# Mr. Keynes and the "Classics"; A Suggested Reinterpretation

Gauti B. Eggertsson[†] Cosimo Petracchi[‡]

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**Abstract**

This paper proposes a resolution to the empirical and theoretical controversy between the Keynesians and the monetarists ("classics") applying the tools developed in macroeconomics over the past fifty years. The controversy dates to Keynes' *General Theory of Employment, Interest and Money* (1936)—famously formalized in Hicks' (1937) article where the IS-LM model is first stated—and has been a subject of several empirical studies although never resulting in a definitive conclusion. We first re-evaluate previous empirical work with more recent data, overturning existing empirical findings. We then resolve the controversy by leveraging the Lucas critique, the microfoundations of macroeconomics, and the application of game theory to model government behavior.

[†]Department of Economics, Brown University; gauti_eggertsson@brown.edu.

[‡]Department of Economics and Finance, Tor Vergata University of Rome; cosimo.petracchi@uniroma2.eu.

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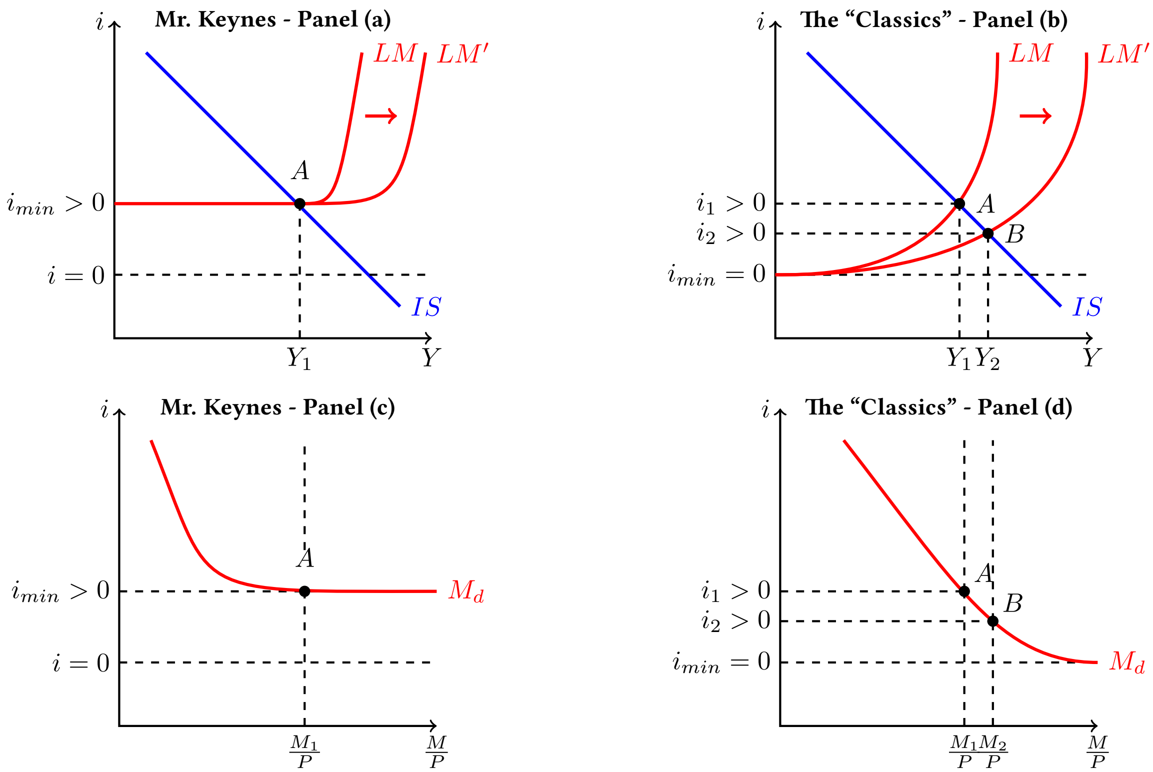



Figure 1: Hicks' suggested interpretation of Mr. Keynes and the "Classics" controversy

# 1 Introduction

This paper applies the Lucas critique, the microfoundations of macroeconomics, and the positive theory of government behavior proposed by Kydland and Prescott (1977) and Barro and Gordon (1983) to study one of the oldest and most famous empirical and theoretical controversy in macroeconomics—the one between the Keynesians and the monetarists. We suggest that more recent data along with the theoretical tools developed over the last fifty years—rational expectations, microfoundations of macroeconomics, and the application of game theory to model government behavior—resolve the controversy.

At a broad level, the main contribution of this paper is to highlight how the scientific revolution in macroeconomics, over the past half a century, allow us to synthesize the early literature and discriminate between different schools of thought. The resolution of the controversy is ultimately startling, and unlike what the participants of the debate anticipated, since it favors neither school of thought: the predictions of either camp can be supported depending on the assumed monetary policy regime, where a policy regime is defined as in Sargent (1982).[1]

[1]Somewhat surprisingly, given the familiarity of the analytic tools we apply, as well as the prominence of the debate in macroeconomics, we are not aware of any work attempting to resolve this classic controversy using more recent data and the modern tools of macroeconomic theory.

Panels (a) and (b) of Figure 1 are arguably the most influential diagrams in macroeconomics. They were published in 1937 in a paper of the same title as this one, except we have replaced "Interpreted" with "Reinterpreted." Panel (a) is John Hicks' interpretation of John M. Keynes' *General Theory of Employment, Interest and Money*. Panel (b) is his characterization of the view of the "classics". Figure 1, and the basic math underlying it, is the backbone of undergraduate macroeconomics teaching to this day. It is known as the IS-LM model.

The key difference between the Keynesian view, Panel (a), and the view of the "classics", Panel (b), is that according to the former view, printing money, which shifts out the LM curve (from $LM$ to $LM'$), has no effect on either output or prices at some positive *long-term* interest rate ($i > 0$). This is because at this interest rate (Point A), the LM curve becomes flat. Meanwhile, according to the "classics", the LM curve remains steep at Point A, and thus monetary policy remains effective. The position that Hicks attributed to the "classics" is essentially the same as the one later associated with the monetarists.[2]

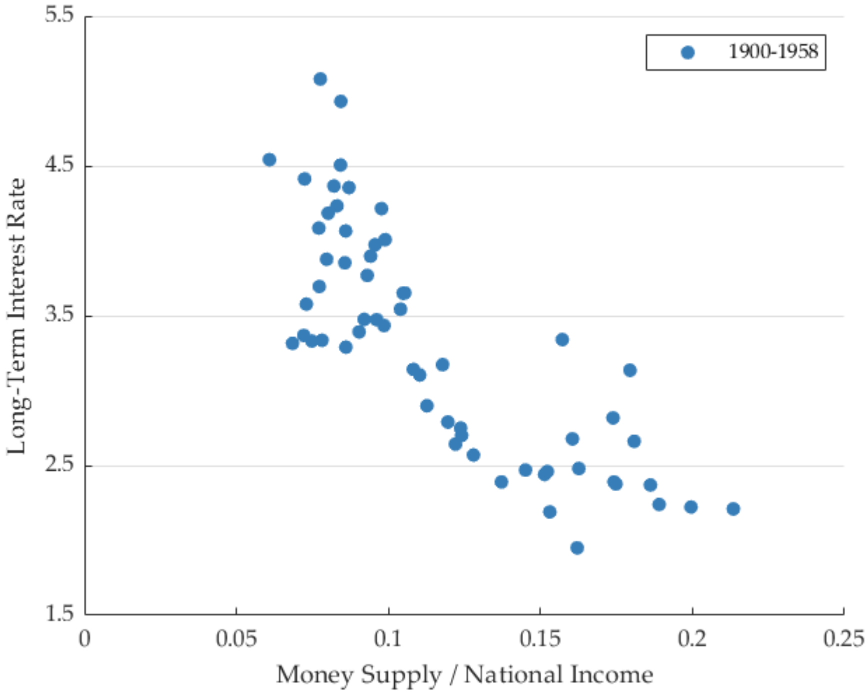
The Keynesian view, Panel (c), predicts that at a certain level of money supply—again denoted by Point A—increasing the money supply (on the x-axis) has no effect on long-term interest rates, even if at Point A long-term interest rates are still positive. But according to the monetarists, Panel (d), there is no such point until all interest rates—short and long—reach zero, at which point money becomes a dominant asset.[3] The monetarists argued that such an environment had never existed in the United States. Long-term interest rates on US corporate bonds, for example, have always been positive in US economic history, both during the Great Depression and the Financial Crisis of 2008. The same applies to long-term interest rates on US government bonds.

We get to the resolution of this controversy, both the theoretical and empirical one, in four steps. First, in Section 2, we survey and replicate the statistical evidence about the existence of a liquidity trap in the literature using the econometric tests by Pifer (1969) and White (1972). This literature ultimately came to the conclusion that there was no evidence for the existence of a liquidity trap using data from 1900-1958. Our first empirical contribution is to show that once we add data from the Great Inflation of the 1970s and the Financial Crisis of 2008, the evidence suggests that a liquidity trap exists *in the sense defined by these authors*.

The statistical power of the more recent data is most easily seen by considering Figure 2. The top panel of Figure 2 is a scatter diagram of long-term government-bond interest rates plotted against the ratio of the money supply to national income in the period 1900-1958, the sample period in this literature. As the top panel of Figure 2 reveals, long-term interest rates fluctuated within a relatively narrow band between 1900-1958. Moreover, the relationship appears close to linear and showing little tendency to asymptote at a low long-term interest rate.

[2]Milton Friedman (1970), arguably the founding father of monetarism, casts the debate between Keynesians and monetarists in the same terms as Hicks, suggesting that "I regard the description of our position that 'money is all that matters for changes in nominal income and for short-run changes in real income' as an exaggeration but one that gives the right flavor of our conclusion." He then highlights that the assumed money-demand elasticity (the slope of the LM curve at Point A) is the fundamental difference between the two schools of thought.

[3]Consistent with this, leading monetarists often argued that for a liquidity trap to be a true trap for policy makers, the entire yield curve would need to be flat, which they suggested had never been observed. See Brunner and Meltzer (1968) for an exposition of this argument, reviewing both theory and empirics. See also Eisner (1963, pp. 532-33).



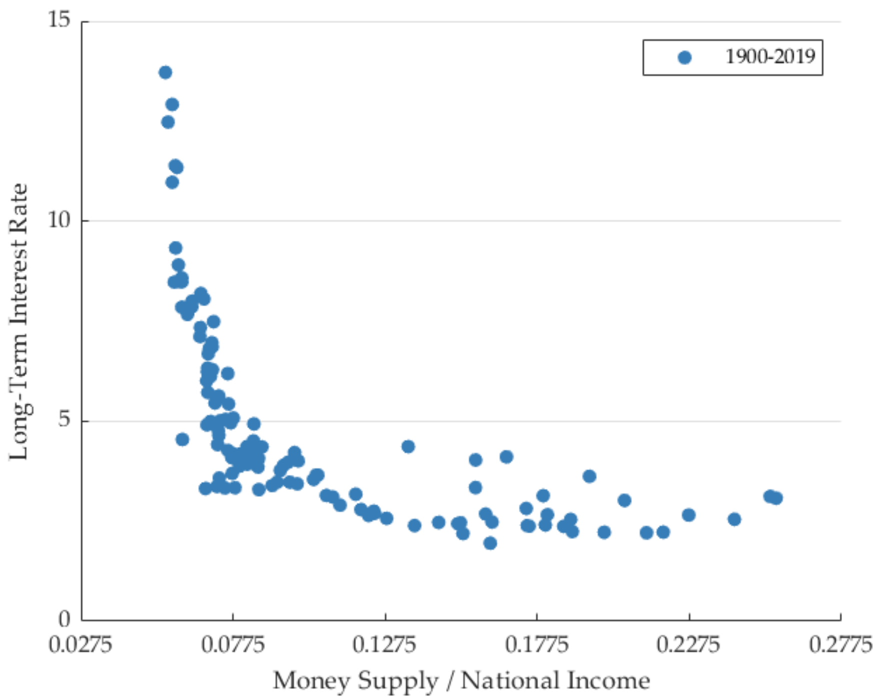




Figure 2: Long-Term Government-Bond Interest Rates versus Money Supply over National Income

In light of this, it is not surprising that the literature found no evidence for a bound on long-term interest rates that was estimated to be statistically different from zero. We confirm this empirical conclusion using the same data from the period 1900-1958. Consider now the bottom panel of Figure 2, representing the data from 1900 to 2019. With the addition of the more recent years, from 1959 to 2019, a curve emerges with obvious non-linearities. Importantly, as the Federal Reserve increased the money supply following the Financial Crisis of 2008, long-term interest rates stopped declining and the curve asymptotes, as predicted by the Keynesians, at a positive long-term interest rate which is estimated to be statistically different from zero around 2 percent, depending on the details of the specification. Has then the controversy been resolved in favor of the Keynesians? According to our theoretical analysis, the answer is no.[4]

The second step of our argument relies on a structural reinterpretation of the econometric tests in light of Lucas (1976). We build a stripped-down dynamic stochastic general equilibrium (DSGE) model. Assuming what we call a Keynesian policy regime the model implies the intersection of the IS and LM curves shown in Panel (a) of Figure 1. In contrast, if we assume what we term a monetarist policy regime, the model replicates the intersection of the IS and LM curves shown in Panel (b) of Figure 1. The key difference between the two monetary policy regimes is that under the Keynesian policy regime, any increase in the money supply is expected to be temporary, while under the monetarist policy regime, any increase in the money supply is expected to be permanent. Accordingly, the relationship between the supply of money supply and the long-term interest rates depends on the policy regime.

