FIN 580: Homework 3

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Outline

Data Preprocessing

- As in past assignments, converted given price series to annualized volatility and took log, after removing days in which asset price didn't change
- Days were randomly labeled as train or test, so we defined daily volatility as the volatility of the five minute returns from 0:00 to 23:55
- Similarly weekly vol as volatility of 5 min returns over 5 day span

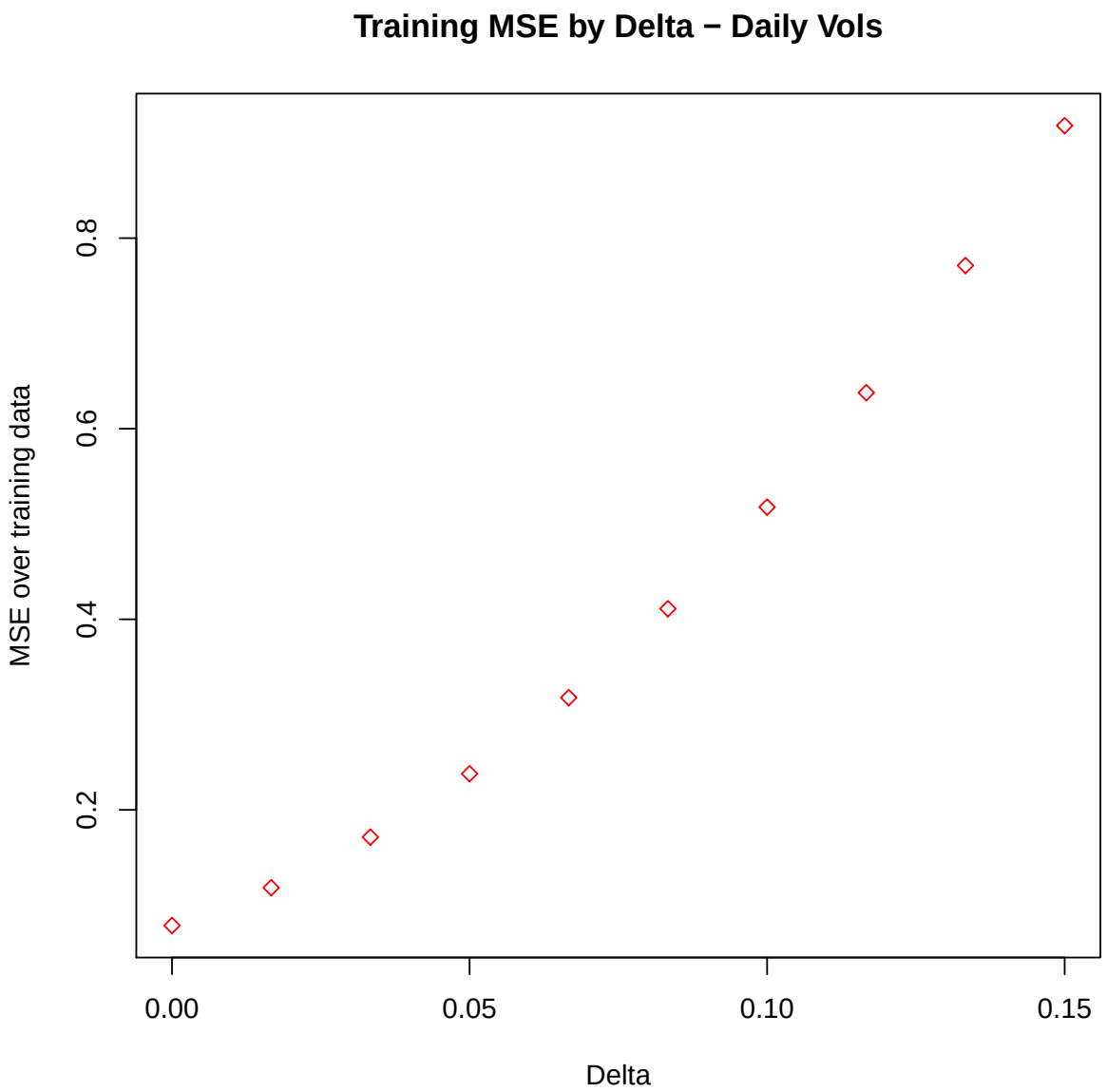
Key takeaways - Logistic Regression

- Weekly vols lead to lower MSES across the board
- Generally lower values of Δ lower MSE - although the shape of the curve is parabolic but heavily skewed
- MSES tend to remain similar across currencies by Δ
- For moving averages daily returns require larger windows - likely because they have a lower signal/noise ratio
- Including moving average returns and volatilities decreases testing error - most significantly in the daily case
  - The optimal window for returns averaging is much higher than for volatilities implying returns are less sticky
  - In this forecaster in the weekly case the optimal Δ is 0 - meaning we have essentially a random walk model

Model setup

$$\sigma_{t+1} = \sigma_t(1 + y_t\Delta)$$
$$\implies y_t = \frac{1}{\Delta}\frac{1}{\sigma_t}(\sigma_{t+1} - \sigma_t)$$

- We optimize over a range of Δs from [0,0.15] with 10 Δs
- To optimize we fit logistic regressions on the training data using each Δ and record the MSE
- We use the Δ of the model with the lowest MSE on the training data to predict testing data and record MSE


Training MSE by Delta − Daily Vols
MSE over training data
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

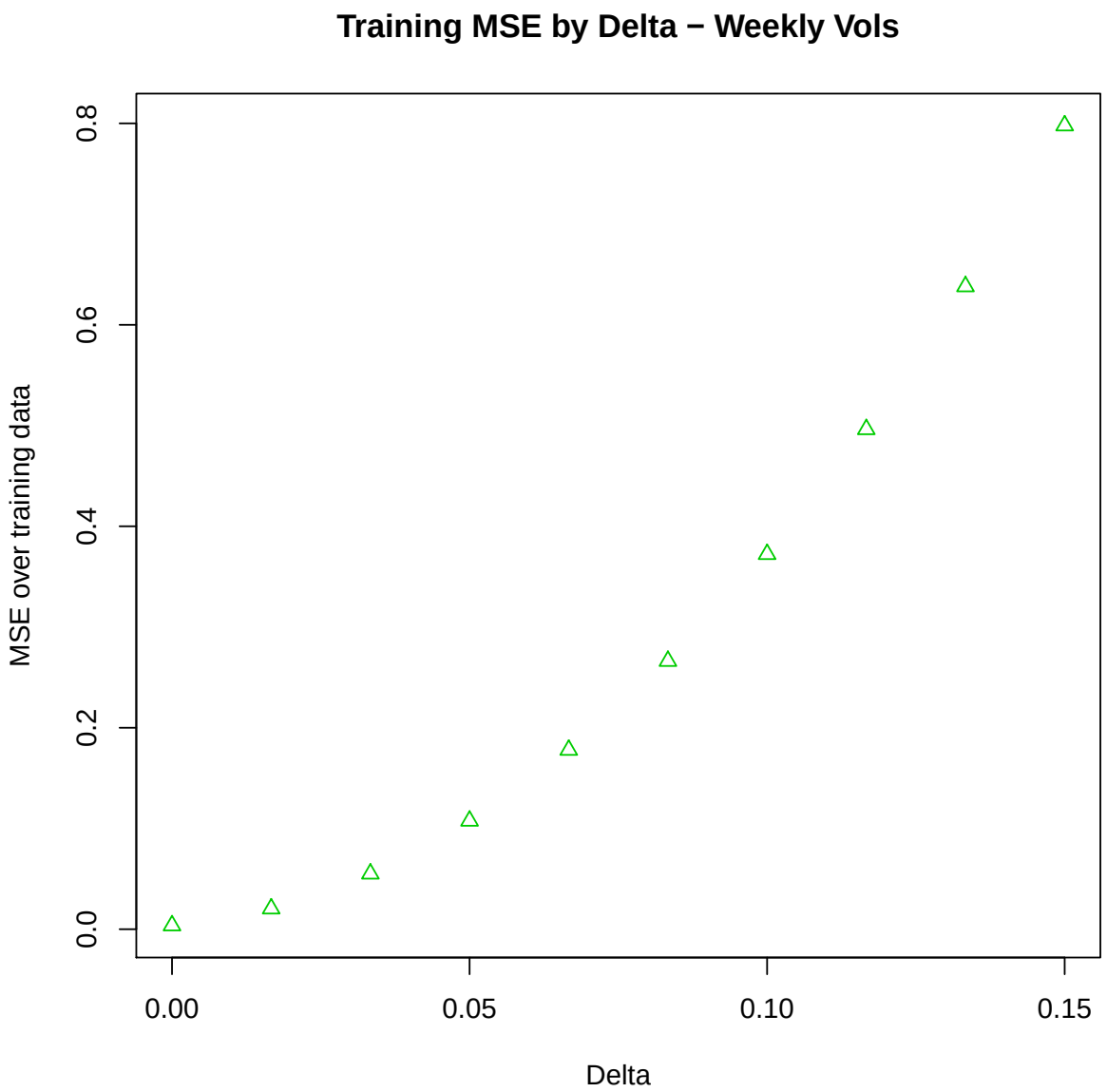

Training MSE by Delta − Weekly Vols
MSE over training data
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

Table: Training MSEs by Delta for Daily Vols

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.057	0.111	0.102	0.087	0.096	0.079	0.061	0.058	0.056
0.02	0.093	0.161	0.147	0.132	0.143	0.114	0.097	0.093	0.086
0.03	0.142	0.225	0.208	0.190	0.204	0.161	0.145	0.140	0.128
0.05	0.203	0.303	0.282	0.264	0.280	0.221	0.206	0.200	0.183
0.07	0.278	0.397	0.370	0.351	0.370	0.293	0.280	0.272	0.248
0.08	0.366	0.505	0.472	0.454	0.476	0.378	0.367	0.356	0.326
0.1	0.467	0.627	0.589	0.570	0.596	0.475	0.466	0.454	0.416
0.12	0.581	0.765	0.719	0.701	0.732	0.585	0.578	0.563	0.517
0.13	0.708	0.916	0.864	0.847	0.882	0.708	0.702	0.685	0.630
0.15	0.848	1.083	1.022	1.007	1.047	0.843	0.840	0.819	0.755

Table: Training MSEs by Delta for Weekly Vols

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.004	0.004	0.005	0.003	0.004	0.005	0.003	0.003	0.002
0.02	0.021	0.022	0.023	0.021	0.022	0.021	0.019	0.019	0.016
0.03	0.055	0.059	0.060	0.059	0.060	0.053	0.052	0.052	0.045
0.05	0.107	0.116	0.115	0.115	0.117	0.103	0.102	0.102	0.091
0.07	0.177	0.192	0.189	0.190	0.193	0.170	0.169	0.168	0.154
0.08	0.264	0.287	0.282	0.284	0.289	0.254	0.253	0.252	0.232
0.1	0.369	0.401	0.393	0.397	0.404	0.354	0.354	0.352	0.326
0.12	0.491	0.535	0.522	0.529	0.539	0.472	0.473	0.469	0.437
0.13	0.630	0.687	0.671	0.680	0.693	0.606	0.608	0.602	0.563
0.15	0.788	0.859	0.837	0.850	0.867	0.758	0.761	0.753	0.706

