

Signature properties of water: Their molecular electronic origins

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Water challenges our fundamental understanding of emergent materials properties from a molecular perspective. It exhibits a uniquely rich phenomenology including dramatic variations in behavior over the wide temperature range of the liquid into water's crystalline phases and amorphous states. We show that many-body responses arising from water's electronic structure are essential mechanisms harnessed by the molecule to encode for the distinguishing features of its condensed states. We treat the complete set of these many-body responses nonperturbatively within a coarse-grained electronic structure derived exclusively from single-molecule properties. Such a "strong coupling" approach generates interaction terms of all symmetries to all orders, thereby enabling unique transferability to diverse local environments such as those encountered along the coexistence curve. The symmetries of local motifs that can potentially emerge are not known a priori. Consequently, electronic responses unfiltered by artificial truncation are then required to embody the terms that tip the balance to the correct set of structures. Therefore, our fully responsive molecular model produces, a simple, accurate, and intuitive picture of water's complexity and its molecular origin, predicting water's signature physical properties from ice, through liquid–vapor coexistence, to the critical point.

subcritical water | intermolecular interactions | many-body dispersion | coarse-grained model | electronic responses

Water is a ubiquitous yet unusual substance exhibiting anomalous physical properties for a liquid and forming many crystalline ices and (at least) two distinct amorphous states of different density (1). As the biological solvent, it is critical that water molecules form a liquid over a very wide range of temperatures (2) and pressures (3, 4) to support life under a wide variety of conditions. Indeed, water's simple molecular structure, a three-atom, two-species moiety, yields a surprisingly rich phenomenology in its condensed phases.

It is well-known that many signature properties of water have their molecular origin in the hydrogen-bonding interactions between molecules (5, 6). These directional networks are also the source of enhanced molecular polarization in the liquid state relative to the gas (7). In addition, there is speculation that dispersion interactions which arise from quantum-mechanical fluctuations of the charge density are also an important factor in the equilibrium properties of the ambient liquid (8, 9). The question of the ranges of temperature and density where these interactions influence observable properties is important for the construction of a conceptually simple but broadly transferable physical model linking molecular and condensed phase properties with the minimum of additional assumptions. Liquid water exhibits anomalies at both extremes of temperature—including a point of maximum density near freezing, an unusually high critical temperature relative to other hydrides, and significant changes in physical properties along the coexistence curve—thereby presenting a unique challenge for predictive simulation and modeling (10–13).

Today, simulation and modeling are considered the third pillar of the scientific method, together with analytical theory and experiment. Studies of condensed phase molecular systems via the

combination of multiscale descriptions and statistical sampling have led to insights into physical phenomena across biology, chemistry, physics, materials science, and engineering (14). Significant progress now requires novel predictive models with reduced empirical input that are rich enough to embody the essential physics of emergent systems and yet simple enough to retain intuitive features (14).

Typically, atomistic models of materials are derived from a common strategy. Interactions are described via a fixed functional form with long-range terms taken from low orders of perturbation theory and are parameterized to fit the results of *ab initio* computations on test systems (both condensed and gas phase) and/or physical properties of systems of interest (15, 16). This approach presumes that the physics incorporated by the functional form is transferable outside the parameterization regime. In addition, first principles methods efficient enough to treat the condensed phase will also miss diagrams leading to truncation (e.g., local density functional theory neglects dispersion). If the truncation scheme is inappropriate for the problem of interest, predictions can go significantly awry (17, 18). Clearly, this is likely to be true for water with the directional, locally polarizing, H-bonded network of its liquid being disrupted in the less-associated gas phase. Consequently, the key physics underpinning a fully predictive model of water has yet to be identified.

