Stochastic inference

with deterministic

spiking neurons

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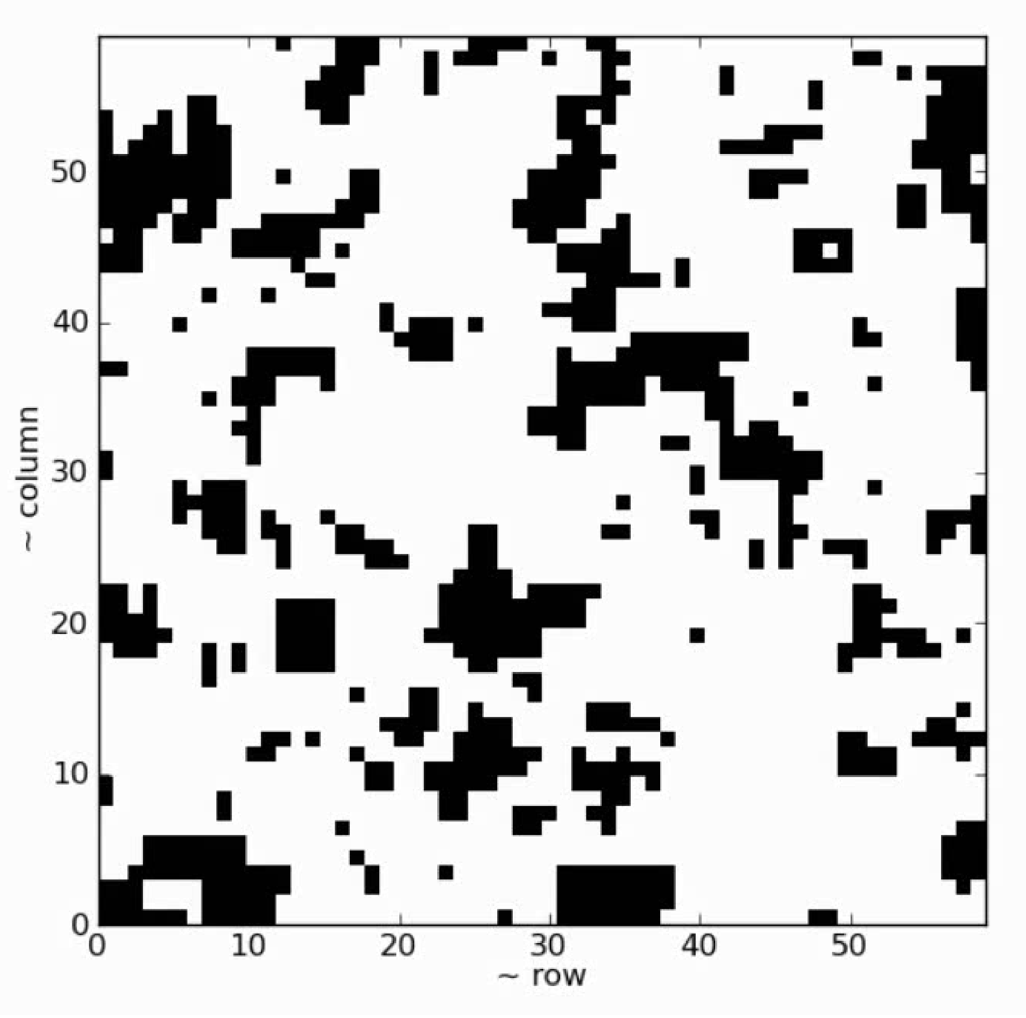
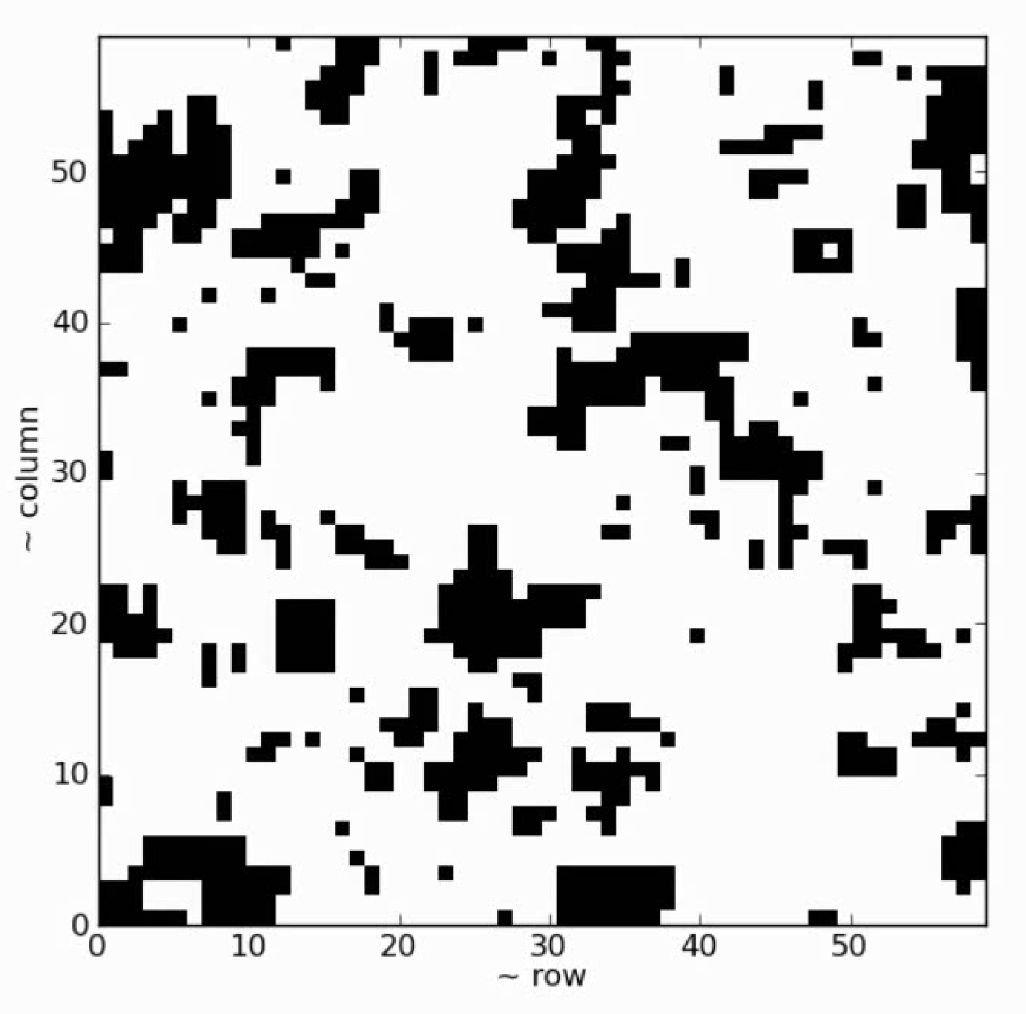
Ensemble dynamics



Ensemble dynamics



Ising model

$$H(\boldsymbol{\sigma}) = -\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i b_i\sigma_i$$

$$p_\beta(\boldsymbol{\sigma}) = \frac{e^{-H(\boldsymbol{\sigma})/k_B T}}{Z_\beta}$$

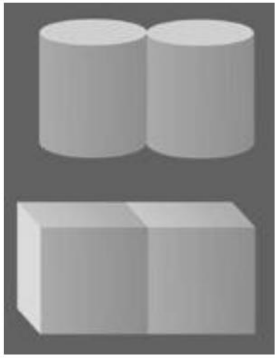
Glauber dynamics

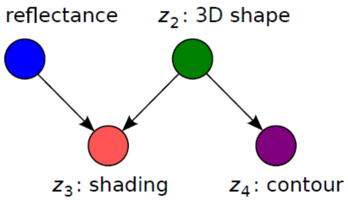
$\rightarrow$ sampling !

