APPENDIX - FOR ONLINE PUBLICATION

Appendix: Back to the 1980s or Not? The Drivers of Inflation and Real Risks in Treasury Bonds

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A Supply Side Microfoundations

A.1 Final good

A final consumption good is produced by a representative perfectly competitive firm from a continuum of differentiated goods $Y_{i,t}$:

$$Y_t = \left(Y_{i,t}^{\frac{\epsilon_p - 1}{\epsilon_p}} \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}. \tag{A1}$$

The constant $\epsilon_p > 1$ is the elasticity of substitution across intermediate goods. The resulting demand for the differentiated good i is downward-sloping in its product price $P_{i,t}$:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_p}. \tag{A2}$$

The aggregate price level is given by

$$P_t = \left(\int_0^1 P_{i,t}^{-(\epsilon_p - 1)} di \right)^{-\frac{1}{\epsilon_p - 1}}. \tag{A3}$$

A.2 Intermediate good producers

Intermediate goods firm i produces according to a Cobb-Douglas production function with constant returns to scale

$$Y_{i,t} = A_t N_{i,t}, \tag{A4}$$

where productivity equals A_t and N_t is the supply of the aggregate labor index. Each firm takes the downward-sloping demand schedule as given (A3) and may therefore choose a different amount of the aggregate labor index. With the final good equation (A1) aggregate output equals

$$Y_t = A_t N_t \tag{A5}$$

where

$$N_t = \int_0^1 N_{i,t} di. \tag{A6}$$

The aggregate resource constraint is simple because there is no real investment and consumption equals output:

$$C_t = Y_t. \tag{A7}$$

Following Lucas (1988) we assume that productivity depends on past skills gained by all agents, and depends on past market labor, n_{t-1}:

$$a_t \;\; = \;\; \nu + a_{t-1} + (1-\phi)n_{t-1}, \tag{A8}$$

where $0 \leq \phi \leq 1$ and $\nu > 0$ are constants. The assumption (A8) ensures that potential output increases with past output. The process(A8) can equivalently be interpreted as a simple endogenous capital stock, similarly to Woodford (2003, Chapter 5), if a fixed proportion of market labor each period is used to produce investment goods with a constant-returns-to-scale technology, and the total amount of labor is scaled accordingly.

Intermediate firm i' real profit in period t equals

$$Pr_{i,t} \;\; = \;\; \frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}N_{i,t}, \tag{A9}$$

subject to the production function (A4), demand for differentiated goods (A2), and taking the wage W_t as given.

A.3 Employment agency

There is a continuum of monopolistically competitive households, each of which supplies a differentiated labor service, $L_{h,t}$, to the production sector. A representative employment agency aggregates households' labor hours according to a CES production technology with elasticity of substitution $\epsilon_w > 1$:

$$N_t \;\; = \;\; \left(\int_0^1 L_{h,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{A10}$$

The agency produces the aggregate labor index, N_t, taking each household's wage rate, $W_{h,t}$ as given, and then sells it to the production sector at the unit cost W_t. The profit maximization of the employment agency is:

$$max_{L_{h,t}} W_t \left(\int_0^1 L_{h,t}^{\epsilon_w - 1}\epsilon_w\right) - \int_0^1 W_{h,t}L_{h,t}dh, \tag{A11}$$

which yields the following demand schedule for the labor hours of household h:

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w} N_t. \tag{A12}$$

The wage index faced by intermediary producers is then given by

$$W_t = \left(\int_0^1 W_{h,t}^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}. \tag{A13}$$

A.4 Labor-leisure choice

Following the classic model of Greenwood et al. (1988), we assume that total consumption consists of a combination of market consumption and home production, given by:

$$C_{h,t}^{home} = A_t \left(1 - \frac{L_{h,t}^{1+\eta}}{1+\eta}\right) \tag{A14}$$

Home production has decreasing returns to scale as in Campbell and Ludvigson (2001), and the parameter η determines the elasticity of market labor supply. Household h's utility depends on market and home good consumption and the corresponding external habit levels H_t and H_t^{home}:

$$U_{h,t} = \frac{\left((C_{h,t} - H_t) + \left(C_{h,t}^{home} - H_t^{home}\right)\right)^{1-\gamma} - 1}{1-\gamma} \tag{A15}$$

We assume that home good habits are shaped by the aggregate consumption of home goods, so $H_t^{home} = C_t^{home}$ and in equilibrium home goods drop out of the utility function because all households end up choosing the same labor supply in equilibrium. Home production nonetheless matters for the wage-setting first-order condition, which depends on the marginal change in utility from choosing an off-equilibrium path labor supply. External market habit is described by the surplus consumption dynamics in the main paper.

A.5 Price- and wage-setting

We consider the simplified case with flexible product prices but sticky wages. Wage-setting frictions take the form of Rotemberg (1982). Specifically, we assume that wage-setters face a quadratic cost if they raise wages faster than past inflation. The indexing to past inflation is analogous to the indexing assumption in Smets and Wouters (2007) and Christiano et al.

(2005). The cost of re-setting wages for household h in terms of aggregate output equals

$$Cost^h = \frac{\gamma_w}{2}\left(\frac{W_{h,t}}{W_{h,t-1}}/\frac{W_{t-1}}{W_{t-2}} - 1\right)^2 Y_t. \tag{A16}$$

We assume that wage-setting costs get rebated to households lump-sum, i.e. aggregate consumption is unaffected.

A.6 Profit first-order condition

Because product prices are flexible, intermediate firm i's profit becomes

$$Pr_{i,t} = Y_t\left(\left(\frac{P_{i,t}}{P_t}\right)^{-(\epsilon_p-1)} - \frac{W_t}{P_t A_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_p}\right). \tag{A17}$$

Taking the first-order condition with respect to the relative price $\frac{P_{i,t}}{P_t}$ gives

$$\frac{P_{i,t}}{P_t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{W_t}{P_t A_t}. \tag{A18}$$

Because in equilibrium all firms end up choosing the same price, we have that the real wage equals

$$\frac{W_t}{P_t} = \frac{\epsilon_p - 1}{\epsilon_p}A_t. \tag{A19}$$

This means that due to partially monopolistic competition the real wage is compressed by a constant fraction relative to productivity and equilibrium profits of intermediary i are exactly proportional to aggregate output:

$$Pr_{i,t} = \frac{1}{\epsilon_p}Y_t. \tag{A20}$$

This is good because a consumption claim is the same as a claim to firm profits.

A.7 Wage-setting first-order condition with flexible wages

To derive the wage-setting first-order condition, we first start by understanding what happens if wages are flexible. In this case, the first-order condition equals:

$$
\begin{aligned}
0 &= \frac{d\left(C_{h,t}+C_{h,t}^{home}\right)}{d\left(W_{h,t}/W_t\right)} && \text{(A21)}\\
&= \frac{d}{d\left(W_{h,t}/W_t\right)}\left[\frac{W_{h,t}}{P_t}\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w}N_t-\frac{A_t}{1+\eta}\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w(1+\eta)}N_t^{(1+\eta)}\right], && \text{(A22)}\\
&= \left[(-\epsilon_w+1)\frac{W_t}{P_t}N_t\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w}+\epsilon_w A_t\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w(1+\eta)-1}N_t^{(1+\eta)}\right] && \text{(A23)}
\end{aligned}
$$

Because all wage-setters choose the same flexible-wage wage, we can set $W_{h,t} = W_t$. It then follows that the flexible-wage real wage increases proportionately with productivity and increases with the total amount of labor supplied

$$
\begin{aligned}
\frac{W_t^{flex}}{P_t} &= \frac{\epsilon_w}{\epsilon_w-1}A_tN_t^{\eta}, && \text{(A24)}\\
&= \frac{\epsilon_w}{\epsilon_w-1}A_t^{1-\eta}Y_t^{\eta} && \text{(A25)}
\end{aligned}
$$

A.8 Sticky wage first-order condition

In the derivation of the wage Phillips curve we use the operator $\tilde{E}_t$ to denote the partially adaptive inflation expectations of wage-setters. With the quadratic wage-setting cost (A14) the first-order condition for wage-setting becomes

$$
\begin{aligned}
0 = {} & \frac{d\left(C_{h,t}+C_{h,t}^{home}\right)}{d\left(\frac{W_{h,t}}{W_t}\right)}-\gamma_w\left(\frac{W_{h,t}}{W_{h,t-1}}\frac{W_{t-2}}{W_{t-1}}-1\right)\frac{W_t}{W_{h,t-1}}\frac{W_{t-2}}{W_{t-1}}Y_t\\
&+\gamma_w\tilde{E}_tM_{h,t+1}\left(\frac{W_{h,t+1}}{W_{h,t}}\frac{W_{t-1}}{W_t}-1\right)\frac{W_{h,t+1}}{W_{h,t}}\frac{W_t}{W_{h,t}}\frac{W_{t-1}}{W_t}Y_{t+1},
\end{aligned}
\tag{A26}
$$

Since there is symmetry (i.e. all households face the same problem), we can drop the h index when solving for the aggregate wage.

