Information choice and economic complexity

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October 8th, 2021

Research question

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How should you choose what information to analyze when the economy is ex-ante complicated?

Alternatively, how much information is it rational to ignore?

Information choice

Information choice is the study of how investors choose what things they wish to know about.

In asset pricing, we like to think about how information choice impacts things like

- Expected returns
- Risk premia
- Welfare (?)

I use “information choice” and “attention” synonymously.

My hypothesis

I am trying to find out whether "complex" economies cause the basic findings of information choice models to fall apart.

Common findings:

- When a "common component" of payoffs or returns becomes more volatile, more attention goes to the common component. Common component here might be the business cycle or market returns.
- Increased attention reduces excess volatility, while decreased attention increases excess volatility.
- Prices are informative about payoffs because prices are a function of investor attention.

Hypothesis 1: Price informativeness

> *The literature: Prices give investors useful information about payoffs.*

Prices may be not informative at all, since investors with different information can interperet prices radically differently.

Importantly, this could happen as the entropy of prior payoffs goes *down*!

Hypothesis 2: Biased expectations

> *The literature: Gaussian payoffs – all investors are reasonably good at guessing payoffs.*

Investors allocate attention in a way that can systematically mislead them about the state of the economy. I.e. a large mass of investors will be *very* disappointed (or pleased) when they find out the true payoffs.

Hypothesis 3: Returns and volatility

> *The literature: More attention means less volatility and lower expected returns.*

Might not be the case in situations with large amounts of disagreement! Even if more attention is given to an asset, disagreement could drive up expected returns and excess volatility.

My hypothesis

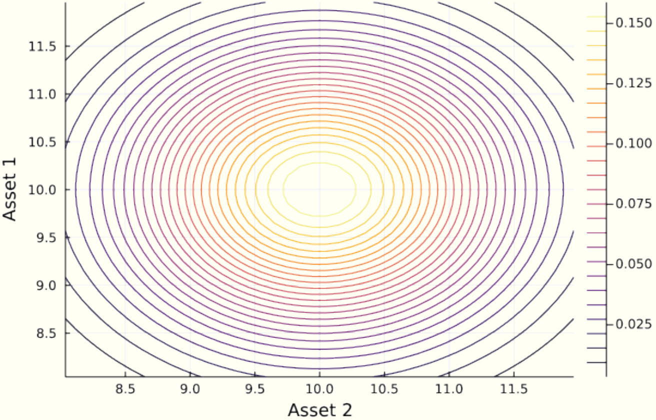
In short – the **findings of information choice models might flip in complex economies**!

Brief literature review

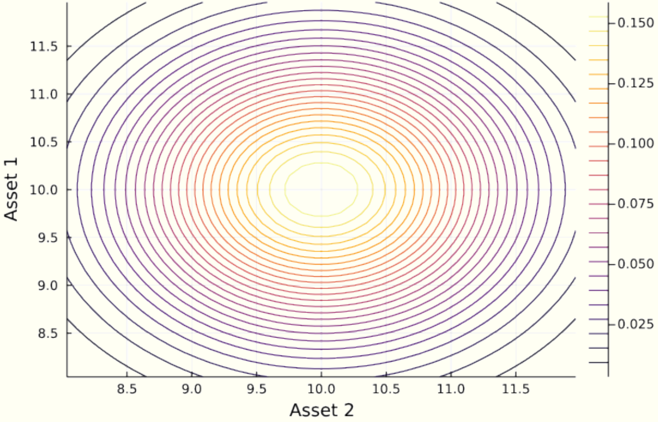
- Theory
  - Kacperczyk et al. (2016) is the model I follow closest, though I do away with a lot of their simplifying assumptions.
  - Peng and Xiong (2006) looks at category learning and attention.
  - Slow learning and model ambiguity can explain all kinds of variance phenomena (Ghaderi et al., 2021)
- Empirics
  - Kholhas and Walther (2021) asymmetric attention to procyclical variables can explain underreaction + extrapolation.
  - Cziraki et al. (2021) local/nonlocal bias in attention allocation causes abnormalities in returns.
  - Hirshleifer and Sheng (2021) shows that macro news triggers higher attention in firm-level securities.