The strategy adopted here captures water's properties from an atomistic perspective by incorporating individual water molecules, designed to respond with full many-body character, which can assemble to form condensed phases, as depicted in Fig. 1 *A–C*. This is a radical departure from standard approaches given above (11, 12). It is enabled by recent work (19–21) which has shown that it is possible to represent the complete hierarchy of long-range responses within a many-body system using a coarse-grained projection of the electronic structure onto an interacting set of

Significance

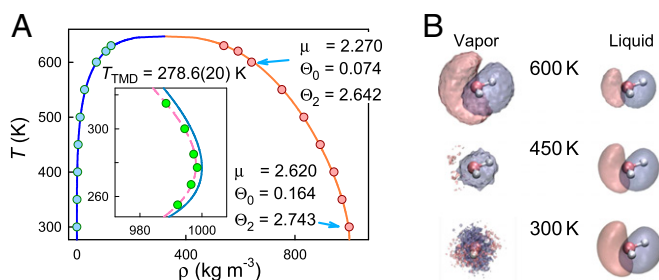
Water is one of the most common substances yet it exhibits anomalous properties important for sustaining life. It has been an enduring challenge to understand how a molecule of such apparent simplicity can encode for complex and unusual behavior across a wide range of pressures and temperatures. We reveal that embedding a complete hierarchy of electronic responses within the molecule allows water's phase behavior and signature properties to emerge naturally even within a simple model. The key result is a simple and accurate, prediction of liquid–gas phase equilibria from freezing to the critical point thus establishing a direct link between molecular and condensed phase properties and a sound physical basis for a conceptually simple but broadly transferable model for water.

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water molecules to their local environment, depicted as the changes in the molecular electronic structure of Fig. 2B and in the tetrahedral coordination of Fig. 4, *Insets*. Obtaining the correct balance between the local structure and electronic structure changes over a wide temperature range 300 K < T < 600 K is a prerequisite for prediction of these key properties.

The nontrivial electronic structure rearrangements that occur in water's condensed phase along coexistence are also reflected in its dielectric responses such as the permittivity given in Fig. 4 from experiment and the QDO model. Fig. 2*B* shows evolution of water's electronic structure with temperature from which induced electrostatic moments can be determined. The electrostatic moments are key to generating the correct dielectric behavior as the dielectric permittivity can be directly related to the fluctuations in the total dipole moment. The molecular dipole in the ambient liquid, for instance, is enhanced to $\mu = 2.6$ D from $\mu = 1.855$ D of the free molecule, a 40% increase. Both the dipole and quadrupole moment tensors in the ambient liquid predicted by the QDO model agree with recent high-level quantum-chemical estimates (27). Along the coexistence curve, there is an almost linear but modest depolarization of the water molecule whereby the dipole moment is reduced from $\mu = 2.6$ D to $\mu = 2.3$ D. The high-temperature moment is still considerably larger than that of the gas phase ($\mu = 1.855$ D), demonstrating that significant environmental correlations leading to molecular polarization are present, even in the disordered environment of the near-critical liquid (Fig. 2*B*) where proportionally more cohesion is provided by dispersion forces. The dipole moment under ambient conditions is consistent with other predictions (within 10%) (7, 28, 29), noting that the definition of a molecular dipole requires a model as an appropriate operator cannot be averaged over the many-body wavefunction to generate it.

We further find that the molecular properties which appear to be necessary to predict water's gas and vapor densities at a wide range of state points also generate realistic thermodynamics. This is evidenced by the temperature dependence along coexistence of the vaporization enthalpy, $\Delta H_{\text{vap}}(T)$, as shown in Fig. 4 in comparison with the reference (The International Association for the Properties of Water and Steam; IAPWS) equation of state (23). At lower temperatures the hydrogen bonding and dipolar forces dominate, whereas at higher temperatures, van der Waals forces

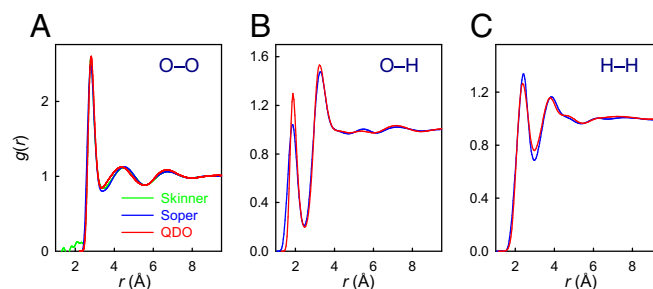
are a larger contribution. Capturing this fundamental force rebalancing demands an electronically responsive model.

