Large Bets and Stock Market Crashes

Albert S. Kyle and Anna A. Obizhaeva

University of Maryland and New Economic School

Microstructure Exchange
Online Seminar
November 16, 2020

Two Types of Market Crashes

There are two types of market crashes:

- **Banking Crises and Sovereign Defaults**: Associated with collapse of the banking system, exchange rate crises, currency collapse, and bouts of high inflation. Documented by Reinhart and Rogoff(2009);
- **Stock Market Crashes**: Crashes or panics triggered by execution of large "bets." Are short-lived if followed by appropriate government policy. They are hard to be explained in the context of traditional paradigm.

Basic Idea

Market microstructure invariance can be used to explain stock market crashes:

- Market microstructure invariance generates universal predictions about "bet size" and "price impact."
- Using portfolio transition data, Kyle and Obizhaeva (2016) fits distribution of bet size, market impact cost, and bid-ask spread costs, to markets for individual stocks.
- These parameters are extrapolated to the entire stock market, the parameter estimates for individual stocks generate reasonable predictions about price declines and bet size for stock market crashes.

Market Crashes Triggered by Bets

We consider **five market crashes** triggered by large bets.

1929 Market Crash: Margin calls resulted in massive selling of stocks and reductions in loans to finance margin purchases.

1987 Market Crash: "Portfolio Insurers" sold large quantities of stock index future contracts, as documented in The Brady Commission report (1988).

2008 SocGén: Société Gén ´erale liquidated billions of Euros in stock index future positions accumulated by rogue trader Jérôme Kerviel.

Market Crashes Triggered by Bets

1987 George Soros: Three days after the 1987 crash, the futures market declined by 20% at the open. George Soros had executed a large sell order and later sued his broker for an excessively expensive order execution.

2010 Flash Crash: A joint study by the CFTC and SEC identified approximately $4 billion in sales of futures contracts by one entity as a trigger for the event.

Conventional Wisdom: CAPM Intuition

Miller, Scholes, Fama, Leland and Rubinstein: Conventional wisdom holds that prices react to changes in fundamental information, not to the price pressure resulting from trades by individual investors.

In competitive markets, investors have minimal private information and their trades have minuscule price impact.

The CAPM implies the demand for market indices is very elastic.

Conventional Wisdom: CAPM Intuition

Merton H. Miller (1991) wrote about the 1987 crash:

"Putting a major share of the blame on portfolio insurance for creating and overinflating a liquidity bubble in 1987 is fashionable, but not easy to square with all relevant facts . . . . No study of price-quantity responses of stock prices to date supports the notion that so large a price decrease (about 30 percent) would be required to absorb so modest (1 to 2 percent) a net addition to the demand for shares."

The conventional wisdom usually assumes that trading **one percent of market capitalization** moves prices by **one percent**.

Conventional Wisdom: Market Efficiency

The Brady report says about the 1929 crash:

"To account for the contemporaneous 28 percent decline in price, this implies a price elasticity of 0.9 with respect to trading volume which seems unreasonably high."

The conventional wisdom usually assumes that trading **five or ten percent of daily trading volume** has price impact **close to zero**.

Conventional Wisdom: Math

The expected log-percentage market impact $\Delta \ln P$ from buying or selling Q shares with N shares outstanding is

$$\Delta \ln P \approx \frac{\Delta P}{P} = \frac{Q}{N}.$$

$$\Delta \ln P \approx \frac{\Delta P}{P} = \frac{Q}{250 \cdot V},$$

since market capitalization is approximately equal to 250 times daily volume.

Conventional Wisdom and Invariance

We disagree with conventional wisdom:

- Large trades, even those known to have no information content such as the margin sales of 1929 or the portfolio insurance sales in 1987, do have large effect of prices.
- Selling pressure of **1% of market capitalization** can lead to decline in index prices of **20-50%**.
- Selling pressure of **10% of average daily volume** can lead to decline in index prices of **2-3%**.

Animal Spirits and Invariance

Keynes (1936), Shiller and Akerlof (2009): Animal spirits holds that price fluctuations occur as a result of random changes in psychology, which may not be based on information or rationality.