Complexity

Complexity

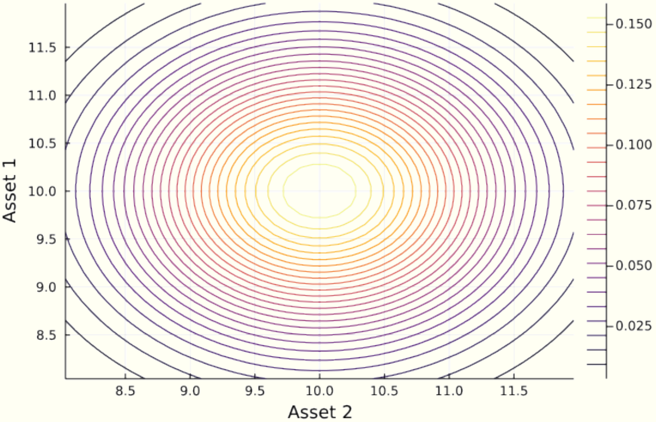


Traditional models of information choice use unimodal joint densities with good properties. The above is the case used in Kacperczyk et al. (2016).



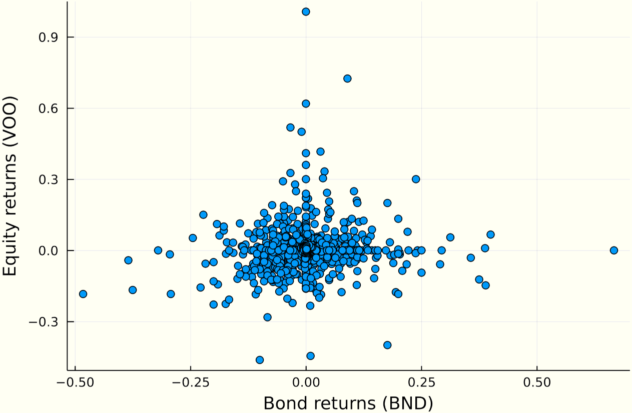
Gaussian variables are the highest entropy distribution for a given mean and standard deviation, meaning they are the least "structured".

Complexity



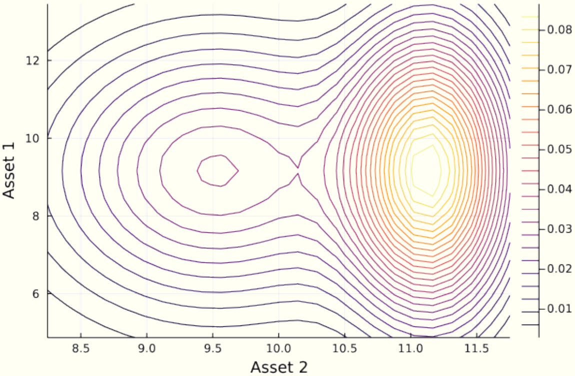
Less structure means a boring blob of investors are generally happy with any minimum-variance outcome!

Complexity

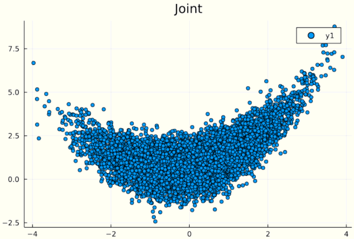


What if payoffs were more "complicated"? Pictured is the daily bond/equity observed returns from 2000-2021.

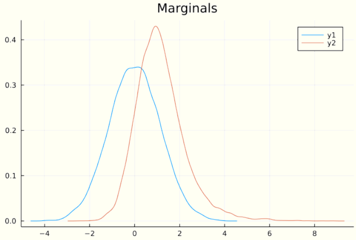
Some hypothetical payoff densities



The banana

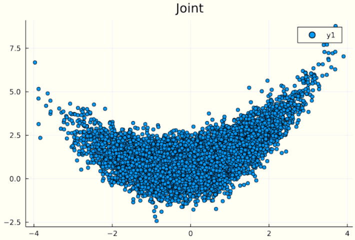


The banana's marginal densities



Note that the marginals look pretty close to actual stock return densities, even though the joint distribution is strange.

The banana takeaway



How do you allocate attention if you know you have a complicated joint density?

The framework

The framework

I mostly follow Kacperczyk et al. (2016), which adds attention to the multiasset noisy rational expectations model of Admati (1985).

States of the economy

The economy has a *state*, denoted with s. A *state* describes the mean and variance of the payoffs in that state, i.e. μ_H is the mean payoff in the good state, while μ_L is the mean payoff in the bad state.

Investors do not observe the state but must infer it.

States of the economy

The density function for the state is:

$$P(s) = \begin{cases} \pi & \text{if } s = H \\ 1 - \pi & \text{if } s = L \end{cases}$$

The assets

We have n assets that have payoffs

$$f = \mu_s + \epsilon$$

where

$$f_i = \mu_{i,s} + z_i \tag{1}$$

$$z \mid s = [z_1, z_2, \ldots, z_n]' \sim \mathcal{N}(0, \Sigma_s) \tag{2}$$

$$f \mid s \sim \mathcal{N}(\mu_s, \Sigma_s) \tag{3}$$

What is this density?

