled with NA values for that range:

```
> ts["/2018Q2"]
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2015 NA NA
2016 1 2 3 4
2017 5 6 7 8
```

Period selection can also be applied on the left hand side of an assignment, in the target timeseries. The object must exist before applying selection.

```
> rts1 <- regts(1:8, start = "2010Q1", end = "2011Q4")
> rts1["2011Q2"] <- 2
> rts1["/2010Q3"] <- 99
> rts1
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2010 99 99 99 4
2011 5 2 7 8
```

The period of the timeseries can also be extended:

```
> rts1["2009Q1/2009Q2"] <- rts1["2010Q3/2010Q4"]
> rts1
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2009
                        NA
       99
              4
                   NA
2010
       99
             99
                   99
                          4
2011
                    7
                          8
        5
              2
```

The missing quarters (2009Q3 and 2009Q4) are filled in with NA values.

Period selection for multivariate regts

Selecting a period for a multivariate regts is quite similar. You can select one or more periods, or a combination of period(s) and columns:

All the extended elements become NA.

NA

Select a combination of period and columns:

```
> rts["2011Q4/", c("a", "b")]
a b
```

Selection can also take place in the target timeseries:

```
a b c x

2011Q1 1 5 9 1

2011Q2 2 6 10 1

2011Q3 3 7 11 1

2011Q4 4 8 12 1

2012Q1 99 99 99 99
```

2012

2011Q4 4 8

NA

4 Reading and writing timeseries

Some common used file types to store data in R are R data, csv or xls(x) files. Inside R projects, preferably use R data files. Writing and reading R objects to and from R data files is fast, easy and efficient. If data has to be exported, you can use for example csv or Excel files.

4.1 Rds and RData file

The main R data filetypes are Rds and RData. Both filetypes are R-readable binary files.

Rds stores a single R object. If you want to write more objects you can combine them in a list and write that list to the file. An R object can be stored to a Rds file by the saveRDS function in the base package:

```
> saveRDS(rts, file = "output.rds")
```

The standard extension is .rds.

An object saved in a .rds file can be reloaded by the readRDS function, and you can assign the contents of an .rds file, possibly under a different name.

```
> series <- readRDS("output.rds")
```

The result is an R object.

Another native R data type is RData. Functions save and load are available to save and restore single or multiple objects to and from a RData file. When loading, the objects are restored with the same name they had when saved (the load function has no return variable). This can be annoying. Also it is not clear what the result of the load function is.

4.2 External files

Package regts includes functions for easily reading and writing timeseries to and from csv or Excel files.

4.2.1 Reading and writing csv files

With the read_ts_csv function a regts variable can be read from a csv file. This function has several handy arguments such as column and decimal separators, skipping rows or columns and a name function. It employs function fread of the data.table package.

In most cases it succeeds in finding the correct frequency and determining how the timeseries are stored: rowwise or columnwise. If not, extra input arguments can be given. Consider the following csv file

```
,a_mn___,afk__c_wn,afk__f_r_wn,afk__l_wn,afk__r_wn
2010Q1,
            6.83,
                     23.4,
                                    19.3,
                                               3.84,
2010Q2,
            6.98,
                     23.0,
                                    19.8,
                                               3.45,
                                                         19.35
2010Q3,
            6.65,
                     23.4,
                                    19.5,
                                               3.54,
                                                         19.83
2011Q1,
            6.21,
                     23.2,
                                    19.8,
                                               3.85,
                                                         19.53
```

Notice that the periods are in the first column, the other columns contain the timeseries. The following command will do:

```
> read_ts_csv("test.csv")
```

	amn	afkc_wn	afk_{f_rwn}	afk_lwn	afk_r_wn
2010Q1	6.83	23.4	19.3	3.84	19.38
201002	6.98	23.0	19.8	3.45	19.35

2010Q3	6.65	23.4	19.5	3.54	19.83
2010Q4	NA	NA	NA	NA	NA
2011Q1	6.21	23.2	19.8	3.85	19.53

The function has an argument rowwise. We could have used rowwise = FALSE to indicate that the timeseries are stored columnwise. But in this case it is not needed, the period column is recognised by the function.

Sometimes the csv file is transposed, as in the following example:

```
; 2017Q1; 2017Q2; 2017Q4
A__MN_BE;
             6.83;
                      6.8;
                              6.71
AC_FWN ;
           23.34;
                    23.24;
                             23.20
BD_WN___;
           19.35;
                    19.38;
                             19.58
BG_WN___;
            3.84;
                     3.84;
                              3.54
                    19.58;
                            19.35
C__WN___;
           19.38;
```

Now the periods are in the first row (column headers), the next rows contain the timeseries. Note that ; is the field separator and that quarter 2017Q3 is missing in the file.

```
> read_ts_csv("transpose.csv")
```

```
A_MN_BE AC_FWN BD_WN___ BG_WN___ C__WN___
2017Q1
            6.83
                  23.34
                            19.35
                                       3.84
                                                19.38
                                                19.58
2017Q2
            6.80
                  23.24
                            19.38
                                       3.84
2017Q3
              NA
                     NA
                               NA
                                         NA
                                                   NA
2017Q4
            6.71
                  23.20
                            19.58
                                       3.54
                                                19.35
```

The sep argument is not needed, separator; (as well as, and tab) is recognised by the function. By default the separator is determined automatically.

Since regts (ànd ts) objects always have a continuous period¹, the missing quarters are filled with NA values.

The timeseries names in this file are in uppercase. You can convert them while reading:

```
> read_ts_csv("transpose.csv", name_fun = tolower)
```

```
a_mn_be ac_fwn bd_wn___ bg_wn___
                                             c__wn___
2017Q1
            6.83
                  23.34
                            19.35
                                       3.84
                                                19.38
2017Q2
            6.80
                  23.24
                            19.38
                                       3.84
                                                19.58
2017Q3
              NA
                     NA
                               NA
                                         NA
                                                   NA
2017Q4
            6.71
                  23.20
                            19.58
                                       3.54
                                                19.35
```

Files may be more complicated then the examples shown above. Possibly there are comments, extra rows or columns, or redundant information in the file. This usually does not cause problems: read_ts_csv can skip these redundant rows. If necessary, use arguments skiprow and skipcol.

