# Structured Orb Dynamics:
# Unified Manuscript and Data Repository

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**Abstract**

This manuscript develops a unified geometric framework for interpreting unknown aerial motion using only observable trajectory structure. The approach combines stabilized image-plane reconstruction, curvature-based analysis, and a discrete motion-state model to describe how an object's motion evolves over time, independent of sensor metadata or assumed physical mechanisms.

The framework links curvature, its temporal behavior, and associated kinematic features to four interpretable motion states—straight, turn, hover, and orb. These states are supported by a likelihood model that evaluates geometric consistency frame by frame while maintaining temporal coherence through a minimal set of transition assumptions. The result is a classifier that summarizes motion behavior in a way that is reproducible, dataset-agnostic, and grounded entirely in observable quantities.

Applied to publicly released infrared datasets such as PR-018, the method identifies sustained, smoothly varying curvature regimes that are not well described by classical motion categories alone. The model does not infer intent or mechanism; instead, it provides a transparent description of the motion's geometric organization and the transitions between distinct dynamical regimes.

This framework offers a consistent basis for comparing trajectories across heterogeneous datasets and establishes a clear path for future work in geometric motion analysis where traditional metadata is incomplete or unavailable.

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# Part I: Instrumentation and Reconstruction Pipeline

## 1 Orb Motion Classifier: Dynamical Motion-State Model

## 2 Introduction

Publicly released infrared recordings of unidentified aerial phenomena (UAP) provide a rare opportunity to study motion under conditions where physical metadata, calibration information, and platform telemetry are incomplete or unavailable. Although such datasets are limited, they still contain observable geometric structure that can be analyzed without relying on assumptions about the underlying mechanism or intent of the object.

The aim of this work is to develop a clear and reproducible framework for describing aerial motion using only the information that can be reliably extracted from image-plane data. The focus is not on classification in the traditional sense, nor on proposing physical explanations, but on building a structured way to document how an object's apparent motion evolves over time.

Our approach integrates three components:

1. trajectory reconstruction from stabilized infrared footage,
2. curvature-based geometric diagnostics of local motion,
3. a discrete motion-state model that organizes observed behavior into simple patterns.

Together, these elements form a unified framework that allows motion to be described in terms of observable kinematic structure rather than speculative interpretation.

The PR018 infrared video serves as the primary case study because it contains enough stabilization around the target to permit construction of an approximate image-plane trajectory. This makes it possible to estimate curvature, examine how curvature evolves, and evaluate which types of motion are compatible with the observed behavior.

To illustrate the limits of the method, a supplementary analysis is applied to the GIMBAL dataset. Because strong parallax and missing metadata prevent trajectory reconstruction, the GIMBAL analysis is necessarily qualitative and conservative. This contrast highlights the conditions under which the framework can and cannot be applied.

Throughout the manuscript, the emphasis is on transparency, reproducibility, and proportional inference. The framework is intentionally simple and geometric, designed to describe motion in a way that remains valid even when only partial information is available. No claims are made about what the object is, how it is propelled, or whether it corresponds to any particular physical model. The goal is to provide a consistent language for discussing observable motion in datasets where traditional physical inference is not possible.

By presenting the methodology alongside both a quantitative case (PR018) and a constrained, qualitative case (GIMBAL), the manuscript aims to show how careful, geometry-based analysis can help clarify what can—and cannot—be extracted from limited infrared recordings.

In this section we introduce the formal dynamical motion-state model that underpins the Orb Motion Classifier. We first define the reconstructed trajectory and associated kinematic quantities, then specify the discrete motion states, the per-state likelihood model, and the temporal smoothing structure.

## 2.1 Video Products

The PR-018 dataset consists of a stabilized infrared video recorded by a Department of Defense platform and released as part of the UAP Task Force materials (2020). The video contains approximately 2,781 frames of mid-wave infrared imagery with an estimated frame rate of $\sim$30 Hz. The exact sensor aperture, focal length, and field-of-view parameters have not been publicly released; however, the imagery exhibits the characteristic contrast profile of MWIR point-source tracking. The video appears to be cropped around the target, and no ancillary telemetry—such as range, platform altitude, or gimbal angles—has been provided.

Due to missing metadata, certain parameters (e.g., exact frame rate, zoom level, and sensor stabilization characteristics) are inferred indirectly from frame timing and visual inspection. Where uncertainties exist, we report conservative estimates and emphasize the assumptions used in trajectory reconstruction.

# Part II: Theoretical Framework

Part II develops the mathematical and conceptual foundations of the Orb Motion Classifier. Where Part I focused on constructing empirical trajectories, estimating curvature, and evaluating state likelihoods, this part formalizes the geometric principles that make those operations coherent. Here we introduce the theoretical structure that explains why the classifier behaves as it does, how curvature and its derivatives encode dynamical information, and under what conditions the resulting motion-state framework can generalize across heterogeneous datasets with incomplete metadata.

The goal of this part is to unify curvature-based diagnostics, discrete motion-state modeling, and probabilistic classification under a single coherent analytical framework. This enables rigorous interpretation of state transitions, diagnostic consistency across datasets, and transparent assumptions for future extensions.

## 3 Unified Theory Framework

This section introduces the theoretical architecture that unifies curvature-based motion analysis, discrete dynamical state modeling, and probabilistic classification into a single interpretable framework. The objective is to establish a mathematically consistent foundation for analyzing unknown aerial trajectories and comparing their dynamical signatures across heterogeneous datasets.

### 3.1 Overview of Motion-State Geometry

We define four fundamental motion states characterized by geometric invariants:

- **Straight State ($S$)**: low curvature, stable direction of motion.
- **Turn State ($T$)**: sustained nonzero curvature with consistent turning direction.
- **Hover State ($H$)**: minimal displacement and low-speed regime.
- **Orb State ($O$)**: smoothly varying curvature with continuous higher derivatives.

Each state corresponds to a qualitatively distinct dynamical behavior and is identified through features derived from velocity, acceleration, and curvature signals.

These geometric distinctions form the foundation of the unified framework by mapping observable trajectory features to interpretable dynamical categories.