Application to a Solid Polymorph: Proton-Ordered Ice II. Lastly, we apply the QDO approach in the context of a solid polymorph, the proton-ordered ice II—to avoid the complications arising from sampling disordered proton positions in this first application to a crystal (30). Ice II forms at pressures above ≈ 0.25 GPa and, when cooled below $T = 120$ K, it is metastable upon recovery to atmospheric pressure. Its structure is characterized by two distinct hexagonal rings of water molecules as shown in Fig. 5A together with a representation of the electronic rearrangement around the water molecules predicted by our coarse-grained electronic structure. The dimensions of the hexagonal unit cell of ice II are a and $b = a$, which control the size of the rings, and c , which controls the spacing between them. Fig. 5B shows the temperature dependence of both a and c , compared with the experimental data of Fortes et al. (31) over the temperature range $100 \text{ K} < T < 160 \text{ K}$. The hydrogen bonding, electrostatics, and many-body polarization roughly control a , whereas many-body dispersion allows c to emerge correctly.

Discussion

A simple model for water consisting of a rigid molecular frame decorated with point charges together with a coarse-grained electronic structure and a short-range pair potential, parameterized to a minimal set of monomer and dimer properties only, has been shown to predict the phase properties of liquid water over a wide range of thermodynamic states where the molecule remains, to good approximation, intact. The key insight is that such a minimal model can capture water's phase equilibria, dielectric properties along coexistence, its critical constants, a proton-ordered ice structure, and water's TMD. This suggests that the model contains the essential physics required to describe water over a wide variety of conditions. It remains to consider why the model is successful and to elaborate on the key physical principles this success reveals.

In assessing the QDO model's unique transferability, we begin with the basic principles underlying its construction. Embedding a QDO to treat long-range forces via a coarse-grained electronic structure and solving the QDO within a strong coupling approach (nonperturbatively) guarantees that interactions of all symmetry are present to all orders albeit within Gaussian statistics. Strong coupling solutions are important in physical models because they are unbiased by any a priori choice of dominant symmetry, order of interaction, or other such considerations. If the underlying model is sufficient, then predictive power across a variety of environments becomes possible. In general there are many configurations (free-energy basins) differing by minor free-energy differences in a complex system, and a strong coupling solution allows the dominant motifs for a given environment to emerge naturally as opposed to giving minor players artificial precedence due to



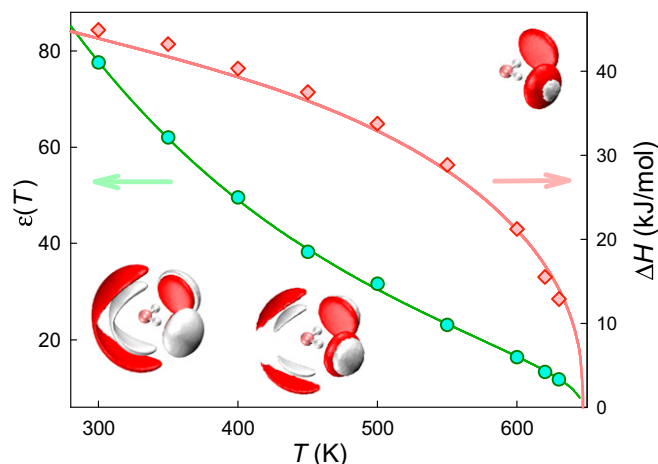


Fig. 4. Water properties along the coexistence line. The lines are the reference IAPWS equation of state (23); the symbols are the results of our simulations with the errors of the order of symbol size. Circles and left axis (indicated by arrows), relative static dielectric permittivity of water along the binodal; diamonds and right axis, enthalpy of vaporization of water. (*Insets*) Disruption of tetrahedral structure with increase of temperature along the binodal ($T = 300, 450,$ and 600 K are shown) is illustrated.

truncation or symmetry selection imposed by the model builder and/or by a perturbative solution. In the absence of such a strong coupling approach, transferability becomes problematic because it is generally not known a priori which terms must be included explicitly.

To illustrate the value of a strong coupling approach, a representation of the evolution of the Drude particle charge distribution in the molecular frame along coexistence is presented in Fig. 2B. Within a dipole polarizable model (which is sometimes referred to as a “polarizable model” in the literature), the induced charge distribution can only exhibit dipole character by construction. The QDO model generates induced multipoles beyond dipole as can be clearly seen by the contours of the distributions and hence richer physics. Water’s quadrupole moment tensor is in fact changed by the environment (by about 10% in principal components magnitude) and the QDO model is capable of capturing this effect (as well as all higher-order induced moments) within the approximation of Gaussian statistics (Fig. 2A).

