$$\boxed{\begin{array}{c}\text{Cross polarisation NV-NV} \\ \text{à champ mag. faible.}\end{array}}$$

Lukin's paper (choi et al.), SM, cf {Kucsko et al. SM}

$$H_{ij}^{dd} = -\frac{J_0}{r^3}\Big[g_{ij}^{+}\left(S_i^x S_j^x + S_i^y S_j^y\right)$$

$$+ h_{ij}^{-}\left(S_i^x S_j^y - S_i^y S_j^x\right) + q_{ij}\, S_i^z S_j^z$$

$$+ g_{ij}^{-}\left(S_i^x S_j^x - S_i^y S_j^y\right)$$

$$+ h_{ij}^{+}\left(S_i^x S_j^y + S_i^y S_j^x\right)\Big]$$

$$\begin{cases} S^x = \frac{1}{\sqrt{2}}\left(|-1\rangle\langle 0| + |0\rangle\langle 1| + h.c\right) = S^+ + S^- \\ S^y = \frac{-i}{\sqrt{2}}\left(|-1\rangle\langle 0| + |0\rangle\langle 1| - h.c\right) = -i\left(S^+ - S^-\right) \end{cases}$$

- $S_i^x S_j^x + S_i^y S_j^y = (S_i^+ + S_i^-)(S_j^+ + S_j^-) - (S_i^+ - S_i^-)(S_j^+ - S_j^-)$
$$= 2S_i^+ S_j^- + 2S_i^- S_j^+$$
$$= \frac{2}{2}\left[|-1\rangle\langle 0|_i + |0\rangle\langle 1|_i\right] \otimes \left[|0\rangle\langle -1|_j + |1\rangle\langle 0|_j\right] + \left[|0\rangle\langle -1|_j + |1\rangle\langle 0|_j\right] \otimes \left[|-1\rangle\langle 0|_j + |0\rangle\langle 1|_j\right]$$
$$= |-1\,0\rangle\langle 0\,-1| + |0\,1\rangle\langle 1\,0| + \underbrace{\varepsilon}_{|0\,0\rangle\langle 1\,-1|} + h.c$$

- $S_i^x S_j^x - S_i^y S_j^y = 2\left(S_i^+ S_j^+ + S_i^- S_j^-\right)$
$$= \left(|-1\rangle\langle 0|_i + |0\rangle\langle 1|_i\right) \otimes \left(|-1\rangle\langle 0|_j + |0\rangle\langle 1|_j\right)$$
$$= \underbrace{|-1\,0\rangle\langle 0\,1| + |0\,-1\rangle\langle 1\,0| + h.c}_{+h.c\ +\varepsilon}$$

- $\cancel{S_i^x S_j^y - S_i^y S_j^x = (|-1\rangle\langle 0| + |0\rangle\langle 1|) \otimes (|-1\rangle\langle 0| + |0\rangle\langle 1|)}$

$$S_i^x S_j^y - S_i^y S_j^x = -i(S_i^+ + S_i^-)(S_j^+ - S_j^-) + i(S_i^+ - S_i^-)(S_j^+ + S_j^-)$$
$$= +2i\, S_i^+ S_j^- - 2i\, S_i^- S_j^+$$
$$= \frac{2i}{2}\left(|-1\rangle\langle 0|_i + |0\rangle\langle 1|_i\right) \otimes \left(|0\rangle\langle -1|_j + |1\rangle\langle 0|_j\right) + h.c$$
$$= i\left(|-1 0\rangle\langle 0 -1| + |01\rangle\langle 10|\right) + h.c.$$

$$S_i^x S_j^y + S_i^y S_j^x = -2i\, S_i^+ S_j^+ + 2i\, S_i^- S_j^-$$
$$= -\frac{2i}{2}\left(|-1\rangle\langle 0|_i + |0\rangle\langle 1|_i\right) \otimes \left(|-1\rangle\langle 0|_j + |0\rangle\langle 1|_j\right) + h.c$$
$$= -i\left(|-1 0\rangle\langle 0 1| + |0 -1\rangle\langle 1 0|\right) + h.c$$