We disagree: Large crashes are neither random nor unpredictable; they are often discussed before crashes occur. The flash crashes were unpredictable, but prices rapidly mean-reverted.

Market Microstructure Invariance

Invariance suggests that the business time is faster for active stocks and slower for inactive stocks.

- **For active stocks** (with high trading volume and high volatility), trading games are played at a **fast pace**.
- **For inactive stocks** (low trading volume and low volatility), trading games are played at a **slow pace**.

Other than the speed at which they are played, trading games are the same!

Business Speed

Business time relates to the speed of bet arrival. Let H denote the length of a "business day" it is inversely proportional to the rate at which bets arrive. Define "trading activity" as

$$W = P \cdot V \cdot \sigma.$$

Dimensional analysis suggests

$$H \sim W^{-2/3} \sim (P \cdot V \cdot \sigma)^{-2/3}.$$

Invariance: Math

$$\frac{Q}{V \cdot H} \stackrel{d}{=} Z^* \quad \text{is invariant} \quad \rightarrow \quad \frac{Q}{V} \quad = H \cdot Z^*.$$

$$\left(\frac{W^*}{W}\right)^{2/3} \cdot Z^*.$$

$$\frac{\Delta P}{P \cdot \sigma \cdot \sqrt{H}} = f(Z^*) \quad \text{is invariant} \quad \rightarrow \quad \frac{\Delta P}{P} \quad = \sigma \cdot \sqrt{H} \cdot f(Z^*).$$

$$\frac{\Delta P}{P} = \alpha \cdot \sigma \cdot \sqrt{H} \cdot \left(\frac{Q}{V \cdot H}\right)^{\beta}.$$

where $\quad H \sim W^{-2/3} \sim (P \cdot V \cdot \sigma)^{-2/3}.$

Invariance: Conjectures

Market microstructure invariance is the following two conjectures:

- The distribution of standard deviations of dollar gains and losses on bets is the same across markets, *when standard deviation is measured in units of business time*.

- The expected dollar transactions costs of executing similar bets are constant across markets, *when similar bets are defined as bets transferring the same dollar risks per unit of business time*.

Invariance: Proof

I. Since $V = \gamma \cdot \bar{Q}$, trading activity W can be written as

$$W := P \cdot V \cdot \sigma = \gamma \cdot \bar{Q} \cdot P \cdot \sigma.$$

II. Invariance conjectures dollar risk $P \cdot \sigma$ transferred by a bet of $\bar{Q}$ per units of business time H is invariant,

$$\bar{Q} \cdot P \cdot \sigma \cdot \sqrt{H} = \frac{\bar{Q} \cdot P \cdot \sigma}{\sqrt{\gamma}} = \text{const.}$$

In terms of $a := \bar{Q} \cdot P \cdot \sigma$ and $b := \gamma$:

$$a \cdot b = W \qquad \text{and} \qquad a \cdot b^{-1/2} = \text{const.}$$

Hence, average bet size $\bar{Q}$ and bet arrival rate $\gamma = 1/H$:

$$\bar{Q} \cdot P \cdot \sigma = a \sim W^{1/3} \qquad \text{and} \qquad \gamma = 1/H = b \sim W^{2/3}.$$

Invariance: Summary of Theory

- "Market Microstructure Invariance: Empirical Hypotheses" (Ecma, 2016): Empirical conjectures and tests.
- "Market Microstructure Invariance: A Dynamic Equilibrium Model": Dynamic equilibrium model of speculative trading in which liquidity constrained investors seek to profit from trading on signals with invariant cost.
- "Dimensional analysis, leverage neutrality, and market microstructure invariance": Physicists' approach, apply dimensional analysis (consistency of units, Buckingham π-theorem)
- "Adverse Selection and Liquidity: From Theory to Practice": A meta-model.

Testing - Portfolio Transition Data

The empirical implications of the three proposed models are tested using a proprietary dataset of **portfolio transitions**.

