

Exploring the Role of Language in Two Systems for Categorization

Kayleigh Ryherd, PhD
University of Connecticut, 2019

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Doctor of Philosophy Dissertation

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1 General Introduction

Categories help us organize the world. They help us predict and hypothesize about category members, helping us quickly select the most appropriate response for each situation. We also rely on language during these processes. As Lupyan (2012) puts it, language augments our thought. For categories, language provides structure in the form of category labels, but language also affects how we think about and even perceive the categories themselves. Thus any thorough investigation of how we learn categories must consider the role of language.

Indeed, many theoretical frameworks of category learning involve language – some even reference it right in the name. For example, a key theory in perceptual category learning, COVIS, stands for "competition between verbal and implicit systems" (Ashby et al., 1998). Similarly, a theory put forth by Minda and colleagues is called "A theory of verbal and nonverbal category learning" (Minda & Miles, 2010). However, to date most theories of category learning that consider language primarily determine whether language has an influence on systems for category learning, rather than further defining the role language plays in these systems.

Thus, the current work seeks to both define a theory of category learning and explore the role language has in this theory. In this review I will synthesize multiple approaches to category learning, all of which have some type of dual-systems model. Following the synthesis, I will review relevant literature that provides suggestions as to how language might be involved in category learning in a dual-systems model. Through these efforts, I will provide a theoretical framework and hypotheses for this dissertation.

1.1 Dual-systems model for category learning

Multiple theories converge on the idea that there are two systems for category learning. In this section, I will first describe a generalized dual-systems model that pulls threads from all of these theories and then go on to describe how each theory fits into the overarching framework.

1.1.1 Proposed model

The proposed model involves two systems for category learning. The first, which I title the **associative system**, uses associative mechanisms in an iterative manner to learn distributions of features. This system is best suited for learning multidimensional *similarity-based* categories such as natural kinds, where it is difficult to describe necessary and sufficient rules for inclusion. Similarity-based categories have features that are correlated and probabilistic, such that a category instance may not have all of the category-relevant features but tends to have some distribution of them. For example, Manx cats do not have tails, a typical feature of cats, but are still undeniably members of the category *cat*. Thus, the associative system must be able to extract the most frequent pattern of features over many instances in order to learn a category.

The second one is called the **hypothesis-testing system**. This system uses a more explicit learning method to test and adjust hypotheses about category boundaries. This method relies on selection of relevant features rather than representation of a distribution of feature probabilities. As such, it is most suited for learning rule-based categories, which typically have one or a few easily verbalizable rules for inclusion. For example, the *ad hoc* category *things to be sold at the garage sale* has a simple rule for inclusion that perfectly separates members from non-members.

Thus, we have two systems for category learning, each one ideal for learning a different type of category (similarity-based vs. rule-based). In the upcoming sections, I will describe theoretical and empirical evidence for a dual-systems model from five different approaches to category learning. I will show how each approach informs the current theoretical framework.

1.1.2 COVIS

COVIS stands for Competition between Verbal and Implicit Systems and is a prominent theoretical framework for perceptual category learning first proposed by Ashby et al. in 1998. This framework provides a dual-systems model that is grounded in neuropsychological data, allowing it to suggest neurobiological underpinnings for the two systems. It is important to note from the beginning that this framework is mostly concerned with perceptual categories, which are defined as "a collection of similar objects belonging to the

same group” (Ashby & Maddox, 2005, p. 151). This is in contrast to concepts, which they define as groups of related ideas. Thus, this approach focuses on categorizing objects that can be encountered and perceived in the real world.

As can be inferred from the title, the two category learning systems in COVIS are the **verbal** and **implicit** systems. The verbal system is COVIS’ answer to our hypothesis-testing system. It is a declarative learning system that uses a hypothesis-testing method to learn category rules, typically for rule-based stimuli. Under COVIS, rule-based stimuli must have inclusion rules that are easy to describe verbally. Typically, rule-based stimuli used by Ashby and colleagues have a single rule for inclusion or two rules combined by ”and” or ”or.” When a rule-based category involves multiple dimensions, decisions about each dimension are made separately, and these decisions are used to evaluate the logical operators. In other words, each dimension is considered on its own before their combination. These guidelines for rule-based categories ensure that an explicit, verbalizable hypothesis-testing method can be used to learn them. When learning a new category, the verbal system holds candidates for category inclusion rules in working memory, which are tested as stimuli are encountered. Over time, the hypotheses are tested and switched until they reflect the optimal strategy for categorization.