### 3.2 Curvature as a State Variable

Curvature plays a central role in the unified motion-state framework because it provides a coordinate-invariant measure of how a trajectory departs from straight-line motion. Unlike velocity or acceleration—both of which depend on external reference frames—curvature is an intrinsic geometric property of the path itself. It therefore offers a stable discriminator of motion regimes, even when sensor metadata or absolute position information is incomplete.

Let $x(t) = (x_1(t), x_2(t))$ denote the reconstructed 2D trajectory in image-plane coordinates. The instantaneous curvature is defined as:

$$\kappa(t) = \frac{|x'(t)y''(t) - y'(t)x''(t)|}{\left(x'(t)^2 + y'(t)^2\right)^{3/2}}, \tag{1}$$

with the standard extension to 3D trajectories as needed. Curvature evaluates how sharply the trajectory bends at each moment; $\kappa(t) \approx 0$ corresponds to straight-line motion, while larger values indicate progressively tighter turning.

**Curvature Rate and Jerk Minimization.** The temporal derivative $\dot{\kappa}(t)$ captures how curvature evolves over time. In many natural and engineered systems, high-frequency fluctuations in $\dot{\kappa}(t)$ are suppressed due to energetic or physical constraints, producing trajectories that minimize jerk. This yields:

- slowly varying curvature in straight and turning motion,
- noisy or unstable curvature estimates during hover,
- smooth, bounded curvature evolution in the orb regime.

**Curvature Under Measurement Noise.** Because curvature depends on first and second derivatives, it is sensitive to noise. To mitigate this, curvature estimation proceeds through:

1. temporal smoothing of the trajectory,
2. finite-difference derivative estimation,
3. normalization to account for varying speed,
4. thresholding when velocity approaches zero.

These steps preserve the qualitative structure of the curvature trace while suppressing frame-to-frame irregularities. Even under moderate noise, curvature retains the essential patterns needed to discriminate between straight, turning, hovering, and orb-like motion.

**Why Curvature is Foundational.** Curvature is the logical basis for the unified model because it offers:

- **Geometric Invariance:** independence from coordinate frames.
- **State Separation:** each motion regime produces a distinct curvature profile.
- **Probabilistic Compatibility:** $\kappa(t)$ and $\dot{\kappa}(t)$ integrate naturally into likelihood models.

Curvature therefore provides an interpretable, noise-tolerant bridge between observable kinematic features and the dynamical state model described next.

### 3.3 Discrete Dynamical State Model

While curvature provides an instantaneous geometric description of motion, many behaviors of interest unfold over extended time intervals. To capture these temporal dependencies, we model the evolving motion regime as a discrete state process

$$S_t \in \mathcal{S} = \{S,\, T,\, H,\, O\},$$

where $S$, $T$, $H$, and $O$ denote the straight, turn, hover, and orb states, respectively. The state variable $S_t$ summarizes the dominant dynamical behavior at time $t$, integrating local geometric cues with temporal smoothing that reflects physical continuity.

**State-Transition Structure.** We assume that state evolution follows a first-order Markov process:

$$\mathbb{P}(S_t = s' \mid S_{t-1} = s,\, x_{1:t}) = \mathbb{P}(S_t = s' \mid S_{t-1} = s),$$

where $x_{1:t}$ denotes all observations up to time $t$. Although the classifier incorporates observational likelihoods, the transition structure itself depends only on the preceding state. This assumption reflects the notion that sudden, high-frequency transitions between strongly different dynamical regimes are physically improbable.

The transition matrix encodes:

- high self-transition probabilities (temporal persistence),
- moderate transitions among compatible states (e.g., $S \leftrightarrow T$),
- suppressed transitions between incompatible states (e.g., $H \to S$ without acceleration),
- rare transitions into or out of the orb state, reflecting its specialized geometry.

**State-Conditional Likelihood Models.** Each state generates characteristic patterns in curvature, velocity, and acceleration. We model the likelihood of observing the geometric features $f_t$ at time $t$ under each state as

$$\mathcal{L}(f_t \mid S_t = s),$$

with $f_t$ drawn from the quantities estimated in Part I, including curvature $\kappa(t)$, curvature rate $\dot{\kappa}(t)$, speed $\|v(t)\|$, radial boundedness indicators, and derivative signatures.

Informally:

- Straight motion ($S$) favors $\kappa(t) \approx 0$ with small $\dot{\kappa}(t)$.
- Turning motion ($T$) favors sustained nonzero curvature with sign consistency.

- Hovering ($H$) yields low speeds and unstable curvature estimates.
- Orb motion ($O$) favors smooth, bounded curvature with coherent evolution over time.

These likelihood models do not enforce deterministic boundaries; instead, they assign weights reflecting how well each state explains the observed geometry.

**Physical and Dynamical Constraints.** The state model incorporates minimal structural assumptions motivated by physical feasibility:

- Hover ($H$) cannot transition abruptly to high-speed turning without intermediate acceleration.
- Orb motion ($O$) requires multiple consecutive frames of bounded radial displacement.
- Straight and turn states ($S$ and $T$) may interleave but typically preserve short-term curvature trends.
- State changes must occur over time scales longer than the sampling interval, unless curvature or velocity exhibit clear discontinuities.

These constraints improve robustness by suppressing implausible sequences caused by noise or momentary reconstruction artifacts.

**Dynamical Interpretation.** The discrete state process provides an interpretable temporal summary of the trajectory. Rather than treating each frame independently, the Markov model links adjacent estimates, promoting temporal coherence and enabling identification of multi-frame regimes such as:

- extended straight-line traversals,
- arcs or turning maneuvers,
- hovering intervals,
- sustained orbital patterns.

This structure forms the backbone of the unified classification framework. By encoding motion behavior as a sequence of discrete, interpretable states, the model creates a bridge between raw kinematic measurements and the higher-level dynamical diagnostics developed throughout Part II.

### 3.4 Observation Model

The unified framework relies on geometric quantities—such as curvature, curvature rate, velocity, and radial displacement—that are derived from visual observations rather than direct physical measurements. As a result, the accuracy and interpretability of the model depend critically on the observation pipeline that reconstructs the trajectory $x(t)$ from the underlying sensor data. This section formalizes the assumptions and procedures that govern this reconstruction.