A.9 Log-linearizing the first-order wage-setting condition

Denoting the flexible wage steady-state output by $\bar{Y}_t$, we have that

$$
\bar{Y}_t = A_t\bar{N}, \tag{A27}
$$

where the flexible-wage labor supply solves

$$\frac{\epsilon_p - 1}{\epsilon_p} = \frac{\epsilon_w}{\epsilon_w - 1}\bar{N}^{\eta}. \tag{A28}$$

Using lower case for logs and hats to denote deviations from the flexible-wage equilibrium, the log output gap equals

$$\begin{aligned} x_t \equiv \hat{y}_t &= n_t - \bar{n}, && \text{(A29)} \\ &= \hat{n}_t. && \text{(A30)} \end{aligned}$$

The steady-state stochastic discount factor equals

$$\bar{M}_{t,t+1} = \beta exp\left(-\gamma g\right). \tag{A31}$$

For convenience we define the constant

$$\beta_g \equiv \beta exp(-(\gamma - 1)g). \tag{A32}$$

Letting $\pi^w_t = \log \frac{W_t}{W_{t-1}}$ denote nominal log wage inflation and taking a first-order approximation around $\pi^w = 0$, expression (A26) simplifies to:

$$\begin{aligned} 0 &= (-\epsilon_w + 1)\frac{W_t}{P_t}N_t + \epsilon_w A_t N_t^{(1+\eta)} - \gamma_w\left(\pi^w_t - \pi^w_{t-1}\right)Y_t \\ &\quad + \gamma_w Y_t \tilde{E}_t M_{t+1}\left(\pi^w_{t+1} - \pi^w_t\right)\frac{Y_{t+1}}{Y_t} && \text{(A33)} \end{aligned}$$

Re-arranging:

$$\begin{aligned} \epsilon_w - 1 &= \epsilon_w A_t \frac{P_t}{W_t}N_t^{\eta} - \gamma_w\left(\pi^w_t - \pi^w_{t-1}\right)\frac{P_t}{W_t}\frac{Y_t}{N_t} \\ &\quad + \beta\gamma_w \frac{P_t}{W_t}\frac{Y_t}{N_t}\tilde{E}_t M_{t+1}\left(\pi^w_{t+1} - \pi^w_t\right)\frac{Y_{t+1}}{Y_t}, && \text{(A34)} \\ &= \epsilon_w A_t \frac{P_t}{W_t}N_t^{\eta} - \gamma_w\left(\pi^w_t - \pi^w_{t-1}\right)\frac{P_t}{W_t}\frac{Y_t}{N_t} \\ &\quad + \gamma_w \frac{P_t}{W_t}\frac{Y_t}{N_t}\beta_g \tilde{E}_t\left(\pi^w_{t+1} - \pi^w_t\right), && \text{(A35)} \end{aligned}$$

where in the last step we dropped second-order terms in M_{t+1} and output growth interacted with wage inflation. We next substitute the production function into (A35):

$$(\epsilon_w - 1)\frac{W_t}{A_t P_t} = \epsilon_w N_t^{\eta} - \gamma_w\left(\pi_t^w - \pi_{t-1}^w\right) + \beta^g \gamma_w \tilde{E}_t\left(\pi_{t+1}^w - \pi_t^w\right), \tag{A36}$$

$$\tag{A37}$$

giving the wage Phillips curve

$$\pi_t^w = \frac{1}{1+\beta_g}\pi_{t-1}^w + \frac{\beta^g}{1+\beta_g}\tilde{E}_t\pi_{t+1}^w + \gamma_w^{-1}\left(\epsilon_w N_t^{\eta} - (\epsilon_w - 1)\frac{W_t}{A_t P_t}\right). \tag{A38}$$

Note that the term in parentheses is the wedge between the real productivity-adjusted wage and workers' productivity-adjusted disutility of labor. Because we have flexible product prices the real productivity-adjusted wage is constant and we can substitute in from (A19):

$$\pi_t^w = \frac{1}{1+\beta_g}\pi_{t-1}^w + \frac{\beta^g}{1+\beta_g}\tilde{E}_t\pi_{t+1}^w + \gamma_w^{-1}\left(\epsilon_w N_t^{\eta} - (\epsilon_w - 1)\frac{\epsilon_p - 1}{\epsilon_p}\right) \tag{A39}$$

In the flexible-wage equilibrium the term in parentheses is zero, giving the first-order log-linearization

$$\begin{aligned}\epsilon_w N_t^{\eta} - (\epsilon_w - 1)\frac{\epsilon_p - 1}{\epsilon_p} &= \epsilon_w \bar{N}^{\eta} exp(\eta \hat{n}_t) - (\epsilon_w - 1)\frac{\epsilon_p - 1}{\epsilon_p}, &\text{(A40)}\\ &\approx \epsilon_w \bar{N}^{\eta}\eta \hat{n}_t, &\text{(A41)}\\ &= \epsilon_w \bar{N}^{\eta}\eta \hat{y}_t &\text{(A42)}\end{aligned}$$

We therefore obtain the standard log-linearized wage Phillips curve

$$\pi_t^w = \frac{1}{1+\beta_g}\pi_{t-1}^w + \frac{\beta^g}{1+\beta_g}\tilde{E}_t\pi_{t+1}^w + \kappa \hat{y}_t, \tag{A43}$$

where the constant κ equals

$$\kappa = \gamma_w^{-1}\epsilon_w \bar{N}^{\eta}\eta \tag{A44}$$

Substituting in the adaptive inflation expectations assumption

$$\tilde{E}_t\pi_{t+1}^w = (1-\zeta)E_t\pi_{t+1}^w + \zeta\pi_{t-1}^w, \tag{A45}$$

gives the wage Phillips curve

$$\pi_t^w = \rho^{\pi}\pi_{t-1}^w + f^{\pi}E_t\pi_{t+1}^w + \kappa \hat{y}_t, \tag{A46}$$

where

$$
\begin{aligned}
\rho^{\pi} &= \frac{1}{1+\beta_g} + \zeta - \frac{1}{1+\beta_g}\zeta, && \text{(A47)}\\
f^{\pi} &= 1-\rho^{\pi}. && \text{(A48)}
\end{aligned}
$$

Phillips curve shocks to (A46), $v_{\pi,t}$, arise from making the degree of monopolistic wage-setting frictions ϵ_w or the marginal cost of providing labor outside the home η time-varying.

A.10 Price inflation

Product prices equal

$$
P_t = \frac{\epsilon_p}{\epsilon_p - 1}\frac{W_t}{A_t}, \tag{A49}
$$

so log price inflation equals (up to a constant)

$$
\begin{aligned}
\pi_t^p &= \pi_t^w - \Delta a_t, && \text{(A50)}\\
&= \pi_t^w - (1-\phi)\hat{y}_{t-1}, && \text{(A51)}
\end{aligned}
$$

where the log deviation of real GDP from potential is the output gap, i.e. $x_t = \hat{y}_t$.

B Solution

In the absence of demand shocks the lagged output gap does not enter as a separate state variable because x_{t-1} can be expressed as a linear combination of the time-t state vector Y_t. This is no longer possible in the presence demand shocks, thereby adding x_{t-1} as a new state variable for asset prices relative to Campbell, Pflueger and Viceira (2020).

B.1 Solving for macroeconomic dynamics

The full macroeconomic dynamics are determined by the Euler equation, the wage Phillips curve (A46) and the monetary policy rule, as well as the short-rate Fisher equation $r_t = i_t - E_t\pi_{t+1}^p$, the relationship between price and wage inflation (A51). The Euler equation is given by

$$
x_t = f^x E_t x_{t+1} + \rho^x x_{t-1} - \psi\left(i_t - E_t\pi_{t+1}^p\right) + v_{x,t}, \tag{A52}
$$

where

$$\rho^x = \frac{\theta_2}{\phi - \theta_1}, \tag{A53}$$

$$f^x = \frac{1}{\phi - \theta_1}, \tag{A54}$$

$$\psi = \frac{1}{\gamma(\phi - \theta_1)}, \tag{A55}$$

$$\theta_2 = \phi - 1 - \theta_1. \tag{A56}$$

The wage Phillips curve is given by

$$\pi_t^w = \rho^\pi \pi_{t-1}^w + f^\pi E_t \pi_{t+1}^w + \kappa x_t + v_{\pi,t}, \tag{A57}$$

The monetary policy rule is given by

$$i_t = \rho^i i_{t-1} + (1 - \rho^i) \left(\gamma^x x_t + \gamma^\pi \pi_t^p \right) + v_{i,t}, \tag{A58}$$

where $v_{x,t} = \frac{1}{\gamma(\phi-\theta_1)} \xi_t$ denotes the demand shock; $v_{\pi,t}$ is the supply shock; and $v_{i,t}$ is the monetary policy shock.

We want to find a solution of the form

$$Y_t = B Y_{t-1} + \Sigma v_t, \tag{A59}$$

where the matrix B is $[3 \times 3]$, the matrix Σ is $[3 \times 3]$, and we work with the state vector

$$Y_t = [x_t, \pi_t^w, i_t]', \tag{A60}$$

and the shock vector

$$v_t = [v_{x,t}, v_{\pi,t}, v_{i,t}]'. \tag{A61}$$

Using the relationship (A51), we can write the macroeconomic dynamics in terms of the state vector Y_t:

$$Y_{1,t} = f^x E_t Y_{1,t+1} + \rho^x Y_{1,t-1} - \psi \left(Y_{3,t} - E_t Y_{2,t+1} + (1 - \phi) Y_{1,t} \right) + v_{x,t}, \tag{A62}$$

$$Y_{2,t} = f^\pi E_t Y_{2,t+1} + \rho^\pi Y_{2,t-1} + \kappa Y_{1,t} + v_{\pi,t}, \tag{A63}$$

$$Y_{3,t} = \rho^i Y_{3,t-1} + (1 - \rho^i) \left(\gamma^x Y_{1,t} + \gamma^\pi Y_{2,t} - \gamma^\pi (1 - \phi) Y_{1,t-1} \right) + v_{i,t}. \tag{A64}$$

We can write this in matrix form:

$$0 = FE_tY_{t+1} + GY_t + HY_{t-1} + Mv_t,$$

where the matrices F, G and H are given by

$$F = \begin{bmatrix} \frac{f^x}{1+\psi(1-\phi)} & \frac{\psi}{1+\psi(1-\phi)} & 0 \\ 0 & f^\pi & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} -1 & 0 & -\frac{\psi}{1+\psi(1-\phi)} \\ \kappa & -1 & 0 \\ (1-\rho^i)\gamma^x & (1-\rho^i)\gamma^\pi & -1 \end{bmatrix},$$

$$H = \begin{bmatrix} \frac{\rho^x}{1+\psi(1-\phi)} & 0 & 0 \\ 0 & \rho^\pi & 0 \\ -(1-\rho^i)(1-\phi)\gamma^\pi & 0 & \rho^i \end{bmatrix}.$$

