Whoever Has Will Be Given More: Information Sharing in Financial Markets

Itay Goldstein, Yan Xiong and Liyan Yang*

February 24, 2021

Abstract

This paper studies information sharing between strategic investors who are privately informed about asset fundamental with different precision levels. We find that a coarsely informed investor would always share her information "as is" if her counterparty investor is well informed about the fundamental. By doing so, the coarsely informed investor invites the well informed investor to trade against her information, thereby offsetting her informed order flow and reducing the price impact. In equilibrium, the coarsely informed investor gains from the information sharing and the well informed investor loses from it. Our model offers insights for financial phenomena such as information networks in financial markets and institutional investors' trading strategies based on sentiment.

Keywords: Information sharing, communication, sentiment, asset pricing

JEL: D82, G14, G18

*Goldstein: University of Pennsylvania, itayg@wharton.upenn.edu. Xiong: Hong Kong University of Science and Technology, yanxiong@ust.hk. Yang: Rotman School of Management, University of Toronto, liyan.yang@rotman.utoronto.ca. We thank Swaminathan Balasubramaniam, Snehal Banerjee, Ing-Haw Cheng, Jason Donaldson, Phil Dybvig, Hanming Fang, Chong Huang, Naveen Khanna, Richard E. Kihlstrom, Mina Lee, Jiasun Li, Michael Sockin, Günter Strobl, Vladimir Vladimirov, Chaojun Wang, Yao Zeng, and the conference/seminar participants at the 2020 Greater China Area Finance Conference, the 2020 Virtual Finance Theory Seminar, Renmin University of China, ShanghaiTech University, Tsinghua University, University of Chinese Academy of Sciences, University of Technology Sydney, and Zhejiang University.

1 Introduction

In financial markets, some well-known proverbs such as “a barking dog never bites” and “a loaded wagon makes no noise” vividly describe the situation in which market participants who own superior information carefully hide and trade on it. In other words, a dog that makes a good bite is the one that is the most silent. Meanwhile, these proverbs suggest that the market participants who make the most noise (a barking dog) rarely express true insights. While it is relatively rare to observe the investment genius share their insights in the public, there has been a great number of investment opinions and analysis shared on the social media such as Twitter, Seeking Alpha, and Facebook Posts. Then, why does a barking dog bark? Is that just noise?

Information sharing also appears to be commonplace among professional investors. For example, Shiller and Pound (1986) provide survey evidence that a majority of institutional investors in the NYSE attribute their recent trades to discussions with peers. Hong, Kubik, and Stein (2005) find that a mutual fund manager’s trading is similar to other fund managers located in the same city and interpret this finding in terms of information spread by word of mouth. More recently, investment conferences as a new type of industry event have become popular. In these conferences, professional investors pitch their investment ideas to the wide audiences including activists, fundamental equity funds, investment advisors, and sell-side analysts; these presented investment ideas are closely followed in the financial media and on investment blogs (Luo, 2018). Why do these professional investors share their investment ideas? In this investor community, who shares information with whom?

Some common intuition for an investor’s information-sharing behavior holds that by sharing her privileged information, the investor can manipulate markets (Benabou and Laroque, 1992), or accelerate price discovery towards the direction that is in favor of her existing positions (Ljungqvist and Qian, 2016). However, such intuition implies that only investors with known superior information have incentives to share their information, which cannot explain why information sharing is such a widespread phenomenon in financial markets.

In this paper, we propose that even an investor with coarse information has a strategic motive to genuinely share her information. This novel theory complements the existing explanations for investors’ information-sharing behavior. It can be further applied to understand the role of social media in financial markets, the nascent sentiment trading strategy implemented by institutional investors, and the formation of and information transmission within a information network.

To study a strategic investor's information-sharing incentives in financial markets, we adopt a standard Kyle (1985) framework and extend it by considering two investors endowed with private information of different precision and allowing for the possibility of information sharing between them. The market consists of noise traders, competitive market makers, and two risk-neutral rational investors. A single risky asset is traded in the financial market. One investor perfectly learns about the fundamental of the asset and we refer to him as the insider. The other investor only observes a noisy signal of the fundamental and we refer to her as the coarsely informed investor. Investors can share their own private information with each other; for example, the coarsely informed investor can choose whether to share her information, and if so, how much to share with the insider. The rest of the model is standard: the two investors trade on their respective endowed information and the shared information, if any; they then submit market orders to maximize their expected profits; and the orders are executed by market makers at the conditional expected value of the asset given the total order flows.

The central finding of the paper is that in financial markets information can transmit from the less informed investor to the more informed one; that is, the insider never shares his information whereas the coarsely informed investor always genuinely shares her information. Such information-sharing behavior has further consequences for the involved investors' profits and market quality. Specifically, after the information sharing, the coarsely informed investor makes higher profits whereas the insider becomes worse off, and market liquidity worsens but both market efficiency and total trading volume increase.

Why is the coarsely informed investor willing to share her information "as is"? The key driving force for her information-sharing behavior is the novel *trading-against-error effect*. This crucially hinges on the fact that the insider is able to identify the error in the coarsely informed investor's endowed information. After observing the shared information, the insider tends to trade against the error in the shared information, thereby offsetting the coarsely informed investor's informed order flow and reducing its price impact. Specifically, the insider well understands the coarsely informed investor's trading strategy and can calculate her trading demand that is not justified on the basis of the asset fundamental. For example, if the error component in the shared information is positive, the coarsely informed investor tends to overly buy or inadequately short the risky asset. From the perspective of the insider, however, this is pure noise.

If, say, the coarsely informed investor overly buys the asset, after observing the shared information, the insider knows that the asset price is pushed too high so that he optimally refrains from buying too many units of the asset. Alternatively, if the coarsely informed investor inadequately shorts the risky asset because of the noise component in her endowed information, the insider knows that the asset price is not low enough and he will short more of the asset. In either case, the insider trades against the shared information, corrects the mispricing, and makes profits accordingly. In some sense, the insider provides liquidity to the coarsely informed investor. Overall, the trading-against-error effect is so strong that it encourages the coarsely informed investor to not only share her information but also does it genuinely.

Further, the more genuine the shared information, the more accurately the insider can calculate the coarsely informed investor's demand that is driven by the error in her endowed information, the more aggressively the insider trades against the shared information, and the less price impact triggered by the coarsely informed investor's order flow. Consequently, the trading-against-error effect is so strong that the coarsely informed investor not only shares her information, but does so truthfully.

Next, why is the insider never sharing his information? Different from that the insider trades against the information shared by the coarsely informed investor, the coarsely informed investor always trades alongside the insider's shared information. This is because for the coarsely informed investor, any piece of information shared by the more informed investor is instrumental for her to make better forecast of the asset fundamental. Therefore, for the insider any information sharing can only dissipate his informational advantage and erode his profits, which prevents him from sharing his information. In this way, information can only from the coarsely informed investor to the insider.

Such information sharing has further consequences for the profits of the involved investors and market quality. We find that relative to the economy without information sharing, when to share information is permitted, the coarsely informed investor makes higher profits whereas the insider becomes worse off. As analyzed above, the coarsely informed investor benefits from the trading-against-error effect in information sharing. Nonetheless, why is the insider worse off? While the insider gains by detecting the error component in the shared information, trading against it, and correcting the mispricing accordingly, he loses because of the more competitive pressure from the coarsely informed investor and the more aggressive pricing by market makers. First, with her order flow partially offset, the coarsely informed investor is less concerned about the price impact and engages in more aggressive trading accordingly. As such, the insider is forced to trade less aggressively on his fundamental information. Second, a the insider trades against the shared information (and thus the error in the coarsely informed investor's endowed information), the two investors' aggregate order flow becomes more correlated with the fundamental. Faced with an effectively more informed investor side, market makers raise the price impact to manage the increased adverse-selection risk, thereby decreasing market liquidity and reducing both investors' profits. Taken together, both effects hurt the insider, ironically leaving him worse off despite the additional piece of free information.

As for market quality, we find that relative to the economy without information sharing, when to share information is permitted, market liquidity is lower whereas market efficiency and total trading volume are higher. Again, the key lies in the fact that the insider tends to trade against the shared information. Because this trading-against-error effect can reduce the noise in the two investors' aggregate order flow, market makers raise the price impact to manage the increasing adverse-selection risk, resulting in a lower market liquidity. Meanwhile, less noise in the total order flow suggests that it is more correlated with the asset fundamental. Therefore, market efficiency improves; that is, asset prices contains more fundamental information. Information sharing is also associated with more total trading volume; specifically, while the insider's trading volume decreases after information sharing, both the coarsely informed investor's and market makers' trading volume increases, and overall the total trading volume increases.

Finally, we consider four extensions of the baseline model and show that the novel trading-against-error effect robustly exists in various extensions. First, as long as the more informed investor can relatively accurately sift the error component in the shared information, the coarsely informed investor would like to genuinely share her information so as to benefit from the counterparty offsetting her informed order flow. In other words, information can be transmitted from a coarsely informed investor to a relatively well (though not perfectly) informed investor. Second, in the baseline model, because the insider becomes worse off after the information sharing, if possible, he would like to commit not to reading the shared information. However, we find that in the presence of multiple insiders, even if they are able to make such a commitment, in equilibrium all insiders may choose to read and trade against the shared information. This constitutes a prisoner's dilemma for the insiders because they would have been better off if they together commit to not using the information shared by the coarsely informed investor. Third, even if information acquisition is costly for the coarsely informed investor, she still has incentives to fully share her private information, invites the insider to trade against it, and makes higher profits accordingly. Fourth, even though the shared information may be leaked to the public during the communication process, we find that as long as market makers have low capabilities to interpret the information and the insider has superior ability to do so, the coarsely informed investor still has incentives to share her private information.

Overall, our theory provides a novel perspective of information-sharing behavior in financial markets and sheds new light on the related phenomena. First, as for the barking-dog questions raised in the beginning, we argue that the investment opinions expressed on the social media such as Twitter may not be mere noise, but instead can represent the true information owned by the posters. By making their information observable to the well informed investors such as hedge funds, these social media investors can have their order flow partially offset. Indeed, as the machine learning technology advances, analyzing the sentiment on the social media becomes feasible and gains popularity among hedge funds. Our theory further suggests that such a sentiment-based trading strategy may not be good for the well informed hedge funds as they can become worse off after trading against the sifted sentiment (error). Second, the two investors in our model can represent the very basic component of any information network in financial markets. In this sense, our theory provides an answer to such fundamental questions as how information network is formed and who shares information with whom.

Related Literature. Previous research has identified other possible reasons for why investors share their information. For example, insiders can use privileged information to manipulate markets (Benabou and Laroque, 1992). Ljungqvist and Qian (2016) suggest that in the face of noise trader risk, arbitrageurs with short positions may reveal their information to accelerate price correction, thereby circumventing limits to arbitrage. The idea that information revelation can be used to accelerate price correction is particularly relevant for investors with short-term incentives (Kovbasyuk and Pagano, 2015; Liu, 2017; Schmidt, 2019). In addition, by injecting noise into the spread information, an investor gains advantage over uninformed followers (Van Bommel, 2003) and commits to aggressive trading to other informed investors (Indjejikian, Lu, and Yang, 2014); by disclosing a mixture of fundamental information and her position, an investor induces market makers to move the asset price in a manner favorable to her (Pasquariello and Wang, 2016).

We contribute to this literature by offering a complementary explanation for information sharing/revelation in financial markets. Our explanation is unique in the following aspects. First, in our model information transmits from the *less* informed investor to the *more* informed one, whereas in all the other explanations the direction of information flow is the opposite. The key underlying this insight is that the information receiver tends to trade against the shared information, whereas in the existing explanations the receiver's trading is aligned with what the shared information suggests. This unique direction of information flow demonstrates that information sharing can be a widespread phenomenon in financial markets; that is, not only well informed investors, but also the ones with (very) coarse information, would be willing to share their information. Second, in our model the information sender is better off whereas the receiver becomes worse off after the information sharing. However, in the existing explanations, both should make profits from the information sharing at the expense of third parties (e.g., Indjejikian, Lu, and Yang, 2014). Third, our explanation does not require that the information sender owns initial positions or has short-term incentives. Unlike other explanations in which the investor "talks for her book" (e.g., Pasquariello and Wang, 2016; Schmidt, 2019), in our model the investor does not have any book yet and she instead reveals information to help build it.

Our paper is also related to the large literature on information transmission in financial markets. Starting from Admati and Pfleiderer (1986, 1988, 1990), there have been studies on how the informed agent monetizes her private information by selling it (e.g., Allen, 1990; Naik, 1997; Cespa, 2008; García and Sangiorgi, 2011). Fishman and Hagerty (1995) rationalizes the sales of information by arguing that via it informed traders can commit to aggressive trading, thereby forcing other informed traders to trade less aggressively. Biais and Germain (2002) study how to structure a combination of proprietary trading with indirect information sales (setting up a fund) to increase the overall profits from proprietary trades and fund trades. In addition to the transmission of information for a fee, the informed agents in financial markets may disclose their private information for various reasons such as fear of negative inferences (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981) and elimination of screening (Glode, Opp, and Zhang, 2018). Again, one commonality of this literature is that information transmits from the more informed agents to the less informed ones, and our model uniquely predicts that information can flow in the opposite direction.

Another strand of related literature studies noise/supply information in financial markets. Ganguli and Yang (2009) examine investors' incentives of acquiring information about the fundamental and the noise/supply in a static model, whereas Farboodi and Veldkamp (2020) consider a dynamic setting and study how financial data technology affects agents' information choices, trading strategies, and market outcomes. Other related works include Madrigal (1996), Cao, Lyons, and Evans (2003), Paul and Rytchkov (2018), among others. We contribute to this literature by focusing on the noise component in the investor's private information, rather than that in the asset supply, and we highlight its effect on investors' information-sharing behavior. Further, in our setting, the well informed investors do not actively search for the noise/supply information (e.g., Farboodi and Veldkamp, 2020), instead such information is voluntarily shared by the coarsely informed investor.

Our paper is also broadly related to the literature on communication and information network in financial markets. DeMarzo, Vayanos, and Zwiebel (2003) propose a model in which individuals with bounded rationality are subject to persuasion bias and fail to account for repetition in the information they receive; as such, the influence of an individual on group opinions depends not only on accuracy but also on her connectedness. Han and Hirshleifer (2016) study how the process by which ideas are transmitted affects active versus passive investment behavior. We contribute to this literature by identifying a novel information-sharing incentive and exploring its implications for market quality.

2 Model Setup

We consider a Kyle-type model (Kyle, 1985) and extend its analysis to allow for information sharing between investors. The economy has three dates, $t = 0, 1, 2$. Figure 1 describes the timeline of the economy. There is a single risky asset with a date-2 liquidation value $\tilde{v}$, where $\tilde{v} \sim N(0, 1)$.[1] The risky asset can be interpreted as a listed firm's stock. The financial market operates on date 1, and it is populated by three groups of agents: competitive market makers, noise traders, and two heterogeneously informed rational investors. As standard in the literature, market makers set the price based on the weak market-efficiency rule and noise traders submit exogenous random market orders. There are two rational investors that own private information about the fundamental of the risky asset and their information is of different precision. On $t = 0$, information can be shared between the two rational investors.

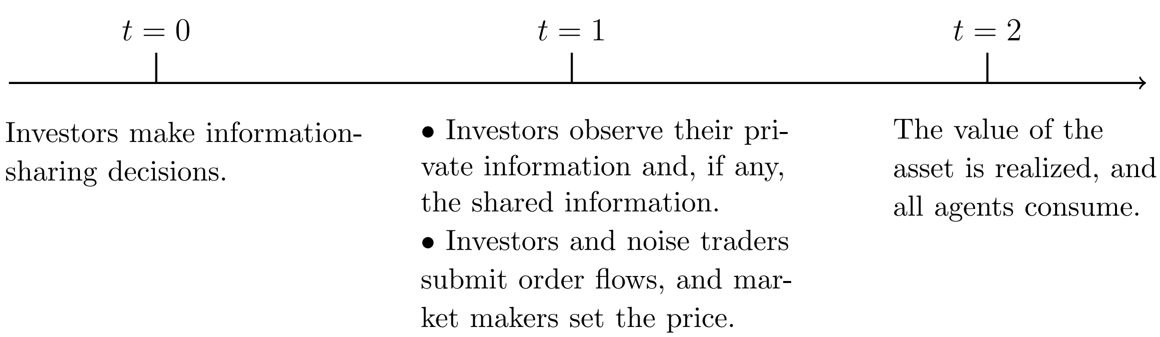


Figure 1: Timeline

We denote the two rational investors H and L. Investor H owns more precise information about the fundamental and he can be a corporate executive or a sophisticated hedge fund manager that possesses high-quality information about the firm's fundamental. In the baseline model, we assume that investor H can privately observe $\tilde{v}$ (i.e., an insider) and we relax this assumption in Section 4.1. Investor L is coarsely informed about the

[1]The normalization that $\tilde{v}$ has a zero mean and a unit standard deviation is without loss of generality. Instead, if we assume $\tilde{v} \sim N(\bar{v}, \sigma_v^2)$, then all our results would hold as long as we reinterpret the information precisions as signal-to-noise ratios.

fundamental, and she can only observe a private noisy signal as follows:

$$\tilde{y} = \tilde{v} + \tilde{e}, \text{ where } \tilde{e} \sim N(0, \rho^{-1}). \tag{1}$$

The parameter $\rho \in (0, +\infty)$ governs the quality of L's private information. If $\rho \to 0$, investor L is almost uninformed about the asset fundamental, whereas if $\rho \to \infty$ investor L knows the fundamental as precisely as investor H. Investor L can represent investors that actively collect information but are still coarsely informed.