Under the Keynesian policy regime the relationship between long-term interest rates and money supply takes the form shown in Panel (c) of Figure 1. Any increase in the money supply has no effect on the long-term interest rates. Our theory suggests that this happens under the condition that the short-term nominal interest rate is at the effective lower bound (ELB). In contrast, under the monetarist policy regime, this relationship takes the form shown in Panel (d) of Figure 1. In this case an increase in the supply of money always decreases long-term interest rates, even once the short-term one is at the ELB, provided the long-term interest rates are positive. The reason is that a permanent increase in the money supply implies a reduction in expected future short-term interest rates. Because long-term interest rates depend on current and expected future short-term interest rates, the long-term ones decline.

The theoretical analysis thus demonstrates that the claims of either the Keynesians or the monetarists can be supported by alternative assumptions about the underlying monetary policy regime. The Keynesians are right in saying that increasing the money supply today is irrelevant once the short-term interest rate is zero, if one implicitly assumes that the central bank cannot commit to future monetary expansions by raising the money supply today.

[4] While the traditional literature on money demand focused on the existence of a liquidity trap, a more recent literature on the stability of money demand emerged following Lucas (1988). The latter literature is not subject to a Lucas critique, because it considers the short-term interest rate, not long-term ones, as the relevant opportunity cost of holding money. If there is a zero lower bound on the nominal interest rate, then money demand defined in this way will always asymptote at the zero, under the assumption there is no storage cost of holding cash. Yet, since this literature does not consider the effect of the money supply on long-term interest rates, or more generally the effect of money on aggregate demand, it does not directly address the controversy between the Keynesians and the monetarists.

Meanwhile, the monetarists are correct in saying that long-term interest rates can always be lowered (as long as they are positive) by increasing the money supply today if one implicitly assumes that by doing so the central bank credibly signals loose future monetary policy. But is there nothing more that can said on this issue? Two obvious questions arise: First, can these two policy regimes be microfounded using established theories of government behavior? Second, if so, can we use the data to discriminate between which theory of the government best describes the data?

The third step of our argument is to propose a positive theory of the Keynesian and monetarist policy regimes, grounded in economic theory. As in Kydland and Prescott (1977) and Barro and Gordon (1983) we posit a government that maximizes social welfare. Importantly, the private sector understands this maximization procedure, and formulates expectations accordingly in a model-consistent way, as proposed by Barro and Gordon (1983). We show that under the assumption of discretion, that is, the government treats "bygones as bygones," the Keynesian policy regime emerges, rationalizing Figure 1, Panels (a) and (c). We then illustrate how a monetarist policy regime can emerge, rationalizing Figure 1, Panels (b) and (d). In sharp contrast to the Keynesian policy regime, the monetarist policy regime relies on the policy commitment. The main conclusion is therefore close in spirit to the central message of both Kydland and Prescott (1977) and Barro and Gordon (1983) which emphasize rules over discretion due to the benefits of policy commitments to improve social welfare relative to policy conducted under pure discretion provided, of course, that these commitments are properly formulated.

The fourth and final step in our proposed resolution of the controversy is to leverage the theoretical framework to show that under two identifying assumptions the data depicted in the bottom panel of Figure 2 traces out a non-structural "quasi-money demand" curve that can be estimated. The quasi-money demand is not structural because it depends upon the assumed policy regime. Our second empirical contribution is to propose a simple alternative empirical approach to that of the earlier literature. The new empirical approach complements the earlier literature and confirms the new empirical finding in Section 2. Our alternative empirical approach has the advantage, however, that the estimated equations flow directly from the theoretical model, hence clarifying the exact conditions needed for identification. Moreover, the theoretical model offers a sharper interpretation of the empirical findings.

The alternative empirical strategy reinforces the results in the first part of the paper—long-term nominal interest rates stop declining in response to increases in the supply of money once the ELB on short-term nominal interest rate is binding. Based on the theoretical analysis, our interpretation of this finding is different from the earlier literature. This empirical result does not prove that monetary policy is powerless as the previous literature claimed to be testing. Instead, it suggests that expectations about future short-term interest rates evolved during this time period *as if* the supply of money was set in a way that is consistent with the Keynesian policy regime. Yet, as can be understood through the prism of the model, this finding does not say that monetary policy is powerless or "trapped". It only says that long-term nominal interest rates stopped responding to static increases in the supply of the stock of money once the ELB became binding.

There remains a fundamental role for monetary policy despite this empirical finding. However, contra the monetarists, the key to successful monetary policy is not static variation in some monetary aggregate, which then feeds into the long-term interest nominal rates and aggregate demand via the quasi-money demand equation. Instead, monetary policy is successful only via the management of expectations about future monetary policy actions once nominal interest rates are no longer constrained by the ELB, which has been a major theme of the modern literature.[5]

# 2 Empirical Estimates for the Existence of the Liquidity Trap

## 2.1 Background

The literature testing for the existence of a liquidity trap typically focuses on US data from the period 1900-1958. This literature originated in an article written by Bronfenbrenner and Mayer (1960), who conclude that "neither the data nor theoretical considerations give any reason for expecting a liquidity trap." This article was followed by objections from Eisner (1963), a rebuttal by Bronfenbrenner and Mayer (1963), and an article by Meltzer (1963*b*), who concludes that "the evidence lends little or no support to the trap." Meltzer's finding was interpreted as suggesting that central banks could always stimulate demand by increasing high-powered money.

As the literature became more mature, starting with Pifer (1969) and following with White (1972), formal statistical tests were employed. The overall conclusion, however, remained the same: the data did not provide evidence in favor of a liquidity trap. Below we replicate this literature and show that more recent data overturn the results.

## 2.2 Estimation Method and Data

The strategy the literature converged upon is best explained by considering a demand function for money:

$$M_t = \frac{\gamma Y_t^{\omega}}{(i_t - i_{min})^{-\alpha}} e^{\xi_t}. \tag{1}$$

Here, $M_t$ is money, $Y_t$ is income, $i_t$ is the long-term interest rate, $i_{min}$ is the rate of interest below which long-term interest rates cannot fall, and $\xi_t$ is an exogenous disturbance.

The empirical question is whether $i_{min}$ is statistically different from zero, but recall from the discussion above that the Keynesian hypothesis was that $i_{min} > 0$ (Figure 1c). Authors interpreted the result that $i_{min}$ was not statistically different from zero as saying the data did not support the existence of a liquidity trap (Figure 1d).

Leaving aside, for now, the classic identification problem, which we return to in the last section before the conclusion, there are several issues that need to be confronted in estimating Equation (1).[6]

[5] Our conclusion does not contradict the hypothesis, for example, that monetary policy played an important role to prevent a more severe recession in response to the Financial Crisis of 2008, either through unconventional monetary policy or via forward guidance.

[6] We leave the discussion of the identification issue to Section 6 once it can be understood more clearly in the context of a structural model.

First, what is the relevant measure of money? Two common measures are M1 (currency in circulation outside of banks as well as bank deposits and close substitutes) and the monetary base (all currency in circulation as well as non-borrowed bank reserves at the Federal Reserve). Second, what is the relevant security for measuring long-term interest rates: long-term corporate or long-term government bonds? Third, what is the relevant income measure? Authors used gross domestic product (GDP), some measure of total assets, or both.

Rather than taking a stance on these questions, we show that the results hold for any combination of these choices.[7] Following the earlier literature, we first present the results using M1 as the measure of money, corporate bonds to measure long-term interest rates, GDP as the measure of income, and an additional measure of total assets. We then extend the results using government bonds to measure long-term interest rates. Lastly, Appendix C documents the results using the monetary base as an alternative measure of money.

We follow Pifer (1969) by formulating the baseline test for whether $i_{min}$ is statistically different from zero, using a nonlinear two-step maximum likelihood estimation. First, for each given $i_{min}$, we run the following regression which is obtained by taking the natural logarithm of Equation (1) and, second, we choose the $i_{min}$ that maximizes the likelihood function:

$$log(M_t) = log(\gamma) + \omega log(Y_t) + \alpha log(i_t - i_{min}) + \xi_t. \tag{2}$$

We then follow White (1972) in extending Pifer's method by considering a more general regression:

$$\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \xi_t. \tag{3}$$

Equation (3) generalizes the specification of Equation (2), which is a special case in which $\lambda \to 0$.[8]

## 2.3 Empirical Results

The first column in Table 1 shows the results for our replication of Pifer (1969), both with and without total assets as an additional explanatory variable, focusing on the sample 1900-1958. This results in an estimate of $i_{min}$ as 2.24. As White (1972) stresses, however, if one considers the more general functional form of Equation (3), the result is no longer statistically significant. White's result is replicated in the second and third columns of Table 1. As shown there, the point estimate of $i_{min}$ is 1.70; however, the 95 percent confidence interval runs from -0.94 to 2.21. This literature thus arrived at the conclusion that there was no empirical evidence for a liquidity trap in the US data.[9]

[7] All the data sources are report in Table 4 in Appendix A.

[8] See Appendix B.1 for the details of Pifer's nonlinear two-step maximum likelihood estimation; importantly, we follow Eisner (1971) to have correct standard errors for the estimate of $i_{min}$. See Appendix B.2 for the details of White's nonlinear two-step maximum likelihood estimation.

[9] These results played an important intellectual backbone to the advise many economists, such as Allan Meltzer, gave Japan in the late 90s that the problem of the zero lower bound on the short-term nominal interest rate could be easily solved by "print money big time".

Table 1: Estimates of $i_{min}$ and $\lambda$ using M1 and Long-Term Corporate Interest Rates

| Time Period | (1900-1958) | | | (1900-2019) | | |
|---|---|---|---|---|---|---|
| Specification | Pifer (1969) | White (1972) | | Pifer (1969) | White (1972) | |
| | $i_{min}$ | $\lambda$ | $i_{min}$ | $i_{min}$ | $\lambda$ | $i_{min}$ |
| $i_t$, $Y_t$ | 2.17<br>(0.01, 2.33)[1]<br>(-1.50, 2.34)[2] | 0.18<br>(-0.09, 0.35)[3]<br>(-0.16, 0.39)[4] | 2.33<br>(0.67, 2.34)[3]<br>(-1.50, 2.34)[4] | 2.30<br>(2.17, 2.33)[1]<br>(2.09, 2.34)[2] | -0.10<br>(-0.16, -0.05)[3]<br>(-0.17, -0.03)[4] | 2.25<br>(2.02, 2.32)[3]<br>(1.93, 2.33)[4] |
| $i_t$, $Y_t$, $A_t$ | 2.24<br>(1.94, 2.32)[1]<br>(1.69, 2.33)[2] | -0.36<br>(-0.60, -0.12)[3]<br>(-0.66, -0.05)[4] | 1.70<br>(-0.94, 2.21)[3]<br>(-1.50, 2.27)[4] | 2.30<br>(2.17, 2.34)[1]<br>(2.09, 2.34)[2] | -0.24<br>(-0.28, -0.19)[3]<br>(-0.30, -0.18)[4] | 2.23<br>(2.10, 2.29)[3]<br>(2.05, 2.30)[4] |

[1] 95%, and [2] 99% confidence intervals, see Hayashi (2000, pp. 52-53); [3] 95%, and [4] 99% confidence intervals, see Greene (2018, p. 554).