Bayesian inference & learning

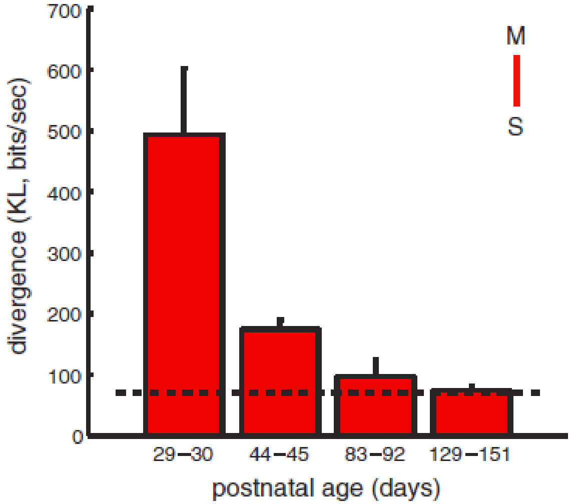
decision problem $\leftrightarrow$ inference problem

p (reality | observations)





Knill & Kersten (1991)

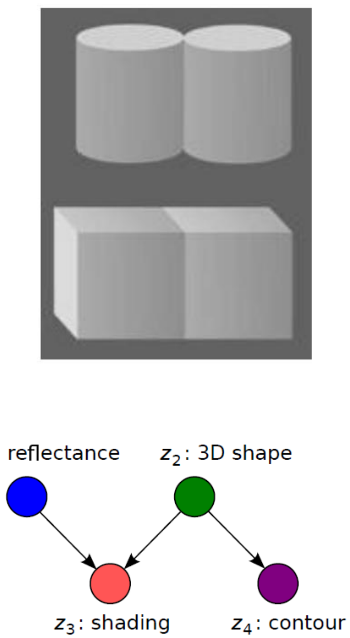


Berkes et al. (2011)

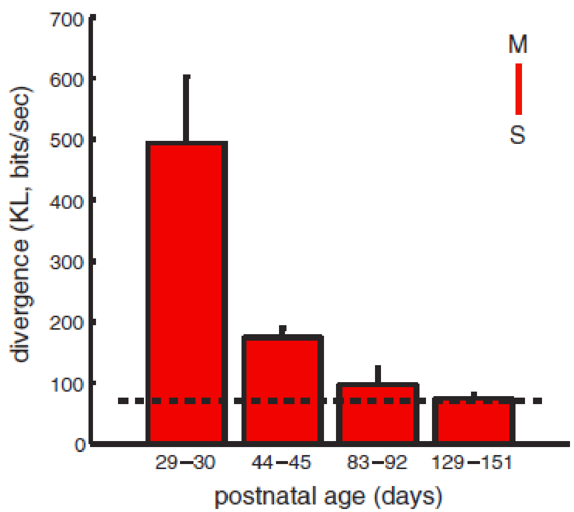
Bayesian inference & learning

decision problem ↔ inference problem

p (reality | observations)



Knill & Kersten (1991)



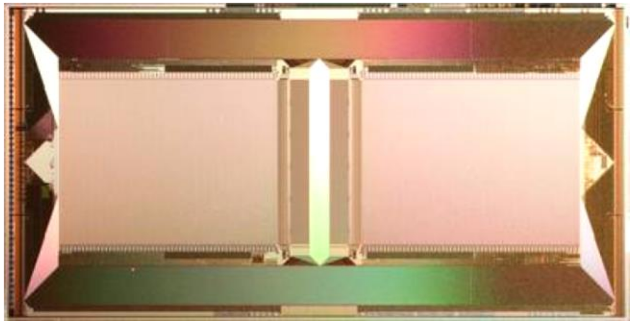
Berkes et al. (2011)

Matthew et al. (ca. 85)

Why spikes ?

understand biology

exploit efficient hardware

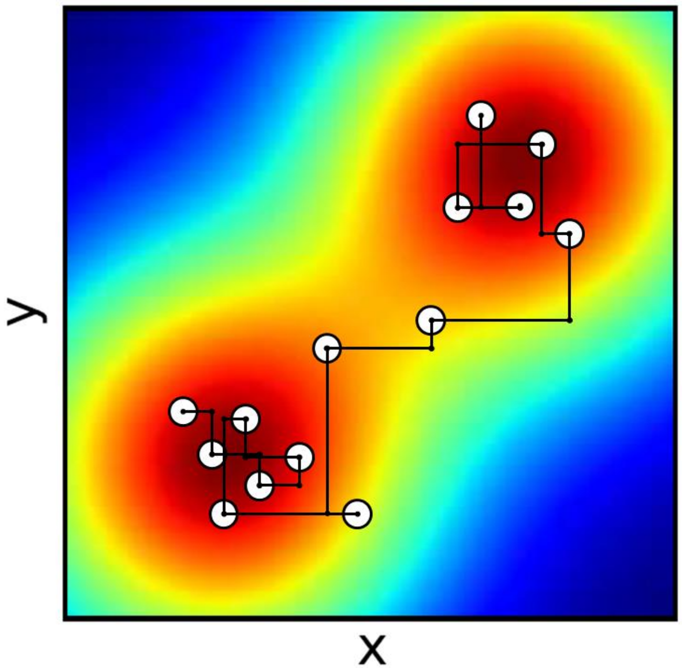


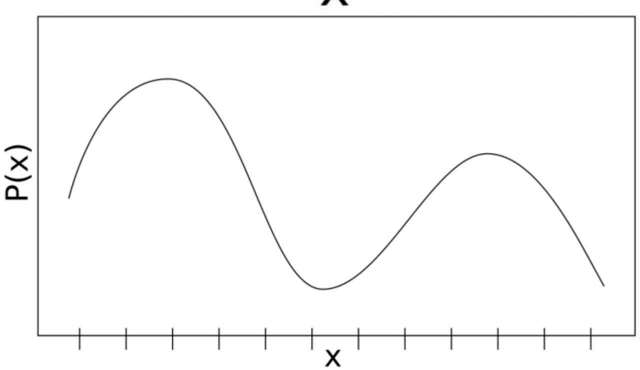
low power, high speed, ...