- Portfolio transition occurs when an old (legacy) portfolio is replaced with a new (target) portfolio during replacement of fund management or changes in asset allocation.

- Our data includes 2,680+ portfolio transitions executed by a large vendor of portfolio transition services over the period from 2001 to 2005.

- Dataset reports executions of 400,000+ orders with average size of about 4% of ADV.

Portfolio Transitions and Bet Sizes

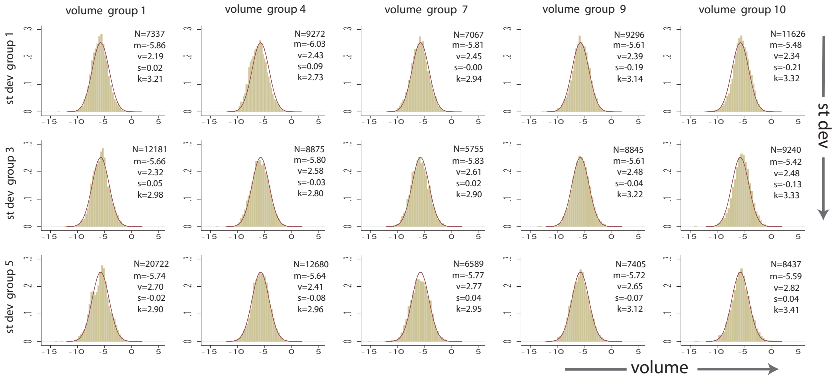
Kyle and Obizhaeva (2016) use portfolio transition data to measure distribution of bet size. Assume portfolio transition trades are representative "bets".

According to invariance hypothesis,

$$\ln\left(\frac{\tilde{Q}}{V \cdot H}\right) \sim \ln\left(\frac{\tilde{Q}}{V} \cdot W^{2/3}\right)$$

is **invariant** across stocks and time.

Distributions of Order Sizes



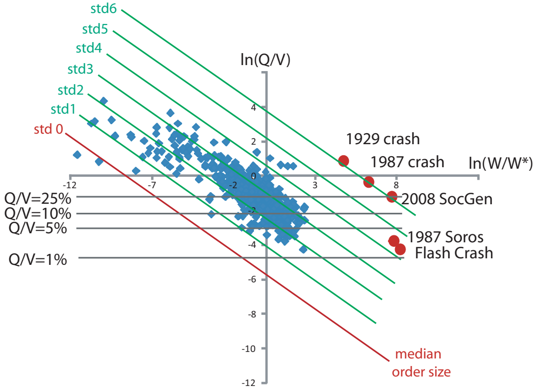
Trading game invariance works well for **entire distributions** of order sizes. Distributions are appr. **log-normal** $(\mu = -5.69, \sigma^2 = 2.50)$.

Invariance: Summary of Empirics

- Portfolio transitions: Bet size and transactions costs
- S&P 500 E-mini futures contracts: Trade size
- Korean stock market: Switching points
- U.S. Stock Market: Trade size
- Russian and U.S. Stock Market: Trade size and bid-ask spreads
- Reuters news articles: Frequency for companies

Implication for Market Crashes

Order of 5% of daily volume is "normal" for a typical stock and "unusually large" for the market.



Conventional intuition that order equal to 5% of average daily volume will not trigger big price changes in indices is wrong!

Implication: Transactions Cost Formula

For direct estimate, invariance suggests a simple formula for calculation of expected price impact cost for any order of X shares for any security with a current stock price P dollars, expected trading volume V shares per calendar day, and daily volatility σ:

$$\ln\left(1+\frac{\Delta P(X)}{P}\right) = \bar{\lambda}/10^4 \cdot \left(\frac{P\cdot V}{40\cdot 10^6}\right)^{1/3} \cdot \left(\frac{\sigma}{0.02}\right)^{4/3} \cdot \frac{X}{(0.01)V}.$$

where $\frac{1}{2}\bar{\lambda} = 2.89$ (standard error 0.195) is calibrated based on **portfolio transition trades** in Kyle and Obizhaeva (2016). Order for 1% of daily volume in benchmark stock has price impact of $\bar{\lambda} = 5.78$ bp.

