Continuous Matrix Product States for Bose/Fermi Fields in One Dimension

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Abstract

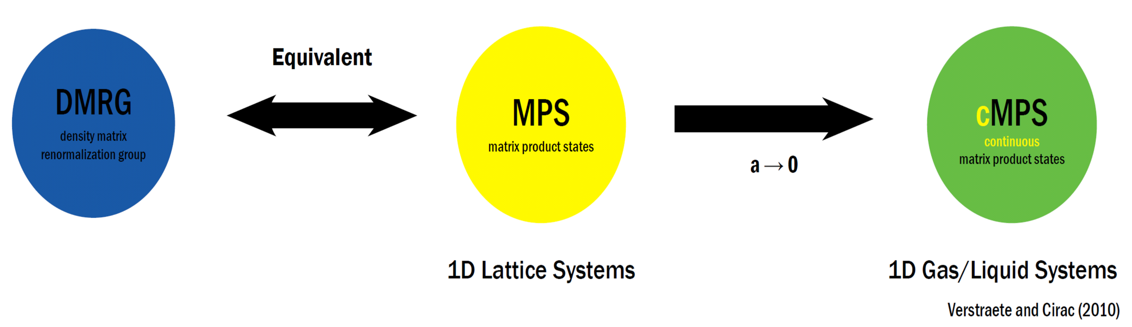
One can describe quantum systems in one spatial dimension through the use of continuous Matrix Product States (cMPS). It has been shown that cMPS can be used to find solutions of one-dimensional quantum field models of interacting bosons and fermions. My goal was to reproduce existing results by testing various optimization processes in order to develop a framework with which to use cMPS to explore the physics of Bose-Fermi mixtures.

What is cMPS?

- Matrix Product States (MPS) can be used to describe properties of quantum systems and were developed from The Numerical Renormalization Group (NRG) and the Density Matrix Renormalization Group (DMRG) [2]
- MPS in certain extensions of DMRG were then adapted by Verstraete and Cirac to describe quantum field theories in a model which they defined as "continuous Matrix Product States" (cMPS) [2]
- cMPS is based on repeated Singular Value Decomposition (SVD)

Why use cMPS?

- Typically, a Quantum Field Theory (QFT) is computationally approached by first discretizing the problem, then applying a lattice-based algorithm, and finally extrapolating the solution back to the continuum
- cMPS circumvents discretization
- It is often more accurate than other solutions
- cMPS gives additional complimentary results when compared with analytical methods such as the Bethe Ansatz



cMPS ansatz

What is Julia?

- High-level environment
- Open source
- Designed for high-performance using LLVM
- Utilizes Multiple dispatch
- Uses JIT Compilation

[1]

Bosonic Gas Case Study

- One can consider a one-dimensional interacting gas of bosons and study it with cMPS

$$H = \int_0^L dx\ (\partial_x \hat{\psi}_b^\dagger \partial_x \hat{\psi}_b + c\ \hat{\psi}_b^\dagger \hat{\psi}_b^\dagger \hat{\psi}_b \hat{\psi}_b)$$

- Q and R are complex, D x D matrices where D represents the bond-dimension

$$|\chi\rangle = Tr_{aux}[\mathcal{P} e^{\int_0^L dx [Q(x)\otimes \hat{\mathbb{1}} + R(x)\otimes \hat{\psi}^\dagger(x)]}]\,|\Omega\rangle$$

$$\langle \chi | \chi \rangle = Tr(e^{TL}) \qquad T = Q\otimes \mathbb{1} + \mathbb{1}\otimes \bar{Q} + R \otimes \bar{R}$$

Translational Invariance $\Rightarrow$ $Q(x) = Q \qquad R(x) = R$

- Expectation values in cMPS are given by:

$$\langle \partial_x \hat{\psi}_b^\dagger \partial_x \hat{\psi}_b \rangle = Tr[e^{TL}(iq_b R_b + [Q, R_b]) \otimes (-iq_b \bar{R}_b + [\bar{Q}, \bar{R}_b])]$$

$$\langle \hat{\psi}_b^\dagger \hat{\psi}_b^\dagger \hat{\psi}_b \hat{\psi}_b \rangle = Tr[e^{TL}(R_b^2 \otimes \bar{R}_b^2)]$$

$$\langle \hat{\psi}_b^\dagger \hat{\psi}_b \rangle = Tr[e^{TL}(R_b \otimes \bar{R}_b)]$$

- Density is set with Lagrange constraints

Lagrange Constraints $\Rightarrow$ $C_P(\langle \hat{\psi}_b^\dagger \hat{\psi}_b \rangle - 1)^2$