... and maybe also gain a computational advantage

Neural sampling

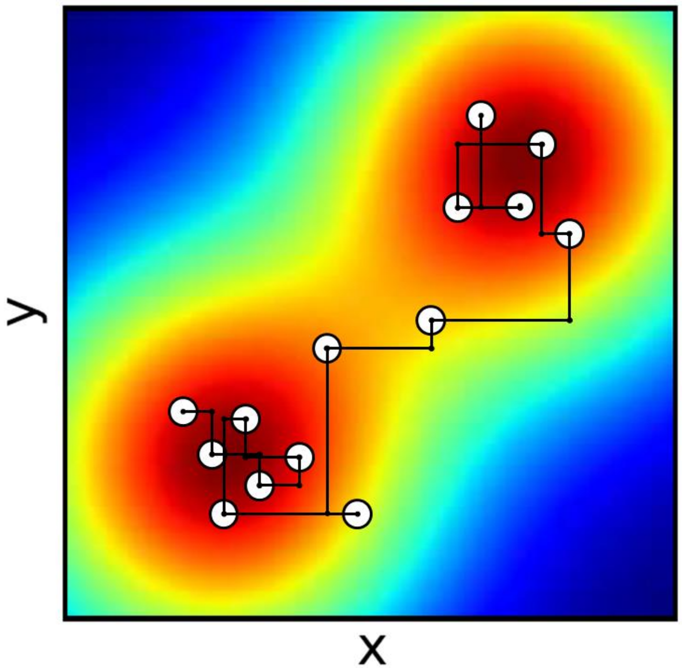
(Gibbs) sampling

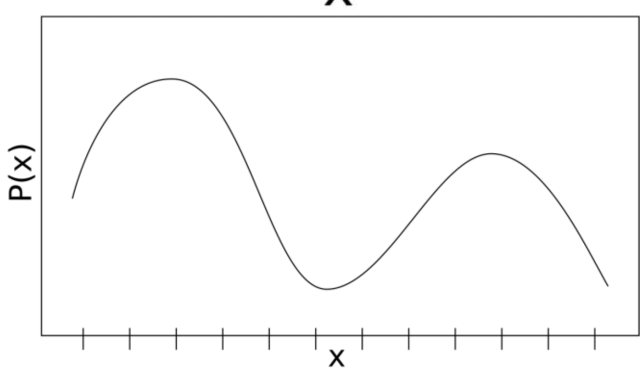




Neural sampling

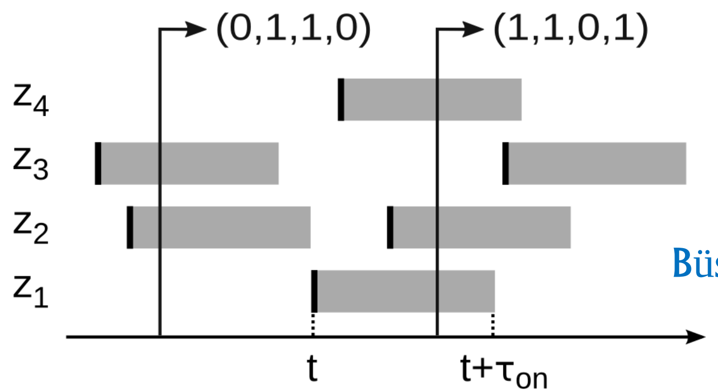
(Gibbs) sampling





assumptions: $z_k \in \{0,1\}$

$z_k = 1 \;\Leftrightarrow\;$ neuron has spiked in $[t-\tau, t)$



Büsing et al. (2011)

information transfer between neurons:

$$u_k = \log \frac{p(z_k = 1 \,|\mathbf{z}_{\setminus k})}{p(z_k = 0 \,|\mathbf{z}_{\setminus k})}$$

$$\Leftrightarrow \qquad \nu_k = p(z_k = 1) = \frac{1}{1+\exp(-u_k)}$$

response/activation function

The LIF activation function

noise source: Poisson spike trains $\Rightarrow$ membrane as Ornstein-Uhlenbeck process

$$du(t) = \Theta \cdot [\mu - u(t)]dt + \sigma\, dW(t)$$

Brunel & Sergi (1998)

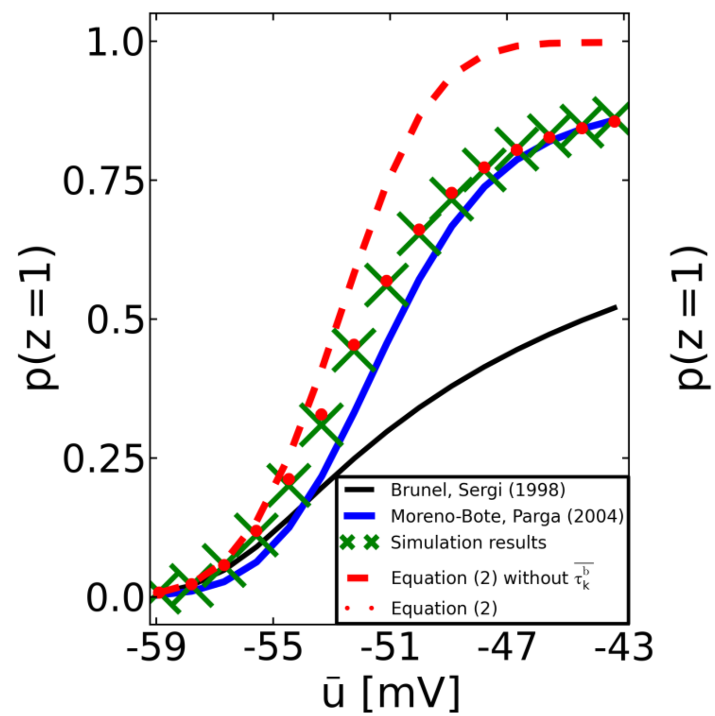
assumption: $\tau_{\mathrm{syn}} \ll \tau_{\mathrm{m}}$

$$\langle T \rangle = \tau\sqrt{\pi} \int_{\rho - \mu/\sigma}^{\vartheta_{\mathrm{eff}} - \mu/\sigma} [1 + \operatorname{erf} x] \exp(x^2)\, dx$$

Moreno-Bote & Parga (2004)

assumption: $\tau_{\mathrm{ref}} \approx \tau_{\mathrm{syn}} \gg \tau_{\mathrm{m}}$

$$\nu = \int_{\vartheta}^{\infty} \left(\tau_{\mathrm{m}} \frac{\varrho - \tilde{u}}{\vartheta - \tilde{u}} \right)^{-1} p(\tilde{u})\, d\tilde{u}$$