Formally, this is what is known as a Gaussian mixture model. Gaussian mixture models can be used to approximate any joint density[1]

The density function is

$$P(f) \sim \pi \mathcal{N}(f \mid \mu_H, \Sigma_H) + (1-\pi)\mathcal{N}(f \mid \mu_L, \Sigma_L)$$

[1] If you have ever used kernel density estimation, this is technically what is happening behind the scenes.

Asset supply

The n assets have an uncertain supply. Sometimes noise traders show up to trade, so the traders cannot perfectly forecast how many shares need to be net bought or sold.

Denote this stochastic supply with

$$x \sim \mathcal{N}(\overline{x}, \Sigma_x)$$

Timing

1. Investors allocate attention and then observe signals, if they are informed.
2. Investors choose asset portfolios and the market clears.
3. Everyone receives the final payoff $q_j' f$.

The investors

There is a unit mass of investors $j \in [0, 1]$. Each investor has CARA utility of the form

$$U_{j2} = E_j[\exp\{-\rho W_j\}]$$

for terminal wealth

$$W_j = rW_0 + q_j'(f - pr)$$

where W_0 is initial wealth, q_j' is the asset quantity purchased, f is the realized stochastic payoffs, and p is the asset price vector. The risk-free rate of r is normalized to 1 to reduce notation.

Private signals

Some portion of investors receive private signals of the true payoff f. Their signals take the form

$$\eta_j \sim \mathcal{N}(f, \Sigma_{\eta,j}), \tag{4}$$

$$\text{or } \eta_j = f + \epsilon_j, \quad \epsilon_j \sim \mathcal{N}(0, \Sigma_{\eta_j}) \tag{5}$$

Investors can control how precise this signal is at the beginning of the game by allocating attention.

$$\Sigma_{\eta_j} = \begin{bmatrix} K_{1j}^{-1} & 0 & \dots & 0 \\ 0 & K_{2j}^{-1} & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & K_{nj}^{-1} \end{bmatrix}$$

Subject to the constraints that $K_i j \geq 0$ (no forgetting) and $\sum K_i j \leq K$ (finite attention).

The information choice problem

The information choice problem is the one I want to focus on, but it's hard to get to!

Ultimately, I want to know the gradients of the attention allocation function

$$\Sigma^*_{\eta_j}(\pi, \mu_H, \mu_L, \Sigma_H, \Sigma_L)$$

with respect to the prior payoff parameters.

Portfolio choice

The portfolio choice problem is quite easy. Assume that investors have their signals η_j and can see prices p. Then they must maximize expected terminal utility:

$$\begin{aligned} \underset{q_j}{\text{maximize}} \quad & U_{j2} = E_j[\exp\{-\rho W_j\} \mid \eta_j, p] \\ \text{subject to} \quad & W_j = rW_0 + q_j'(f - pr) \end{aligned} \tag{6}$$

Portfolio choice

Solving this yields the (maybe familiar) exact quantity:

$$q_j = \frac{1}{\rho}\big(\hat{s}_j\hat{\Sigma}_{H,j} + (1-\hat{s}_j)\hat{\Sigma}_{L,j}\big)^{-1}\big(\hat{s}_j\hat{\mu}_{H,j} + (1-\hat{s}_j)\hat{\mu}_{L,j}\big) \qquad (7)$$

for posterior state beliefs after observing p and η_j.

Prices

Prices are the tricky bit in this economy. In other models, you can conjecture a linear price as a function of the true payoffs f and the supply x:

$$p = A + Bf + Cx$$

Kacperczyk et al. (2016) and others have exact forms for these matrices.

Prices

I don't! It's been very difficult even with a simple density to get closed form prices, partly because everyone's posterior beliefs about the state ($\hat{s}_j$) move around with f.

This means that prices p are linear in f and x, but the parameters A, B, and C are non-linear functions of f and x.

Tricky!

Solving for prices numerically

I can solve for prices numerically by conjecturing A, B, and C such that the market clearing condition is satisfied:

$$\int_j q_j(p) = x$$

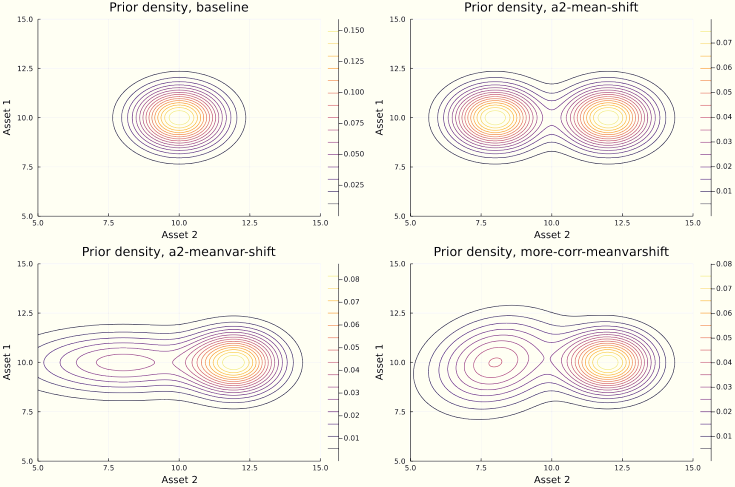
Example economies

Four economies

Two asset economies for easy plotting. I set $\pi = 1/2$ and $K = 10$ in all cases.

	μ_H	μ_L	Σ_H	Σ_L
1	$[10, 10]$	$[10, 10]$	I	I
2	$[10, 12]$	$[10, 8]$	I	I
3	$[10, 12]$	$[10, 8]$	I	$[1, 0; 0, 5]$
4	$[10, 12]$	$[10, 8]$	I	$[2, 0.4; 0.4, 2]$

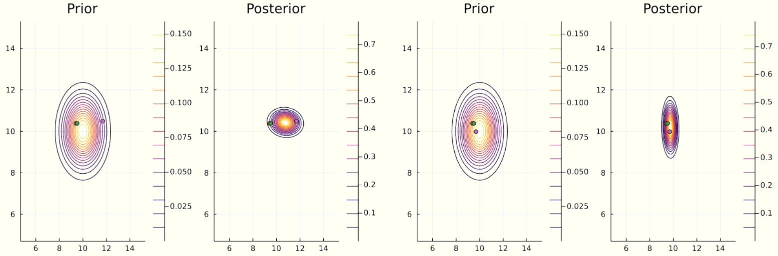
Prior payoffs



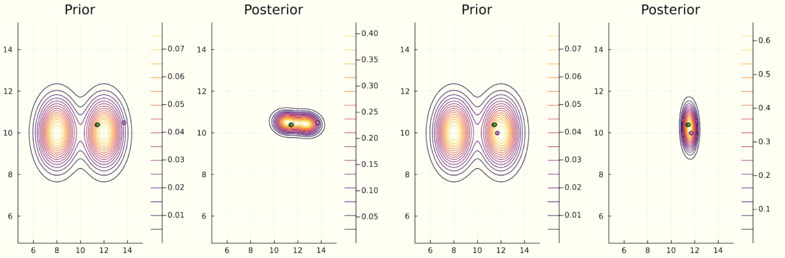
The investors

To examine how disparate attention impacts each of these economies, I force half the investors to place 90% of their attention on asset 1, and the other half to focus 90% on asset 2.

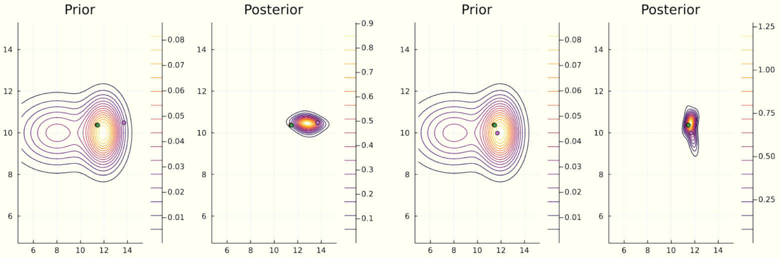
Baseline (investor posterior)



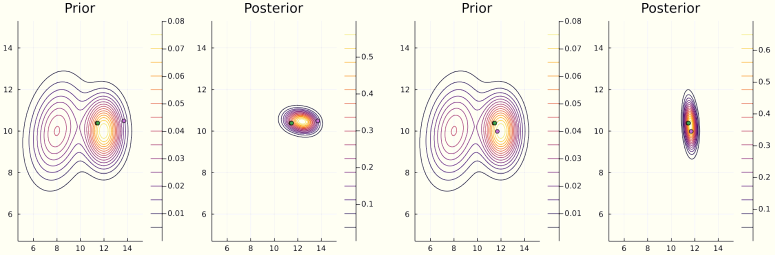
Mean shift (investor posterior)



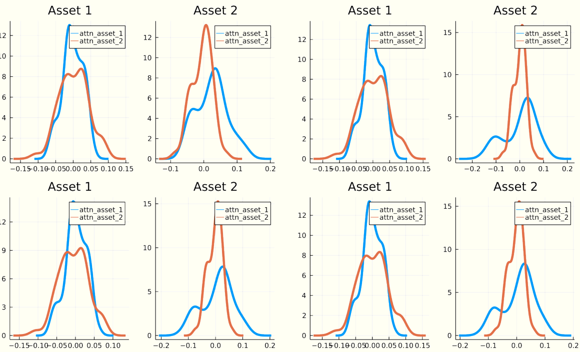
Mean and variance shift (investor posterior)