Stock Market Crashes: Implementation Issues

To apply microstructure invariance, several implementation issues need to be discussed:

- **Boundary of the market:** Different securities and futures contracts, traded on various exchanges, may share the same fundamentals or be correlated. How to aggregate estimates across economically related markets? How to identify market boundaries?
- **Permanent vs. transitory price impact** Invariance formula assumes that orders are executed in some "natural" units of time. If execution is speeded up, then invariance formulas may underestimate price impact.
- **Inputs:** Invariance formulas requires expected volume and expected volatility as inputs. Expected volume and volatility may be higher than historical levels during extreme events.

Stock Market Crashes: Implementation Issues

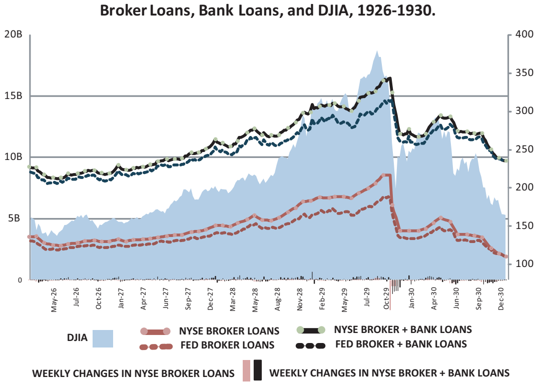
- **Changes in market mechanisms:** Estimates are based on portfolio transitions during 2001-2005, but applied to the entire period from 1929 to 2010. Changes in technologies, electronic handling of orders, reduction in tick size could have changed "deep parameters" of trading games.

- **Other considerations:** Invariance formula predicts impact of sales by particular group of traders. Other events may influence prices at the same time, including arrival of news and trading by other traders.

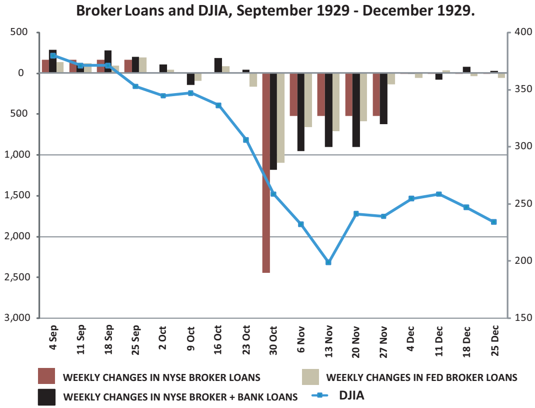
1929 Stock Market Crash: Facts

- In 1920s, many Americans became heavily invested into stocks (as in late 1990s), with a significant portion of investments made in margin accounts.
- To finance margin accounts, brokers relied on broker loans, pooling purchased securities to pledge as collateral (similar to shadow banking system in 2000s).
- Lenders were banks (except for NY banks after 1927), investment trusts, corporations, and foreign institutions.
- After doubling in value during the two years prior to Sept 1929, the Dow fell by 9% before Oct 24, 1929. This decline led to liquidations of stocks in margin accounts.
- During Oct 24 through Oct 30, **the Dow fell by 25%**. The slide continued for three more weeks. From Sept 25 to Dec 25, **the Dow fell by 48%**.

1929 Stock Market Crash



1929 Stock Market Crash



10/23-10/30: Margin sales of $1.181 billion.

09/25-12/25: Margin sales of $4.348 billion.

1929 Stock Market Crash

Facts about 1929 stock market crash:

- **Volatility** was about 2.00%.
- **Trading volume** was **$342.29 million** per day.
- Prior to 1935, the volume reported on the ticker did not include “odd-lot” transactions and “stopped-stock” transactions (about 30% percent of the “reported” volume), so adjust reported volume by 10/7.
- Inflation makes 1929 dollar worth more than 2001-2005 dollar: $1 in 1929 to $9.42 in 2005.

  from Sept 25 to Dec 25, margin sales were equal to
- During 10/24-10/29, the Dow declined by **24%** from 305.85 to 230.07. During 9/25-12/25, the Dow declined by **34%** from 305.85 to 230.07.