On $t = 0$, the two rational investors simultaneously make their information-sharing decisions to maximize their respective expected trading profits. Specifically, investor H can share a garbled signal with investor L as follows:

$$\tilde{s}_H = \tilde{v} + \tilde{\varepsilon}_H, \text{ where } \tilde{\varepsilon}_H \sim N\left(0, \tau_H^{-1}\right),$$

whereas investor L's shared information is as follows:

$$\tilde{s}_L = \tilde{y} + \tilde{\varepsilon}_L, \text{ where } \tilde{\varepsilon}_L \sim N\left(0, \tau_L^{-1}\right).$$

The precisions of the shared information τ_H and τ_L are controlled by H and L, respectively, and can range between 0 and $+\infty$; that is, $\tau_i \in [0, +\infty]$, where $i \in \{H, L\}$. If $\tau_i = 0$, investor i's shared information is not informative at all, or equivalently investor i is not sharing any private information. If $\tau_i = +\infty$, then investor i shares the private information "as is."

Trading occurs on $t = 1$. Let $\tilde{p}$ denote the date-1 price of the risky asset in the financial market. Conditional on the endowed private information, as well as the shared information if any, investor $i \in \{H, L\}$ places market order $\tilde{x}_i$ to maximize the expected trading profits as follows:

$$E[\tilde{x}_i(\tilde{v} - \tilde{p})|\mathcal{F}_i], \tag{2}$$

where $\mathcal{F}_i$ indicates investor i's information set: $\mathcal{F}_H = \{\tilde{v}, \tilde{s}_H, \tilde{s}_L\}$ and $\mathcal{F}_L = \{\tilde{y}, \tilde{s}_H, \tilde{s}_L\}$. Noise traders place market order $\tilde{u}$, where $\tilde{u} \sim N(0, \sigma_u^2)$ (with $\sigma_u > 0$) and $\tilde{u}$ is independent of all other random shocks. Therefore, the total order flow faced by market makers are

$$\tilde{\omega} = \tilde{x}_H + \tilde{x}_L + \tilde{u}. \tag{3}$$

Then competitive market makers set price $\tilde{p}$ according to the weak-efficiency rule,

$$\tilde{p} = E[\tilde{v}|\tilde{\omega}]. \tag{4}$$

3 Information Sharing

In this section, we first characterize the two investors' optimal trading strategies on $t = 1$ given their information-sharing decisions, and then move backward to solve for their optimal information-sharing strategies on $t = 0$.

3.1 Trading on $t = 1$

Given any pair of the two investors' information-sharing strategy (τ_H, τ_L) on $t = 0$, we solve for their optimal trading strategies and market makers' equilibrium pricing rule on $t = 1$.

We consider a linear pricing rule for market makers $\tilde{p} = \lambda\tilde{\omega}$, where the total order flow $\tilde{\omega}$ is specified by equation (3), and linear trading rules for the two investors: $\tilde{x}_H = \alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{s}_L$ and $\tilde{x}_L = \beta_y\tilde{y} + \beta_H\tilde{s}_H + \beta_L\tilde{s}_L$, where coefficients $\alpha_v, \alpha_H, \alpha_L, \beta_y, \beta_H$, and β_L are endogenously determined. The coefficients α_v and β_y respectively represent the trading aggressiveness of investor H and investor L when they make decisions based on their endowed information. The coefficients $\alpha_H, \alpha_L, \beta_H$, and β_L capture the strategic interaction between the two investors when trading on the shared information.

With the information set $\mathcal{F}_H = \{\tilde{v}, \tilde{s}_H, \tilde{s}_L\}$, investor H's posterior beliefs about the value of the risky asset, investor L's endowed information, and noise trading are respectively as follows:

$$E[\tilde{v}|\mathcal{F}_H] = \tilde{v},\ E[\tilde{y}|\mathcal{F}_H] = \tilde{v} + \frac{\tau_L}{\rho + \tau_L}(\tilde{s}_L - \tilde{v}),\ \text{and } E[\tilde{u}|\mathcal{F}_H] = 0.$$

Then, investor H's conditional expected trading profits in (2) can be expressed as follows:

$$E[\tilde{x}_H\left(\tilde{v} - \tilde{p}\right)|\mathcal{F}_H] = \tilde{x}_H\left(\tilde{v} - \lambda\left(\tilde{x}_H + \beta_y\left(\tilde{v} + \frac{\tau_L}{\rho + \tau_L}(\tilde{s}_L - \tilde{v})\right) + \beta_H\tilde{s}_H + \beta_L\tilde{s}_L\right)\right). \tag{5}$$

Maximizing investor H's profits yields his optimal trading rule, $\tilde{x}_H = \alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{s}_L$, with

$$\alpha_v = \frac{\rho + \tau_L - \lambda\rho\beta_y}{2\lambda(\rho + \tau_L)},\ \alpha_H = -\frac{\beta_H}{2},\ \text{and } \alpha_L = -\frac{\beta_L}{2} - \frac{\beta_y\tau_L}{2(\rho + \tau_L)}. \tag{6}$$

For investor L, given her information set $\mathcal{F}_L = \{\tilde{y}, \tilde{s}_H, \tilde{s}_L\}$, we can express her conditional expected trading profits as follows:

$$E[\tilde{x}_L(\tilde{v} - \tilde{p})|\mathcal{F}_L] = \tilde{x}_L\left(\frac{\rho\tilde{y} + \tau_H\tilde{s}_H}{1 + \rho + \tau_H} - \lambda\left(\tilde{x}_L + \alpha_v\frac{\rho\tilde{y} + \tau_H\tilde{s}_H}{1 + \rho + \tau_H} + \alpha_H\tilde{s}_H + \alpha_L\tilde{s}_L\right)\right). \tag{7}$$

Again, maximizing the profits yields investor L's optimal trading rule, $\tilde{x}_L = \beta_y \tilde{y} + \beta_H \tilde{s}_H + \beta_L \tilde{s}_L$, with

$$\beta_y = \frac{\rho(1-\lambda\alpha_v)}{2\lambda(1+\rho+\tau_H)}, \ \beta_H = -\frac{\alpha_H}{2} + \frac{(1-\lambda\alpha_v)\tau_H}{2\lambda(1+\rho+\tau_H)}, \text{ and } \beta_L = -\frac{\alpha_L}{2}. \tag{8}$$

Equations (6) and (8) are the reaction functions, which jointly determine the equilibrium values of $(\alpha_v, \alpha_H, \alpha_L, \beta_y, \beta_H, \beta_L)$ as shown in the following lemma.

Lemma 1 *Given the two investors' information-sharing strategies (τ_H, τ_L) and market makers' pricing rule λ, investors' equilibrium trading strategies on $t = 1$ are characterized by $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_H \tilde{s}_H + \alpha_L \tilde{s}_L$ and $\tilde{x}_L = \beta_y \tilde{y} + \beta_H \tilde{s}_H + \beta_L \tilde{s}_L$ with*

$$\begin{aligned}
\alpha_v &= \big(2\tau_H\,(\tau_L+\rho) + 2(\rho+1)\tau_L + \rho(\rho+2)\big)\Omega^{-1}, \\
\alpha_H &= -2\tau_H\,(\tau_L+\rho)(3\Omega)^{-1}, \\
\alpha_L &= -2\rho\tau_L(3\Omega)^{-1}, \\
\beta_y &= \rho\,(\tau_L+\rho)\Omega^{-1}, \\
\beta_H &= 4\tau_H\,(\tau_L+\rho)(3\Omega)^{-1}, \\
\beta_L &= \rho\tau_L(3\Omega)^{-1},
\end{aligned}$$

where $\Omega = \lambda\left(4\tau_H\,(\tau_L+\rho) + 4(\rho+1)\tau_L + \rho(3\rho+4)\right).$

Lemma 1 states that while investor H trades in the same direction as suggested by his endowed information ($\alpha_v > 0$), he trades against not only his own shared information ($\alpha_H < 0$) but also the one shared by investor L ($\alpha_L < 0$). On the contrary, investor L trades alongside all pieces of information she has access to; that is, $\beta_y > 0$, $\beta_H > 0$, and $\beta_L > 0$. What determines the two investors' different trading rules?

First, it is intuitive that each investor's trading should be in the same direction as suggested by their own endowed information; that is, $\alpha_v > 0$ and $\beta_y > 0$. The endowed information is informative about the fundamental value of the risky asset. So when the signal indicates a positive (negative) return, the investor tends to buy (sell) the risky asset.

Second, while investor L's trading is aligned with the information shared by investor H, investor H trades against the information shared by investor L, namely, $\beta_H > 0$ and $\alpha_L < 0$. As the less informed side, when investor L receives the information shared by investor H, though the signal is noisy, investor L can still employ the information to make better forecasts of the asset fundamental. So her trading direction is aligned with what investor H's shared information suggests. However, since investor H has already been well informed of the asset fundamental, the information shared by investor L is of no additional use to him in forecasting the fundamental. Still, investor H uses this information. Why? Note that investor L's trading follows her endowed information $\tilde{y}$. Since $\tilde{y} = \tilde{v} + \tilde{e}$ is only a noisy signal of the asset fundamental, by trading on it, investor L trades not only "correctly" on the fundamental $\tilde{v}$, but also "incorrectly" on the error $\tilde{e}$. The latter noise trading can move asset price away from the fundamental value and if investor H could detect it, he would always have incentives to trade against it and make profits accordingly. For investor H, investor L's shared information exactly serves this purpose. Specifically, with the fundamental information $\tilde{v}$ and the information shared by investor L, investor H can infer the error in investor L's endowed information as follows:

$$E[\tilde{e}|\mathcal{F}_H] = \frac{\tau_L}{\rho + \tau_L} \left(\tilde{s}_L - \tilde{v} \right). \tag{9}$$

Since the inference $E[\tilde{e}|\mathcal{F}_H]$ always shares the same sign as that of $\tilde{s}_L$, it appears that investor H trades against $\tilde{s}_L$, namely, $\alpha_L < 0$. We refer to this novel effect as the *trading-against-error effect*. To further see the intuition, let's consider an illustrative example in which investor L buys the risky asset. When $\tilde{e} > 0$, investor L tends to buy an additional amount of the asset than is justified on the basis of the fundamental value. Understanding that this trading is merely driven by error, investor H will sell an additional amount $\alpha_L E[\tilde{e}|\mathcal{F}_H]$ of the risky asset, which can partly offset investor L's trading demand based on the error $\tilde{e}$. As such, investor L's informed order flows are partially canceled, which enables her to execute her trade at a better price and lose less from the trading on the error $\tilde{e}$. In some sense, investor H "provides" liquidity to investor L. As will be shown later, this effect proves central to our information-sharing results.

Third, while investor H trades against the information shared by himself ($\alpha_H < 0$), investor L's trading is aligned with her own shared information ($\beta_L > 0$). As shown by the best response of investor H in equations (6) and that of investor L in equations (8), the coefficient of one investor's trading demand on the own shared information crucially depends how the counterparty investor trades on it, namely, $\alpha_H = -\frac{\beta_H}{2}$ and $\beta_L = -\frac{\alpha_L}{2}$. Let's examine investor H's trading strategy first. By sharing the garbled information $\tilde{s}_H$ with investor L, investor H understands that investor L trades in the same direction as suggested by $\tilde{s}_H$ as it helps her better predict the asset fundamental. Meanwhile, trading on $\tilde{s}_H$ suggests that investor L's trading injects the added noise $\tilde{\varepsilon}_H$ into the price, which induces investor H to trade against $\tilde{s}_H$ to correct the overshot price. Therefore, as $\beta_H > 0$, it must follow that $\alpha_H < 0$. Similarly, investor L understands that investor H's trading against her shared information $\tilde{s}_L$ also incorporates her added noise $\tilde{\varepsilon}_L$ into the price and thus has the incentive to correct it to make profits. As analysed above, since investor H trades against investor L's shared information $\tilde{s}_L$, investor L ends up trading in the same direction as suggested by her own shared information; that is, as $\alpha_L < 0$, we have $\beta_L > 0$.

Next, after receiving the total order flow from the two investors and noise traders, market makers set the price for the risky asset: $\tilde{p} = \lambda\tilde{\omega}$. Based on the weak-efficiency rule (4) and investors' optimal trading rules as specified by Lemma 1, the equilibrium pricing rule can be expressed as a function of τ_H and τ_L as follows:

$$\lambda(\tau_H, \tau_L) = \frac{\sqrt{\begin{array}{r}\rho\tau_L\left(8\tau_H\left(8\tau_H + 14\rho + 17\right) + \rho(48\rho + 113) + 72\right) + 4\tau_L^2\left(\tau_H + \rho + 1\right)\left(8\tau_H + 8\rho + 9\right) \\ +4\rho^2\tau_H\left(8\tau_H + 12\rho + 17\right) + 9(\rho(2\rho + 5) + 4)\rho^2\end{array}}}{3\sigma_u\left(4\tau_L\left(\tau_H + \rho + 1\right) + \rho\left(4\tau_H + 3\rho + 4\right)\right)}. \tag{10}$$

The following proposition summarizes the subgame equilibrium on $t = 1$.

Proposition 1 (Trading) *Given investors' information-sharing strategy (τ_H, τ_L), on $t = 1$ the asset price is $\tilde{p} = \lambda\tilde{\omega}$, where λ is specified by equation (10), and investors' equilibrium trading rules are specified by Lemma 1.*

3.2 Information Sharing at $t = 0$

Now we study the optimal information sharing between the two rational investors on $t = 0$. The following proposition characterizes the two investors' optimal information-sharing strategies.

Proposition 2 (Information sharing) *In equilibrium, investor H does not share any of his information whereas investor L shares her information "as is." That is, $\tau_H^* = 0$ and $\tau_L^* = +\infty$.*

Proposition 2 states that with information sharing permitted information won't flow from the more informed investor (investor H) to the less informed one (investor L); quite surprisingly, it transmits in the opposite direction from the less informed investor to the more informed one.

We first explain why investor L would like to share her information genuinely. We ask how investor L's information-sharing decision τ_L affects her expected profits given τ_H. Inserting the two investors' optimal trading rules as specified by Lemma 1 into investor L's conditional expected profits (7) and taking expectation yields her unconditional expected trading profits as follows:

$$\pi_L = \lambda E[\tilde{x}_L^2] = \frac{1}{4\lambda} E\Big[E\big[\tilde{v} - \lambda(\alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{s}_L)\big|\tilde{y}, \tilde{s}_H, \tilde{s}_L\big]\Big]^2.$$

We then use the chain rule to decompose how τ_L affects investor L's information-sharing incentive, which is summarized by the following equation:

$$\begin{aligned}\frac{\mathrm{d}\pi_L}{\mathrm{d}\tau_L} = & \underbrace{\frac{\partial \pi_L}{\partial \alpha_L}\frac{\partial \alpha_L}{\partial \tau_L}}_{\text{trading-against-error effect } >0} + \underbrace{\frac{\partial \pi_L}{\partial \alpha_v}\frac{\partial \alpha_v}{\partial \tau_L}}_{\text{competition } <0} + \underbrace{\frac{\partial \pi_L}{\partial \alpha_H}\frac{\partial \alpha_H}{\partial \tau_L}}_{<0} \\ & + \underbrace{\frac{\partial \pi_L}{\partial \lambda}\frac{\partial \lambda}{\partial \tau_L}}_{\text{liquidity } <0} + \underbrace{\frac{\partial \pi_L}{\partial \tau_L}}_{<0} > 0, \end{aligned} \tag{11}$$

where

$$\begin{cases} \frac{\partial \pi_L}{\partial \alpha_L} = -\frac{1}{3} < 0, \\ \frac{\partial \alpha_L}{\partial \tau_L} = -\frac{2\rho^2(4\tau_H+3\rho+4)}{3\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} < 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_L}{\partial \alpha_v} = -\frac{4\tau_H(\tau_L+\rho)+\rho(4\tau_L+3\rho)}{3(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))} < 0, \\ \frac{\partial \alpha_v}{\partial \tau_L} = \frac{2\rho^2(\tau_H+\rho+1)}{\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} > 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_L}{\partial \alpha_H} = -\frac{1}{3} < 0, \\ \frac{\partial \alpha_H}{\partial \tau_L} = \frac{2\rho^2\tau_H}{3\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} > 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_L}{\partial \lambda} = -\frac{\left(\begin{array}{r}16\tau_H\left((7\rho+4)\rho\tau_L+(4\rho+2)\tau_L^2+(3\rho+2)\rho^2\right)+32\tau_H^2(\tau_L+\rho)^2 \\ +\rho\left((48\rho+59)\rho\tau_L+32(\rho+1)\tau_L^2+9(2\rho+3)\rho^2\right)\end{array}\right)}{9\lambda^2(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} < 0, \\ \frac{\partial \lambda}{\partial \tau_L} = \frac{\rho^2(20\tau_H(\tau_L+\rho)+20(\rho+1)\tau_L+\rho(27\rho+20))}{18\lambda\sigma_u^2(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^3} > 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_L}{\partial \tau_L} = -\frac{\rho^2}{9\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} < 0. \end{cases}$$

As shown in equation (11), the effects of τ_L on investor L's profits (and thus incentives of information sharing) can be classified into three groups: (i) the one through its effect on the rival investor H's trading rules (α_L, α_v, and α_H), (ii) the one through its effect on market makers' pricing rule (λ), and (iii) the direct effect. The overall effect is positive so that L would like to share all her information. We then discuss these effects one by one.

First and most importantly, the key that underlies investor L's information-sharing behavior is the trading-against-error effect. Specifically, as L shares more precise information, H is able to infer the error in her endowed information more accurately (see equation (9)), thereby trading more aggressively against the information shared by L, i.e., $\frac{\partial \alpha_L}{\partial \tau_L} < 0$ (note that $\alpha_L < 0$). Recall that L's trading on the error $\tilde{e}$ can move prices away from the fundamental. Therefore, L's informed order flows are offset more due to this more aggressive trading-against-error effect, thereby gaining L better execution price and higher profits, i.e., $\frac{\partial \pi_L}{\partial \alpha_L}\frac{\partial \alpha_L}{\partial \tau_L} > 0$. Interestingly, as evident in equation (11), this is the only positive force that induces L to share her information, which suggests that it must be very strong to overturn all the other negative forces that discourage L's information sharing.