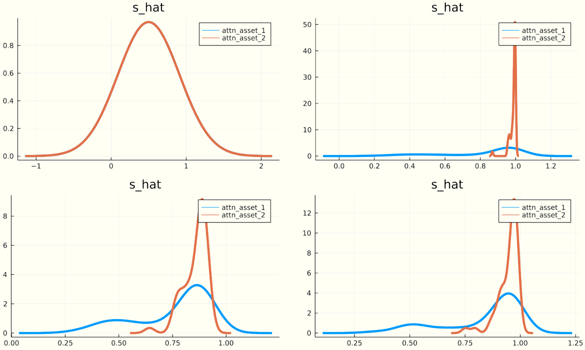
Mean, correlation, and variance shift (investor posterior)



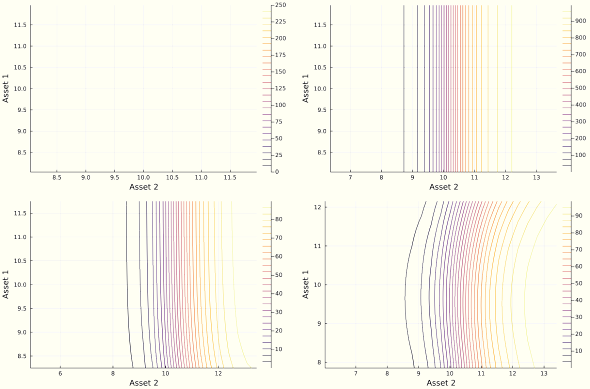
What do investors think expected returns are $(E[f/p-1])$?



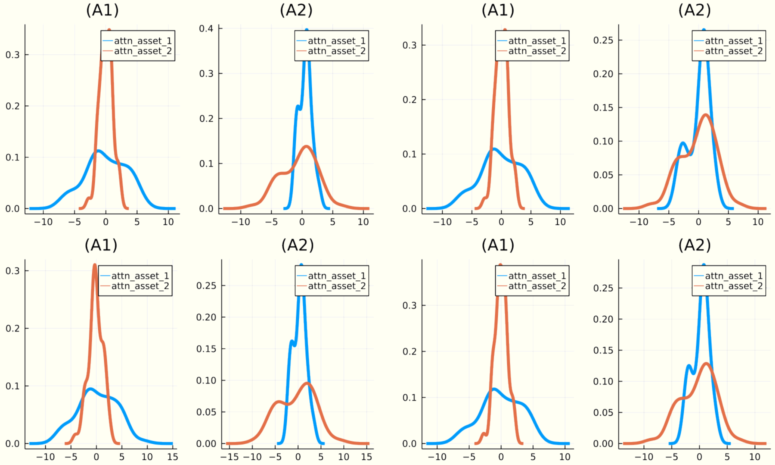
What do investors think the state is $(\hat{s}_j)$?



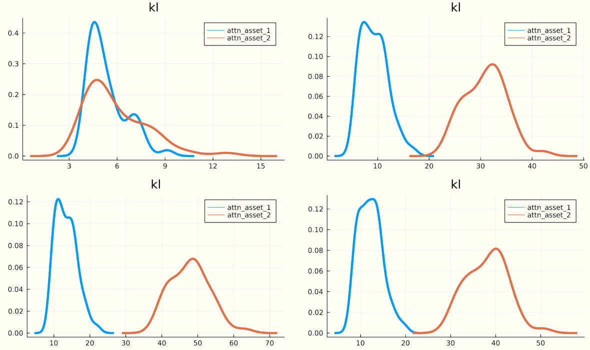
How much disagreement is there ($\mathrm{Var}[\hat{s}_j]$)?



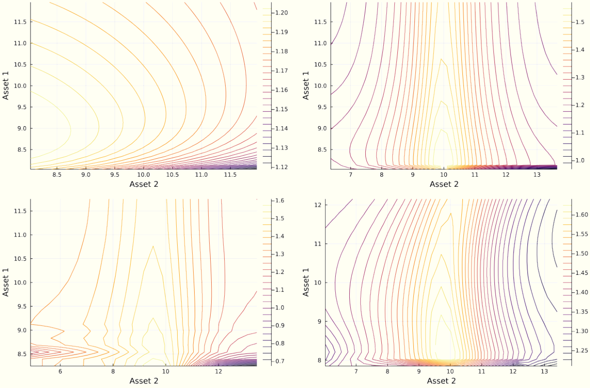
What do investors buy or sell (q_j^*)?