1929 Stock Market Crash

Invariance formula implies decline of **49.22%** during 10/24-10/30,

$$1 - \exp\left[- \frac{5.78}{10^4} \cdot \left(\frac{488.98 \cdot 10^6 \cdot 9.42}{(40)(10^6)} \right)^{1/3} \cdot \left(\frac{0.0200}{0.02} \right)^{4/3} \cdot \frac{1.181 \cdot 10^9}{(0.01)(488.98 \cdot 10^6)} \right].$$

Invariance formula implies decline of **91.75%** during 09/25-12/25,

$$1 - \exp\left[- \frac{5.78}{10^4} \cdot \left(\frac{488.98 \cdot 10^6 \cdot 9.42}{(40)(10^6)} \right)^{1/3} \cdot \left(\frac{0.0200}{0.02} \right)^{4/3} \cdot \frac{4.348 \cdot 10^9}{(0.01)(488.98 \cdot 10^6)} \right].$$

Invariance suggests **margin sales** should have had a larger market impact than the actual price changes of **24%** during 10/24-10/30 and **34%** during 9/25-12/25.

1929 Stock Market Crash - Robustness

	Months Preceding 24 October 1929					
N:	1	2	3	4	6	12
ADV (in 1929-$M)	488.98	507.08	479.65	469.45	4425.47	429.06
Daily Volatility	0.0200	0.0159	0.0145	0.0128	0.0119	0.0111
Sales 10/24-10/30 (%ADV)	242%	233%	246%	252%	278%	275%
Price Impact 10/24-10/30	**49.22%**	38.67%	36.05%	32.04%	31.05%	28.72%
Sales 9/25-12/25 (%ADV)	1270%	1225%	1295%	1323%	1460%	1448%
Price Impact 9/25-12/25	**91.75%**	83.47%	80.71%	75.87%	74.56%	71.25%

The actual price changes were **24%** during 10/24-10/30 and **34%** during 9/25 and 12/25. The conventional wisdom predicts price decline of **1.36%** and **4.99%**, respectively.

1987 Stock Market Crash: Facts

- **Volatility** during crash was about 1.35%.
- **Trading volume** on October 19 was **$20 billion** ($10.37 billion futures plus $10.20 billion stock).
- From Wednesday to Tuesday, **portfolio insurers** sold **$14 Billion** ($10.48 billion in the S&P 500 index futures and $3.27 billion in the NYSE stocks in 1987 dollars).
- Inflation makes 1987 dollar worth more than 2001-2005 dollar: $1 in 1987 to $1.54 in 2005.
- From Wednesday to Tuesday, **S&P 500 futures** declined from 312 to 185, a decline of **40%** (including bad basis). **Dow** declined from 2500 to 1700, a decline of **32%**.

1987 Stock Market Crash

Our market impact formula implies decline of **19.12%**,

$$1 - \exp\left[-5.78/10^4 \cdot \left(\frac{(10.37 + 10.20) \cdot 10^9 \cdot 1.54}{40 \cdot 10^6} \right)^{1/3} \cdot \left(\frac{0.0135}{0.02} \right)^{4/3} \cdot \frac{(10.48 + 3.27) \cdot 10^9}{(0.01)(10.37 + 10.20) \cdot 10^9} \right]$$

Invariance suggests **portfolio insurance selling** had market impact smaller than the actual price change of **32%** in stock market and **40%** in futures market.

1987 Stock Market Crash - Robustness

	Months Preceding 14 October 1987					
N:	1	2	3	4	6	12
S&P 500 ADV (1987-$B)	10.37	11.29	11.13	10.12	10.62	9.85
NYSE ADV (1987-$B)	10.20	10.44	10.48	10.16	10.04	9.70
Daily Volatility	0.0135	0.0121	0.0107	0.0102	0.0112	0.0111
Sell Orders as % ADV	66.84%	63.28%	63.65%	67.82%	66.53%	70.33%
Price Impact of Sell Orders	**19.12%**	16.20%	14.00%	13.59%	15.10%	15.60%
Price Impact of Imbalances	**15.75%**	13.30%	11.47%	11.13%	12.39%	12.80%

The actual price change was **32%** in stock market and **40%** in futures market. The conventional wisdom predicts price declines of **0.51%** for portfolio insurers' order imbalances and **0.63%** for their sales.