Further, τ_L also affects investor H's trading on his endowed information $\tilde{v}$ and his own shared information $\tilde{s}_H$, and both effects discourage L from sharing information. Specifically, as L shares more precise information, H trades more aggressively on his endowed information, i.e., $\frac{\partial \alpha_v}{\partial \tau_L} > 0$. This is because by observing L's shared information, H not only knows the fundamental perfectly, but also his rival investor better, which gains him more competitive advantage. This effect reduces L's information-sharing incentives $\frac{\partial \pi_L}{\partial \alpha_v}\frac{\partial \alpha_v}{\partial \tau_L} < 0$. Moreover, as L shares more precise information, H uses less of his own shared information, i.e., $\frac{\partial \alpha_H}{\partial \tau_L} > 0$ (note that $\alpha_H < 0$). In the extreme case $\tau_H = 0$, this effect is shut down completely, i.e., $\alpha_H = 0$.

Next, as L shares more precise information, market makers raise the price impact accordingly, i.e., $\frac{\partial \lambda}{\partial \tau_L} > 0$, which reduces market liquidity and thus hurts the investor, i.e., $\frac{\partial \pi_L}{\partial \lambda}\frac{\partial \lambda}{\partial \tau_L} < 0$. This effect works on top of the trading-against-error effect. When L's shared information becomes more precise, H trades against the error $\tilde{e}$ more aggressively, thereby leaving the aggregate order flow from the two investors more correlated with the fundamental. Thus, market makers respond by increasing price impact to better manager adverse selection risk and market liquidity worsens accordingly. Ultimately, L's profits can be eroded.

Finally, τ_L also affects L's profits directly, independent of the channels through the rival investor's trading strategies and market makers' pricing rule. This direct effect arises from L's added noise $\tilde{\varepsilon}_L$. Intuitively, as L shares more precise information, the term $\tilde{\varepsilon}_L$ becomes less volatile and its effect on L's profits diminished.

We then investigate investor H's information-sharing incentives. Similarly, we can derive H's unconditional trading profits as follows:

$$\pi_H = \lambda E[\tilde{x}_H^2] = \frac{1}{4\lambda} E\Big[E\big[\tilde{v} - \lambda(\beta_y \tilde{y} + \beta_H \tilde{s}_H + \beta_L \tilde{s}_L)|\tilde{v}, \tilde{s}_H, \tilde{s}_L\big]\Big]^2,$$

and use the chain rule to decompose the effect of τ_H on his profits. Having established that L would like to genuinely share her information, we now fix $\tau_L = \infty$ and explore H's information-sharing behavior. The following equation decomposes the effect τ_H on

H's profits:

$$\frac{\mathrm{d}\pi_H}{\mathrm{d}\tau_H} = \underbrace{\frac{\partial \pi_H}{\partial \beta_H}\frac{\partial \beta_H}{\partial \tau_H}}_{\text{information leakage} <0} + \underbrace{\frac{\partial \pi_H}{\partial \beta_y}\frac{\partial \beta_y}{\partial \tau_H}}_{>0} + \underbrace{\frac{\partial \pi_H}{\partial \beta_L}\frac{\partial \beta_L}{\partial \tau_H}}_{>0} + \underbrace{\frac{\partial \pi_H}{\partial \lambda}\frac{\partial \lambda}{\partial \tau_H}}_{\text{liquidity}>0} + \underbrace{\frac{\partial \pi_H}{\partial \tau_H}}_{<0} < 0, \tag{12}$$

where

$$\begin{cases} \frac{\partial \pi_H}{\partial \beta_H} &= -\frac{1}{3} < 0, \\ \frac{\partial \beta_H}{\partial \tau_H} &= \frac{\rho+1}{3\lambda(\tau_H+\rho+1)^2} > 0, \end{cases} \qquad \begin{cases} \frac{\partial \pi_H}{\partial \beta_y} &= -\frac{1}{3} < 0, \\ \frac{\partial \beta_y}{\partial \tau_H} &= -\frac{\rho}{4\lambda(\tau_H+\rho+1)^2} < 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_H}{\partial \beta_L} &= -\frac{1}{3} < 0, \\ \frac{\partial \beta_L}{\partial \tau_H} &= -\frac{\rho}{12\lambda(\tau_H+\rho+1)^2} < 0, \end{cases} \qquad \begin{cases} \frac{\partial \pi_H}{\partial \lambda} &= -\frac{8\tau_H+8\rho+9}{36\lambda^2(\tau_H+\rho+1)} < 0, \\ \frac{\partial \lambda}{\partial \tau_H} &= -\frac{1}{72\lambda\sigma_u^2(\tau_H+\rho+1)^2} < 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_L}{\partial \tau_L} = -\frac{1}{36\lambda(\tau_H+\rho+1)^2} < 0. \end{cases}$$

The key effect of H's information sharing on his own profits is the information leakage effect, which strongly reduces his incentives to share information, i.e., $\frac{\partial \pi_H}{\partial \beta_H}\frac{\partial \beta_H}{\partial \tau_H} < 0$. With superior information about the asset fundamental, H owns an informational advantage over L. So, any piece of information sharing with L can only dissipate investor H's informational advantage. Recall that L uses H's shared information to better inform her trading decisions by trading alongside it, i.e., $\beta_H > 0$. With more precise information shared by H, more information is leaked to L and she trades more aggressively on it, i.e., $\frac{\partial \beta_H}{\partial \tau_H} > 0$. This greatly erodes H's competitive advantage and reduces his trading profits.

At the same time, as H shares more precise information, his shared information crowds out L's endowed information $\tilde{y}$ and her shared information $\tilde{s}_L$ in her trading rules, that is, $\frac{\partial \beta_y}{\partial \tau_H} < 0$ and $\frac{\partial \beta_L}{\partial \tau_H} < 0$. And both effects encourage H to share his information, i.e., $\frac{\partial \pi_H}{\partial \beta_H}\frac{\partial \beta_y}{\partial \tau_H} > 0$ and $\frac{\partial \pi_H}{\partial \beta_L}\frac{\partial \beta_L}{\partial \tau_H} > 0$. This is because if H's shared information becomes more precise, L trades more on this information and uses less her other information ($\tilde{y}$ and $\tilde{s}_L$) accordingly. Now investor H not only has an informational advantage in the asset fundamental but also gains a strategic advantage by knowing his rival investor's trading rule better. Thus, H's profits improve through these two channels accordingly.

As the liquidity effect in L's information-sharing incentives, τ_H also affects H's profits through its effect on pricing rule. However, different from the effect of τ_L, as H shares more precise information, market makers reduces price impact, that is, $\frac{\partial \lambda}{\partial \tau_H} < 0$. Why do market makers respond differently to the two investors' information sharing behavior? As the more informed investor, when H shares more his information with L, the two investors' private information becomes more homogeneous, which induces the two investors to compete with each other more aggressively. This intense inter-investor competition reveals more information to market makers, reducing their informational disadvantage and adverse selection risk. As a response, market makers decreases price impact and the improved market liquidity benefits investor H.

Finally, the analysis of the direct effect of τ_H on H's profits is the same as the direct effect in L's information-sharing incentives. Overall, as the more informed investor, the information leakage effect greatly hurts investor H and he simply refrains from revealing any of his information to investor L.

To sum, by sharing information, L invites H to trade against her shared information, offsetting her informed order flow and gaining her a better execution price. On the other hand, any piece of information sharing can greatly dissipate H's informational advantage. Taken together, the strong trading-against-error effect encourages L to not only share her information, but do so "as is," and the strong information-leakage effect discourages H from information sharing.

Having established the equilibrium information-sharing strategy, according to Proposition 1 we can characterize the asset price and investors' trading strategies along the equilibrium path. The following corollary summarizes the results.

Corollary 1 *In equilibrium, the asset price is $\tilde{p} = \lambda^* \tilde{\omega}$, where*

$$\lambda^* = \frac{\sqrt{9+8\rho}}{6\sigma_u\sqrt{1+\rho}}. \tag{13}$$

Investors H and L submit market orders $\tilde{x}_H = \alpha_v^* \tilde{v} + \alpha_L^* \tilde{y}$ and $\tilde{x}_L = (\beta_y^* + \beta_L^*)\tilde{y}$, respectively, where

$$\alpha_v^* = \frac{3\sigma_u\sqrt{1+\rho}}{\sqrt{9+8\rho}}, \ \alpha_L^* = -\frac{\rho\sigma_u}{\sqrt{(1+\rho)(9+8\rho)}}, \ \text{and } \beta_y^* + \beta_L^* = \frac{2\rho\sigma_u}{\sqrt{(1+\rho)(9+8\rho)}}, \tag{14}$$

and their respective unconditional trading profits are as follows:

$$\pi_H^* = \frac{(9+4\rho)\sigma_u}{6\sqrt{(1+\rho)(9+8\rho)}} \ \text{and } \pi_L^* = \frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}}. \tag{15}$$

3.3 Implications

In this section, we examine the effect of information sharing on investors' profits and market quality by comparing the equilibrium outcomes with a benchmark economy without information sharing ($\tau_H = \tau_L = 0$). According to Proposition 1, we immediately obtain the following corollary which summarizes the equilibrium in the benchmark economy (note the superscript 0 represents the benchmark).

Corollary 2 *Suppose there is no information sharing: $\tau_H = \tau_L = 0$. In equilibrium, the asset price is $\tilde{p} = \lambda^0 \tilde{\omega}$, where*

$$\lambda^0 = \frac{\sqrt{4 + 5\rho + 2\rho^2}}{(4 + 3\rho)\sigma_u}. \tag{16}$$

Investors H and L submit market orders $\tilde{x}_H = \alpha_v^0 \tilde{v}$ and $\tilde{x}_L = \beta_y^0 \tilde{y}$, respectively, where

$$\alpha_v^0 = \frac{(2 + \rho)\sigma_u}{\sqrt{4 + 5\rho + 2\rho^2}} \text{ and } \beta_y^0 = \frac{\rho\sigma_u}{\sqrt{4 + 5\rho + 2\rho^2}}, \tag{17}$$

and their respective unconditional expected profits are as follows:

$$\pi_H^0 = \frac{(2 + \rho)^2\sigma_u}{(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}} \text{ and } \pi_L^0 = \frac{\rho(1 + \rho)\sigma_u}{(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}}. \tag{18}$$

When there is no information transmission between the two strategic investors, they trade on their own private information to maximize their respective trading profits, taking into account the competition between them and the optimal response of market makers. Thanks to the more precise private information, investor H owns an informational advantage over investor L and trades more aggressively on his private information. That is, $\alpha_v^0 > \beta_y^0$. As such, investor H makes higher trading profits: $\pi_H^0 > \pi_L^0$.

The implications of information sharing on the two investors' profits and market quality are summarized in the following proposition.

Proposition 3 *Compared with the economy without information sharing, when information sharing is permitted,*

(i) *Investor L is better off whereas investor H is worse off and their combined profits are higher; that is, $\pi_L^* > \pi_L^0$, $\pi_H^* < \pi_H^0$, and $\pi_H^* + \pi_L^* > \pi_H^0 + \pi_L^0$.*

(ii) *Market liquidity is lower whereas market efficiency and total trading volume are higher; that is, $\lambda^* > \lambda^0$, $m^* > m^0$, and $TV^* > TV^0$.*

We first investigate how information sharing affects the two investors' trading profits. Part (i) of Proposition 3 summarizes the results. Why is investor L better off with information sharing? As analyzed above, this is due to the novel trading-against-error effect. In the benchmark without information sharing, investor L gains from trading on the fundamental component but loses from trading on the error component. Nonetheless, the investor cannot distinguish the two components and have to trade on them simultaneously. With information sharing permitted, investor H's trading against $\tilde{e}$ helps hide L's informed order flow, thereby benefiting L. For example, when $\tilde{e} > 0$, investor L tends to buy an additional amount of $(\beta_y^* + \beta_L^*)\tilde{e}$ than is justified on the basis of the fundamental value $\tilde{v}$. Meanwhile, since investor H trades against L's shared information ($\alpha_L^* < 0$), he tends to sell an additional amount $|\alpha_L^*|\tilde{e}$ of the risky asset, which can partly offset investor L's trading demand. As such, investor L can execute her order at a better price and lose less from the trading on the error in her private information.

Yet, how can investor H become worse off after receiving more information? With more informed order flow offset, investor L becomes less concerned about the losses associated with the trading on the error and in turn trades more aggressively. Mathematically, recall that with information sharing permitted investor L's trading strategy is $\tilde{x}_L = (\beta_y + \beta_L)\tilde{y}$ and the trading aggressiveness is captured by the coefficient $\beta_y + \beta_L$. A direct comparison of investor L's trading aggressiveness in the main model with information sharing permitted (see equations (14)) with that in the benchmark economy without information sharing (see equations (17)) yields that $\beta_y^* + \beta_L^* > \beta_y^0$; that is, L trades more aggressively after sharing her information. Meanwhile, investor H is forced to trade less aggressively on his endowed information, that is, $\alpha_v^* + \alpha_L^* < \alpha_v^0$. Therefore, ironically, investor H becomes worse off after the information sharing.

The whole investor side makes higher profits after information sharing ($\pi_H^* + \pi_L^* > \pi_H^0 + \pi_L^0$) because with information sharing the two investors can better internalize the competition between them. One numerical example indicates that, given $\rho = 1$ and $\sigma_u = 1$, by sharing information, investor L's profits increases by 32.7% whereas investor H's profits drop by 4.1%; meanwhile, two investors' total profits increase by 2.6%.

Having established that investor H makes lower profits with the shared information, it is intuitive that H would be better off if he could commit not to using the received information; that is, when contemplating the optimal trading rule, investor H commits that $\alpha_L = 0$. But without additional assumptions, can he credibly make this commitment? No. This is evident from investor H's optimal trading strategy as specified by equations (14). Specifically, knowing that investor L trades on information $\tilde{y}$ and being able to filter out the error in this information, investor H always has the tendency to trade against it, correct the price, and make profits. In other words, after receiving the information from investor L, investor H cannot help using this piece of information despite its negative consequences for his profits. We discuss the extended economy in which H has such a commitment power in Section 4.2.

Next, how is information sharing affecting market quality? We examine market liquidity, market efficiency (price discovery), and trading volume. Part (ii) of Proposition 3 summarizes the results. Market liquidity is measured by the Kyle's λ, which measures the effect of noise trading on prices, and so it is an inverse measure of market depth: more liquid markets have a smaller λ. A straightforward comparison of the equilibrium λ^* as specified by (13) and that in the benchmark λ^0 as specified by (16) reveals that when information sharing is permitted market liquidity decreases; that is, $\lambda^* > \lambda^0$. This is because with investor L's private information shared with investor H, investor H trades against the error in the shared information, thereby reducing the noise in the total order flow. As such, market makers increase the price impact to manage the increasing adverse-selection risk, which dampens market liquidity.

Following the literature (e.g. Kyle, 1985), We measure market efficiency (price discovery) by the precision of the asset payoff conditional on its price, i.e., $m \equiv Var\left(\tilde{v}|\tilde{p}\right)^{-1}$. Intuitively, when the price aggregates a great deal of information, the residual uncertainty of the fundamental $\tilde{v}$ conditional on the price $\tilde{p}$ is low, and thus market efficiency is high. According to corollaries 1 and 2, it is easy to show that market efficiency improves when information sharing is permitted; that is, $m^* > m^0$. As argued above, after information sharing, the total order flow becomes more correlated with the fundamental. Accordingly, the price can aggregate more information about the fundamental. Therefore, while what L shares is noisy information, market efficiency still improves.

Finally, following Vives (2010), we measure total volume traded, denoted by TV, by the sum of the expected absolute value of the demands coming from the different agents in the model divided by 2.[2] Thus, the total volume traded is given by

$$TV = \frac{1}{2}\Big(E\left[|\tilde{x}_H| + |\tilde{x}_L| + |\tilde{\omega}| + |\tilde{u}|\right]\Big).$$

We find that information sharing is associated with higher total trading volume, that is, $TV^* > TV^0$. Specifically, after information sharing is permitted, since L trades more aggressively and H is forced to trade less aggressively, L's trading volume increases whereas H's decreases, that is, $E\left[|\tilde{x}_L|\right]$ increases but $E\left[|\tilde{x}_H|\right]$ decreases. In the net, market maker's trading volume increases, namely, $E\left[|\tilde{\omega}|\right]$ increases.

4 Extensions

In this section, we consider four extensions of the baseline model to show the robustness and generality of our key insight. That is, a coarsely informed investor can have a strategic incentive to genuinely share her information with the well informed investor.

[2]Our result remains robust under alternative measure of trading volume, e.g., $TV = \frac{1}{2}E[|\tilde{\omega}|]$ (Bernhardt and Miao, 2004).

4.1 Information Sharing Between Imperfectly Informed Investors

In the baseline model, we assume that investor H has perfect information about the asset fundamental. In this section, we relax this assumption and consider the more general case in which information can be transmitted between imperfectly informed investors.

Assume that there are two investors, denoted by 1 and 2, who are endowed with private information about the asset fundamental of potentially different precision. Specifically, investor i, where $i \in \{1, 2\}$, receives private information as follows:

$$\tilde{y}_i = \tilde{v} + \tilde{e}_i, \quad \tilde{e}_i \sim N(0, \rho_i^{-1}) \text{ and } \rho_i \in (0, +\infty].$$

For example, if $\rho_1 > \rho_2$, investor 1 is more informed about the fundamental than investor 2, and if $\rho_1 = \rho_2$ their information is of the same precision. The baseline model is nested by assuming an investor's information precision to infinity, $\rho_i = +\infty$, and letting the other investor's information precision be $\rho_j > 0$, where $i, j \in \{1, 2\}$ and $i \neq j$,

On $t = 0$, investor i can decide whether to share a piece of garbled information to the other investor:

$$\tilde{s}_i = \tilde{y}_i + \tilde{\varepsilon}_i, \text{ where } \tilde{\varepsilon}_i \sim N(0, \tau_i^{-1}) \text{ and } \tau_i \in [0, +\infty].$$

Investor i chooses her information-sharing strategy τ_i to maximize her expected trading profits. All the other setups remain the same as in the baseline model.