Use function write_ts_csv to store a regts timeseries in a csv file. By default the timeseries are stored rowwise (i.e. the period is written in the first row).

```
> rts <- regts(matrix(1:8, ncol = 2), start = "2016Q1", names = c("a","b"))
> write_ts_csv(rts, "result.csv")
```

You may want to write the data in transposed form, with a different separator:

```
> write_ts_csv(rts, "result_transposed.csv", rowwise = FALSE, sep = "&")
```

¹regts and ts objects are regular timeseries. Packages xts and zoo provide also irregularly spaced timeseries objects. Then periods can be missing.

4.2.2 Reading and writing Excel files

Special functions have been developed for regts objects

- read_ts_xlsx (employing the read_excel function in package readxl) for reading an Excel file, both for xls and xlsx format
- write_ts_xlsx for writing an xlsx file employing package xlsx

Like the corresponding csv function, read_ts_xlsx has arguments for skipping rows or columns, orientation and frequency. Besides that you can specify a sheet, a range and how to recognise na-strings. Consider the next xlsx file:

Tabel

names	code	2017Q1	2017Q2	2017Q3	2017Q4
AMN_BE		6.83	6.8	NA	6.71
AC_FWN		23.34	23.24	23.20	23.68
BD_WN		19.35	19.38	19.58	19.68
BG_WN		3.84	3.84	3.54	3.74
CWN		19.38	19.58	19.35	19.69

When reading this file a regts object is returned:

```
> read_ts_xlsx("series.xlsx", skiprow = 2, name_fun = tolower, na_string = c("","NA"))
```

```
a_mn_be ac_fwn bd_wn__ bg_wn__ c_wn__
2017Q1
                           19.35
                                              19.38
           6.83
                 23.34
                                     3.84
2017Q2
           6.80
                 23.24
                           19.38
                                     3.84
                                              19.58
2017Q3
                 23.20
                           19.58
                                     3.54
                                              19.35
             NA
2017Q4
           6.71
                 23.68
                           19.68
                                     3.74
                                              19.69
```

The function skips the first 2 lines, analyses where the period begins, converts the timeseries names to lowercase and recognises that empty cells and cells with text 'NA' are not available.

Function write_ts_xlsx is used for writing a regts object to an xlsx file. It creates or opens an Excel workbook depending on argument append. Appending to a non existent file means creating the file. You can specify the sheetname (by default Sheet1) and whether you want to write labels. An example:

```
> write_ts_xlsx(rts, "data.xlsx", sheet_name = "extra", append = TRUE)
```

In this case a sheet 'extra' is appended to the xlsx file.

4.2.3 Problems when reading

Sometimes data in a file needs to be processed before it can be converted to a timeseries. This happens when a file has a deviant format.

In that case try reading information as a data frame type. Most functions in R for reading a csv return a data frame. Examples: read.csv, read.csv2 and read.table in package utils. Functions, read_xls(x) and read_excel in package readxl or read.xlsx (in package xlsx) for reading Excel files return some sort of data frame.

After some processing (e.g. selecting and/or deleting (parts of) rows and columns) the data frame can be converted to a regts as explained in section 11.5.

5 Labels

A label is a way to document an object or the elements of an object. Labels in base R exist but only for objects. In the regts package each timeseries in a regts object can be equipped with a label. Labels can be defined when the series are created and they are preserved in most timeseries functions.

As you can see the labels are not shown automatically when the timeseries are printed. With the View function you can view timeseries and their labels in RStudio. The result looks like this:

	child	adult	senior
	Age 0-17	Age 18-65	Age 65+
2017Q1	1	6	3
2017Q2	3	4	0
2017Q3	2	8	9

The labels can also be retrieved with the ts_labels function:

```
> ts_labels(t1)
```

```
child adult senior
"Age 0-17" "Age 18-65" "Age 65+"
```

In combination with the <- operator, the ts_labels command can be used to assign labels to, or remove labels from an existing regts:

```
> tx <- regts(data, start = "2017Q1")
> ts_labels(tx) <- c("Trans X", "Label X")
> ts_labels(tx)
```

```
[1] "Trans X" "Label X"
> ts_labels(tx) <- NULL  # remove all labels
> ts_labels(tx)
```

NULL

For updating one or more timeseries labels in a regts object, the function update_ts_labels is provided:

```
> t1_update <- update_ts_labels(t1, labels = c(adult = "Age 18-67", senior = "Age 67+"))
```

```
child adult senior "Age 0-17" "Age 18-67" "Age 67+"
```

Argument labels is a named character vector with the column names as names and the labels as values.

5.1 Labels in files

In this section we discuss how to read timeseries provided with labels from a csv or xls(x) file, or how to write timeseries including labels to a file. Also the situation is viewed where label information is available on a separate file.

5.1.1 Reading timeseries with labels from a file

It is easy to read timeseries with labels from a csv or xls(x) file, provided that the labels are placed before or after the timeseries names. Read the following Excel file including the labels:

```
2016Q1 2016Q2 2016Q3 2016Q4
        label
name
bbpwn
       "dbp"
                        6.83
                                6.98
                                       6.65
bet_wn "taxes"
                        23.4
                                23.0
                                       23.4
                                              23.2
clt_wn "expenses"
                       19.3
                                19.8
                                       19.5
                                              19.8
hl__hr "productivity"
                       3.84
                                3.45
                                       3.54
                                              3.86
a__mn_ "labour"
                                19.35 19.83 19.54
                        19.38
> series <- read_ts_xlsx("tslabels.xlsx", labels = "after")
```

Now check the labels:

> ts_labels(series)

bbpwn	bet_wn	${\tt clt_wn}$	hlhr	amn_
"bbp"	"taxes"	"expenses" "pro	oductivity"	"labour"

Labels and timeseries can also be stored columnwise:

	arbeidsvolume	bbp	belasting	coll.lasten
	$a_mmn_$	bbp_m_wn	bet_c_wn	clt_c_wn
2010Q1	6.83	23.4	19.3	3.84
2010Q2	6.98	23.0	19.8	3.45
2010Q3	6.65	23.4	19.5	3.54
2010Q4	6.21	23.2	19.8	3.85

Read this file with result tseries and look at the labels:

5.1.2 Reading labels from a separate file

Suppose we have a file with label information. This label file contains a list of timeseries names and the corresponding labels:

```
NAMES, LABELS
a, label_a
b, label_b
d, label_d
c, label_c
```

There is also a regts object which contains two series: c and b.

```
> ts <- regts(matrix(1:4, ncol=2), names = c("c","b"), start = "2016")
```

We want to add some labels from this file to the timeseries using the function update_ts_labels. Input for this function is a named character vector containing the labels.