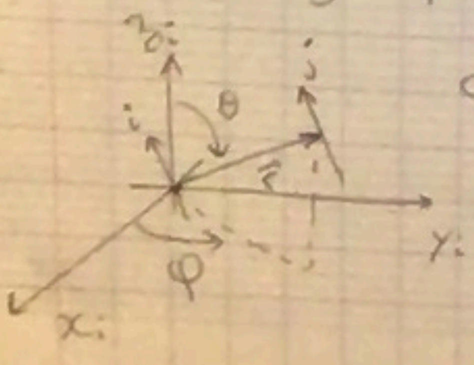
$$\Rightarrow H_{ij}^{dd} = -\frac{J_0}{r^3}\left[(g_{ij}^+ + i h_{ij}^+)\left(|-1 0\rangle\langle 0 -1| + |01\rangle\langle 10| + h.c.\right)\right]$$
$$- \frac{J_0}{r^3}\left[(g_{ij}^- - i h_{ij}^+)\left(|-1 0\rangle\langle 0 1| + |0 -1\rangle\langle 10| + h.c\right)\right]$$
$$- \frac{J_0}{r^3} g_{ij} S_i^z S_j^z$$

dans la base i

$\boxed{\text{Scalars :}}$ $\vec{r} = (\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta)$

$$\to g_{ij}^+ = \frac{1}{2}\left[3(\hat{r}\cdot\hat{x}_i)(\hat{r}\cdot\hat{x}_j) - \hat{x}_i\cdot\hat{x}_j + 3(\hat{r}\cdot\hat{y}_i)(\hat{r}\cdot\hat{y}_j) - \hat{y}_i\cdot\hat{y}_j\right]$$

⊛ Même groupe ($x_i = x_j \ldots$)



$$g_{ij}^+ = \frac{1}{2}\left[3\cos^2\varphi \sin^2\theta + 3\sin^2\varphi \sin^2\theta - 2\right]$$
$$= \frac{1}{2}\left[3\sin^2\theta - 2\right] = \frac{1}{2}\left[3 - 2 - 3\cos^2\theta\right]$$
$$= \frac{1}{2}\left[1 - 3\cos^2\theta\right]$$

$$\rightarrow h_{ij}^{-} = \frac{1}{2}\left[3(r\cdot x_i)(r\cdot y_j) - x_i y_j - 3(r\cdot y_i)(r\cdot x_j) + y_i x_j\right]$$

* Même groupe $= 0$

$$\rightarrow g_{ij}^{-} = \frac{1}{2}\left[3(r\cdot x_i)(r\cdot x_j) - x_i x_j - 3(r\cdot y_i)(r\cdot y_j) + y_i\cdot y_j\right]$$

* Même groupe :

$$= \frac{1}{2}\left[3\cos^2\varphi\sin^2\theta - 3\sin^2\varphi\sin^2\theta\right]$$

$$= \frac{3}{2}\sin^2\theta\cos 2\varphi$$

$$\rightarrow h_{ij}^{+} = \frac{1}{2}\left[3(r\cdot x_i)(r\cdot y_j) - x_i\cdot y_j + 3(r\cdot y_i)(r\cdot x_j) - y_i\cdot x_j\right]$$

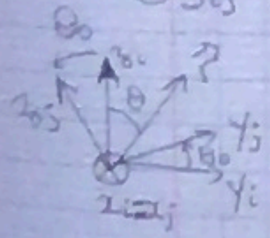
$\hat{m}$ groupe

$$\hookrightarrow = \frac{1}{2}\left[3\times 2(\cos\varphi\sin\theta)(\sin\varphi\sin\theta) - 0\right]$$

$$= 3\times 2\sin 2\varphi\sin^2\theta$$

---

* g_{ij}^{+} pour 2 groupes de NV différents $\boxed{\vec{y_j} = \cos\theta_0\,\vec{y_i} + \sin\theta_0\,\vec{z_i}}$

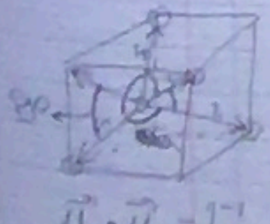


$$= \frac{1}{2}\left[3(\cos\varphi\sin\theta)(\cos\varphi\sin(\theta\ldots)) - 1 + 3(\sin\varphi\sin\theta)\,\cancel{\sin\varphi\sin(\theta+\theta_0)} + \cos\theta_0\right]$$

$\hookrightarrow \times (\sin\varphi\sin\theta\cos\theta_0 + \cos\theta\sin\theta_0)$

$-\cos\theta_0$

$$\cancel{= \frac{1}{2}\left[3\sin\theta\sin(\theta+\theta_0) - 1 - \cos\theta_0\right]}$$