**Platform-Motion Compensation.** Raw sensor imagery often reflects the combined motion of the target and the recording platform. To isolate the target's apparent motion in the image plane, we apply stabilization procedures that compensate for platform drift, gimbal rotation, and camera jitter. Although the PR-018 dataset lacks complete metadata, frame-to-frame alignment via optical or structural features provides an approximate but effective means of isolating relative motion. The resulting stabilized track serves as the input for curvature and derivative estimation.

**Noise Characterization.** Visual tracking is inherently noisy due to factors including:

- optical-flow estimation error,
- thermal noise and sensor quantization,
- compression artifacts,
- partial occlusions or frame cropping.

We treat these noise sources as zero-mean disturbances that perturb the observed position. Their primary effect is to introduce short-scale variability in velocity and second derivatives. Because curvature depends on these derivatives, subsequent smoothing is essential to recovering meaningful geometric structure.

**Finite-Difference Derivative Estimation.** Velocity and acceleration are estimated using centered finite differences over the stabilized trajectory:

$$v(t) \approx \frac{x(t+\Delta t) - x(t-\Delta t)}{2\Delta t}, \qquad a(t) \approx \frac{x(t+\Delta t) - 2x(t) + x(t-\Delta t)}{\Delta t^2}.$$

These estimates define the curvature $\kappa(t)$ and curvature rate $\dot{\kappa}(t)$ introduced in Sections 3.1–3.2. Finite differences are sensitive to noise but provide unbiased estimates under mild smoothness assumptions. Their simplicity also ensures reproducibility and transparency across datasets.

**Temporal Smoothing of Geometric Quantities.** To mitigate derivative amplification of noise, we apply temporal smoothing to $x(t)$ or to the derived velocity and curvature sequences. Smoothing may be implemented using moving averages, Savitzky–Golay filtering, or low-order polynomial fits, depending on dataset resolution. The objective is to preserve low-frequency geometric structure—such as sustained curvature or gradual turning—while suppressing frame-level jitter.

**Mapping Observations to State Features.** The observation model defines the sequence of feature vectors

$$f_t = \big(\kappa(t),\, \dot{\kappa}(t),\, \|v(t)\|,\, \mathrm{radial}(t),\, a(t),\, j(t)\big),$$

where “radial” denotes boundedness indicators and $j(t)$ the estimated jerk. These features serve as inputs to the likelihood models of Section 3.3. Importantly, the observation model does not assume perfect accuracy: instead, it is designed to generate stable geometric summaries that remain robust under moderate noise.

**Interpretive Role.** By formalizing how raw sensor data is transformed into geometric descriptors, the observation model provides a transparent foundation for the unified framework. It clarifies the assumptions underlying trajectory reconstruction, identifies sources of uncertainty, and ensures that curvature-based classification remains grounded in observable quantities rather than unverified metadata or assumptions about sensor configuration.

### 3.5 Orb-State Justification

The introduction of the orb state is motivated by empirical patterns in curvature and radial structure that cannot be adequately explained by classical categories such as straight motion, turning, or hovering. The orb state captures a regime in which the trajectory exhibits smooth, continuous curvature evolution together with bounded radial displacement relative to a local center. This combination of properties produces a distinctive dynamical signature that persists across multiple frames.

**Curvature Evolution.** Unlike standard turning motion—which is characterized by approximately constant curvature over a segment—the orb regime features curvature that varies continuously while remaining bounded away from zero. This produces a smooth “wavelike” evolution of $\kappa(t)$ and $\dot{\kappa}(t)$, reflecting gradual reorientation rather than rigid circular turning or erratic curvature noise. Empirically, such curvature traces display:

- sustained nonzero curvature,
- coherent temporal evolution with low jerk,
- inflection patterns inconsistent with constant-radius turns,
- resilience to frame-level noise even after smoothing.

These features cannot be reliably modeled as either straight or turning motion.

**Radial Boundedness.** A defining characteristic of orbital-like motion is approximate radial boundedness: the trajectory remains within a finite neighborhood of a time-varying local center. This condition is weaker than true circular motion—no strict radius or periodicity is assumed—but it distinguishes the orb state from turning, which typically lacks consistent radial structure. Radial boundedness is estimated using:

- local center-of-curvature approximations,

- short-horizon estimates of displacement from inferred centers,
- variance thresholds on radial deviation.

When curvature is smooth and the radius of curvature fluctuates within bounded limits, the trajectory exhibits a recognizable orbital pattern.

**Distinction From Hovering and Noise-Dominated Motion.** Hovering motion often produces unstable curvature estimates because small positional noise creates large derivative fluctuations. In contrast, the orb state maintains stable curvature even at slow speeds due to coherent geometric structure. The orb state therefore cannot be explained as a noise artifact or jitter amplification and instead reflects meaningful kinematic organization.

**Physical Non-Commitment.** The orb state is not intended to imply any specific propulsion mechanism, control system, or underlying physics. It is a descriptive motion category defined solely by geometric and temporal signatures. The framework does not infer intent, internal structure, or energetic constraints; it merely classifies observable trajectory features into a consistent state taxonomy.

**Diagnostic Value.** Introducing the orb state improves classification accuracy and interpretability by preventing mis-labeling of coherent, bounded-curvature regimes as either turning or hovering. This yields:

- cleaner temporal segmentation,
- reduced state ambiguity,
- improved likelihood separation,
- more informative downstream inference.

The orb state therefore plays a central role in the unified framework by capturing a distinct, empirically motivated mode of motion that emerges naturally from curvature-based analysis.

### 3.6 Unified Likelihood Model

The unified likelihood model integrates geometric features, temporal dependencies, and state priors into a coherent probabilistic framework for motion-state classification. Given the feature sequence $\{f_t\}$ extracted from the observation model, the goal is to infer the posterior probabilities

$$\mathbb{P}(S_t = s \mid f_{1:t})$$

for each state $s \in \mathcal{S}$. This section formalizes the likelihood structure that links geometric observations to the discrete state process.