The label file contains the labels for the two timeseries. Variable dflabel is read from the file as a data frame using function read.csv:

```
> dflabel <- read.csv("label.csv")
> dflabel

NAMES LABELS
1    a label_a
2    b label_b
3    d label_d
4    c label c
```

With the dflabel variable a named character vector labels is constructed. The names can be set with the names function:

```
> labels <- dflabel[[2]]
> names(labels) <- dflabel[[1]]
> labels
```

```
a b d c label_a label_b label_d label_c Levels: label_a label_b label_c label_d
```

The variable labels contains more labels than necessary. Also the ordering of the labels is not the same as in the column names of the regts object. This is not a problem, the function update_ts_labels takes care of all that:

```
> ts <- update_ts_labels(ts, labels)</pre>
```

This is the result (of View(ts)):

```
c b label_c label_b 2016 1 3 2017 2 4
```

5.1.3 Writing timeseries with labels to file

Functions write_ts_csv and write_ts_xlsx write timeseries to file automatically including the labels. By default they are written after the names. An example:

Specify labels = "before" to write the labels before the names and specify labels = "no" to prevent the labels being written.

5.2 Labels in dataframes

How to handle labels when converting a data frame to a regts or vice versa and when transposing a data frame is described in section 11.

6 Operators and functions for regts

Since the regts class is an extension of the ts class all functionality from this class is available. All arithmetic (+, -, *, /, ^, %%, %/%), logical (&&, ||, !), relational (==, !=, <, <=, >, >=) and subsetting ([]) operators applicable for a ts variable, can be used for a regts timeseries. Some examples:

When a binary arithmetic operator is used on two timeseries objects, the intersection of the periods is taken:

```
> tx <- regts(1:6, start = "2016Q1")
> ty <- regts(1:6, start = "2017Q1")
> tx + ty
```

```
Qtr1 Qtr2
2017 6 8
```

You can use all functions applicable to a ts class timeseries. Some examples:

```
> tsin <- sin(tx)
> tlog <- log(tx)
> diff(tx)
> plot(tx)
```

Combining timeseries

As we have seen all functions that are available for ts are also available for regts. The cbind function (an S3 generic function) is one of them: it binds two or more objects with a common frequency. With the cbind function it is easy to combine several univariate timeseries to a multivariate timeseries, or a combination of univariate and multivariate timeseries to a new multivariate timeseries.

Yet the extension for regts has special features: arguments union and suffixes.

The argument union effects the period range. With union = TRUE, the default value, the union of the period ranges of the joined objects is taken, otherwise it is the intersection. An example:

```
> rt1 <- regts(1:4, start = "2016Q1")
> rt2 <- regts(1:4, start = "2016Q2")
> cbind(rt1, rt2, union = TRUE)
```

```
rt1 rt2
2016Q1 1 NA
2016Q2 2 1
2016Q3 3 2
2016Q4 4 3
2017Q1 NA 4
```

```
> cbind(rt1, rt2, union = FALSE)
rt1 rt2
```

2016Q2 2 1 2016Q3 3 2 2016Q4 4 3

For union = TRUE the result is padded with NA's if needed.

The suffixes argument is obligatory if the joined objects have overlapping column names:

```
> rt1 <- regts(data1, start = "2016Q1", names = c("b", "a", "c"))
> rt2 <- regts(data, start = "2016Q1", names = c("d", "b"))
> cbind(rt1, rt2, suffixes = c("_1", "_2"))
```

The implementation of cbind.regts is also different from function cbind.ts (which is an alias for ts.union) because of the fact that available column names are always preserved.

7 Multiple or chain calculations

Sometimes you have a 'chain' of calculations, where the left hand side of an equation is used in the right hand side of a following equation. For example:

```
> a <- regts(1:4, start = "2010Q2")
> b <- a + 1
> c <- b / a
> d <- lag(a, -1)
> cbind(a, b, c, d)
```

```
a b c d
2010Q2 1 2 2.000000 NA
2010Q3 2 3 1.500000 1
2010Q4 3 4 1.333333 2
2011Q1 4 5 1.250000 3
2011Q2 NA NA NA A
```

The result is a multivariate regts.

The code is less transparent when the chain is evaluated using a multivariate timeseries:

```
> xts <- regts(data, start = "2010Q2", names = c("a", "x"))
> xts[, "b"] <- xts[, "a"] + 1
> xts[, "c"] <- xts[, "b"] / xts[, "a"]
> xts["2010Q3/2011Q2", "d"] <- lag( xts[, "a"], -1)
> xts
```

```
a x b c d
2010Q2 1 5 2 2.000000 NA
2010Q3 2 6 3 1.500000 1
2010Q4 3 7 4 1.333333 2
2011Q1 4 8 5 1.250000 3
2011Q2 NA NA NA NA NA 4
```

Note that when timeseries d is created you also have to select the lagged period. Otherwise the xts period is used. The reason is that in an assignment the period is not aligned.