Fermionic Gas Case Study

- One can consider a one-dimensional interacting gas of fermions and study it with cMPS

$$H_{GY} = \int_0^L dx \sum_{\sigma=\uparrow,\downarrow} (\partial_x \hat{\psi}_\sigma^\dagger \partial_x \hat{\psi}_\sigma - 2\Delta \hat{\psi}_\sigma^\dagger \hat{\psi}_{\bar{\sigma}}^\dagger \hat{\psi}_{\bar{\sigma}} \hat{\psi}_\sigma)$$

$$\hat{f} = \hat{H}_{GY}/L - \mu(\hat{n}_\uparrow + \hat{n}_\downarrow) - h(\hat{n}_\uparrow - \hat{n}_\downarrow)$$

- Rup and Rdn are formulated similarly to R in the bosonic case, now representing up or down spin-1/2 fermions

$$|\chi\rangle = Tr_{aux}[\mathcal{P} e^{\int_0^L dx [Q(x)\otimes \hat{\mathbb{1}} + \Sigma_\sigma R_\sigma(x)\otimes \hat{\psi}^\dagger(x)]}]\,|\Omega\rangle$$

$$T(x) \equiv Q(x)\otimes \mathbb{1} + \mathbb{1}\otimes \bar{Q}(x) R_\uparrow(x) + R_\downarrow(x) \otimes \bar{R}_\downarrow(x) + R_\uparrow(x)\otimes \bar{R}_\uparrow(x) \equiv T$$

Translational Invariance $\Rightarrow$ $Q(x) = Q \qquad R_\sigma(x) = R_\sigma e^{iq_\sigma x}$

Constraints $\Rightarrow$ $\{R_\uparrow, R_\downarrow\} = 0 \qquad R_\sigma^2 = 0$

- Expectation values in cMPS are given by:

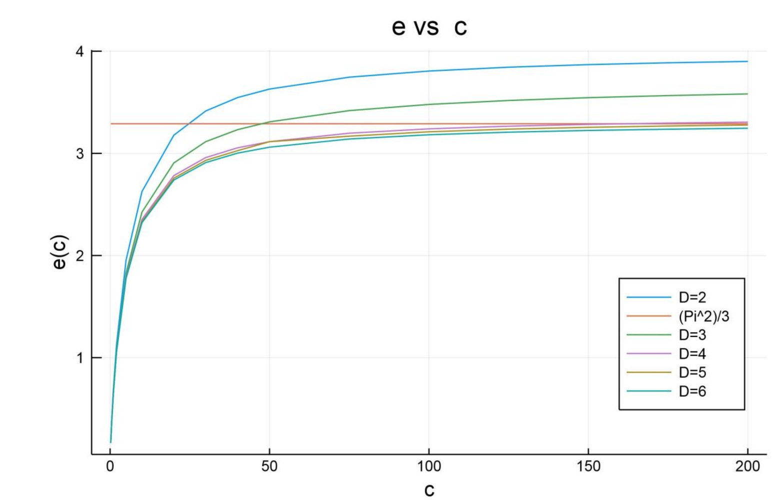
$$\langle \partial_x \hat{\psi}_\sigma^\dagger \partial_x \hat{\psi}_\sigma \rangle = Tr\{e^{TL}(iq_\sigma R_\sigma + [Q, R_\sigma]) \otimes (-iq_\sigma \bar{R}_\sigma + [\bar{Q}, \bar{R}_\sigma])\}$$

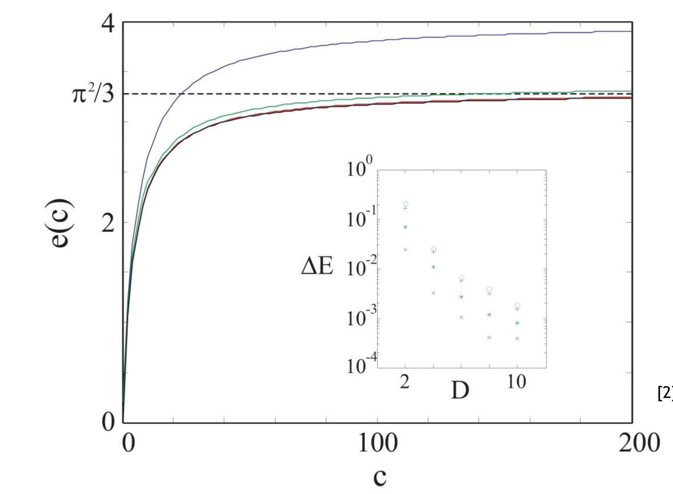
$$\langle \hat{\psi}_\sigma^\dagger \hat{\psi}_{\bar{\sigma}}^\dagger \hat{\psi}_{\bar{\sigma}} \hat{\psi}_\sigma \rangle = Tr[e^{TL}(R_\sigma R_{\bar{\sigma}} \otimes \bar{R}_\sigma \bar{R}_{\bar{\sigma}})]$$

$$\langle \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma \rangle = Tr\{e^{TL}(R_\sigma \otimes \bar{R}_\sigma)\}$$