Who learns the most $(KL(\hat{f}_j \,||\, f))$?



In what states are investors the most uncertain $(\hat{H}(\hat{f}_j \parallel f))$?



Prices

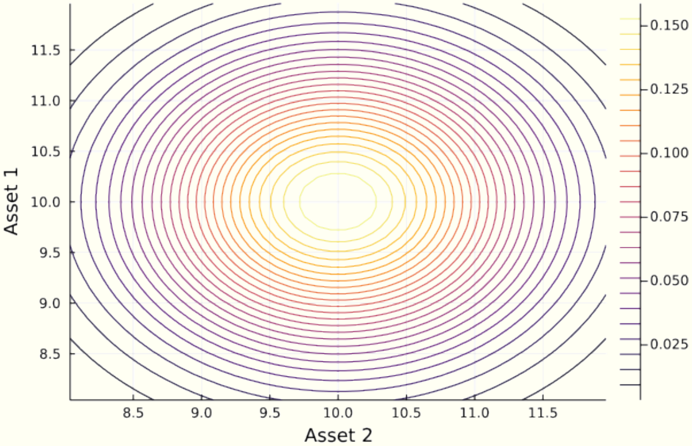
Prices

My estimation procedure estimates the matrices A, B, and C:

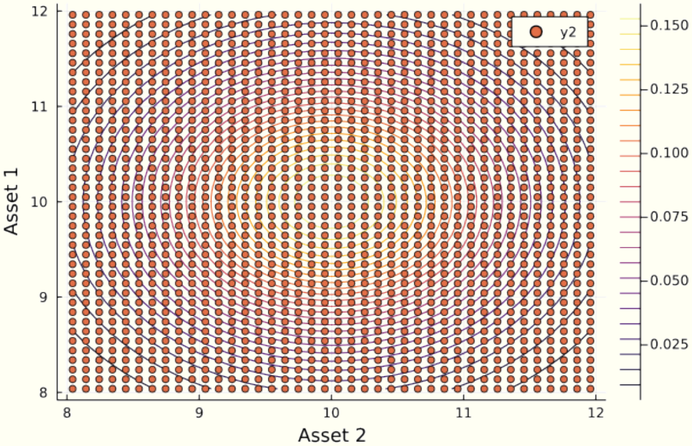
$$p = A + Bf + Cx$$

Since I do not (yet) have a closed form expression for this, I form a 50×50 grid of possible draws of the payoffs f, fixing supply $x = \overline{x}$.

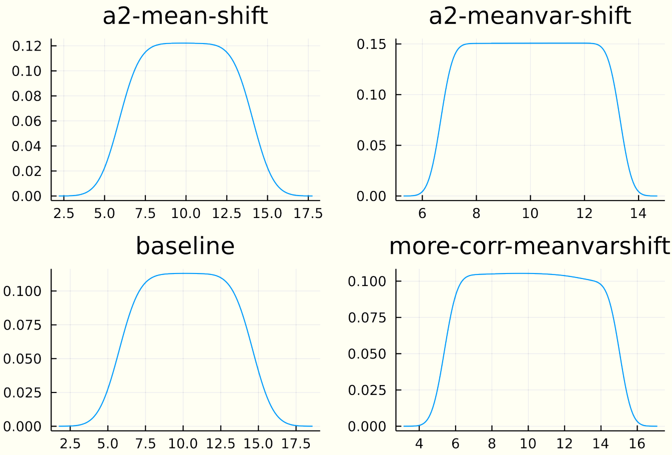
For each f, I calculate the equilibrium conditions and store them. This lets me plot out the approximate pricing function!



Asset 1
Asset 2
11.5
11.0
10.5
10.0
9.5
9.0
8.5
8.5
9.0
9.5
10.0
10.5
11.0
11.5
0.150
0.125
0.100
0.075
0.050
0.025



Prices



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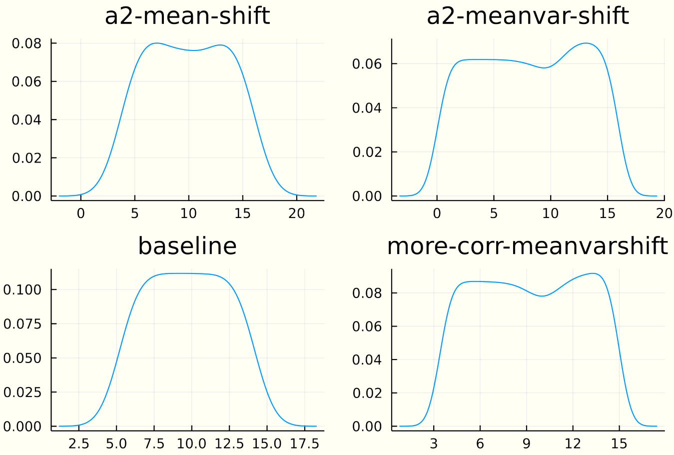

