Spring 2011
EXST 7039
Assignment 3

1. The file `Bleed.txt`, in `RSplidaAlpha\RSplida_text_data`, contains field failure data on failures in aircraft engine bleed systems (each aircraft has one such system) from a fleet of 2256 military aircraft. Use these data to compute the Kaplan-Meier product limit estimator out to 500 hours of operation. Set this up in a table and do the computation without the aid of a computer, unless you do your own programming (e.g., in R or an Excel spreadsheet). Show the table outlining the computations as part of your solution. You can use available software (e.g. JMP or RSPLIDA) to check your answers.

2. The random variable T has a cdf $\Pr(T \leq t) = 1 - \exp\left(-t/\eta\right).$

   (a) Derive an expression for the pdf of T.

   (b) Derive an expression for the hazard function of T

   (c) Derive an expression for $\mathrm{E}(T)$.

   (d) Derive an expression for $\mathrm{Var}(T)$.

   (e) Derive an expression for the p quantile of T.

3. The natural logarithm of a Weibull random variable has a smallest extreme value distribution. Starting with the Weibull distribution in the traditional parameterization (η and β), show this. Note this can be done in terms of the cdf or the pdf. Try to do it both ways.

4. The reciprocal of a Weibull random variable follows a Fréchet distribution. Starting with the Weibull cdf in the traditional parameterization, derive an expression for the Fréchet cdf.

5. The reciprocal of a loglogistic random variable also has a loglogistic distribution. Show this. Can you make a more general statement about the reciprocal of certain positive random variables?

6. A bearing life T, measured in thousands of cycles, has a distribution that can be described by a linearly increasing hazard function with a slope of θ.

(a) Derive an expression for the cdf of T.

(b) Derive an expression for the pdf of T.

(c) Derive an expression for the quantile function of T.

(d) Provide an expression of the scale parameter of this distribution, as a function of θ.

(e) How would the cdf change if T were measured in units of millions of cycles?

(f) Suppose that the median $t_{0.50} = 200$ thousand cycles. Compute the probability that the bearing will fail between 20 and 30 thousand cycles, given that the bearing survives through 20 thousand cycles. Compute the probability that the bearing will fail between 50 and 60 thousand cycles, given that the bearing survives through 50 thousand cycles.

(g) What do the computations in part (6f) above and the shape of the hazard function say about the nature of the failure mechanism for the bearings?