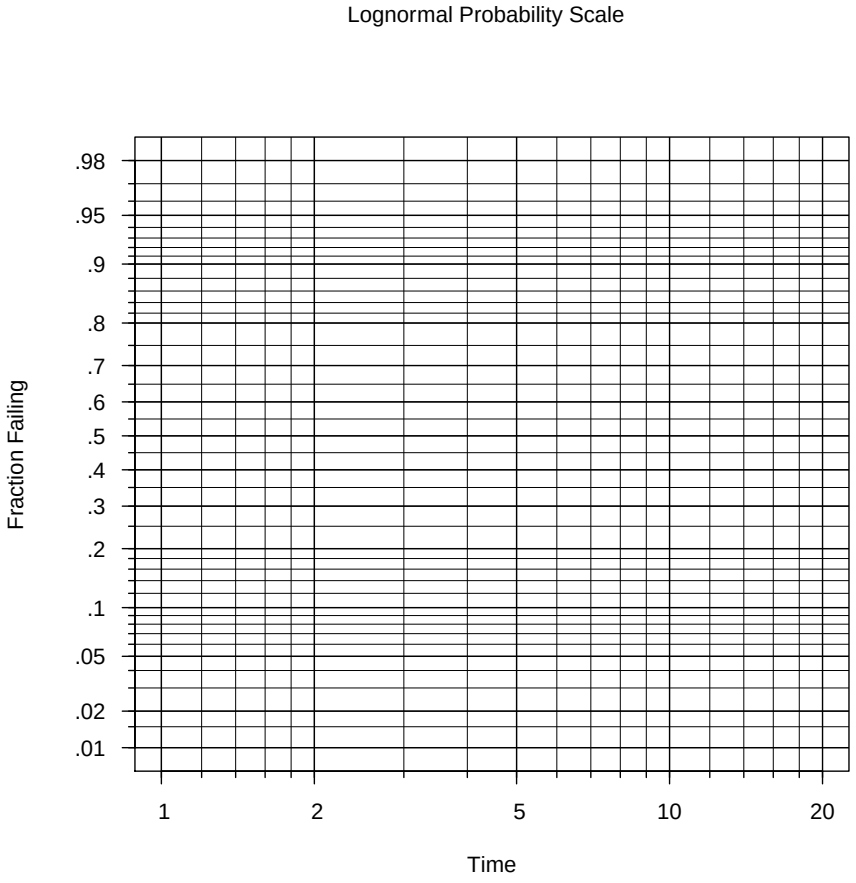
EXST7039 PART 2, SECOND EXAM Fall 2011

Name: _____

This Exam is Individual Work. No Collaboration is Allowed

1. (15 points).
   Consider the probability paper given in Figure 1. Do the following:

Figure 1: Lognormal Probability Plot

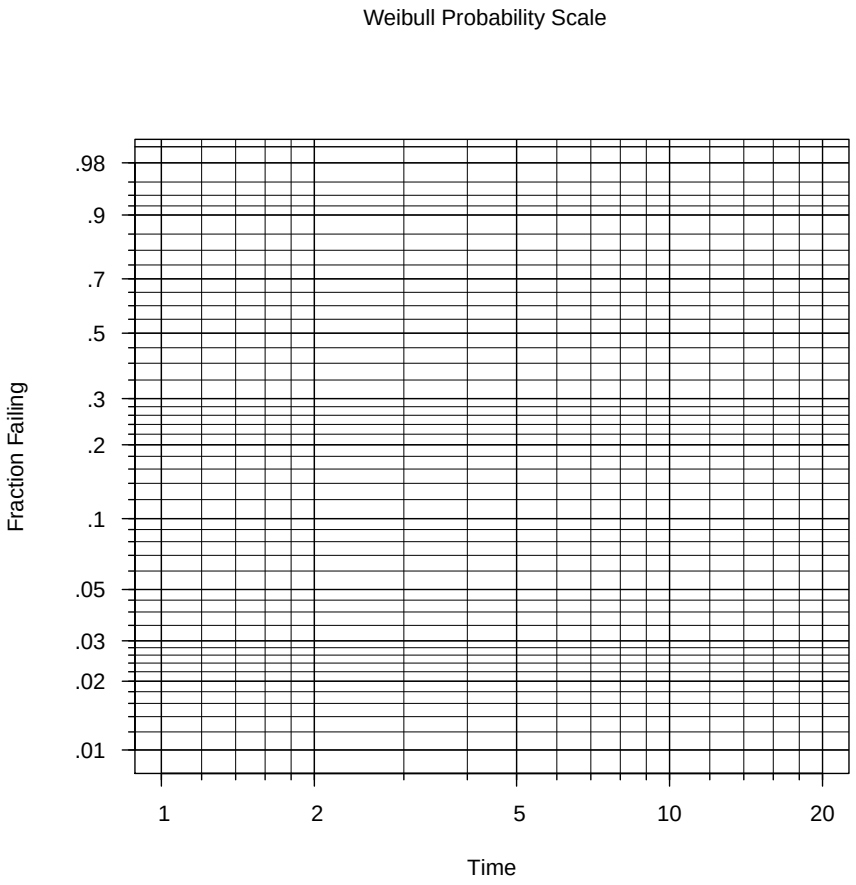


(a) Plot on the paper the LOGNOR($\exp(\mu) = 5, \sigma = 1$). Explain clearly the process to plot the line.

(b) Plot on the paper the LOGNOR($\exp(\mu) = 5, \sigma = 2$). Explain clearly the process to plot the line.

2. (15 points).
Consider the probability paper given in Figure 2. Do the following:

Figure 2: Weibull Probability Plot



(a) Plot on the paper the WEIB($\eta = 10, \beta = 1$). Explain clearly the process to plot the line.

(b) Plot on the paper the WEIB($\eta = 10, \beta = 2$). Explain clearly the process to plot the line.

3. (20 points).
Consider a data set with two observations.

Time	Status
1	Fail
2	Censored

(a) Write the likelihood of the data for a lognormal model.

(b) Write the likelihood of the data for a Weibull model.

(c) Use JMP to fit the data. Provide parameter estimates and a plot of the likelihood contour surface.

4. (30 points).
   A component has two independent failure modes, say Mode 1 and Mode 2, respectively. The independent failure modes can be modeled using the Weibull distributions WEIB(η_1, β_1) and WEIB(η_2, β_2), respectively.

   Define by T the life of the component when the two failure modes are active. From a set of failure time data, the following estimates were obtained from JMP:

$$\widehat{\eta}_1 = 30, \widehat{\beta}_1 = 1$$
$$\widehat{\eta}_2 = 40, \widehat{\beta}_2 = 2$$

   Using the estimates above, do the following:

   (a) Provide an estimate of the survival function $S_T(t)$ of the component.

   (b) Provide an estimate of the cdf $F_T(t)$ of the component.

(c) Obtain an expression for the hazard estimate of T. Your answer must be in function of the hazard functions $h_1(t)$ and $h_2(t)$ for failures from Mode 1 and Mode 2, respectively.

(d) Plot the hazard function of T. Make relevant comments.

5. Use the ShockAbsorber data with failure modes to work on this question.

   (a) Fit the data without using the information on failure modes.

(b) Use a Weibull model for the two modes of failure and the information on failure mode to estimate failure time when the two modes of failure are acting.

(c) Compare the two models fitted above. Explain throughly differences or similarities you observe in the two model fits. For this, it would be convenient to provide graphical comparisons for the failure probabilities and hazard function estimates from the two models.