The matrix M is $[3 \times 3]$ and equals:

$$M = \begin{bmatrix} \frac{1}{1+\psi(1-\phi)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A65}$$

Following Uhlig (1999), we solve for the generalized eigenvectors and eigenvalues of the matrix Ξ with respect to the matrix Δ, where

$$\Xi = \begin{bmatrix} -G & -H \\ I_3 & 0_3 \end{bmatrix}, \tag{A66}$$

$$\Delta = \begin{bmatrix} F & 0_3 \\ 0_3 & I_3 \end{bmatrix} \tag{A67}$$

To obtain a solution, we then pick three generalized eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with generalized eigenvectors $[\lambda z_1', z_1']'$, $[\lambda_2 z_2', z_2]'$, and $[\lambda_3 z_3', z_3']'$. We denote the diagonal matrix of these eigenvalues by $\Lambda = diag\,(\lambda_1, \lambda_2, \lambda_3)$, and the matrix of the lower $[3 \times 1]$ portion of the eigenvectors by $\Omega = [z_1, z_2, z_3]$. The corresponding solutions for B and Σ are then given by:

$$B = \Omega\Lambda\Omega^{-1}, \tag{A68}$$

$$\Sigma = -\left[FB + G\right]^{-1} M. \tag{A69}$$

For both our calibrations, there exist exactly three generalized eigenvalues with absolute value less than one, and we pick the non-explosive solution corresponding to these three eigenvalues.

B.2 Rotated state vector

Our state space for solving for asset prices is five-dimensional: It consists of $\tilde{Z}_t$, which a scaled version of Y_t, the surplus consumption ratio relative to steady-state $\hat{s}_t$, and the lagged output gap x_{t-1}.

We next describe the definition of $\tilde{Z}_t$. To simplify the numerical implementation of the asset pricing recursions, we require that shocks to the scaled state vector $\tilde{Z}_t$ are independent standard normal and that the first dimension of the scaled state vector is perfectly correlated with consumption innovations. This rotation facilitates the numerical analysis, because it is easier to integrate over independent random variables. Aligning the first dimension of the scaled state vector with output gap innovations (and hence surplus consumption innovations) helps, because it allows us to use a finer grid to integrate numerically over this crucial dimension over which asset prices are most non-linear.

If the scaled state vector equals $\tilde{Z}_t = AY_t$ for some invertible matrix A, the dynamics of $\tilde{Z}_t$ are given by:

$$\tilde{Z}_t = AY_t, \tag{A70}$$

$$\tilde{Z}_{t+1} = \underbrace{ABA^{-1}}_{\tilde{B}}\tilde{Z}_t + \underbrace{A\Sigma v_{t+1}}_{\epsilon_{t+1}}. \tag{A71}$$

We hence want a matrix, A, such that

$$Var\left(\epsilon_{t+1}\right) = A\Sigma\Sigma_v\Sigma' A', \tag{A72}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{A73}$$

Finding such a matrix A should in general be possible, because the matrix M and therefore $\Sigma\Sigma_v\Sigma'$ generally have rank three. We require that the first dimension of ϵ_{t+1} is perfectly correlated with the consumption shock. We can therefore find the three rows of A using the following steps:

1. Set $A_1 = \frac{e_1}{\sqrt{e_1\Sigma\Sigma_v\Sigma' e_1'}}$.

2. We use the MATLAB function *null* to compute the null space of $A_1\Sigma\Sigma_v\Sigma'$. Let n_2 denote the first vector in $null\,(A_1\Sigma\Sigma_v\Sigma')$. We then define the second row of A as the normalized version of n_2:

$$A_2 \;=\; \frac{n_2}{\sqrt{n_2\Sigma\Sigma_v\Sigma' n_2'}}. \tag{A74}$$

3. Let n_3 denote the first vector in $null\,(A_1\Sigma\Sigma_v\Sigma', A_2\Sigma\Sigma_v\Sigma')$. We then define the third row of A as the normalized version of n_3:

$$A_3 \;=\; \frac{n_3}{\sqrt{n_3\Sigma\Sigma_v\Sigma' n_3'}}. \tag{A75}$$

It is then straightforward to verify that equation (A73) holds for

$$A \;=\; \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}. \tag{A76}$$

B.3 Asset pricing recursions

Before deriving the recursions for the numerical asset pricing computations, we derive a convenient form for the dynamics of the log surplus consumption ratio. We use e_i to denote a row vector with 1 in position i and zeros elsewhere. The matrix

$$\Sigma_M \;=\; e_1\Sigma \tag{A77}$$

denotes the loading of consumption innovations onto the vector of shocks v_t, where e_1 is a basis vector with a one in the first position and zeros everywhere else. The volatility of consumption surprises equals:

$$\sigma_c^2 \;=\; \Sigma_M\Sigma_v\Sigma_M'. \tag{A78}$$

To simplify notation, we define $\hat{s}_t$ as the log deviation of surplus consumption from its steady state. The dynamics of $\hat{s}_t$ are:

$$\hat{s}_t \;=\; s_t - \bar{s}, \tag{A79}$$

$$\hat{s}_t \;=\; \theta_0\hat{s}_{t-1} + \theta_1 x_{t-1} + \theta_2 x_{t-2} + \lambda(\hat{s}_{t-1})\varepsilon_{c,t}, \tag{A80}$$

where with an abuse of notation we write:

$$\lambda(\hat{s}_t) = \lambda_0\sqrt{1-2\hat{s}_t}-1, \hat{s}_t \leq s_{max}-\bar{s}, \tag{A81}$$

$$\lambda(\hat{s}_t) = 0, \hat{s}_t \geq s_{max}-\bar{s}. \tag{A82}$$

The steady-state surplus consumption sensitivity equals:

$$\lambda_0 = \frac{1}{\bar{S}}. \tag{A83}$$

In our calculations of bond prices, we repeatedly substitute out expected log SDF growth, which equals:

$$\begin{aligned} E_t\left[m_{t+1}\right] &= \log\beta - \gamma E_t\Delta\hat{s}_{t+1} - \gamma E_t\Delta c_{t+1}, & \text{(A84)} \\ &= -r_t + \xi_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t), & \text{(A85)} \\ &= -(e_3 - e_2 B + (1-\phi)e_1)Y_t + \xi_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t) & \text{(A86)} \end{aligned}$$

We often combine this with $r_t = \bar{r} + (e_3 - e_2B)Z_t$ and $\hat{r}_t = (e_3 - e_2B)Z_t$.

Including the constant, consumption growth is given by:

$$\Delta c_{t+1} = g + x_{t+1} - \phi x_t. \tag{A87}$$

The steady state real short-term interest rate at $x_t = 0$ and $s_t = \bar{s}$ is the same as in Campbell and Cochrane (1999):

$$\bar{r} = \gamma g - \frac{1}{2}\gamma^2\sigma_c^2/\bar{S}^2 - \log(\beta). \tag{A88}$$

The updating rule for the log surplus consumption ratio can then be written in terms of the state variables as:

$$\hat{s}_{t+1} = \theta_0\hat{s}_t + \theta_1 e_1 A^{-1}\tilde{Z}_t + \theta_2 x_{t-1} + \lambda(\hat{s}_t)\varepsilon_{c,t+1}. \tag{A89}$$

B.3.1 Recursion for zero-coupon consumption claims

We now derive the recursion for zero-coupon consumption claims in terms of state variables $\tilde{Z}_t$, $\hat{s}_t$ and x_{t-1}. Let P^c_{nt}/C_t denote the price-dividend ratio of a zero-coupon claim on consumption at time $t+n$. The outline of our strategy here is that we first derive an analytic expression for the price-dividend ratio for P^c_{1t}/C_t. For $n \geq 1$ we guess and verify recursively that there exists a function $F_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$, such that

$$\frac{P^c_{nt}}{C_t} = F_n\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{A90}$$

The ex-dividend price-consumption ratio for a claim to all future consumption is then given by

$$\frac{P_t}{C_t} = F\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right), \tag{A91}$$

where we define

$$F\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right) = \sum_{n=1}^{\infty} F_n\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{A92}$$

We now derive the recursion of zero-coupon consumption claims in terms of state variables $\tilde{Z}_t$ and $\hat{s}_t$. The one-period zero coupon price-consumption ratio solves:

$$\frac{P^c_{1,t}}{C_t} = E_t\left[\frac{M_{t+1}C_{t+1}}{C_t}\right] \tag{A93}$$

We simplify

$$\begin{aligned}\frac{M_{t+1}C_{t+1}}{C_t} &= \beta\exp(-\gamma E_t\Delta\hat{s}_{t+1} - (\gamma-1)E_t\Delta c_{t+1} \\ &\quad -\gamma(\hat{s}_{t+1} - E_t s_{t+1}) - (\gamma-1)(c_{t+1} - E_t c_{t+1})).\end{aligned}$$

Using the notation $f_n = \log(F_n)$, this gives the log one-period price-consumption ratio as:

$$\begin{aligned}f_1\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right) &= \log\beta - \gamma[(\theta_0-1)\hat{s}_t + \theta_1 x_t + \theta_2 x_{t-1}] - (\gamma-1)[g + E_t x_{t+1} - \phi x_t] \\ &\quad +\frac{1}{2}\left(\gamma\lambda(\hat{s}_t) + (\gamma-1)\right)^2\sigma_c^2, \qquad \text{(A94)} \\ &= \log\beta - (\gamma-1)g - e_1[(\gamma\theta_1 - \gamma\phi + \phi)I + (\gamma-1)B]A^{-1}\tilde{Z}_t \\ &\quad -\gamma(\theta_0-1)\hat{s}_t - \gamma\theta_2 x_{t-1} + \frac{1}{2}\left(\gamma\lambda(\hat{s}_t) + (\gamma-1)\right)^2\sigma_c^2 \qquad \text{(A95)}\end{aligned}$$