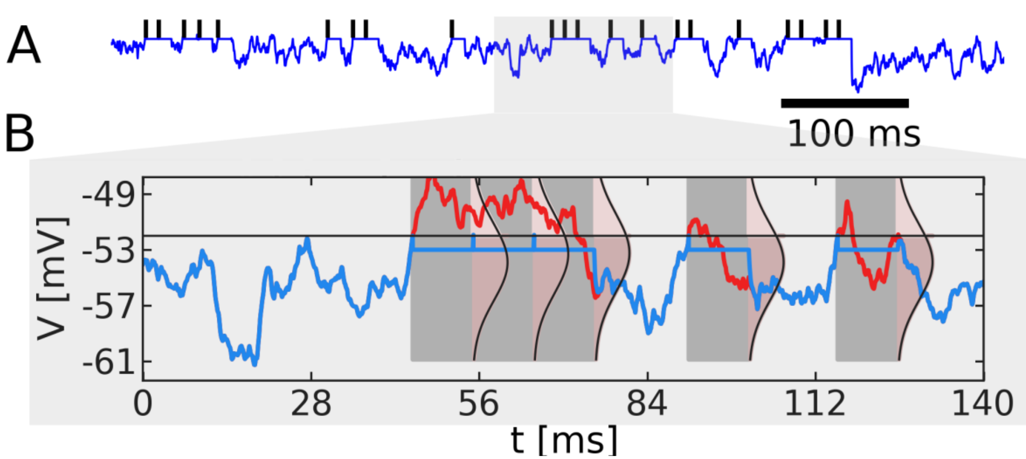
The membrane autocorrelation propagation

$$p(z_k = 1) = \frac{t_{k,\,\text{refractory}}}{t_{\text{total}}} = \frac{\sum_n P_n\, n\, \tau_{\text{ref}}}{\sum_n P_n \cdot \left(n\, \tau_{\text{ref}} + \sum_{k=1}^{n-1} \overline{\tau_k^b} + T_n\right)}$$

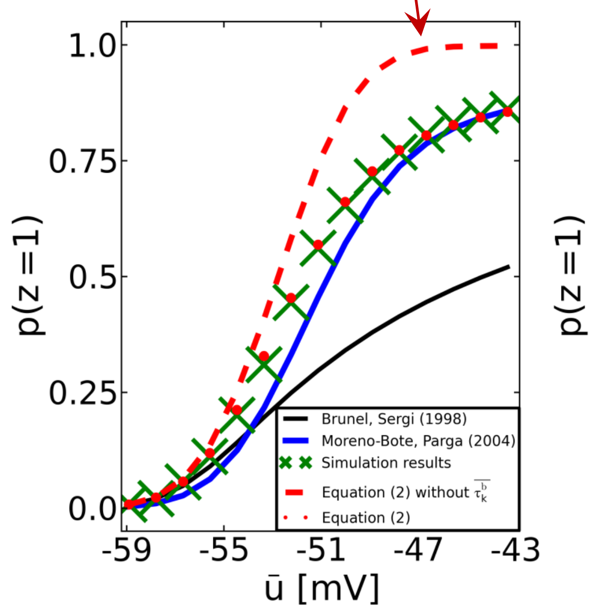
$$P_n = \left(1 - \sum_{i=1}^{n-1} P_i\right) \cdot \int_{V_{\text{thr}}}^{\infty} dV_{n-1}\; p(V_{n-1}|V_{n-1} > V_{\text{thr}}) \left[\int_{-\infty}^{V_{\text{thr}}} dV_n\; p(V_n|V_{n-1})\right]$$

$$T_n = \int_{V_{\text{thr}}}^{\infty} dV_{n-1}\; p(V_{n-1}|V_{n-1} > V_{\text{thr}}) \left[\int_{-\infty}^{V_{\text{thr}}} dV_n\; p(V_n|V_{n-1}) \langle FPT(V_{\text{thr}}, V_n)\rangle\right]$$

$$\overline{\tau_k^b} = \int_{\vartheta}^{\infty} du_k\; \tau_{\text{eff}} \ln\left(\frac{\varrho - u_k}{\vartheta - u_k}\right) p(u_k|u_k > \vartheta, u_{k-1})$$

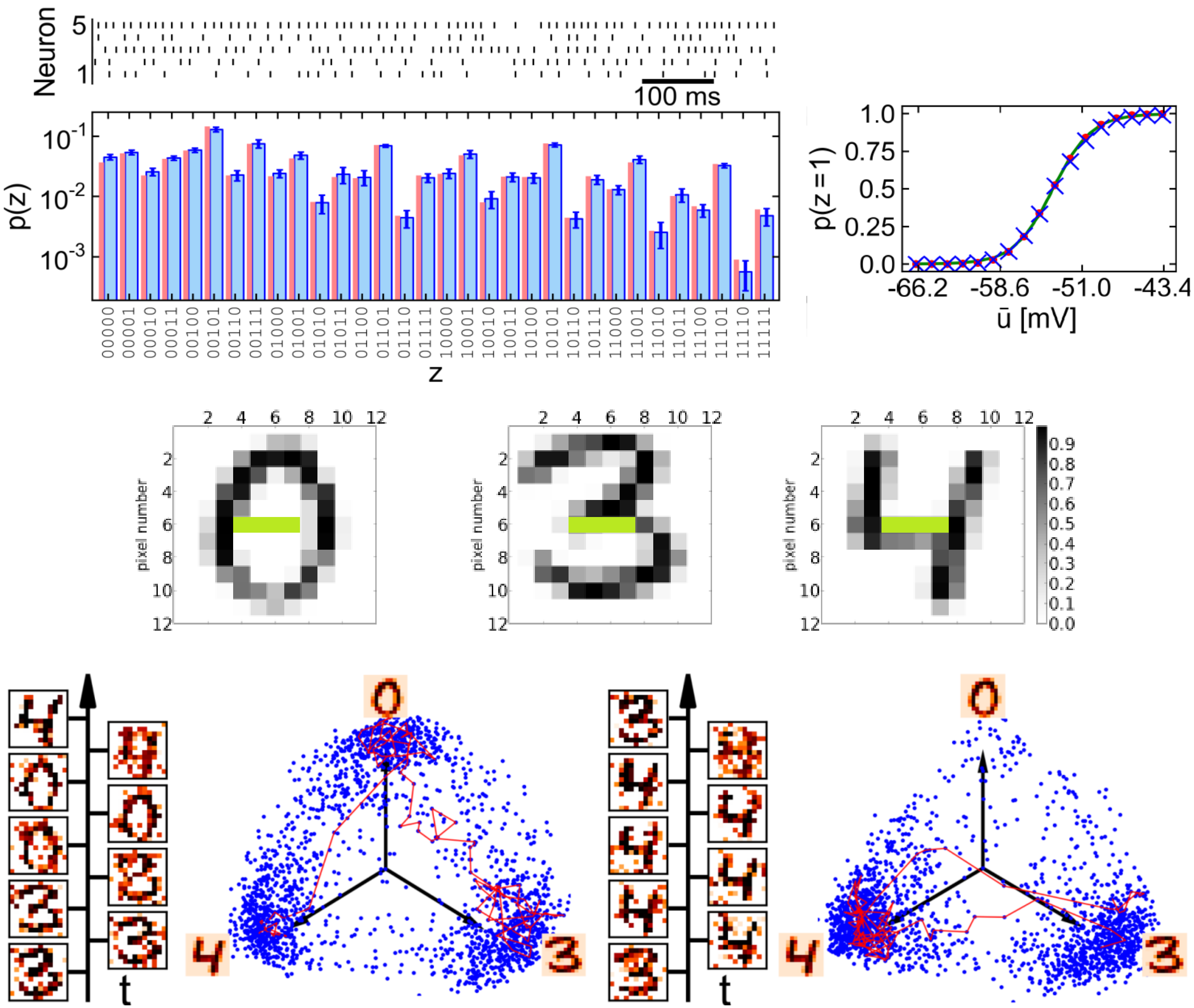


HCS !