The implicit system from COVIS is most similar to our associative system. Like the associative system, it uses incremental learning to find category boundaries. It is most ideal for learning information-integration categories, which are like similarity-based categories but have also some specific guidelines. Information-integration categories are defined by some combination of dimensions, like some rule-based categories. However, while each dimension can be considered separately in rule-based categories, all dimensions must be considered simultaneously for information-integration categories. Information-integration category membership depends on both the values associated with each dimension as well as the relationship between these values. Information-integration category boundaries are difficult or impossible to describe verbally. COVIS suggests that the implicit system relies on an information stream that connects stimuli, motor responses, and feedback to learn category membership.

1.1.3 Dimensionality

1.1.4 Statistical Density

1.1.5 Verbal/nonverbal

1.1.6 Taxonomic/thematic

1.2 Vocabulary/labels and category learning

1.3 Executive function and category learning

2 Experiment 1

2.1 Methods

2.1.1 Participants

Data was collected from 236 undergraduate psychology students at the University of Connecticut (161 Female, 67 Male, mean age = 18.94). Data for the category learning task was lost for 7 subjects due to technical errors. Thus, the final sample size was 229. Each subject was placed into one of six groups. Each group completed two blocks of the category learning task in a specific order. For more details, see Table 2. Unequal group sizes result from lost data due to technical errors.

Table 1. Group sizes for each order

Effect	Group	<i>N</i>
1	1	40
	2	38
2	3	39
	4	39
3	5	36
	6	37

2.1.2 Category Learning Task

This task measures learning of dense and sparse categories and is based off of a paradigm from previous research (Kloos & Sloutsky, 2008). Participants learn novel categories of items in four possible conditions in a 2 x 2 design. The first manipulation is learning type (supervised vs. unsupervised). In *supervised* learning, participants learn the categories by being instructed on the relevant features (e.g., All friendly aliens have big noses.). Images of the relevant features are provided along with the descriptions. In *unsupervised* learning, participants learn the categories by viewing sixteen instances of the category.

The second manipulation is category type (sparse vs. dense). Category type is measured by statistical density, which ranges from zero (where all features vary freely) to one (where all features co-occur perfectly). It is based on a comparison between within- and between-category entropy (Sloutsky, 2010). All categories in this experiment have seven dimensions. The *sparse* categories cohere on a single dimension, while the other dimensions vary freely (density = .25). In contrast, the *dense* categories cohere on six of the seven dimensions (density = .75). The seventh dimension is allowed to vary freely. For more details on how density was calculated, see Appendix A. Stimuli for each of the four blocks are different. See Fig. 1 for examples of the experimental manipulations.

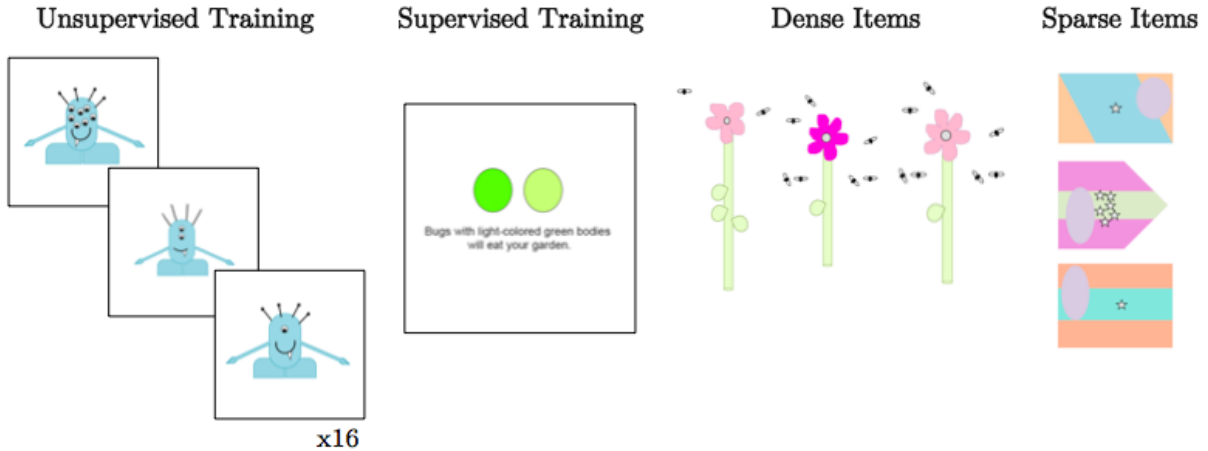


Figure 1. Examples of learning type and category type manipulations for category learning experiment.