In this case another method can be preferable. First convert the multivariate regts to a list of univariate timeseries with the as.list function. Then apply the within function.

```
> xts <- regts(data, start = "2010Q2", names = c("a", "x"))
> list <- as.list(xts)
> list2 <- within(list, {
+     b <- a + 1
+     c <- b / a
+     d <- lag(a, -1)
+ })</pre>
```

The result is a list. The timeseries in this list can be combined with a combination of functions cbind and do.call. Note the reverse ordering of the columns.

```
> do.call(cbind, list2)

a x d c b
2010Q2 1 5 NA 2.000000 2
2010Q3 2 6 1 1.500000 3
2010Q4 3 7 2 1.333333 4
2011Q1 4 8 3 1.250000 5
2011Q2 NA NA 4 NA NA
```

The elements in the result list all have type regts, so the result is a multivariate regts.

If another numeric or logical type is added to this list, it is also converted to a regts by cbind:

```
> list3 <- within(list2, {
+     e <- 0
+     f <- TRUE
+ })
> do.call(cbind, list3)
```

```
a x d c b f e 2010Q2 1 5 NA 2.000000 2 1 0 2010Q3 2 6 1 1.500000 3 1 0 2010Q4 3 7 2 1.333333 4 1 0 2011Q1 4 8 3 1.250000 5 1 0 2011Q2 NA NA A NA NA 1 0
```

The elements e and f are repeated for the whole period and the value TRUE is coerced to numeric. 2

8 Update function

Sometimes you want to use a multivariate timeseries to update another multivariate timeseries. Function update_ts updates the columns of a timeseries with the values in the columns of another timeseries with the same name.

Of course the two timeseries must have the same frequency but their period ranges may be different (non overlapping periods will be set to NA). For the result timeseries the union of the ranges is taken. There are three different methods to deal with NA values. Possible methods for this function update_ts are:

²You could also add a character variable, i.e. g <- "a". Now the result will remain a regts, but all elements of the timeseries will be coerced to character data, which is probably not what you want.

- upd use all of the second timeseries to update the first timeseries
- updna only replace NA values in the first timeseries with values from the second timeseries
- updval only use valid values in the second timeseries to update the first timeseries

Of course only the common columns in both timeseries are updated. Extra columns in the second timeseries aer added to the result, depending on the method and the content of that columns.

Some examples:

```
> series1 <- regts(matrix(rep(1,6), ncol = 3), start = "2017Q1", names = c("a", "b", "c"))
> series2 <- regts(matrix(rep(2,4), ncol = 2), start = "2017Q1", names = c("a", "d"))
> series1["2017q2", "b"] <- NA
> series2[, "c"] <- NA
> update_ts(series1, series2, method = "upd")
       a b c d
2017Q1 2 1 NA 2
2017Q2 2 NA NA 2
> update_ts(series1, series2, method = "updna")
       a bcd
2017Q1 1 1 1 2
2017Q2 1 NA 1 2
> update_ts(series1, series2, method = "updval")
       a bcd
2017Q1 2 1 1 2
2017Q2 2 NA 1 2
```

In the first result as much values as possible are updated. A difference between the second and the last result: variable "a" is left unchanged, because it is not "NA", or updated, because of valid values in the second series.

9 Differences between multivariate timeseries

Comparing two multivariate timeseries with a large number of columns can be quiet tedious, therefore function tsdif is created. It compares two multivariate timeseries objects which must have the same frequency. The function calculates differences between columns with the same name, they do not have to be in the same order. The actual function used to compute the differences can be specified, by default normal differences are computed. Also argument tol can be specified: all differences smaller than tol will be ignored. By default tol is 0.

```
> rt1 <- regts(matrix(1:12, ncol = 3), start = "2016Q1", names = c("a", "b", "c"))
> # create timeseries with slight differences, larger and smaller than the tolerance factor
> rt2 <- rt1
> rt2[,"a"] <- rt1[,"a"] + 0.01
> rt2[,"b"] <- rt1[,"b"] + 1e-6
> rt2
```

```
2016Q1 1.01 5.000001 9
2016Q2 2.01 6.000001 10
2016Q3 3.01 7.000001 11
2016Q4 4.01 8.000001 12
```

```
tsdif timeseries comparison result

Compared timeseries: rt1 and rt2

Difference tolerance: 0.001

Names of timeseries with differences:

[1] "a"

Differences (the first 4 rows and 1 columns)

a

2016Q1 -0.01

2016Q2 -0.01

2016Q3 -0.01

2016Q4 -0.01

The function shows a limited report of the result, but it's result is a list with several components. Some of them are shown in the next example.

> difvar <- tsdif(rt1, rt2, tol = 1e-3)

> difvar$equal

[1] FALSE
```

[1] "a"

> difvar\$tol

> difvar\$difnames

> tsdif(rt1, rt2, tol = 1e-3)

[1] 0.001

In this exampl equal is FALSE, because there are differences larger than the tolerance tol. The names of the common columns with differences larger than the tolerance appear in diffnames. Component dif is a regts with the computed differences of these columns. If there are no differences dif becomes NULL. In this case the difference between columns a is equal for all quarters: 0.01. The tolerance argument tol is actually an input argument but added to the list of components for the sake of completeness.

In the previous example the two objects had the same column names. Now we look at an example where the column names are different. We create a new timeseries rt3, an extension of timeseries rt1 with an extra column:

```
> rt3 <- rt1
> rt3[, "d"] <- regts(9:12, start = "2016Q1")
> tsdif(rt1, rt3)
```

tsdif timeseries comparison result

Compared timeseries: rt1 and rt3

No differences found

Missing timeseries in rt1:

[1] "d"

> tsdif(rt1, rt3)\$missing_names1

```
[1] "d"
```

The missing_names1 component contains the column names in the second timeseries that are missing in the first timeseries, in this case variable d. There are no missing_names2.

```
> tsdif(rt1, rt3)$equal
```

[1] FALSE

```
> tsdif(rt1, rt3)$dif
```

NULL

All computed differences are smaller than or equal to the tolerance factor, so component dif is NULL. Component equal is FALSE; it is only TRUE if both component dif is NULL and there are no missing names.

Let's compare two columns of regts objects rt1 and rt2: columns b differ less than the tolerance factor and columns c are equal. Now the result is equal:

```
> dif_bc <- tsdif(rt1[, c("b","c")], rt2[, c("c","b")], tol = 1e-5)
> dif_bc$equal
```

[1] TRUE

As you can see the ordering of the columns makes no difference.