Table 2: Estimates of $\lambda$ and $i_{min}$ using M1

| Specification | $\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \delta \frac{A_t^\lambda - 1}{\lambda} + u_t$ | |
|---|---|---|
| | $\lambda$ | $i_{min}$ |
| Government Interest Rates | -0.19<br>(-0.27, -0.08)[1]<br>(-0.29, -0.04)[2] | 1.72<br>(1.36, 1.87)[1]<br>(1.20, 1.89)[2] |
| Corporate Interest Rates | -0.24<br>(-0.28, -0.19)[1]<br>(-0.30, -0.18)[2] | 2.23<br>(2.10, 2.29)[1]<br>(2.05, 2.30)[2] |

[1] 95%, and [2] 99% confidence intervals, see Greene (2018, p. 554). The estimated coefficients with government interest rates are: *constant* = -1.69 (0.261), $\hat{\omega}$ = 0.80 (0.093), $\hat{\alpha}$ = -0.10 (0.008), $\hat{\delta}$ = 0.49 (0.141); standard errors are in square brackets. The estimated coefficients with corporate interest rates are: *constant* = -3.09 (0.202), $\hat{\omega}$ = 0.50 (0.066), $\hat{\alpha}$ = -0.08 (0.004), $\hat{\delta}$ = 1.21 (0.113); standard errors are in square brackets.

The last column of Table 1 shows that later data overturn this result, as confidence intervals are greatly reduced. The point estimate is 2.23 with a 99 percent confidence interval from 2.05 to 2.30. Table 1 focuses on corporate-bond interest rates, whereas we show in Table 2 that the results are unchanged if government bonds are used to measure long-term interest rates. Table 5 in Appendix C presents the results using the monetary base, instead of M1, as the measure of money and showing that results are robust to this alternative measure.

Thus, data from the second half of the twentieth century and the early part of the twenty-first seems to resolve the clash: the Keynesians won. Or did they? That is the question we address next from a theoretical perspective. Our answer is negative. The second half of the twentieth century also witnessed theoretical advances, with the emergence of microfoundations for economic decisions and expectation formation and the incorporation of game theory to model government behavior. Indeed, a more modern analysis shows that the LM curve of the form envisioned by either the Keynesians or the monetarists can emerge under different assumptions about the monetary policy regime.

# 3 A Minimalistic DSGE IS-LM Model: A Classic Lucas Critique

This section presents a minimalistic dynamic stochastic general equilibrium (DSGE) IS-LM model to generate the Hicks' diagrams under different assumptions about the underlying monetary policy regime. The analysis demonstrates that that the statistical test of Section 2 is subject to the classic Lucas critique.[10]

The model can be summarized by the following three equations, detailed microfoundations are in Appendix D. All variables are expressed in terms of log deviation from steady state, with $\hat{Y}_t$ denoting output, $\hat{\imath}_t$ the gross short-term risk-free nominal interest rate, $\hat{M}_t$ the money supply, $\pi_t$ inflation, and $\hat{r}^e_t$ is an exogenous disturbance. The IS equation, derived from the household optimization problem, is

$$\hat{Y}_t = \delta \hat{Y}_{t+1} - \sigma\delta(\hat{\imath}_t - \mathbb{E}_t \pi_{t+1} - \hat{r}^e_t), \tag{IS}$$

where $0 < \delta \leq 1$ is a discounting term and $\sigma > 0$ is the inverse of the coefficient of relative risk aversion. The term $\delta$ arises due to, for example, relative wealth in the utility function as in Michaillat and Saez (2021), overlapping generations as in Eggertsson, Mehrotra, and Robbins (2019), deviation from full rationality as in Gabaix (2020), heterogeneity between borrowers and spenders as in Bilbiie (2021), or the heterogeneous agent new Keynesian (HANK) literature following Kaplan, Moll, and Violante (2018).[11]

Inflation and output satisfy a simple Phillips-curve:

$$\pi_t = \kappa \hat{Y}_t. \tag{AS}$$

where, $\kappa > 0$. We adopt a simpler version of the Phillips curve than is common in the the literature—in which a forward-looking term appears on the right-hand side—to easier the exposition.[12] This equation can be derived under the assumption that firms are monopolistically competitive with a fixed fraction of firms indexing their prices to the previous period's aggregate price level while the remaining fraction set their prices optimally (see Appendix D).

[10] See Benati (2008) for another example of how the Lucas critique applies to (macro) empirical results.

[11] We provide an explicit example of wealth in the utility function in Appendix D, following Michaillat and Saez (2021).

[12] Nothing substantive depends on this assumption as the model is focused on the demand side with price dynamics playing an auxiliary role.

Consider the case in which $\kappa \to 0$ so that prices are completely fixed and $\pi_t = 0$. This is the assumption Hicks (1937) makes when he introduces the IS-LM model.[13] The LM equation is then

$$\hat{M}_t \geq \eta_y \hat{Y}_t - \eta_i \hat{\imath}_t + \epsilon_t^d \text{ and } \hat{\imath}_t \geq i_{elb}, \tag{LM / ELB}$$

where $\eta_y > 0$, $\eta_i > 0$, and at least one of the inequalities holds with an equality at any given time. $\epsilon_t^d$ represents a money demand shock; we set this shock at zero for now and return to it in Section 6. Since the variables are expressed as log deviations from steady state, $i_{elb}$ is negative assuming that the short-term interest rate cannot go below zero. This equation is derived by assuming that real money balances provide transaction services for the household and appear directly in the utility function (Appendix D). At some point, however, households have enough liquidity to satisfy all their transaction needs (that is, they are satiated with the satiation point being denoted by $M^*$), at which point the first inequality is slack and $\hat{\imath}_t = i_{elb}$. Policy maker sets $\hat{M}_t$ via open market operations in short-term risk-free government bonds, and, through this, the short-term nominal interest rate.

The IS equation can be forwarded to yield

$$\hat{Y}_t = -\sigma \mathbb{E}_t \sum_{j=0}^{\infty} \delta^j (\hat{\imath}_{t+j} - \pi_{t+j+1} - \hat{r}^e_{t+j}), \tag{4}$$

illustrating that output depends not just on the current real interest rate but also on the expected future path of the real interest rate. Introducing $\delta$ allows us to consider ELB episodes of arbitrary duration.[14]

We consider the following assumption about the exogenous disturbance:

*Assumption 1.* At time 0, $\hat{r}^e_t = \hat{r}^e_S < 0$; it then reverts to steady state with a fixed transition probability $\mu$ in each of the following periods so that $\hat{r}^e_t = 0$. The stochastic period in which the shock reverts to steady state is denoted $T$. Once the shock reaches steady state, it stays there forever. We call the periods $t < T$ the short run, denoted $S$, and the periods $t \geq T$ the long run, denoted $L$.

One non-conventional property of Hicks assumption, that $\kappa \to 0$, is that money is not neutral in the long run, which goes against the current consensus in the monetary literature. Long-run monetary non-neutrality, however, is not critical for the result. Instead, what matters is that monetary policy still retains power in the period immediately following the reversal of $\hat{r}^e_t$ to steady state.[15]

We consider two types of monetary policy regimes:

*Assumption 2.* Under the *Keynesian policy regime*, then $\hat{M}_t = \hat{M}_L = 0$ for $t \geq T$ and $\hat{M}_t = \hat{M}_S$ for $t < T$ is a

[13] Nothing substantive that follows depends on this assumption, but the more general case is however useful for the discussion in Section 5.

[14] As stressed by Eggertsson, Mehrotra, and Robbins (2019), a permanent reduction in the natural rate of interest cannot be considered in a standard representative-agent dynamic stochastic general equilibrium (DSGE) model. Yet, it is of economic interest to consider the case of a permanent liquidity trap to connect to the earlier literature.

[15] One can alternatively write the model with short run ($t < T$), medium run (defined as the stochastic period $T$), and long run ($t > T$) stipulating that the long-run money is neutral. The result remain unchanged using this extension, even if the algebra needs modest adjustment.

policy choice.

*Assumption 3.* Under the *monetarist policy regime*, then $\hat{M}_t = \hat{M}_S = \hat{M}_L$ for any $t$ is a policy choice.

Under the Keynesian policy regime, if there is a monetary expansion today the central bank is expected to reverse it as soon as economic conditions improve (when the shock $\hat{r}^e_S$ reverts to steady state). In contrast, in the case of the monetarist policy regime, any short-run monetary expansion is expected to be permanent.

Under assumptions 1–3, the endogenous variables take on the same value in all periods $t < T$, denoted by the subscript $S$, and the same value for $t \geq T$, which we denote with the subscript $L$. The expectation of future output, conditional on $t < T$, is then $\mathbb{E}_t \hat{Y}_{t+1} = \mu \hat{Y}_L + (1-\mu)\hat{Y}_S$.

The short-run IS equation is then:

$$\hat{Y}_S = \frac{\mu\delta}{1-\delta(1-\mu)}\hat{Y}_L - \frac{\sigma\delta}{1-\delta(1-\mu)}\hat{\imath}_S + \frac{\sigma\delta}{1-\delta(1-\mu)}\hat{r}^e_S, \tag{5}$$

while the LM equation is

$$\hat{M}_S \geq \eta_y \hat{Y}_S - \eta_i \hat{\imath}_S \text{ and } \hat{\imath}_S \geq i_{elb}. \tag{6}$$

Under the assumption that the LM equation holds with equality in the long run, we can solve for output in the long run to yield

$$\hat{Y}_L = \frac{\delta\sigma/\eta_i}{1-\delta+\sigma\delta(\eta_y/\eta_i)}\hat{M}_L. \tag{7}$$

The IS and LM equations derived above are portrayed in Figure 3. Panel (a) shows the solution under the assumption that $\hat{r}^e_S < i_{elb}$ so that the ELB is binding, while Panel (b) shows the intersection of the two curves when the ELB is not binding—that is, $\hat{r}^e_S = 0 > i_{elb}$. While a shift in the LM curve will increase output when there is no shock, it will do nothing once the shock brings the IS curve to the flat region of the LM curve, as in Panel (a).

At a superficial level, these figures look exactly like the figures in the introduction, with Mr. Keynes corresponding to Panel (a) in Figure 1 and the "classics" to Panel (b) in Figure 1. It would thus seem the data give a straightforward way to discriminate between the two views. Panel (a) suggests that the Keynesian regime reigns when the short-term nominal interest rate is zero.[16]

Yet this clearly is not the correct interpretation of the controversy. Keynes and Hicks were well aware that the short-term interest rate had been close to zero during the 1930s in several countries and could not go below it.

The Keynesian theory of liquidity demand, however, depends on the long-term interest rates. Hicks (1937) argues that the lower bound on the short-term interest rate, in turn, implies that the long-term interest rate also has a bound and that this bound is greater than zero.[17]

[16]Short-term nominal interest rates were close to zero from 1931 to 1947 and from 2008 until December 2015. According to this reading, the economy operated according to the Keynesian logic during the Great Depression and the Great Recession, but according to the "classical" logic in other periods.

