Analysis of the Statistical Uncertainty on Monte Carlo W Decay Events

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Abstract

The mass of the W boson forms a key parameter in the Standard Model, and an accurate experimental determination of this value is important in verifying the internal consistency of the model. Statistical fluctuations in the distribution of Monte Carlo simulated W decay events can introduce uncertainty in the W mass measurement, since limits on computational power restrict the number of Monte Carlo events. However, it is possible to make use of the theory behind W decay events to understand and reduce the statistical fluctuation in the decay parameters by parameterizing the W decay cross section in terms of the so called angular coefficients $A_0 ... A_7$, which reveal how the cross section should be theoretically distributed. A smoothing process which reweights the noisy Monte Carlo data to this ideal can reveal how the uncertainty on parameters of interest may be changed by removing the problem of data scarcity. These parameters of interest are the so called reconstructed parameters: the transverse momentum p_T^l and pseudorapidity η^l of leptons produced in the W decay. This smoothing process is itself a computationally intense one which required processing around 5 million simulated W decay events. However, application of the smoothing revealed that the statistical uncertainties on these reconstructed parameters were unchanged by it. This indicates that smoothing based on the angular coefficients would not be useful in decreasing the uncertainty on the W mass measurement.

Introduction

LHC collisions at center of mass energy $\sqrt{s} = 13$ TeV allow for quark production of the W bosons of the form $\bar{u} + d \to W^-$ and $u + \bar{d} \to W^+$ to leading order in QCD [6]. The branching ratios of the W decay are dominated by the hadronic decay channel $W \to q + \bar{q}$ where q is any quark, with this being the mode of 68% of all W decays [3]. However, the channels simulated in the Monte Carlo data are actually the leptonic ones, with the decays $W \to e + \bar{\nu_e}$, $W \to \mu + \bar{\nu_\mu}$, and $W \to \tau + \bar{\nu_\tau}$ each accounting for about 10% of all decays [3]. When we parameterize the leptonic decay, we do so in the Collins-Sopper reference frame, as shown in Figure 1.

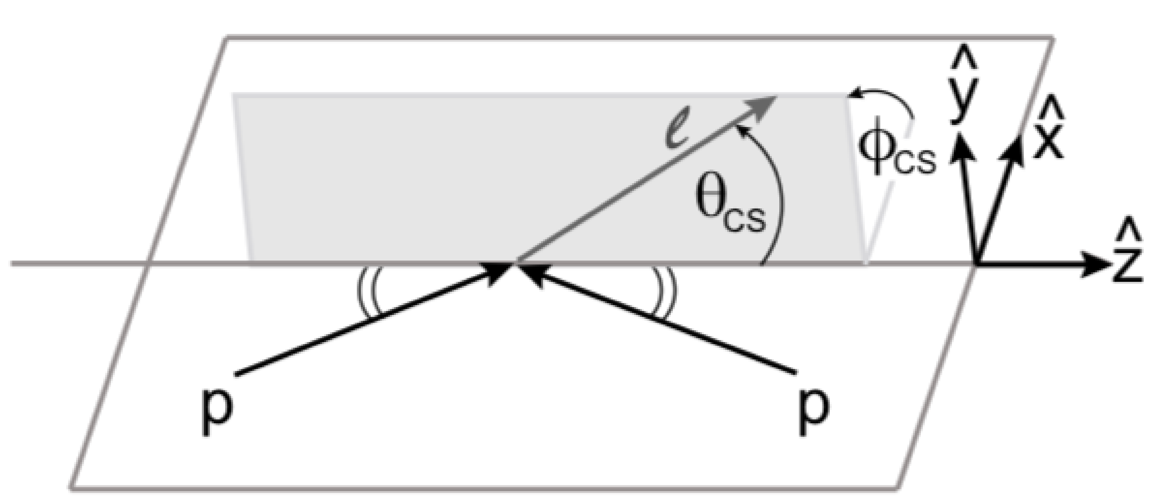

Fig. 1: The Collins-Sopper reference frame

The Monte Carlo simulated decays are completely parameterized by the angular variables ϕ_{CS} and θ_{CS} as well as the transverse momentum p_T^W, rapidity y^W and invariant mass m^W of the parent W. We show the explicit dependence of the total differential cross section on ϕ_{CS} and $\cos\theta_{CS}$ by using the eight angular coefficients $A_0 ... A_7$ and factoring out the unpolarized differential cross section $d\sigma^{U+L}$ as follows [2].