10 period and period_range

When the same period is used several times, it is preferable to define a variable containing this period. This could be a text string containing a period but package regts also defines special classes representing periods: the period class for a single period and the period_range class for a range of periods. With these classes is it possible to manipulate the period or the period_range, as shown further on in this section.

10.1 period

A period object can be created with the period function.

```
> prd1 <- period("2016Q4")
> prd1
```

[1] "2016Q4"

```
> prd2 <- period("2015-2", frequency = 4)
> prd2
```

```
[1] "2015Q2"
```

In the last example the frequency has to be specified. "2015-2" is ambiguous. Here "2" denotes the second quarter but it could also be for instance a month indicator or a half year. The frequency argument is required if you want to define a period with a frequency other than 1, 4 or 12, since there is no special format for these frequencies.

Special functions have been designed for the period class e.g. to extract a subperiod or the year:

```
> get_subperiod(prd1)
```

[1] 4

```
> get_year(prd2)
```

[1] 2015

period objects can be used as input for the start and end arguments of the regts function.

```
> ts <- regts(1:10, start = prd1)
```

When printed, a period variable seems to be nothing more than a character string. With the str function we can show the internal representation of a period object:

```
> str(period("2016Q4"))
```

```
Class 'period' atomic [1:1] 8067
..- attr(*, "frequency")= num 4
```

The internal representation of period 2016Q4 is the number of quarters since the beginning of the Christian era, starting with 0 (8067 = 2016 * 4 + 3).

10.2 period_range

There is also a period_range, an object that represents an interval of periods. Such a range can be created in the following ways:

```
> range <- period_range("2014Q4", "2017Q4")
> range
```

[1] "2014Q4/2017Q4"

```
> period_range("2014Q4/2017Q4")
```

[1] "2014Q4/2017Q4"

The lower or upper bound of the range may be undetermined:

```
> period_range("2017Q1", NULL) # No upper bound
```

[1] "2017Q1/"

A period_range variable can be used in the regts function, specifying the period argument:

```
> range1 <- period_range("2016Q1", "2016Q3")
> rts <- regts(matrix(1:18, ncol = 3), period = range1, names = c("a", "b", "c"))
> rts
```

```
a b c
2016Q1 1 7 13
2016Q2 2 8 14
2016Q3 3 9 15
```

A period_range variable can also be obtained from a timeseries:

```
> get_period_range(rts)
```

[1] "2016Q1/2016Q3"

The length of a period_range can be determined by the nperiod function:

```
> nperiod(range1)
```

[1] 3

Functions start_period and end_period are used to retrieve the first and last period in a period_range:

```
> start_period(range1)
[1] "2016Q1"
> end_period(range1)
[1] "2016Q3"
> range2 <- period_range(start_period(range1)+1, end_period(range1)-1)</pre>
> ts <- regts(data, period = range2)</pre>
```

10.3 Shifting period(s)

With operators + and - the period can be shifted. Some examples for period:

```
> period("2016Q4") + 1
[1] "2017Q1"
> p1 <- period("2015Q1")
> p1-5
[1] "2013Q4"
And for a period_range:
> period_range("2016Q4", "2017Q4") + 1
[1] "2017Q1/2018Q1"
> period_range(p1, p1+4)
                                     # a `period` as input
[1] "2015Q1/2016Q1"
```

Note that one of the operands must be numeric³.

Selection with period(_range) objects 10.4

In section 3 we have shown how periods can be selected with a string. A period can also be used to select or modify a period of a timeseries object. Consider the following (previously defined) timeseries:

```
> rts
      ab c
2016Q1 1 7 13
2016Q2 2 8 14
2016Q3 3 9 15
```

Use a period to select and modify the timeseries.

```
> prd1 <- period("2016Q2")</pre>
> rts[prd1]
       ab c
2016Q2 2 8 14
> rts[prd1] <- 3
> rts
```

³Command period("2015Q1") + period("2015Q2") results in an error message: 'Arithmetic operation on two periods is not allowed.

```
ab c
2016Q1 1 7 13
2016Q2 3 3 3
2016Q3 3 9 15
```

A period_range can be used to select and modify multiple periods:

```
> # define a period range and make selections
> range <- period_range("2016Q2", "2016Q3")
> var <- rts[range]</pre>
> rts[range, c("a", "b")]
       a b
2016Q2 3 3
2016Q3 3 9
> # define another period range and create a new timeseries column
> smpl <- period_range("2016Q1", "2017Q1")
> rts[smpl, "xx"] <- 2</pre>
> rts
        a b c xx
2016Q1 1 7 13 2
2016Q2 3 3 3 2
2016Q3 3 9 15 2
2016Q4 NA NA NA 2
2017Q1 NA NA NA 2
```

Note that because timeseries xx is created with an extended period, all timeseries in (multivariate regts) object rts are extended.

11 Conversion between regts and data frame

In R a data frame is a much-used type for storing data tables or vectors. It is therefore useful to convert a data frame object to a regts and vice versa. In this section we explain how this can be done. We also discuss how to handle labels. And we give an example how to read information from a deviant file.

data frame to regts 11.1

Suppose we have a data frame with (numerical) data. It is possible to use such a data frame as input data for the regts function:

```
> df <- data.frame(a = 1:3, b = 4:6, c = 7:9)
> ts <- regts(df, start = "2017Q1")
                                                 # create a multivariate regts
> ts
       a b c
2017Q1 1 4 7
2017Q2 2 5 8
2017Q3 3 6 9
```

The function regts employs the data.matrix function in the base package to convert a data frame to a numeric matrix⁴. By default the column names of the data frame are used for the timeseries names.

⁴Columns with text strings are converted to NA

As with matrix input, it is also possible to specify names or add labels. Specifying names is already described in section 2.2. Labeling of data frames will be discussed in section 11.3.

