Cheat sheet for linewidths

8 août 2022

1 Fourier transform of exponential decays

1.1 Fourier transform convention for cyclical frequencies

Forward convention (going from t to f) :

$$H(f) = \int_{-\infty}^{+\infty} h(t) e^{-2i\pi f t} dt \tag{1}$$

Reverse convention (going from f to t) :

$$h(t) = \int_{-\infty}^{+\infty} H(f) e^{+2i\pi f t} df \tag{2}$$

1.2 Fourier transform convention for angular frequencies

Forward convention (going from t to ω) :

$$H(\omega) = \int_{-\infty}^{+\infty} h(t) e^{-i\omega t} dt \tag{3}$$

Reverse convention (going from f to t) :

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\omega) e^{+i\omega t} d\omega \tag{4}$$

1.3 Single exponential decay (Lorentzian)

We assume $h(t) \propto e^{-t/T_2}$ for $t > 0$. To compute the Fourier transform we need to assume that $h(t) \propto e^{-|t|/T_2}$.

$$H(f) \propto \frac{1}{1 + (2\pi f T_2)^2} \tag{5}$$

1.4 Squared exponential decay (Gaussian)

We assume $h(t) \propto e^{-(t/T_2)^2}$

$$H(f) \propto e^{-(\pi f T_2)^2} \tag{6}$$

2 HWHM and T_2 in the f basis

2.1 Gaussian

For a Gaussian $e^{\frac{(x-\mu)^2}{2\sigma^2}}$, HWHM=$\sigma\sqrt{2\ln 2} \approx 1.18\sigma$.

$$T_2 = \frac{1}{\pi\sigma\sqrt{2}} = \frac{\sqrt{\ln 2}}{\pi(\text{HWHM})} \tag{7}$$

2.2 Lorentzian

For a Lorentzian $\frac{1}{1+(\frac{(x-\mu)^2}{\sigma})^2}$, HWHM=$\sigma$

$$T_2 = \frac{1}{2\pi(\text{HWHM})} \tag{8}$$