[17]To quote Hicks (1937): "In an extreme case, the shortest short-term rate may perhaps be nearly zero. But if so, the long-term rate must lie

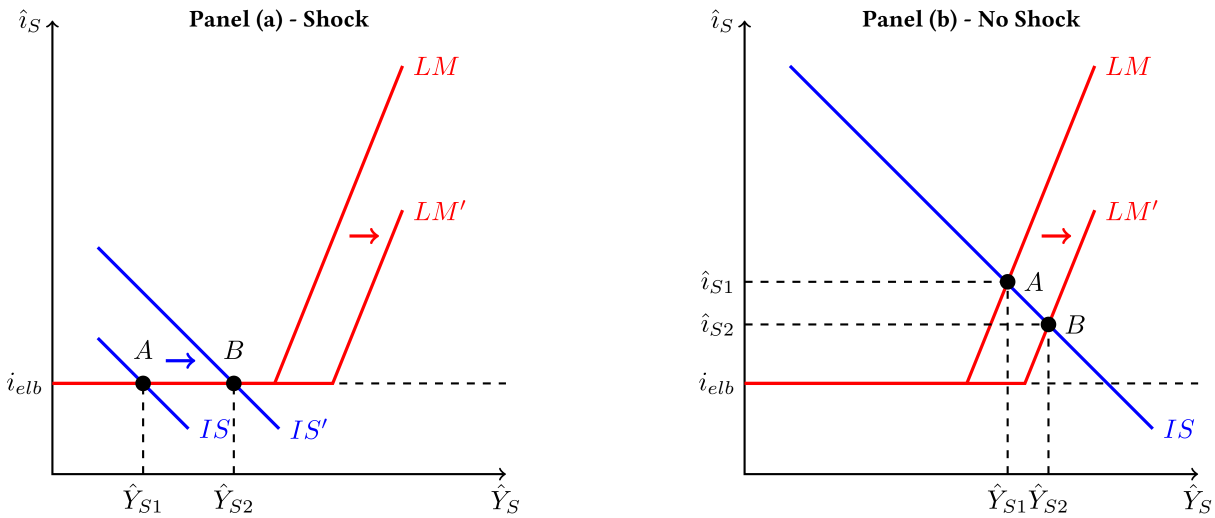



Figure 3: A Minimalistic DSGE IS-LM Model

Hence, to resolve the controversy, one needs to account for the behavior of the long-term interest rate and the extent to which it is insensitive to an expansion in the money supply. Before getting there, however, it is useful to understand a key difference between the model we have just derived, and the classic Hicks' IS-LM model.

As we can see in Equation (4), the IS equation depends on the entire expected future path of the short-term nominal interest rate. Also, the model reveals in Equation (5) that output depends on expected future output, or $\hat{Y}_L$, which in turn is determined by the expectation about the long-term money supply determined by Equation (7).

Panel (a) of Figure 3 shows the effect of a monetary expansion in the short run when there is a shock that puts the intersection of the IS and LM curves at the flat part of the LM curve. What is the effect of a monetary expansion—that is, $\hat{M}_S \uparrow$—assuming either the Keynesian or the monetarist policy regime? Under both monetary policy regimes, the LM curve shifts rightward from $LM$ to $LM'$.

Under the Keynesian policy regime, this is the end of the story. Under the monetarist policy regime, however, the increase in $\hat{M}_S$ also signals a future monetary expansion—that is, $\hat{M}_L \uparrow$—once the ELB is no longer binding. This, however, has no effect on the LM curve. Instead, it shifts out the IS curve because it increases expectations about future income, thus stimulating spending today, as shown by the rightward shift of the IS curve from $IS$ to $IS'$.

It is straightforward to show that the equilibrium is characterized by

$$\hat{Y}_S = \frac{1}{1-\delta(1-\mu)} \left[ \frac{(\mu\delta)(\sigma\delta/\eta_i)}{1-\delta+\sigma(\eta_y/\eta_i)} \hat{M}_L + \sigma\delta(\hat{r}^e_S - i_{elb}) \right], \tag{8}$$

above it, for the long rate has to allow for the risk that the short rate may rise during the currency of the loan, and it should be observed that the short rate can only rise, it cannot fall. This does not only mean that the long rate must be a sort of average of the probable short rates over its duration, and that this average must lie above the current short rate."

with $\hat{M}_L = 0$ under the Keynesian policy regime and $\hat{M}_L = \hat{M}_S$ under the monetarist policy regime. Thus, the monetary expansion is effective under the monetarist regime, while it is not under the Keynesian one.

The effectiveness of the monetary expansion under the monetarist regime is not coming from the short-run money supply increase per se. Instead, it is due to the signal it sends about a permanent monetary expansion in the future, which implies lower future short-term interest rates and higher future output.

# 4 Resolving the Controversy

## 4.1 Long-Term Interest Rates

While the IS equation is a pricing equation for one-period debt, the model in Section 3 can be used to price any asset, such as a loan of infinite duration, whose price is the relevant interest rate according to Keynes and Hicks.

To capture the idea that the long-term interest rate is the price of loan of "infinite duration", define the long-term interest rate as the implied yield, denoted by $i_t^l$, of a perpetuity whose coupon payment declines geometrically at a rate $\rho$. The implied duration of such as bond is given by $(1-\rho\beta)^{-1}$ so by appropriate choice of $\rho$ we can approximate a bond of arbitrary duration. The case in which $\rho = 0$ then correspond to one period risk-free bond, while $\rho = 1$ is a classic consol. This slight generalization of the classic consol allows us to simplify the analytics considerably, by an appropriate choice of $\rho$. Appendix D details the pricing of this bond and how its yield is defined.

The long-term interest rate defined by this consol, is up to a first order approximated by

$$\hat{\imath}_t^l = (1 - \rho\beta/\delta)\mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{\rho\beta}{\delta}\right)^j \hat{\imath}_{t+j}, \tag{9}$$

where $\beta$ is the time-discount factor of the representative household. As suggested by this expression, the long-term interest rate is a weighted average of current and future short-term interest rates. To simplify the analytics, we choose $\rho = \delta^2/\beta$ so that the long-term interest rate corresponds to the yield on a bond with duration $(1-\delta)^{-1}$. [18]

Under these assumptions the IS equation in the short run can be written as

$$\hat{Y}_S = -\frac{\sigma\delta}{1-\delta}(\hat{\imath}_S^l - \hat{r}_S^{e,l}), \tag{10}$$

where $\hat{r}_t^{e,l} \equiv (1-\delta)\mathbb{E}_t \sum_{j=0}^{\infty} \delta^j \hat{r}_{t+j}^e$. Now, one can rewrite the LM equation in terms of the long-term interest rate. As we have documented Hicks, and the literature that followed, assumed that money demand depended on the long-term interest rate. As we will show, however, the formulation of the LM equation in terms of the long-term interest rate takes different shape depending on the assumed monetary policy regime. Accordingly, we will refer to

[18] If we were to further assume that $\delta^2 = \beta$ then this bond is a classic consol.

the resulting relationships as “quasi-money demand” with one corresponding to the Keynesian policy regime while the other corresponds to the monetarist policy regime.

To derive a Keynesian LM curve—that is, a quasi-money demand expressed in terms of the long-term interest rate—we first write the long-term interest rate in the short run as

$$\hat{i}^l_S = \frac{1-\delta}{1-\delta(1-\mu)}\hat{i}_S + \frac{\delta\mu}{1-\delta(1-\mu)}\hat{i}_L. \tag{11}$$

Under the Keynesian policy regime, the interest rate turns to its steady state in the long run so that $\hat{i}_L = 0$. Using Equation (11), the bound on the long-term nominal interest rate is therefore

$$\hat{i}^l_S \geq i^l_{elb,K} = \frac{1-\delta}{1-\delta(1-\mu)}i_{elb}. \tag{12}$$

Note that $i^l_{elb,K} \geq i_{elb}$, the lower bound on the long-term interest rate is higher than the bound on the short-term interest rate if $\mu > 0$. Intuitively, suppose the short-term interest rate cannot go below zero. Equation (12) then says that the long-term interest rate is bounded since, under the Keynesian policy regime, people’s expectations about future short-term interest rates once the shock is over are fixed.

Substituting Equation (11) into the LM equation allows us to derive a Keynesian quasi–LM curve,

$$\hat{M}_S \geq \eta_y\hat{Y}_S - \eta_i\alpha^K_i\hat{i}^l_S \text{ and } \hat{i}^l_S \geq i^l_{elb,K}, \tag{13}$$

where $\alpha^K_i \equiv \frac{1-\delta(1-\mu)}{1-\delta}$. This is not a structural equation, but a theoretical relationship derived conditional on the Keynesian policy regime.

Consider now a quasi-LM relationship in the case of a monetarist policy regime. The key difference is that in this case $i_L$ is no longer fixed at its steady-state value because any increase in the money supply is expected to be permanent—that is, $\hat{M}_L = \hat{M}_S$. Combining the long-run IS and LM equations, we obtain

$$\hat{i}_L = -\frac{1}{\eta_i + [\sigma\delta/(1-\delta)]\eta_y}\hat{M}_S \text{ if } \hat{M}_S < M^* \text{ and } \hat{i}_L \geq i_{elb}, \tag{14}$$

where $M^*$ is the money-satiation point in the long run and $\hat{i}_L = i_{elb}$ if $\hat{M}_S \geq M^*$. This implies that, under the monetarist policy regime, the ELB is only binding when both $i_S$ and $i_L$ reach their lower bound. Accordingly, from Equation (11) we see that the lower bound on the long-term interest rate under the monetarist regime is the same as that on the short-term interest rate—that is, $i_{elb}$.

Using the expression for $\hat{i}_L$ in Equation (14), and using Equation (11) to substitute $\hat{i}^l_S$ for $\hat{i}_S$, in the LM equation we obtain a monetarist quasi–LM curve in terms of the long-term interest rate, given by

$$\hat{M}_S \geq \eta_y \alpha_y^M \hat{Y}_S - \eta_i \alpha_i^M \hat{\imath}_S^l \text{ and } \hat{\imath}_S^l \geq i_{elb}, \tag{15}$$

where $\alpha_y^M \equiv \frac{(1-\delta)\eta_i+\sigma\eta_y}{[1-\delta(1-\mu)]\eta_i+\sigma\eta_y}$ and $\alpha_i^M \equiv \frac{1-\delta(1-\mu)}{1-\delta}\alpha_y^M$.

There are two key differences between the LM curves under the two monetary policy regimes. First, the slope of the LM curve is steeper under the monetarist regime. Second, the bound on the long-term interest rate is lower under the monetarist policy regime and coincides with the bound on the short-term interest rate.

## 4.2 Discussion

The IS and LM equations under the two monetary policy regimes are shown in Figure 4, which replicates the original figure from Hicks (1937). The analysis, however, makes clear that the money-demand equations written in terms of the long-term interest rate are not structural, in the sense of Lucas (1976), but instead depend upon the assumed monetary policy regime. Under what condition is the flat region of the LM curve reached under the two monetary policy regimes?

In the Keynesian policy regime, a sufficient condition is that the current short-term interest rate, $i_S$, reaches the lower bound. In this monetary policy regime, the expectation of the future short-term interest rate once the shock is over, $i_L$, is fixed at the steady-state value. This implies that under the Keynesian policy regime, the term structure will always be upward sloping.

Importantly, however, the lower bound on the long-term interest rate, $i^l_{elb,K}$, depends on the expected duration of the shock that gives rise to the ELB. Hence, if the shock is expected to last longer, then this will automatically reduce $i^l_{elb,K}$ and make the term structure flatter.

In the monetarist regime, however, both $i_S$ and $i_L$ need to reach the ELB for the policy makers to find themselves at the flat region of the LM curve. This implies not only that the short-term interest rate needs to reach zero, but that the entire term structure is flat.

How should one interpret our empirical results in light of the theoretical analysis? The first major empirical implication of the theory concerns Panels (c) and (d) in Figure 4. According to the model, the LM curve estimated in Section 2 is not structural, in the sense of Lucas (1976), but depends upon the assumed monetary policy regime. If the monetary policy regime is Keynesian, then the quasi-LM curve asymptotes at $i^l_{elb,K}$; if it is monetarist, then the quasi-LM curve asymptotes at $i_{elb}$. This analysis then suggests the following reformulation of the Keynesian and monetarist perspectives.