This task is within-subjects. Based on the group they were placed into, participants completed two of the four possible learning-category type combinations. In this experiment, I tested three main order effects. First, I tested order effects for the matching conditions (unsupervised-dense and supervised-sparse). The second order effect used unsupervised-dense and supervised-dense blocks. Finally, the third order effect tested the same sparse stimuli, testing unsupervised-dense and supervised-sparse blocks. This design led to six possible order groups that each participant could be placed into. See Table 2 for a summary.

Table 2. Block orders for statistical density task

Effect	Group	Block 1	Block 2
1	1	Unsupervised-dense	Supervised-sparse
	2	Supervised-sparse	Unsupervised-dense
2	3	Unsupervised-dense	Supervised-dense
	4	Supervised-dense	Unsupervised-dense
3	5	Unsupervised-sparse	Supervised-sparse
	6	Supervised-sparse	Unsupervised-sparse

In each block, participants were introduced to the task through a short cover story. They were told to learn which items go with a certain property (e.g., which aliens are friendly). Crucially, no labels were attached to the categories (e.g., some aliens are Ziblets). Then, participants completed a training block (either supervised or unsupervised). After training, participants completed 40 test trials (16 target, 16 distractor, 8 catch), following the design of Kloos & Sloutsky (2008). In each trial, participants saw a single item and used the keyboard to indicate whether the item matched the category they had just learned (e.g., if the alien is friendly). Catch items looked significantly different than both the target and competing categories, so participants should have always rejected them as members of the learned category. This experiment was presented using PsychoPy v.1.84.2 (Peirce, 2007).

2.1.3 Behavioral Measures

I used multiple assessments to test participants’ language ability. The choice of assessments was based on the epiSLI criteria for language impairment (Tomblin et al., 1996), which includes comprehension, expression, vocabulary, grammar, and narrative. I adapted these requirements from a kindergarten population to a college-aged population. The epiSLI criteria have been shown to be robust for diagnosis of specific language impairment (SLI). In addition, other studies of language impairment more broadly have adapted a similar multidimensional approach to measuring language ability, sometimes including measures of phonological skills (Catts et al., 2006). Thus, using assessments that the many domains of language outlined in epiSLI criteria will allow me to get a fuller picture of individual differences in language ability. See Table 3 for a summary of the assessments and which domains of the epiSLI criteria they cover. The specific tests used in this experiment are detailed below.

Test of word reading efficiency (TOWRE) phonemic decoding subtest. TOWRE is a test of nonword fluency (Torgesen et al., 1992). This test is a part of the comprehension aspect of epiSLI, since the comprehension measure is reading-based. In the TOWRE, individuals have 45 seconds to read as many nonwords as possible. The nonwords become longer and more difficult as the list goes on.

Woodcock Johnson-III word attack (WA) subtest. This task measures nonword decoding ability (Woodcock et al., 2001). Like the TOWRE, it is helpful for measuring the comprehension aspect of epiSLI. However, while the TOWRE measures word fluency, this task measures decoding accuracy. Participants read a list of nonwords out loud at their own pace.

Computerized reading comprehension. This test covers the comprehension and narrative aspects of epiSLI. This computerized reading comprehension test is based on the Kaufman Test of Educational Achievement (KTEA) reading comprehension subtest (Kaufman & Kaufman, 2004). To create this test, I copied the passages and questions contained in the KTEA reading comprehension subtest into E-Prime (Schneider et al., 2002) for presentation on a computer. I created multiple choice answers for the KTEA questions that did not already have them. In this task, participants read short expository and narrative texts and answered multiple-choice comprehension questions about them. Some questions are literal, while others require participants to make an inference. Because this task is a modified version of the KTEA, I use raw scores in analysis rather than standardized scores based on the KTEA norms.