Next, we solve for f_n, $n \geq 2$ iteratively. Note that:

$$\frac{P^c_{nt}}{C_t} = \mathbb{E}_t\left[\frac{M_{t+1}C_{t+1}}{C_t}\frac{P^c_{n-1,t+1}}{C_{t+1}}\right] = \mathbb{E}_t\left[\frac{M_{t+1}C_{t+1}}{C_t}F_{n-1}\left(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t\right)\right] \tag{A96}$$

This gives the following expression for f_n:

$$\begin{aligned} f_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \log\Big[\mathbb{E}_t\Big[\exp\Big(\log\beta - (\gamma-1)g - e_1[(\gamma\theta_1 - \gamma\phi + \phi)I + (\gamma-1)B]A^{-1}\tilde{Z}_t \\ &\quad -\gamma(\theta_0 - 1)\hat{s}_t - \gamma\theta_2 x_{t-1} - (\gamma(1+\lambda(\hat{s}_t)) - 1)\sigma_c\epsilon_{1,t+1} \\ &\quad +f_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\Big]\Big]. \end{aligned} \tag{A97}$$

Here, $\epsilon_{1,t+1}$ denotes the first dimension of the shock ϵ_{t+1}.

B.3.2 Recursion for zero-coupon bond prices

We use $P^{\$}_{n,t}$ and $P_{n,t}$ to denote the prices of nominal and real n-period zero-coupon bonds. The strategy is to develop analytic expressions for one- and two-period bond prices. We then guess and verify recursively that the prices of real and nominal zero-coupon bonds with maturity $n \geq 2$ can be written in the following form:

$$P_{n,t} = B_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}), \tag{A98}$$

$$P^{\$}_{n,t} = B^{\$}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}), \tag{A99}$$

where $B_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$ and $B^{\$}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$ are functions of the state variables. As discussed in the main paper, we assume that the short-term nominal interest rate contains no risk premium, so the one-period log nominal interest rate equals $i_t = r_t + E_t\pi_{t+1}$. Taking account of the constants, one-period bond prices equal:

$$P^{\$}_{1,t} = \exp(-Y_{3,t} - \bar{r}), \tag{A100}$$

$$P_{1,t} = \exp(-Y_{3,t} + \mathbb{E}_t Y_{2,t+1} - (1-\phi)Y_{1,t} - \bar{r}). \tag{A101}$$

We next solve for longer-term bond prices including risk premia. Substituting in (A100) into the bond-pricing recursion gives:

$$P^{\$}_{2,t} = \exp(-\xi_t)\mathbb{E}_t\left[M_{t+1}P^{\$}_{1,t+1}\exp(-Y_{2,t+1} + (1-\phi)Y_{1,t})\right] \tag{A102}$$

$$= \exp(-\xi_t)\mathbb{E}_t\left[M_{t+1}\exp(-Y_{3,t+1} - Y_{2,t+1} + (1-\phi)Y_{1,t} - \bar{r})\right]. \tag{A103}$$

We can now verify that the two-period nominal bond price takes the form (A99):

$$B_2^{\$}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = \exp\left(E_t\left(m_{t+1} - \xi_t - Y_{3,t+1} - Y_{2,t+1} + (1-\phi)Y_{1,t}\right) - \bar{r}\right) \times \mathbb{E}_t\left[\exp\left(\left(-\gamma\left(\lambda(\hat{s}_t)+1\right)\Sigma_M - \underbrace{[(e_2+e_3)\Sigma]}_{v_{\$}}\right)v_{t+1}\right)\right]. \tag{A104}$$

Here, we define the vector $v_{\$}$ to simplify notation. Taking logs, substituting out for $E_t m_{t+1}$, and using the definition for the sensitivity function $\lambda(\hat{s}_t)$, we get:

$$\begin{aligned} b_2^{\$} &= -e_3[I+B]A^{-1}\tilde{Z}_t + \frac{1}{2}v_{\$}\Sigma_v v_{\$}\prime \\ &\quad +\gamma\left(\lambda(\hat{s}_t)+1\right)\Sigma_M\Sigma_v v_{\$}' - 2\bar{r}. \end{aligned} \tag{A105}$$

The closed-form solution for the two-period real bond price becomes

$$P_{2,t} = \exp\left(E_t\left(m_{t+1} - \xi_t - Y_{3,t+1} - (1-\phi)Y_{1,t+1} + Y_{2,t+2}\right) - \bar{r}\right) \times \mathbb{E}_t\left[\exp\left((-\gamma(\lambda(\hat{s}_t)+1)\Sigma_M - \underbrace{(e_3 + (1-\phi)e_1 - e_2 B)\Sigma)}_{v_r}v_{t+1}\right)\right] \tag{A106}$$

We define the vector v_r to simplify notation. Taking logs, substituting out for $E_t m_{t+1}$, and using the definition for $\lambda(\hat{s}_t)$ gives:

$$b_2(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = -v_r\left[I+B\right]A^{-1}\tilde{Z}_t + \frac{1}{2}v_r\Sigma_v v_r' + \gamma\left(\lambda(\hat{s}_t)+1\right)\Sigma_M\Sigma_v v_r' - 2\bar{r}. \tag{A107}$$

For $n \geq 3$, we repeatedly substitute out for $E_t m_{t+1}$ to obtain the following recursion for nominal and real bond prices, respectively:

$$\begin{aligned} B_n^{\$}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \mathbb{E}_t\left[\exp\left(m_{t+1} - \xi_t - Y_{2,t+1} + (1-\phi)Y_{1,t} + b_{n-1}^{\$}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, B^{\$}x_t)\right)\right] \\ &= \mathbb{E}_t\left[\exp\left(-\bar{r} - e_3 A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t)\right.\right. \\ &\quad \left.\left. -\gamma(1+\lambda(\hat{s}_t))\sigma_c\epsilon_{1,t+1} - e_2 A^{-1}\epsilon_{t+1} + b_{n-1}^{\$}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\right)\right]. \end{aligned} \tag{A108}$$

The value function iteration for real bond prices then becomes

$$
\begin{aligned}
B_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \mathbb{E}_t\left[\exp\Big(m_{t+1} - \xi_t + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\right] \\
&= \mathbb{E}_t\left[\exp\Big(-\bar{r} - (e_3 - e_2 B + (1-\phi)e_1)A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t)\right. \\
&\quad \left. -\gamma(1+\lambda(\hat{s}_t))\sigma_c\epsilon_{1,t+1} + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\right]. \qquad \text{(A109)}
\end{aligned}
$$

B.3.3 Computing returns

The log return on the consumption claim equals:

$$
\begin{aligned}
r^c_{t+1} &= \log\left(\frac{P^c_{t+1} + C_{t+1}}{P^c_t}\right), \qquad \text{(A110)} \\
&= \Delta c_{t+1} + \log\left(\frac{1 + \frac{P^c_{t+1}}{C_{t+1}}}{\frac{P^c_t}{C_t}}\right). \qquad \text{(A111)}
\end{aligned}
$$

Real and nominal log bond yields equal:

$$
\begin{aligned}
y_{n,t} &= -\frac{1}{n} b_{n,t}, \qquad \text{(A112)} \\
y^{\$}_{n,t} &= -\frac{1}{n} b^{\$}_{n,t}. \qquad \text{(A113)}
\end{aligned}
$$

Real log bond returns equal:

$$
r_{n,t+1} = b_{n-1,t+1} - b_{n,t}. \qquad \text{(A114)}
$$

Nominal log bond returns equal:

$$
r^{\$}_{n,t+1} = b^{\$}_{n-1,t+1} - b^{\$}_{n,t}. \qquad \text{(A115)}
$$

Real and nominal bond log excess returns then equal:

$$
\begin{aligned}
xr_{n,t+1} &= r_{n,t+1} - r_t, \qquad \text{(A116)} \\
xr^{\$}_{n,t+1} &= r^{\$}_{n,t+1} - i_t. \qquad \text{(A117)}
\end{aligned}
$$

B.3.4 Levered stock prices and returns

We note that the price of the levered equity claim is δP_t^c, so the price-dividend ratio equals:

$$\frac{P_t^\delta}{D_t^\delta} \;=\; \delta \frac{C_t}{D_t^\delta} \frac{P_t^c}{C_t}. \tag{A118}$$

Using the expression

$$D_{t+1}^\delta \;=\; P_{t+1}^c + C_{t+1} - (1-\delta) P_t^c \exp(r_t) - \delta P_t^c, \tag{A119}$$

and

$$P_t^\delta \;=\; \delta P_t^c \tag{A120}$$

gives the gross return on levered stocks:

$$\begin{aligned}
\left(1 + R_{t+1}^\delta\right) &= \frac{D_{t+1}^\delta + P_{t+1}^\delta}{P_t^\delta}, && \text{(A121)}\\
&= \frac{1}{\delta} \frac{P_{t+1}^c + C_{t+1} - (1-\delta) P_t^c \exp(r_t)}{P_t^c}, && \text{(A122)}\\
&= \frac{1}{\delta} \left(1 + R_{t+1}^c\right) - \frac{1-\delta}{\delta} \exp(r_t)\,. && \text{(A123)}
\end{aligned}$$

Log stock excess returns then equal:

$$xr_{t+1}^\delta \;=\; r_{t+1}^\delta - r_t. \tag{A124}$$

To mimic firms' dividend smoothing in the data, we report simulated moments for the price of equities dividend by dividends smoothed over the past 64 quarters:

$$P_t^\delta / \left(\frac{1}{64} (D_t^\delta + D_{t-1}^\delta + \ldots + D_{t-63}^\delta) \right). \tag{A125}$$

B.4 Risk-premium decomposition

We use the superscript ^{rn} for risk-neutral, superscript ^{cf} for cash flow, and ^{rp} for risk premium. Risk-neutral valuations are expected cash flows discounted with the risk-neutral discount factor, given by:

$$M_{t+1}^{rn} \;=\; \exp(-(r_t - \xi_t)). \tag{A126}$$

Note that since we are not interested in risk-neutral bond and stock prices, but only a decomposition of returns, multiplying M^{rn}_{t+1} by a constant discount rate does not matter. For any zero-coupon claim it would shift risk-neutral returns merely by a constant and therefore leave our decomposition into risk-neutral and risk-premium components unaffected. For a claim to all future consumption or stock returns, a constant discount rate could theoretically shift the weights between nearer-term consumption claims and longer-term consumption claims, and therefore change risk-neutral returns. However, since consumption growth is stationary we have found that this makes very little different to risk-neutral stock returns in any of our numerical applications.