We numerically verify that the investor with superior information does not share her information; that is, if $\rho_i > \rho_j$, $\tau_i = 0$ for $i, j \in \{1, 2\}$ and $i \neq j$. The intuition is the same as that in the baseline model. Without information sharing the more informed investor owns an informational advantage over the other investor and hence makes higher profits. Any piece of shared information can only dissipate the informational advantage of the more informed investor. In the following analysis, we take it as given that the more informed investor does not share her information and examine the information-sharing behavior of the less informed investor. The following proposition summarizes the equilibrium information-sharing strategy of the less informed investor in this extended economy. Figure 2 graphically illustrates it.

Proposition 4 *Consider two investors endowed with private information with different precision. Assume that investor i is the more informed investor and she does not share her information, where $i \in \{1, 2\}$, i.e., $\tau_i = 0$.*

(i) *If $\rho_i \geq \hat{\rho}_i \equiv 2(\rho_j + 1)$, where $j \in \{1, 2\}$ and $j \neq i$, investor j fully shares her information with investor i; that is, $\tau_j = +\infty$;*

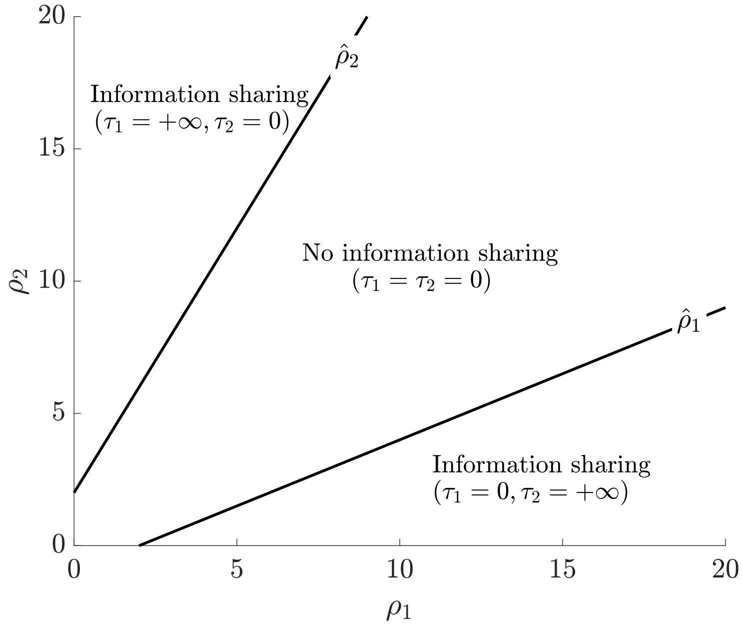


Figure 2: Information sharing between partially informed investors

(ii) *Otherwise, investor j does not share her information; that is, $\tau_j = 0$.*

Proposition 4 shows that a coarsely informed investor would like to share her information only when her counterparty investor has sufficiently precise information about the fundamental. Moreover, if the investor decides to share her information, she will share it "as is." For all values of σ_u, Figure 2 plots the two investors' information-sharing behavior against the precision of their endowed information ρ_1 and ρ_2. Specifically, if the precision of investor i's endowed information exceeds some threshold ($\rho_i > \hat{\rho}_i$), the other investor j would like to share all her information with investor i, where $i, j \in \{1, 2\}$ and $i \neq j$. Otherwise, if the two investors' information precision levels are close, there can not be any information sharing between them. Recall that in the baseline model investor L would like to share her information with investor H because of the trading-against-error effect. This effect crucially relies on the fact that the more informed investor is able to identify the error in the received information, and this is feasible only when the more informed investor owns sufficiently more precise information about the fundamental.

To present the mechanism mathematically, we examine the two investors' optimal trading strategies. Without loss of generality, assume that $\rho_1 \geq \rho_2$. For ease of exposition, we only consider whether investor 2 would like to genuinely share her information $\tilde{y}_2$ to investor 1. After the information sharing, the two investors' trading strategies are respectively as follows: $\tilde{x}_1 = \alpha_y \tilde{y}_1 + \alpha_2 \tilde{y}_2$ and $\tilde{x}_2 = \beta_2 \tilde{y}_2$, with

$$\begin{aligned}
\alpha_y &= \frac{\rho_1}{2\lambda(1+\rho_1+\rho_2)} > 0, \\
\alpha_2 &= \frac{\rho_2(2+2\rho_2-\rho_1)}{6\lambda(1+\rho_1)(1+\rho_1+\rho_2)} < 0 \text{ iff } \rho_1 > 2(1+\rho_2), \\
\beta_y &= \frac{\rho_2}{3\lambda(1+\rho_2)} > 0.
\end{aligned}$$

First, consistent with Lemma 1, an investor tends to trade alongside her endowed information; that is, $\alpha_y > 0$ and $\beta_y > 0$. Second, investor 1 (the information receiver) trades against the information shared by investor 2 (the information sender) if and only if investor 1 owns sufficiently precise information about the fundamental, namely, $\alpha_2 < 0$ if and only if $\rho_1 > \hat{\rho}_1 \equiv 2(1+\rho_2)$. If, however, investor 1's own information is not that precise and investor 2 still shares her information, then instead of trading against the shared information, investor 1 tends to trade alongside the shared information ($\alpha_2 > 0$). In this case, the information shared by investor 2 helps investor 1 better forecast the fundamental, thereby eroding investor 2's competitive advantage and thus making greater profits at the expense of investor 2. Therefore, in this case investor 2 will refrain from sharing any of her information.

4.2 Multiple Insiders

In the baseline model, although investor H becomes worse off after being shared with information, he cannot commit to not trading on it. In this section, we relax this assumption and allow the insider H to commit to not receiving this information. We show that as long as there is a large number of insiders it can be an equilibrium in which all insiders trade against the shared information, despite the fact that they would be better off if all of them committed to not receiving the information. In other words, the insiders can be trapped in a prisoner's dilemma.

In the extended economy, there are two groups of investors: (i) a number M of insiders, denoted by $H_1, ..., H_M$ where $M > 1$ is an integer, who privately observe $\tilde{v}$; and (ii) investor L who only privately observes a noisy signal $\tilde{y}$ about the asset fundamental as specified by (1). As in Section 4.1, we take it as given that insiders do not share their information (which is verified numerically) and explore the information-sharing incentives of investor L. For simplicity, we assume that if investor L would like to share her information, she shares it truthfully; that is, the precision of investor L's shared information $\tilde{s}_L$ is either zero or infinity: $\tau_L \in \{0, \infty\}$. All the other model setups remain the same as in the baseline model. The following proposition summarizes the equilibrium outcomes in this extended economy.

Proposition 5 *In the presence of M multiple insiders, the following two statements must be true.*

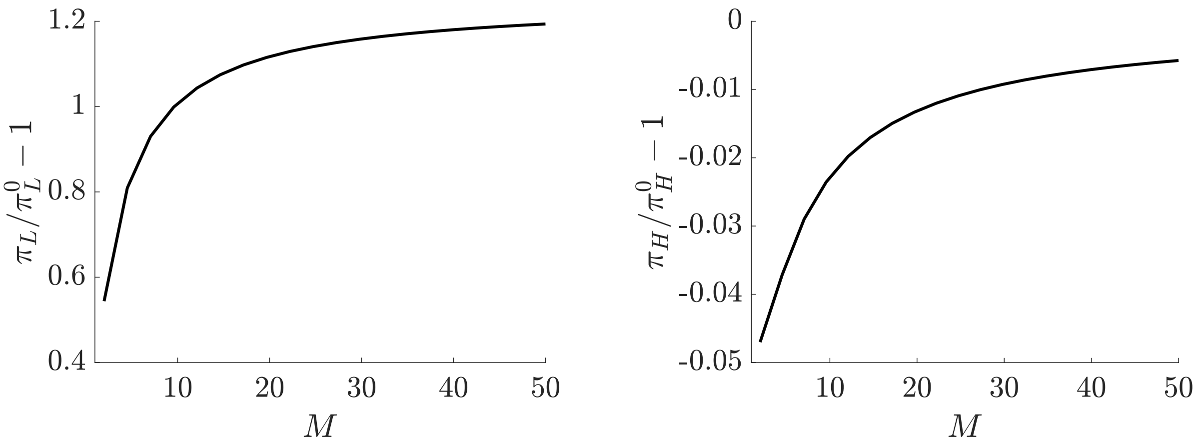
(i) *When $M > 3$, the following cannot be an equilibrium: every insider commits to not receiving L's shared information.*

(ii) *There exists a constant $\hat{M} > 0$ such that when $M > \hat{M}$, the following equilibrium always exists: all insiders choose to trade against L's shared information. However, the insiders' profits would be higher if they all committed not to receiving L's shared information.*

In the baseline model with a single insider, we have shown that the insider is better off by not trading against the shared information. In other words, if the insider has commitment power, he would commit not to receiving the shared information. Will this be true if there are multiple insiders? Part (i) of Proposition 5 shows that it is not when there are more than three insiders. The intuition is as follows. When there is a single insider, he fully internalizes the negative effects of trading against the shared information and the resulting increase in price impact, thereby optimally refraining from receiving the shared information. However, when there are multiple insiders, one insider can deviate to receiving the information and privately enjoy its incremental value without fully taking into account its impact on investor L's trading and market makers' price setting.

Further, in the presence of multiple insiders, it is always an equilibrium in which every insider chooses to receive L's shared information. We can analytically prove this when M is sufficiently large, as given by Part (ii) of Proposition 5. Meanwhile, these insiders are trapped in a prisoner's dilemma as they would be better off if they all committed not to receiving the information shared by investor L, which couldn't be an equilibrium as shown by Part (i) of Proposition 5. We next numerically discuss that this prisoner's dilemma type of equilibrium should be a robust feature of the multi-insider economy in Figure 3.

First, as the number of insiders increases, investor L has a stronger strategic motive to share her private information. To see this, we compute the profit change of investor L from the benchmark economy without information sharing to the extended economy in which L genuinely shares her information and all insiders trade against it. Panel (a) of Figure 3 plots the profit change of investor L against the number M of insiders. We find that consistently for all values of M, sharing information can improve L's profits. Further, as M grows, the profit improvement ratio keeps increasing. That is, investor L has a stronger motive to share her information as M increases. This is because with more insiders trading against the shared information, investor L's informed order flow can be offset more; consequently, investor L trades more aggressively on her own information and makes higher profits. In fact, it can be shown that if M goes to infinity, L's profit improvement ratio approaches $\frac{3+2\rho}{1+\rho^2}$, which is the upper limit of the profit improvement ratio for L via information sharing. For example, if investor L owns very coarse information (i.e., $\rho \to 0$), this limit reaches 3; that is, by sharing her information to a sufficiently large number of insiders, the coarsely informed investor L's profits can triple in the best scenario.



(a) L's profit change

(b) An insider's profit change

Figure 3: Multiple insiders

Second, as M increases, the insiders, though worse off after information sharing, incur fewer losses. Panel (b) of Figure 3 plots the profit change of an insider from the benchmark economy without information sharing to the extended economy in which L genuinely shares her information and all insiders trade against it. We find that for $M \geq 2$ all insiders' profits drop after they trade against the shared information. In addition, while the insiders keep suffering a loss after information sharing, the loss decreases as M increases. This is because in the presence of multiple insiders, although as in the baseline model investor L trades more aggressively after sharing her information, the incremental aggressiveness is smaller if L is faced with a large number of insiders. Therefore, the loss of an average insider decreases.

4.3 Costly Information Acquisition for Investor L

In the baseline model, the two investors are endowed with their respective private information and we find that the less informed investor L would like to fully share her information with the well informed investor H. One natural question arises: will investor

L still share her information if the information needs to be acquired at a non-negligible cost?

In this section, we endogenize investor L's information acquisition. Assume that before the two investors' information sharing on $t = 0$, to acquire information of precision ρ investor L needs to incur a cost according to a linear cost function, $c \cdot \rho$, where c is a positive constant.[3] Investor L chooses the precision ρ to maximize her expected trading profits net of the information-acquisition cost. We then study the effect of information sharing on investor L's information-acquisition incentives by comparing this extended model to its benchmark economy in which there is endogenous information production but no information sharing (i.e., investor L can produce information and $\tau_H = \tau_L = 0$).

In the benchmark economy in which there is no information sharing and investor L can decide how much information to produce, based on equation (18), investor L's expected trading profits net of the information-acquisition cost can be expressed as follows:

$$\pi_L^0 - c \cdot \rho = \frac{\rho(1+\rho)\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} - c \cdot \rho.$$

Maximizing the net profits yields investor L's optimal information-acquisition decision, ρ^0, which is uniquely determined by the following equation:

$$\frac{c}{\sigma_u} = \frac{32+84\rho+69\rho^2+19\rho^3}{2(4+3\rho)^2(4+5\rho+2\rho^2)^{3/2}}. \tag{19}$$

When information sharing is permitted, according to Section 3 we know that in equilibrium investor L shares her information "as is" whereas investor H does not share any information. Based on equation (15), investor L's expected profits net of the information-acquisition cost can be calculated as follows:

$$\pi_L^* - c \cdot \rho = \frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}} - c \cdot \rho.$$

Again, maximizing the net profits yields the optimal information-acquisition decision, ρ^*, which is uniquely determined by the following equation:

$$\frac{c}{\sigma_u} = \frac{18+17\rho}{3(9+17\rho+8\rho^2)^{3/2}}. \tag{20}$$

Then, a comparison of ρ^* with ρ^0 yields the following Proposition.

[3]The linear information-acquisition cost is assumed for tractability. Linearity in precision can be an analog of the case with discrete sampling with a constant cost per independent sample. Such an assumption is commonly made in the literature (e.g., Verrecchia, 1982; Kim and Verrecchia, 1991; Myatt and Wallace, 2002).

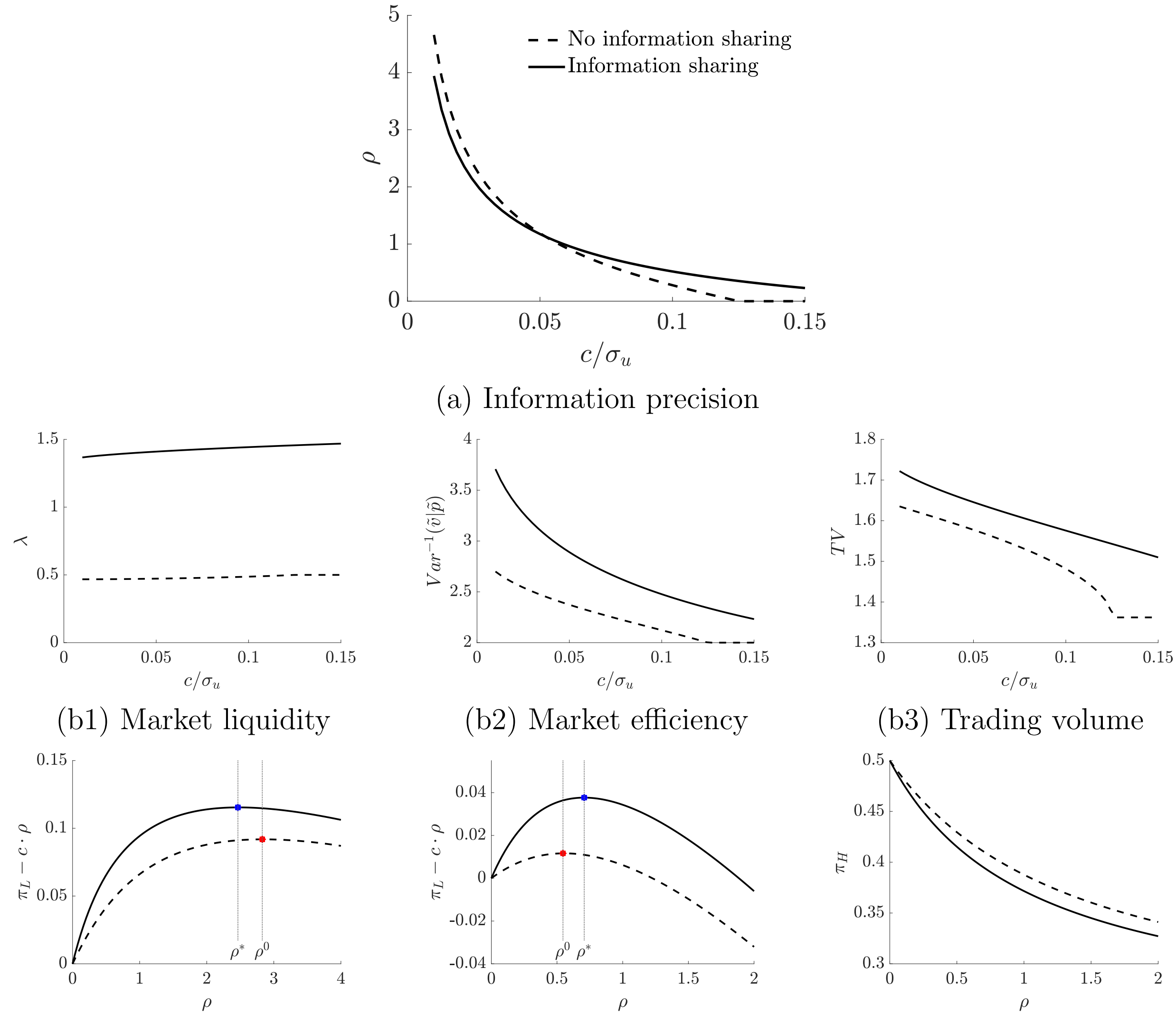
Proposition 6 *Assume that information acquisition is costly for investor L. There exists a constant $\hat{c}$, where $\hat{c} \approx 0.0520$, such that relative to the economy without information sharing, when information sharing is permitted, if $c/\sigma_u > (<) \, \hat{c}$, investor L acquires more (less) information; that is, $\rho^* > (<) \, \rho^0$.*

Relative to the benchmark economy without information sharing, when information sharing is permitted, investor L's information acquisition is determined by the following trade off. On the one hand, as discussed above, by sharing information with investor H, investor L can better hide her informed order flows, thereby trading more aggressively even though her information remains as noisy as before. This trading-against-error effect depresses L's incentives of acquiring information. On the other hand, with higher trading profits after sharing her information, investor L can afford to acquire more information about the fundamental and make more informed trading decisions. This in turn encourages investor L to acquire more information.