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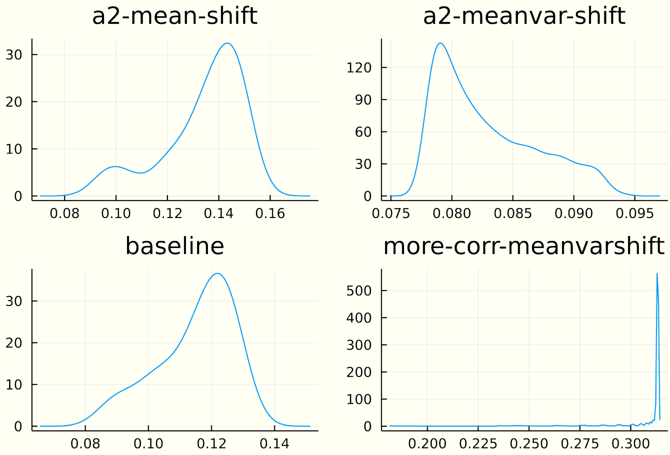

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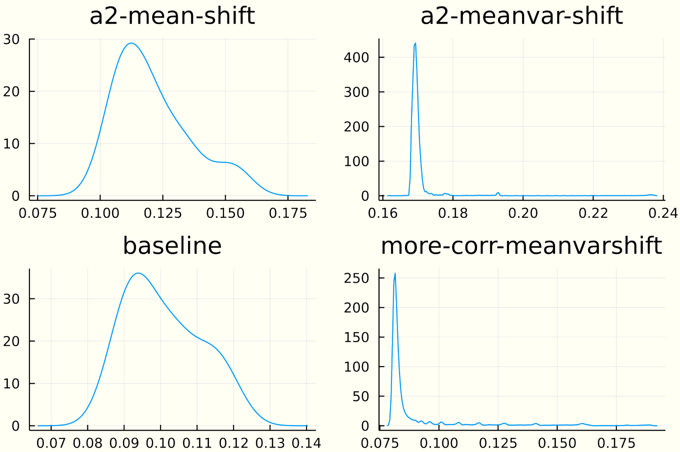

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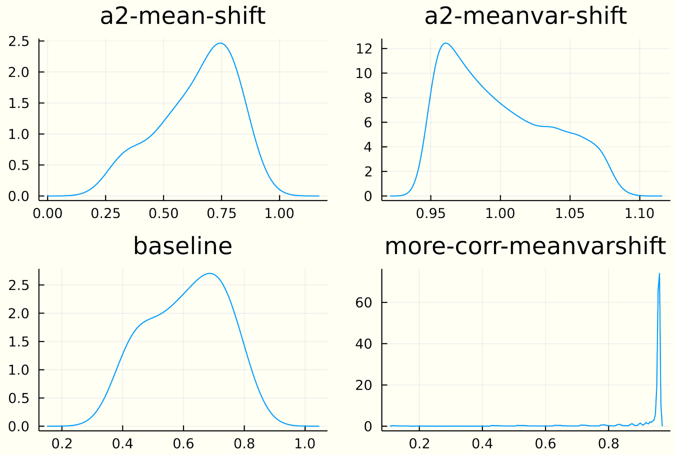

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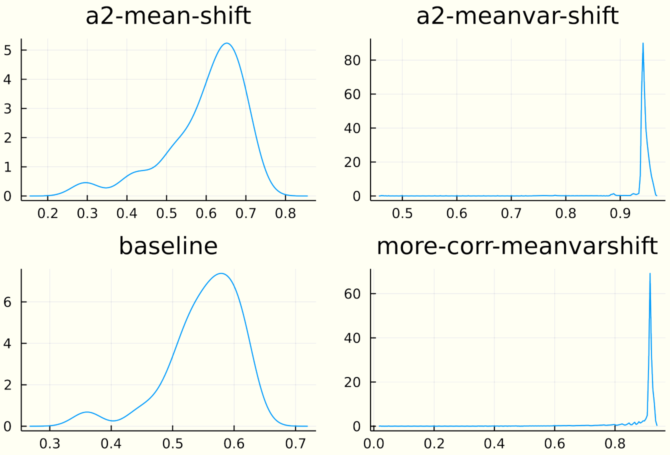