Petrovici & Bill et al. (2013)

(Fully visible) LIF-based Boltzmann machines

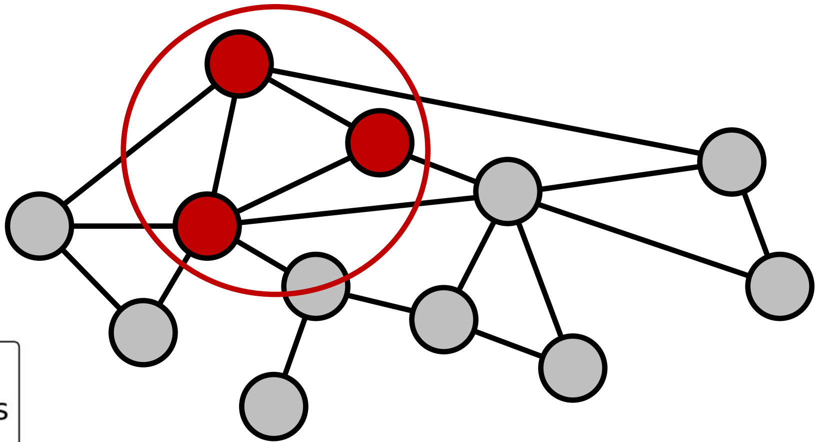


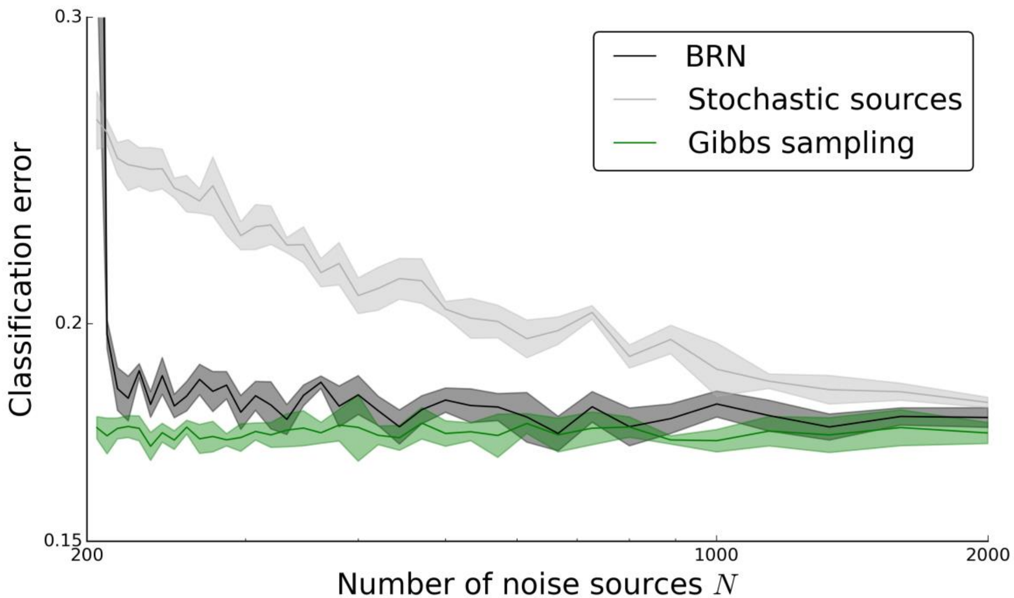
Petrovici & Bill et al. (2013)

… so where does the noise come from ?

- in simulations: from independent Poisson sources
- more realistic: from being embedded in larger networks
- but: noise correlations hurt !

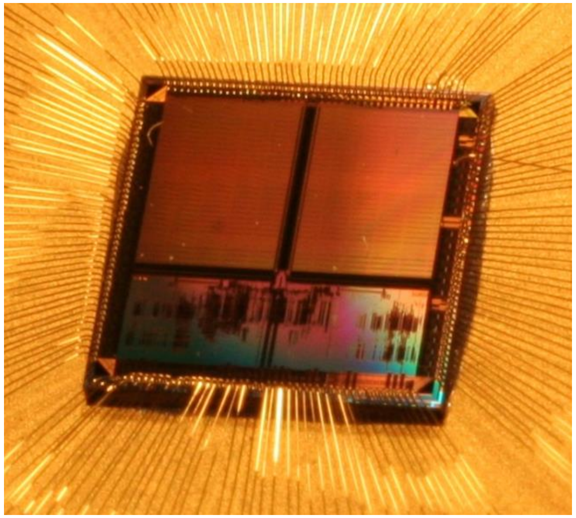
  → decorrelation by lateral inhibition





Jordan et al. (2015)

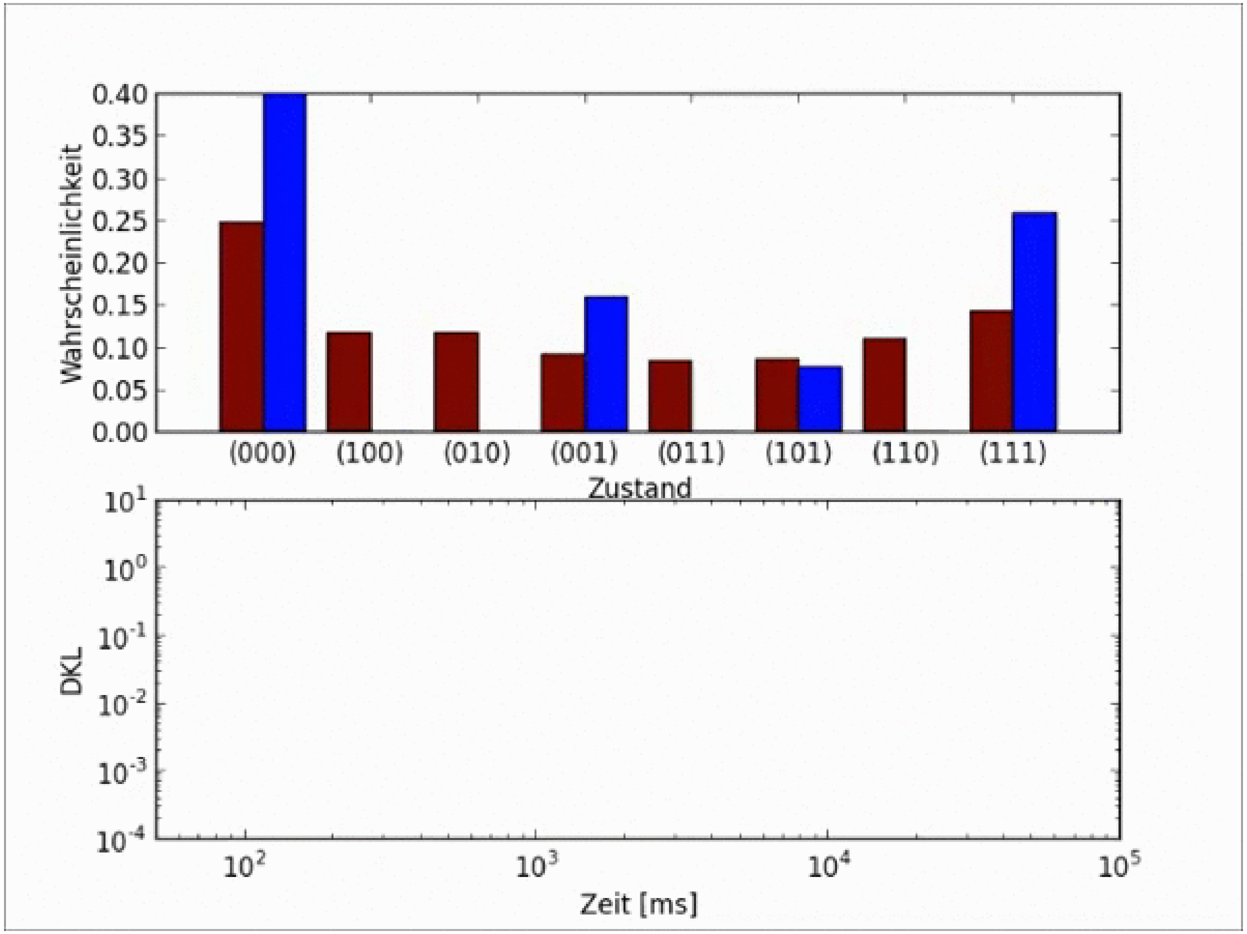
LIF sampling on accelerated neuromorphic hardware