* $h_{ij}^{-} = \cancel{\ldots\sin\theta\sin\varphi\ldots\times 0}$?

$$\frac{1}{2}\left[3\cos\varphi\sin\theta\left[\sin\varphi\sin\theta\cos\theta_0 + \cos\theta\sin\theta_0\right] - 3\cos\varphi\sin\theta\left[\sin\varphi\sin\theta\right]\right]$$

$$\vec{u}_1\cdot\vec{u}_2 = \begin{vmatrix}-1\\-1\\-1\end{vmatrix}\cdot\begin{vmatrix}1\\1\\-1\end{vmatrix} = -1 = 3\cos\theta_0 \Rightarrow \theta_0 = a\cos\left(-\frac{1}{3}\right)$$

$\hookrightarrow \pi -$

$$= \frac{1}{2}\left[3\,\frac{1}{2}\sin 2\varphi\sin^2\theta\cos\theta_0 + 3\,\frac{1}{2}\cos\varphi\sin 2\theta\sin\theta_0 - 3\,\frac{1}{2}\sin 2\varphi\sin^2\theta\right]$$

$$= \frac{1}{2}\left[\frac{3}{2}\sin 2\varphi\sin^2\theta(\cos\theta_0 - 1) + \frac{3}{2}\cos\varphi\ldots\right]$$

$\cos\theta)$

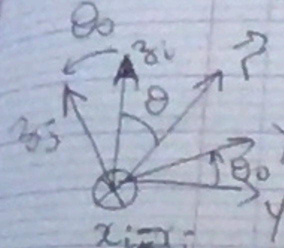
$y_j]$

$\sin^2\theta$

$\cos^2\theta]$

$$= 3 \times 2 \sin 2\varphi \sin^2\theta$$

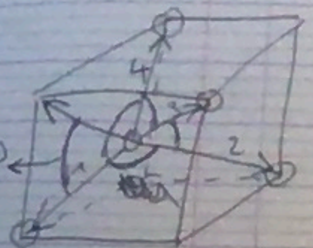
(*) g_{ij}^{+} pour 2 groupes de NV differents $\boxed{\vec{y_j} = \cos\theta_0 \vec{y_i} + \sin\theta_0 \vec{z_i}}$



$$= \frac{1}{2}\Big[3(\cos\varphi\sin\theta)(\cos\varphi\sin(\theta\ldots)) - 1$$
$$+3(\sin\varphi\sin\theta)\ldots$$
$$\hookrightarrow \times (\sin\varphi\sin\theta\cos\theta_0 + \cos\theta\sin\theta_0)$$

$$\cancel{= \frac{1}{2}\Big[3\sin\theta\sin(\theta+\theta_0) - 1 - \cos\theta_0\Big]}$$

$-\cos\theta_0$



θ^0

(*) $h_{ij}^{-} =$

$$\frac{1}{2}\Big[3\cos\varphi\sin\theta[\sin\varphi\sin\theta\cos\theta_0 + \cos\theta\sin\theta_0]$$
$$-3\cos\varphi\sin\theta[\sin\varphi\sin\theta]\Big]$$

$$\vec{u_1}\cdot\vec{u_2} = \begin{vmatrix}-1\\-1\\-1\end{vmatrix} \cdot \begin{vmatrix}1\\1\\-1\end{vmatrix} = -1$$
$$= 3\cos\theta_0 \quad\Big] \Rightarrow \theta_0 = \operatorname{acos}\left(-\frac{1}{3}\right)$$
$$\hookrightarrow \pi -$$

$$= \frac{1}{2}\Big[3\,\frac{1}{2}\sin 2\varphi\sin^2\theta\cos\theta_0 + 3\,\frac{1}{2}\cos\varphi\sin 2\theta\sin\theta_0$$
$$-3\,\frac{1}{2}\sin 2\varphi\sin^2\theta\Big]$$

$$= \frac{1}{2}\Big[\frac{3}{2}\sin 2\varphi\sin^2\theta(\cos\theta_0 - 1) + \frac{3}{2}\cos\varphi\sin 2\theta\sin\theta_0\Big]$$

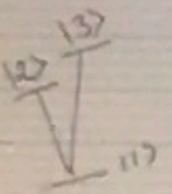
niveaux δ des spins des fluct.