Proposition 6 formalizes the above trade off faced by investor L when making information-acquisition decisions. The relative strength of the two forces depends on the primitives of the model, namely, the cost of acquiring information and the noise trading volatility. Ceteris paribus, investor L's net gains of information acquisition decrease with the information-acquisition cost c and increases with the noise trading volatility σ_u, so c and σ_u have opposite effects on investor L's incentives to acquire information. Nevertheless, the optimal information acquisition only depends on c/σ_u.

If c/σ_u is low, investor L has acquired a great deal of information. She is less concerned about improving the forecasting ability of the fundamental, but cares more about hiding her informed order flows. When information sharing is permitted, investor L would like to induce investor H to trade against her information and help her offset the order flow. Therefore, investor L is less incentivized to acquire information. However, if c/σ_u is high, investor L only acquires a limited amount of information. When information sharing is permitted, with the higher profits, investor L would like to produce more information to further enhance her trading decision-making. Panel (a) of Figure 4 graphically illustrates the information-acquisition result.

Then, how will investor L's information-acquisition behavior affect the market quality? We plot market liquidity, market efficiency, and trading volume across the extended economy and its benchmark economy in panels (b1)–(b3) of Figure 4, respectively. We find that regardless of investor L's information acquisition, the economy with information sharing is always featured with lower market liquidity, higher market efficiency, and higher trading volume. Again, with investor L sharing her information, investor H tends to trade against it, which reduces the noise in the total order flow and induces market



(a) Information precision

(b1) Market liquidity (b2) Market efficiency (b3) Trading volume

(c1) L's net profits ($c = 0.02$) (c2) L's net profits ($c = 0.08$) (c3) H's total profits

Figure 4: Costly information acquisition for investor L

makers to raise the price impact. Therefore, market liquidity decreases. Further, despite that investor L may acquire less information when information sharing is permitted, the intensive trading by the two investors render the total order flow more correlated with the fundamental, which always improves market efficiency. Finally, L's aggressive trading after information sharing raises total trading volume.

Next, to take a further look at investor L's optimal information-acquisition decisions, we plot how L's information production ρ affects her expected trading profits net of the information-acquisition cost $\pi_L - c \cdot \rho$ in panels (c1) and (c2) of Figure 4 under the cost $c = 0.02$ and $c = 0.08$, respectively. The other parameter is $\sigma_u = 1$. The solid (dashed) line denotes the economy with (without) information sharing. Consistent with Proposition 6, with a fixed σ_u, if c is low (high), investor L acquires less (more) information when information sharing is permitted, namely, $\rho^* < (>) \rho^0$. More importantly, Panels (c1) and (c2) show that allowing investor L's endogenous information acquisition can only reinforce her gains from the information sharing. In other words, when information sharing is permitted, by choosing ρ^* investor L makes higher profits than those in the situation where the information precision is exogenously given. Finally, regardless of L's information-acquisition cost c or her information production ρ, relative to the benchmark economy without information sharing, investor H always makes lower profits, as shown in Panel (c3) of Figure 4.

4.4 Public Shared Information

The insight in the baseline model hinges on the fact that rational investors can privately communicate with each other, that is, the shared information is not revealed to the public (market makers). However, in reality, information may be leaked during this communication process. In this section, we allow the shared information to be public and show that our key insights remain robust as long as market makers cannot interpret the shared information precisely and investor H has better ability to process the shared information than market makers.

To simplify the analysis, we assume that investor L either refrains from sharing information or shares all of her information: $\tau_L \in \{0, +\infty\}$. Consider the following extended economy. As in the baseline model, investors control the precision of their respective shared information, but the information becomes public. Following Myatt and Wallace (2002), we introduce "receiver noise" to capture receivers' different capabilities in interpreting the same information. Specifically, for L's shared information $\tilde{s}_L$, investor H and market makers observe

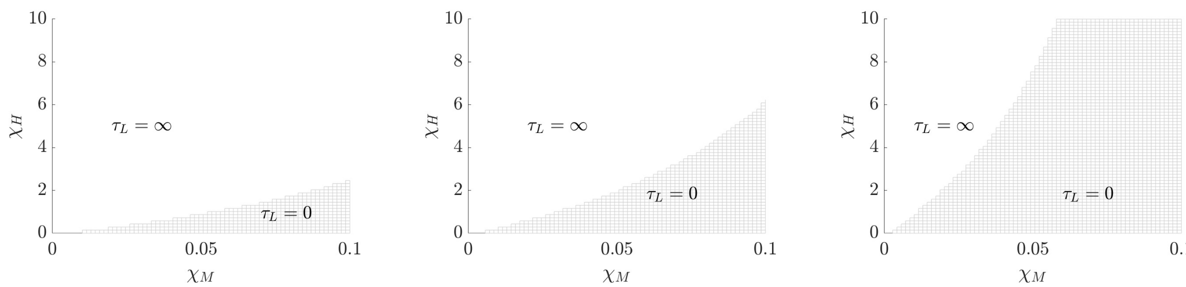
$$\tilde{q}_H = \tilde{s}_L + \tilde{\xi}_H \text{ and } \tilde{q}_M = \tilde{s}_L + \tilde{\xi}_M,$$

respectively, where $\tilde{\xi}_H \sim N(0, \chi_H^{-1})$, $\tilde{\xi}_M \sim N(0, \chi_M^{-1})$, and $\tilde{s}_L$, $\tilde{\xi}_H$, and $\tilde{\xi}_M$ are mutually independent. Similarly, for H's shared information $\tilde{s}_H$, investor L and market makers observe

$$\tilde{z}_L = \tilde{s}_H + \tilde{\eta}_L \text{ and } \tilde{z}_M = \tilde{s}_H + \tilde{\eta}_M,$$

respectively, where $\tilde{\eta}_L \sim N(0, \kappa_L^{-1})$, $\tilde{\eta}_M \sim N(0, \kappa_M^{-1})$, and $\tilde{s}_H$, $\tilde{\eta}_L$, and $\tilde{\eta}_M$ are mutually independent. Thus, the baseline model is nested with $\kappa_L = \chi_H = \infty$ and $\kappa_M = \chi_M = 0$.

We use Figure 5 to numerically characterize investor L's information sharing behavior in this extended economy. We assume that the investors have the same ability to interpret each other's shared information ($\kappa_L = \chi_H$) and market makers have the same ability to interpret the two investors' information ($\kappa_M = \xi_M$); the other parameter is $\sigma_u = 1$. We find that as in the baseline model, H never shares any of his information. Therefore, we only plot L's information sharing behavior in Figure 5: the shaded area indicates the region in which L does not share her information ($\tau_L = 0$) and the blank area is the one in which L would like to share her information ($\tau_L = \infty$). As long as χ_M is small and χ_H is sufficiently higher than χ_M, investor L is willing to share her information despite the potential information leakage to market makers. Again, by sharing her private information and inviting H to trade against it, investor L has her order flow partially offset and obtains a better execution price. If market makers can only observe a noisy version of this signal and investor H has superior ability in interpreting it, for L the benefit of information sharing overshadows the loss from information leakage to the public. Moreover, in the three panels plotted in Figure 5, as L owns more precise information (ρ increases), she is less likely to share her information. This is because with the less error in the endowed information, L benefits less from the trading-against-error effect in information sharing.



(a) $\rho = 3$ (b) $\rho = 5$ (c) $\rho = 10$

Figure 5: Public shared information ($\sigma_u = 1, \kappa_L = \chi_H, \kappa_M = \chi_M$)

5 Applications

5.1 Why does a barking dog bark?

It is not uncommon to observe investment opinions expressed on the social media such as Twitter, Seeking Alpha, and Facebook Posts. Yet, as suggested by the proverb "a barking dog never bites," these barking dogs (posters) should rarely possess true insights, thereby rendering (most of) the barks noise. Then, why does a barking dog bark?

One intuitive explanation for why investors share their information in financial markets is that the information disclosure helps speed up price discovery, thereby overcoming the noise trader risk. For example, Ljungqvist and Qian (2016) find that by releasing detailed reports after building short positions, investors manage to correct the mispricing and close their positions. In this way, they circumvent the short-sale constraints and noise trader risk. Alternatively, some investors spread information to manipulate the market (e.g., Benabou and Laroque, 1992). For such short-and-disclose strategy and manipulation strategy to work, reputation and the associated credibility are the key. That is, the barking dog really bites. While some big asset managers such as Black Rock, Pimco, and State Street regularly publish reports with strategies and views of the assets they are investing in, it is hard to argue that the masses of investors who share their investment views on Twitter own such reputation. Some other common intuition for the observed widespread information-sharing behavior holds that the individual investors share investment ideas for economic reward, social recognition, self-esteem, or to develop individual skills (Estellés-Arolas and González-Ladró-De-Guevara, 2012).

Our analysis offers a novel and complementary explanation for why the masses of investors with coarse information would like to share it truthfully on the social media. Section 4.4 provides a setting which enables us to map this situation into our framework. Specifically, the investors who share their investment opinions on the social media (such as Tweet writers) are represented by the coarsely informed investor L in our model, whereas the investors who extract investment signals from the social media (such as hedge funds who actively analyze the sentiment in Twitter) are represented by the well informed investor(s) in our setting. While the investment opinion is public information, the uninformed market participants (e.g., market makers) are not as skilled as the well informed investor(s) in analyzing the contents on the social media and understanding the information there. As such, the information expressed on the social media can be seen as being only transmitted from the coarsely informed investors to the well informed ones. Therefore, our theory rationalizes this information-sharing behavior as follows: by sharing noisy yet truthful information on the social media, the general investors invite the well informed investors to trade against their shared information, which partially offsets their informed order flow, and gains them a better execution price.

Our paper thus suggests that the coarsely informed investors can have a more strategic motive for information sharing. This insight is useful in interpreting a stream of empirical studies that examine the ability of information contained in the social media to predict financial market movements. The claims about the predicatibility of such information are mixed. For example, Chen et al. (2014) find that the views expressed in Seeking Alpha articles and commentaries predict stock returns over the ensuing three months and earnings surprises. However, Tumarkin and Whitelaw (2001), Antweiler and Frank (2004), Das and Chen (2007) fail to detect strong relationship between opinion transmitted through the social media and stock returns. As suggested by Antweiler and Frank (2004), we need first understand why people post messages on the social media outlet and to properly answer this question requires a theory of communication that contains a financial market. We provide such a theory and it points out that some coarsely informed investors do have the incentive to truthfully share their investment opinions. Because the opinions contain information about the fundamental, it is useful to predict stock returns; in particular, this explains why the views expressed in Seeking Alpha can predict returns over as long as a three-year horizon (Chen et al., 2014). Meanwhile, these investment opinions, though truthful, are noisy and thus their predictability can be undermined.

In addition, our model sheds new light on the increasingly popular trading strategies based on the sentiment extracted from the social media. For example, a growing number of hedge funds are buying the data feeds from Dataminr, which applies advanced analytics to the entire Twitter "fire hose" to detect events likely to move the market ("How investors are using social media to make money," December 7, 2015, Fortune). Our model suggests that for investors that are not well informed, such sentiment might better inform their trading decisions and increase trading profits. However, if the investors per se have been well informed about the fundamental of a firm, an industry, or the economy, then subscription to the data feeds can be a prisoner's dilemma for these investors. That is, by filtering out the noise in the coarse information on the social media and trading against it, these well-informed investors offset the order flow from the coarsely informed investors and can be forced to trade less aggressively on their own information, thereby losing competitive advantage and incurring a loss.

5.2 Communication and information network

Word of mouth communication also plays an important role among professional investors in information transmission in financial markets (e.g., Shiller and Pound, 1986; Hong, Kubik, and Stein, 2005; Luo, 2018). Given the enormous amount of resources spent on acquiring information, why do investors share their valuable private information? Who shares information with whom?

The answers to these questions are fundamental to our understanding of information networks and their implications for financial markets. Our theory offers a parsimonious framework to answer the two questions. Specifically, even in possession of coarse information, an investor would like to share her information with the well informed investors to induce them to offset her informed order flow. The more genuine the shared information, the higher the benefits for the coarsely informed investor. Therefore, the focal coarsely informed investor is willing to truthfully share her information. Meanwhile, the direction of the information flow is unique here in that it transmits from the less informed side to the more informed one, whereas in the common explanations, the information flows in the opposite direction. This novel insight thus helps explain why information sharing can be such a widespread phenomenon in financial markets; that is, even coarsely informed investors would like to share their investment opinions.

Further, as shown by Proposition 3, while the coarsely informed investor gains from sharing her information, the well informed investor loses from trading against the shared information. Therefore, if the well informed investor can choose, he might only communicate with other well informed investors but refuse to talk to the coarsely informed investors. As the very basic component of any information network, according to our theory a pair of investors is a stable relationship if it consists of investors with similar information quality. Casual observations suggest that professional fund managers usually establish a core group of friends of similar background and the community is relatively stable (Cohen, Frazzini, and Malloy, 2008, 2010; Cohen and Malloy, 2010).

Meanwhile, Section 4.2 suggests that when there are multiple well informed investors, all of them may end up checking and trading against the information shared by the coarsely informed investor. This is a prisoner's dilemma because they could have been better off if they together refused the shared information. Interestingly, this suggests that for the increasingly popular investment conferences (Luo, 2018), even though the expressed investment ideas might not be that great, an well informed investor (e.g., a prestigious fund manager) attending it can induce other well informed ones to participate because otherwise the non-participant well informed investors are left at an information disadvantage. This result potentially explains why such investment conferences are gaining popularity.

6 Conclusion

This paper proposes that an investor with imperfect information may voluntarily reveal her information to a well informed investor. Being able to sift the error from the fundamental, the more informed investor tends to trade against the shared information. In this way, the investor that shares her information can have her informed order flows partially offset and enjoy better execution price (trading-against-error effect). By contrast, the well informed investor never shares his information because any piece of information sharing can only dissipate his informational advantage and erodes his profits accordingly. Further, after information sharing, the less informed investor becomes less concerned with the error in her information and trades more aggressively on it. The well informed investor is forced to trade less aggressively despite the superior information, which ultimately erodes his profits. Market liquidity worsens whereas both market efficiency and trading volume increase after information sharing. Our model offers a novel explanation for why investors share their information in financial markets. Our explanation is unique in that it is less informed investor that releases her information to the more informed investor.

Appendix

Proof of Lemma 1

See the main text.

Proof of Proposition 1-2 and Corollary 1-2

Most of the proof is presented in the main text. We here only verify that investor H does not share any information ($\tau_H = 0$) and investor L shares all of her information ($\tau_L = +\infty$). Equation (11) can be rewritten as the following:

$$\frac{\partial \pi_L(\tau_H,\tau_L)}{\partial \tau_L} = \rho^2\sigma_u \times \frac{\begin{pmatrix} 4\tau_H\left(2445\rho^2+6374\rho+4128\right)\rho^2\tau_L + 4\tau_H\left(2856\rho^2+6907\rho+4128\right)\rho\tau_L^2 \\ +64\tau_H\left(69\rho^2+155\rho+86\right)\tau_L^3 + 4\tau_H\left(693\rho^2+1947\rho+1376\right)\rho^3 \\ +1472\tau_H^3\left(\tau_L+\rho\right)^3 + 16\tau_H^2\left(\tau_L+\rho\right)^2\left((276\rho+310)\tau_L+\rho(219\rho+310)\right) \\ +\left(2772\rho^3+10803\rho^2+14072\rho+6048\right)\rho^2\tau_L \\ +4\left(876\rho^3+3187\rho^2+3823\rho+1512\right)\rho\tau_L^2 + 32(\rho+1)^2(46\rho+63)\tau_L^3 \\ +9\left(81\rho^3+335\rho^2+476\rho+224\right)\rho^3 \end{pmatrix}}{\begin{pmatrix} \begin{pmatrix} 6\rho\tau_L\left(8\tau_H\left(8\tau_H+14\rho+17\right)+\rho(48\rho+113)+72\right) \\ +24\tau_L^2\left(\tau_H+\rho+1\right)\left(8\tau_H+8\rho+9\right) \\ +24\rho^2\tau_H\left(8\tau_H+12\rho+17\right)+54(\rho(2\rho+5)+4)\rho^2 \end{pmatrix}^{3/2} \\ \times\left(4\tau_H\left(\tau_L+\rho\right)+4(\rho+1)\tau_L+\rho(3\rho+4)\right)^2 \end{pmatrix}} > 0.$$

Therefore, investor L will optimally choose $\tau_L = +\infty$. Similarly, equation (12) can be simplified to the following:

$$\frac{\partial \pi_H(\tau_H,\tau_L)}{\partial \tau_H} = -2\sigma_u\left(\tau_L+\rho\right)^2 \times \frac{\begin{pmatrix} 4\tau_H\begin{pmatrix} \left(1713\rho^2+4450\rho+2856\right)\rho^2\tau_L + \left(2184\rho^2+5021\rho+2856\right)\rho\tau_L^2 \\ +8\left(114\rho^2+233\rho+119\right)\tau_L^3 + \left(441\rho^2+1293\rho+952\right)\rho^3 \end{pmatrix} \\ +1216\tau_H^3\left(\tau_L+\rho\right)^3 + 16\tau_H^2\left(\tau_L+\rho\right)^2\left((228\rho+233)\tau_L+\rho(159\rho+233)\right) \\ +2\left(882\rho^3+3495\rho^2+4532\rho+1944\right)\rho^2\tau_L + 4\left(636\rho^3+2225\rho^2+2561\rho+972\right)\rho\tau_L^2 \\ +16(\rho+1)^2(76\rho+81)\tau_L^3 + 9\left(45\rho^3+202\rho^2+292\rho+144\right)\rho^3 \end{pmatrix}}{\begin{pmatrix} 3\begin{pmatrix} \rho\tau_L\left(8\tau_H\left(8\tau_H+14\rho+17\right)+\rho(48\rho+113)+72\right) \\ +4\tau_L^2\left(\tau_H+\rho+1\right)\left(8\tau_H+8\rho+9\right) \\ +4\rho^2\tau_H\left(8\tau_H+12\rho+17\right)+9(\rho(2\rho+5)+4)\rho^2 \end{pmatrix}^{3/2} \\ \times\left(4\tau_H\left(\tau_L+\rho\right)+4(\rho+1)\tau_L+\rho(3\rho+4)\right)^2 \end{pmatrix}} < 0.$$

Therefore, investor H will not share any of his information: $\tau_H = 0$. QED.

