

Stochastic variables:

- State variable $s \sim \text{Bernoulli}(\pi)$. $P(s = H) = \pi$, $P(s = L) = 1 - \pi$.
- Risk factor payoffs $\tilde{f} \mid s \sim N(\Gamma^{-1}\mu_s, \Sigma_s)$
- Risk factor supply $x \sim N(\bar{x}, \sigma_x I)$
- Private signals $\eta_j \mid s \sim N(z, \Sigma_{\eta_j})$
- Price signal $\eta_p \mid s \sim N(z, \Sigma_p)$

Joint density:

$$P(s, \tilde{f}, x, \eta_j, \eta_p) = P(\tilde{f} \mid s)P(\eta_j \mid s)P(\eta_p \mid s)P(s)P(x)$$

Posterior density:

$$P(s, \tilde{f} \mid x, \eta_j, \eta_p) = \frac{P(x, \eta_j, \eta_p \mid s, \tilde{f})P(s, \tilde{f})}{P(x, \eta_j, \eta_p)}$$

Unknown values:

- $E_j[\tilde{f} - \tilde{p}r \mid H]$
- $E_j[\tilde{f} - \tilde{p}r \mid L]$
- $V_j[\tilde{f} - \tilde{p}r \mid H]$
- $V_j[\tilde{f} - \tilde{p}r \mid L]$
- $P(H \mid \eta_p, \eta_j)$
- $P(L \mid \eta_p, \eta_j)$
- $E_j[\tilde{f} \mid \eta_p, \eta_j]$
- $V_j[\tilde{f} \mid \eta_p, \eta_j]$
- \tilde{p}

Portfolio choice problem:

$$U_{2j} = \max_{\tilde{q}_j} \rho E_j[W_j] - \frac{\rho^2}{2} V_j[W_j]$$

Optimal quantity:

$$\begin{aligned}
\tilde{q}_j &= \frac{1}{\rho} \left(P(H)\Sigma_H + P(L)\Sigma_L \right)^{-1} \left(P(H)E_j[\tilde{f} \mid H] + P(L)E_j[\tilde{f} \mid L] - \tilde{p}r \right) \\
&= \frac{1}{\rho} V_j[\tilde{f}]^{-1} (E_j[\tilde{f}] - \tilde{p}r)
\end{aligned}$$

Ex-ante expected utility:

$$\begin{aligned}
U_{1j} &= E \left[\rho E_j[W_j] - \frac{\rho^2}{2} V_j[W_j] \right] \\
&= \pi E \left[\rho E_j[W_j \mid H] - \frac{\rho^2}{2} V_j[W_j \mid H] \right] \\
&\quad + (1 - \pi) E \left[\rho E_j[W_j \mid L] - \frac{\rho^2}{2} V_j[W_j \mid L] \right] \\
&= \rho r W_0 \\
&\quad + \rho \tilde{q}'_j \left(\pi E_j[\tilde{f} - \tilde{p}r \mid H] + (1 - \pi) E_j[\tilde{f} - \tilde{p}r \mid L] \right) \\
&\quad - \frac{\rho^2}{2} \tilde{q}'_j \left(\pi V_j[\tilde{f} - \tilde{p}r \mid H] + (1 - \pi) V_j[\tilde{f} - \tilde{p}r \mid L] \right) \tilde{q}_j
\end{aligned}$$

Information choice and state uncertainty

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A strand of economic literature studies how limited attention and cognitive constraints guide economic choices. In these papers, researchers examine how prices can be used to aggregate private signals observed by attention-constrained investors. Signals usually¹ take the form of the true payoff with additive Gaussian noise, $\eta_j = \tilde{f} + \epsilon_j$, for investor j , signal η_j , payoff vector \tilde{f} , and noise term ϵ_j .

I examine how endogenous information choice with attention limitations can lead investors to choose distinct portfolios when signals inform investors about both the underlying economic state and asset payoffs in those states. Allowing risk averse investors to select signals that are informative about state and payoffs jointly produces **substantially different** portfolios from standard noisy rational equilibrium models, as well as shifting the conclusions of canonical models of information choice where signals inform investors *only* about payoffs.

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1 Model framework

Much of my notation and model structure follows from Kacperczyk et al. (2016), which introduce attention constraints and information choice to the multiasset noisy rational equilibrium model of Admati (1985).

The model has three periods. In time 1, informed investors allocate their attention across n signals. At time 2, all investors construct portfolios. At time 3, all investors receive payoffs.

I assume, as in Kacperczyk et al. (2016), that there are n risky assets with an arbitrary factor structure given by principal components. Assets $1, 2, \dots, n$ represent specific assets with idiosyncratic shocks. The key difference between my paper and Kacperczyk et al. (2016) is that the economic state is a stochastic variable that is not known by investors. The economy is in state $s \in H, L$, where $s = H$ represents a “good” state with probability π and $s = L$ represents a “bad” state with probability $1 - \pi$. The density of s is written

¹A noteworthy exception is Breon-Drish (2015), which allows signals and payoff distributions to vary to a greater degree – namely, that the density of the payoff conditional on a private signal ($P(\tilde{f} \mid \eta_j)$, in terms of Kacperczyk et al. (2016)) be a member of the exponential family of distributions.

$$P(s) = \begin{cases} \pi & \text{if } s = H \\ 1 - \pi & \text{if } s = L \end{cases}$$

Importantly, asset payoffs are governed by two different stochastic processes conditional on state. Payoffs of the n assets are written

$$f_i = \mu_{i,s} + z_i \quad (1)$$

$$z = [z_1, z_2, \dots, z_n]' \sim \mathcal{N}(0, \Sigma_s) \quad (2)$$

$$f \sim \mathcal{N}(\mu_s, \Sigma_s) \quad (3)$$

That is, both the mean payoff vector $\mu_s = [\mu_{1,s}, \mu_{2,s}, \dots, \mu_{n,s}]'$ and the $n \times n$ variance-covariance matrix of payoff shocks Σ_s are functions of the unobserved economic state. When investors are allowed to receive signals about the underlying shocks z , those same signals will allow investors to assign a probability to the underlying state and the associated payoff structure.²

Investors have mean-variance preferences over terminal wealth W_j , with a risk-aversion coefficient ρ . Expected utility is a function of risk-free rate r , initial wealth W_0 , asset quantities q_j , asset payoffs f , and asset prices p .

$$U_j = \rho E_j[W_j] + \frac{\rho^2}{2} \text{Var}_j[W_j] \quad (5)$$

for law of motion on wealth $W_j = rW_0 + q_j'(f - pr)$.

²Kacperczyk et al. (2016) utilize a transformation of asset payoffs to the corresponding risk factor payoffs – in my case, the eigen-decomposition $\Sigma_s = \Gamma_s \Lambda_s \Gamma_s'$ for $s \in H, L$ yields Arrow-Debreu synthetic securities on risk factors:

$$\tilde{f} \mid s \sim \mathcal{N}(\Gamma_s^{-1} \mu_s, \Lambda_s) \quad (4)$$

Unfortunately, I cannot proceed with the Kacperczyk et al. (2016) solution method, which requires an additional transformation of risk factor prices $\tilde{p} = \Gamma^{-1}p$ and risk factor quantities $\tilde{q} = \Gamma^{-1}q$. My model

<https://math.stackexchange.com/questions/195911/calculation-of-the-covariance-of-gaussian-mixtures>