Soros's Trades in 1987: Facts

Facts about Soros's trades after 1987 stock market crash:

- **Volatility** prior to October 22 was about 8.63%.
- **Trading volume** prior to October 22 was **$13.52 billion** in futures.
- At the open of October 22, 1987, George Soros sold **2,400 contracts of S&P 500 futures** at a limit price of 200. A broker oversold **651 contracts**. Later in the morning, a pension plan sold **2,478 contracts**.
- Inflation makes 1987 dollar worth more than 2001-2005 dollar: $1 in 1987 to $1.54 in 2005.
- Price declined by **22%** from 258 at close of October 21, 1987, to 200 and then rebounded, over the next two hours, to the levels of the previous day's close.
- Soros sued a broker for tipping off other traders and executing order at too low prices.

Soros's Trades in 1987

Our market impact formula implies decline of **7.21%**,

$$1 - \exp\Big[-\frac{5.78}{10^4} \cdot \left(\frac{13.52 \cdot 10^9 \cdot 1.54}{40 \cdot 10^6}\right)^{1/3} \cdot \left(\frac{0.0863}{0.02}\right)^{4/3} \cdot \frac{309.60 \cdot 10^6}{(0.01)(13.52 \cdot 10^9)}\Big].$$

Invariance suggests somewhat smaller price impact relative to the actual price change of **22%**.

Soros's Trades in 1987 - Robustness

	Months Preceding 22 October 1987					
N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-$B)	13.52	11.72	11.70	10.99	10.75	10.04
Daily Volatility	0.0863	0.0622	0.0502	0.0438	0.0365	0.0271
2,400 contracts as %ADV	2.29%	2.64%	2.65%	2.82%	2.88%	3.08%
Price Impact A	**7.21%**	5.18%	3.92%	3.42%	2.73%	1.93%
Price Impact B	**9.07%**	6.54%	4.96%	4.32%	3.45%	2.45%
Price Impact C	**15.83%**	11.53%	8.80%	7.70%	6.17%	4.40%

(A) 2,400 contracts; (B) $2,400 + 651$ contracts; (C) $2,400 + 651 + 2,478$ contracts. The actual price change was **22%**. The conventional wisdom predicts price declines of **0.01%**, **0.02%**, and **0.03%**, respectively.

Fraud at Société Générale, January 2008: Facts

- From Jan 21 to Jan 23, a fraudulent position of Jérôme Kerviel had to be liquidated: **€30 billion** in Euro STOXX50 futures, **€18 billion** in DAX futures, and **€2 billion** in FTSE futures.
- **Trading volume** was **€69.51 billion** in seven largest European exchanges and **€110.98 billion** in ten most actively traded Euro pean index futures.
- **Volatility** was about **1.10%** per day in Stoxx TMI.
- Inflation makes 2008 dollar worth less than 2001-2005 dollar: \$1 in 2008 to \$0.92 in 2005.
- Bank has reported exceptional losses of **€6.3 billion**, which were attributed to "adverse market movements" between Jan 21 and Jan 23. Broad European index Stoxx TMI declined by **9.44%** from 316.73 on January 18 to its lowest level of 286.82 on January 21. Many European markets experienced worst price declines.

Liquidation of Kerviel's Positions in 2008

Our market impact formula implies decline of **12.37%**,

$$1 - \exp\Big[-\frac{5.78}{10^4} \cdot \Big(\frac{180.49 \cdot 1.4690 \cdot 0.92 \cdot 10^9}{40 \cdot 10^6} \Big)^{1/3} \Big(\frac{0.0011}{0.02} \Big)^{4/3} \frac{50}{(0.01)180.49} \Big].$$

Invariance suggests price impact similar in magnitude to the actual price change of **9.44%**.