B.4.1 Risk-neutral zero-coupon bond prices

We use analogous recursions to solve for risk-neutral bond prices. One-period risk-neutral bond prices are given exactly as before by equations (A100) and (A101). For $n > 1$, we guess and verify that the prices of real and nominal risk-neutral zero-coupon bonds with maturity n can be written in the following form

$$
\begin{aligned}
P^{rn}_{n,t} &= B^{rn}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}), && \text{(A127)} \\
P^{\$,rn}_{n,t} &= B^{\$,rn}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}). && \text{(A128)}
\end{aligned}
$$

for some functions $B^{rn}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$ and $B^{\$,rn}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$.

We derive the two-period risk-neutral nominal bond price analytically:

$$
\begin{aligned}
P^{\$,rn}_{2,t} &= \exp(-\xi_t)\mathbb{E}_t\left[M^{rn}_{t+1}P^{\$,rn}_{1,t+1}\exp(-Y_{2,t+1} + (1-\phi)Y_{1,t})\right] && \text{(A129)} \\
&= \exp(-r_t)\mathbb{E}_t\left[\exp(-Y_{3,t+1} - Y_{2,t+1} + (1-\phi)Y_{1,t} - \bar{r})\right]. && \text{(A130)}
\end{aligned}
$$

We can hence verify that the two-period risk-neutral nominal bond price takes the form (A99)

$$
b^{\$,rn}_2 = -e_3\left[I + B\right]A^{-1}\tilde{Z}_t + \frac{1}{2}v_{\$}\Sigma_v v_{\$}\prime - 2\bar{r} \qquad \text{(A131)}
$$

Here, the vector $v_{\$}$ is identical to the case with risk aversion. Comparing expressions (A131) and (A105) shows that they agree when $\gamma = 0$. We similarly solve for 2-period real bond prices in closed form:

$$P_{2,t}^{rn} = \exp\left(-Y_{3,t} + \mathbb{E}_t Y_{2,t+1} - (1-\phi)Y_{1,t} - \bar{r}\right) \times \exp\left(\mathbb{E}_t\left(-Y_{3,t+1} + \mathbb{E}_{t+1} Y_{2,t+2} + (1-\phi)Y_{1,t+1} - \bar{r}\right)\right) \times \mathbb{E}_t\left[\exp\left(-\underbrace{(e_3 + (1-\phi)e_1 - e_2 B)\Sigma}_{v_r} v_{t+1}\right)\right]. \tag{A132}$$

The vector v_r is again identical to the case with risk aversion. Taking logs gives:

$$b_2^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = -(e_3 + (1-\phi)e_1 - e_2 B)\left[I + B\right] A^{-1}\tilde{Z}_t + \frac{1}{2} v_r \Sigma_v v_r' - 2\bar{r}. \tag{A133}$$

We note that the risk-neutral bond prices (A133) and bond prices with risk aversion (A107) are identical when the utility curvature parameter γ equals zero.

For $n \geq 3$ the n-period risk neutral nominal and real bond prices satisfy the following recursions, respectively:

$$B_n^{\$,rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = \mathbb{E}_t\left[\exp\left(-\bar{r} - e_3 A^{-1}\tilde{Z}_t - e_2 A^{-1}\epsilon_{t+1} + b_{n-1}^{\$}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\right)\right], \tag{A134}$$

$$B_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = \mathbb{E}_t\left[\exp\left(-\bar{r} - (e_3 + (1-\phi)e_1 - e_2 B)A^{-1}\tilde{Z}_t + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\right)\right] \tag{A135}$$

B.4.2 Risk-neutral zero-coupon consumption claims

Next, we derive recursive solutions for the risk-neutral prices of zero-coupon consumption claims. Let $P_{nt}^{c,rn}/C_t$ denote the risk-neutral price-dividend ratio of a zero-coupon claim on consumption at time $t+n$. The risk-neutral price-consumption ratio of a claim to the entire stream of future consumption equals:

$$\frac{P_t^{c,rn}}{C_t} = \sum_{n=1}^{\infty} \frac{P_{nt}^{c,rn}}{C_t}. \tag{A136}$$

For $n \geq 1$, we guess and verify there exists a function $F_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1})$, such that

$$\frac{P_{nt}^{c,rn}}{C_t} = F_n^{rn}\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{A137}$$

We start by deriving the analytic expression for F_1^{rn}. The one-period risk-neutral zero-coupon price-consumption ratio solves

$$\begin{aligned}\frac{P_{1,t}^{c,rn}}{C_t} &= \mathbb{E}_t\left[M_{t+1}^{rn}\frac{C_{t+1}}{C_t}\right] && \text{(A138)}\\ &= \exp(-r_t+u_t)\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] && \text{(A139)}\\ &= \exp(-\gamma\mathbb{E}_t x_{t+1}+\gamma(\phi-\theta_1)x_t-\gamma\theta_2 x_{t-1}-\bar{r})\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] && \text{(A140)}\end{aligned}$$

Using (A87) to substitute for consumption growth, we can derive the following analytic expression for f_1^{rn}:

$$f_1^{rn}(\tilde{Z}_t,\hat{s}_t,x_{t-1}) = g-\bar{r}-e_1[(\gamma\theta_1-\gamma\phi+\phi)I-(1-\gamma)B]A^{-1}\tilde{Z}_t-\gamma\theta_2 x_{t-1}+\frac{1}{2}\sigma_c^2. \tag{A141}$$

Next, we solve for f_n, $n\geq 2$ iteratively:

$$\frac{P_{nt}^{c,rn}}{C_t} = \exp\left(-\gamma\mathbb{E}_t Y_{1,t+1}+\gamma(\phi-\theta_1)Y_{1,t}-\gamma\theta_2 x_{t-1}-\bar{r}\right)\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}F_{n-1}^{rn}\left(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t\right)\right] \tag{A142}$$

This gives the following expression for f_n^{rn}:

$$\begin{aligned}f_n^{rn}(\tilde{Z}_t,\hat{s}_t,x_{t-1}) = \; & \log\left[\mathbb{E}_t\left[\exp\left(-\gamma\mathbb{E}_t Y_{1,t+1}+\gamma(\phi-\theta_1)Y_{1,t}-\gamma\theta_2 x_{t-1}-\bar{r}\right.\right.\right.\\ & +g-\phi Y_{1,t}+\mathbb{E}_t Y_{1,t+1}+\sigma_c\epsilon_{1,t+1}\\ & \left.\left.\left.+f_{n-1}^{rn}(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t)\right)\right]\right]. \qquad \text{(A143)}\end{aligned}$$

Finally, we re-write $f_{n,t}^{rn}$ as an expectation involving $f_{n-1,t+1}^{rn}$, the state variables $\tilde{Z}_t$, and period $t+1$ shocks:

$$\begin{aligned}f_n^{rn}(\tilde{Z}_t,\hat{s}_t,x_{t-1}) = \; & \log\left[\mathbb{E}_t\left[\exp\left(e_1[(1-\gamma)B-(\phi-\gamma(\phi-\theta_1))I]A^{-1}\tilde{Z}_t-\gamma\theta_2 x_{t-1}\right.\right.\right.\\ & \left.\left.\left.+g-\bar{r}+\sigma_c\epsilon_{1,t+1}+f_{n-1}^{rn}(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t)\right)\right]\right]. \qquad \text{(A144)}\end{aligned}$$

B.5 Risk-neutral returns

We plug risk-neutral price-consumption ratios and bond prices into equations (A111) through (A117). This gives risk-neutral returns on the consumption claim, risk-neutral log excess bond returns, and risk-neutral bond yields. We then substitute risk-neutral returns on the consumption claim into (A123)-(A124) to obtain risk-neutral log excess stock returns.

C Microfoundations for bond preference shock

In this Section, I show that the bond preference shock ξ_t can be mapped into either a safety shock, similar to the international finance literature, or expected growth shock in line with the growing literature that expectations of future economic activity are a major driver of stock prices and the economy (e.g. Beaudry and Portier (2006), Bordalo et al. (2020)).

C.1 Safety shock

For simplicity, I follow Bianchi and Lorenzoni (2021) and model private loans as intermediated intermediaries with time-varying convex intermediation costs. A model of government bonds in the utility function as in Krishnamurthy and Vissing-Jorgensen (2012) or Kekre and Lenel (2020) would act similarly, driving a wedge between the loan rate faced by private households and the government bond yield.