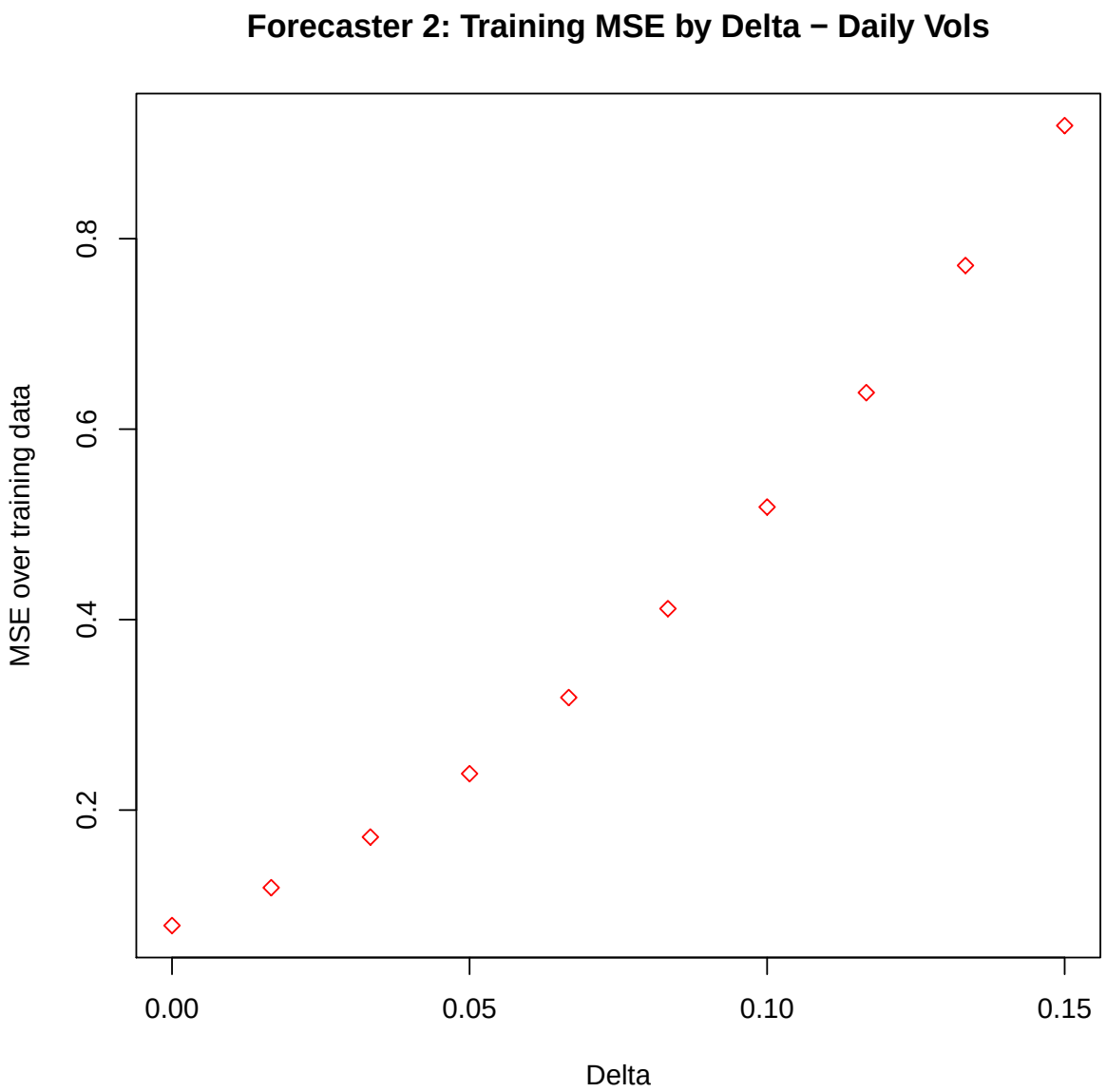
Table: Testing MSES for optimal Delta

	Daily Testing	Weekly Testing
AUD	0.057	0.004
CAD	0.111	0.004
CHF	0.102	0.005
EUR	0.087	0.003
GBP	0.096	0.004
JPY	0.079	0.005
NOK	0.061	0.003
NZD	0.058	0.003
SEK	0.056	0.002
Average	0.079	0.004

Model setup

$$\sigma_{t+1} = \sigma_t(1 + y_t\Delta) + r_t$$
$$\implies y_t = \frac{1}{\Delta}\frac{1}{\sigma_t}(\sigma_{t+1} - r_t - \sigma_t)$$

- As in Forecaster 1 we optimize over Δ using the same range and the training data
- We then report the results using the optimized Δ on the testing data


Forecaster 2: Training MSE by Delta − Daily Vols
MSE over training data
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

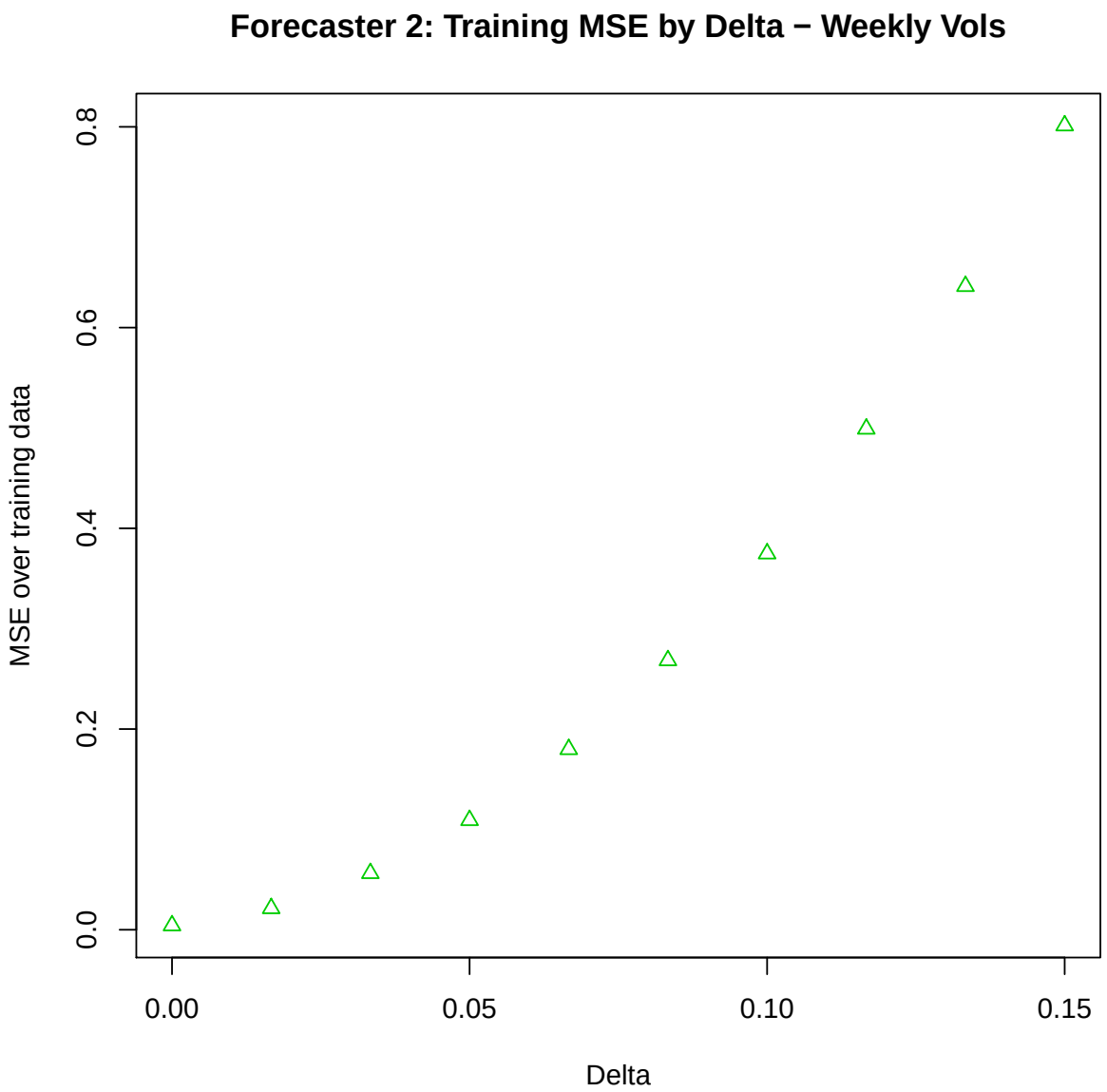

Forecaster 2: Training MSE by Delta − Weekly Vols
MSE over training data
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

Table: Training MSEs by Delta for Daily Vols

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.058	0.111	0.101	0.087	0.097	0.079	0.062	0.059	0.056
0.02	0.094	0.161	0.147	0.132	0.143	0.113	0.097	0.094	0.087
0.03	0.142	0.225	0.207	0.191	0.204	0.161	0.146	0.141	0.129
0.05	0.204	0.304	0.281	0.264	0.280	0.220	0.207	0.201	0.183
0.07	0.279	0.397	0.369	0.352	0.371	0.293	0.281	0.273	0.249
0.08	0.367	0.505	0.472	0.454	0.476	0.377	0.368	0.358	0.327
0.1	0.468	0.628	0.588	0.571	0.597	0.475	0.467	0.455	0.416
0.12	0.582	0.765	0.718	0.702	0.732	0.585	0.579	0.564	0.518
0.13	0.710	0.917	0.863	0.847	0.882	0.707	0.703	0.686	0.631
0.15	0.850	1.084	1.022	1.007	1.047	0.842	0.841	0.821	0.756

Table: Training MSEs by Delta for Weekly Vols

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.005	0.004	0.005	0.004	0.004	0.005	0.004	0.004	0.003
0.02	0.023	0.023	0.023	0.022	0.022	0.020	0.020	0.021	0.017
0.03	0.058	0.061	0.060	0.059	0.060	0.053	0.053	0.055	0.047
0.05	0.110	0.118	0.115	0.116	0.118	0.103	0.104	0.105	0.094
0.07	0.181	0.195	0.189	0.191	0.194	0.169	0.171	0.172	0.156
0.08	0.268	0.290	0.281	0.285	0.290	0.253	0.256	0.256	0.235
0.1	0.374	0.405	0.392	0.399	0.406	0.353	0.358	0.357	0.330
0.12	0.497	0.539	0.521	0.531	0.541	0.471	0.477	0.474	0.441
0.13	0.637	0.692	0.669	0.682	0.695	0.605	0.613	0.609	0.568
0.15	0.795	0.864	0.836	0.853	0.869	0.756	0.766	0.760	0.711