„Spikey“ chip

Schemmel et al. (2006)

Pfeil et al. (2013)

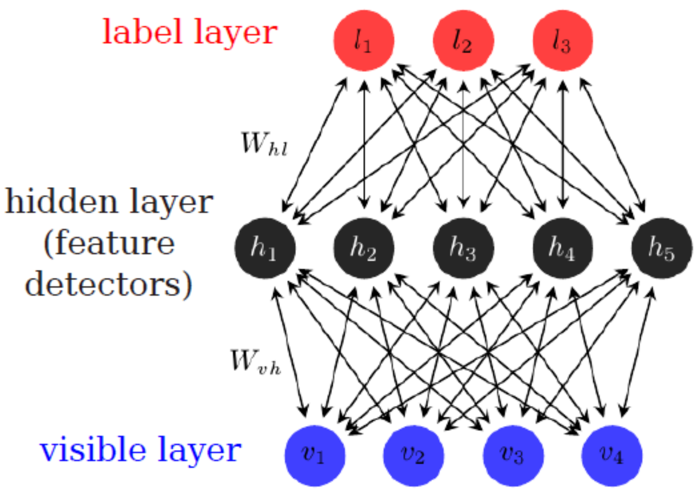


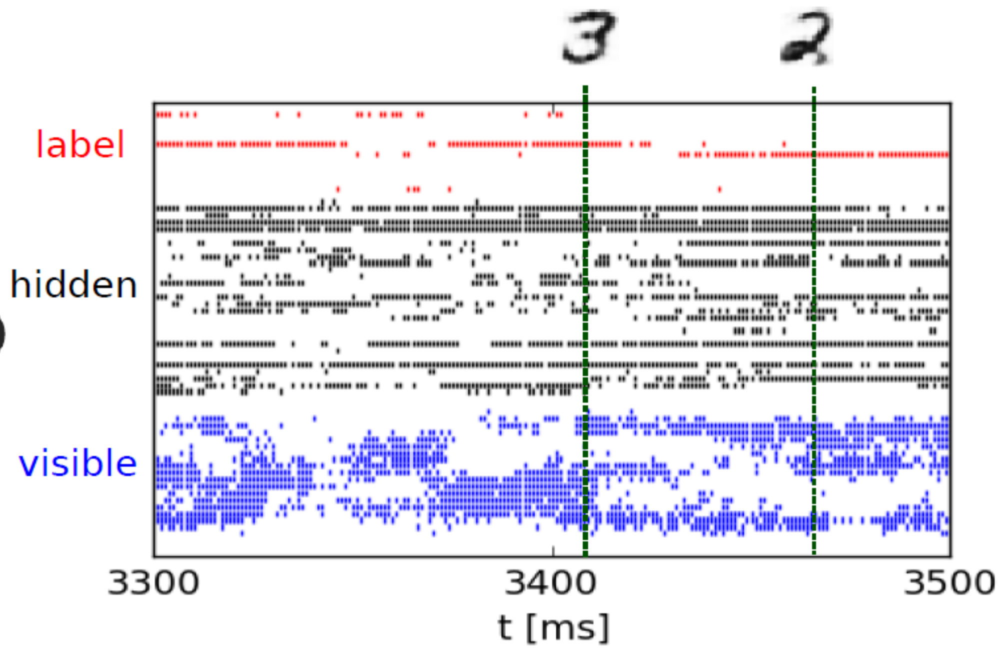
bio: 100 s → software simulation: 10 s

bio: 100 s → neuromorphic emulation: 10 ms

Petrovici & Stöckel et al. (2015)

Deep learning architectures



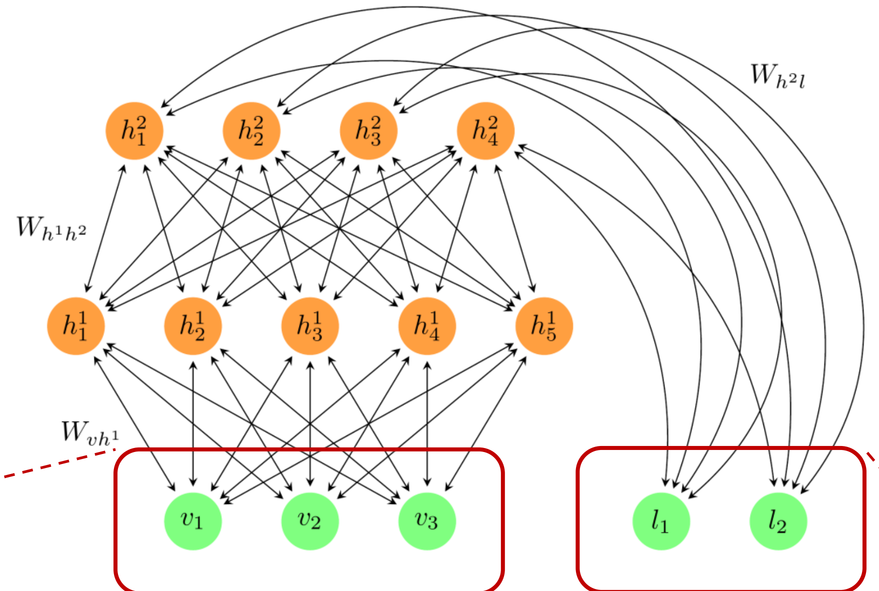


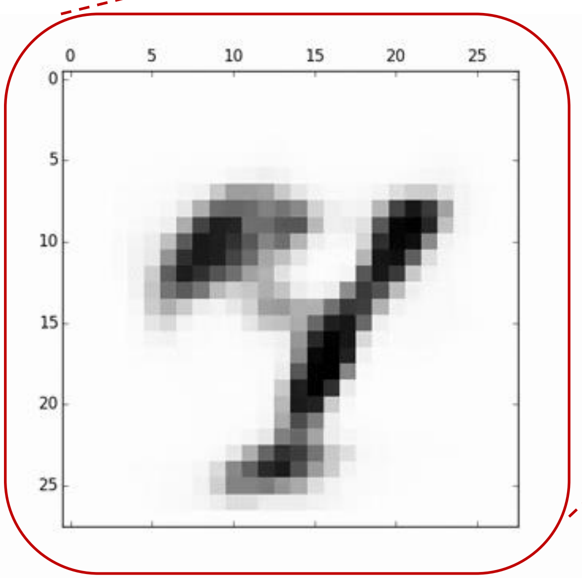
Leng & Petrovici et al. (2016)