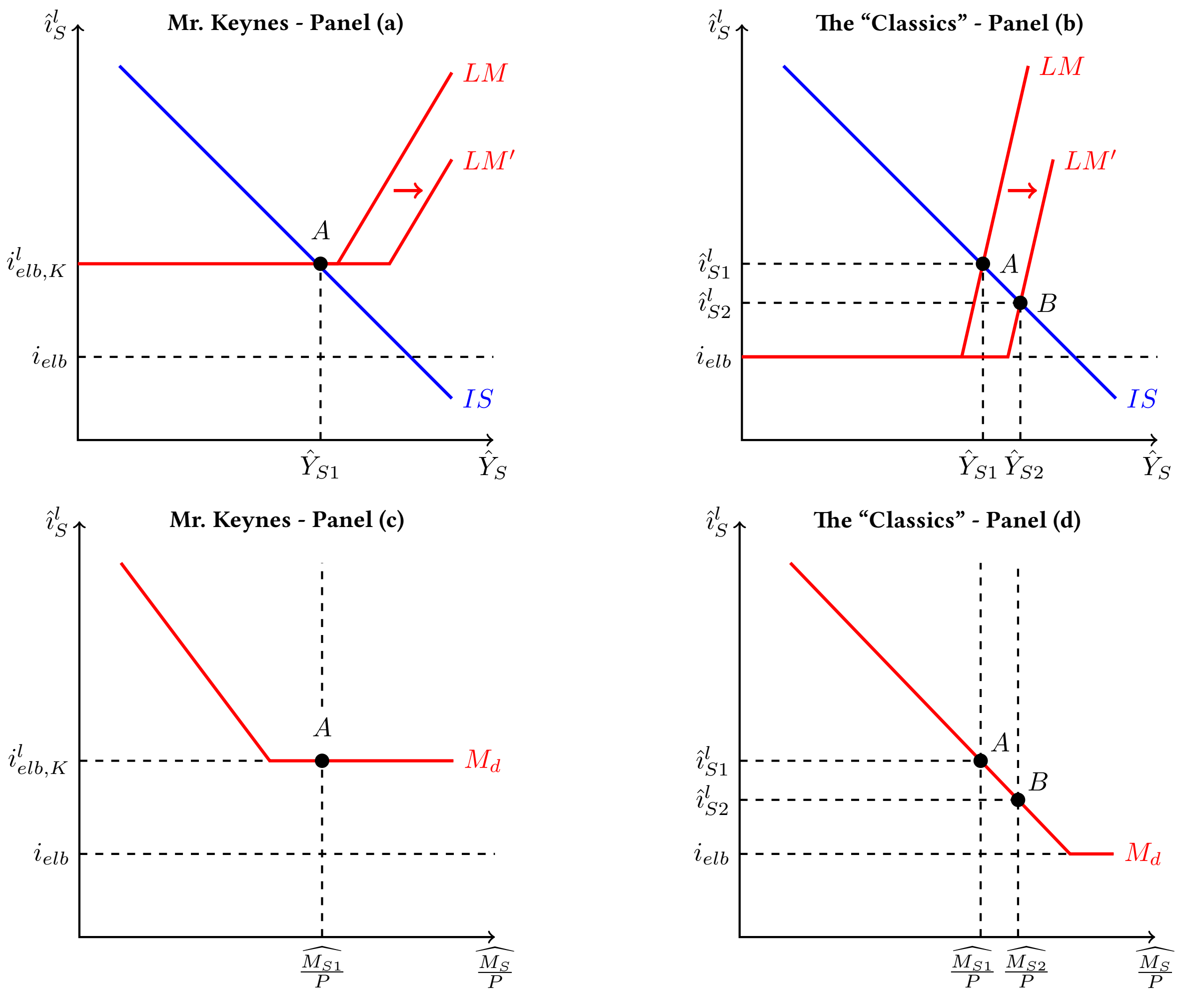



Figure 4: Suggested Reinterpretation

The Keynesians asserted that long-term interest rates could not be lowered further once the short-term interest rate reached zero. This assertion implicitly relies upon the assumption that the central bank cannot commit to a future monetary expansion: an increase in the money supply today does not signal future monetary expansion. Hence increases in the money supply, beyond a certain point, leave long-term interest rates unchanged.

Meanwhile, in contrast, the monetarists asserted that long-term interest rates could always be lowered even when the short-term interest rate reached zero provided that long-term interest rates were still positive. To do so the central bank simply needed to increase the money supply. This assertion implicitly assumes that the central bank, by increasing the money supply today, is credibly signaling loose monetary policy in the future.

### 4.3 Corporate Bonds

All the relationships above were derived under the assumption that the measure of interest rate is a risk-free bond rate (typically measured by government bond rates). The same relationships can be derived using an interest rate that incorporates a risk of default (typically measured by corporate bond rates) which we denote by $i_t^d$.

Consider a loan contract according to which there is a $\omega_{t+1}$ probability of default in period $t+1$. As we show in Appendix D, this interest rate satisfies

$$\hat{\imath}_t^d = \hat{\imath}_t + \mathbb{E}_t \hat{\omega}_{t+1},$$

where $\mathbb{E}_t \hat{\omega}_{t+1}$ is the risk premia in deviation from steady state.

The same quasi-LM curve can now be derived by adjusting both the interest rates and the lower bound by the risk premia. The model now predicts that the lower bound on the long-term risky interest rate is higher than for the risk-free rate. The exact same analysis, however, applies in this case.

## 5 Interpretation of the Monetary Policy Regimes

We now turn to the interpretation of the monetary policy regimes, using the approach of Kydland and Prescott (1977) and Barro and Gordon (1983). Consider a government authority which maximizes the following social welfare criterion subject to the IS, AS, LM, and ELB constraints, taking expectations as given:[19]

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda \hat{Y}_t^2). \tag{16}$$

First, consider the maximization problem of the government authority in the long run, once the ELB is no longer binding. The assumption in Barro and Gordon (1983) is that the government authority will disregard any previous policy announcements while, at the same time, taking expectations as a given function of any state variables—that is optimal policy under discretion. Since the model has no endogenous state variables, the maximization problem of the government authority boils down to a static maximization problem. Assuming an optimal policy under discretion as in Barro and Gordon (1983), the first-order condition of the government authority's maximization problem is then:

$$\pi_t + \frac{\lambda}{\kappa} \hat{Y}_t = 0. \tag{17}$$

This condition, together with the AS equation, implies that $\pi_t = \hat{Y}_t = 0$ for $t \geq T$. This in turn implies that $\hat{M}_t = 0$ for $t \geq T$. We can thus write $\pi_L = \hat{Y}_L = \hat{M}_t = 0$ for $t \geq T$. This is exactly the assumption we have under the Keynesian policy regime ($\hat{M}_L = 0$). Hence a natural microfoundation for the Keynesian policy regime is that it

[19] This social welfare criterion can be derived via a second-order approximation of the household utility function.

corresponds to the optimal policy under discretion, in the same sense as proposed by Kydland and Prescott (1977) and Barro and Gordon (1983)—that is, the case in which the government cannot commit to future policy.

What is a plausible interpretation of the monetarist policy regime? It is well known in the literature that the optimal policy under commitment is to commit to future expansion of inflation and output, or equivalently commitment to higher future money supply. The monetarist regime is then one example of a policy regime in which the government can make a credible commitment about future policies. More explicitly, it presumes that a monetary expansion in the short run necessarily implies a permanent increase in the money supply and a commitment to future inflation.[20]

# 6 Empirically Identifying Quasi-Money Demand

The early literature estimating the money demand, surveyed in Section 2, assumed there exists a well-defined money demand that is "stable." This is the fundamental identifying assumption of the monetarists. As stressed by Friedman (1956), 'quantity theorists accept the empirical hypothesis that the demand for money is highly stable.' Here, the term "stable" refers to the assumption that shocks to the money demand are not "too important" as long as the time period under consideration is sufficiently long. Indeed, seasonal fluctuation in money demand were already well recognized at the time, which is why the literature focused on data at annual frequencies.[21]

Estimating quasi-money demand presents a classic identification problem: we only observe the intersection of demand and supply which generates data on prices and quantities over time. Yet, movements in either demand or supply can change prices and quantities. Nevertheless, in this section, we show two assumptions under which the quasi-money demand is empirically identified. We illustrate how identification is achieved by showing a simple alternative regression to the ones proposed in the earlier literature. While the regression is less flexible in its functional form than those in Section 2, it clarifies how the quasi-money demand can be empirically identified as it is directly derived from the theoretical model in Section 3 under the two identifying assumptions.

The monetarist claim that money demand was "stable", suggested that variations in money demand do not play an important role, at least when considering data at annual frequency. Thus, the literature we replicated in Section 2 implicitly assumed that it was instead the supply that was moving around over time. This, the argument went, suggested the data traces out the demand for money. In the theoretical model of Section 3, this argument can be used to empirically identify the *quasi-money demand* by making two assumptions. First, suppose the the natural rate of interest $\hat{r}_t^e$ is the primary shock perturbing the economy. Second, suppose the money supply is determined according to optimal policy under discretion. This second assumption implies that the money supply is set to stabilize

[20]To be clear, however, the monetarist policy regime is only one example of a monetary policy regime where the policy maker can signal a future monetary expansion.

[21]We also focus on data at annual frequencies in Section 2. Moreover, in this spirit, Robert Lucas in his Nobel prize lecture took this approach one step further: he averaged annual inflation rates over thirty years and contrasted them to the growth rates of money over the same time period.

the output gap whenever possible—$\hat{Y}_{S,t} = 0$—this is what we termed the *Keynesian policy regime*.

Relative to our earlier assumption, we incorporate two additional sources of shocks—in addition to the stochastic process for the natural rate of interest $\hat{r}^e_{S,t}$—to show how to identify the quasi-money demand: a money demand shock, $\epsilon^d_{S,t}$, and a time-varying transition probability, $\mu_t$, conditional on the ELB being binding. The money demand shock—$\epsilon^d_{S,t}$—follows the same two-state Markov process as the natural rate of interest—$\hat{r}^e_{S,t}$—reverting back with probability $\mu_t$ to the absorbing state given by the steady state. We assume, however, that the exogenous variables are not correlated with one another in the short run, but satisfy the martingale property—that is, $\mathbb{E}_t \hat{r}^e_{S,t+1} = \hat{r}^e_{S,t}$, $\mathbb{E}_t \epsilon^d_{S,t+1} = \epsilon^d_{S,t}$, and $\mathbb{E}_t \mu_{t+1} = \mu_t$.

Following the same steps as before, we obtain the expression for the Keynesian quasi–LM curve as in Section 3. The important difference is that the variables vary stochastically. If $\hat{i}^l_{S,t} \geq i^l_{elb,K}$ then:[22]

$$\hat{M}_{S,t} = \eta_y \hat{Y}_{S,t} - \eta_i \alpha_i^K \hat{i}^l_{S,t} + \epsilon^d_{S,t}, \tag{18}$$

where $\alpha_i^K \equiv \frac{1-\delta(1-\mu)}{1-\delta}$. Since $\hat{Y}_{S,t} = 0$, we can re-arrange the equation to obtain:

$$\hat{i}^l_{S,t} = -(\eta_i \alpha_i^K)^{-1} \hat{M}_{S,t} + (\eta_i \alpha_i^K)^{-1} \epsilon^d_{S,t}. \tag{19}$$

This equation is only satisfied if $\hat{i}^l_{S,t} \geq i^l_{elb,K}$. However, $i^l_{elb,K}$ is no longer fixed and it is instead given by:

$$i^l_{elb,K} = \frac{1-\delta}{1-\delta(1-\mu_t)} i_{elb}, \tag{20}$$

which may vary because we allow for $\mu_t$ to vary with time. Equation (20) implies that the ELB on the long-term nominal interest rate depends on the expected duration of the ELB, on the short-term rate, governed by $\mu_t$. If the expected duration of binding ELB in the short-run increases, the ELB on the long rate would decreases. If people expect the ELB on the short-term rate to be permanently binding, then the ELB on the short-term and long-term rates coincide.

Since $\hat{i}^l_{S,t} \equiv \log(1 + i^l_{S,t}) - \log(1 + \bar{i}^l)$, then equations (19) and (20) can be summarized by a simple regression:

$$\log(1 + i^l_t) = \beta_0 + \beta_1 D_t \log(M_t / Y_t) + \beta_2 D_t + \xi_t, \tag{21}$$

Here, $i^l_t$ is the long-term nominal interest rate in levels, $M_t$ is money in dollar value, $Y_t$ is nominal income, and $\xi_t$ is a exogenous disturbance. One identifying assumption is that the central bank targets the output gap. The role of nominal income is to express the supply of money in real terms.