The QDO model also captures many-body dispersion beyond the dipole approximation and to all orders (two-body, three-body, etc). These many-body terms, more important in the gas phase (high temperatures, low density) where the local environment is more spherical (21) and at surfaces where symmetry is broken, can tip the balance and yield improved model predictions although they themselves are not intrinsically large. Barker demonstrated (17, 32) that it is insufficient to truncate the dispersion series at the pairwise level for noble gases as three-body terms are critical to phase equilibria and surface tension. Recently, the importance of many-body dispersion in aspirin polymorphism has also been revealed (18). By contrast, the radial distribution function in dense systems is not strongly affected, being driven by packing. These higher-order dispersion terms are less critical in ambient water where typical models truncate at the pairwise induced dipole-induced dipole level but, for the transferability across the phase diagram presented here, they matter. In other work (33), we have shown the QDO water model generates the surface tension along coexistence with high accuracy. This, along with all of the other physics (dielectric properties along coexistence and the surface tension predicted by the same model) is difficult to achieve outside a strong coupling approach.

The QDO water model works well without expending enormous effort in its parameterization because the QDO and the strong coupling solution used provide the long-range interactions

(to all orders), thereby allowing the remainder (short-ranged repulsion) to be treated efficaciously within a pairwise approximation. Also, the large delocalized charge distribution of the quantum oscillator removes limitations arising from insufficient out-of-plane density in simple model electrostatics (27). Last, we have taken care to treat accurately the low-order moments (up to quadrupole) of the gas-phase charge distribution as their neglect has been identified as an important issue (27). It should be noted that the QDO model as currently constructed will fail (*i*) when the molecule dissociates (*ii*) at very high pressure, before dissociation, where three-body short-range repulsive terms will need to be added following, for example, the seminal work of Madden and co-workers (34). Rare events relying on molecular flexibility like certain dynamical processes around proteins (35) will also be outside of the predictive capability.

To provide further context for our findings, we note that early (and current) water models (5, 10) typically have fixed charges embedded in a rigid frame, fixed pairwise induced dipole-induced dipole dispersion terms (pairwise dispersion in the dipole approximation), and an oxygen-centered repulsion. Among the most important and challenging tests of model physics is the prediction of liquid-vapor equilibria as a function of temperature and of other physical properties along the coexistence curve. Here, the performance of this class of empirical model shows considerable variation. In general, the liquid-vapor coexistence curve for the most common water models (parameterized for the ambient liquid) severely underestimates the experimental density of the liquid branch with increasing temperature (36). The predictions suggest that either additional cohesive forces not contained in these models become more relevant under nonambient conditions or/and some aspects of the original model parameterization are inappropriate under circumstances where the H-bond network is reconfigured or weakened. More importantly, even the more successful models of this class do not immediately expose the relevant physics required to produce a better description of the balance of interactions governing liquid-vapor coexistence. The addition of polarizability in the dipole limit, for example, does not produce a substantial improvement. As a result, there has been no clear consensus as to the optimum parameterization strategy nor agreement on the extent to which it is reasonable to distort molecular properties of the isolated molecule to improve the description of the liquid under ambient conditions or other state points.

The above discussion suggests that a more complete representation of the electronic structure is required. However,

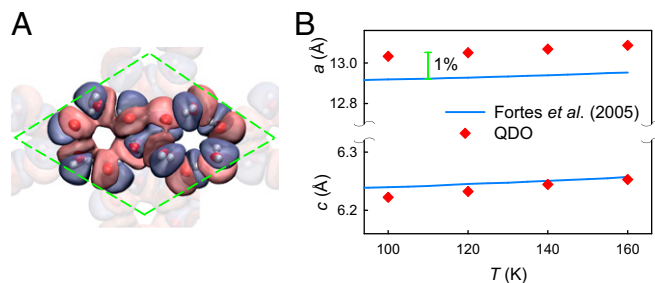


Fig. 5. Simulation of the proton-ordered ice II. (A) Characteristic hexagonal rings in ice II as viewed along the c axis of the hexagonal cell along with the coarse-grained electronic structure generated by our model. Regions of electron density increase are given in pink, whereas regions of depletion are given in blue. The distribution is defined as the difference between the total Drude oscillator density in the system and density of the unperturbed oscillators positioned on the molecular sites. (B) Unit cell dimensions of ice II crystals in the hexagonal settings as a function of temperature. The diamonds are the results of our simulations in the isobaric–isothermal ensemble at $T = 100, 120, 140$, and 160 K; the error bars are smaller than the symbols. The solid lines represent the experimental data obtained from ref. 31 for deuterated ice II.