$$\frac{d\sigma}{dp_T^W dy^W dm^W d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^W dy^W dm^W}\Big(1+\cos^2\theta + \frac{1}{2}A_0(1-3cos^2\theta) + A_1 \sin 2\theta cos\phi \quad (1)$$

$$+\frac{1}{2}A_2 \sin^2\theta\cos 2\phi + A_3 \sin\theta\cos\phi + A_4\cos\theta + A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta\sin\phi\Big)$$

Noticing that each angular coefficient A_i in Equation 1 is multiplied by a harmonic polynomial $P_i(\cos\theta, \phi)$, we used the orthogonality of these polynomials to find each A_i as a weighted average or moment of $P_i(\cos\theta, \phi)$, denoted $< P_i >= \Sigma_i P_i w_i / \Sigma_i w_i$ for cross sectional weights w_i.

$$< \frac{1}{2}(1-3cos^2\theta) >= \frac{3}{20}\left(A_0 - \frac{2}{3}\right) \; ; \; < \sin 2\theta\cos\phi >= \frac{1}{5}A_1 \; ; \; < \sin^2\theta \cos 2\phi >= \frac{1}{10}A_2 \; ;$$

$$< \sin\theta\cos\phi >= \frac{1}{4}A_3 \; ; \; < \cos\theta >= \frac{1}{4}A_4 \; ; \; < \sin^2\theta\sin 2\phi >= \frac{1}{5}A_5 \; ;$$

$$< \sin 2\theta \sin\phi >= \frac{1}{5}A_6 \; ; \; < \sin\theta\sin\phi >= \frac{1}{4}A_7 \quad (2)$$

We calculated $A_0..A_7$ directly from the decay data, giving the smooth cross section of Equation 1. We then used this to smooth the noisy Monte Carlo cross section by a reweighting method.

Data

We used Monte Carlo data generated from the MADGRAPH5 AMC event generator at next to leading order. The non-angular parameters are broken into 6 bins of p_T^W and 3 bins of $|y^W|$, while the m^W distribution of the data is so well localized that we grouped all events into one bin, for a total of 18 bins. The number of events per bin is roughly equal.

We first calculated the angular coefficients for each bin of p_T^W and $|y^W|$ according to Equation 2. We calculated the uncertainty on each angular coefficient directly from each corresponding moment $< f >$ by treating the data as Poisson distributed, allowing us to approximate the uncertainties on the sums as follows.

$$\sigma_f = |f|\sqrt{\left(\frac{\sigma_A}{\mathrm{A}}\right)^2 + \left(\frac{\sigma_B}{\mathrm{B}}\right)^2} \; ; \; \sigma_A = \sum_j (P_i(\cos\theta_j, \phi_j)w_j)^2 \; ; \; \sigma_B = \sum_j (w_j)^2$$

Because of the relative scarcity of data, we verified this uncertainty with the bootstrapping method. This is a process of randomly resampling with replacement the original data set to create a new one which is clearly distinct, but still reflects the original sample and population. We bootstrapped the data many times to create many distinct samples, from which an uncertainty can be computed as the square root of the variance.