In the previous example we had to specify the period of the result timeseries. Sometimes the period is already contained in the data frame, e.g. in the row names. Consider the following data frame:

Now the function as.regts can be used to convert a data.frame to a regts:

Function as.regts assumes that the period is stored in the row names. If the periods are in a column of the data frame, then the time_column argument can be used. You can specify the column name or number of the data frame in which the period is stored. Specifying 0 means the period is contained in the row names of the data frame.

Function as.regts uses the function period to convert the 'period text' (like 2015Q3) to a period. This function can handle several period formats: 2016, 2016Q4, 2016.4q, 2016/4Q, 2016-4q, 2016_q4, ... where no distinction is made between 'q' and 'Q'.

If the period is ambiguous, use the frequency argument:

```
> # data frame with indicisive period in row names
> df <- data.frame(a = 1:3, b = 4:6)
> rownames(df) <- c("2015-3", "2015-4", "2016-1")
> ts <- as.regts(df, frequency = 4)</pre>
```

If the data frame contains periods in another, non standard, format, it is possible to specify your own function for the conversion of a text to a period. Use the fun argument, if necessary with extra arguments.

11.2 regts to data frame

The reverse conversion from a regts to a data frame can be achieved with function as.data.frame. The as.data.frame function has been extended for regts types. Variable ts has been previously defined:

> as.data.frame(ts)

```
a b
2015Q3 1 4
2015Q4 2 5
2016Q1 3 6
```

The results look similar to ts. The timeseries names have become the data frame column names and the periods show up in the row names. Of course the classes are different.

11.3 Labels when converting data frame to regts and vice versa

Base R does not have facilities for adding labels to a data frame. However the Hmisc package has introduced a function label for adding label attributes to data frames columns and to retrieve them. Package regts uses this function to preserve the labels, when converting a regts object with labels to a data frame or vice versa.

Consider the following data frame:

```
> df <- data.frame(a = 1:3, b = 6:8, c = 10:12)
> rownames(df) <- c("2017", "2018", "2019")</pre>
```

Now add labels to the columns of the data frame. First load the library:

```
> library(Hmisc)
> label(df) <- list(a = "aap", b = "noot", c = "mies")</pre>
```

If you want to see the labels in the result, retrieve them with the label function⁵.

> label(df)

```
a b c "aap" "noot" "mies"
```

When this data frame with labels is converted to a multivariate regts you can display the labels as usual employing the ts_labels function:

```
> tt <- as.regts(df)
> ts_labels(tt)
```

```
a b c "aap" "noot" "mies"
```

Similarly, if a regts object with labels is converted to a data frame, then the timeseries labels are used to create the label attributes of the data frame.

```
a b
"Series_a" "Series_b"
```

11.4 Transposing a data frame (including labels)

Sometimes it can be necessary to transpose a data frame. Package regts includes a transpose function: transpose_df. Variable df in the next example is defined in the previous section.

⁵You can also view the whole data frame including labels with the View function in Rstudio.

```
> df
     a b
2017 1 6 10
2018 2 7 11
2019 3 8 12
> dft <- transpose_df(df)
> dft
  label 2017 2018 2019
           1
                 2
    aap
           6
                 7
                      8
b
  noot
  mies
          10
                11
                     12
```

As you can see, if a data frame contains labels, the transpose_df function puts the labels in the first column of the result data frame. The labels have now become data and the resulting data frame contains no labels.

However, function transpose_df has an argument label_column. If you specify this argument, then the values in the selected column are used to create labels:

```
> df2 <- transpose_df(dft, label_column = "label")
> df2
    a b c
2017 1 6 10
2018 2 7 11
2019 3 8 12
> label(df2)
    a    b    c
"aap" "noot" "mies"
```

11.5 Example: reading timeseries from a deviant file

As discussed in section 4.2 package regts provides functions for directly reading timeseries from csv or Excel files (read_ts_csv and read_ts_xlsx). However, sometimes these functions cannot be used, because the file has a deviant layout. In that case try reading the information as a form of data frame and convert this data frame to a regts timeseries. An example file:

	kt	soort	type	2018	2019	2020	2021
AOW heffing door beperkt indexeren							
	lasten	Lh	Fiscaal	3	3	3	3
	trans	Lh	Fiscaal	1	1	1	1
	kas	Lh	Fiscaal	1	1	1	1
Willekeurige afschrijving investeri	ngen						
	lasten	Inkh	Fiscaal				
	trans	Inkh	Fiscaal	0	0	0	0
	kas	Inkh	Fiscaal	-2.325			
Vervallen vrijstelling MRB auto's >	25jr						
	lasten	Vb	Fiscaal				
	trans	Vb	Fiscaal	0	0	0	0
	kas	Vb	Fiscaal	-7			

The timeseries are stored rowwise, but there are extra comment lines with a description of the three variables below. We want to use these descriptions to create labels for the timeseries. If we didn't need them, there would be no problem, comment rows are normally skipped when using functions read_ts_csv and

read_ts_xlsx. A more serious problem is that there are no unique variable names. Therefore we want to combine columns 'soort' and 'kt' to create these names.

Read this file with the read_excel function. The result of this function is a tibble, a special sort of data frame.

	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<db1></db1>	<db1></db1>	<db1></db1>	<db1></db1>
1	AOW heffing door beperkt indexeren	<na></na>	<na></na>	<na></na>	NA	NA	NA	NA
2	<na></na>	lasten	Lh	Fiscaal	3.00	3.	3.	3.00
3	<na></na>	trans	Lh	Fiscaal	1.00	1.	1.	1.00
4	<na></na>	kas	Lh	Fiscaal	1.00	1.	1.	1.00
5	Willekeurige afschrijving investeringen	<na></na>	<na></na>	<na></na>	NA	NA	NA	NA
6	<na></na>	lasten	Inkh	Fiscaal	NA	NA	NA	NA
7	<na></na>	trans	Inkh	Fiscaal	0.	0.	0.	0.
8	<na></na>	kas	Inkh	Fiscaal	-2.33	NA	NA	NA
9	Vervallen vrijstelling MRB auto's>25jr	<na></na>	<na></na>	<na></na>	NA	NA	NA	NA
10	<na></na>	lasten	Mb	Fiscaal	6.00	6.	6.	0.
11	<na></na>	trans	Mb	Fiscaal	6.00	6.	6.	NA
12	<na></na>	kas	Mb	Fiscaal	6.00	6.	6.	0.600