Table: Testing MSES for optimal Delta

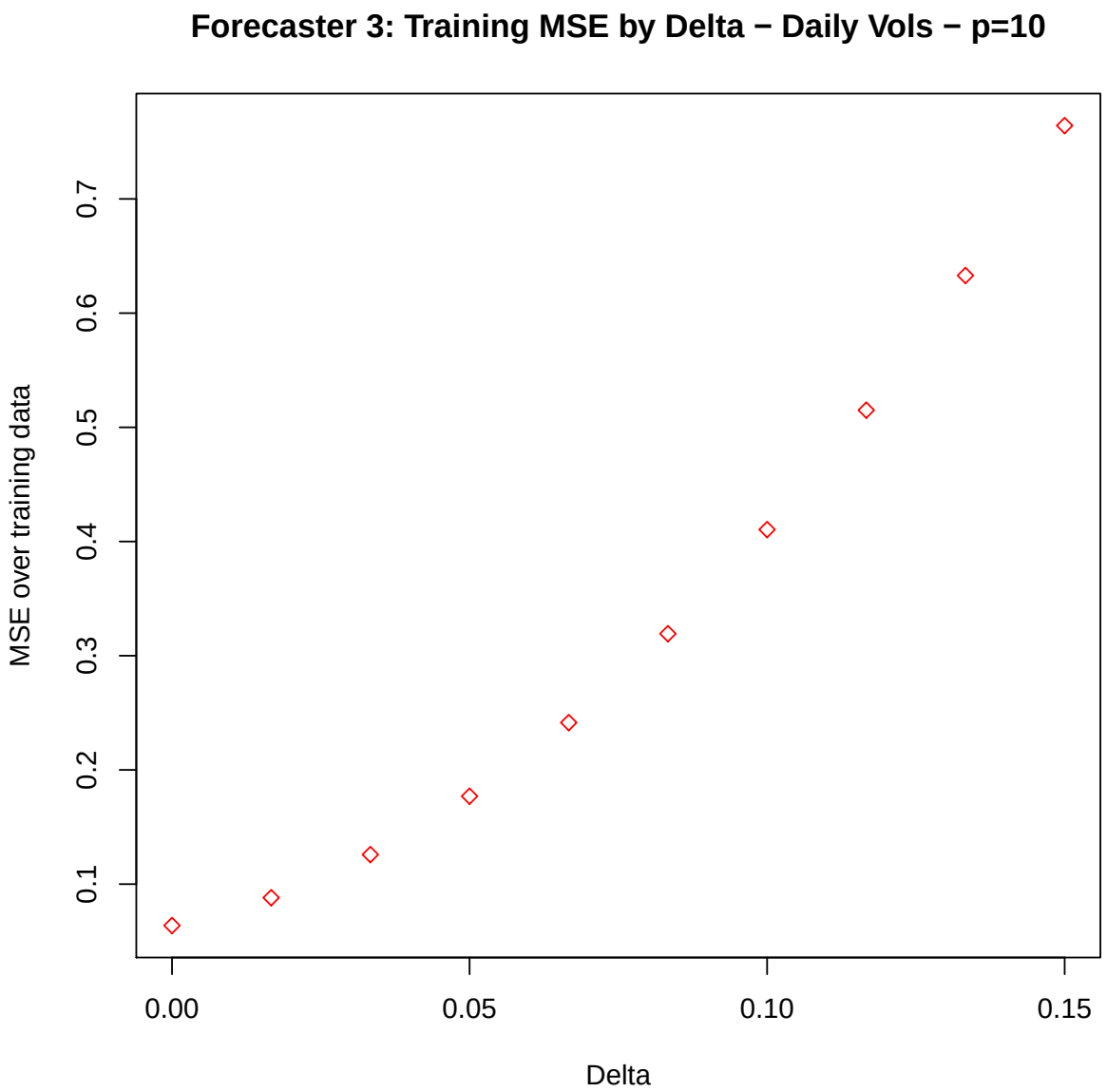
	Daily Testing	Weekly Testing
AUD	0.057	0.004
CAD	0.111	0.004
CHF	0.102	0.005
EUR	0.087	0.003
GBP	0.096	0.004
JPY	0.079	0.005
NOK	0.061	0.003
NZD	0.058	0.003
SEK	0.056	0.002
Average	0.079	0.004

Model set up

$$\sigma_{t+1} = \left(\frac{1}{p} \sum_{i=t-p+1}^{t} \sigma_i \right) (1 + y_t \Delta)$$

$$\implies y_t = \frac{1}{\Delta} \frac{1}{\sum_{i=t-p+1}^{t} \sigma_i} \left(\sigma_{t+1} - \sum_{i=t-p+1}^{t} \sigma_i \right)$$

- To optimize p and Δ we fit over all possible Δs (same range as previously) and over $p \in \{3, 5, 10\}$ using training data
- We then use the optimal p and Δ to get MSE on testing data


Forecaster 3: Training MSE by Delta − Daily Vols − p=10
MSE over training data
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.05
0.10
0.15
Delta

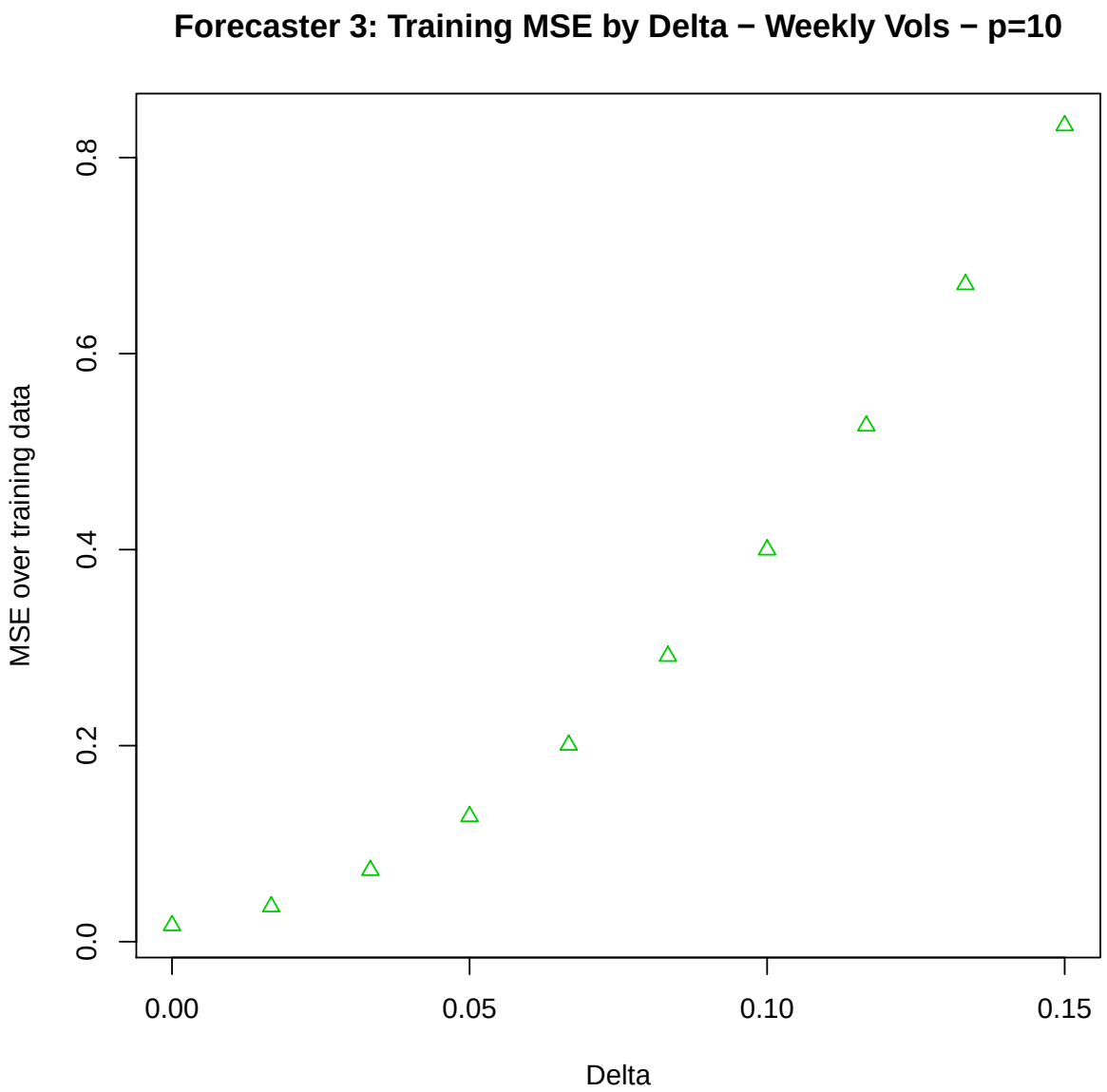
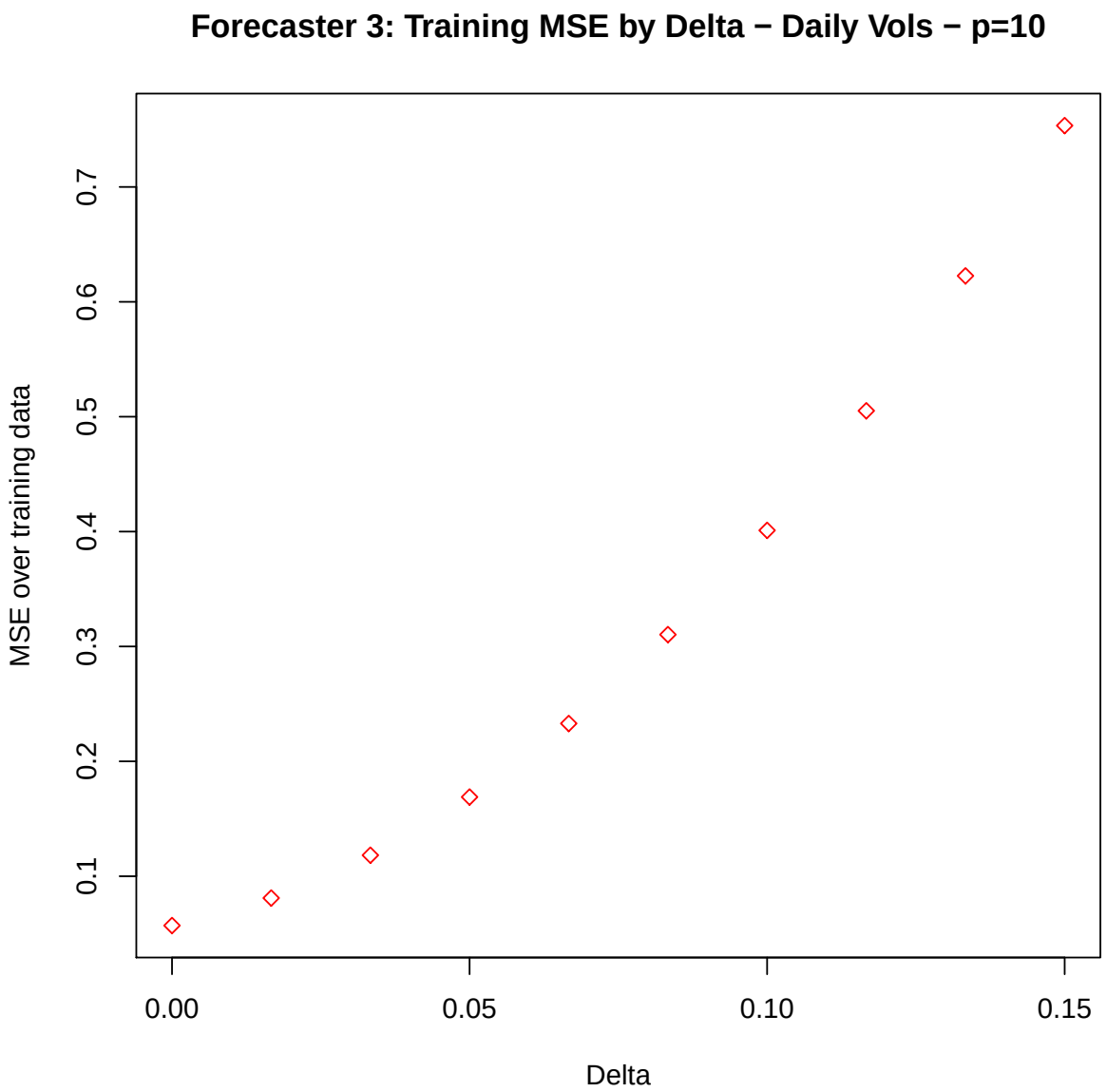