[22] Conditional on $t < T$, we once again have that $\mathbb{E}_t \hat{Y}_{t+1} = \mu_t \hat{Y}_L + (1 - \mu_t)\hat{Y}_{S,t} = (1 - \mu_t)\hat{Y}_{S,t}$ by using the martingale property.

Table 3: Empirical Estimates

| | $\beta_0$ | $\beta_1$ | $\beta_2$ |
|---|---|---|---|
| Government Interest Rates | 1.34 | -0.88 | -1.74 |
| | (1.23, 1.45)[1] | (-1.00, -0.76)[1] | (-2.05, -1.43)[1] |
| | (1.19, 1.48)[2] | (-1.03, -0.72)[2] | (-2.15, -1.34)[2] |

[1] 95% and [2] 99% confidence intervals are in square brackets.

We use a dummy variable, $D_t$, that takes the value 0 when the ELB is binding, in which case Equation (20) applies, and the value 1 when the ELB is not binding, in which case Equation (19) applies.

Table 3 reports the regression results using the data from the bottom panel of Figure 2 that showed the relevant data for the period 1900-2019.[23] The main finding is an estimate of $\beta_0$ which is statistically different from zero with a 99 percent confidence interval from 1.19 to 1.48. This parameter is the constant of the regression when the ELB is binding (i.e., $D_t = 0$). It represents the estimated value of $i^l_{elb,K}$ which is in log units. Converted to long-term nominal interest rate, it suggests an ELB on the long term interest rate of 2.82 percent with a 99 percent confidence interval from 2.29 to 3.39 percent. This is consistent with the estimated $i_{min}$, the "floor" for the long-term nominal interest rates, using the regressions proposed in the earlier literature.[24]

The top panel of Figure 5 graphically presents the piecewise linear regression line (in logs), using the estimated values from Table 3. This is useful to understand why the two identifying assumptions correctly identify the quasi-money demand. The key identifying assumptions are that the natural rate of interest—$\hat{r}^e_t$—is the main shock perturbing the economy, and that the money supply is set so as to offset these shocks. If there are only shocks to the natural rate of interest, and if the central bank offsets them with monetary policy (as implied by the Keynesian policy regime), then the regression will exactly trace out the quasi-money demand implied by the model as shown by the black dots.

Consider money demand shocks—$\epsilon^d_{S,t}$—and its effect on the estimated quasi-money demand. On upward sloping part of the quasi-money demand, a shock to money demand results in the data moving away from the estimated red line. The band around the black dot on the far left of the figure shows one standard deviation of the residuals of the estimated regression. The residuals of the regression when $D_t = 1$ represent shocks to money demand according to the model. A small standard deviation provides suggestive evidence in favor of the hypothesis that shocks to money demand are sufficiently small for the quasi-money demand to be correctly identified.

[23] We pose that the ELB is binding when the effective federal funds rate is below 0.25, corresponding to 1934-1936, 1938-1941, and 2009-2015.

[24] As reported in Appendix C, the estimate of $i_{min}$, the "floor" for long-term nominal interest rates using long-term government bonds, is 1.69 percent. The lower estimate, using the approach of the earlier literature, is primarily driven by the fact that the lowest value of the long-term nominal interest rates, observed in the data, plays a critical role in the estimation procedure.

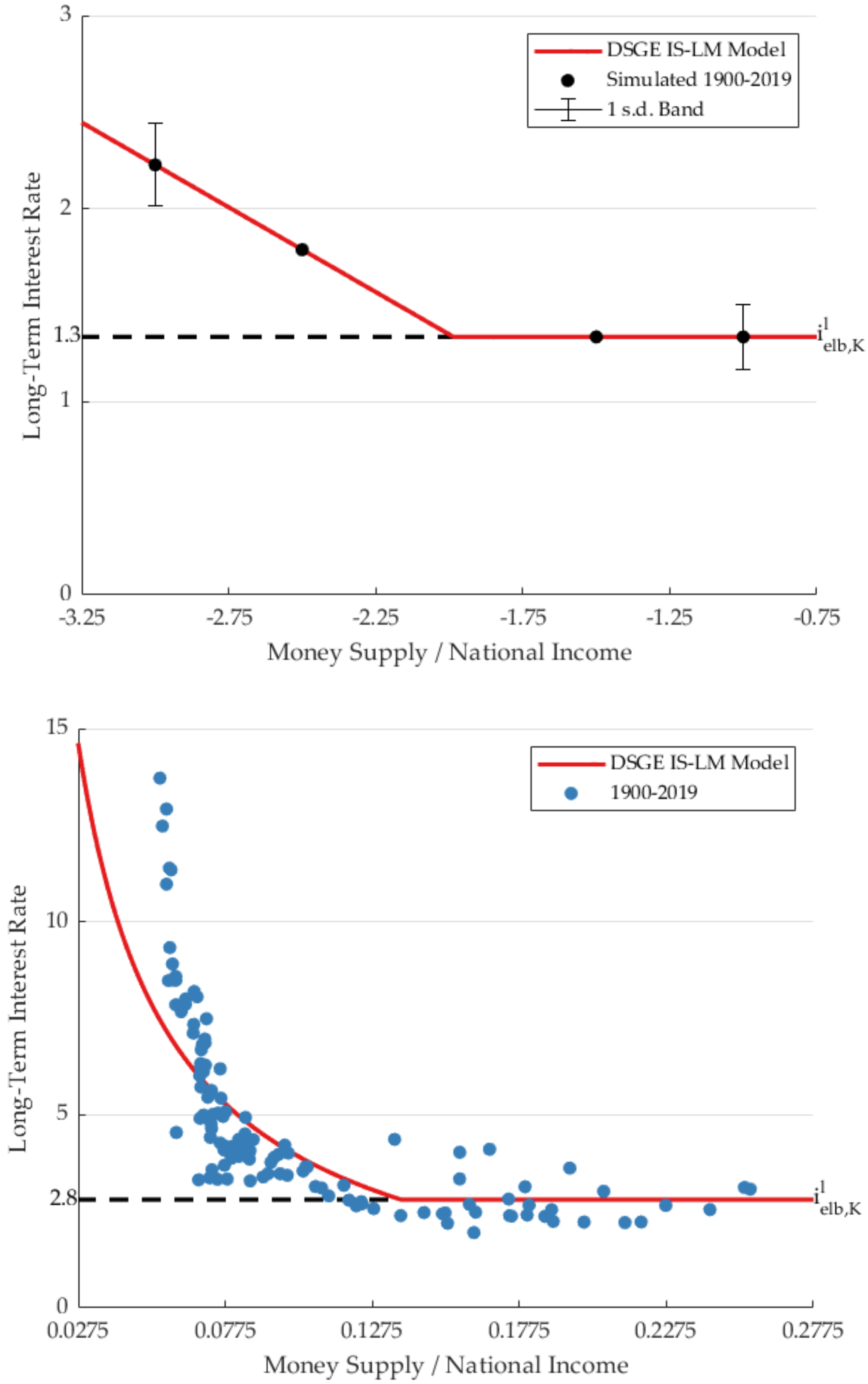



Figure 5: Long-Term Government-Bond Interest Rates versus Money Supply over National Income

Consider now the flat part of the the quasi-money demand. At this segment, demand for money is indeterminate, because money and government bonds with zero interest rate are perfect substitutes. This means that a shock to the demand of money has no effect. As evident from Equation 20, however, changes in peoples beliefs about the duration of the ELB being binding, for the short-term nominal interest rate, moves the estimated $i^l_{elb,K}$, i.e., the lower bound of the long-term nominal interests rate. The residuals of the estimated regression when $D_t = 0$ now have the theoretical interpretation of representing changes in the expected duration of the ELB. The band on the far right of the figure shows one standard deviation of the residual of the regression when $D_t = 0$.

The second part of Figure 5, presents the piecewise non-linear regression line (instead of its logarithmic counterpart), implied by the estimation, against the actual data. The fit is challenged by the restriction of a logarithmic functional form of the regression, which is more restrictive than the estimation procedure in Section 2. This restriction does not fully account for the steepness of the curve once inflation rose to double digits in the 1970s.[25]

The model analysis, however, fundamentally changes the interpretation of the finding that $i^l_{elb,K}$ is estimated to be greater than zero. That this number is positive is not evidence of that monetary policy is incapable of stimulating aggregate demand. Instead, it is evidence of a Keynesian policy regime which implies that even if the supply of money is increased at the ELB, markets expect the supply of money to be contracted as soon as as the natural rate of interest recovers. Accordingly, we think the most plausible interpretation of our empirical finding is that they suggest that expectations were consistent with a Keynesian monetary policy regime. Large increases in money supply at the ELB do not trigger significantly higher expectation of higher future money supply, and accordingly, do not in isolation provide any additional increase in demand, contrary to the monetarist contention.[26]

Suppose, instead, that $i^l_{elb,K}$ is not statistically different from zero. What would be the interpretation of that finding? The earlier literature suggested it meant that "there is no evidence in favor of a Keynesian liquidity trap." In the last section, we suggested that one rationalization of the monetarist perspective is what we termed a monetarist regime. Yet it is worth clarifying, that an econometrics finding that $i^l_{elb,K}$ is not statistically different from zero is *not* evidence in favor of a monetarist regime relative to a Keynesian one. To see this, consider Equation (20). We have already pointed out that $i^l_{elb,K}$ may collapse close to zero while still maintaining the assumption of a Keynesian policy regime. This happens if markets expect that the duration of the ELB for short-term nominal interest rate to be very persistent. In this case, an econometrician cannot reject the null hypothesis that the $i^l_{elb,K}$ is zero.

[25]A higher order approximation of the model may capture this non-linearity. We leave this for future work, as it is not essential to the interpretation of the results.

[26]This, of course, does not exclude the possibility that the central bank might have changed expectations about future interest rates via forward guidance of future interest rates.

# 7 Conclusion

A narrative emerged following the 2008 Financial Crisis that the standard IS-LM model was “good enough for government work” to use a popular saying. The argument was that despite the revolution in macroeconomics over the last half a century, the Hicksian IS-LM model provides a sufficient policy framework—one that, in some cases, is even superior to modern analysis, typically based upon dynamic stochastic general equilibrium (DSGE) models.[27]

This narrative is not totally far-fetched. After all, those wedded to the old paradigm correctly anticipated, following the Financial Crisis of 2008, that budget deficits would not lead interest rates to soar, that the massive increase in monetary aggregates would not lead to inflation, that an increase in government spending would lead to more than a one-to-one increase in output, and that the turn of some governments during this period to fiscal austerity would be counterproductive. None of these predictions were obvious, and many economists armed with more modern modeling frameworks were led to take the opposite view, which arguably was less in tune with how things turned out. While the empirical resolution to some of these questions is far from settled, several prominent commentators promoted this narrative to suggest macroeconomics had gone widely off track over the last half a century.

Yet this narrative glosses over a fundamental issue: the IS-LM model, as proposed by Hicks, left unanswered a fundamental question of Keynes' *General Theory of Employment, Interest and Money.* Is monetary policy unable to stimulate demand in a recession, while fiscal policy retains its power (per Keynes/Keynesians)? Or is monetary policy effective at fighting recessions, while fiscal policy is not (per “classics”/monetarists)? Hicks' analysis was consistent with either answer and suggested that which scenario applies depends on the elasticity of money demand. Moreover, several authors later argued in a series of articles that an empirical test could be brought to bear on this question, using Hicks original IS-LM framework. The literature converged on the view that there was no evidence of a liquidity trap based upon the IS-LM framework, using data from 1900 to 1958.

The main contribution of this paper is to give a clear illustration of the general value of the revolution in macroeconomics over the last fifty years. It is hard to think of a better example than Hicks' original formalization of Keynes' *General Theory of Employment, Interest and Money* to make this point. In Hicks' analysis, there is no way of determining what, exactly, is the correct measure of interest rates, how short-term interest rates are related to long-term ones, how the risk-free interest rate is related to risky ones, and so on. Even more distressingly, it is not even obvious how one can resolve these controversies in principle.

