Self-spin cooling the librational mode of a levitating diamond

Previous work considered the strong coupling regime where the trapping frequency is larger than the decoherence rate of a single atom. Here, we consider the intermediate, weak-coupling regime where the microwave controls the spin-dependent force, using many spins. We show, using linear response theory, that spin-cooling can be observed using current technologies.

Explanation.

I. SUSCEPTIBILITY OF THE DIAMOND LIBRATIONAL MODE IN A PAUL TRAP

The equation of motion for the librational mode $\theta(t)$ is

$$I\ddot{\theta} \;=\; I\omega_\theta^2\theta - I\gamma\dot{\theta} + \Gamma_T \tag{1}$$

where I is the moment of inertia, ω_θ is the angular frequency for that mode, γ is the damping rate due to collisions with the background gaz and $\Gamma_T(t)$ is the associated Langevin torque.

Fourier Transforming this equation yields

$$\theta(\omega) = \chi(\omega)\Gamma_T(\omega)$$

where

$$\chi(\omega) = \frac{1}{I(\omega^2 - \omega_\theta^2 + i\omega\gamma)}$$

The Langevin torque obeys the relation

$$\langle \Gamma_T(\omega)\Gamma_T(\omega')\rangle = 2\pi\delta(\omega+\omega')S_T(\omega)$$

where

$$S_T(\omega) = -\frac{2kT}{\omega}\mathrm{Im}(\frac{1}{\chi(\omega)})$$

is the Langevin torque spectrum. Using the susceptibility for the librational mode, one finds

$$S_T(\omega) = 2kT\gamma I$$

.

The librational spectrum is then be found to be

$$\begin{aligned} S_\theta(\omega) \;&=\; |\chi(\omega)|^2 S_T(\omega) && (2)\\ &=\; \frac{2\gamma kT}{I((\omega_\theta^2-\omega^2)^2+\gamma^2\omega^2)} && (3)\end{aligned}$$

II. SPIN TORQUE

The spin torque is maximal when the magnetic field and NV center form an angle $\pi/4$.

A. Diagonalisation with a general B field angle

We are interested in the strongest possible variation of the splittings with angle, so we look for changes when the diamond rotates with a rotation axis perpendicular to the B-field.

The NV hamiltonian reads

$$\hat{H}_{\mathrm{NV}} = D\hat{S}_z^2 + \gamma_e B(S_x\cos(\theta) + S_z\sin(\theta)) \tag{4}$$

We suppose that $\gamma B \ll D$ so that $H_B = \gamma_e B(S_x\cos(\theta) + S_z\sin(\theta))$ can be treated as a perturbation. The eigenstates of $D\hat{S}_z^2$ are $|\pm1\rangle$, which are degenerate, and $|0\rangle$.

The perturbed energy ϵ_0 of $|0\rangle$ due to the transverse B field is

$$\epsilon_0 = \sum_{m_s=\pm1}\frac{|\langle 0|H_B|\pm1\rangle|^2}{-\epsilon^0_{\pm1}} = -\frac{(\gamma_e B)^2}{D}\cos^2\theta$$

The perturbed energy $\epsilon_{\pm1}$ of $|\pm1\rangle$ due to the transverse and longitudinal parts of the B field can be found by solving

$$(\epsilon_1 - D)\alpha_1 = \sum_{m_s=\pm1}\Big[\langle 1|H_B|\pm1\rangle + \frac{\langle 1|H_B|0\rangle\langle 0|H_B|\pm1\rangle}{\epsilon^0_{\pm1}}\Big]\alpha_{\pm1}$$

$$(\epsilon_{-1} - D)\alpha_{-1} = \sum_{m_s=\pm1}\Big[\langle -1|H_B|\pm1\rangle + \frac{\langle -1|H_B|0\rangle\langle 0|H_B|\pm1\rangle}{\epsilon^0_{\pm1}}\Big]$$

which comes down to finding the determinant of the two by two matrix

$$M = \begin{pmatrix}\Delta\epsilon - \alpha - \beta & \beta\\ \beta & \Delta\epsilon + \alpha - \beta\end{pmatrix} \tag{5}$$

where $\alpha = \gamma_e B\sin\theta$
$\beta = \frac{(\gamma_e B)^2}{2D}\cos^2\theta$ and $\Delta\epsilon = \epsilon - D$.
One finds

$$\Delta\epsilon_{\pm1} = \beta \pm \sqrt{\alpha^2+\beta^2}$$

.