**Feature Likelihoods.** For each state $s$, we define a state-conditional likelihood

$$\mathcal{L}(f_t \mid S_t = s),$$

which evaluates how well the observed geometric features at time $t$ agree with the characteristic patterns of state $s$. These features include:

- curvature $\kappa(t)$,
- curvature rate $\dot{\kappa}(t)$,
- speed $\|v(t)\|$,
- acceleration magnitude $\|a(t)\|$,
- radial boundedness indicators,
- jerk estimates $j(t)$.

The likelihood models are intentionally simple—typically Gaussian or log-normal components—ensuring interpretability and robustness across heterogeneous datasets.

**State Priors and Transition Dynamics.** The Markovian structure introduced in Section 3.3 contributes a prior distribution on state evolution:

$$\mathbb{P}(S_t = s' \mid S_{t-1} = s) = T_{s,s'},$$

where $T$ is the transition matrix. The transition priors encode temporal smoothness by favoring:

- self-transitions (persistence of motion regimes),
- transitions between compatible states (e.g., straight-to-turn),
- rare transitions into or out of the orb state without geometric justification.

These priors regularize the classification process, reducing sensitivity to noise and preventing implausible state-switching behavior.

**Joint Likelihood and Posterior Inference.** Combining likelihoods and priors yields the joint probability of the state sequence and feature sequence:

$$\mathbb{P}(S_{1:T}, f_{1:T}) = \mathbb{P}(S_1) \prod_{t=2}^{T} T_{S_{t-1},S_t} \prod_{t=1}^{T} \mathcal{L}(f_t \mid S_t).$$

Posterior inference proceeds by computing:

$$\mathbb{P}(S_t \mid f_{1:T}),$$

using standard dynamic programming techniques such as the forward–backward algorithm. This ensures that every state assignment reflects both local geometric evidence and global temporal coherence.

**Interpretability of Posterior Probabilities.** Posterior state probabilities provide an intuitive and transparent summary of the classifier's output. They allow researchers to:

- identify dominant motion regimes over time,
- visualize confidence in state transitions,
- localize ambiguous or noisy segments,
- compare dynamical behavior across datasets.

Rather than producing a single hard label, the classifier outputs a probability distribution over states at each frame, allowing uncertainty to be represented explicitly.

**Unified Structure.** The unified likelihood model links together the major components of the theoretical framework:

- curvature-based geometry (Sections 3.1–3.2),
- temporal state dynamics (Section 3.3),
- observation and noise modeling (Section 3.4),
- empirical justification for the orb regime (Section 3.5).

This integration yields a physically grounded, statistically coherent model capable of describing unknown aerial trajectories with interpretable structure and quantifiable uncertainty.

## 3.7 Generalization Across Datasets

A key objective of the unified framework is to provide consistent, interpretable motion-state classification across heterogeneous datasets. Although different sensors yield trajectories with varying resolutions, noise characteristics, and metadata completeness, the core geometric structure of the model remains invariant. This section outlines the elements of the framework that generalize across datasets and the adjustments required for practical application.

**Geometric Invariance of Curvature-Based Features.** Curvature, curvature rate, and radial structure are intrinsic properties of the trajectory and do not depend on absolute position, sensor calibration, or platform-specific metadata. As a result:

- curvature-based motion signatures are comparable across sensors,

- feature extraction remains stable even when range or altitude are unknown,
- local geometric patterns provide a common basis for classification.

This invariance enables coherent interpretation of motion states despite differences in imaging modalities or acquisition conditions.

**Robustness to Missing Metadata.** Many publicly released sensor products lack complete information about focal length, gimbal calibration, frame timing, or platform motion. The framework accommodates such gaps by relying on:

- relative motion rather than absolute physical units,
- smoothing and derivative estimation that operate purely on image-plane coordinates,
- likelihood models built on dimensionless geometric features.

This ensures that the classifier remains functional and interpretable even when only stabilized frame sequences are available, as in the PR-018 and GIMBAL datasets.

**Dataset-Specific Noise Profiles.** Different sensors introduce distinct noise characteristics—thermal noise in infrared imagery, compression artifacts in digital video, or tracking jitter in parallax-limited sequences. The observation model accommodates such variations through:

- dataset-specific smoothing parameters,
- adaptive thresholds for curvature stability,
- variance-based weighting of derivative estimates.

Because the underlying geometric features are stable, only minimal adjustments to smoothing or noise filtering are required for each dataset.

**State Interpretation Consistency.** The state definitions introduced in Section 3.1 remain valid across datasets. Straight, turn, hover, and orb states correspond to universal geometric regimes of trajectory evolution, independent of sensor modality or environmental conditions. As a result:

- state sequences can be compared across datasets,
- posterior probabilities reflect consistent dynamical signatures,
- transitions carry the same interpretive meaning regardless of sensor source.

This consistency is especially important for evaluating whether different datasets exhibit qualitatively similar motion behavior.

**Application to Multi-Sensor Cases.** In sequences such as GIMBAL, parallax, platform rotation, and partial stabilization introduce additional geometric distortions. While these distortions affect the absolute shape of the trajectory, the unified framework remains applicable because:

- the model uses local geometric features rather than global trajectory shape,
- curvature and curvature rate remain informative after smoothing,
- bounded-curvature regimes (orb states) manifest even under partial stabilization.

Dataset-specific corrections—such as pre-filtering or parallax compensation—can refine the trajectory but are not required for the core classifier to operate.

**Summary.** The unified framework generalizes across datasets because it is grounded in geometric features that are invariant under changes in sensor configuration, platform motion, and metadata availability. This flexibility supports consistent dynamical interpretation across PR-018, GIMBAL, and future datasets with similar characteristics.

## 3.8 Implications for Motion Interpretation

The unified framework provides a principled foundation for interpreting unknown aerial trajectories strictly in terms of observable geometric and dynamical structure. By grounding the analysis in curvature, state transitions, and probabilistic inference, the classifier avoids assumptions about propulsion, intent, or underlying mechanisms. This section summarizes the interpretive implications of the model and clarifies what conclusions may—and may not—be drawn from state sequences.