▶ $\Gamma_{ij} = 2 \sum_{\alpha\beta} |C^{ij}_{\alpha\beta}|^2 \, \mathrm{Re}\left[S^{\alpha\beta}(\omega_{ij}) \right]$

⚠ niveaux des spins $i,j \in \{1,2,3\}$

avec $S^{\alpha\beta}(\omega) = \begin{cases} \frac{1}{3} \frac{1}{i(\omega + \omega_{\alpha\beta}) - 2\gamma_f} \; ; \; \alpha \neq \beta \\ \frac{1}{9}\left(\frac{1}{i\omega} + \frac{2}{i\omega - 3\gamma_f} \right) ; \; \alpha = \beta \end{cases}$

et $C^{ij}_{\alpha\beta} = \langle i\alpha | H^{dd} | j\beta \rangle$



spin "normal"
fluctuateur

$$H^{dd} = H_{11} + H_{22} + H_{33} + H_{44} \quad \text{(same)}$$
$$+ H_{12} + H_{13} + \dots \quad \text{(diff)}$$
$$+ H_{21} + H_{23} + \dots$$
$$+ H_{31} + H_{32} + \dots = \sum_{(m,n)>0}^{4} H_{mn}$$

La plupart ~~des H_{mn}~~ disparaissent en champ mag fort (non degenerescence $\rightarrow$ faible cross pol)

▶ Calcul des H_{mn} : Pour Γ_{21} (i.e $\Gamma_{|1\rangle,|0\rangle}$)

$$C^{21}_{\alpha\beta} = \langle 2\alpha | H^{dd} | 1\beta \rangle$$

$$= -\frac{J_0}{r^3}(g^+ + ih^-) \langle 2\alpha | \left(|-1\rangle\langle 0-1| + |01\rangle\langle 01| + h.c. \right) |1\beta\rangle$$

$|0\rangle$

$$- \frac{J_0}{r^3}(g^- - ih^+) \langle 2\alpha | \left(|-10\rangle\langle 01| + |0-1\rangle\langle 01| + h.c. \right) |1\beta\rangle$$

$|0\rangle$

~~$\langle 2\alpha | H$~~

1er terme : $\langle -1\alpha | \left(|-10\rangle\langle 0-1| \right) |0\beta\rangle \rightarrow \delta_{\alpha 0}\delta_{\beta,-1}$
$+ \langle -1\alpha | \left(|01\rangle\langle 01| \right) |0\beta\rangle \rightarrow 0$
$+ \langle -1\alpha | \left(|0-1\rangle\langle -10| \right) |0\beta\rangle \rightarrow 0$
$+ \langle -1\alpha | \left(|10\rangle\langle 01| \right) |0\beta\rangle \rightarrow 0$

2^{nd} terme : $\langle -1\alpha|(|-10\rangle\langle 011|)|0\beta\rangle \to \delta_{\alpha 0}\delta_{\beta 1}$
$+\langle -1\alpha|(|0-1\rangle\langle 10|)|0\beta\rangle \to 0$
$+\langle -1,\alpha|(|01\rangle\langle -10|)|0\beta\rangle \to 0$
$+\langle -1\alpha|(|10\rangle\langle 0-1|)|0\beta\rangle \to 0$

$$C^{21}_{\alpha\beta} = -\frac{J_0}{r^3}\left[g^+\delta_{\alpha 0}\delta_{\beta -1} + (g^- - ih^+)\delta_{\alpha 0}\delta_{\beta 1}\right]$$

$$* \quad Re\left[S^{\alpha\beta}(\omega)\right] = \frac{1}{3}\frac{2\gamma_f}{(\omega+\omega_{\alpha\beta})^2+(2\gamma_f)^2} \qquad \text{car } \alpha \neq \beta$$

$$\Rightarrow \Gamma_{21} = \left(\frac{J_0^2}{r^3}\right)^2 \frac{\eta^2}{\gamma_f}$$

avec $\eta^2 = \frac{2}{3}\left[|g^+|^2\frac{2\gamma_f^2}{(\omega_{-1,0}+\omega^f_{0,-1})^2+4(\gamma_f)^2} + (|g^-|^2+|h^+|^2)\frac{2\gamma_f^2}{(\omega_{-1,0}+\omega^f_{01})^2+(2\gamma_f)^2}\right]$

① En présence d'1 fort champ mag. $\omega_{-10}+\omega^f_{01} \gg \gamma_f$ et donc

$$\eta^2 = \frac{2}{3}|g^+|^2\frac{2\gamma_f^2}{(4\gamma_f)^2} \quad \text{(négligeant l'élarg. inhom.)}$$

moyenne sur les orientations de η

$$\int Prob^{same}(\eta)\,\eta\, d\eta = \sqrt{\frac{2}{3}}\left[\frac{1}{4\pi}\iint_{\theta,\varphi}\sin\theta\, d\theta\, d\varphi\, |g^+|\right]\frac{1}{\sqrt{2}}$$

Numeriquement $\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta\left|\frac{1}{2}(1-3\cos^2\theta)\right|d\theta$

$= 0.385$

$\simeq \frac{2}{3\sqrt{3}}$ (choix et a! SSS!)