current ab initio models sufficiently tractable for condensed phase applications neglect dispersion completely or include truncated approximations to the dispersion series. They do not, therefore, generate the balance of forces required to reproduce water's signature properties (37). On the other hand, a fully developed electronic structure incorporated at the level of simplicity afforded by a single embedded quantum oscillator, which can be solved nonperturbatively and efficiently, appears to be sufficient. Together with the model's fixed charge density (approximating the isolated molecule charge distribution), the QDO treatment generates the changing balance of textbook hydrogen-bonding, polar, and van der Waals forces required to account for the key properties of water across its vapor, liquid, and solid phases.

Given the results presented here and that our formulation is readily extensible to other materials and biological systems, we discuss the prospects for the QDO class of electronically coarse-grained models to yield scientific insight into the emergence of complexity from the molecular perspective across the physical and life sciences. We highlight the following general considerations: (i) Models of the type we illustrate here are intuitive and transparent, having properties defined entirely in terms of isolated molecules—unbiased toward any thermodynamic state point or condensed phase. (ii) The resulting interactions are rich, containing the complete hierarchy of many-body inductive and dispersive forces as they are solved in strong coupling. (iii) They are capable of properly generating the balance between hydrogen-bonding, electrostatic, and van der Waals interactions; this, together with point (i), offers a promising foundation for a highly versatile description of matter, linking molecular physics to material properties. (iv) The simplification of the electronic problem inherent in the QDO model permits linear scale sampling methods to be used (38). The formulation can thus be applied with no loss of accuracy to large systems, allowing the molecular-scale exploration of a wide variety of important scientific phenomena at greater length scales of organization including hydrophobic hydration and drying (39), ionic solvation as well as Hofmeister effects, and biological processes such as protein–protein association.

Materials and Methods

The Model and the Simulation Method. The QDO-based multiscale Hamiltonian for a system with N water molecules is

$$\begin{aligned} H &= \sum_i T_i^{(\text{rigid})} + \phi^{(\text{Coul})}(\mathbf{R}) + \sum_{j>i} \phi^{(\text{rep})}(R_{Oij}) + E_0^{(\text{Drude})}(\mathbf{R}), \\ \hat{H}^{(\text{Drude})} \psi_0(\mathbf{r}, \mathbf{R}) &= E_0^{(\text{Drude})}(\mathbf{R}) \psi_0(\mathbf{r}, \mathbf{R}), \\ \hat{H}^{(\text{Drude})} &= \sum_i \left(\hat{T}_i + \frac{m_D \omega_D^2}{2} (\mathbf{r}_i - \mathbf{R}_{ci})^2 \right) + \phi^{(\text{Coul})}(\mathbf{r}, \mathbf{R}). \end{aligned} \quad [1]$$

Here, $T_i^{(\text{rigid})}$ is the classical rigid-body kinetic energy of water molecule i ; $9N$ -dimensional vector \mathbf{R} represents the coordinates of all of the water molecules in the system; $\phi^{(\text{Coul})}(\mathbf{R})$ is the intermolecular Coulomb interaction energy between the fixed monomer gas-phase charge distributions only; $\phi^{(\text{rep})}(R_{Oij})$ is an oxygen-centered pairwise repulsion. The quantities $E_0^{(\text{Drude})}(\mathbf{R})$ and $\psi_0(\mathbf{r}, \mathbf{R})$ are the ground-state Born–Oppenheimer energy surface and wavefunction of the QDO electronic structure, respectively; \mathbf{r}_i and \hat{T}_i are the position and quantum kinetic energy of the oscillator centered on molecule i . The oscillators centered at \mathbf{R}_{ci} are characterized by parameters $\{m_D, \omega_D, q_D\}$. Finally, $\phi^{(\text{Coul})}(\mathbf{r}, \mathbf{R})$ is intermolecular Coulomb interaction between the Drude particles of a negative charge of $-q_D$, at position \mathbf{r} , their centers of oscillation with a positive charge of $+q_D$, at position \mathbf{R}_c , and the fixed monomer charge distribution. Regularization of the Coulomb interaction is discussed elsewhere (20). All model parameters are given in Table 1.