Forecaster 3: Training MSE by Delta − Weekly Vols − p=10
MSE over training data
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

Table: Training MSEs by Delta for Daily Vols, p=3

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.056	0.079	0.078	0.074	0.073	0.068	0.049	0.054	0.043
0.02	0.078	0.107	0.105	0.102	0.101	0.089	0.072	0.076	0.062
0.03	0.114	0.150	0.147	0.145	0.144	0.122	0.108	0.111	0.093
0.05	0.162	0.208	0.203	0.202	0.201	0.167	0.156	0.157	0.136
0.07	0.224	0.280	0.273	0.273	0.274	0.226	0.217	0.217	0.191
0.08	0.298	0.367	0.356	0.359	0.361	0.296	0.291	0.288	0.257
0.1	0.386	0.469	0.454	0.459	0.463	0.379	0.377	0.372	0.335
0.12	0.487	0.585	0.566	0.574	0.579	0.475	0.476	0.469	0.425
0.13	0.601	0.716	0.692	0.703	0.711	0.583	0.587	0.578	0.526
0.15	0.727	0.861	0.832	0.846	0.857	0.704	0.711	0.699	0.640

Table: Training MSEs by Delta for Weekly Vols, p=3

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.020	0.015	0.018	0.015	0.016	0.027	0.013	0.016	0.008
0.02	0.040	0.035	0.039	0.036	0.036	0.046	0.031	0.035	0.024
0.03	0.078	0.075	0.077	0.076	0.075	0.082	0.066	0.070	0.056
0.05	0.133	0.133	0.135	0.135	0.134	0.136	0.118	0.122	0.104
0.07	0.206	0.211	0.211	0.213	0.212	0.206	0.187	0.191	0.168
0.08	0.297	0.308	0.306	0.311	0.310	0.293	0.274	0.276	0.248
0.1	0.405	0.424	0.419	0.427	0.427	0.397	0.377	0.379	0.344
0.12	0.530	0.559	0.551	0.562	0.564	0.518	0.498	0.498	0.457
0.13	0.674	0.713	0.701	0.716	0.720	0.656	0.635	0.634	0.585
0.15	0.834	0.887	0.870	0.889	0.895	0.811	0.790	0.787	0.730


Forecaster 3: Training MSE by Delta − Daily Vols − p=10
MSE over training data
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.05
0.10
0.15
Delta

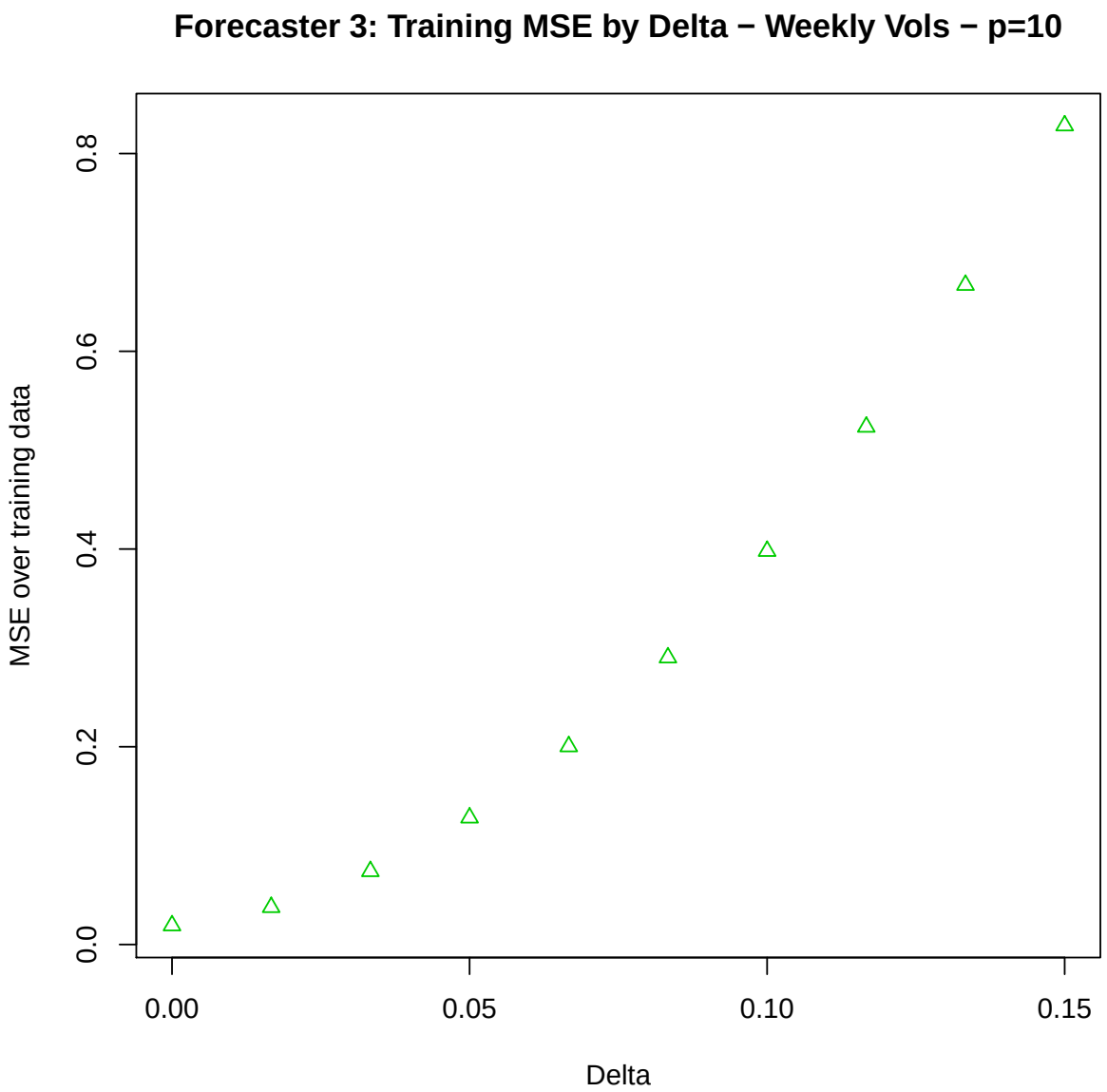
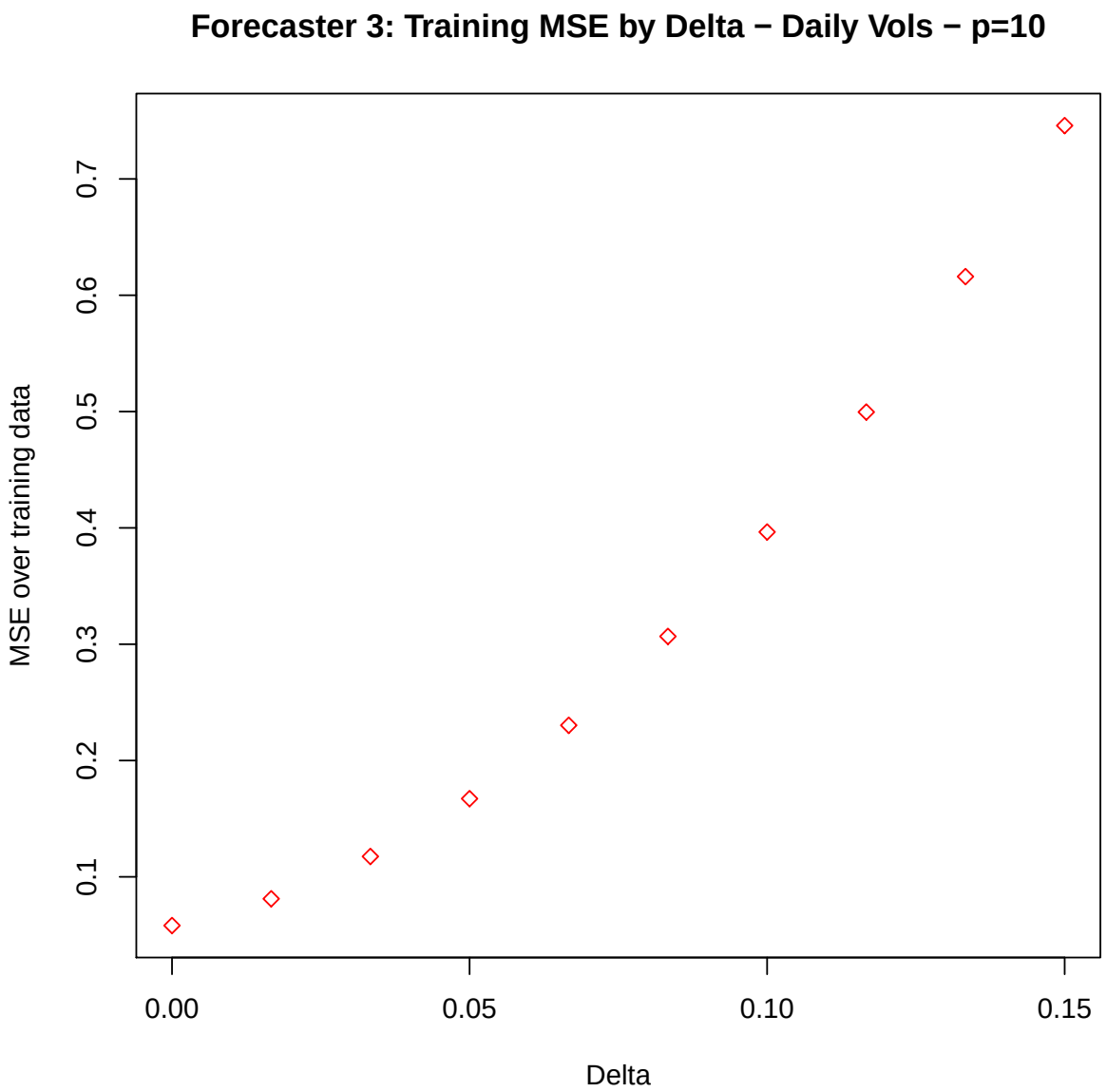