② En champ nul

$$\eta = \frac{1}{\sqrt{3}}\left(|g^+|^2 + |g^-|^2 + |h^+|^2\right)^{\frac{1}{2}}$$

$$\iint_{\theta\varphi} \left[|g^+|^2 + |g^-|^2 + |h^+|^2\right]^{1/2} = 2{,}6778$$

③ Γ_{12} $(\Gamma_{|0\rangle|-1\rangle})$

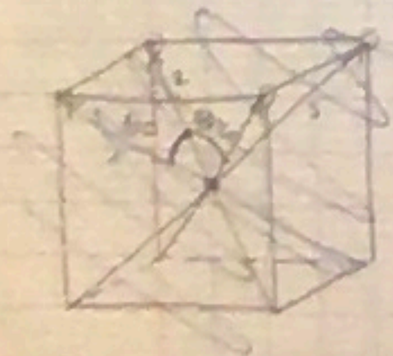
$$C_{\alpha\beta}^{|0\rangle|-1\rangle} = g^+ \delta_{\alpha 0}\delta_{\beta -1} + (g^- - ih^+)\delta_{\alpha 1}\delta_{\beta 0}$$

$$\Rightarrow \eta^2 = \frac{2}{3}\left[|g^+|^2 \frac{2\gamma_f}{(\omega_{0-1} + \omega_{-10}^f)^2 + 4\gamma_f^2} + (|g^-|^2 + |h^+|^2)\frac{2\gamma_f}{(\omega_{0-1} + \omega_{10}^f)^2 + 4\gamma_f^2}\right]$$

$$\Rightarrow \Gamma_{12} = \Gamma_{21}$$

et $\Gamma_{|0\rangle|+1\rangle} = \Gamma_{|0\rangle|-1\rangle}$

- Calcul des H_{mn}



① $\boxed{H_{12}}$

pour $\gamma_{|0\rangle|-1\rangle}$

$$\eta^2 = \frac{2}{3}\left[(|g^+|^2 + |h^-|^2)\frac{2\gamma_f}{(\omega_{0-1} + \omega_{-10}^f)^2 + 4\gamma_f^2} + (|g^-|^2 + |h^+|^2)\frac{2\gamma_f}{(\omega_{0-1} + \omega_{10}^f)^2 + 4\gamma_f^2}\right]$$

① En champ fort

$$\eta = \sqrt{\frac{2}{3}}\underbrace{\left(|g^+|^2+|h^-|^2\right)^{\frac{1}{2}}}_{0.6507\ (\theta=109°)}\left[\frac{2\gamma f^2}{(\omega_{0-1}+\omega_{-1,0}^{f})^2+4\gamma f^2}\right]^{\frac{1}{2}}$$

② En champ nul :

$$\eta^{diff} = \sqrt{\frac{1}{3}}\left(|g^+_1|^2+|h^-_1|^2+|g^-_1|^2+|h^+_1|^2\right)^{\frac{1}{2}}$$

③ En general

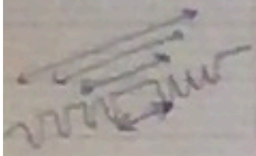
$$\overline{\eta}^{diff} = \sqrt{\frac{2}{3}}\iint_{\theta\varphi}\left[(|g^+|^2+|h^-|^2)\mathcal{L}(\Delta_{11})+(|g^-|^2+|h^+|^2)\mathcal{L}(\Delta_{-1})\right]^{\frac{1}{2}}$$

$$\overline{\eta}^{same} = \sqrt{\frac{2}{3}}\iint\left[|g^+|^2\ \mathcal{L}(\Delta_{11})+|g^-|^2+|h^+|^2\mathcal{L}(\Delta_{-1})\right]^{\frac{1}{2}}$$

~~Le plus simple - Pour traiter le cas general est de prendre la $\langle g \rangle$ pour $\vec{B}$~~ à 4 orientation