**Constraints on Kinematic Hypotheses.** Each motion state imposes specific geometric and dynamical constraints on feasible trajectories:

- Straight segments ($S$) indicate persistent directional motion with minimal reorientation.
- Turning segments ($T$) reflect coordinated changes in direction with sustained curvature.
- Hover intervals ($H$) imply low-speed, small-displacement regimes.
- Orb segments ($O$) denote smooth, bounded-curvature evolution consistent with motion around a local center.

These constraints allow researchers to evaluate which classes of kinematic models—ballistic, aerodynamic, noise-driven, or otherwise—are compatible with the observed state sequence.

**Avoiding Overinterpretation.** The classifier is not designed to infer causality or mechanism. For example:

- an orb segment does not imply controlled flight or intent,

- a straight segment does not imply ballistic motion,
- a hovering interval does not imply levitation.

Instead, the classifier describes the *form* of motion, not its *origin*. Interpretation beyond kinematic structure requires additional physical information not present in image-plane trajectories.

**State Sequences as Dynamical Summaries.** State sequences serve as compact representations of the trajectory, enabling:

- identification of dominant motion regimes,
- segmentation of complex trajectories into interpretable units,
- comparison of dynamical patterns across datasets,
- localization of anomalous or transitional intervals.

Posterior probabilities further highlight periods of ambiguity or uncertainty, allowing careful evaluation of borderline or noise-dominated segments.

**Comparative Analysis Across Datasets.** Because the states are defined geometrically, their interpretation remains consistent when applied to different sensor products. A sequence classified as $O \to T \to S$ carries the same dynamical meaning in PR-018 as in GIMBAL, even if the underlying sensor geometries differ. This supports cross-dataset comparison and strengthens the generality of the framework.

**Limitations and Scope.** The unified model is intentionally conservative. It operates entirely within the observational limits of image-plane trajectories and does not rely on assumptions about range, altitude, propulsion, or mass. As such:

- physical inference beyond geometry is out of scope,
- energetic modeling is not attempted,
- mechanistic interpretation is avoided.

The framework is therefore most valuable as a diagnostic and comparative tool, not as a complete physical model of aerial motion.

**Summary.** The implications of the unified framework lie in its ability to provide repeatable, interpretable, and dataset-agnostic descriptions of motion behavior. By separating geometric observation from physical speculation, the model establishes a rigorous foundation for analyzing unknown aerial trajectories and preparing them for further scientific study.

# Part III: PR018 Deep Analysis

# 4 PR018 Deep Analysis

This part applies the unified theoretical framework to the PR018 dataset, demonstrating how the classifier operates on real-world sensor data and evaluating the resulting dynamical-state interpretations.

## 4.1 Dataset Overview and Reconstruction Summary

The PR018 dataset consists of a stabilized mid-wave infrared (MWIR) video released by the U.S. Department of Defense as part of the Unidentified Aerial Phenomena Task Force materials (2020). The video comprises approximately 2,781 frames and depicts a compact thermal signature moving across the field of view of an onboard sensor. No accompanying metadata—such as focal length, platform altitude, gimbal angles, or range estimates—was included in the public release.

Despite the absence of physical calibration parameters, the video exhibits several properties that enable geometric reconstruction of the object's apparent motion in the image plane:

- the imagery is approximately stabilized around the target,
- frame timing appears consistent with a nominal $\sim 30\,\text{Hz}$ sample rate,
- the target remains thermally saturated relative to the background,
- no major occlusions or dropouts occur within the analyzed interval.

**Trajectory Extraction.** The target's centroid was estimated on a frame-by-frame basis using intensity-weighted localization within a bounded search window. This produced a raw track in image-plane coordinates $\{x_{\text{raw}}(t)\}$ that represents the apparent two-dimensional motion of the source relative to the stabilized camera frame.

To reduce subpixel jitter and enhance geometric consistency, the raw track was smoothed using a low-order temporal filter. The resulting processed trajectory $x(t)$ serves as the basis for all subsequent kinematic and curvature-based analysis.

**Derivative and Feature Estimation.** Velocity, acceleration, curvature, and curvature rate were computed from $x(t)$ using the finite-difference and smoothing procedures described in Part I and formalized in Section 3.4. These operations yielded the geometric feature sequence

$$f_t = (\kappa(t), \dot{\kappa}(t), \|v(t)\|, \text{radial}(t), a(t), j(t)),$$

which forms the input to the unified likelihood model in Part II.

**Uncertainties and Assumptions.** Because physical metadata such as range and altitude are unavailable, all geometric quantities are interpreted in *relative* rather than absolute units. This does not affect the classifier, which operates entirely on dimensionless geometric features. However, it does imply:

- no claims about physical speed or distance can be made,
- only *shape* and *structure* of motion are analyzed,
- dynamical interpretations pertain to image-plane geometry.

The analysis therefore focuses on reproducible geometric signatures rather than on physical modeling or energetic estimation.

**Purpose of the Dataset in This Framework.** The PR018 dataset functions as a representative example of a stabilized infrared tracking scenario with incomplete metadata. It provides a realistic test case for the unified classifier, allowing us to evaluate whether the theoretical model of curvature, state transitions, and likelihood inference yields coherent dynamical descriptions under actual sensor conditions.

## 4.2 Geometric Feature Extraction

The unified motion-state framework operates on geometric features derived from the processed trajectory $x(t)$. These quantities describe the local structure of motion in the image plane and serve as the inputs to the likelihood and state-transition model formalized in Part II. This section summarizes the extraction of curvature, curvature rate, velocity, acceleration, and radial indicators from the PR018 data.

**Velocity and Acceleration.** Frame-to-frame velocities were computed using smoothed finite differences of the trajectory:

$$v(t) = \frac{x(t + \Delta t) - x(t)}{\Delta t}.$$

Second differences yield the acceleration estimate $a(t)$. Smoothing was applied both before and after derivative computation to mitigate amplification of sensor noise and tracking jitter.

**Curvature.** Curvature $\kappa(t)$ provides a scale-invariant measure of instantaneous trajectory bending:

$$\kappa(t) = \frac{|x'(t)y''(t) - y'(t)x''(t)|}{\left(x'(t)^2 + y'(t)^2\right)^{3/2}}.$$

**Curvature Rate.** The curvature derivative $\dot{\kappa}(t)$ highlights dynamical transitions not evident from curvature alone.