Forecaster 3: Training MSE by Delta − Weekly Vols − p=10
MSE over training data
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

Table: Training MSEs by Delta for Daily Vols, p=5

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.054	0.066	0.067	0.064	0.061	0.065	0.045	0.051	0.040
0.02	0.076	0.094	0.094	0.092	0.089	0.086	0.067	0.073	0.058
0.03	0.111	0.137	0.135	0.134	0.131	0.119	0.102	0.107	0.089
0.05	0.160	0.195	0.190	0.190	0.188	0.164	0.149	0.154	0.131
0.07	0.221	0.267	0.259	0.260	0.259	0.222	0.209	0.213	0.185
0.08	0.295	0.353	0.342	0.345	0.346	0.292	0.282	0.285	0.251
0.1	0.383	0.455	0.439	0.445	0.447	0.375	0.368	0.369	0.329
0.12	0.483	0.571	0.550	0.559	0.563	0.471	0.466	0.466	0.418
0.13	0.597	0.701	0.675	0.687	0.694	0.579	0.576	0.575	0.519
0.15	0.723	0.846	0.814	0.829	0.840	0.699	0.699	0.696	0.632

Table: Training MSEs by Delta for Weekly Vols, p=5

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.024	0.017	0.021	0.017	0.018	0.031	0.015	0.020	0.009
0.02	0.043	0.037	0.040	0.038	0.037	0.049	0.033	0.038	0.025
0.03	0.080	0.075	0.078	0.077	0.075	0.085	0.067	0.072	0.056
0.05	0.135	0.133	0.135	0.135	0.133	0.137	0.118	0.123	0.103
0.07	0.207	0.210	0.210	0.213	0.211	0.207	0.187	0.192	0.167
0.08	0.296	0.306	0.304	0.309	0.307	0.293	0.273	0.276	0.246
0.1	0.403	0.422	0.417	0.425	0.423	0.396	0.375	0.378	0.342
0.12	0.528	0.556	0.548	0.559	0.559	0.516	0.495	0.497	0.454
0.13	0.670	0.710	0.697	0.713	0.714	0.653	0.632	0.632	0.582
0.15	0.830	0.883	0.865	0.885	0.888	0.807	0.786	0.784	0.726


Forecaster 3: Training MSE by Delta − Daily Vols − p=10
MSE over training data
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.05
0.10
0.15
Delta

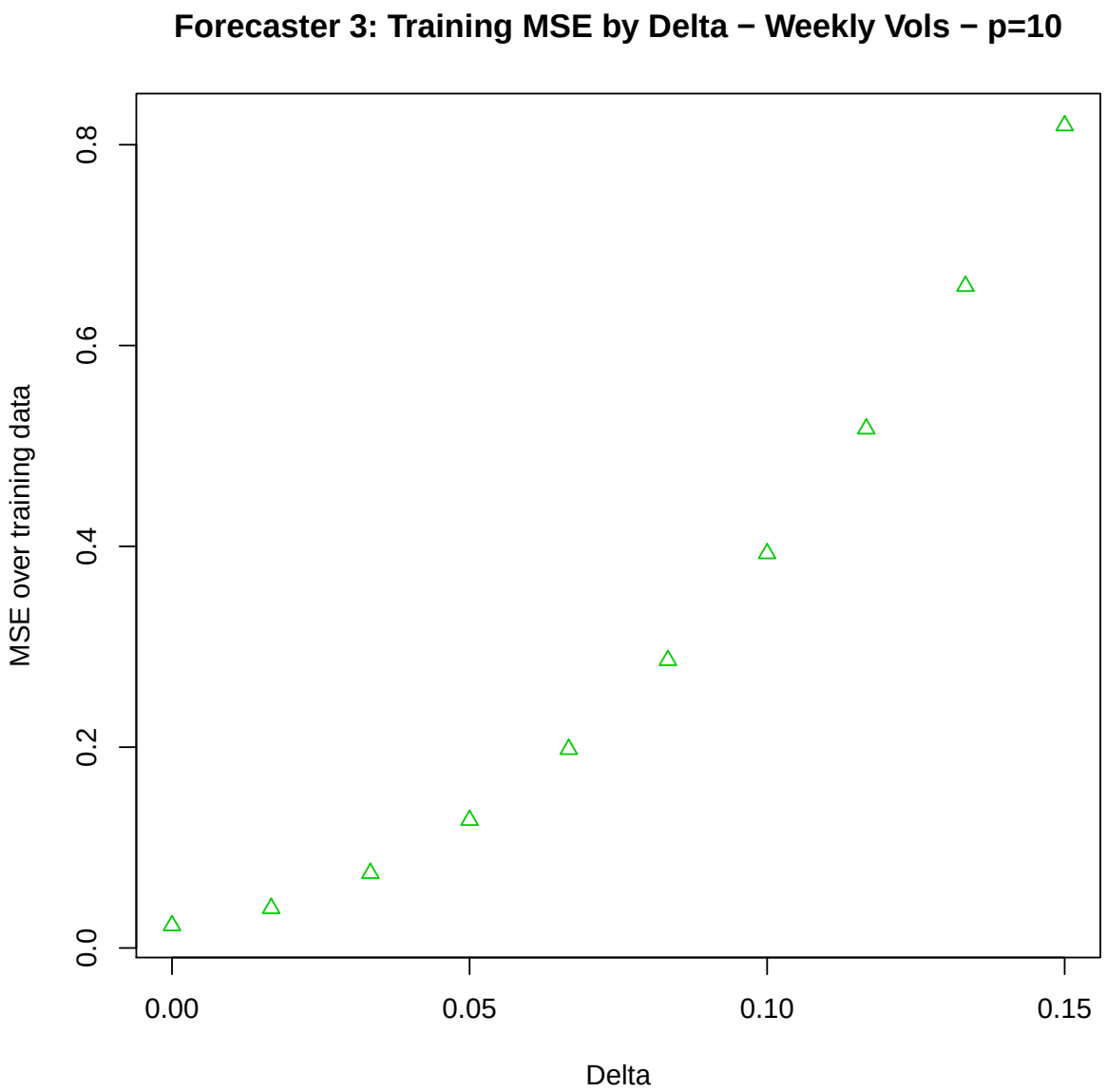

Forecaster 3: Training MSE by Delta − Weekly Vols − p=10
MSE over training data
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
Delta

Table: Training MSEs by Delta for Daily Vols, p=10

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.058	0.066	0.068	0.066	0.061	0.065	0.047	0.053	0.040
0.02	0.079	0.093	0.094	0.092	0.087	0.084	0.068	0.075	0.059
0.03	0.114	0.134	0.133	0.133	0.128	0.116	0.103	0.109	0.089
0.05	0.161	0.190	0.187	0.188	0.184	0.160	0.150	0.155	0.130
0.07	0.222	0.260	0.255	0.257	0.254	0.217	0.209	0.214	0.184
0.08	0.296	0.345	0.337	0.341	0.340	0.286	0.281	0.285	0.249
0.1	0.382	0.445	0.433	0.439	0.440	0.368	0.366	0.368	0.326
0.12	0.482	0.559	0.543	0.552	0.555	0.462	0.464	0.464	0.415
0.13	0.595	0.688	0.667	0.679	0.684	0.569	0.574	0.572	0.516
0.15	0.721	0.831	0.806	0.820	0.829	0.689	0.696	0.693	0.628

Table: Training MSEs by Delta for Weekly Vols, p=10

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
0	0.030	0.020	0.024	0.020	0.020	0.033	0.019	0.025	0.011
0.02	0.048	0.038	0.042	0.039	0.038	0.049	0.035	0.041	0.026
0.03	0.083	0.075	0.078	0.077	0.075	0.082	0.068	0.075	0.056
0.05	0.136	0.131	0.134	0.135	0.131	0.132	0.119	0.125	0.103
0.07	0.207	0.207	0.207	0.211	0.207	0.199	0.186	0.192	0.165
0.08	0.295	0.302	0.300	0.306	0.303	0.283	0.271	0.276	0.244
0.1	0.401	0.415	0.410	0.420	0.418	0.384	0.372	0.376	0.339
0.12	0.524	0.548	0.540	0.553	0.552	0.501	0.491	0.494	0.450
0.13	0.664	0.701	0.688	0.705	0.706	0.636	0.627	0.628	0.577
0.15	0.823	0.872	0.854	0.877	0.879	0.788	0.780	0.779	0.720

Table: Testing MSES for optimal Delta and p

	Daily Testing, p=5	Weekly Testing, p=3
AUD	0.056	0.020
CAD	0.079	0.015
CHF	0.078	0.018
EUR	0.074	0.015
GBP	0.073	0.016
JPY	0.068	0.027
NOK	0.049	0.013
NZD	0.054	0.016
SEK	0.043	0.008
Average	0.064	0.016

Model setup

$$\sigma_{t+1} = \left(\frac{1}{p}\sum_{i=t-p+1}^{t}\sigma_i\right)(1+y_t\Delta) + \left(\frac{1}{q}\sum_{i=t-p+1}^{t} r_i\right)$$

$$\implies y_t = \frac{1}{\Delta}\frac{1}{\left(\frac{1}{p}\sum_{i=t-p+1}^{t}\sigma_i\right)}\left(\sigma_{t+1} - \frac{1}{q}\sum_{i=t-p+1}^{t} r_i - \frac{1}{p}\sum_{i=t-p+1}^{t}\sigma_i\right)$$

- Fit model over same sequence of Δs
- Also fit over $p \in \{3, 5, 10\}$ and $q \in \{3, 5, 10\}$
- Due to large number of combinations we do not output graphs and tables for all the combinations over the training data - we simply output the optimal parameters and the testing MSEs for that combination

Table: Testing MSES for optimal Delta and p

	Daily Testing 0 5 10	Weekly Testing 0 3 10
AUD	0.054	0.020
CAD	0.066	0.015
CHF	0.067	0.018
EUR	0.064	0.015
GBP	0.061	0.016
JPY	0.065	0.027
NOK	0.045	0.013
NZD	0.051	0.017
SEK	0.040	0.008
Average	0.057	0.017

Optimal parameters

- Daily: $Delta$=0.017,p=5,q=10
- Weekly: $Delta$=0,p=3,q=10

Overview

- Modeled daily and weekly volatilities as $\sigma_{t+1} = \sigma_t(1 + \Delta y_t)$, where $y_t \in \{-1, +1\}$
- Trained an SVM to predict labels (ie y_t) using R package 'e1071'
  - Model 1 : $y(\sigma_t)$
  - Model 2: $y(\sigma_t, r_t)$
  - Model 3: $y\left(\frac{1}{p}\sum_{k=t-p+1}^{t}\sigma_t\right)$
  - Model 4: $y\left(\frac{1}{p}\sum_{k=t-p+1}^{t}\sigma_t, \frac{1}{p}\sum_{k=t-p+1}^{t} r_t\right)$
- Having fit the SVM, optimized in sample over Δ and the lag parameters, p and q.

