Stochastic variables:

- State variable $s \sim \text{Bernoulli}(\pi)$. $P(s = H) = \pi$, $P(s = L) = 1 - \pi$.

- Risk factor payoffs $\tilde{f} \mid s \sim N(\Gamma^{-1}\mu_s, \Sigma_s)$

- Risk factor supply $x \sim N(\overline{x}, \sigma_x I)$

- Private signals $\eta_j \mid s \sim N(z, \Sigma_{\eta_j})$

- Price signal $\eta_p \mid s \sim N(z, \Sigma_p)$

Joint density:

$$P(s, \tilde{f}, x, \eta_j, \eta_p) = P(\tilde{f} \mid s)P(\eta_j \mid s)P(\eta_p \mid s)P(s)P(x)$$

Posterior density:

$$P(s, \tilde{f} \mid x, \eta_j, \eta_p) = \frac{P(x, \eta_j, \eta_p \mid s, \tilde{f})P(s, \tilde{f})}{P(x, \eta_j, \eta_p)}$$

Unknown values:

- $E_j[\tilde{f} - \tilde{p}r \mid H]$

- $E_j[\tilde{f} - \tilde{p}r \mid L]$

- $V_j[\tilde{f} - \tilde{p}r \mid H]$

- $V_j[\tilde{f} - \tilde{p}r \mid L]$

- $P(H \mid \eta_p, \eta_j)$

- $P(L \mid \eta_p, \eta_j)$

- $E_j[\tilde{f} \mid \eta_p, \eta_j]$

- $V_j[\tilde{f} \mid \eta_p, \eta_j]$

- $\tilde{p}$

Portfolio choice problem:

$$U_{2j} = \max_{\tilde{q}_j} \rho E_j[W_j] - \frac{\rho^2}{2}V_j[W_j]$$

Optimal quantity:

$$\tilde{q}_j = \frac{1}{\rho}\left(P(H)\Sigma_H + P(L)\Sigma_L\right)^{-1}\left(P(H)E_j[\tilde{f} \mid H] + P(L)E_j[\tilde{f} \mid L] - \tilde{p}r\right)$$

$$= \frac{1}{\rho}V_j[\tilde{f}]^{-1}(E_j[\tilde{f}] - \tilde{p}r)$$

Ex-ante expected utility:

$$U_{1j} = E\left[\rho E_j[W_j] - \frac{\rho^2}{2}V_j[W_j]\right]$$

$$= \pi E\left[\rho E_j[W_j \mid H] - \frac{\rho^2}{2}V_j[W_j \mid H]\right]$$

$$+ (1-\pi)E\left[\rho E_j[W_j \mid L] - \frac{\rho^2}{2}V_j[W_j \mid L]\right]$$

$$= \rho r W_0$$

$$+ \rho \tilde{q}_j'\left(\pi E_j[\tilde{f} - \tilde{p}r \mid H] + (1-\pi)E_j[\tilde{f} - \tilde{p}r \mid L]\right)$$

$$- \frac{\rho^2}{2}\tilde{q}_j'\left(\pi V_j[\tilde{f} - \tilde{p}r \mid H] + (1-\pi)V_j[\tilde{f} - \tilde{p}r \mid L]\right)\tilde{q}_j$$

Information choice and state uncertainty

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August 19, 2021

A strand of economic literature studies how limited attention and cognitive constraints guide economic choices. In these papers, researchers examine how prices can be used to aggregate private signals observed by attention-constrained investors. Signals usuall[1] take the form of the true payoff with additive Gaussian nois, $\eta_j = \tilde{f} + \epsilon_j$, for investor j, signal η_j, payoff vector $\tilde{f}$, and noise term ϵ_j.

I examine how endogenous information choice with attention limitations can lead investors to choose distinct portfolios when signals inform investors about both the underlying economic state and asset payoffs in those states. Allowing risk averse investors to select signals that are informative about state and payoffs jointly produces substantially different portfolios from standard noisy rational equilibrium models, as well as shifting the conclusions of canonical models of information choice where signals inform investors *only* about payoffs.

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1   Model framework

Much of my notation and model structure follows from Kacperczyk et al. (2016), which introduce attention constraints and information choice to the multiasset noisy rational equilibrium model of Admati (1985).

The model has three periods. In time 1, informed investors allocate their attention across n signals. At time 2, all investors construct portfolios. At time 3, all investors receive payoffs.

I assume, as in Kacperczyk et al. (2016), that there are n risky assets with a single factor structure. Assets $1, 2, \ldots, n-1$ represent specific assets with idiosyncratic shocks, while the nth asset is a composite asset with only a common shock. The key difference between my paper and Kacperczyk et al. (2016) is that the economic state is a latent variable. The economy is in state $s \in H, L$, where $s = H$ represents a "good" state with probability π and $s = L$ represents a "bad" state with probability $1 - \pi$. The density of s is written

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[1]A noteworthy exception is Breon-Drish (2015), which allows signals and payoff distributions to vary to a greater degree – namely, that the density of the payoff conditional on a private signal ($P(\tilde{f} \mid \eta_j)$, in terms of Kacperczyk et al. (2016)) be a member of the exponential family of distributions.

$$P(s) = \begin{cases} \pi & \text{if } s = H \\ 1 - \pi & \text{if } s = L \end{cases}$$

Importantly, asset payoffs are governed by two different stochastic processes conditional on state. Payoffs of the n assets are written

$$f_i = \mu_i(s) + b_i z_n + z_i, \quad i \in \{1, \ldots, n-1\} \tag{1}$$

$$f_n = \mu_n(s) + z_n \tag{2}$$

$$z = [z_1, z_2, \ldots, z_n]' \sim \mathcal{N}(0, \Sigma(s)) \tag{3}$$

That is, both the mean payoff vector $\mu(s) = [\mu_1(s), \mu_2(s), \ldots, \mu_n(s)]'$ and the $n \times n$ variance-covariance matrix of payoff shocks $\Sigma(s)$ are functions of the unobserved economic state. When investors are allowed to receive signals about the underlying shocks z, those same signals will allow investors to assign a probability to the underlying state and the associated payoff structure.