Proof of Proposition 3

Based on Corollary 1-2, we know that

$$\pi_H^* - \pi_H^0 = \frac{1}{6}\sigma_u \left(\frac{9+4\rho}{\sqrt{9+17\rho+8\rho^2}} - \frac{6(2+\rho)^2}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} \right) < 0,$$

where the inequality follows because

$$\begin{aligned}\left((9+4\rho)(4+3\rho)\sqrt{4+5\rho+2\rho^2}\right)^2 - \left(6(2+\rho)^2\sqrt{9+17\rho+8\rho^2}\right)^2& \\ = -\rho\left(1296+3044\rho+2611\rho^2+970\rho^3+132\rho^4\right) < 0.&\end{aligned}$$

Similarly,

$$\pi_L^* - \pi_L^0 = \frac{1}{3}\rho\sigma_u \left(\frac{2}{\sqrt{9+17\rho+8\rho^2}} - \frac{3(1+\rho)}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} \right) > 0,$$

where the inequality follows because

$$\begin{aligned}\left(2(4+3\rho)\sqrt{4+5\rho+2\rho^2}\right)^2 - \left(3(1+\rho)\sqrt{9+17\rho+8\rho^2}\right)^2& \\ = 175+389\rho+293\rho^2+75\rho^3 > 0.&\end{aligned}$$

Further, according to Corollary 1-2,

$$\lambda^* - \lambda^0 = \frac{1}{6\sigma_u}\left(\frac{\sqrt{9+17\rho+8\rho^2}}{1+\rho} - \frac{6\sqrt{4+5\rho+2\rho^2}}{4+3\rho} \right) > 0,$$

where the inequality holds because

$$\left((4+3\rho)\sqrt{9+17\rho+8\rho^2}\right)^2 - \left(6(1+\rho)\sqrt{4+5\rho+2\rho^2}\right)^2 = \rho\left(21\rho^2+41\rho+20\right) > 0.$$

For market efficiency, in the benchmark economy without information sharing,

$$(m^0)^{-1} = 1 - \frac{(\alpha_v^0+\beta_y^0)^2}{\alpha_v^0+\beta_y^0)^2+(\beta_y^0)^2/\rho+\sigma_u^2} = \frac{2+\rho}{4+3\rho},$$

and in equilibrium,

$$(m^*)^{-1} = 1 - \frac{(\alpha_v^*+\beta_y^*-\alpha_L^*)^2}{(\alpha_v^*+\beta_y^*-\alpha_L^*)^2+(\beta_y^{*0}-\alpha_L^*)^2/\rho+\sigma_u^2} = \frac{32\rho^2+93\rho+36}{153\rho^2+225\rho+72}.$$

A direct comparison yields $(m^*)^{-1} < (m^0)^{-1}$. Therefore, $m^* > m^0$.

Finally, we discuss trading volume. The trading volume of H in the benchmark economy and that in equilibrium are respectively $TV_H^0 = \frac{(2+\rho)\sigma_u}{\sqrt{2\pi}\sqrt{4+5\rho+2\rho^2}}$ and $TV_H^* = \frac{\sqrt{9+4\rho}\sigma_u}{\sqrt{2\pi}\sqrt{9+8\rho}}$, and it can be shown that $TV_H^* < TV_H^0$. Similarly, the trading volume of L in the benchmark economy and that in equilibrium are respectively $TV_L^0 = \frac{\sqrt{\rho(1+\rho)}\sigma_u}{\sqrt{2\pi}\sqrt{4+5\rho+2\rho^2}}$ and $TV_L^* = \frac{\sqrt{\rho(1+\rho)}\sigma_u}{\sqrt{2\pi}\sqrt{(9+8\rho)(1+\rho)}}$, and it can be shown that $TV_L^* > TV_L^0$. Further, the trading volume of market makers in the benchmark economy and that in equilibrium are respectively $TV_M^0 = \frac{\sqrt{(1+\rho)(4+3\rho)}\sigma_u}{\sqrt{\pi}\sqrt{4+5\rho+2\rho^2}}$ and $TV_M^* = \frac{\sqrt{3(6+11\rho)}\sigma_u}{\sqrt{2\pi}\sqrt{9+8\rho}}$, and it can be shown that $TV_M^* > TV_M^0$. Last, to prove the total trading volume increases after information sharing ($TV^* > TV^0$), given that the trading volume of market makers increases ($TV_M^* > TV_M^0$), it suffices to show that $TV_H^* + TV_L^* > TV_H^0 + TV_L^0$. Define

$$f(\rho) \equiv (TV_H^* + TV_L^*) - \left(TV_H^0 + TV_L^0\right) = \frac{\sigma_u}{\sqrt{2\pi}} \left(\frac{2\sqrt{\rho} + \sqrt{9+4\rho}}{\sqrt{9+8\rho}} - \frac{2+\rho+\sqrt{\rho(1+\rho)}}{\sqrt{4+5\rho+2\rho^2}} \right).$$

Solving $f(\rho) = 0$ yields the unique real root $\rho = 0$. This suggests that for all $\rho \in (0, +\infty)$, $f(\rho)$ shares the same sign. With a randomly picked positive number, i.e., $\rho = 1$, we find that $f(1) = \frac{\sigma_u}{\sqrt{2\pi}} \left(\frac{2+\sqrt{13}}{\sqrt{17}} - \frac{3+\sqrt{2}}{\sqrt{11}} \right) \approx 0.0286 \cdot \frac{\sigma_u}{\sqrt{2\pi}} > 0$. Therefore, $f(\rho) > 0$ for $\rho > 0$, that is, the total trading volume increases after information sharing. QED.

Proof of Proposition 4

Without loss of generality, we consider the case in which investor 1 is endowed with information of higher precision than investor 2, that is, $\rho_1 \geq \rho_2$. We take it as given that investor 1 as the more informed investor does not share her information. We consider a linear pricing rule for market makers $\tilde{p} = \lambda\tilde{\omega}$ and linear trading strategies for the two investors: $\tilde{x}_1 = \alpha_y \tilde{y}_1 + \alpha_2 \tilde{s}_2$ and $\tilde{x}_2 = \beta_y \tilde{y}_2 + \beta_2 \tilde{s}_2$.

After information sharing, investor 1's information set is $\mathcal{F}_1 = \{\tilde{v}, \tilde{s}_2\}$. With the information set, investor 1's posterior beliefs about the fundamental value, investor 2's information, and noise trading are

$$\begin{aligned} E(\tilde{v}|\mathcal{F}_1) &= \frac{\rho_1(\rho_2+\tau_2)\tilde{y}_1 + \rho_2\tau_2\tilde{s}_2}{(1+\rho_1+\rho_2)\tau_2 + \rho_2(1+\rho_1)}, \\ E(\tilde{y}_2|\mathcal{F}_1) &= \frac{\rho_1\rho_2\tilde{y}_1 + (1+\rho_1+\rho_2)\tau_2\tilde{s}_2}{(1+\rho_1+\rho_2)\tau_2 + \rho_2(1+\rho_1)}, \\ E(\tilde{u}|\mathcal{F}_1) &= 0. \end{aligned}$$

Then investor H's conditional trading profits can be expressed as follows:

$$\pi_1 = \tilde{x}_1 \left(E\left(\tilde{v}|\mathcal{F}_1\right) - \lambda \left(\tilde{x}_1 + \beta_y E\left(\tilde{y}_2|\mathcal{F}_1\right) \right) + \beta_2 \tilde{s}_2 \right). \tag{A1}$$

Maximizing investor H's profits yields the optimal trading strategy: $\tilde{x}_1 = \alpha_y \tilde{y}_1 + \alpha_2 \tilde{s}_2$, with

$$\alpha_y = \frac{\rho_1 \left(\tau_2 + \rho_2 \left(1 - \lambda \beta_y\right)\right)}{2\lambda \left(\left(\rho_1 + 1\right) \tau_2 + \rho_2 \left(\rho_1 + \tau_2 + 1\right)\right)},$$
$$\alpha_2 = -\frac{\beta_2}{2} - \frac{\tau_2 \left(\lambda \left(\rho_1 + \rho_2 + 1\right) \beta_y - \rho_2\right)}{2\lambda \left(\left(\rho_1 + 1\right) \tau_2 + \rho_2 \left(\rho_1 + \tau_2 + 1\right)\right)}.$$

Similarly,we can derive the trading strategy of investor 2 $\tilde{x}_2 = \beta_y \tilde{y}_2 + \beta_2 \tilde{s}_2$, with

$$\beta_y = \frac{1}{2\lambda} \frac{1 - \lambda \alpha_y}{1 + \rho_2},$$
$$\beta_2 = -\frac{\alpha_2}{2}.$$

With the two investors best-response trading strategies we can derive their optimal trading rules as functions of τ_2 and λ as follows:

$$\alpha_y = \frac{1}{\lambda} \frac{\rho_1 \left(2\rho_2 \left(\tau_2 + 1\right) + \rho_2^2 + 2\tau_2\right)}{\rho_2^2 \left(3\rho_1 + 4\tau_2 + 4\right) + 4\rho_2 \left(\rho_1 \left(\tau_2 + 1\right) + 2\tau_2 + 1\right) + 4 \left(\rho_1 + 1\right) \tau_2},$$
$$\alpha_2 = \frac{1}{3\lambda} \frac{2\rho_2 \left(-\rho_1 + 2\rho_2 + 2\right) \tau_2}{\rho_2^2 \left(3\rho_1 + 4\tau_2 + 4\right) + 4\rho_2 \left(\rho_1 \left(\tau_2 + 1\right) + 2\tau_2 + 1\right) + 4 \left(\rho_1 + 1\right) \tau_2}$$
$$\beta_y = \frac{1}{\lambda} \frac{\rho_2 \left(\left(\rho_1 + 2\right) \tau_2 + \rho_2 \left(\rho_1 + 2\tau_2 + 2\right)\right)}{\rho_2^2 \left(3\rho_1 + 4\tau_2 + 4\right) + 4\rho_2 \left(\rho_1 \left(\tau_2 + 1\right) + 2\tau_2 + 1\right) + 4 \left(\rho_1 + 1\right) \tau_2}$$
$$\beta_2 = -\frac{1}{3\lambda} \frac{\rho_2 \left(-\rho_1 + 2\rho_2 + 2\right) \tau_2}{\rho_2^2 \left(3\rho_1 + 4\tau_2 + 4\right) + 4\rho_2 \left(\rho_1 \left(\tau_2 + 1\right) + 2\tau_2 + 1\right) + 4 \left(\rho_1 + 1\right) \tau_2},$$

where $\lambda > 0$ is determined by the following equation:

$$\lambda = \frac{\sqrt{\begin{array}{r} \rho_1^2 \left(\rho_2^3 \left(48\tau_2 + 45\right) + \rho_2^2 \left(32\tau_2^2 + 113\tau_2 + 36\right) + 4\rho_2 \tau_2 \left(17\tau_2 + 18\right) + 18\rho_2^4 + 36\tau_2^2\right) \\ +\rho_1 \left(\rho_2^4 \left(48\tau_2 + 45\right) + 8\rho_2^3 \left(8\tau_2^2 + 23\tau_2 + 9\right) + 4\rho_2^2 \left(41\tau_2^2 + 52\tau_2 + 9\right) + 8\rho_2 \tau_2 \left(17\tau_2 + 9\right) + 36\tau_2^2\right) \\ +4\rho_2 \left(\rho_2 + 1\right) \left(\rho_2^2 \left(8\tau_2^2 + 17\tau_2 + 9\right) + \rho_2 \tau_2 \left(16\tau_2 + 17\right) + 8\tau_2^2\right) \end{array}}}{3 \left(\rho_2^2 \left(3\rho_1 + 4\tau_2 + 4\right) + 4\rho_2 \left(\rho_1 \left(\tau_2 + 1\right) + 2\tau_2 + 1\right) + 4 \left(\rho_1 + 1\right) \tau_2\right) \sigma_u}.$$

Inserting the optimal trading strategies into investor 2' profit function and taking expectations yields her unconditional profits $\pi_2(\tau_2; \rho_1, \rho_2) \equiv E[\pi_2]$.

As in Proposition 2, if $\rho_1 \to +\infty$, we can show that

$$\frac{\partial \pi_2(\tau_2; +\infty, \rho_2)}{\partial \tau_2} > 0.$$

That is, investor 2 shares her information "as is."

If $\rho_1 = \rho_2$, we observe that for any ρ_2

$$\frac{\partial \pi_2(\tau_2;\rho_2,\rho_2)}{\partial \tau_2} = -\rho_2^3\left(\rho_2+2\right)\sigma_u$$
$$\times \frac{\begin{pmatrix} 9\rho_2^8\left(1688\tau_2+2477\right)+6\rho_2^7\left(4576\tau_L^2+20014\tau_2+13365\right) \\ +8\rho_2^6\left(1600\tau_2^3+24488\tau_L^2+47824\tau_2+18621\right) \\ +4\rho_2^5\left(22976\tau_2^3+140793\tau_2^2+156913\tau_2+37512\right) \\ +4\rho_2^4\left(63544\tau_L^3+208679\tau_2^2+139650\tau_2+19332\right) \\ +8\rho_2^3\left(44254\tau_2^3+84085\tau_2^2+31958\tau_2+1980\right) \\ +16\rho_2^2\tau_2\left(16534\tau_2^2+17457\tau_2+2942\right) \\ +16\rho_2\tau_2^2\left(6311\tau_2+2914\right)+2511\rho_2^9+15392\tau_2^3 \end{pmatrix}}{\begin{pmatrix} 6\left(8\rho_2^2\left(\tau_2+1\right)+4\rho_2\left(3\tau_2+1\right)+3\rho_2^3+4\tau_2\right)^2 \\ \times\rho_2\begin{pmatrix} \rho_2^4\left(96\tau_2+90\right)+\rho_2^3\left(128\tau_2^2+365\tau_2+144\right) \\ +8\rho_2^2\left(41\tau_2^2+52\tau_L+9\right)+4\rho_2\tau_2\left(67\tau_2+35\right)+18\rho_2^5+68\tau_2^2 \end{pmatrix}^{3/2} \end{pmatrix}} < 0.$$

Therefore, investor 2 does not share her information, i.e., $\tau_2 = 0$.