Nelson-Denny vocabulary subtest. The Nelson-Denny vocabulary sub-test is a written assessment of vocabulary (Brown et al., 1981). This test covers the vocabulary aspect of epiSLI. This test has been used in multiple studies of college-aged adults and provides sufficient variability for individual difference investigations in this population (e.g., Boudewyn et al. 2015; Stafura & Perfetti 2014). In this test, participants are asked to choose the word closest to a target vocabulary word.

Clinical Evaluation of Language Fundamentals recalling sentences subtest. I will use the recalling sentences subtest from the Clinical Evaluation of Language Fundamentals (CELF; Semel et al. 2006; Stafura & Perfetti 2014). This test covers the grammar and expression aspects of epiSLI. In this

subtest, participants hear sentences and are asked to repeat them. Scoring is based on how many errors the participant makes in their repetition.

Finally, I used Set II of Raven’s Advanced Matrices to measure nonverbal IQ (Raven, 1998). In this task, participants see a grid containing eight images and an empty space. The images are arranged in the grid according to some rule or rules. Participants must choose one of eight additional images that fits in the empty space.

Table 3. Assessments of language and their corresponding epiSLI domains.

Test	epiSLI Criteria
TOWRE	Comprehension (decoding aspect)
WA	
Computerized Reading Comprehension	Comprehension, narrative
ND Vocab	Vocabulary
CELF RS	Grammar, expression

2.2 Procedure

Each participant completed the category learning task as well as all of the behavioral measures. TOWRE, WA, and CELF were audio-recorded to allow for offline scoring. To allow multiple subjects to be run in a single timeslot, some participants received tasks they could complete on their own (category learning, ND, Computerized reading comprehension, Raven’s) first while others completed tasks with the experimenter first (WA, CELF, TOWRE). All together, the seven tasks took approximately one hour.

2.3 Results

For all analyses shown below, accuracy was converted to d' values (Macmillan & Creelman, 2004) using the R package **neuropsychology** (Makowski, 2016). Correction for extreme values was done following (Hautus, 1995). Following prior research, all blocks where 5 or fewer catch items were correctly rejected were dropped from analysis. This resulted in 22 total missing blocks (out of 458 total), including both blocks from a single subject in group 5.

2.3.1 Order Effect 1: Matching Conditions

The first analysis investigated order effects for blocks in which the learning type (supervised vs. unsupervised) and category type (sparse vs. dense) both engaged the same category learning system (hypothesis testing vs. associative). Participants completed supervised-sparse and unsupervised-dense blocks.

I used linear mixed-effects models to examine the effects of block and order on accuracy at test. The relationship between accuracy and block/order showed significant variance in intercepts across participants $SD = 0.26$. Adding block and order as fixed effects significantly increased model fit, $\chi^2(2) = 13.04$, $p = 0.001$. Adding the interaction between block and order further improved model fit $\chi^2(1) = 6.05$, $p = 0.014$. Thus, the final model had fixed effects of block, order, and the interaction between block and order as well as random intercepts for subject.

The final model revealed three significant effects. First, there was not a significant main effect of block,

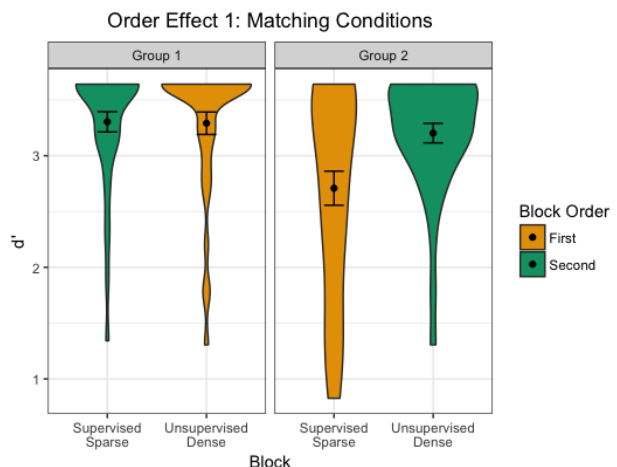


Figure 2. Accuracy (d') for each block completed by each group for the first order effect. Colors indicate which block was encountered first by each group. Points indicate means with error bars reflecting standard error. Shaded portions represent the distribution of accuracy values; wider portions indicate more subjects with that accuracy value.