We wish to find the sweet spot where the level splitting varies the most with angle. The greatest variation can be found between the ground and lowest excited state where

$$\Delta = 3\beta - \sqrt{\alpha^2+\beta^2}$$

For simplicity, we approximate the maximum slope to at around $\pi/4$. Using $\cos(\pi/4+\delta) = \sqrt{2}/2(\cos\delta - \sin\delta) \approx 1/\sqrt{2}(1-\delta)$, for instance, we get

$$\Delta = A + C\delta$$

where

$$A = 3/4(\gamma_e B)^2/D - \gamma_e B/\sqrt{2}$$

$$C = \frac{\gamma_e B}{\sqrt{2}}(1 - \frac{3\gamma_e B}{\sqrt{2}D})$$

That is $\frac{\gamma_e B}{\sqrt{2}}$

B. The associated NV center spin frequency shift

In the limit where the B field is weaker than the the zero field splitting D we get

$$\Delta = \omega - 2\pi D - \frac{2\pi\gamma_e^2}{D}B^2.$$

Due to an intersystem crossing in the excited state of the NV centers, the state $|0\rangle$ is brighter then the state $|1\rangle$ upon green laser optical illumination. This means that the state $|g\rangle$ will be brighter than the state $|d\rangle$ (since it "contains more $|0\rangle$", albeit with a reduced ESR contrast). This provides a means to read out the Zeeman splitting by scanning the microwave around the resonance (this is the essence of Electron spin resonance, ESR)

To measure the rotation, the microwave frequency is set so that $\Delta \approx 0$.

If the magnet rotates about the axis y a time dependent frequency shift will be seen by the NV and observed on the ESR.

$$\Delta(t) = \gamma_e B\theta(t)$$

.

III. QUASI-STEADY STATE DYNAMICS

A. Rotating a levitating diamond using NV spins

A diamond with main principal axis I is levitating with a Paul trap induced rotational angular frequency ω_θ. The angular motional Hamiltonian takes the form

$$H_{\text{meca}} = \frac{1}{2}I\omega_\theta^2\hat{\theta}^2 \tag{6}$$

The angular position of the particule in the trap can be shifted due to a torque induced by spins in the presence of a magnetic field. This will in turn shift the ESR peaks. The NV hamiltonian reads

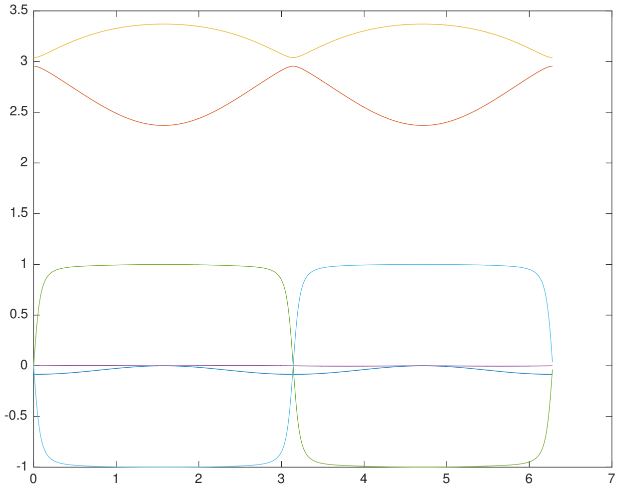

FIG. 1: Splitting of the dressed levels (in GHz) as a function of angle, in radian for $\gamma B = 1$ GHz.

$$\hat{H}_{\text{NV}} = \hbar D\hat{S}_z^2 + \hbar N\gamma B(S_x\cos\theta + S_z\sin\theta) \tag{7}$$

We wish to find the spot for the microwave pump where the level splitting varies the most with angle. Looking at Fig. 1, the greatest variation can be found between the ground and lowest excited state. The first spot is close to $\theta = \pi/4$, where the energy is linear in θ and the slope is around γB. The eigen-energy is changed to

$$E = \frac{1}{2}I\omega_\theta^2\theta^2 + \hbar N\gamma B\theta + Cte$$

The center of the angular confinement is thus shifted by

$$\delta\theta = \frac{\hbar N\gamma B}{I\omega_\theta^2}$$

This angular shift can be measured using another NV orientation. For such a read-out, using the correspondance $\delta\nu = \gamma B\delta\theta$ (if the read orientation is close in frequency to the pumped one) we get

$$\delta\nu = \frac{\hbar N(\gamma B)^2}{I\omega_\theta^2}$$

The experiment could be done in CW by comparing the angular shift when the microwave pump is tuned in and out of the transition. In the pulsed regime it would take too much time ($2\pi/\omega_\theta \approx 5$ ms) for the particle to turn after changing the spin state compared to the coherence time.

Including the imperfect polarisation $p_0 \approx 80\%$ in the ground state. The end formula for the shift on the probed transition is

$$\delta\nu = \frac{\hbar N p_0 (\gamma B)^2}{I\omega_\theta^2}.$$

We need a particle that is larger than 2 μm in diameter, for stable angular detection, easy injection of single particles, insensitivity to external forces. We also need as much NVs as possible on a single monocrystal, be it attached to another particle or not.