Households can borrow or invest in the private loan market and the stock market, but they cannot directly invest in the government bond market. The private loan market is intermediated by a sequence of short-lived intermediaries, who require repayment after one period. Households are able to take loans analogous to n-period nominal or inflation indexed bonds at time t, which require a time $t+1$ nominal repayment of $P^{\$}_{n-1,t+1}$ or $P_{n-1,t+1}exp(\pi_t)$ to the intermediary. If households wish to hold these n-period nominal or inflation-indexed private loans to maturity, they can roll them over using a sequence of n short-lived intermediaries. The assumption of short-lived intermediaries simplifies the exposition, as it ensures that shocks to intermediation capacity are not priced.

Let $Q_{n,t}$ and $Q^{\$}_{n,t}$ denote the quantity of n-period private loans intermediated. Intermediaries are assumed to be able to go long and short in the government bond market and in the private loan market. So if intermediaries are at all risk averse, it is optimal to completely hedge the period $t+1$ cash flows in period $t+1$. Similarly to Bianchi and Lorenzoni (2021), generation t intermediaries are assumed to face convex intermediation costs in the quantity intermediated

$$\frac{1}{\omega_t}\Omega\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right), \tag{A145}$$

where ω_t is a shock to intermediation capacity. The net profits of a generation t intermediaries are given by

$$\sum_{n=1}^{\infty}\left(\hat{P}_{n,t}-P_{n,t}\right)Q_{n,t}+\sum_{n=1}^{\infty}\left(\hat{P}^{\$}_{n,t}-P^{\$}_{n,t}\right)Q^{\$}_{n,t}-\frac{1}{\omega_t}\Omega\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right). \quad \text{(A146)}$$

The intermediary's first-order condition for intermediating the n-period real private loan becomes

$$\hat{P}_{n,t}-P_{n,t} = \frac{1}{\omega_t}\Omega'\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)\hat{P}_{n,t}. \quad \text{(A147)}$$

The intermediary's first-order condition for the n-period nominal private loan becomes

$$\hat{P}^{\$}_{n,t}-P^{\$}_{n,t} = \frac{1}{\omega_t}\Omega'\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)\hat{P}^{\$}_{n,t}. \quad \text{(A148)}$$

Simplifying further to quadratic adjustment costs of the form $\Phi(x)=\frac{c}{2}x^2$ these expressions simplify to

$$P_{n,t} = \left(1-\frac{c\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)}{\omega_t}\right)\hat{P}_{n,t}, \quad \text{(A149)}$$

$$= \left(1-\frac{c\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)}{\omega_t}\right)E_t\left[M_{t+1}P_{n-1,t+1}\right], \quad \text{(A150)}$$

$$P^{\$}_{n,t} = \left(1-\frac{c\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)}{\omega_t}\right)\hat{P}^{\$}_{n,t}, \quad \text{(A151)}$$

$$= \left(1-\frac{c\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)}{\omega_t}\right)E_t\left[M_{t+1}exp(-\pi_t)P^{\$}_{n-1,t+1}\right], \quad \text{(A152)}$$

where the last step follows from households' first order condition with respect to the inflation-indexed and nominal n-period private loan. Defining $exp(-\xi_t)=\left(1-\frac{c\left(\sum_{n=1}^{\infty}\hat{P}_{n,t}Q_{n,t}+\sum_{n=1}^{\infty}\hat{P}^{\$}_{n,t}Q^{\$}_{n,t}\right)}{\omega_t}\right)$, the bond asset pricing recursions (14) in the main paper follow.

C.2 Expected growth shock

An alternative for the bond preference shock is as a shock to expected growth, that is not subsequently not realized. Beaudry and Portier (2006) provide empirical evidence that shocks to expectations about future growth are an important driver of business cycle fluctuations and stock returns. Assume that $v_{a,t}$ is an iid shock to expected productivity growth:

$$E_t \Delta a_{t+1} = (1-\phi)x_t + v_{a,t}. \tag{A153}$$

Here, $v_{a,t}$ is a shock to expected productivity, that is subsequently not realized. Let $\tilde{E}$ denote the subjective expectations including this expected productivity shock, and E denote the expectations of a rational outside observer. Then the bond pricing Euler equation for a real n-period bond becomes

$$\begin{aligned}
P_{n,t} &= \beta \tilde{E}_t \left[exp\left(-\gamma \Delta c_{t+1} - \gamma \Delta s_{t+1}\right) P_{n-1,t+1} \right], && \text{(A154)} \\
&= exp\left(-\gamma v_{a,t}\right) \tilde{E}_t \left[exp\left(-\gamma \left(x_{t+1} - \phi x_t\right) - \gamma \Delta s_{t+1}\right) P_{n-1,t+1} \right], && \text{(A155)} \\
&= exp(-\gamma v_{a,t}) E_t \left[exp\left(-\gamma \left(x_{t+1} - \phi x_t\right) - \gamma \Delta s_{t+1}\right) P_{n-1,t+1} \right], && \text{(A156)} \\
&= exp(-\gamma v_{a,t}) E_t \left[M_{t+1} P_{n-1,t+1} \right], && \text{(A157)}
\end{aligned}$$

where M_{t+1} is the SDF in terms of ex-post realized consumption and surplus consumption. Similarly, for an n-period nominal bond

$$P_{n,t} = exp(-\gamma v_{a,t}) E_t \left[M_{t+1} exp(-\pi_{t+1}) P^{\$}_{n-1,t+1} \right]. \tag{A158}$$

Defining $exp(-\xi_t) = exp(-\gamma v_{a,t})$ the bond pricing recursions in equation (14) in the main paper go through.

All that remains to check is that stock pricing recursions also go through. For this, I consider a levered claim to consumption with leverage parameter $\frac{1}{\delta}$ as in Abel (1990). The main paper uses a slightly more complicated model for leverage with very similar properties, while also keeping dividends and consumption cointegrated. The advantage of the simple Able (1990) leverage specification is that it can show that stock prices are exactly unchanged, whereas with the cointegrated leverage specification stock prices might change slightly, though due to its similarity to the Abel (1990) specification any changes are plausibly quantitatively insignificant.

The value function iteration for a levered consumption claim then equals:

$$\frac{P^{\delta,c}_{n,t}}{C_t^{1/\delta}} = \tilde{E}_t \left[exp\left(-\gamma \Delta c_{t+1} - \gamma \Delta s_{t+1}\right) \left(\frac{C_{t+1}}{C_t}\right)^{1/\delta} \frac{P^{\delta,c}_{n-1,t+1}}{C_{t+1}^{1/\delta}} \right]. \tag{A159}$$

If $\gamma = \frac{1}{\delta}$, as in the 1980s calibration in the main paper, any shocks to expected potential output growth drop out from the right-hand-side

$$\begin{aligned}
\frac{P^{\delta,c}_{n,t}}{C_t^{1/\delta}} &= \tilde{E}_t \left[exp\left(-\gamma \Delta s_{t+1}\right) \frac{P^{\lambda,c}_{n-1,t+1}}{C^{\delta}_{t+1}} \right], && \text{(A160)}\\
&= E_t \left[exp\left(-\gamma \Delta s_{t+1}\right) \frac{P^{\lambda,c}_{n-1,t+1}}{C^{\delta}_{t+1}} \right], && \text{(A161)}\\
&= E_t \left[M_{t+1} \left(\frac{C_{t+1}}{C_t}\right)^{1/\delta} \frac{P^{\delta,c}_{n-1,t+1}}{C^{1/\delta}_{t+1}} \right], && \text{(A162)}
\end{aligned}$$

i.e. the standard pricing equation for a levered consumption claim takes the same form as in the main paper with Abel (1990) leverage.

To show that the simplification to Abel (1990) leverage might be a concern, I show that the model properties for bond-stock betas are unchanged in the special case with unit EIS, i.e. i.e. $\gamma = 1$, where the asset pricing Euler equation for the unlevered consumption claim is exactly identical to equation (15) in the main paper, as can easily be seen by setting $\gamma = \delta = 1$ in equation (A162). Table A2 shows that setting $\gamma = 1$ and/or $\phi = 1$ leaves the asset pricing properties in Table 2 in the main paper unchanged. Setting $\phi = 1$ is also informative, as in this case price inflation equals wage inflation in equation (26) in the main paper, so there is no concern that shocks to expected productivity growth could lead to a wedge between price and wage inflation.

D Additional Model Results

D.1 Varying Adaptiveness of Wage-Setter Inflation Expectations

Because the calibration procedure separately calibrates the adaptive inflation expectations parameter ζ holding all other parameters constant, it is important to check that the macroeconomic moments used to calibrate the volatilities of shocks and monetary policy parameters do not suffer unreasonably from changing ζ. Because the 2001.Q2-2019.Q4 calibration uses $\zeta = 0$ this is not a concern here. Figure A1 compares the model fit to macroeconomic data with $\zeta = 0$ and $\zeta = 0.6$ for both subperiods.

The right plots in Figure A1 show that the parameterization with $\zeta = 0$ (as in the 2000s calibration in the main paper) does fit the macroeconomic data better than an alternative version with adaptive inflation expectations and $\zeta = 0.6$. In particular, setting $\zeta = 0.6$ in the 2000s calibration no longer matches the positive output gap-inflation relationship that is visible in the data in the top-left panel. The reason is that $\zeta = 0.6$ implies a strongly backward-looking Phillips curve and high inflation inertia. Because wage-setters' inflation expectations are slow to move, a positive demand shock has very little impact on inflation through the Phillips curve when $\zeta = 0.6$. This plot therefore visually shows that there is no reason to deviate from rational inflation expectations for the 2000s calibration, and that rational wage-setter inflation expectations help match the change from "stagflations" in the 1980s to high inflation recessions in the 2000s.