$b = -0.52$, $SE = 0.32$, $t(74) = -1.60$, $p = 0.12$. This model also showed a significant main effect of order, $b = -0.59$, $SE = 0.16$, $t(141) = -3.76$, $p = 0.0002$. Finally, there was a significant interaction between block and order, $b = 0.51$, $SE = 0.20$, $t(72) = 2.48$, $p = 0.016$. This interaction was broken down by conducting two separate models for each of the orders (unsupervised-dense first and supervised-sparse first). These analyses showed that when the associative system was engaged first (unsupervised-dense first), there was no significant main effect of block, $b = -0.0064$, $SE = 0.11$, $t(34) = -0.057$, $p = 0.95$. When the hypothesis testing system was used first (supervised-sparse first), there is a significant effect of block, $b = 0.50$, $SE = 0.17$, $t(37) = 2.92$, $p = 0.0059$. Inspection of means shows that when participants complete the supervised-sparse block first, performance on the supervised-sparse block is lower than in the unsupervised-dense block (see Figure 2).

2.3.2 Order Effect 2: Dense Stimuli

The second order effect analysis compared groups 3 and 4. All participants learned only dense categories, with the order of learning types differing between groups. Again, I used linear-mixed effects models to investigate the effects of block and order on accuracy at test. The variance in intercepts across participants had a standard deviation of 0.62. Adding the fixed effects to the model did not significantly improve fit $\chi^2(2) = 0.24$, $p = 0.89$. Inspection of coefficients confirmed this finding. Block was not a significant predictor of accuracy, $b = 0.03$, $SE = 0.10$, $t(145) = 0.27$, $p = 0.79$. Similarly, order was not a significant predictor of accuracy, $b = -0.04$, $SE = 0.10$, $t(145) = -0.11$, $p = 0.68$. Thus, accuracy at test on dense categories was similar regardless of training type or block order (see Figure 3).



Figure 3. Accuracy (d') for each block completed by each group for the second order effect.

2.3.3 Order Effect 3: Sparse Stimuli

The third order effect investigated differences in learning sparse categories based on learning type order, using data from groups 5 and 6. I used the same type of linear mixed-effect models as the prior two order effects. Random intercepts for subject had a standard deviation of 0.11. Adding block as a fixed effect significantly increased model fit, $\chi^2(1) = 59.44$, $p < 0.00001$. Adding order to this model did not further improve model fit $\chi^2(1) = 0.05$, $p = 0.82$. Thus, the final model had a fixed effects of block as well as random intercepts for subject, but no fixed effect of order or interaction between block and order. This model revealed a significant main effect of block, $b = -1.14$, $SE = 0.13$, $t(72) = -8.91$, $p < 0.00001$. Inspection of means showed that participants exhibited better performance in supervised-sparse blocks than in unsupervised-dense blocks (see Figure 4).

2.3.4 Exploratory Order Analyses

An interesting feature of this experimental design is that both manipulations (learning type and category type) push individuals towards a certain category learning system. Supervised and sparse blocks encourage use of the hypothesis-testing system, while unsupervised and dense blocks evoke the associative system. Thus, mismatch blocks (i.e., unsupervised-sparse and supervised-dense) have conflicting information on which category learning system to use and thus likely are less effective at evoking that system. To investigate this possibility, I completed some exploratory analyses.

First, I compared unsupervised-dense blocks completed by groups 2 and 4. Group 2 completed a supervised-sparse (matching, hypothesis-testing) block before their supervised-dense block, while group 4 completed a supervised-dense (mismatch, hypothesis-testing) block before their unsupervised-dense block. If matching blocks more strongly evoke the category learning system and the hypothesized order effect (where

activating the hypothesis-testing system first interferes with later use of the associative system) holds, then performance in group 2 on the unsupervised-dense block should be worse than performance in group 4 on the same block. A two-sample t -test indicated that this hypothesis did not hold – the two groups had equivalent performance ($t(73) = -0.62, p = 0.54$).

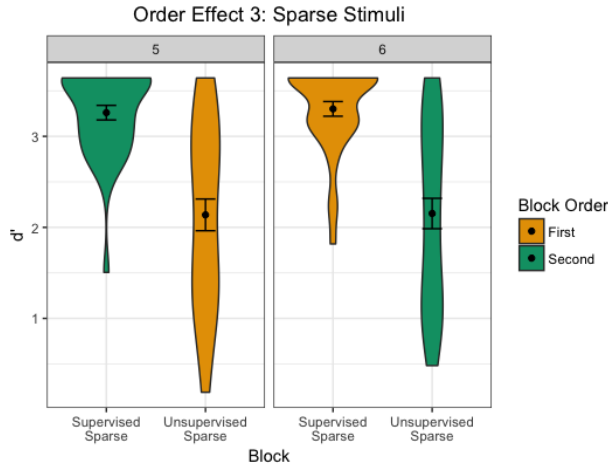


Figure 4. Accuracy (d') for each block completed by each group for the third order effect.