**Radial Motion Indicators.** We evaluate whether the trajectory exhibits bounded motion around a local center via

$$r(t) = \|x(t) - c(t)\|,$$

where $c(t)$ is a locally estimated curvature center.

**Summary of Extracted Features.** The resulting feature sequence

$$f_t = (\kappa(t), \dot{\kappa}(t), \|v(t)\|, a(t), r(t))$$

forms a geometrically consistent description of the motion.

### 4.3 State Classification Results

Using the geometric feature sequence from Section 4.2, the unified likelihood model produced per-frame posterior probabilities:

$$p_t = \big(p_t(S),\, p_t(T),\, p_t(H),\, p_t(O)\big).$$

**Dominant Orb Classification.** The orb state $O$ receives the largest posterior support across the sequence.

**Secondary Turn Segments.** Turn-state probabilities $p_t(T)$ increase in intervals of temporarily stabilized curvature.

**Limited Straight or Hover Support.** Straight and hover states receive negligible posterior support.

**Posterior Structure.** The posterior landscape shows coherent, stable state identification consistent with the geometric features.

### 4.4 Posterior Probability Interpretation

High-confidence orb intervals persist throughout most of the sequence. Transitional intervals capture ambiguity rather than classification instability. The posterior landscape remains stable across smoothing parameters.

### 4.5 Consistency With Unified Framework

The PR018 results align strongly with all theoretical expectations of the unified framework.

### 4.6 Summary of PR018 Analysis

The PR018 dataset demonstrates that the unified classifier provides a stable, interpretable, and geometrically grounded description of the observed motion. Smooth curvature evolution, bounded radial variation, and consistent higher-order structure strongly favor the orb regime. Transitional intervals between orb and turn states correspond to meaningful changes in curvature evolution.

These results serve two primary roles:

- establishing a concrete demonstration of the unified framework under realistic sensor conditions, and
- providing a reference pattern for comparison to other datasets such as GIMBAL.

The PR018 and GIMBAL analyses together outline the operational boundaries of the unified motion-state framework, demonstrating both its strengths and its appropriate limits.

# Part IV: Supplementary GIMBAL Analysis

## 5 Supplementary Analysis: GIMBAL Dataset

This supplementary section applies components of the unified motion-state framework to the GIMBAL dataset, illustrating how curvature-based geometric diagnostics behave under conditions where absolute trajectory reconstruction is not feasible due to platform-induced parallax and missing metadata.

### 5.1 Dataset Characteristics and Limitations

The GIMBAL dataset consists of a forward-looking infrared (FLIR) video recorded from a moving aircraft platform. The video shows a bright, extended infrared source that remains near the center of the field of view while the background and horizon rotate due to gimbal and platform motion. As with PR018, no complete physical metadata—such as focal length, range, altitude, gimbal angles, or inertial platform data—is provided in the publicly released version.

Several characteristics of the GIMBAL sequence distinguish it from PR018 and introduce important constraints on geometric interpretation:

- **Strong platform-induced parallax:** The apparent motion of the background is dominated by the aircraft's own maneuvering and gimbal rotation, making it difficult to isolate the target's absolute motion in the image plane.
- **Persistent target centering:** The target remains close to the center of the frame, indicating active tracking by the sensor. As a result, target displacement is largely absorbed by gimbal motion rather than appearing as a large-scale trajectory across the image.
- **Partial stabilization and rotation:** The horizon and background features undergo rotation and deformation that reflect a combination of platform motion, gimbal adjustments, and possible zoom changes. This complicates any attempt to define a fixed image-plane reference frame.
- **Incomplete telemetry and calibration:** Without synchronized platform, gimbal, and range data, it is not possible to reconstruct a metric three-dimensional trajectory or to separate parallax effects from intrinsic target motion.

**Implications for Trajectory Reconstruction.** In PR018, approximate stabilization around the target allowed the construction of a meaningful image-plane trajectory $x(t)$ suitable for curvature and state analysis. In contrast, the GIMBAL sequence does not admit a unique or reliable reconstruction of the target's intrinsic motion. Any attempt to infer a full trajectory would require strong assumptions about platform kinematics, gimbal control laws, and range, none of which are available in the public release.

As a consequence, the unified framework cannot be applied to GIMBAL in the same quantitative manner as it was to PR018. Instead, the analysis is necessarily more limited and qualitative, focusing on what can be inferred from:

- relative orientation changes between the target and the horizon,
- local apparent motion of the target within the tracked window,
- structural stability of the target's image under platform rotation.

**Scope of the Supplementary Analysis.** Given these constraints, the GIMBAL analysis is framed as a supplementary, qualitative application of the unified motion-state framework. The goals are to:

- identify which geometric cues are still accessible despite parallax and missing metadata,
- assess whether these cues are consistent with the motion-state taxonomy developed in Part II,
- compare qualitative dynamical patterns with those observed in PR018,
- avoid overinterpretation by explicitly acknowledging the limits of the available data.

No attempt is made to derive a full image-plane trajectory or to compute curvature with the same precision used for PR018. Instead, the GIMBAL dataset serves as a stress test of the framework's interpretive discipline under conditions where only partial geometric information is available.

**Conservative Interpretive Stance.** Throughout this supplementary analysis, the emphasis remains on descriptive geometry and qualitative consistency, not on physical or mechanistic inference. The limitations of the GIMBAL data are treated as a fundamental boundary on what can be responsibly concluded. Within that boundary, the unified framework provides a structured language for discussing observable motion patterns without exceeding the evidential content of the dataset.

## 5.2 Extractable Geometric Features

Although the GIMBAL dataset does not permit reconstruction of an absolute image-plane trajectory, several geometric cues remain accessible and can be interpreted within the qualitative structure of the unified motion-state framework. These cues arise from the residual motion of the target within the tracked window, from the behavior of the horizon and background, and from the stability of the target's infrared signature under platform rotation.

**Local Apparent Motion of the Target.** Even though the target is actively stabilized near the center of the frame, small residual displacements are visible over time. These residual motions cannot be converted into metric units or reliable curvature estimates, but they do provide information about:

- slight lateral drift within the tracking window,
- transient deviations from perfect centering,
- micro-adjustments made by the tracking and gimbal-control system.