Liquidation of Kerviel's Positions - Robustness

	Months Preceding January 18, 2008					
N:	1	2	3	4	6	12
Stk Mkt ADV (2008-€B)	69.51	66.51	67.37	67.01	66.73	66.32
Fut Mkt ADV (2008-€B)	110.98	114.39	118.05	117.46	127.17	121.26
Daily Volatility	0.0110	0.0125	0.0121	0.0117	0.0132	0.0111
Order as %ADV	27.70%	27.64%	26.97%	27.11%	25.79%	26.66%
Price Impact	**12.37%**	14.48%	13.67%	13.21%	14.79%	12.14%
Total Losses (2008-€B)	3.19	3.76	3.54	3.42	3.85	3.13
Losses/Adj A (2008-€B)	5.50	6.07	5.85	5.73	6.16	5.44
Losses/Adj B (2008-€B)	7.81	8.38	8.16	8.04	8.47	7.75

Adj A and Adj B are adjustments for losses during 12/31/2007 through 01/18/2008. The actual price change was **9.44%** in Stoxx Europe TMI. The reported losses were **€6.3 billion** relative to value on 12/31/2007. The conventional wisdom predicts price decline of **0.43%**.

Liquidation of Kerviel's Positions - DAX, Stoxx 50, FTSE 100

	Months Preceding January 18, 2008					
N:	1	2	3	4	6	12
EURO STOXX 50 (2008-€B)	55.19	54.02	54.64	53.75	57.88	52.32
Daily Volatility	0.0098	0.0110	0.0098	0.0095	0.0112	0.0099
Euro Stoxx 50 Order as %ADV	54.36%	55.54%	54.90%	55.81%	51.83%	57.33%
Price Impact	**13.82%**	16.15%	14.00%	13.63%	15.86%	14.47%
DAX (2008-€B)	32.40	31.86	33.01	32.40	35.55	35.80
Daily Volatility	0.0100	0.0108	0.0096	0.0090	0.0100	0.0098
Order as %ADV	55.56%	56.49%	54.53%	55.56%	50.63%	50.28%
Price Impact	**12.34%**	13.63%	11.55%	10.83%	11.62%	11.30%
FTSE 100 (2008-£B)	7.34	7.87	7.73	7.74	8.01	7.21
Daily Volatility	0.0109	0.0138	0.0124	0.0119	0.0137	0.0110
Order as %ADV	27.24%	25.41%	25.88%	25.84%	24.97%	27.76%
Price Impact	**4.75%**	6.16%	5.43%	5.12%	6.05%	4.86%
Total Losses (2008-€B)	3.35	3.86	3.31	3.17	3.62	3.35
Losses/Adj A (2008-€B)	5.66	6.17	5.62	5.48	5.93	5.66
Losses/Adj B (2008-€B)	7.97	8.48	7.92	7.79	8.24	7.97

DAX declined by **11.91%**; Euro Stoxx50 by **10.50%**; FTSE100 by **4.65%**

Integrated vs. Separate Markets

Financial markets are **integrated**. Many European markets experienced large declines during Jan 18 through Jan 22 with rapid recoveries by Jan 24.

- The **Spanish index IBEX 35** dropped by **7.54%**, the biggest one-day fall in the history of the index (since 1992).
- The **Italian index FTSE MIB** fell by **10.11%**.
- The **Swedish index OMXS 30** fell by **8.63%**.
- The **French index CAC 40** fell by **11.53%**.
- The **Dutch index AEX** fell by **10.80%**.
- The **Swiss Market Index** fell by **9.63%**.

Similar patterns were observed during the 1987 market crash. How to aggregate estimates across economically related markets is a question for the future research.