Deep learning architectures

Training of RBMs/DBMs on MNIST:

- contrastive divergence
- coupled adaptive tempering

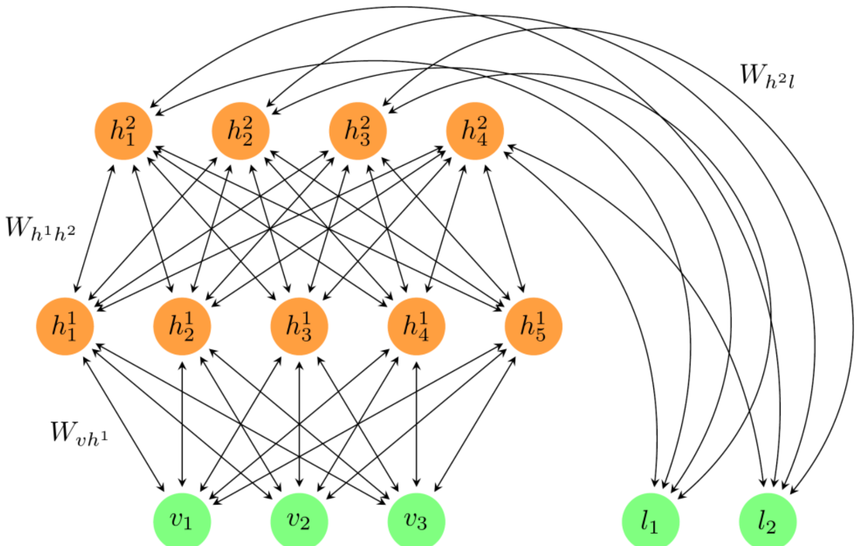


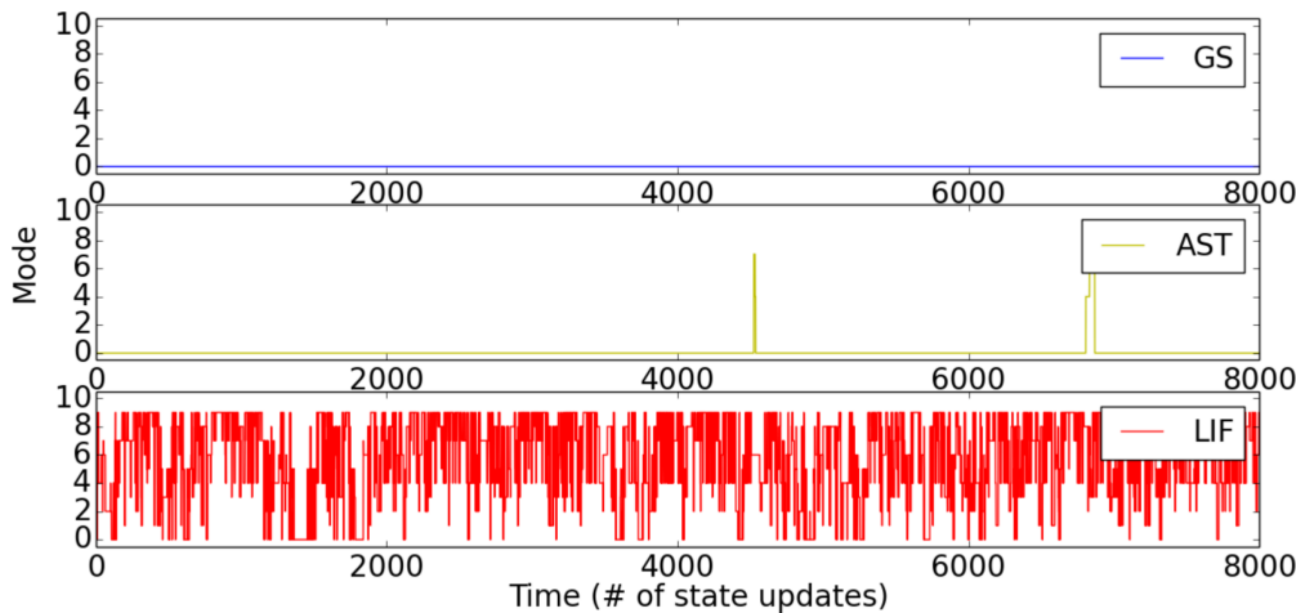


96.9 % correct classification

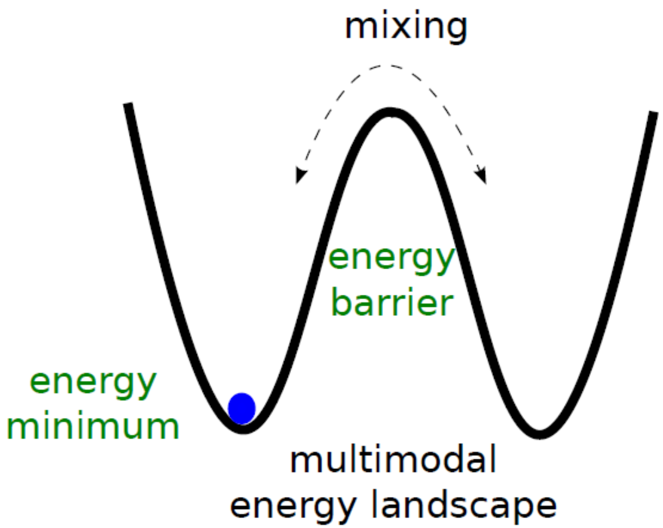
Leng & Petrovici et al. (2016)