So, based on the fact that $\frac{\partial \pi_2(\tau_2;+\infty,\rho_2)}{\partial \tau_2} > 0$ and $\frac{\partial \pi_2(\tau_2;\rho_2,\rho_2)}{\partial \tau_2} < 0$, by intermediate value theorem, there must exist $\hat{\rho}_1$ such that when $\rho_1 = \hat{\rho}_1$ investor 2 is indifferent between sharing information with investor 1 or not for any ρ_2. Further,

$$\frac{\partial \pi_2(\tau_2;\rho_1,\rho_2)}{\partial \tau_2} = \sigma_u\rho_2(2+2\rho_2-\rho_1)\frac{\Psi_1}{\Psi_2},$$

where

$$\Psi_1 = -\rho_1^4 \begin{pmatrix} 9\rho_2^6 (164\tau_2 + 209) + 24\rho_2^5 \left(218\tau_2^2 + 647\tau_2 + 381\right) \\ +4\rho_2^4 \left(944\tau_2^3 + 7516\tau_2^2 + 10745\tau_2 + 3195\right) \\ +4\rho_2^3 \left(4192\tau_2^3 + 15487\tau_2^2 + 11646\tau_2 + 1476\right) \\ +4\rho_2^2\tau_2 \left(7028\tau_2^2 + 13707\tau_2 + 4428\right) \\ +144\rho_2\tau_2^2 (146\tau_2 + 123) - 243\rho_2^7 + 5904\tau_2^3 \end{pmatrix}$$

$$- 2\rho_1^3 \begin{pmatrix} 18\rho_2^7 (211\tau_2 + 403) + 9\rho_2^6 \left(672\tau_2^2 + 3166\tau_2 + 2231\right) \\ +\rho_2^5 \left(4576\tau_2^3 + 45296\tau_2^2 + 84766\tau_2 + 31446\right) \\ +2\rho_2^4 \left(13200\tau_2^3 + 61622\tau_2^2 + 61823\tau_2 + 12726\right) \\ +8\rho_2^3 \left(7503\tau_2^3 + 19954\tau_2^2 + 11052\tau_2 + 1035\right) \\ +4\rho_2^2\tau_2 \left(16858\tau_2^2 + 25119\tau_2 + 6210\right) \\ +72\rho_2\tau_2^2 (521\tau_2 + 345) + 1377\rho_2^8 + 8280\tau_2^3 \end{pmatrix}$$

$$- 24 (\rho_2 + 1) \rho_1^2 \begin{pmatrix} 3\rho_2^7 (85\tau_2 + 82) + 3\rho_2^6 \left(118\tau_2^2 + 525\tau_2 + 319\right) \\ +\rho_2^5 \left(200\tau_2^3 + 2752\tau_2^2 + 4889\tau_2 + 1725\right) \\ +\rho_2^4 \left(1560\tau_2^3 + 7814\tau_2^2 + 7433\tau_2 + 1452\right) \\ +4\rho_2^3 \left(981\tau_2^3 + 2530\tau_2^2 + 1299\tau_2 + 111\right) \\ +4\rho_2^2\tau_2 \left(1103\tau_2^2 + 1509\tau_2 + 333\right) \\ +12\rho_2\tau_2^2 (191\tau_2 + 111) + 444\tau_2^3 \end{pmatrix}$$

$$- 32\rho_2 (\rho_2 + 1)^2 \rho_1 \begin{pmatrix} 3\rho_2^5 \left(17\tau_2^2 + 38\tau_2 + 21\right) + \rho_2^4 \left(-86\tau_2^3 + 128\tau_2^2 + 451\tau_2 + 225\right) \\ +\rho_2^3 \left(-42\tau_2^3 + 769\tau_2^2 + 1021\tau_2 + 234\right) \\ +2\rho_2^2\tau_2 \left(195\tau_2^2 + 679\tau_2 + 342\right) \\ +2\rho_2\tau_2^2 (281\tau_2 + 333) + 216\tau_2^3 \end{pmatrix}$$

$$+ 128\rho_2^2 (\rho_2 + 1)^3 (\rho_2 (\tau_2 + 1) + \tau_2)^2 (\rho_2 (17\tau_2 + 18) + 17\tau_2),$$

and

$$\Psi_2 = \left(\rho_2^2 (3\rho_1 + 4\tau_2 + 4) + 4\rho_2 (\rho_1 (\tau_2 + 1) + 2\tau_2 + 1) + 4 (\rho_1 + 1) \tau_2\right)^2$$
$$\begin{pmatrix} \rho_1^2 \left(\rho_2^3 (48\tau_2 + 45) + \rho_2^2 \left(32\tau_2^2 + 113\tau_2 + 36\right) + 4\rho_2\tau_2 (17\tau_2 + 18) + 18\rho_2^4 + 36\tau_2^2\right) \\ +\rho_1 \begin{pmatrix} \rho_2^4 (48\tau_2 + 45) + 8\rho_2^3 \left(8\tau_2^2 + 23\tau_2 + 9\right) \\ +4\rho_2^2 \left(41\tau_2^2 + 52\tau_2 + 9\right) + 8\rho_2\tau_2 (17\tau_2 + 9) + 36\tau_2^2 \end{pmatrix} \\ +4\rho_2 (\rho_2 + 1) \left(\rho_2^2 \left(8\tau_2^2 + 17\tau_2 + 9\right) + \rho_2\tau_2 (16\tau_2 + 17) + 8\tau_2^2\right) \end{pmatrix}^{3/2}.$$

Note that the indifference condition at $\rho_1 = \hat{\rho}_1$ suggests that for any τ_2 we must have $\frac{\partial \pi_2(\tau_2;\rho_1,\rho_2)}{\partial \tau_2} = 0$, which can only be true if $\hat{\rho}_1 = 2(1 + \rho_2)$. Therefore, when $\rho_1 > \hat{\rho}_1 \equiv 2(1 + \rho_2)$, investor 2 will share all her information with investor 1, whereas when $\rho_1 < \hat{\rho}_1 \equiv 2(1 + \rho_2)$, investor 2 does not share any information with investor 1.

The analysis of the case $\rho_1 < \rho_2$ is similar. QED.

Proof of Proposition 5

Note that we focus on the case in which insiders do not share their information and investor L either shares all or none of her information, i.e., $\tau_H = 0$ and $\tau_L \in \{0, +\infty\}$. Assume that among the M insiders, M_1 of them choose to read the information shared by investor L. We consider the following symmetric linear trading strategies; that is, the insider that reads the shared information demands $\tilde{x}_i = \alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{y}$ units of the risky asset, where $i \in \{1, ..., M_1\}$, and the insider that commits to not receiving the shared information demands $\tilde{x}_k = \alpha_{v_2}\tilde{v}$ units of the risky asset, where $k \in \{M_1 + 1, ..., M\}$. We also consider a linear pricing rule for market makers $\tilde{p} = \lambda\tilde{\omega}$.

Consider insider $i \in \{1, ..., M_1\}$ that reads the shared information. With the information set $\{\tilde{v}, \tilde{y}\}$, her conditional expected profits are as follows:

$$E[\tilde{x}_i(\tilde{v} - \tilde{p})] = \tilde{x}_i \left(\tilde{v} - \lambda \left(\tilde{x}_i + (M_1 - 1)(\alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{y}) + (M - M_1)\alpha_{v_2}\tilde{v} + \beta\tilde{y}\right)\right).$$

Maximizing the profits yields insider i's optimal trading rule $\tilde{x}_i = \alpha_{v_i}\tilde{v} + \alpha_{L_i}\tilde{y}$ with

$$\alpha_{v_i} = \frac{1}{2\lambda}\left(1 - (M_1 - 1)\lambda\alpha_{v_1} - (M - M_1)\lambda\alpha_{v_2}\right) \text{ and } \alpha_{L_i} = -\frac{1}{2}\left((M_1 - 1)\alpha_{L_1} + \beta\right). \tag{A2}$$

For insider $k \in \{M_1 + 1, ..., M\}$ that commits to not receiving the shared information, with the information set $\{\tilde{v}\}$, her conditional expected profits are as follows:

$$E[\tilde{x}_k(\tilde{v} - \tilde{p})] = \tilde{x}_k \left(\tilde{v} - \lambda \left(\tilde{x}_k + M_1(\alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{v}) + (M - M_1 - 1)\alpha_{v_2}\tilde{v} + \beta\tilde{v}\right)\right).$$

Maximizing the profits yields insider k's optimal trading rule $\tilde{x}_k = \alpha_{v_k}\tilde{v}$ with

$$\alpha_{v_k} = \frac{1}{2\lambda}\left(1 - \beta\lambda - M_1\lambda(\alpha_{L_1} + \alpha_{v_2}) - (M - M_1 - 1)\lambda\alpha_{v_2}\right). \tag{A3}$$

For investor L, her conditional expected trading profits are as follows:

$$E[\tilde{x}_L(\tilde{v} - \tilde{p})] = \tilde{x}_L \left(\frac{\rho}{1+\rho}\tilde{y} - \lambda\left(\tilde{x}_L + M_1\left(\alpha_{v_1}\frac{\rho}{1+\rho}\tilde{y} + \alpha_{L_1}\tilde{y}\right) + (M - M_1)\alpha_{v_2}\frac{\rho}{1+\rho}\tilde{y}\right)\right).$$

Maximizing investor L's profits yields her optimal trading strategy $\tilde{x}_L = \beta\tilde{y}$, with

$$\beta = -\frac{M_1}{2}\alpha_{L_1} + \frac{\rho}{2\lambda(1+\rho)}\left(1 - M_1\lambda\alpha_{v_1} - (M - M_1)\lambda\alpha_{v_2}\right). \tag{A4}$$

Imposing symmetric equilibrium $\alpha_{v_i} = \alpha_{v_1}$, $\alpha_{L_i} = \alpha_{L_1}$, and $\alpha_{v_k} = \alpha_{v_2}$, the interaction of the reaction functions (A2)-(A4) yields the optimal trading strategies as specified below:

$$\alpha_{v_1} = \frac{(2 + M_1)(1 + \rho)}{\lambda\left((1 + M)(2 + M_1) + (2 + M)(1 + M_1)\rho\right)}, \tag{A5}$$

$$\alpha_{L_1} = -\frac{\rho}{\lambda\left((1 + M)(2 + M_1) + (2 + M)(1 + M_1)\rho\right)}, \tag{A6}$$

$$\alpha_{v_2} = \frac{2 + M_1 + \rho + M_1\rho}{\lambda\left((1 + M)(2 + M_1) + (2 + M)(1 + M_1)\rho\right)}, \tag{A7}$$

$$\beta = \frac{(1 + M_1)\rho}{\lambda\left((1 + M)(2 + M_1) + (2 + M)(1 + M_1)\rho\right)}. \tag{A8}$$

Using the weak efficiency rule, market makers' optimal pricing rule is as follows:

$$\lambda = \frac{M_1(\alpha_{v_1} + \alpha_{L_1}) + (M - M_1)\alpha_{v_2} + \beta}{(M_1(\alpha_{v_1} + \alpha_{L_1}) + (M - M_1)\alpha_{v_2} + \beta)^2 + (M_1\alpha_{L_1} + \beta)/\rho + \sigma_u^2}.$$

Inserting the optimal trading strategies into λ we can express the equilibrium pricing rule as follows $\tilde{p} = \lambda\tilde{\omega}$ with

$$\lambda = \frac{\sqrt{M(2 + M_1 + \rho + M_1\rho)^2 + \rho + \rho^2 + M_1\rho(3 + M_1 + (2 + M_1)\rho)}}{(\lambda\left((1 + M)(2 + M_1) + (2 + M)(1 + M_1)\rho\right)\sigma_u}. \tag{A9}$$

Now, inserting the optimal trading rules (A5)-(A8) and the optimal pricing rule (A9) into the investors' expected trading profits and taking expectations yields their respective unconditional profits as follows:

$$\pi_i(M_1, M) = \frac{(1 + \rho)\sigma_u^2\left((2 + M_1)^2 + (1 + M_1)^2\rho\right)}{\lambda\left(M(1 + (1 + M_1)(1 + \rho))^2 + \rho + \rho^2 + M_1\rho(3 + M_1 + (2 + M_1)\rho)\right)}, \tag{A10}$$

$$\pi_k(M_1, M) = \frac{\sigma_u^2\left(1 + (1 + M_1)(1 + \rho)\right)^2}{\lambda\left(M(1 + (1 + M_1)(1 + \rho))^2 + \rho + \rho^2 + M_1\rho(3 + M_1 + (2 + M_1)\rho)\right)}, \tag{A11}$$

$$\pi_L(M_1, M) = \frac{\sigma_u^2\rho(1 + \rho)\left(1 + M_1\right)^2}{\lambda\left(M(1 + (1 + M_1)(1 + \rho))^2 + \rho + \rho^2 + M_1\rho(3 + M_1 + (2 + M_1)\rho)\right)}. \tag{A12}$$

First, we discuss if $M_1 = 0$ (that all insiders commit not to receiving L's shared information) is an equilibrium. When $M_1 = 0$, based on equations (A11), the profits of the insiders are as follows:

$$\pi_k(0, M) = \frac{(2 + \rho)^2\sigma_u}{(2 + 2\rho + M(2 + \rho))\sqrt{M(3 + 2\rho)^2 + \rho(5 + 4\rho)}}.$$

If one insider deviates to receiving the shared information, according to equation (A10), her profits will become

$$\pi_i(1, M) = \frac{(1 + \rho)(9 + 4\rho)\sigma_u}{(3 + 4\rho + M(3 + 2\rho))\sqrt{M(3 + 2\rho)^2 + \rho(5 + 4\rho)}}.$$

Therefore, $M_1 = 0$ is not an equilibrium if $\frac{\pi_i(1,M)}{\pi_k(0,M)} > 1$.Further, we know that

$$\frac{\partial}{\partial M}\left(\frac{\pi_i(1, M)}{\pi_k(0, M)}\right) = \frac{\rho\pi_i(1, M)}{\pi_k(0, M)} \frac{\begin{pmatrix} M^2\left(18\rho^4 + 121\rho^3 + 303\rho^2 + 335\rho + 138\right) \\ +M\left(56\rho^4 + 299\rho^3 + 577\rho^2 + 469\rho + 132\right) \\ +2(\rho + 1)^2\left(24\rho^2 + 56\rho + 33\right) \end{pmatrix}}{\begin{pmatrix} 2(M(\rho + 2) + 2(\rho + 1))\left(M(\rho + 2)^2 + \rho(\rho + 1)\right) \\ \times(M(2\rho + 3) + 4\rho + 3)\left(M(2\rho + 3)^2 + \rho(4\rho + 5)\right) \end{pmatrix}} > 0$$

and when $M = 4$,

$$\frac{\pi_i(1,4)}{\pi_k(0,4)} = \frac{2(\rho+1)(3\rho+5)(4\rho+9)\sqrt{5\rho^2+17\rho+16}}{3(\rho+2)^2(4\rho+5)\sqrt{20\rho^2+53\rho+36}} > 1,$$

where the inequality holds because

$$\begin{aligned}&\left(2(\rho+1)(3\rho+5)(4\rho+9)\sqrt{5\rho^2+17\rho+16}\right)^2 - \left(3(\rho+2)^2(4\rho+5)\sqrt{20\rho^2+53\rho+36}\right)^2\\ &\quad = \rho\left(240\rho^6+2744\rho^5+13071\rho^4+33004\rho^3+46336\rho^2+34132\rho+10260\right) > 0.\end{aligned}$$

Therefore, if $M > 4$, we must have $\frac{\pi_i(1,M)}{\pi_k(0,M)} > 1$; that is, $M_1 = 0$ cannot be an equilibrium.

Second, we discuss if $M_1 = M$ (that all insiders receive L's shared information) can be an equilibrium. When $M_1 = M$, based on equation (A10), the profits of the insiders are as follows:

$$\pi_i(M,M) = \frac{\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)\sigma_u}{\left(M^2+3M+2\right)\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}}.$$

If one insider deviates to not receiving the shared information, according to equation (A11), her profits will become

$$\pi_k(M-1,M) = \frac{(M\rho+M+1)^2\sigma_u}{\left(M^2(\rho+1)+2M(\rho+1)+1\right)\sqrt{M^3(\rho+1)^2+M^2\left(\rho^2+3\rho+2\right)+M(\rho+1)-\rho}}.$$

Therefore, $M_1 = M$ is an equilibrium if $\frac{\pi_k(M-1,M)}{\pi_i(M,M)} < 1$. Further,

$$\begin{aligned}\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) &= \frac{\rho\pi_k(M-1,M)}{\pi_i(M,M)}\\ &\times\frac{\begin{pmatrix} 4M^{10}(\rho+1)^4+M^9(\rho+1)^3(23\rho+31)+M^8(\rho+1)^2\left(29\rho^2+89\rho+64\right)\\ -M^7(\rho+1)^2\left(97\rho^2+244\rho+143\right)-M^6\left(397\rho^4+2092\rho^3+4005\rho^2+3326\rho+1016\right)\\ -M^5\left(624\rho^4+3715\rho^3+7907\rho^2+7213\rho+2397\right)\\ -M^4\left(532\rho^4+3590\rho^3+8551\rho^2+8651\rho+3166\right)\\ -M^3\left(258\rho^4+1966\rho^3+5321\rho^2+6128\rho+2551\right)\\ -M^2\left(68\rho^4+562\rho^3+1785\rho^2+2484\rho+1246\right)\\ -M\left(8\rho^4+62\rho^3+260\rho^2+523\rho+340\right)-2\left(3\rho^2+25\rho+20\right)\end{pmatrix}}{\begin{pmatrix} 2\left(M^2+3M+2\right)(M\rho+M+1)\\ \times\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)\\ \times\left(M^3(\rho+1)+M^2(3\rho+4)+M(3\rho+4)+\rho\right)\\ \times\left(M^3(\rho+1)^2+M^2\left(\rho^2+3\rho+2\right)+M(\rho+1)-\rho\right)\end{pmatrix}}.\end{aligned}$$

Therefore, as $M \to +\infty$, $\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) > 0$. Together with the fact that $\lim_{M\to 1}\frac{\pi_k(M-1,M)}{\pi_i(M,M)} = 1$, we know that there exits $\hat{M}_1 > 0$ such that when $M > \hat{M}_1$, $\frac{\pi_k(M-1,M)}{\pi_i(M,M)} < 1$; that is, there exists an equilibrium in which all insiders use the shared information.

Finally, we show that while $M_1 = M$ can be an equilibrium, it is always dominated by $M_1 = 0$ in terms of the insiders' profits. That is, the insiders would have been better off if they committed not to receiving the shared information.

$$\frac{\pi_i(0,M)}{\pi_k(M,M)} = \frac{(M^2+3M+2)(\rho+2)^2}{(M(\rho+2)+2\rho+2)(M^2(\rho+1)+2M(\rho+2)+\rho+4)} \times \frac{\sqrt{\rho(M^2(\rho+1)+M(2\rho+3)+\rho+1)+M(M\rho+M+\rho+2)^2}}{\sqrt{M(\rho+2)^2+\rho(\rho+1)}}.$$

We know that

$$\frac{\partial}{\partial M}\frac{\pi_i(0,M)}{\pi_k(M,M)} = \rho(\rho+2)^2 \times \frac{\begin{pmatrix} -M^7(\rho+1)^2\left(4\rho^2+15\rho+14\right) \\ -M^6\left(22\rho^4+119\rho^3+242\rho^2+217\rho+72\right) \\ -M^5\left(33\rho^4+165\rho^3+314\rho^2+268\rho+86\right) \\ +M^4\left(37\rho^4+185\rho^3+368\rho^2+336\rho+116\right) \\ +M^3\left(181\rho^4+805\rho^3+1372\rho^2+1060\rho+312\right) \\ +M^2\left(237\rho^4+949\rho^3+1380\rho^2+844\rho+176\right) \\ +2M\rho\left(70\rho^3+247\rho^2+293\rho+116\right)+32\rho(\rho+1)^3 \end{pmatrix}}{\begin{pmatrix} 2(M(\rho+2)+2(\rho+1))^2\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)^2 \\ \times\sqrt{\rho(M^2(\rho+1)+M(2\rho+3)+\rho+1)+M(M\rho+M+\rho+2)^2} \\ \times\left(M(\rho+2)^2+\rho(\rho+1)\right)^{3/2} \end{pmatrix}}.$$

So, there exists a constant $\hat{M}_2 > 0$ such that when $M > \hat{M}_2$, $\frac{\partial}{\partial M}\frac{\pi_i(0,M)}{\pi_k(M,M)} < 0$. Further, as $M \to +\infty$, $\frac{\pi_i(0,M)}{\pi_k(M,M)} \to 1$. When $M = 2$,

$$\frac{\pi_i(0,2)}{\pi_k(2,2)} = \frac{6(\rho+2)^2\sqrt{27\rho^2+59\rho+32}}{(2\rho+3)(9\rho+16)\sqrt{3\rho^2+9\rho+8}} > 1,$$

where the inequality holds because

$$\left(6(\rho+2)^2\sqrt{27\rho^2+59\rho+32}\right)^2 - \left((2\rho+3)(9\rho+16)\sqrt{3\rho^2+9\rho+8}\right)^2 = 23328\rho^8 +288036\rho^7+1531575\rho^6+4575726\rho^5+8389959\rho^4+9652176\rho^3+6788864\rho^2+2660352\rho+442368 > 0.$$

Therefore, when $M > \hat{M}_2$, $\frac{\pi_i(0,M)}{\pi_k(M,M)} > 1$; that is, the insiders would be better off if all of them committed not to receiving the shared information. Overall, when $M > \max\{\hat{M}_1, \hat{M}_2\}$, part (ii) of the proposition holds. QED.