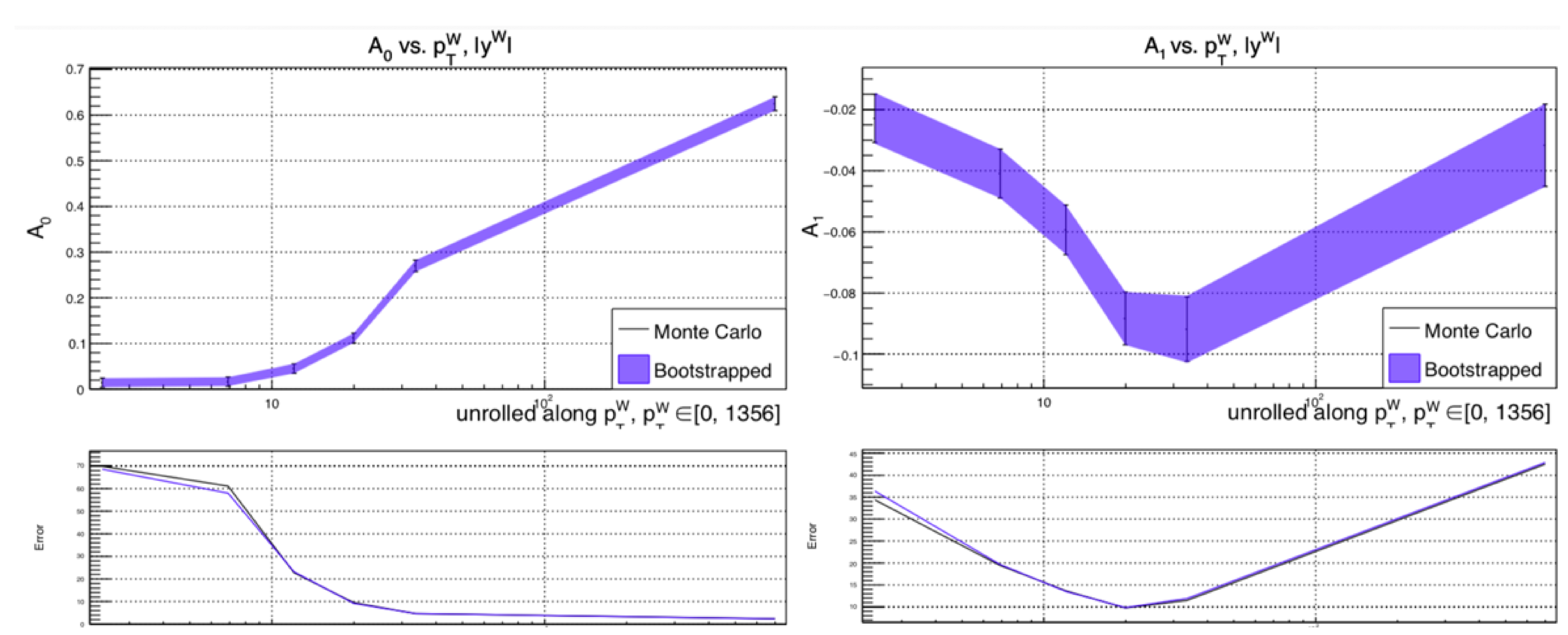

Fig. 2: Angular coefficients A_0 and A_1 as a function of p_T^W. Note the good agreement between the direct and bootstrapped error.

Each point in Figure 2 shows a value of A_i for a specific bin of p_T^W, y^W. We compute the total cross section as in Equation 1 for each such bin.