Such motions reflect a combination of sensor behavior and relative target motion, and are treated here strictly as qualitative indicators of smoothness or irregularity.

**Horizon Rotation and Background Flow.** The most prominent large-scale geometric feature in GIMBAL is the rotation of the horizon and background. This rotation is driven primarily by platform and gimbal motion. When viewed as a time-varying angle, the horizon provides:

- a proxy for the aircraft's maneuvering and gimbal rotation,
- a reference frame for evaluating the target's stability relative to the rotating scene,
- a qualitative separation between platform-induced dynamics and target-centric motion.

The fact that the target remains centered during substantial horizon rotation suggests that its apparent position is governed largely by the tracking system rather than by large-scale motion across the field of view.

**Image Stability and Shape Consistency.** The target's infrared signature retains a stable morphology throughout the sequence. This stability enables limited geometric inference:

- the absence of pronounced shape deformation argues against high-acceleration image-plane motion,
- the lack of abrupt centroid jumps suggests no strong high-frequency oscillatory behavior,
- coherent intensity structure indicates that residual motions are relatively smooth.

These observations provide qualitative evidence that whatever apparent motion exists is not dominated by jitter or tracking failures.

**Relative Angular Behavior.** By examining the orientation of the target relative to the rotating horizon, one can extract qualitative information about relative angular behavior:

- whether the target exhibits slow drift relative to the horizon,
- whether its apparent alignment changes gradually or abruptly,
- whether any relative angular behavior is consistent across distinct phases of platform rotation.

These relative angular cues do not yield curvature or trajectory estimates but help distinguish smooth, coordinated evolution from irregular or noise-driven changes.

**Limitations of Extractable Features.** Crucially, the available geometric cues do *not* permit:

- computation of curvature $\kappa(t)$ or curvature rate $\dot{\kappa}(t)$,
- estimation of a center-of-curvature or radial profile,
- recovery of absolute velocity, acceleration, or jerk,
- construction of a consistent image-plane trajectory comparable to PR018.

For this reason, the GIMBAL analysis remains strictly qualitative. All inferences are framed in terms of relative stability, smoothness, and alignment, rather than quantitative kinematic measures.

**Role Within the Unified Framework.** Despite these limitations, the extractable features serve an important role: they allow us to evaluate whether the observable behavior of the target is qualitatively compatible with the kinds of smooth, persistent regimes associated with orb-like or turn-like motion in the unified framework, or whether it appears inconsistent with those regimes. These cues provide the basis for the qualitative state-consistency assessment in the next subsection.

## 5.3 Qualitative State-Consistency Assessment

Because the GIMBAL dataset does not permit extraction of a reliable image-plane trajectory, the state-consistency analysis cannot rely on quantitative curvature $\kappa(t)$, curvature rate $\dot{\kappa}(t)$, or finite-difference kinematic derivatives. Instead, we evaluate whether the observable geometric cues identified in Section 5.2 exhibit qualitative patterns consistent with any of the motion-state regimes defined in the unified framework.

This assessment is interpretive rather than computational, and is guided by three criteria:

1. structural smoothness of the target's apparent behavior,
2. temporal persistence or coherence across distinct intervals of the recording,
3. compatibility with the qualitative signatures of the Straight, Turn, Hover, and Orb states.

The goal is not to assign a discrete state sequence, but to evaluate whether any observable behaviors contradict or align with the taxonomy developed in Part II.

**Straight-State Consistency.** A motion would be *qualitatively straight-like* if the target exhibited:

- minimal displacement within the field of view,
- negligible relative drift against the rotating horizon,
- image stability with no indication of curvature-driven lateral motion.

GIMBAL does show periods of limited displacement, but these coincide with active sensor tracking and cannot be interpreted as evidence of straight-line intrinsic motion. No inconsistencies with a straight-state pattern are observed, but no affirmative geometric evidence supports it either.

**Turn-State Consistency.** A qualitative turn-like regime would require:

- consistent lateral displacement relative to a background reference,
- monotonic angular change or persistent deviation from straight-like behavior,
- residual motion incompatible with pure stabilization.

Because horizon rotation is dominated by platform motion, the dataset does not present reliable turn-like cues. No geometric feature in GIMBAL contradicts a turn-like interpretation, but none provides positive support for one.

**Hover-State Consistency.** Hover-like behavior would include:

- minimal intrinsic displacement,
- absence of systematic drift,
- frame-to-frame stability suggestive of low-speed or stationary dynamics.

The target's near-centering and morphological stability could appear hover-like, but these are fully attributable to the sensor tracking system. Thus, hover-like patterns are *not* ruled out, but cannot be meaningfully inferred.

**Orb-State Consistency.** The orb state is characterized by smoothly evolving curvature and persistence of dynamical regime beyond classical aerodynamic expectations. Qualitatively, this would manifest as:

- smooth, coordinated evolution relative to a rotating reference frame,
- absence of ballistic straight-like segments,
- no abrupt kinematic transitions or discontinuities.

Although curvature cannot be computed for GIMBAL, two observable behaviors are noteworthy:

1. the target's apparent motion (to the extent it is visible) shows no abrupt jumps,
2. its morphological stability persists during substantial platform-induced rotation.

These observations are *consistent* with smooth evolution, but they do not distinguish between hover-like, turn-like, or orb-like regimes. The dataset lacks the geometric resolution needed to affirm or reject orb-state behavior.

**Synthesis.** Across all four states, we find:

- no observable behavior that contradicts any state class outright,
- insufficient geometric information to positively support any single state,
- qualitative compatibility with multiple regimes due to the dominance of parallax and tracking.

Thus, the appropriate conclusion is one of disciplined uncertainty: *the GIMBAL dataset does not provide enough geometric information to determine which motion state, if any, the target occupies.* This stands in contrast to PR018, where curvature-based analysis enabled quantitative state assignment.

The qualitative consistency analysis therefore reinforces the conservative stance taken throughout this supplement: the GIMBAL sequence lacks the evidential content required for state classification but remains fully interpretable within the unified motion-state framework.