The left plots in Figure A1 show the comparison for the 1980s calibration. As expected the macroeconomic fit deteriorates when moving ζ to match the bond excess return predictability in the data. However, it is informative which macroeconomic moment looks different for $\zeta = 0$ vs. $\zeta = 0.6$. Most importantly, the top-left plot looks very similar under these two parameterizations, so the choice of rational vs. adaptive wage-setter inflation expectations does not affect the model's implication of "stagflationary" dynamics in the 1980s. What changes is mostly the persistence of inflation, which is visible in a more persistent policy rate response to an inflation innovation in the bottom-left plot. A volatile persistent component in inflation for the 1980s is in line with a long-standing econometrics literature that has decomposed inflation into its permanent and transitory components (Stock and Watson (2007)), so this persistence with $\zeta = 0.6$ is in line with outside data not used directly for the calibration.

Additional empirical moments from inflation surveys provide additional external empirical support for partially adaptive inflation expectations in the 1980s and rational inflation expectations in the 2000s, which are calibrated to bond excess return predictability. Table A1 runs the well-known test for the rationality of inflation expectations of Coibion and Gorodnichenko (2015):

$$\pi_{t+3} - \tilde{E}_t \pi_{t+3} = a_0 + a_1 \left(\tilde{E}_t \pi_{t+3} - \tilde{E}_{t-1} \pi_{t+3} \right) + \varepsilon_{t+3}. \tag{A163}$$

Here, a tilde denotes potentially subjective inflation expectations. If expectations are full information rational the forecast error on the left-hand side of (A163) should be unpredictable, and the coefficient a_1 should equal zero. The empirical specification follows Coibion and Gorodnichenko (2015), using the Survey of Professional Forecasters four-quarter and three-quarter GDP deflator inflation forecasts to compute forecast revisions. The first column in Table A1 uses a long sample 1968.Q4-2001.Q1 and confirms their well-known empirical result. An upward revision in inflation forecasts tends to predict positive forecast errors. The second and third columns run the same empirical regressions for the 1979.Q4-2001.Q1 and 2001.Q2-2019.Q4 subperiods. I find that for both subperiods the evidence becomes insignificant. While this is potentially due to the smaller sample size and weaker statistical power, the change in a_1 from 1968.Q4-2001.Q1 to 2001.Q2-2019.Q4 is statistically significant. The last two columns of the table show that the model matches this broad pattern in the predictability of inflation forecast errors documented in the data.[2]

Figure A2 and A3 drill down further into the mechanism, comparing the model macroeconomic impulse responses to demand, supply and monetary policy shocks for different values of ζ. In particular, we see that the inflation response to a supply shock is much less persistent when $\zeta = 0$ than when $\zeta = 0.6$. This change in the inflation response to a supply shock is less important for the properties of the 2000s calibration, which only has a small volatility of supply shocks. But for the 1980s calibration, where the volatility of supply shocks is high, we can see that a higher value of $\zeta = 0.6$ generates a much more persistent inflation process. This strong persistent component in the model inflation dynamics for the 1980s calibration matters for the predictability of bond excess returns. It is the reason that supply shocks move the slope of the term structure in the same direction as bond risk premia in the middle column in the top row in Figure 10 in the main paper.

D.2 Calibration Robustness to Different Parameter Values

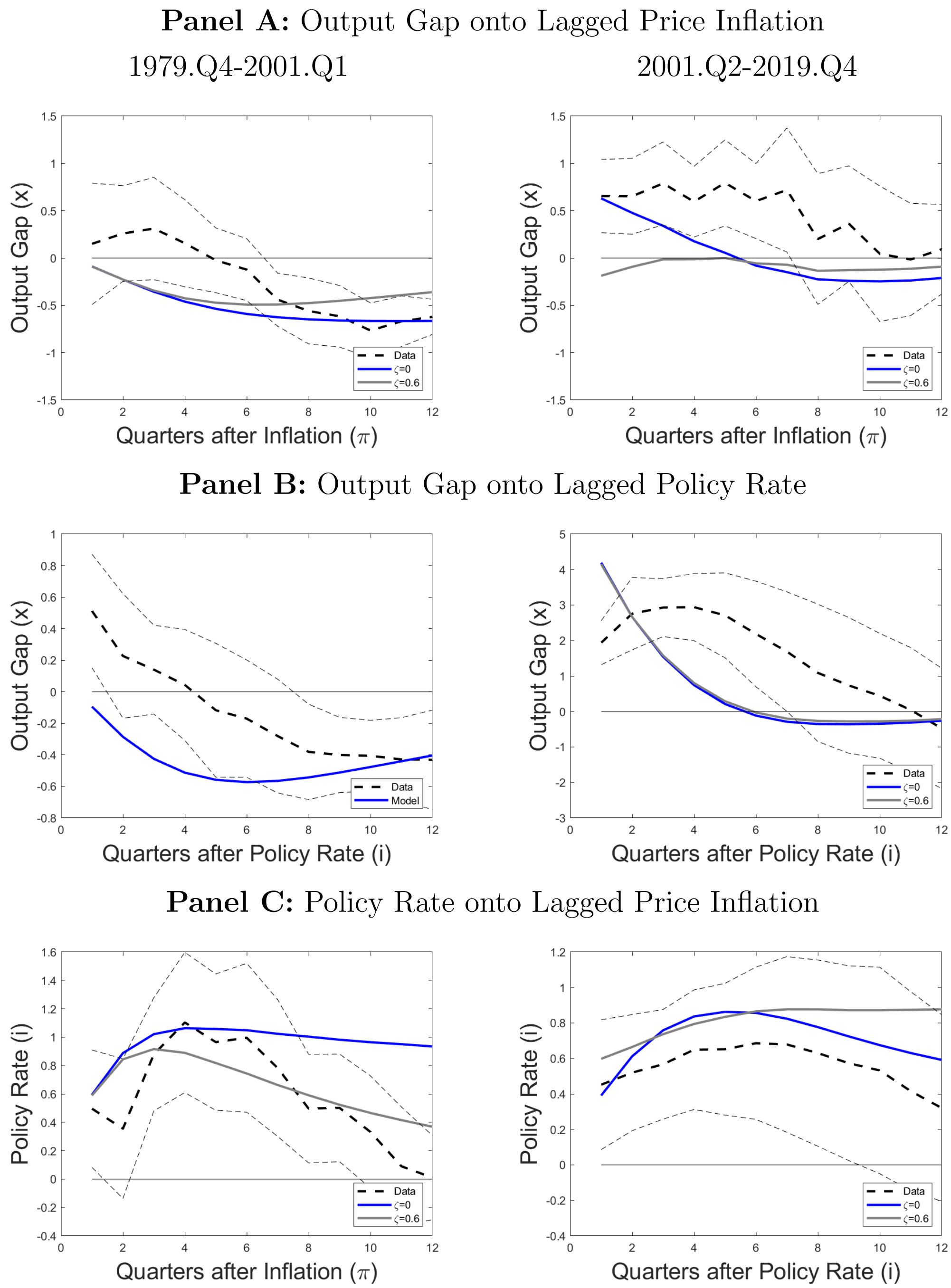
Table A2 shows that the main model implications for bond-stock betas are not sensitive to choosing different values for the utility curvature, the consumption-output gap link ϕ, the adaptiveness of inflation expectations ζ, or the slope of the Phillips curve κ. Implied nominal bond-stock betas are positive for the 1980s calibration and negative for the 2000s calibration and the all moments are qualitatively or even quantitatively unchanged.

The case with $\phi = 1$ is of interest because this switches off the difference between price and wage inflation (equation (26) in the main paper. We see that setting $\phi = 1$ leaves all asset pricing and macroeconomic moments qualitatively and quantitatively unchanged. This tells us that the model properties are unaffected whether we assume sticky wages or sticky prices.

Changing the utility curvature parameter to $\gamma = 1$ implies a positive nominal bond-stock for the 1980s calibration and a negative nominal bond-stock beta for the 2000s calibration. The nominal bond-stock for the 1980s calibration is somewhat smaller in magnitude than in the baseline. While the Campbell-Shiller bond excess return predictability coefficient is still positive and within a 90% confidence interval of the empirical coefficient, it is also smaller

[2] The literature has not reached an agreement on whether inflation expectations have become more or less rational over time. On the one hand, Bianchi, Ludvigson and Ma (2022) find less inflation forecast error predictability post-1995, and Davis et al. (2012) shows that inflation expectations have become less responsive to oil prices shocks in recent decades. However, Coibion and Gorodnichenko (2015) and Maćkowiak and Wiederholt (2015) provide evidence and a model of decreasing attention to inflation as economic volatility declined during the 1990s.

Figure A1: Empirical Output Gap, Inflation, and Policy Rate Dynamics Pre- vs. Post-2001 with Rational Inflation Expectations



This figure is analogous to Figure 2 in the main paper, but it compares $\zeta = 0$ and $\zeta = 0.6$ for both calibrations.

because with a larger EIS monetary policy is more effective at reducing inflation, leading to a less persistent inflation process. While the parameterization in the main paper with $\gamma = 2$

is chosen to replicate the output impulse response to an identified monetary policy shock int he data, the case with $\gamma = 1$ and $\phi = 1$ is of interest because it shows one extremely simple example where preference shocks for bonds are functionally isomorphic to shocks to expected potential output growth.

Setting $\zeta = 0$ (i.e. inflation expectations are rational) changes the Campbell-Shiller excess bond return predictability coefficient to zero for the 1980s calibration, consistent with the comparative statics in 3 in the main paper. At the same time, this change leaves all other moments unchanged. This tells us that ζ is pinned down by the predictability of bond excess returns without affecting any of the other model implications.