The key problem is that the older literature does not have clear criteria for how to answer these questions. In contrast, the modern literature offers a clear way forward. Since modern dynamic stochastic general equilibrium (DSGE) models are derived by fully specifying the environment agents live in and how they make decisions, the researcher can simply ask the agents that live inside the model. The fact that the model we sketched out here is

[27] This argument is, for example, forcefully put forth by Krugman (2018).

derived from microfoundations does allow one to conclusively answer how each variable is related to another and provides a systematic way of documenting how these relationships change depending on the assumptions.

As we have documented, more recent data overturns existing empirical results. This would seem to allow the Keynesians to snatch victory from the monetarists, at least according to the old interpretation of this literature. Yet our theoretical analysis shows that the money-demand equation this literature studied is not structural, to use the language of modern general equilibrium theory. This result, of course, is the major theme of the modern literature on the liquidity trap, see for example Krugman (1998) and Eggertsson and Woodford (2003). The modern treatment of expectations, a critical omission of the IS-LM model readily admitted by Hicks, as well as a positive theory of how monetary policy is determined are the pivotal element needed to understand both the effect of monetary and fiscal policy on aggregate demand, as well as giving a meaningful interpretation of existing empirical evidence.[28] It is difficult to imagine how all these insights could have been obtained without the development of the microfoundations of macroeconomics over past half a century, and the understanding of how to construct positive models of government policy.

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[28] Modigliani (1944, p. 56) also highlights the limits of the IS-LM model and the role of expectations: "In the diagram we have assumed that there is a single rate of interest r, instead of a whole system of rates for loans of different duration. While it may be assumed that in principle all the rates tend to move in the same direction, we must bear in mind that the extent to which a change in the supply of money changes the rates on loans of different maturities depends on the character of interest expectations."

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# Appendix for

# Mr. Keynes and the "Classics"; A Suggested Reinterpretation

# Appendix A - Data Sources

Table 4: Data Sources

| | (1900-1958) | (1959-2019) |
|---|---|---|
| M1 ($M_t$) | *Historical Statistics of the United States*[1] | Federal Reserve Economic Data[1] |
| Corporate Interest Rates ($i_t$) | *Historical Statistics of the United States*[2] | *Historical Statistics of the United States* and Federal Reserve Economic Data[2] |
| National Income ($Y_t$) | *Historical Statistics of the United States and Federal Reserve Economic Data*[3] | Federal Reserve Economic Data[3] |
| Assets ($A_t$) | *A Study of Saving in the United States* and Board of Governors of Federal Reserve System[4] | Board of Governors of Federal Reserve System[4] |
| Government Interest Rates ($i_t$) | Jordà-Schularick-Taylor Macrohistory Database and Federal Reserve Economic Data[5] | Federal Reserve Economic Data[5] |
| Monetary Base ($M_t$) | Jordà-Schularick-Taylor Macrohistory Database[6] | Jordà-Schularick-Taylor Macrohistory Database and Federal Reserve Economic Data[6] |

[1] From 1899 to 1957, Series X 267 in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960—original source of Pifer (1969) and White (1972); for 1958, Series Cj49 in *Historical Statistics of the United States, Millennial Edition Online*, Cambridge University Press, Cambridge (England), 2006; from 1959 to 2019, 'M1SL' Series in Federal Reserve Economic Data.

[2] From 1900 to 1957, Series X 346 (20-year maturity) in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960—original source of Pifer (1969) and White (1972); from 1958 to 1975, Series Cj1241 (20-year maturity) in *Historical Statistics of the United States, Millennial Edition Online*, Cambridge University Press, Cambridge (England), 2006; from 1975 to 2019, 'AAA' Series (20-year-and-above maturity) in Federal Reserve Economic Data.

[3] From 1900 to 1957, Series F 7 in U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957*, Washington, D.C., 1960; from 1958 to 2019, 'NICUR' Series in Federal Reserve Economic Data.

[4] From 1900 to 1949, the series has been constructed following Meltzer (1963*a*), which is the original source of Pifer (1969) and White (1972): "Total assets (A) were taken from Goldsmith, [R. W. Goldsmith, *A study of Savings in the United States*, Princeton University Press, Princeton N.J., 1956], Tables W-9 through W-16, and were available for eight benchmark dates from 1900-1949. The series used in the estimates for 1900-1945 was obtained by taking the ratio of total assets, A, to wealth, G (from Table W-1). Denote this ratio for any base year as $b_0$. Let $b_n$ = the next base year ratio, and let $i = 0, 1, \ldots, n$ be an index of the years between base year dates inclusive of the end points. Then $A_i = [\frac{(n-i)}{n} b_0 + \frac{i}{n} b_n]$."; from 1950 to 2019, the series has been built summing the Series FL102000005.A, FL112000005.A, FL152000005.A, FL212000095.A, FL312000095.A, and FL792000095.A in the Financial Accounts of Board of Governors of Federal Reserve System.

[5] From 1900 to 1953, 'ltrate' Series (10-year maturity) in Jordà-Schularick-Taylor Macrohistory Database; from 1954 to 2019, 'GS20' Series (20-year maturity) in Federal Reserve Economic Data.

[6] From 1900 to 2016, 'narrowm' Series in Jordà-Schularick-Taylor Macrohistory Database; from 2017 to 2019, 'BOGMBASE' Series in Federal Reserve Economic Data.

# Appendix B - Estimation Methods

## B.1 - Estimation Method in Pifer (1969)

Pifer (1969) proposes the following nonlinear two-step maximum likelihood method to estimate $i_{min}$:

$$
\begin{aligned}
\max_{i_{min},\gamma,\omega,\alpha} \quad & \mathcal{L}(y; i_{min}, \gamma, \omega, \alpha) = \max_{i_{min}} \left[ \max_{\gamma,\omega,\alpha} \mathcal{L}(y; \gamma, \omega, \alpha | i_{min}) \right] \\
\textit{s.t.} \quad & \mathcal{L}(y; \gamma, \omega, \alpha | i_{min}) = -e'e, \\
& y = log(M_t) = log(\gamma) + \omega log(Y_t) + \alpha log(i_t - i_{min}) + \xi_t.
\end{aligned}
$$

The first step is constructing a grid for $i_{min}$ and, for each possible value of $i_{min}$, running the above regression to calculate the sum of squared residuals.[29] The second step is minimizing such sum, that is maximizing the likelihood function $\mathcal{L}$, to identify the maximum likelihood estimate of $i_{min}$.

[29] We construct a grid for $i_{min}$ from -1.5 to the minimum value of the series of long-term government-bond (corporate-bond) interest rates, in increments of 0.01.

### B.1 - Estimation Method in White (1972)

White (1972) proposes the following generalization of Pifer 's estimation method to estimate $i_{min}$:

$$\max_{\lambda, i_{min}, \gamma, \omega, \alpha} \quad \mathcal{L}(y; \lambda, i_{min}, \gamma, \omega, \alpha) = \max_{\lambda, i_{min}} \left[ \max_{\gamma, \omega, \alpha} \mathcal{L}(y; \gamma, \omega, \alpha | \lambda, i_{min}) \right]$$

$$\textit{s.t.} \quad \mathcal{L}(y; \gamma, \omega, \alpha | \lambda, i_{min}) = -\frac{T}{2} log(e'e) + (\lambda - 1) \sum_{t=1}^{T} log(M_t),$$

$$y = \frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i_{min})^\lambda - 1}{\lambda} + \xi_t.$$

The first step is constructing a two-dimensional grid for $\lambda$ and $i_{min}$ and, for each possible point in this grid, running the above regression to calculate to calculate the corresponding likelihood function $\mathcal{L}$.[30] The second step is maximizing the latter to identify the maximum likelihood estimate of $i_{min}$.

## Appendix C - Results with the Monetary Base

Table 5: Estimates of $\lambda$ and $i_{min}$ using the Monetary Base

| Specification | $\frac{M_t^\lambda - 1}{\lambda} = \frac{\gamma^\lambda - 1}{\lambda} + \omega \frac{Y_t^\lambda - 1}{\lambda} + \alpha \frac{(i_t - i^{min})^\lambda - 1}{\lambda} + \delta \frac{A_t^\lambda - 1}{\lambda} + u_t$ | |
|---|---|---|
| | $\lambda$ | $i^{min}$ |
| Government Interest Rates | -0.07 | 1.69 |
| | (-0.12, -0.03)[1] | (1.35, 1.84)[1] |
| | (-0.13, -0.02)[2] | (1.21, 1.86)[2] |
| Corporate Interest Rates | -0.13 | 2.13 |
| | (-0.18, -0.08)[1] | (1.84, 2.26)[1] |
| | (-0.19, -0.07)[2] | (1.72, 2.28)[2] |

[1] 95%, and [2] 99% confidence intervals, see Greene (2018, p. 554). The estimated coefficients with government interest rates are: *costant* = -4.16 (0.281), $\hat{\omega}$ = 0.48 (0.124), $\hat{\alpha}$ = -0.35 (0.017), $\hat{\delta}$ = 0.83 (0.138); standard errors are in square brackets. The estimated coefficients with corporate interest rates are: *costant* = -6.47 (0.332), $\hat{\omega}$ = -0.02 (0.130), $\hat{\alpha}$ = -0.26 (0.013), $\hat{\delta}$ = 1.89 (0.169); standard errors are in square brackets.

[30] Respectively, we construct a two-dimensional grid for $\lambda$ and $i_{min}$ from -1.5 to 1.5 (excluding 0) and from -1.5 to the minimum value of the series of long-term government-bond (corporate-bond) interest rates, in increments of 0.01.

# Appendix D - Model Derivation

## D.1 - Households

There is a continuum of identical households of measure 1. Household $j$ solves the following maximization problem:

$$\max_{\{C_t(j),M_t(j),B_t(j),B_t^r(j),B_t^l(j),N_t(j)\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ u\left(C_t(j)\right) + \chi\left(\frac{M_t(j)}{P_t}\right)\xi_t^d + w\left(\frac{A_t(j)-A_t}{P_t}\right) - v\left(N_t(j)\right)\right]\xi_t$$

$$\begin{aligned}
\textit{s.t.} \quad & P_tC_t(j) + M_t(j) + B_t(j) + B_t^r(j) + S_tB_t^l(j) = \\
& = W_tN_t(j) + M_{t-1}(j) + (1+i_{t-1})B_{t-1}(j) + (1-\omega_t)(1+i_{t-1}^r)B_{t-1}^r(j) + \\
& + (1+\rho S_t)B_{t-1}^l(j) + \int_0^1 Z_t(i,j)di, \\
& A_t(j) = M_t(j) + B_t(j) + B_t^r(j) + S_tB_t^l(j), \\
& i_t \geq 0.
\end{aligned}$$

Here, $\beta$ is an intertemporal discount factor, $C_t(j) \equiv \left[\int_0^1 c_t(i,j)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is the aggregate consumption, $M_t(j)$ is the amount of dollars that the household holds at time $t$, $B_t(j)$ is a risk-free nominal bond that pays $i_t$ numbers of dollars at time $t+1$, $B_t^r(j)$ is a nominal bond that pays $i_t^r$ number of dollars at time $t+1$, but with probability $\omega_{t+1}$ it will not be repaid, $B_t^l(j)$ is a perpetuity that pays $p^j$ dollars in period $j+1$ and $S_t$ is its price, $N_t(j)$ is the labor supply that the household offers, $\xi_t^d$ is a money demand shock, and $\xi_t$ is a preference shock.

The function $u$ is the period utility of consumption, it is increasing and concave in its argument and at least twice differentiable.

The function $\chi$ denotes the period utility of holding real money balances, it is increasing and concave in its argument. Define the real money balance $m_t \equiv \frac{M_t}{P_t}$, we assume that there is satiation at $m^*$ so that, the partial derivative of the function $\chi$, $\chi_m(m_t) = 0$ for $m_t \geq m^*$.

The function $w$ represents the period utility that the household has from its asset holding, $A_t(j)$, relative to the aggregate asset holding in the economy $A_t$, as in Michaillat and Saez (2021).