For collecting the labels, select the rows⁶ with label information (not NA).

```
> is_label_row <- !is.na(df[[1]])
> labels <- df[[1]][is_label_row]
> labels
```

- [1] "AOW heffing door beperkt indexeren" "Willekeurige afschrijving investeringen"
- [3] "Vervallen vrijstelling MRB auto's>25jr"

Each label is used three times:

```
> labels <- rep(labels,each = 3)</pre>
```

Remove comment rows by selecting rows with NA in first column. This is the reverse of the selection we applied before.

```
> df <- df[!is_label_row, ]</pre>
```

Next construct lowercase row names from columns soort and kt

```
> df$names <- tolower(paste(df$soort, df$kt, sep = "_"))
> df$names
```

```
[1] "lh_lasten" "lh_trans" "lh_kas" "inkh_lasten" "inkh_trans" "inkh_kas" [7] "mb_lasten" "mb_trans" "mb_kas"
```

Now remove redundant columns 1 to 4 ('X__1', 'kt', 'soort' and 'type') and transpose the data frame with the column argument to set the column names:

```
> df[1:4] <- NULL
> dft <- transpose_df(df, colname_column = "names")</pre>
```

Finally, convert to regts and add the labels.

⁶For selecting a single column by position from a tibble [[is used.

```
> rts <- as.regts(dft)
> ts_labels(rts) <- labels
> rts
```

	lh_lasten	lh_trans	lh_kas	inkh_lasten	inkh_trans	inkh_kas	mb_lasten	mb_trans	mb_kas
2018	3	1	1	NA	0	-2.325	6	6	6.0
2019	3	1	1	NA	0	NA	6	6	6.0
2020	3	1	1	NA	0	NA	6	6	6.0
2021	3	1	1	NA	0	NA	0	NA	0.6

12 Aggregate functions

12.1 Function aggregate

The function aggregate can be used to convert a timeseries to a lower frequency. For example to convert a monthly timeseries to a quarterly timeseries, or a quarterly timeseries to a yearly timeseries.

An example with a monthly timeseries:

```
> regtm1 <- regts(1:24, start = "2016M1")
> regtm1
```

```
    Jan
    Feb
    Mar
    Apr
    May
    Jun
    Jul
    Aug
    Sep
    Oct
    Nov
    Dec

    2016
    1
    2
    3
    4
    5
    6
    7
    8
    9
    10
    11
    12

    2017
    13
    14
    15
    16
    17
    18
    19
    20
    21
    22
    23
    24
```

```
> aggregate(regtm1, FUN = mean, nfrequency = 4)
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2016 2 5 8 11
2017 14 17 20 23
```

And another with a quarterly timeseries:

```
> regt1 <- regts(1:10, start = "2016Q1")
> regt1
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2016 1 2 3 4
2017 5 6 7 8
2018 9 10
```

```
> aggregate(regt1, FUN = sum)
```

When the newlines are removed the result looks like this:

```
regt1 Time Series: Start = 2016 End = 2017 Frequency = 1 [1] 10 26
```

The quarters in 2016 add up to 10, the quarters in 2017 to 26. Note that the 2 quarters in 2018 are ignored. The aggregate function skips the incomplete years at the end.

A special situation occurs when for instance a quarterly timeseries doesn't start in the first quarter:

```
> regt2 <- regts(2:10, start = "2016Q2")
> regt2
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2016 2 3 4
```

```
2017 5 6 7 8
2018 9 10
> aggregate(regt2, FUN = sum)
```

Now the result is:

```
regt2 Time Series: Start = 2017 End = 2017 Frequency = 1 [1] 26
```

The quarters in 2017 add up to 26 as before. But now the information for 2016 is also not complete, and therefore ignored. This is what you would expect: for regts objects the aggregate function skips all years for which not all quarters are present.

Usually the aggregate function works the same for regts and ts, but in this case the result is very different. The same example with a ts timeseries:

```
> t2 <- ts(2:10, start = c(2016,2), frequency = 4)
> aggregate(t2, FUN = sum)
```

The result is:

```
t2 Time Series: Start = 2016.25 End = 2017.25 Frequency = 1 [1] 14 30
```

Now the first year (2016) is not ignored. This leads to a shifted result period (the start is 2016.25) and shifted input (14 = 2 + 3 + 4 + 5), which makes the results peculiar.⁷

So the function aggregate operates differently for a regts or a ts object, if the first period does not start at a subperiod for the new frequency.

12.2 Aggregation for growth timeseries

Another case is the treatment of timeseries that contain absolute, relative or percentage changes. For this case the function aggregate_gr is developed with special aggregation algorithms, the so called 'cumulative growth'-methods. A more detailed description of these methods can be found in the next subsection. This function can be employed for both regts and ts timeseries.

All methods convert input timeseries with high frequency to low frequency outputseries. There are four different type of methods for different types of input timeseries:

The dif1s and dif1 methods assume that the input timeseries is a first difference of length 1 in the input frequency (for dif1s the input is also scaled) and calculate a first difference of length 1 in the output frequency.

The pct and rel methods assume that the input timeseries is a one-period relative or percentage change and calculate the exact relative or percentage change for the output timeseries.

First a timeseries is created containing differences:

```
> xdif <- diff(regts(c(1,3,4,3,6,2,4,1,3,2), start = "2015Q4"))
> xdif
```

```
Qtr1 Qtr2 Qtr3 Qtr4
2016 2 1 -1 3
2017 -4 2 -3 2
2018 -1
```

Two examples of aggregation methods are shown:

```
> aggregate_gr(xdif, method = "dif1s", nfrequency = 1) # result is a yearly timeseries
```

⁷This problem does not occur when a quarterly timeseries ends before the last quarter, or before the end of the quarter in case of a monthly timeseries. Then the last year or quarter is ignored.