We have treated the water molecules in our multiscale Hamiltonian as classical rigid bodies following seminal early work (10). This is a good approximation because the essential physics underlying water's properties is not altered in going from light water, H_2O , to tritiated water, T_2O . The critical constants, for instance, are not strong functions of isotope mass, T_c shifting 5 °C and the temperature of maximum density by 9 °C from H_2O to T_2O (1)—both the molecular polarizability and dispersion coefficients decrease with increasing

isotope mass, counterbalancing quantum effects. In addition, it is known that there is a cancellation of effects as one goes from a classical rigid-body description of the water molecule to a quantum-mechanical, fully flexible molecular entity (37), hence the wide adoption of the classical rigid-body approach (10). The molecular vibrational degrees of freedom being close to ground state have little entropy as in the rigid model, and classical flexible, classical rigid, and quantum flexible models simulated under the same simple force law show the rigid approximation closely matches the quantum result, whereas the classical flexible model exhibits strong deviations (37, 40).

Classical Drude models are limited to dipole polarization and there are many such models in the literature (41–43) which can provide comparisons for our results. This model class, dipole polarizable models, is more computationally efficient than its quantum analog, but the responses are much more limited. We have derived exact expressions for dispersion coefficients and higher-order polarizabilities in the QDO model—all of which depend on Planck's constant and hence vanish in the classical limit (20). This provides a rigorous justification for the first sentence in this paragraph. The classical limit of the QDO, the classical Drude model, would have no dispersion and could not reproduce the dimer curve of Fig. 1 (without the ad hoc addition of pairwise dispersion, quadrupolar polarization, and higher terms).

The Drude oscillator parameters, $\{m_D, \omega_D, q_D\}$, are chosen such that the long-range responses match reference values of the monomer and dimer, i.e., monomer polarizabilities and pair-dispersion coefficients. The values of three fixed point charges, $\{q_H, q_H, q_O = -2q_H\}$, and the position of center of negative charge placed down the molecular bisector at position \mathbf{R}_M , are selected to represent the isolated molecule electrostatics by matching to low-order electrostatic moments; the positive charges are fixed on the positions of the hydrogen atoms. A QDO is tethered to the point $\mathbf{R}_c = \mathbf{R}_M$ which coincides with the position of M site of the transferable potential function, 4-point (TIP4P) model (21). The values of the Gaussian charge widths are given in Table 1.

Short-range repulsion is incorporated as the difference between a reference high-level quantum-chemical dimer potential energy surface [calculated at the CCSD(T) level using aug-cc-pVTZ basis set using ACESIII, Version 3.0.7 (44)], and the QDO dimer ground energy surface computed using norm-conserving diffusion Monte Carlo for the QDO model (45) with 1,000 walkers. The difference is represented as the isotropic pairwise, oxygen-centered term $\phi^{(\text{rep})}(R_{Oij})$ of Eq. 1 and is approximated in the simulation by (45) $\phi^{(\text{rep})}(\mathbf{r}) = \sum_i \kappa_i \exp(-\lambda_i r)$, with parameters given in Table 1.

Molecular Dynamics Implementation with Adiabatic Path Integrals. Finite-temperature condensed phase simulation of the QDO model in the NVT , NpT , and NpT -flex ensembles were performed using the adiabatic path integral molecular dynamics for QDOs (APIMD-QDO) method (19, 38, 46). The technique uses a separation in time scale between the path integral degrees of freedom representing the Drude oscillators and the molecules, to generate motion of the water molecules on the coarse-grained electronic Born–Oppenheimer surface provided by the Drude oscillators. APIMD-QDO scales as $\mathcal{O}(N)$ in the number of molecules. The path integral was discretized using $P=96$ beads. We typically studied a periodic system of $N=300$ water molecules and used 3D-Ewald summation (47) with vacuum boundary conditions (48) (Ewald parameter $\epsilon_{\text{rf}} = 1$) to compute the electrostatic interactions.