Bosonic Case Study Results

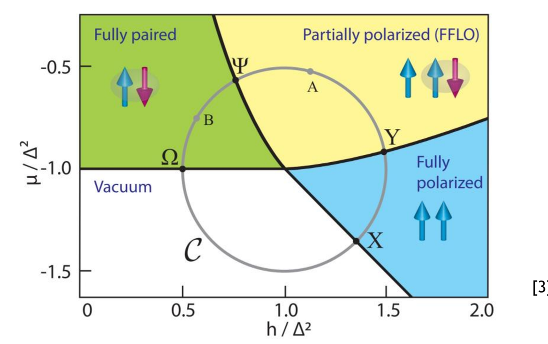
- The expectation value of the Hamiltonian is minimized and compared with exact values
- Reults were obtained using the Nelder-Mead Algorithm

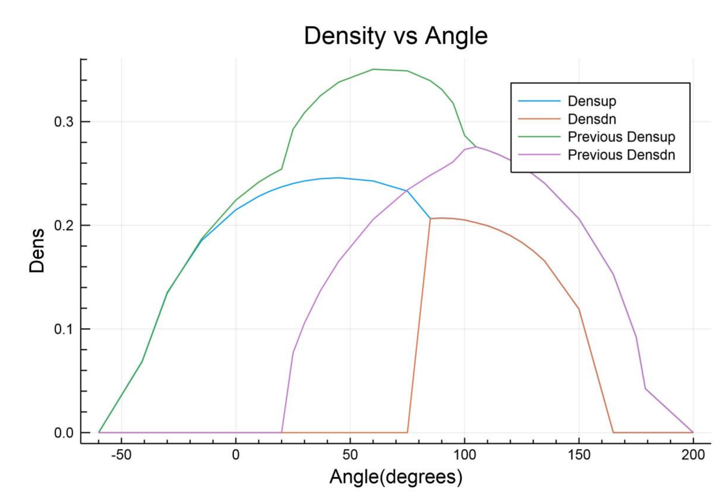


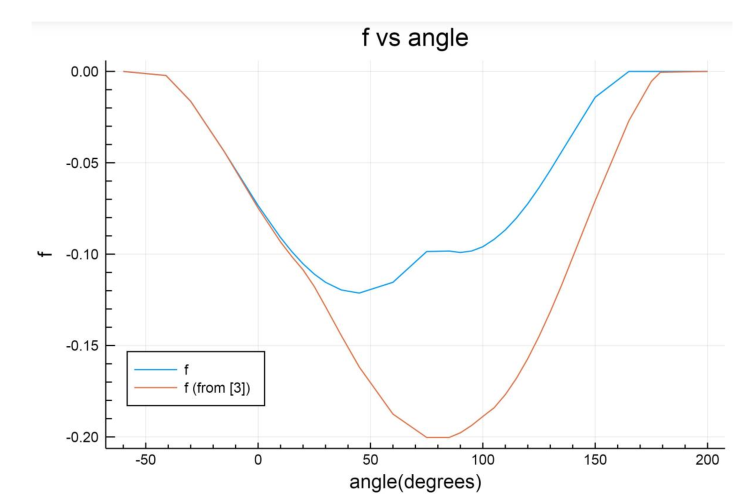


Fermionic Results

- The zero temperature Grand Canonical free energy is minimized to compare with previous results found in [3]
- Results were obtained using the Nelder-Mead Algorithm







Acknowledgements

- First, I would like to thank the Ohio Supercomputing Center for providing the computing time and processing power necessary to achieve the above results. I would also like to thank the OSC help staff for fixing every problem that arose while using the OSC system and website
- Second, I am grateful for the funding provided for this research by the MUSE Fellowship

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[3] Sangwoo S. Chung, Kuei Sun, and C. J. Bolech. Matrix product ansatz for fermi fields in one dimension.Phys. Rev. B, 91:121108, Mar 2015.