I extended this analysis by doing the sample thing for groups 1 and 5, who both completed the supervised-sparse block second. Group 1 completed an unsupervised-dense (match, associative) block before their supervised-sparse block and group 4 completed an unsupervised-sparse (mismatch, associative) block first. Again, a two-sample t -test indicated that the two groups had equivalent performance ($t(69) = 0.36, p = 0.71$).

As an additional check, I looked at two more comparisons. First, I compared the unsupervised-dense blocks for the two groups who completed it first, group 1 and group 3. There should be no difference between these groups on this block, since it was the first block each group encountered. A two-sample t -test confirmed this hypothesis ($t(70) = 0.097, p = 0.92$). I then checked the same thing for the supervised-sparse blocks within groups 2 and

6. Interestingly, these two groups were found to be different ($t(54) = -3.43, p = 0.001$).

2.4 Discussion

2.4.1 Order Effects

The primary analyses looked at three different order effects. Each effect compared different ways to engage the hypothesis testing system before the associative system and vice versa. The first order effect used category learning blocks whose learning type and category type matched (i.e., supervised-sparse/hypothesis-testing, unsupervised-dense/associative). A significant interaction was found between block and order. Individuals who completed the unsupervised-dense/associative block first showed similar performance on both blocks. However, participants who completed the supervised-sparse/hypothesis-testing blocks first showed reduced performance in the supervised-sparse/hypothesis-testing block, with considerable recovery by the time they got to the unsupervised-dense/associative blocks. This result may be spurious, however, because the individuals in group 2 showed atypically low accuracy on their first block (supervised-sparse/hypothesis-testing), as compared to participants in group 6 that received the same block first and exhibited higher performance.

Overall, few order effects were found in accuracy. Most performance was close to ceiling. The main effect of block found in the third order analysis indicated that the unsupervised-sparse block may be more difficult overall. This is consistent with prior between-subjects research showing the worst performance in the unsupervised-sparse condition (Kloos & Sloutsky, 2008).

2.4.2 Individual Differences

3 Experiment 2

3.1 Methods

3.1.1 Participants

3.1.2 Category Learning Tasks

3.1.3 Behavioral Measures

3.2 Procedure

3.3 Results

3.4 Discussion

4 General Discussion

5 Appendix A: Statistical Density Calculations

5.1 Statistical Density Formulae

Statistical density is the method that Sloutsky and colleagues use to define categories (Sloutsky, 2010). Dense categories have multiple intercorrelated features, while sparse categories have few relevant features. Statistical density can vary between 0 and 1. Higher values (closer to 1) are dense, while lower values (closer to 0) are sparse. We calculate statistical density (D) with the following formula, where H_{within} is the entropy within the category and $H_{between}$ is the entropy between the category and contrasting categories.

$$D = 1 - \frac{H_{within}}{H_{between}}$$

To find total entropy (H), we sum entropy due to varying dimension and entropy due to varying relations among dimensions.

$$H = H^{dim} + H^{rel}$$

This equation is the same whether you are calculating within-category entropy or between-category entropy. To find entropy due to dimensions, you use the following formulas, where M is the total number of varying dimensions, w_i is the attentional weight of a particular dimension (assumed to be 1), and p_j is the probability of value j on dimension i .

$$H_{within}^{dim} = \sum_{i=1}^M w_i \left[\sum_{j=0,1} within(p_j \log_2 p_j) \right]$$

$$H_{between}^{dim} = \sum_{i=1}^M w_i \left[\sum_{j=0,1} between(p_j \log_2 p_j) \right]$$

To find entropy due to relations, you use a similar set of formulas, where O is the total number of possible dyadic relations among the varying dimensions, w_k is the attentional weight of a relation (assumed to be 0.5), and p_{mn} is the probability of the co-occurrence of values m and n on dimension k .

$$H_{within}^{rel} = - \sum_{k=1}^O w_k \left[\sum_{\substack{m=0,1 \\ n=0,1}} within(p_{mn} \log_2 p_{mn}) \right]$$

$$H_{between}^{rel} = - \sum_{k=1}^O w_k \left[\sum_{\substack{m=0,1 \\ n=0,1}} between(p_{mn} \log_2 p_{mn}) \right]$$

All categories have 7 dimensions. For dense categories, 6 of these dimensions are correlated. The seventh dimension is allowed to vary randomly. For sparse categories, 6 of the dimensions vary randomly. The seventh dimension is category-relevant and defines the category. All dimensions have two levels (e.g., for hair shape in aliens – curly and straight).