Fitting the SVM

- Linear SVM

$$\min_{w \in n, b \in, s \in m} 1/2||w||^2 + C\sum_{j=1}^{m} s_j$$

$$\text{s.t. } y_j(x_j^T w + b) \geq 1 - s_j, j = 1, \ldots, m$$

$$s_j \geq 0$$

- Kernel SVM
  - We choose to use the radial basis function kernel $k(x, z) := e^{-\frac{\gamma}{2}x - z_2^2}$, giving rise to the kernalized model

$$\max_{\alpha \in m} \mathbf{1}^T \alpha - 1/2 \alpha^T \phi(Z)^T \phi(Z) \alpha$$

$$\text{s.t. } \mathbf{0} \leq \alpha \leq C\mathbf{1}$$

  where $\phi(Z)^T \phi(Z)_{i,j} = y_i y_j k(x_i, x_j)$
- The cost $C > 0$ and, for rbf, γ are hyperparameters which we train via 10-fold cross validation

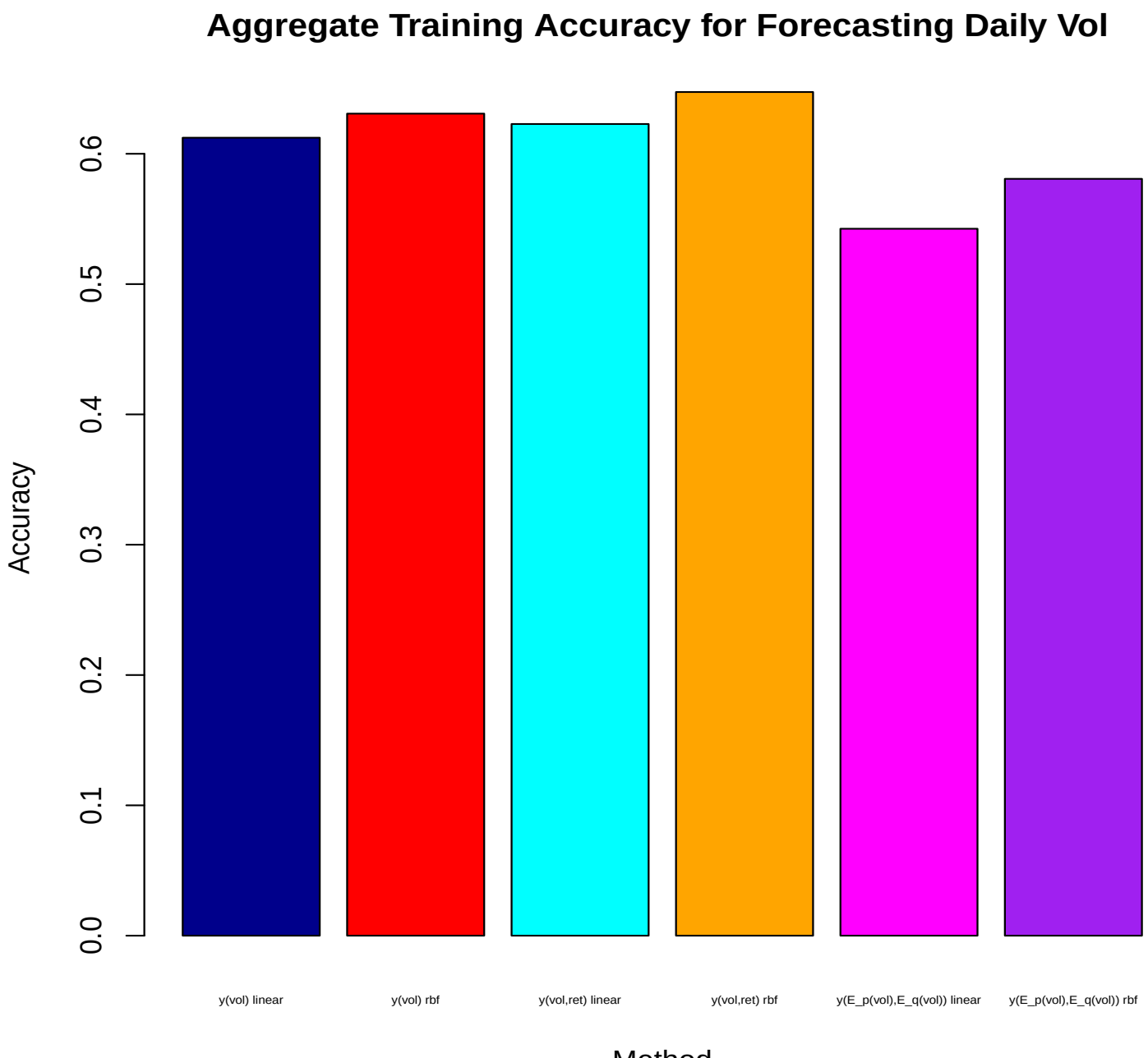
Tuning the parameters

- Tuning Δ
  - Given labels $y_t,\ t = 1, \ldots, T$ from SVM, we seek to solve

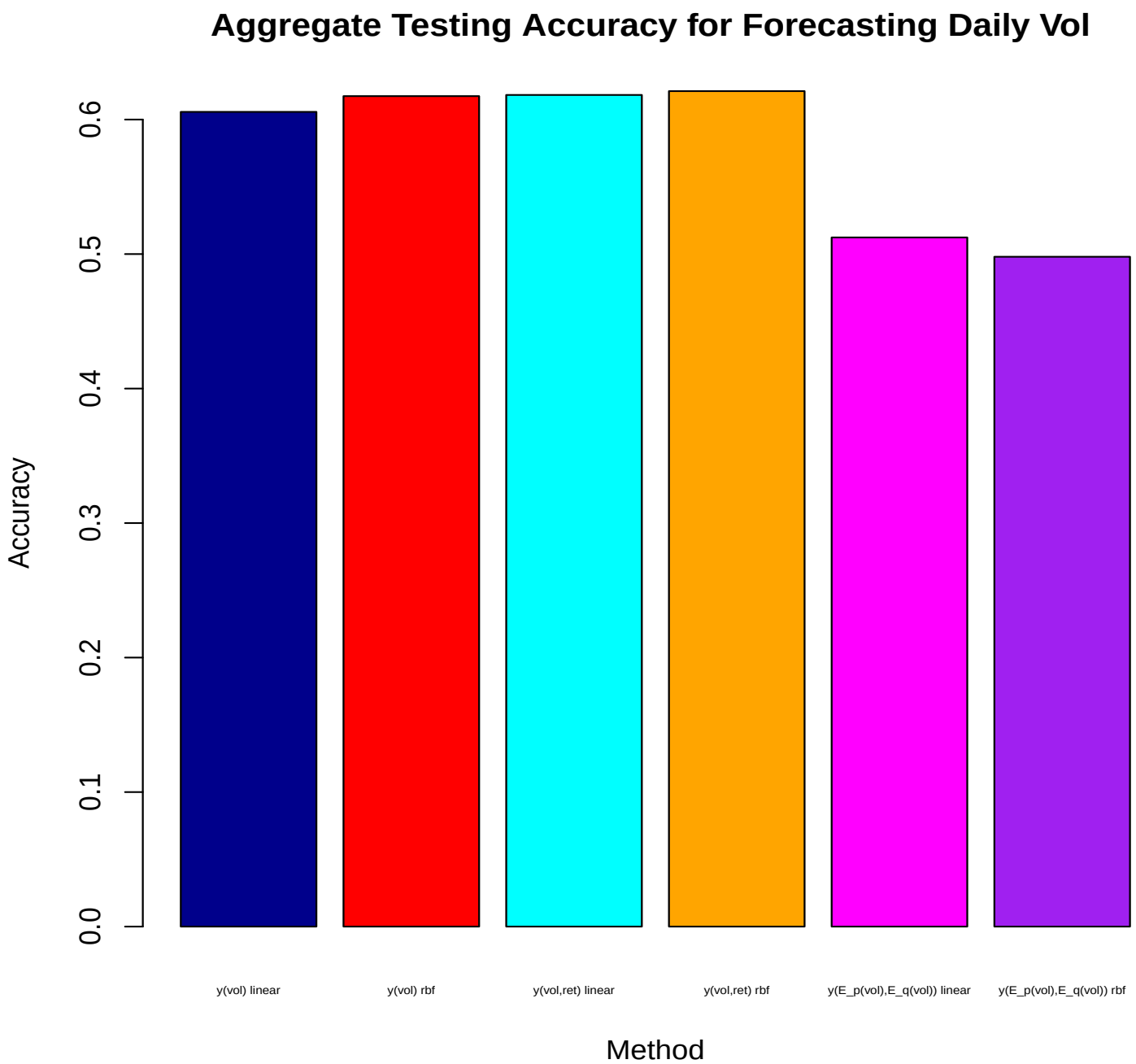
$$\min_{\Delta} \frac{1}{T} \sum_{t=1}^{T} (\sigma_{t+1} - \sigma_t(1 + \Delta y_t))^2$$
$$\text{s.t. } \Delta \geq 0$$

  - Ignoring the nonnegativity constraint, this is equivalent to least-square regression of $\sigma_{t-1} - \sigma_t$ on $y_t \sigma_t$
  - KKT conditions for optimality call for setting $\Delta = \Delta_{LS}$ if $\Delta_{LS} \geq 0$ and $\Delta = 0$ otherwise
- Tuning p and q
  - We do a grid search on $p, q \in \{3, 5, 10\}$