Finally, $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ denotes the aggregate price index, $W_t$ denotes the nominal wage rate, and $Z_t(i)$ are the profits of firm $i$.

In equilibrium, all households hold the same assets so that $A_t(j) = A_t = 0$. We substitute this equilibrium conditon in the optimality conditions below, to simplify the notation and also omit reference to $j$.

The solution to the household maximization problem can be obtained by formulating a Lagrangian problem. Combining the first-order conditions of the Lagrangian problem results in the Euler equation, the money demand equation, the asset pricing equations, and the labor supply equation:

$$u_c(C_t) = \beta(1+i_t)\mathbb{E}_t\left[u_c(C_{t+1})\frac{\xi_{t+1}}{P_{t+1}}\right]\frac{P_t}{\xi_t} + w_A(0), \tag{EE}$$

$$\frac{\chi_m(m_t)}{u_c(C_t)}\xi_t^d \geq \frac{i_t}{(1+i_t)} - \frac{i_t}{(1+i_t)}\frac{w_A(0)}{u_c(C_t)}; i_t \geq 0, \tag{MD}$$

$$u_c(C_t) = \beta(1+i_t^r)\mathbb{E}_t\left[(1-\omega_{t+1})u_c(C_{t+1})\frac{\xi_{t+1}}{P_{t+1}}\right]\frac{P_t}{\xi_t} + w_A(0), \tag{AP1}$$

$$u_c(C_t) = \beta\mathbb{E}_t\left[(1+\rho S_{t+1})u_c(C_{t+1})\frac{\xi_{t+1}}{P_{t+1}}\right]\frac{P_t}{\xi_t} + w_A(0), \tag{AP2}$$

$$\frac{v_N(N_t)}{u_C(C_t)} = \frac{W_t}{P_t}. \tag{LS}$$

## D.2 - Asset Pricing of the Perpetuity

The duration of the perpetuity is defined as

$$D \equiv \frac{\beta\sum_{j=0}^{\infty}(j+1)(\beta\rho)^j}{\beta\sum_{j=0}^{\infty}(\beta\rho)^j}, \tag{22}$$

while its yield at time $t$ is the interest rate $i_t^l$ which solves the equation:

$$S_t = \sum_{j=0}^{\infty}\frac{\rho^j}{(1+i_t^l)^{j+1}}, \tag{23}$$

implying that:

$$1 + i_t^l = S^{-1} + \rho. \tag{AP3}$$

## D.3 - Firms

There is one firm for each good $i$ which faces the demand function $y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$, where $Y_t$ denotes aggregate output. The demand function for each firm is implied by the optimal spending decision of the households across good types. We assume that a fixed fraction of firm $\gamma$ set their prices flexibly, while the remaining fraction $1-\gamma$ index their prices to the past price level.

The flexible-price firm $i$ maximize profits at time $t$:

$$\max_{p_t(i),N_t(i)} \quad Z_t(i) = p_t(i)y_t(i) - W_t N_t(i)$$

$$\textit{s.t.} \quad y_t(i) = N_t(i),$$

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t.$$

We obtain the optimal pricing for firm $i$ from the first-order condition of the profit maximization and, assuming a symmetric equilibrium, so that $p_t(i) = p_t^{flex}$:

$$\frac{p_t^{flex}}{P_t} = \frac{\theta}{\theta-1}\frac{W_t}{P_t}.$$

The firms which index their prices set

$$p_t^{index} = P_{t-1}.$$

The price index at time $t$ can be now written as:

$$P_t = \left[\gamma(p_t^{flex})^{1-\theta} + (1-\gamma)(p_t^{index})^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Since production is linear in labor, aggregate hours are given by

$$N_t = \int_0^1 y_t(i)di = Y_t \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di = Y_t\Delta_t,$$

where $\Delta_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di$.

The labor supply can be now expressed as

$$\frac{v_N(Y_t\Delta_t)}{u_C(C_t)} = \frac{W_t}{P_t}.$$

Using this equation and the expressions for $p_t^{flex}$ and $p_t^{index}$, we obtain the price dispersion:

$$\Delta_t = \gamma\left(\frac{\theta}{\theta-1}\frac{v_N(Y_t\Delta_t)}{u_C(C_t)}\right)^{-\theta} + (1-\gamma)(\Pi_t^{-1})^{-\theta}, \tag{PD}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

Similarly, following the same steps, the price index can be used to state a non-linear Phillips curve:

$$1 = \gamma \left( \frac{\theta}{\theta - 1} \frac{v_N(Y_t \Delta_t)}{u_C(C_t)} \right)^{1-\theta} + (1-\gamma)(\Pi_t^{-1})^{1-\theta}. \tag{PC}$$

## D.4 - Market Clearing

Assume that all production is consumed:

$$Y_t = C_t. \tag{MC}$$

## D.5 - Equilibrium Definition in the Non-Linear Model

An equilibrium is a collection of stochastic processes for $\{C_t, P_t, i_t, i_t^r, S_t, i_t^l, \Delta_t, Y_t\}$ that solve the Euler equation (EE), the money demand equation (MD), the asset pricing equations (AP1, AP2, AP3), the price dispersion equation (PD), the Phillips curve (PC), and the market clearing equation (MC) given the exogenous shocks $\{\xi_t^d, \xi_t, \omega_t\}$ and a policy specification for the sequence $\{M_t\}$.

## D.6 - Steady State

We consider a steady state in which inflation is zero, that is $\overline{\Pi}_t = 1$, and there is no price dispersion, that is $\Delta_t = 1$. Steady-state output, denoted by $\overline{Y}$, then solves:

$$\frac{v_N(\overline{Y})}{u_C(\overline{Y})} = \frac{\theta - 1}{\theta}.$$

Define $\delta \equiv 1 - \frac{w_A(0)}{u_C(\overline{Y})}$, we assume that $u_C(\overline{Y}) > w_A(0) \geq 0$, so that $0 < \delta \leq 1$.

The Euler equation (EE) in steady state implies that

$$1 + \bar{\imath} = \beta^{-1}\delta.$$

We assume that the steady-state interest rate is positive, which implies the restriction that $\delta > \beta$.

Let us denote by $\overline{\omega}$ the "steady-state" value of $\omega_t$; the steady-state risky interest rate is implied by the Euler equation (EE) and the first asset pricing equation (AP1):

$$1 + \bar{\imath}^r = \frac{1 + \bar{\imath}}{1 - \overline{\omega}} = \frac{\beta^{-1}\delta}{1 - \overline{\omega}}.$$

The Euler equation (EE) and the second asset pricing equation (AP2) imply that:

$$\bar{S} = \frac{1}{1 + \bar{\imath} - \rho} = \frac{1}{\beta^{-1}\delta - \rho}.$$

The money demand equation (MD) can be used to solve for the steady-state real money balance $\overline{m}$:

$$\frac{\chi_m(\overline{m})}{u_c(\overline{Y})}\bar{\xi}^d \geq \frac{\bar{\imath}}{(1+\bar{\imath})} - \frac{\bar{\imath}}{(1+\bar{\imath})}\frac{w_A(0)}{u_c(\overline{Y})}; \bar{\imath} \geq 0.$$

Finally, from the second and third asset pricing equations (AP2, AP3), we obtain:

$$(1 + \bar{\imath}^l) = \frac{1 + \rho\bar{S}}{\bar{S}} = (1 + \bar{\imath}).$$

## D.7 - Log-Linear Approximation

We define the elasticity of intertemporal substitution $\sigma \equiv -\frac{u_C(\overline{Y})}{u_{CC}(\overline{Y})\overline{Y}} > 0$ and an approximation of the Euler equation (EE) yields the IS curve:

$$\hat{Y}_t = \delta\hat{Y}_{t+1} - \sigma\delta(\hat{\imath}_t - \mathbb{E}_t\pi_{t+1} - \hat{r}^e_t), \tag{IS}$$

where $\hat{r}^n_t \equiv \hat{\xi}_t - \mathbb{E}_t\hat{\xi}_{t+1}$, with $\hat{\xi}_t \equiv \log\xi_t - \log\overline{\xi}$, and with the other variables defined as $\hat{Y}_t \equiv \log Y_t - \log\overline{Y}$, $\pi_t \equiv \log P_t - \log P_{t-1}$, and $\hat{\imath}_t \equiv \log(1 + i_t) - \log(1 + \bar{\imath})$.

Approximating the Euler equation (EE) and the first asset pricing equation (AP1) yields:

$$\hat{\imath}_t = \hat{\imath}^r_t - \mathbb{E}_t\hat{\omega}_{t+1}, \tag{AP1*}$$

where $\hat{\imath}^r_t = \log(1 + i^r_t) - \log(1 + \bar{\imath}^r)$ and $\hat{\omega}_{t+1} = \frac{\omega_t - \overline{\omega}}{1 - \overline{\omega}}$.

Approximating the Euler equation (EE) and the second asset pricing equation (AP2) yields:

$$\hat{S}_t = \frac{\beta\rho}{\delta}\mathbb{E}_t\hat{S}_{t+1} - \hat{\imath}_t, \tag{AP2*}$$

where $\hat{S}_t = \log S_t - \log\bar{S}$.

Approximating the second asset pricing equation (AP2) yields:

$$\hat{S}_t = -\frac{\delta}{\delta - \beta\rho}\hat{\imath}^l_t, \tag{AP3*}$$

where $\hat{\imath}^l_t = \log(1 + i^l_t) - \log(1 + \bar{\imath}^l)$.

Using the last two equations (AP2*, AP3*), we then obtain a relation between $\hat{\imath}^l_t$ and all the series of short-term risk-free interest rates under the assumption that $\rho < \frac{\delta}{\beta}$:

$$\hat{\imath}_t^l = (1 - \rho\beta/\delta)\mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{\rho\beta}{\delta}\right)^j \hat{\imath}_{t+j}.$$

We define the elasticity of real money balances as $\psi \equiv -\frac{\chi_m(\overline{m})}{\chi_{mm}(\overline{m})\overline{m}} > 0$ and an approximation of the money demand equation (MD) yields the LM curve:

$$\hat{M}_t \geq \eta_y \hat{Y}_t - \eta_i \hat{\imath}_t + \epsilon_t^d \text{ and } \hat{\imath}_t \geq i_{elb}, \tag{LM / ELB}$$

where $\eta_y \equiv \psi\sigma^{-1}\frac{\bar{\imath}}{\bar{\imath}+1-\delta} \geq 0$, $\eta_i \equiv \psi\frac{\beta}{1-\beta} \geq 0$, $\hat{m}_t \equiv \log m_t - \log \overline{m}$, and $\epsilon_t^d = \psi(\log \xi_t^d - \log \overline{\xi}^d)$.

Linearizing the non-linear Phillips curve (PC) yields the Phillips-curve equation:

$$\pi_t = \kappa \hat{Y}_t, \tag{AS}$$

where $\kappa \equiv \frac{\gamma}{1-\gamma}(\sigma^{-1} + \varphi)$ and $\varphi \equiv \frac{v_{NN}(\bar{N})\bar{N}}{v_N(\bar{N})} > 0$ (inverese Frisch elasticity).

Let us define the nominal money growth by $\mu_t \equiv \frac{M_t}{M_{t-1}}$. This definition implies that $m_t \equiv m_{t-1}\mu_t\Pi_t^{-1}$ or expressed with a log-linear approximated equation:

$$\hat{m}_t \equiv \hat{m}_{t-1} + \hat{\mu}_t - \pi_t. \tag{MG}$$

## D.8 - Approximated Equilibrium Definition

An approximated equilibrium is a collection of stochastic processes $\{\hat{Y}_t, \hat{\imath}_t, \pi_t, \hat{\imath}_t^r, \hat{S}_t, \hat{\imath}_t^l, \hat{m}_t\}$ that solve the IS curve (IS), the approximated asset pricing equations (AP1*, AP2*, AP3*), the LM curve (LM / ELB), the Phillips-curve equation (AS), and the nominal money growth equation (MG) given the exogenous shocks $\{\hat{r}_t^e, \epsilon_t^d, \hat{\omega}_t\}$ and a policy specification for the sequence $\{\hat{\mu}_t\}$.