5.2 Statistical Density Calculations – Sparse

First, we calculate the entropy due to dimensions. We have 7 dimensions, so $M = 7$. Between categories (i.e., across all categories), each level of each dimension has a 0.5 probability of being present.

$$H_{between}^{dim} = -7 * 1(2 * 0.5 \log_2 0.5)$$

$$H_{between}^{dim} = -7 \log_2 0.5$$

$$H_{between}^{dim} = 7$$

Within categories, the relevant dimension does not vary – thus it does not contribute to the entropy. Its value goes to zero, leading to the following calculations.

$$\begin{aligned} H_{within}^{dim} &= -6 * 1(2 * 0.5 \log_2 0.5) \\ H_{within}^{dim} &= -6 \log_2 0.5 \\ H_{within}^{dim} &= 6 \end{aligned}$$

To find the entropy due to relations, we start by calculating O .

$$\begin{aligned} O &= \frac{M!}{(M-2)! * 2!} \\ O &= 21 \end{aligned}$$

Between categories, all dyadic relations have the same probability of co-occurrence (0.25). For each relation between dimensions, there are 4 possible combinations of the levels of those dimensions. They're all equally probable. Recall that for relations, we use an attentional weight of 0.5. So, we end up with the following.

$$\begin{aligned} H_{between}^{rel} &= -21 * 0.5(4 * 0.25 \log_2 0.25) \\ H_{between}^{rel} &= -10.5 \log_2 0.25 \\ H_{between}^{rel} &= 21 \end{aligned}$$

Within the target category, 15 of the dyadic relationships don't include the relevant feature. Thus, their probability of co-occurrence is .25. For 6 of the dyadic relations (any including the relevant feature), there is perfect co-occurrence: probability is either 0 or 1. This makes these terms go to zero, because $\log_2 1 = 0$, and anything multiplied by zero is zero.

$$\begin{aligned} H_{within}^{rel} &= -15 * 0.5(4 * 0.25 \log_2 0.25) \\ H_{within}^{rel} &= -7.5 \log_2 0.25 \\ H_{within}^{rel} &= 15 \end{aligned}$$

Now, we use these calculated values to find entropy between and within categories.

$$\begin{aligned} H_{within} &= 6 + 15 \\ H_{within} &= 21 \\ H_{between} &= 7 + 21 \\ H_{between} &= 28 \end{aligned}$$

Finally, we use the within- and between-category entropy to calculate the density.

$$\begin{aligned} D &= 1 - \frac{21}{28} \\ D &= 0.25 \end{aligned}$$

5.3 Statistical Density Calculations – Dense

The between category entropy for dense categories is the same as for sparse categories. $H_{between} = 28$

Next, we will consider within-category entropy due to dimensions. Six of the seven dimensions do not vary, so they do not contribute to the entropy. Their value goes to zero.

$$H_{within}^{dim} = -1 * 1(2 * 0.5 \log_2 0.5)$$

$$H_{within}^{dim} = -\log_2 0.5$$

$$H_{within}^{dim} = 1$$

Entropy due to relations is similar. Within the target category, 6 of the dyadic relationships don't include the relevant feature. Thus, their probability of co-occurrence is .25. For 15 of the dyadic relations, there is perfect co-occurrence, so their values go to zero.

$$H_{between}^{rel} = -6 * 0.5(4 * 0.25 \log_2 0.25)$$

$$H_{between}^{rel} = -3 \log_2 0.25$$

$$H_{between}^{rel} = 6$$

Next, we calculate the within-category entropy.

$$H_{within} = 1 + 6$$

$$H_{within} = 7$$

Finally, we use the within- and between-category entropy to calculate the density.

$$D = 1 - \frac{7}{28}$$

$$D = 0.75$$

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