Simulation of Coexistence Densities and Thermodynamic and Dielectric Properties.

To simulate water along the binodal, the coexistence pressure (or density) is required. We used direct simulation of two coexisting phases in a series of NVT calculations of water forming a slab in the central part of an elongated $L_z > \sqrt{L_x, L_y}$ cell (49), as a Gibbs ensemble method for QDOs has yet to be developed. The coexistence pressures obtained were then used in an independent series of NpT simulations of the bulk phase. Resulting densities were then compared with those obtained from hyperbolic tangent fits to the density profile of the slab (49), to assess consistency. The normal component of the stress tensor, calculated using the virial route (50), agrees, within statistical uncertainty, with experimental values. In all cases, the QDO water system was evolved for a simulation time of at least 1 ns.

The dielectric permittivity, ϵ , of bulk water was estimated using the linear response theory expression for a system with electrostatics computed using Ewald summation (47, 48),

$$\frac{(\epsilon - \epsilon_\infty)(2\epsilon_{\text{rf}} + 1)^2}{(2\epsilon_{\text{rf}} + \epsilon)(2\epsilon_{\text{rf}} + \epsilon_\infty)} = \frac{\langle \bar{\mathbf{M}}_n^2 \rangle - \langle \bar{\mathbf{M}}_n \rangle^2}{3\epsilon_0 V k_B T}. \quad [2]$$

Here, $\bar{\mathbf{M}}_n$ is the total dipole moment of the sample for a given set of molecular positions (see below), ϵ_{rf} is the permittivity of the medium at the asymptotic surface (37, 48), $V = L_x L_y L_z$ is the system volume, k_B is the Boltzmann constant, T is the thermostat temperature, ϵ_0 is the electric

constant, and the angle brackets represent the canonical ensemble average of the molecules on the ground-state energy surface provided by the quantum Drude oscillators at temperature T (performed statistically over at least 1 ns of simulation time). We evaluated the high-frequency limit ϵ_∞ using equation 24 of ref. 48 appropriately modified for our model. In the above calculations of the dielectric permittivity, performed in the NVT ensemble under 3D periodic boundary conditions, we also used $\epsilon_{\text{rf}} = 1$, taking the densities and temperatures from the coexistence simulations.

Within the APIMD-QDO method, we do not have access to the QDO properties for each instantaneous nuclear configuration; to obtain this information the path integral degrees of freedom would have to be formally integrated (or averaged) out. To perform the required average to good approximation on the total dipole moment operator of Eq. 2, the staging dipole moment estimator of ref. 19 is subaveraged over a time interval, $\tau = n\delta t$, where $\delta t = 0.125$ fs is the APIMD-QDO time step and the subaverage is indicated by the notation $\overline{\text{M}}$ in Eq. 2. As the adiabatic separation (how fast the Drude path integral degrees of freedom evolve compared with the physical atoms) is selected such that the time scale of nuclear motion encompasses many correlation times of the path integral sampling, the approach converges rapidly with n , allowing $n = 20$ corresponding to $\tau = 2.5$ fs to be used; the averaging time scale is thus small compared with the Debye relation time of water (ca. 10 ps).

The enthalpy of vaporization, ΔH_{vap} , was calculated in the NpT ensemble using the densities and temperatures from the coexistence simulation. Because we assumed rigid monomer geometry in the QDO model, a temperature-dependent correction term for intra- and intermolecular vibrations (16) has been added.

Simulation of Ice II. To calculate the unit cell dimensions and density of ice II, we simulated a supercell of $N = 324$ water molecules at ambient pressure ($p = 0.1$ MPa) and temperatures ranging from $T = 100$ to 160 K, in steps of 20 K, using periodic boundary conditions and a fully flexible unit cell (NpT -flex ensemble). The supercell was equilibrated for a simulation time of 20 ps and the values reported in Fig. 3B are the result of averaging over a further 100 ps of simulation time. We used vacuum boundary conditions in Ewald summation of electrostatic forces ($\epsilon_{\text{rf}} = 1$), which resulted in minimal finite size effects.

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