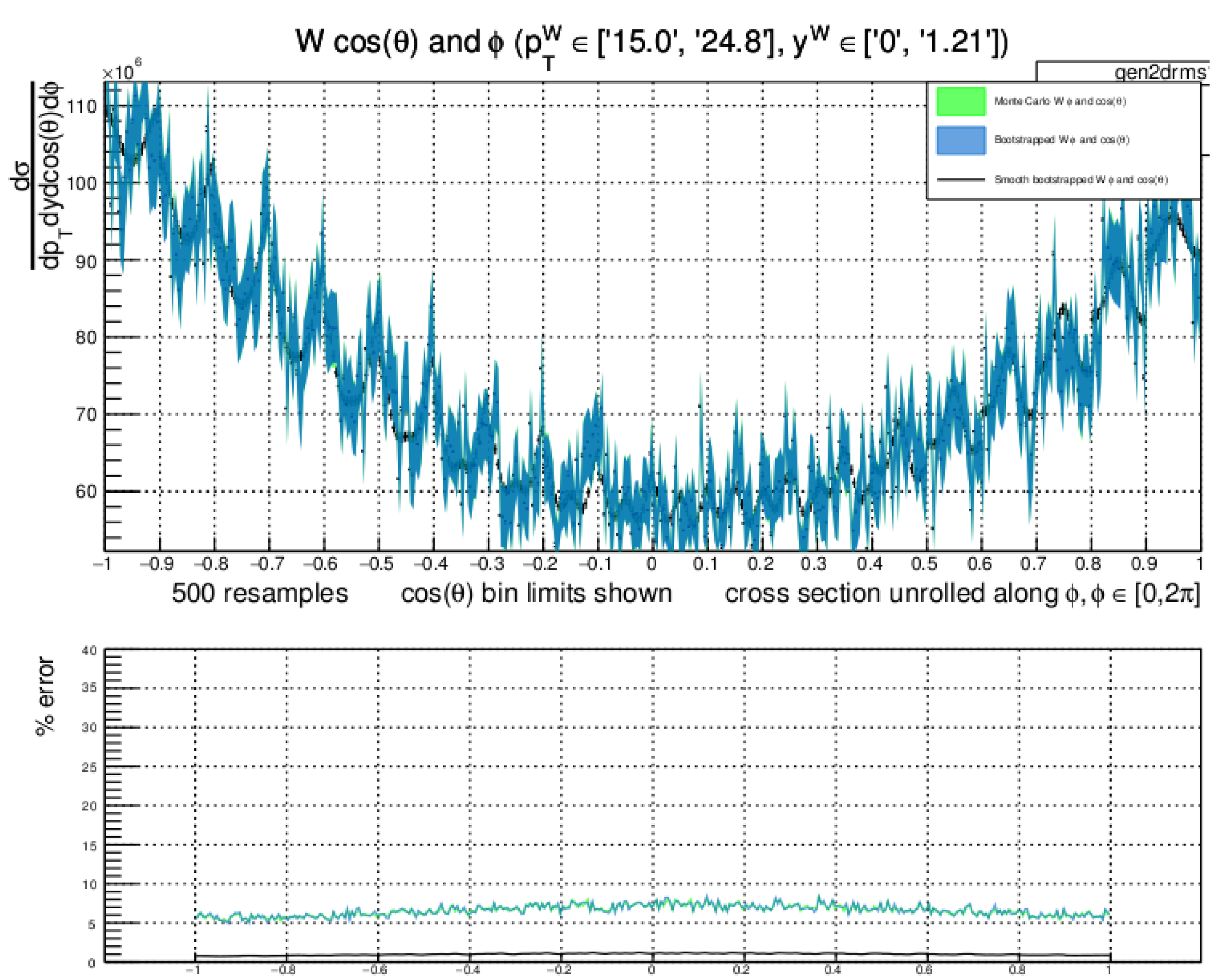

Fig. 3: W decay cross section unrolled along ϕ. The direct error is compared with the smoothed and bootstrapped smoothed errors.

The purpose of calculating this cross section is to smooth the angular parameters ϕ and $\cos\theta$ by means of a reweighting process. This amounts to multiplying each cross sectional weight w_i by a constant so that the cross section matches Equation 1. This inevitably affects the distribution of all parameters of each event.

The error on each parameter depends on the weights and so will be changed by the smoothing. However, any particular reweighting could result in a lower or higher error. We are more interested in whether the smoothing gives us consistently smaller errors, so we again apply bootstrapping. The standard deviation of the distribution of bootstrapped smoothed cross sections is the third error shown in Figure 3.

Of interest to us is whether the error on lepton pseudorapidity η^l and transverse momentum p_T^l can be reduced by smoothing. In the case of the generator parameters, the theory tells us that the bootstrapped smoothed distributions will follow the same smooth curve with reduced uncertainty, since this is precisely what the reweighting is designed to do, but for the reconstructed parameters this may or may not be the case.