The “Flash Crash” of May 6, 2010: Facts

- News media report that a large trader sold **75,000 S&P 500 E-mini contracts**. One contracts represents ownership of about \$58,200 with S&P level of 1,164 on May 5.
- **Trading volume** was **\$132.00 billion** in S&P 500 E-mini futures and **\$161.41 billion** in stock market in 2010 dollars.
- **Volatility** was about **1.07%** per day in the S&P 500 E-mini future. It could be higher due to European debt crisis, e.g., $\sigma = 0.02$
- Inflation makes 2010 dollar worth less than 2001-2005 dollar: \$1 in 2010 to \$0.90 in 2005.
- The E-mini S&P 500 futures price fell from 1,113 at 2:40 p.m. to 1,056 at 2:45 p.m., a decline of **5.12%** over a five-minute period. Pre-programmed circuit breakers stopped futures trading for five seconds. Over the next ten minutes, the market rose by about 5%.

Flash Crash in May 2010

Our market impact formula implies decline of **0.70%**,

$$1 - \exp\Big[-\frac{5.78}{10^4} \cdot \Big(\frac{(132+161)\cdot 0.90 \cdot 10^9}{40 \cdot 10^6}\Big)^{1/3} \cdot \Big(\frac{0.0107}{0.02}\Big)^{4/3} \cdot \frac{75,000 \cdot 50 \cdot 1,164}{0.01 \cdot (132+161)\cdot 10^9}\Big].$$

Invariance suggests somewhat smaller price impact relative to the actual price change of **5.12%**.

Flash Crash in May 2010 - Robustness

	Months Preceding 6 May 2010					
N:	1	2	3	4	6	12
S&P500 Fut ADV (2010 \$B)	132.00	107.49	109.54	112.67	100.65	95.49
Stk Mkt ADV (2010 \$B)	161.41	146.50	142.09	143.03	132.58	129.30
Daily Volatility	0.0107	0.0085	0.0078	0.0090	0.0089	0.0108
Order as %ADV	1.49%	1.72%	1.73%	1.71%	1.87%	1.94%
Price Impact (hist σ)	**0.70%**	0.57%	0.50%	0.61%	0.63%	0.84%
Price Impact ($\sigma = 2\%$)	**1.60%**	1.76%	1.77%	1.75%	1.86%	1.91%

The actual price change of the S&P 500 E-mini futures was **5.12%**. The conventional wisdom predicts price decline of **0.03%**.

Summary of Five Crash Events: Actual and Predicted Price Declines

	Actual	Predicted Invariance	Predicted Conventional	%ADV	%GDP	Frequency
1929 Market Crash	**25%**	**49.22%**	**1.36%**	241.52%	1.136%	once/5,539 years
1987 Market Crash	**40%**	**19.12%**	**0.63%**	66.84%	0.280%	once/716 years
1987 Soros's Trades	**22%**	**7.21%**	**0.01%**	2.29%	0.007%	once/month
2008 SocGén Trades	**9.44%**	**12.37%**	**0.43%**	27.70%	0.401%	once/819 years
2010 Flash Crash	**5.12%**	**0.50%**	**0.03%**	1.49%	0.030%	several/year

Discussion

- **Price impact predicted by invariance is large and similar to actual price changes.**
- **The financial system in 1929 was remarkably resilient.** The 1987 portfolio insurance trades were equal to about 0.28% of GDP and triggered price impact of 32% in cash market and 40% in futures market. The 1929 margin-related sales during the last week of October were equal to 1% of GDP. They triggered price impact of 24% only.

Discussion - Cont'd

- **Speed of liquidation magnifies short-term price effects.** The 1987 Soros trades and the 2010 flash-crash trades were executed rapidly. Their actual price impact was greater than predicted by microstructure invariance, but followed by rapid mean reversion in prices.

- **Market crashes happen too often.** The three large crash events were approximately 6 standard deviation bet events, while the two flash crashes were approximately 4.5 standard deviation bet events. Right tail appears to be fatter than predicted. The true standard deviation of underlying normal variable is not 2.50 but 15% bigger, or far right tail may be better described by a power law.

Early Warning System

Early warning systems may be useful and practical. Invariance can be used as a practical tool to help quantify the systemic risks which result from sudden liquidations of speculative positions.