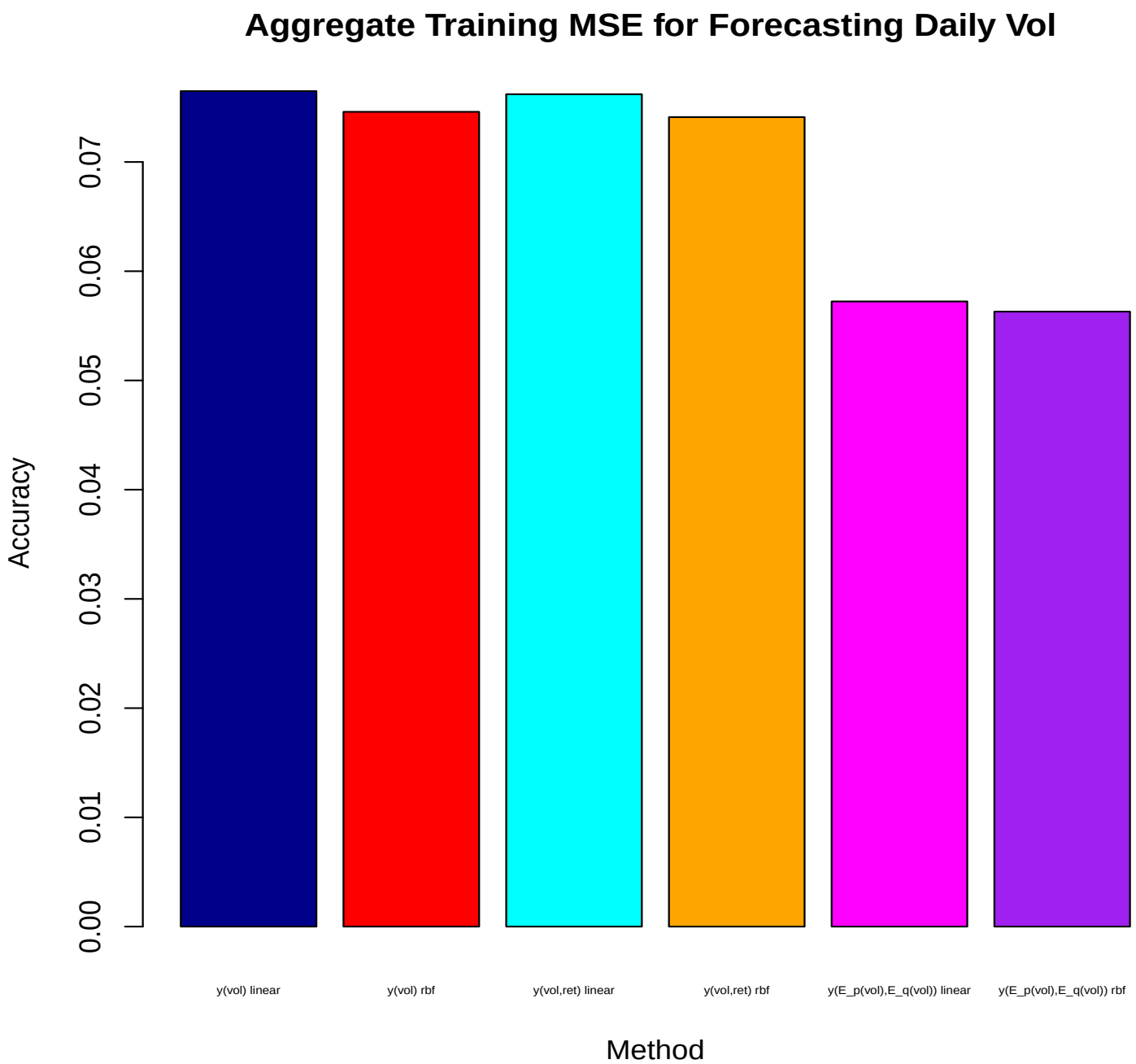
Predicting Daily Volatility - Accuracy



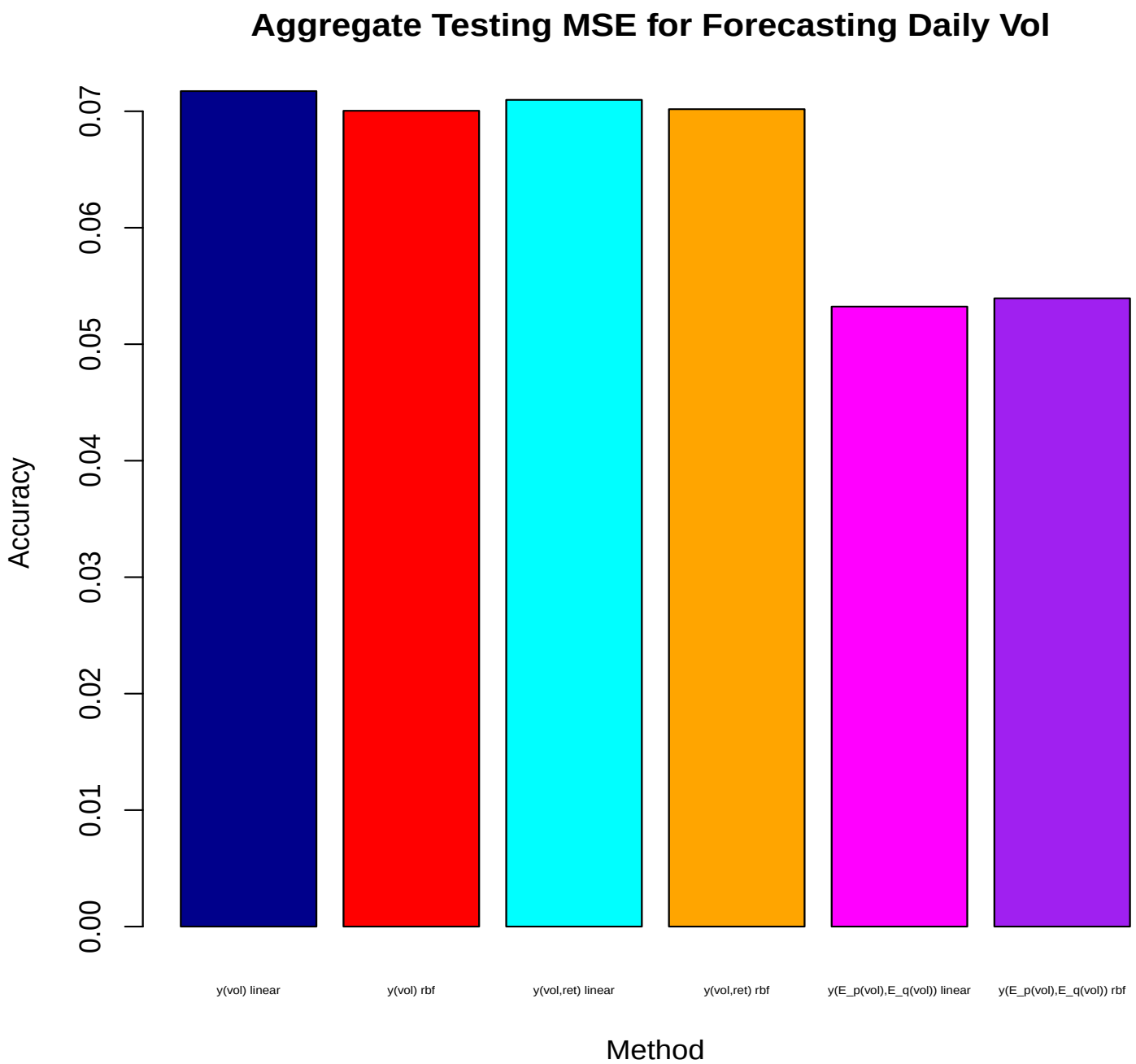
Predicting Daily Volatility - Accuracy



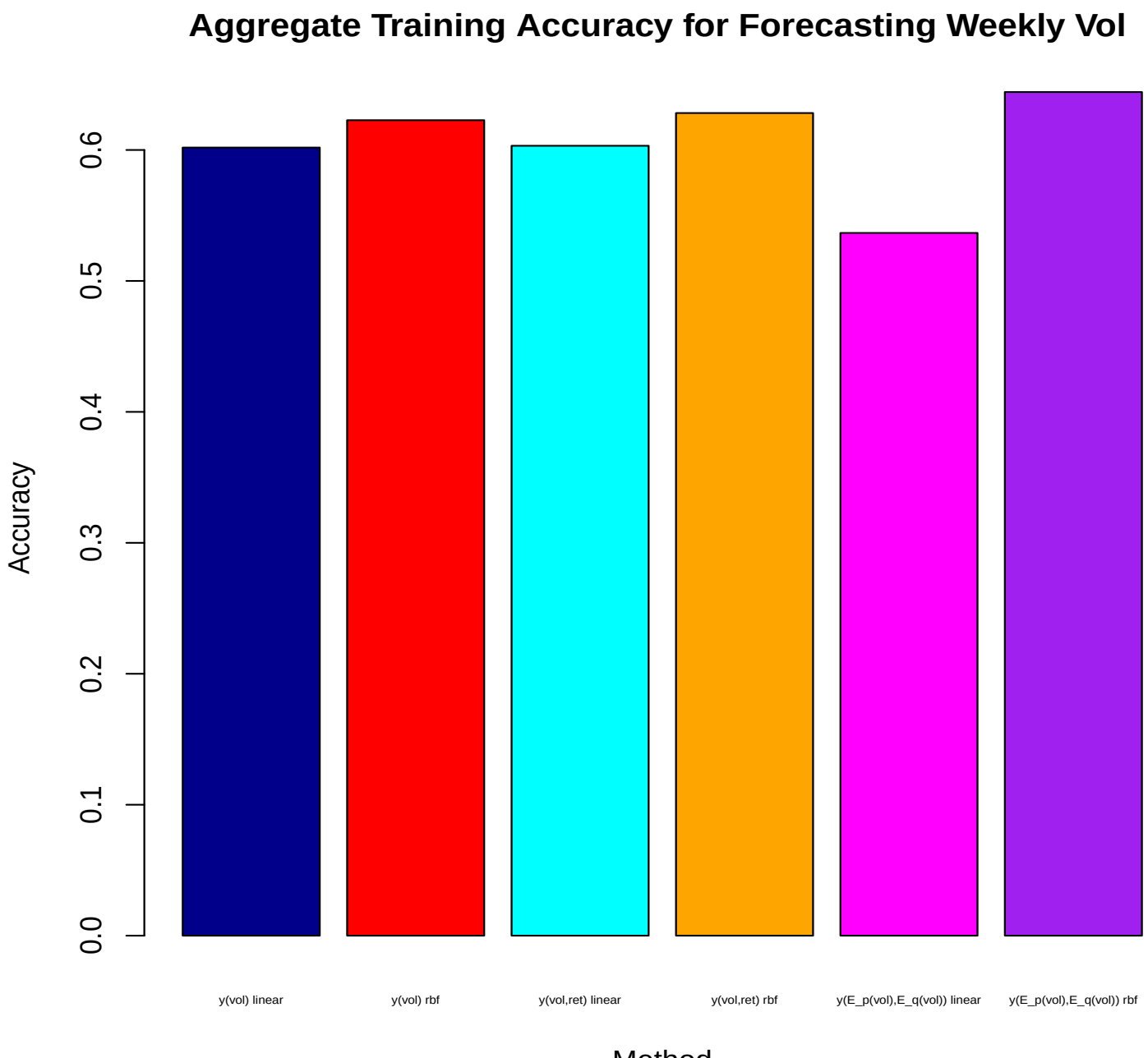
Predicting Daily Volatility - MSE



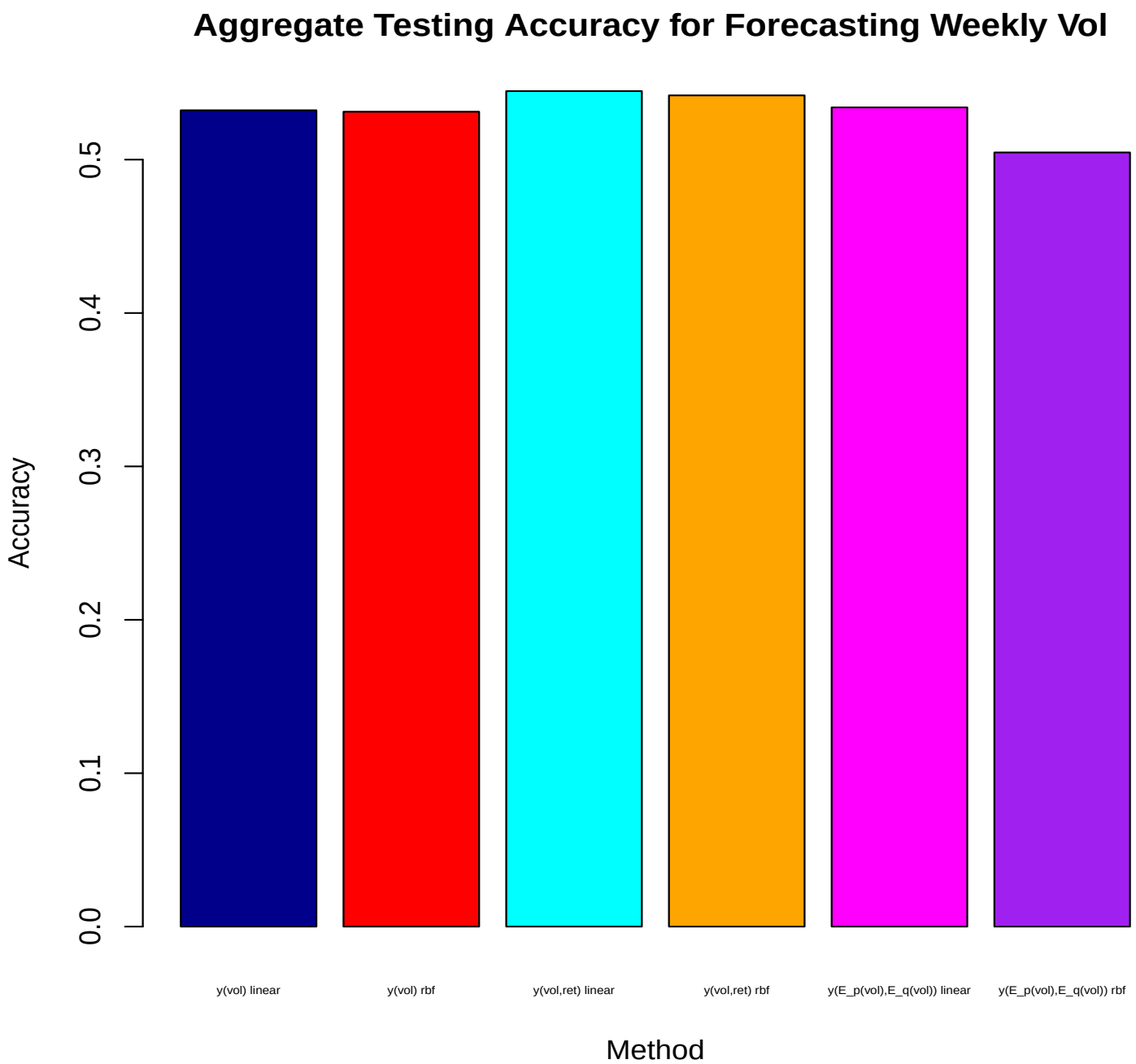
Predicting Daily Volatility - MSE



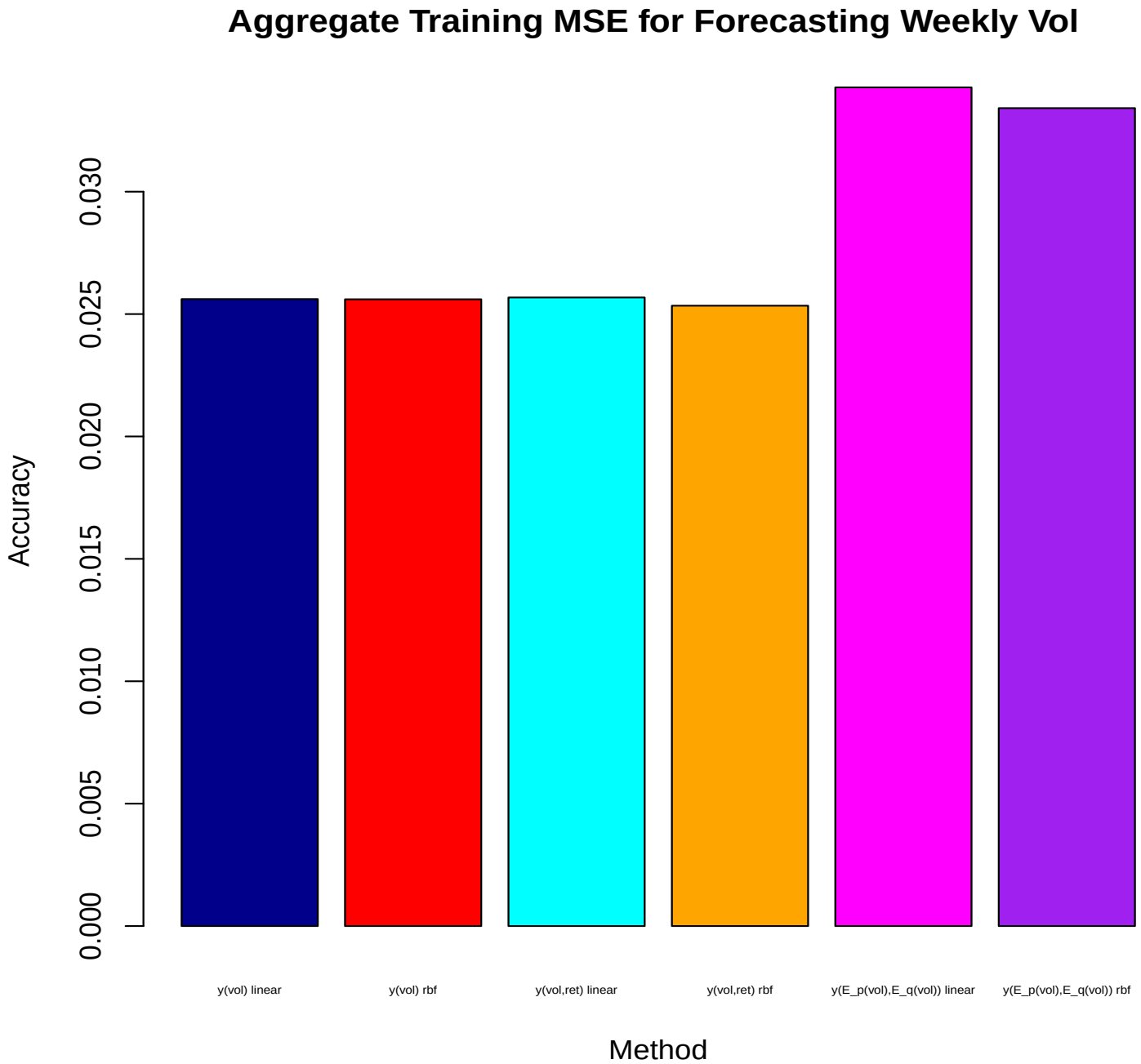
Predicting Weekly Volatility - Accuracy



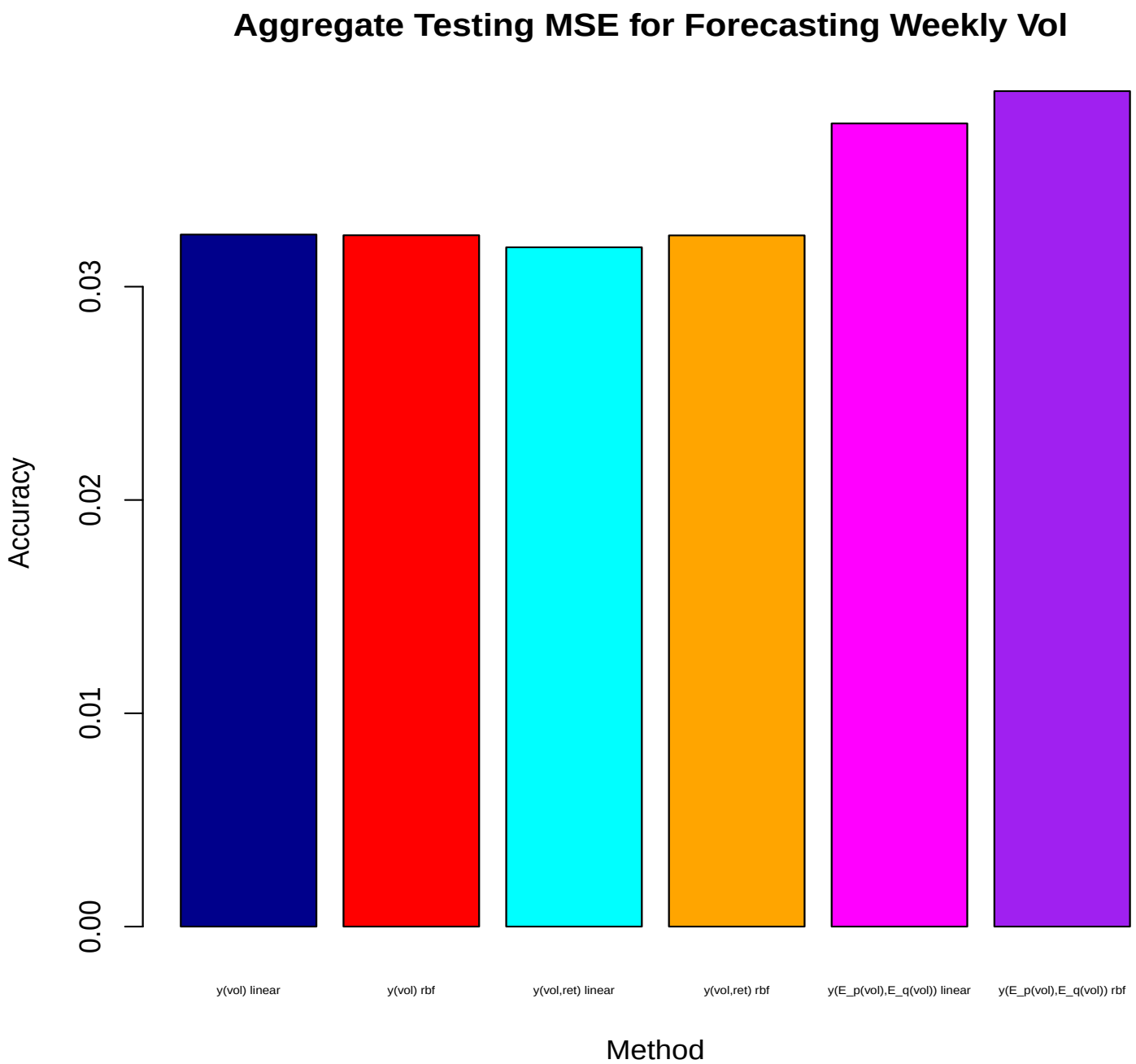
Predicting Weekly Volatility - Accuracy



Predicting Weekly Volatility - MSE



Predicting Weekly Volatility - MSE



Summary

- Non moving average models outperformed for accuracy yet not MSE
- MSE results should be taken with grain of salt, frequently had nonnegativity constraint bind producing $\Delta = 0$
- Weekly results seem to be more sensible.
- As model generally performed best for smallest $|\Delta|$, converging towards random walk of $|\Delta| = 0$
- SVM useful in predicting up or down, though the regression model $\sigma_{t+1} = \sigma_t(1 + \Delta y_t)$ is flawed