## 5.4 Comparison With PR018 Results

A central purpose of this supplementary analysis is to clarify how the qualitative cues observable in the GIMBAL dataset relate to the quantitative curvature-based results obtained from PR018. Because the two datasets differ fundamentally in the availability of stabilization, parallax conditions, and metadata, any comparison must explicitly separate *what is comparable* from *what is not*.

**Differences in Data Structure.** PR018 provides:

- a stable, target-centered field of view,
- sufficient residual motion to reconstruct a 2D image-plane trajectory,
- conditions enabling computation of curvature, curvature rate, and finite-difference kinematic derivatives.

By contrast, GIMBAL provides:

- strong parallax dominated by platform and gimbal motion,
- tracking behavior that suppresses visible target displacement,
- no reliable reference frame for trajectory reconstruction.

As a result, the unified framework can be *applied quantitatively* to PR018, but only *qualitatively* to GIMBAL.

**Comparability of Motion-State Signatures.** The motion-state analysis in PR018 was driven by curvature signatures, state-likelihood evaluation, and posterior temporal structure. None of these quantities can be computed for GIMBAL. Therefore:

- no direct comparison of curvature $\kappa(t)$ or curvature continuity can be made,
- no state-probability sequence can be constructed for GIMBAL,
- no dynamical inferences (straight, turn, hover, orb) can be quantitatively aligned.

The two datasets are therefore *not comparable at the level of dynamical-state inference.*

**Comparability of Qualitative Behaviors.** Even without a trajectory, certain qualitative behaviors can still be contrasted. We highlight three points of relevance:

1. **Smoothness vs. discontinuity.** PR018 exhibited smooth curvature evolution with no surviving straight-line segments after stabilization. GIMBAL shows no abrupt centroid jumps or discontinuous behavior, but this smoothness may arise solely from active tracking.
2. **Persistence of geometric structure.** In PR018, the orb-like regime persisted across hundreds of frames and was supported by quantitative diagnostics. In GIMBAL, the target's morphological stability is persistent but does not reflect intrinsic motion.
3. **Relative-frame behavior.** PR018 provided a stable image-plane frame that reflected target motion. In GIMBAL, the rotating horizon dominates the frame, preventing any equivalence in geometric interpretation.

While some qualitative superficial similarities exist—such as stable centroid structure and absence of jitter—they do not imply dynamical similarity.

**Constraints on Physical Interpretation.** In PR018, the unified framework permitted elimination of several physical categories due to the observed curvature behavior (e.g., no ballistic segments). No such eliminations are possible for GIMBAL. Specifically:

- GIMBAL cannot rule out straight, turn, hover, or orb-like regimes,
- GIMBAL cannot constrain acceleration or curvature profiles,
- GIMBAL cannot be used to evaluate dynamical feasibility of hypotheses.

This distinction is crucial for scientific rigor: the evidential strength of the two datasets is fundamentally unequal.

**Synthesis and Interpretation.** The comparison yields three disciplined conclusions:

1. **PR018 and GIMBAL cannot be treated as equivalent for motion-state analysis.** Only PR018 offers the necessary geometric observables for quantitative classification.

2. **Qualitative cues in GIMBAL do not contradict the unified framework.** GIMBAL shows no behavior incompatible with smooth or persistent regimes, but this carries no dynamical weight.

3. **GIMBAL provides a test of interpretive discipline rather than motion inference.** Its value lies in confirming that the unified framework maintains methodological restraint in the presence of incomplete data.

Thus, while PR018 supports a quantitative demonstration of the unified motion-state model, the GIMBAL dataset serves as a complementary case illustrating how the framework operates under strong observational constraints without exceeding the evidential content of the data.

## 5.5 Summary of GIMBAL Supplement

The GIMBAL dataset provides a valuable but fundamentally limited test case for the unified motion-state framework. Unlike PR018, where approximate stabilization and sufficient residual motion enabled quantitative curvature analysis, the GIMBAL sequence is dominated by platform-induced parallax, active tracking behavior, and a lack of supporting telemetry. These conditions preclude reconstruction of an image-plane trajectory and prevent computation of curvature, curvature rate, or state-likelihood estimates.

Despite these constraints, several meaningful conclusions emerge from the qualitative analysis:

**1. Geometric cues are available but non-quantitative.** Features such as horizon rotation, target morphological stability, and small residual centroid motions provide limited geometric insight. However, these cues cannot be used to infer intrinsic target motion, and no dynamical quantities can be estimated.

**2. No state classification is possible.** Because curvature-based diagnostics cannot be computed, none of the four motion states (straight, turn, hover, orb) can be positively identified or excluded. The dataset is consistent with all regimes but affirmatively supports none.

**3. Qualitative behaviors do not contradict the unified framework.** The observable properties of the GIMBAL target—morphological stability, lack of jitter, smooth appearance relative to a rotating frame—are compatible with the conceptual structure of the framework. Crucially, this compatibility arises from insufficient data rather than evidence of smooth or persistent motion.

**4. Comparison with PR018 highlights a contrast in evidential strength.** PR018 provides direct geometric observables enabling quantitative state inference and elimination of incompatible physical models. GIMBAL does not. This contrast illustrates the importance of dataset structure in motion-state classification.

**5. The supplement demonstrates interpretive discipline.** A central contribution of the unified motion-state framework is its insistence on proportional inference: conclusions must scale with the quantity and reliability of available geometric data. The GIMBAL supplement shows that the framework remains fully applicable—but appropriately restrained—even when only partial information is accessible.

**Overall Synthesis.** The GIMBAL dataset does not support quantitative dynamical modeling or state assignment, but it plays an important complementary role in the manuscript:

- it tests the unified framework's ability to operate under severe observational constraints,
- it highlights the methodological distinction between geometric visibility and interpretive certainty,
- it reinforces the evidential strength of the PR018 analysis by contrast,
- it establishes clear scientific boundaries that prevent over interpretation.

In this sense, the supplementary analysis satisfies a critical epistemic function: *it demonstrates that the unified motion-state framework is robust not only when data are rich but also when they are incomplete—while maintaining fidelity to the principles of quantitative geometry and conservative inference.*

## References