Mixing in deep spiking networks

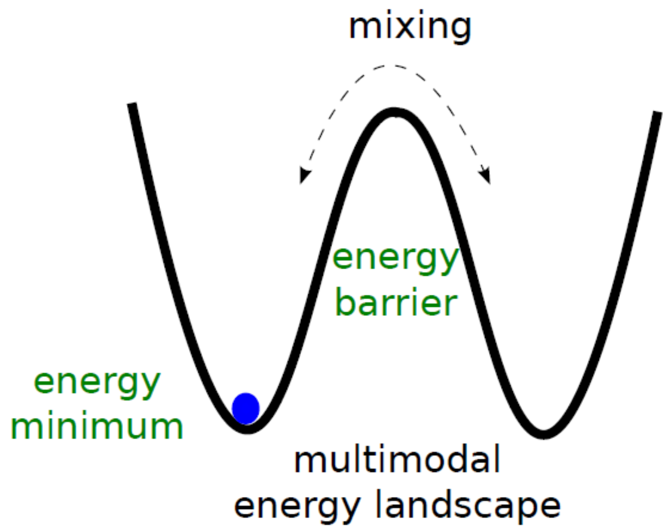




Mixing in deep spiking networks



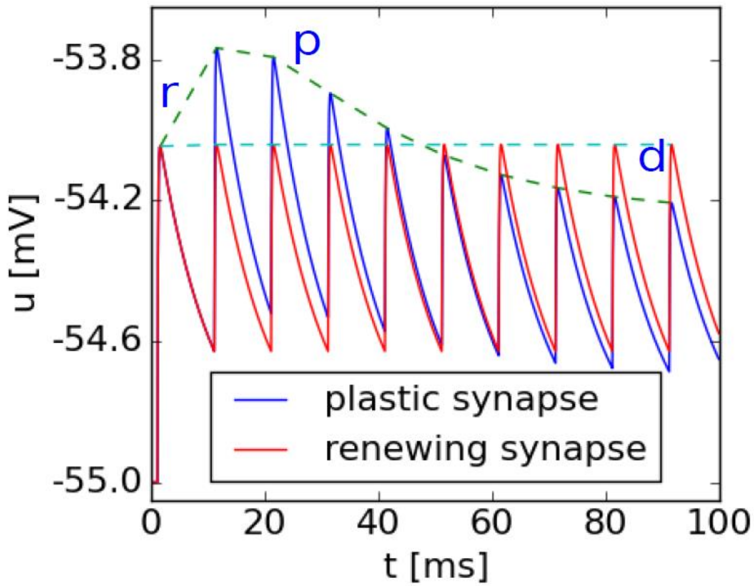
Mixing in deep spiking networks

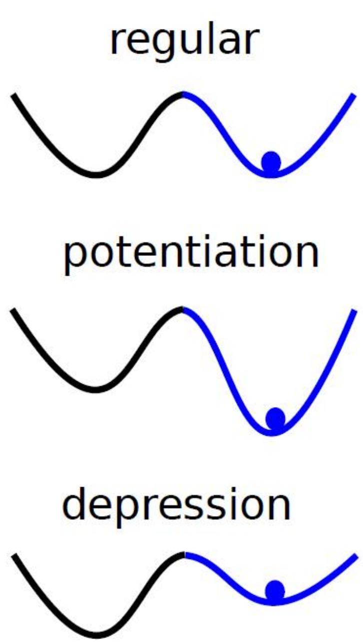


Tsodyks-Markram short-term plasticity

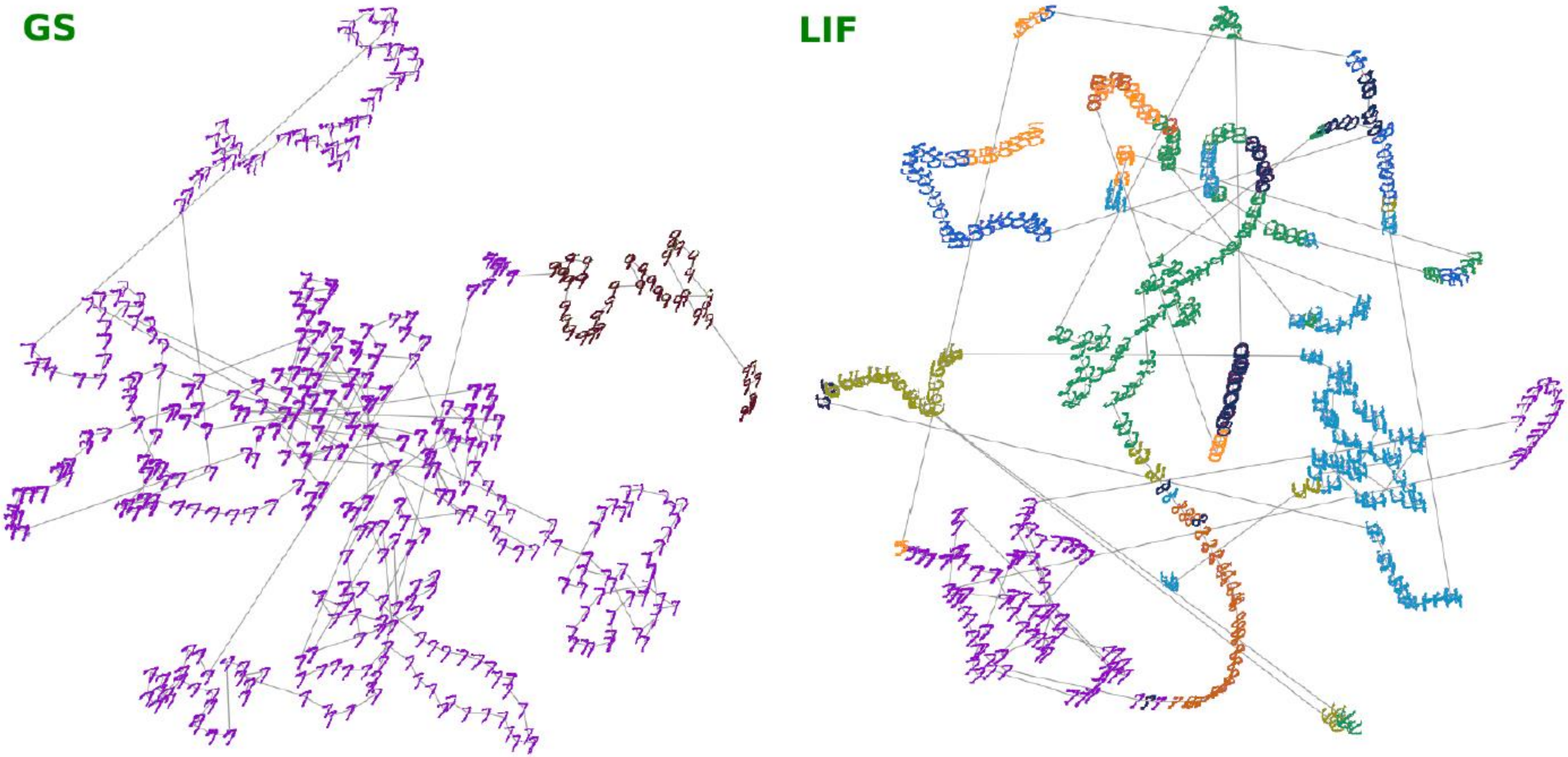
$$\frac{dR}{dt} = \frac{1-R}{\tau_{rec}} - U_{SE}R\delta(t-t_{sp})$$

$$\frac{dU_{SE}}{dt} = -\frac{U_{SE}}{\tau_{facil}} + U_0(1-U_{SE})\delta(t-t_{sp})$$





Spiking networks as superior generative and discriminative models



Leng & Petrovici et al. (2016)

Summary

BIOLOGY

- functional role of HCS
- functional role of STP
- uncorrelated noise is surprisingly easy to generate

HARDWARE

- design of a robust spike-based sampling model
- extremely fast convergence to target distribution

THEORY & MODELS

- model of LIF sampling
- analytical description of LIF activation function
- deep spiking networks as simultaneous discriminative and generative models