~~On peut obtenir~~ formule

Soit $U(\theta_0) = \frac{4}{3}\left(|g^+|^2 + |h^-|^2\right)$



$V(\theta_0) = \frac{4}{3}\left(|g^-|^2 + |h^+|^2\right)$

$$\mathcal{L}(\Delta_{ij}, \Delta m = 1) = \frac{\cancel{4}\gamma f^2}{(\omega_{\cancel{ij}} - \omega_j)^2 + 4\gamma f^2}$$

$$\mathcal{L}(\Delta_{ij}, \Delta m = 2) = \frac{4\gamma f^2}{\omega_i - \omega_j}$$

$\omega_i - \omega_j \rightsquigarrow$ avec $\neq$ spin

pour la m̂ transition $|0\rangle \to |\pm 1\rangle$

$$\bar{\eta}^{ij} = \left\langle \left[U(\theta_0)\mathcal{L}(\Delta_{ij}, \Delta m = 1) + V(\theta_0)\mathcal{L}(\Delta_{ij}, \Delta m = 2)\right]^{\frac{1}{2}} \right\rangle_{\theta, \phi}$$

$$\bar{\eta}_{TOTAL} = \frac{1}{N}\left[\bar{\eta}^{11} + \eta^{22} + \eta^{33} + \eta^{44} + \eta^{12} + \eta^{21} + \eta^{13} + \eta^{31} + \cdots\right] \quad \underline{N = 16}$$

$$= \frac{1}{16} \sum_{i,j=1}^{4} \bar{\eta}^{ij}$$

$B /\!/ \langle 100 \rangle$

$\mathcal{L}(\Delta_{ij}, \Delta m = 1) = 1 \quad \forall_{ij}$ si $B /\!/ \langle 100 \rangle$

$\mathcal{L}(\Delta_{ij}, \Delta m = 2) = \mathcal{L}(\gamma B, \Delta m = 2) \quad \forall_{ij}$ si $B /\!/ \langle 100 \rangle$

$\eta^{ii} = \langle U(\theta_0 = 0) + V(\theta_0 = 0)\mathcal{L}(\gamma B, \Delta m = 2) \rangle$

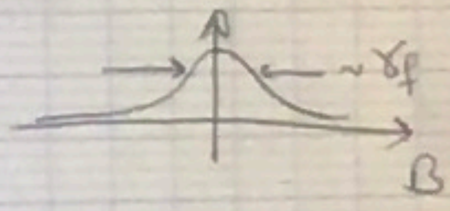
$$\eta^T = \frac{1}{16}\left[\left\langle \left[U(0) + V(0)\mathcal{L}(\gamma B)\right]^{\frac{1}{2}} \right\rangle + \left\langle \left[U(\theta_0) + V(\theta_0)\mathcal{L}(\gamma B)\right]^{\frac{1}{2}} \right\rangle\right]$$

$$g^-(\theta_0) = \frac{1}{2}\Big[3(\cos\varphi\sin\theta)^2 - 1 + \cos\theta_0 - 3(\sin\varphi\sin\theta)(\sin\varphi\sin\theta\cos\theta_0 + \cos\theta\sin\theta_0)\Big]$$

$$h^+(\theta_0) = \frac{1}{2}\Big[3\cos\varphi\sin\theta(\sin\varphi\sin\theta\cos\theta + \cos\theta\sin\theta_0) + 3\sin\varphi\sin^2\theta\cos\varphi\Big]$$

~~[illegible]~~

$B /\!/ \langle 100 \rangle \Rightarrow$ $\boxed{\dfrac{R_1(B=0)}{R_1(B\to\infty)} \simeq 5}$ $(= 4.8037)$



B quelconque $\Rightarrow$ $\boxed{\dfrac{R_1(B=0)}{R_1(B\to\infty)} = 34.8}$ → avec courant identique autour NV ~~[illegible]~~

$R_1(B\to\infty) \to \frac{1}{16}\langle W(0)^2 \rangle$

Vrai si les états propres sont $|\pm 1\rangle, |0\rangle$ Mais en champ nul, le strain les mélange.

Largeur en champ nul + forte à cause de l'élargissement inhomogène dû au strain $W \sim 30\,MHz \to \underline{\Delta \sim 60\,MHz}$

en champ fort $W \sim 10\,MHz \to \underline{\underline{\Delta \sim 15\,MHz}}$