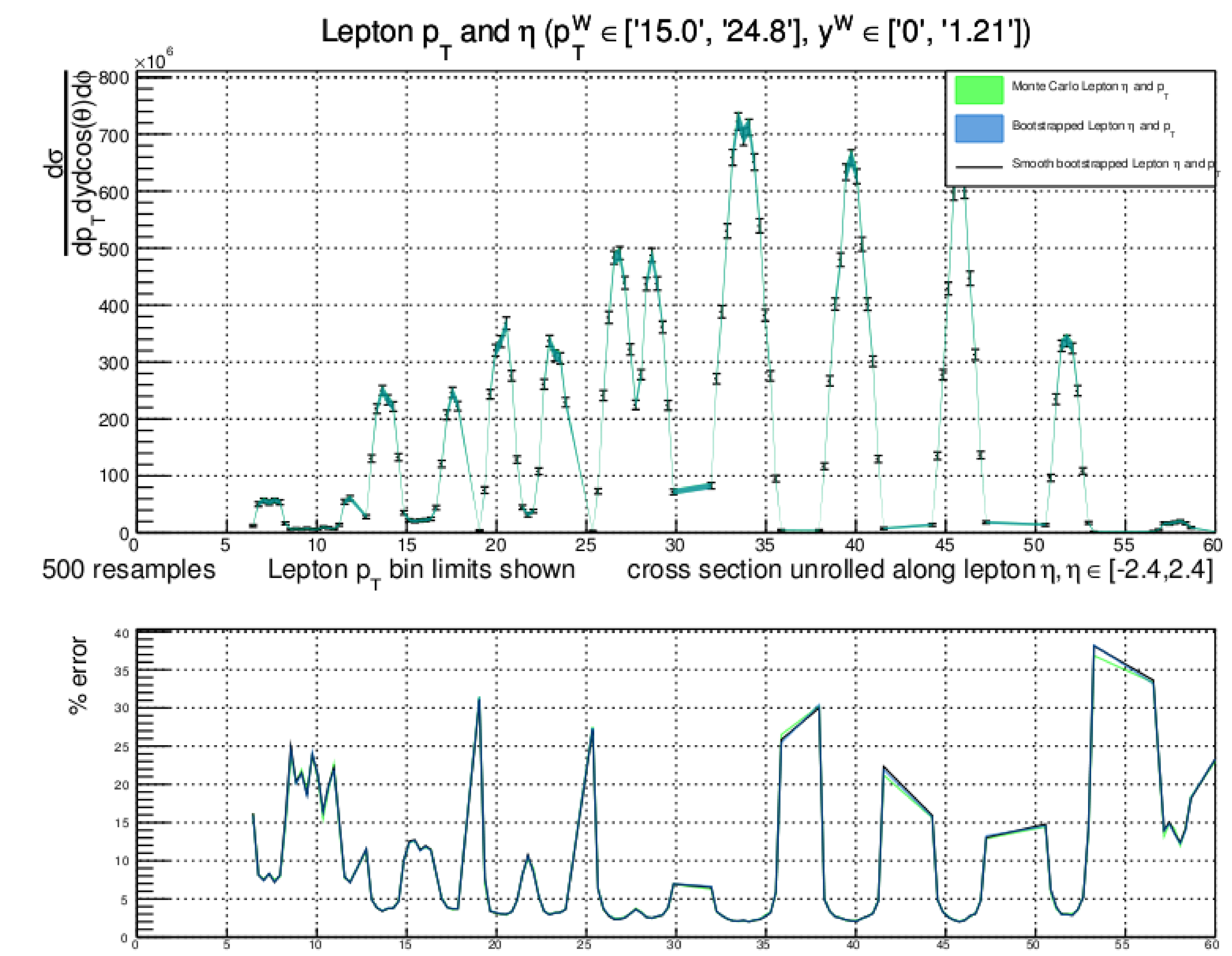

Fig. 4: W decay cross section unrolled along ϕ. The direct error is compared with the smoothed and bootstrapped smoothed errors.

As Figure 4 shows, in actuality the bootstrapped smoothed error is virtually indistinguishable from the direct error.

Results

As the results show, smoothing of the generator parameters of W boson decay does not result in a measurable decrease of the statistical error on the reconstructed parameters η^l and p_T^l. This was contrary to the expected result in which the smoothing process could have compensated for the scarcity of MC data by reducing the statistical fluctuations of the reconstructed parameters. Further investigation could look into the dependence of the error on the number and size of p_T^W and y^W bins as the decision here to use 6 bins of p_T^W and 3 bins of y^W was arbitrary. Another further step would be to bin data by m^W as well instead of lumping everything into the same mass bin as was done here. Because of the interdependence of the generator parameters and reconstructed parameters, reweighting of the latter is expected to affect the former in some form, so changing the binning as described above could reveal an error reduction that was disguised under the current binning.

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