Proof of Proposition 6

Denote the right-hand-side of equations (19) and (20) as $f_{NS}(\rho)$ and $f_S(\rho)$, respectively. It is easy to see that $f'_{NS}(\rho) < 0$ and $f'_S(\rho) < 0$. Therefore, as c/σ_u increases, investor L acquires less information in both cases with and without information sharing; that is, both ρ^* and ρ^0 decrease in c/σ_u. Further, setting $f_{NS}(\rho) = f_S(\rho)$ yields $\rho = \hat{\rho} \approx 1.1307$ and we know that $f_{NS}(\hat{\rho}) = f_S(\hat{\rho}) \equiv \hat{c} \approx 0.0520$. Therefore, the function $f_S(\rho) - f_{NS}(\rho) = 0$ has a unique root at $\rho = \hat{\rho}$.

Since $\lim_{\rho\to 0} f_S(\rho) - f_{NS}(\rho) = 7/72 > 0$, if $c/\sigma_u > \hat{c}$ so that both ρ^0 and ρ^* are small, we know that $\rho^* > \rho^0$. Similarly, since $\lim_{\rho\to\infty} f_S(\rho) - f_{NS}(\rho) = 0$, when $c/\sigma_u < \hat{c}$, both ρ^0 and ρ^* are large and we know that $\rho^* < \rho^0$. QED.

Equilibrium Characterization in Section 4.4

In this extended economy, after L shares her information, H's information set is $\mathcal{F}_H = \{\tilde{v}, \tilde{s}_H, \tilde{q}_H\}$, L's information set is $\mathcal{F}_L = \{\tilde{y}, \tilde{s}_L, \tilde{z}_L\}$, and that of market makers is $\{\tilde{\omega}, \tilde{z}_M, \tilde{q}_M\}$. We focus on a linear pricing rule for market makers $\tilde{p} = \lambda_\omega\tilde{\omega} + \lambda_H\tilde{z}_M + \lambda_L\tilde{q}_M$ and linear trading strategies for the two investors: $\tilde{x}_H = \alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{q}_H$ and $\tilde{x}_L = \beta_y\tilde{y} + \beta_H\tilde{z}_L + \beta_L\tilde{s}_L$.

For investor H, the conditional trading profits are

$$E\left[\tilde{x}_H(\tilde{v}-\tilde{p})|\tilde{v},\tilde{s}_H,\tilde{q}_H\right] = \tilde{x}_H\Big(\tilde{v} - \lambda_\omega\big(\tilde{x}_H + \beta_y E[\tilde{y}|\mathcal{F}_H] + \beta_H E[\tilde{z}_L|\mathcal{F}_H] + \beta_L E[\tilde{s}_L|\mathcal{F}_H]\big) - \lambda_H E[\tilde{z}_M|\mathcal{F}_H] - \lambda_L E[\tilde{q}_M|\mathcal{F}_H]\Big),$$

where

$$\begin{aligned} E[\tilde{y}|\tilde{v},\tilde{s}_H,\tilde{q}_H] &= \frac{\rho(\tau_L+\chi_H)\tilde{v} + \tau_L\chi_H\tilde{q}_H}{\rho(\tau_L+\chi_H)+\tau_L\chi_H}, \\ E[\tilde{s}_L|\tilde{v},\tilde{s}_H,\tilde{q}_H] = E[\tilde{q}_M|\tilde{v},\tilde{s}_H,\tilde{q}_H] &= \frac{\rho\tau_L\tilde{v} + (\rho+\tau_L)\chi_H\tilde{q}_H}{\rho(\tau_L+\chi_H)+\tau_L\chi_H}, \\ E[\tilde{z}_L|\tilde{v},\tilde{s}_H,\tilde{q}_H] = E[\tilde{z}_M|\tilde{v},\tilde{s}_H,\tilde{q}_H] &= \tilde{s}_H. \end{aligned}$$

Maximizing the profits yields investor H's optimal trading strategy: $\tilde{x}_H = \alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{q}_H$ with

$$\alpha_v = \frac{\tau_L\left(\chi_H + \rho\left(-\lambda_L\right) - \rho\beta_L\lambda_w + \rho - \rho\lambda_w\beta_y\right) + \rho\chi_H\left(1-\lambda_w\beta_y\right)}{2\lambda_w\left(\tau_L\left(\chi_H+\rho\right)+\rho\chi_H\right)}, \tag{A13}$$

$$\alpha_H = -\frac{\lambda_H + \beta_H\lambda_w}{2\lambda_w}, \tag{A14}$$

$$\alpha_L = -\frac{\chi_H\left(\lambda_L\left(\tau_L+\rho\right) + \lambda_w\left(\beta_L\left(\tau_L+\rho\right)+\tau_L\beta_y\right)\right)}{2\lambda_w\left(\tau_L\left(\chi_H+\rho\right)+\rho\chi_H\right)}. \tag{A15}$$

Similarly, investor L's conditional trading profits are

$$E\left[\tilde{x}_L(\tilde{v}-\tilde{p})|\tilde{y},\tilde{s}_L,\tilde{z}_L\right] = \tilde{x}_L\Big(E[\tilde{v}|\mathcal{F}_L] - \lambda_\omega\big(\tilde{x}_L + \alpha_v E[\tilde{v}|\mathcal{F}_L] + \alpha_H E[\tilde{s}_H|\mathcal{F}_L] + \alpha_L E[\tilde{q}_H|\mathcal{F}_L]\big) \\ -\lambda_H E[\tilde{z}_M|\mathcal{F}_L] - \lambda_L E[\tilde{q}_M|\mathcal{F}_L]\Big),$$

where

$$\begin{aligned} E[\tilde{v}|\tilde{y},\tilde{s}_L,\tilde{z}_L] &= \frac{\rho(\tau_H+\kappa_L)\tilde{y} + \tau_H\kappa_L\tilde{z}_L}{(1+\rho)(\tau_H+\kappa_L)+\tau_H\kappa_L}, \\ E[\tilde{z}_M|\tilde{y},\tilde{s}_L,\tilde{z}_L] = E[\tilde{s}_H|\tilde{y},\tilde{s}_L,\tilde{z}_L] &= \frac{\rho\tau_H\tilde{y} + \kappa_L(1+\rho+\tau_H)\tilde{z}_L}{(1+\rho)(\tau_H+\kappa_L)+\tau_H\kappa_L}, \\ E[\tilde{q}_H|\tilde{y},\tilde{s}_L,\tilde{z}_L] &= \tilde{s}_L. \end{aligned}$$

Maximizing the profits yields investor L's optimal trading strategy: $\tilde{x}_L = \beta_y\tilde{y} + \beta_H\tilde{z}_L + \beta_L\tilde{s}_L$ with

$$\beta_y = -\frac{\rho\left(\tau_H\left(\lambda_H + \alpha_H\lambda_w + \alpha_v\lambda_w - 1\right) + \kappa_L\left(\alpha_v\lambda_w - 1\right)\right)}{2\lambda_w\left(\kappa_L\left(\tau_H+\rho+1\right) + (\rho+1)\tau_H\right)}, \tag{A16}$$

$$\beta_H = -\frac{\kappa_L\left(\lambda_H\left(\tau_H+\rho+1\right) + \tau_H\left(\alpha_v\lambda_w - 1\right) + \alpha_H\lambda_w\left(\tau_H+\rho+1\right)\right)}{2\lambda_w\left(\kappa_L\left(\tau_H+\rho+1\right) + (\rho+1)\tau_H\right)}, \tag{A17}$$

$$\beta_L = -\frac{\lambda_L + \alpha_L\lambda_w}{2\lambda_w} \tag{A18}$$

Based on equations (A13)- (A18), we can solve for the two investors' optimal trading strategies as functions of λ_ω, λ_H, and λ_L:

The pricing rule for market makers is $\tilde{p} = E[\tilde{v}|\tilde{\omega},\tilde{z}_M,\tilde{q}_M] = \lambda_\omega\tilde{\omega} + \lambda_H\tilde{z}_M + \lambda_L\tilde{q}_M$, where λ_ω, λ_H, and λ_L are computed accordingly.

Based on the above derivation, we are able to solve for the equilibrium at the trading stage as functions of τ_L: $\alpha_v(\tau_L), \alpha_L(\tau_L), \beta_y(\tau_L), \beta_L(\tau_L), \lambda_\omega(\tau_L)$ and $\lambda_L(\tau_L)$. We then submit them into L's profit function and compute unconditional trading profits $\pi_L(\tau_L) \equiv E\big[E\left[\tilde{x}_L(\tilde{v}-\tilde{p})|\tilde{y},\tilde{s}_L\right]\big]$. Finally, investor L's is willing to share her information if $\pi_L(\infty) > \pi_L(0)$.

References

Admati, Anat R., and Paul Pfleiderer. (1986). A monopolistic market for information. *Journal of Economic Theory*, 39(2), 400-438.

Admati, Anat R., and Paul Pfleiderer. (1988). Selling and trading on information in financial markets. *American Economic Review*, 78(2), 96-103.

Admati, Anat R., and Paul Pfleiderer. (1990). Direct and indirect sale of information. *Econometrica*, 901-928.

Allen, Franklin. (1990). The market for information and the origin of financial intermediation. *Journal of Financial Intermediation*, 1(1), 3-30.

Antweiler, Werner, and Murray Z. Frank. (2004). Is all that talk just noise? The information content of internet stock message boards. *Journal of Finance*, 59(3), 1259-1294.

Aragon, George O., and Philip E. Strahan. (2012). Hedge funds as liquidity providers: Evidence from the Lehman bankruptcy. *Journal of Financial Economics*, 103(3), 570-587.

Bartov, Eli, Lucile Faurel, and Partha S. Mohanram. (2018). Can Twitter help predict firm-level earnings and stock returns?. *The Accounting Review*, 93(3), 25-57.

Benabou, Roland, and Guy Laroque. (1992). Using privileged information to manipulate markets: Insiders, gurus, and credibility. *Quarterly Journal of Economics*, 107(3), 921-958.

Bernhardt, Dan, and Jianjun Miao. (2004). Informed trading when information becomes stale. *Journal of Finance*, 59(1), 339-390.

Biais, Bruno, and Laurent Germain. (2002). Incentive-compatible contracts for the sale of information. *Review of Financial Studies*, 15(4), 987-1003.

Cao, H. Henry, Richard K. Lyons, and Martin DD Evans. (2003). Inventory information. *Journal of Business*, 79, 325–364.

Cespa, Giovanni. (2008). Information sales and insider trading with long-lived information. *Journal of Finance*, 63(2), 639-672.

Chen, Hailiang, Prabuddha De, Yu Jeffrey Hu, and Byoung-Hyoun Hwang. (2014). Wisdom of crowds: The value of stock opinions transmitted through social media. *Review of Financial Studies*, 27(5), 1367-1403.

Cohen, Lauren, Andrea Frazzini, and Christopher Malloy. (2008). The small world of investing: Board connections and mutual fund returns. *Journal of Political Economy*, 116(5), 951-979.

Cohen, Lauren, Andrea Frazzini, and Christopher Malloy. (2010). Sell-side school ties. *Journal of Finance*, 65(4), 1409-1437.

Cohen, Lauren, and Christopher J. Malloy. (2010). The power of alumni networks. *Harvard Business Review* 88, no. 10.

Das, Sanjiv R., and Mike Y. Chen. (2007). Yahoo! for Amazon: Sentiment extraction from small talk on the web. *Management science*, 53(9), 1375-1388.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. (1990). Noise trader risk in financial markets. *Journal of Political Economy*, 98(4), 703-738.

DeMarzo, Peter M., Dimitri Vayanos, and Jeffrey Zwiebel. (2003). Persuasion bias, social influence, and unidimensional opinions. *Quarterly Journal of Economics*, 118(3), 909-968.

Estellés-Arolas, Enrique, and Fernando González-Ladró-De-Guevara. (2012). Towards an integrated crowdsourcing definition. *Journal of Information Science*, 38(2), 189-200.

Farboodi, Maryam, and Laura Veldkamp (2020). Long run growth of financial data technology. Forthcoming at *American Economic Review.*

Fishman, Michael J., and Kathleen M. Hagerty (1995). The incentive to sell financial market information. *Journal of Financial Intermediation*, 4(2), 95-115.

Ganguli, Jayant Vivek, and Liyan Yang. (2009). Complementarities, multiplicity, and supply information. *Journal of the European Economic Association*, 7(1), 90-115.

García, Diego, and Francesco Sangiorgi. (2011). Information sales and strategic trading. *Review of Financial Studies*, 24(9), 3069-3104.

Glode, Vincent, Christian C. Opp, and Xingtan Zhang(2018). Voluntary disclosure in bilateral transactions. *Journal of Economic Theory*, 175, 652-688.

Grossman, Sanford J. (1981). The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics*, 24(3), 461-483.

Grossman, Sanford J., and Oliver D. Hart. (1980). Disclosure laws and takeover bids. *Journal of Finance*, 35(2), 323-334.

Han, Bing, and David A. Hirshleifer. (2016). Social transmission bias and active investing. Working Paper, University of Toronto and University of California, Irvine.

Hong, Harrison, Jeffrey D. Kubik, and Jeremy C. Stein. (2005). Thy neighbor's portfolio: Word-of-mouth effects in the holdings and trades of money managers. *Journal of Finance*, 60(6), 2801-2824.

Indjejikian, R., Lu, H., and Yang, L. (2014). Rational information leakage. *Management Science*, 60(11), 2762-2775.

Jylhä, Petri, Kalle Rinne, and Matti Suominen. (2014). Do hedge funds supply or demand liquidity?. *Review of Finance*, 18(4), 1259-1298.

Kaniel, Ron, Gideon Saar, and Sheridan Titman. (2008). Individual investor trading and stock returns. *Journal of Finance*, 63(1), 273-310.

Kim, Oliver, and Robert E. Verrecchia. (1991). Trading volume and price reactions to public announcements. *Journal of Accounting Research* 29(2): 302-321.

Kovbasyuk, Sergey, and Marco Pagano (2015). Advertising arbitrage. Working Paper, Einaudi Institute for Economics and Finance (EIEF) and University of Naples Federico II.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica* 1315-1335.

Lee, S. (2013). Active investment, liquidity externalities, and markets for information. Working Paper, Santa Clara University.

Liu, Ying. (2017). Why do large investors disclose their information? Working Paper, Shanghai University of Finance and Economics.

Ljungqvist, Alexander, and Wenlan Qian. (2016). How constraining are limits to arbitrage?. *Review of Financial Studies*, 29(8), 1975-2028.

Luo, Patrick. (2018). Talking Your Book: Evidence from Stock Pitches at Investment Conferences. Working Paper, Harvard University.

Marmora, Paul, and Oleg Rytchkov (2018). Learning about noise. *Journal of Banking and Finance*, 89, 209-224.

Shiller, Robert J., and John Pound. (1986). Survey evidence on diffusion of interest among institutional investors (No. 794). Cowles Foundation for Research in Economics, Yale University.

Tumarkin, Robert, and Robert F. Whitelaw. (2001). News or noise? Internet postings and stock prices. *Financial Analysts Journal*, 57(3), 41-51.

Madrigal, Vicente.(1996). Non-fundamental speculation. *Journal of Finance*, 51(2), 553-578.

Milgrom, Paul R. (1981). Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 380-391.

Mitchell, Mark, and Todd Pulvino. (2012). Arbitrage crashes and the speed of capital. *Journal of Financial Economics*, 104(3), 469-490.

Myatt, David P., and Chris Wallace. (2012). Endogenous information acquisition in coordination games. *Review of Economic Studies*, 79(1), 340-374.

Naik, Narayan Y. (1997). Multi-period information markets. *Journal of Economic Dynamics and Control*, 21(7), 1229-1258.

Pasquariello, Paolo, and Yifei Wang. Speculation with information disclosure. (2016) Working Paper, University of Michigan.

Schmidt, Daniel. (2018). Stock market rumors and credibility. Forthcoming at *Review of Financial Studies*.

Van Bommel, Jos. (2003). Rumors. *Journal of Finance*, 58(4), 1499-1520.

Verrecchia, Robert E. (1982). Information acquisition in a noisy rational expectations economy. *Econometrica* 1415-1430.

Vives, Xavier. (2010). Information and learning in markets: the impact of market microstructure. Princeton University Press.

Zuckerman, Roy. (2011). Synchronized arbitrage and the value of public announcements. Working Paper, Rutgers Business School.