B. Spin torque

Close to $\pi/4$, the spin torque is thus $\Gamma_{\mathrm{mag}} = \hbar\gamma B S_z$. S_z also depends on θ since when the diamond rotates, the B field projection changes. The microwave can for instance be out of resonance which will change the total magnetization. This mechanism is analogous to the radiation pressure force onto mirrors in a cavity, which depends upon the mirror position. This implies that the effects that are observed in the detuned cavity regime : self-cooling, Kerr non-linearities, bistabilities can also be observed in this system.

C. Bistability

If the scan is performed on times scales of the spin re-polarisarion rate, then bistable behaviour can be observed. The change in magnetisation is now mostly due to the fact that the microwave is scanned at a rate that is on the order of the laser induced depolarisation.

Equation IV A, together with the Bloch equations for a two level system can be used to predict this effect.

Taking a reference phase so that the microwave amplitude is a real number, the Bloch equations read

$$\frac{\partial S_z}{\partial t} = \left(\frac{1}{T_2^*} + i(\Delta + \gamma_e B\theta)\right)S + i\Omega(2S_z - 1) \quad (8)$$

$$\frac{\partial S}{\partial t} = -\gamma_{\mathrm{las}} S_z + i\Omega(S - S^*) \quad (9)$$

Figure X show the result of numerical simulations where the microwave frequency is scanned at a rate γ_{las}

Instabilities ?

IV. SPIN COOLING

A. Cooling mechanism

The magnetic torque from the NV spins at a position $\pi/4$ is given by

$$\Gamma_{\mathrm{mag}} = \hbar\gamma_e B S_z$$

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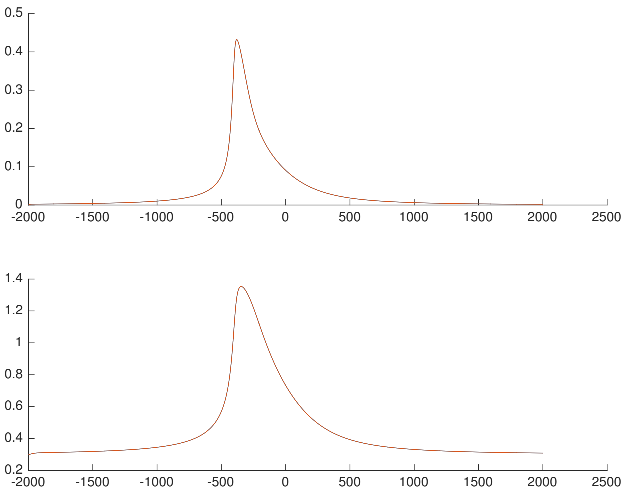

FIG. 2: ESR lineshapes in the presence of bistabilities

This torque modifies the equations of motion IV A for θ, which are now

$$I\ddot{\theta} = I\omega_\theta^2\theta - I\gamma\dot{\theta} + \Gamma_T + \Gamma_{\mathrm{mag}} \quad (10)$$

Following Arcizet al. we can now write :

$$\theta(\omega) = \chi(\omega)(\Gamma_T(\omega) + \Gamma_{\mathrm{mag}}(\omega)) \quad (11)$$

One can also introduce an effective susceptibility χ_{eff} such that

$$\theta(\omega) = \chi_{\mathrm{eff}}\Gamma_T(\omega)$$

In order to find χ_{eff}, one needs a relationship between the angle and the spin population. To do this, we will decompose the spin and angles as a mean value and a fluctuating component. Note that doing this is valid for Gaussian variables. $S_z = \overline{S_z} + \delta S_z$ and $\theta = \overline{\theta} + \delta\theta$. Linearizing the Bloch equations will then allow to get a linear relationship between δS_z and $\delta\theta$

$$\delta S_z = \xi(\omega)\delta\theta$$

which we will derive $\xi(\omega)$ in the next section. Injecting this relation back into Eq. (11), we get

$$\delta\theta = \chi(\omega)\delta\Gamma_T + \chi\hbar\gamma_e B N\xi(\omega)\delta\theta$$

. Self-referenced relationship : feedback.

The power spectral density of the librational motion θ can thus be found to be

$$S_\theta(\omega) = \left|\frac{\chi(\omega)}{1 - \hbar\gamma_e B N\xi\chi(\omega)}\right|^2 S_T(\omega)$$

It is the retarded action of the spin onto the angle that provides the cooling/heating mechanism. It depends on the sign of the spin torque, that is, on the sign of ξ.