2017 -1.5

> aggregate_gr(xdif, method = "dif1")

2017 -6

12.3 The cumulative growth methods

The so called growth timeseries require special methods, a simple averaging of the subperiods in the high frequency observations will not return the correct answer. In this section a more detailed description of the four previously defined frequency conversion methods is given.

In advance some definitions are needed:

We are temporally aggregating a timeseries x with n subperiods to a timeseries X with a lower frequency. In other words we want to convert a timeseries x with a high frequency (monthly, quarterly) to a new timeseries with a lower frequency (quarterly, annual).

Let $x_{t,i}$ stand for the value of x in subperiod i of main period t. The index i takes on values in the range 1..n. We always interpret $x_{t,0}$ to mean $x_{t-1,n}$. The time index t refers to the periods in the time domain of X. Many conversion methods only need the observations subperiod i of main period t. For example the mean method calculates X_t as follows

$$X_t = \sum_{i=1}^{i=n} x_{t,i}/n$$

dif1s method

Define x to be the first difference of z scaled to the output frequency. Thus

$$x_{t,i} = n(z_{t,i} - z_{t,i-1})$$

We can also define $z_{t,i}$ in terms of x as follows

$$z_{t,i} = z_{t,i-1} + x_{t,i}/n$$

= $z_{t-1,n} + \sum_{i=1}^{i} x_{t,i}/n$

Now define Z_t as the level timeseries in time domain t corresponding to z

$$Z_{t} = \sum_{i=1}^{n} z_{t,i}$$

$$= \sum_{i=1}^{n} z_{t-1,n} + \sum_{i=1}^{n} \sum_{j=1}^{i} x_{t,j}/n$$

$$= \sum_{i=1}^{n} z_{t-2,n} + \sum_{i=1}^{n} \sum_{j=1}^{n} x_{t-1,j}/n + \sum_{i=1}^{n} \sum_{j=1}^{i} x_{t,j}/n$$

Then X in any period t can be calculated from

$$X_t = Z_t - Z_{t-1}$$

Since Z_{t-1} can be written as

$$Z_{t-1} = \sum_{i=1}^{n} z_{t-2,n} + \sum_{i=1}^{n} \sum_{j=1}^{i} x_{t-1,j}/n$$

it is easy to show that X_t does not depend on $z_{t-2,n}$. In any practical algorithm $z_{t-2,n}$ can be set to 0.

A compact equation for X_t can be derived by substituting the equations for Z_t and Z_{t-1} into the definition of X_t

$$X_{t} = \sum_{i=1}^{n} \left(\sum_{j=i+1}^{n} x_{t-1,j} + \sum_{j=1}^{i} x_{t,j} \right) / n$$

This expression can be futher simplified by changing the order of summation. For the first summation, we can write

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} x_{t-1,j} = \sum_{j=2}^{n} \sum_{i=1}^{j-1} x_{t-1,j} = \sum_{j=2}^{n} (j-1)x_{t-1,j}$$

Similarly,

$$\sum_{i=1}^{n} \sum_{j=1}^{i} x_{t,j} = \sum_{j=1}^{n} \sum_{i=j}^{n} x_{t,j} = \sum_{j=1}^{n} (n-j+1)x_{t,j}$$

The final equation is given by

$$X_{t} = \left(\sum_{j=2}^{n} (j-1)x_{t-1,j} + \sum_{j=1}^{n} (n-j+1)x_{t,j}\right)/n$$

dif1 method

Define x to be the first difference of z. Thus

$$x_{t,i} = z_{t,i} - z_{t,i-1}$$

We can also define $z_{t,i}$ in terms of x as follows

$$z_{t,i} = z_{t,i-1} + x_{t,i}$$

= $z_{t-1,n} + \sum_{j=1}^{i} x_{t,j}$

Further derivations are analogous to the case for the dif1s method described in the previous section.

rel method

Define x to be the relative change in z. Thus

$$x_{t,i} = (z_{t,i} - z_{t,i-1})/z_{t,i-1}$$

We can also define $z_{t,i}$ in terms of x as follows

$$z_{t,i} = z_{t,i-1}(1+x_{t,i})$$

$$= z_{t-1,n} \prod_{j=1}^{i} (1+x_{t,j})$$

$$= z_{t-2,n} \prod_{j=1}^{n} (1+x_{t-1,j}) \prod_{j=1}^{i} (1+x_{t,j})$$

Now define Z_t as the level time series in time domain t corresponding to z

$$Z_{t} = \sum_{i=1}^{n} z_{t,i}$$

$$= z_{t-1,n} \sum_{i=1}^{n} \prod_{j=1}^{i} (1 + x_{t,j})$$

$$= z_{t-2,n} \prod_{j=1}^{n} (1 + x_{t-1,j}) \sum_{j=1}^{n} \prod_{j=1}^{i} (1 + x_{t,j})$$

Then X in any period t can be calculated from

$$X_t = Z_t/Z_{t-1} - 1$$

Using the expression for Z_{t-1}

$$Z_{t-1} = z_{t-2,n} \sum_{i=1}^{n} \prod_{j=1}^{i} (1 + x_{t-1,j})$$

we obtain

$$X_{t} = \frac{\prod_{j=2}^{n} (1 + x_{t-1,j}) \sum_{i=1}^{n} \prod_{j=1}^{i} (1 + x_{t,j})}{1 + \sum_{i=2}^{n} \prod_{j=2}^{i} (1 + x_{t-1,j})} - 1$$

pct method

Define x to be the percentage change in z. Thus

$$x_{t,i} = 100(z_{t,i} - z_{t,i-1})/z_{t,i-1}$$

We can also define $z_{t,i}$ in terms of x as follows

$$z_{t,i} = z_{t,i-1}(1+0.01x_{t,i})$$
$$= z_{t-1,n} \prod_{j=1}^{i} (1+0.01x_{t,j})$$

Now define Z_t as the level time series in time domain t corresponding to z

$$Z_t = \sum_{i=1}^n z_{t,i}$$

Then X in any period t can be calculated from

$$X_t = 100(Z_t/Z_{t-1} - 1)$$

Further derivations are analogous to the case for the dif1s method described in the previous section.