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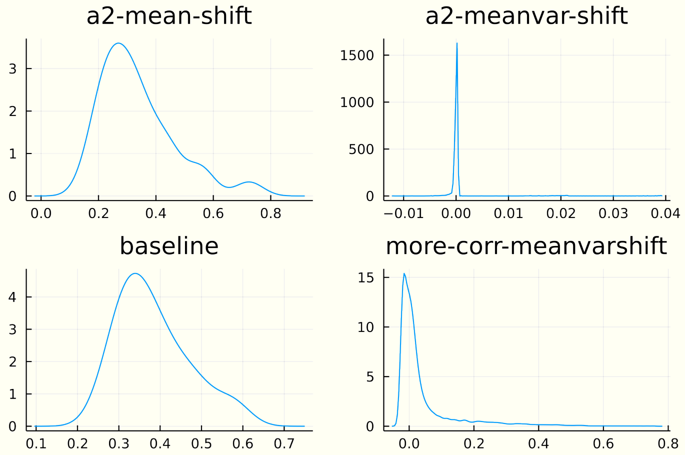

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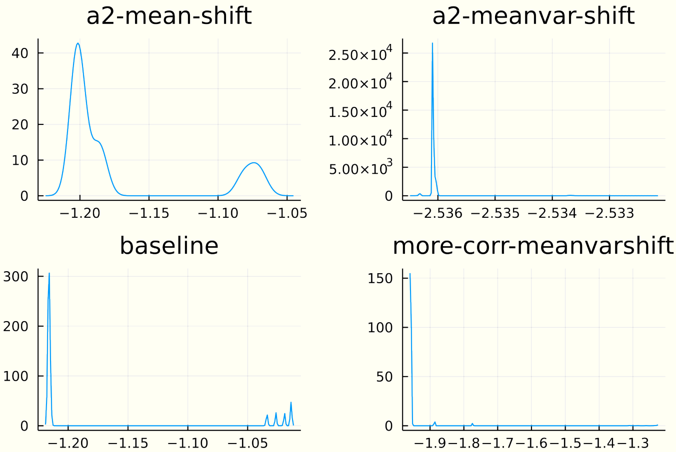

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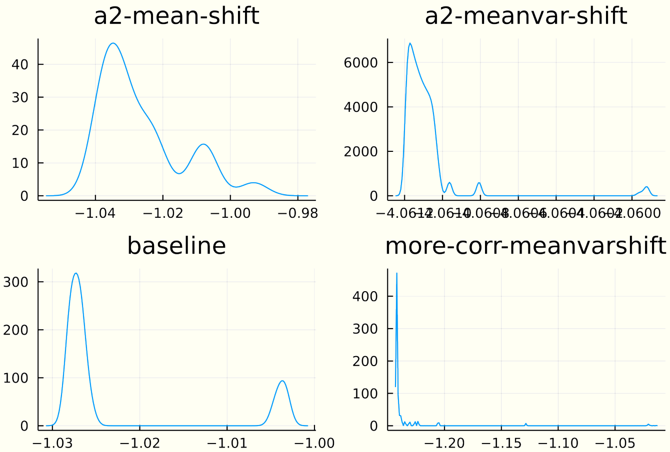

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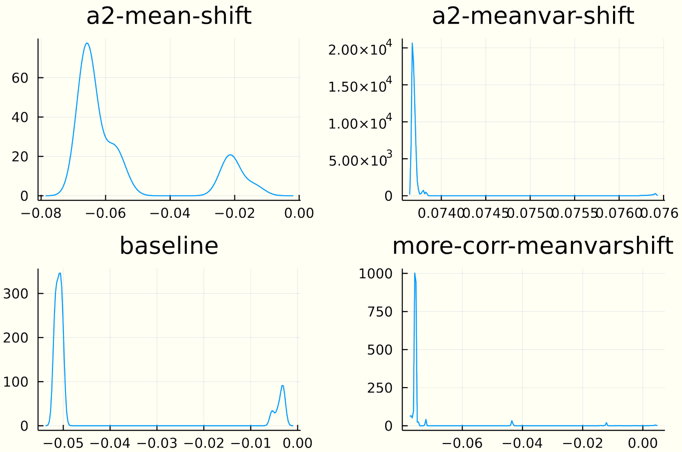

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Next steps

Next steps

1. Solve the ex-ante information choice problem.
2. Solve the hidden attention allocation modification, a mixed strategy on Σ_{η_j}.
3. Estimate the (observed) joint density (f) of many assets. Which assets should we see the most attention on given my understanding of $\Sigma_{\eta_j}(\mu, \mathbf{\Sigma})$?