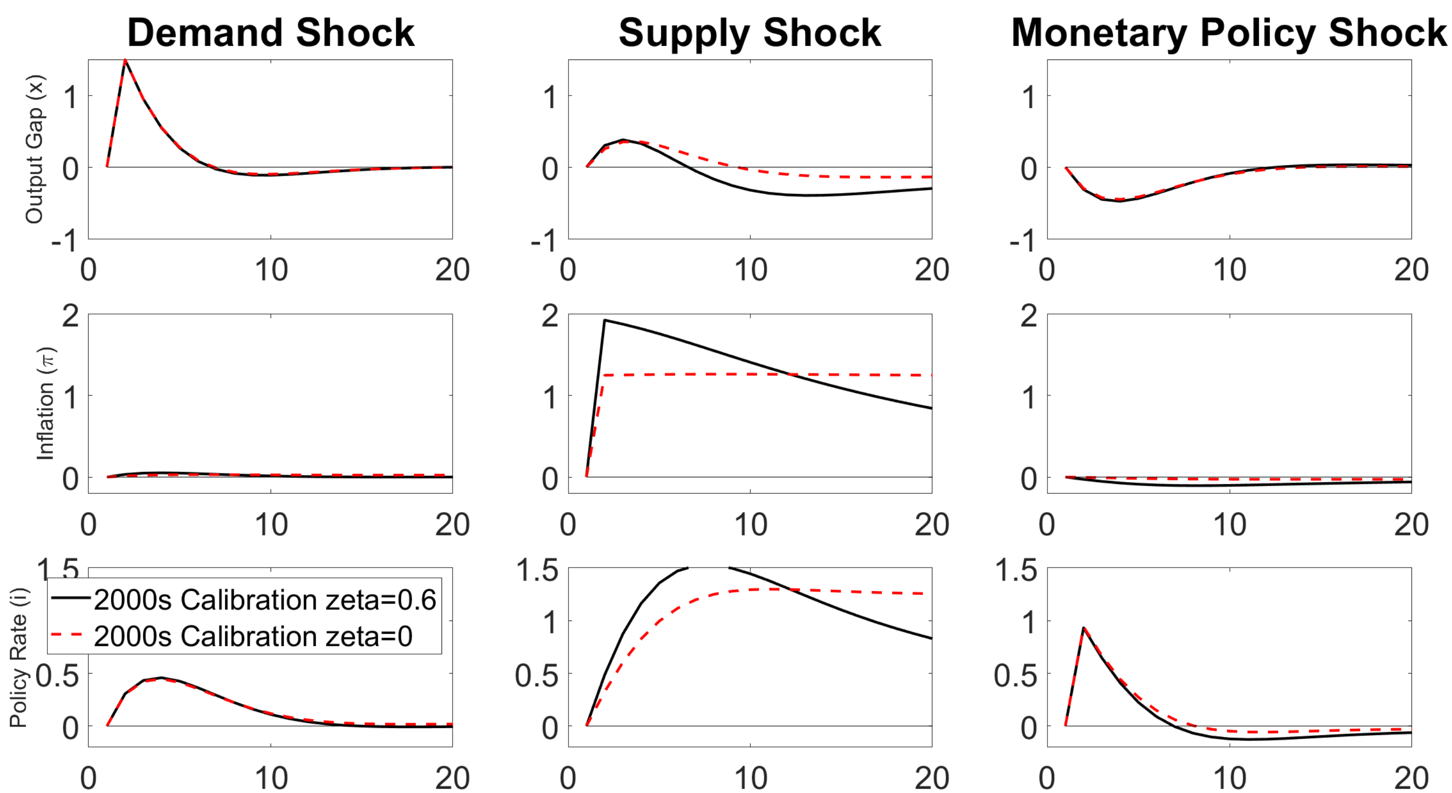
Finally, the rightmost column sets the slope of the Phillips curve to a higher value following Rotemberg and Woodford (1997). The nominal and real bond-stock betas are unchanged from the main calibration and all other moments look similar. The only slight change is in the value of the 1980s Campbell-Shiller bond excess return predictability coefficient, which is smaller for this model parameterization. The reason is that a higher slope of the Phillips curve means that supply shocks lead to a stronger output response, and monetary policy is more powerful in stabilizing inflation by driving down the output gap, leading to a less persistent inflation process. As in the main paper, the Campbell-Shiller bond excess return predictability coefficient is therefore informative for the Phillips curve more broadly. Overall, the implications for nominal and real bond-stock betas are moreover robust to varying the preference, consumption-output gap link, inflation rationality, and Phillips curve parameters.

Table A1: Inflation Forecast Error Regressions by Subperiod

	Data			Model	
$\tilde{E}_t\pi_{t+3} - \tilde{E}_{t-1}\pi_{t+3}$	0.926***	0.433	-0.310	1.43	-0.01
	(0.34)	(0.32)	(0.43)		
Const.	-0.114	-0.795***	-0.046		
	(0.28)	(0.20)	(0.18)		
N	126	87	71		
R-sq	0.09	0.03	0.00		
Sample	1968.Q4-2001.Q1	1979.Q4-2001.Q1	2001.Q2-2019.Q4	1979.Q4-2001.Q1	2001.Q2-2019.Q4

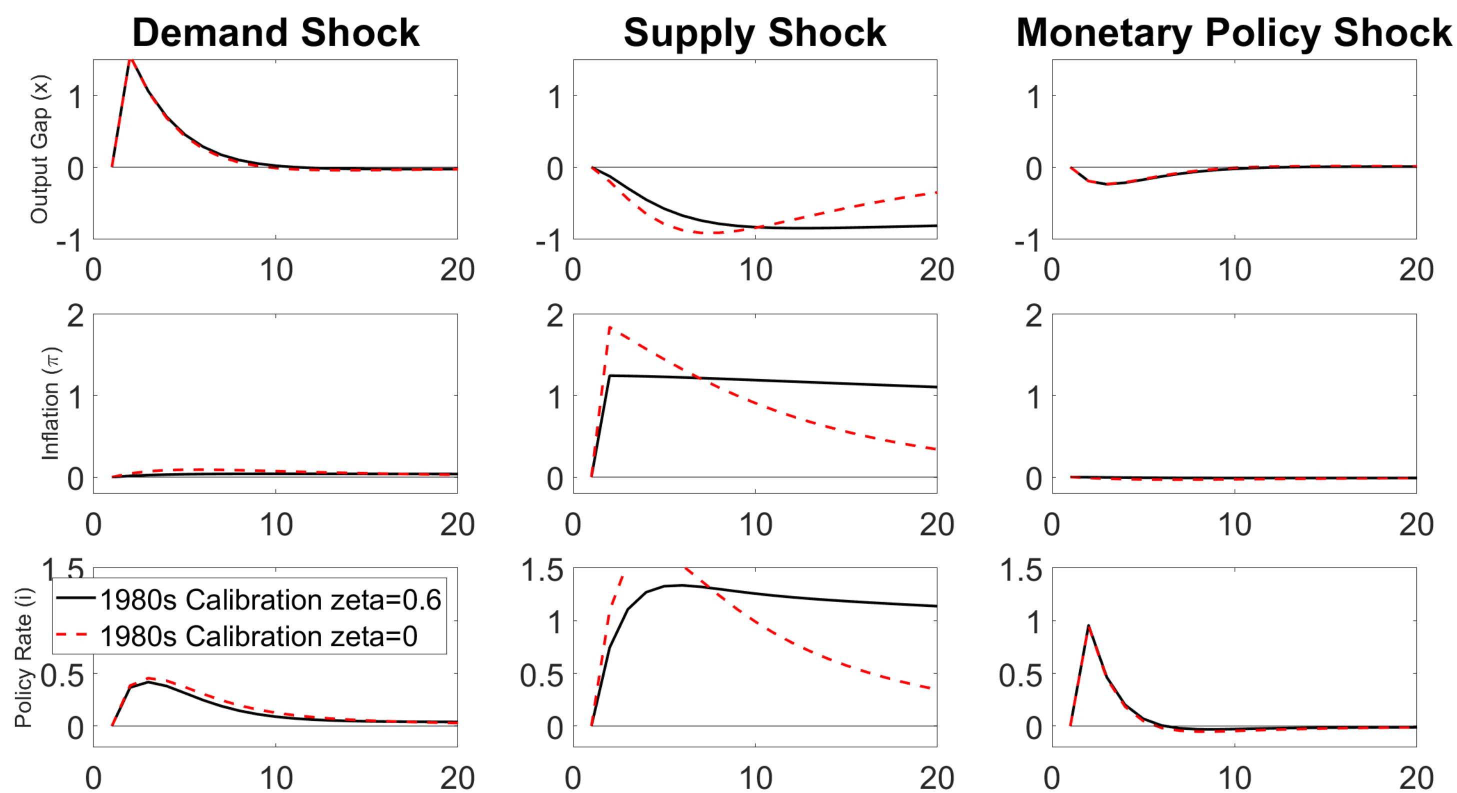
This table estimates Coibion and Gorodnichenko (2015) regressions of the form $\pi_{t+3} - \tilde{E}_t\pi_{t+3} = a_0 + a_1\left(\tilde{E}_t\pi_{t+3} - \tilde{E}_{t-1}\pi_{t+3}\right) + \varepsilon_{t+3}$ using quarterly GDP deflator inflation forecasts from the Survey of Professional Forecasters. Newey-West standard errors with 4 lags in parentheses. Model subjective n-quarter inflation expectations are computed assuming that inflation expectations are a weighted average of rational expectations and past average inflation $\tilde{E}_t\pi_{t+n} = \zeta\pi_{t-n-1\to t-1} + (1-\zeta)E_t\pi_{t+n}$

Figure A2: Model Macroeconomic Impulse Responses by Adaptive Inflation Expectations for 1980s Calibration



This figure compares the structural macroeconomic impulse responses for the 1980s calibration with $\zeta = 0$ and $\zeta = 0.6$.

Figure A3: Model Macroeconomic Impulse Responses by Adaptive Inflation Expectations for 2000s Calibration



This figure compares the structural macroeconomic impulse responses for the 2000s calibration with $\zeta = 0$ and $\zeta = 0.6$.

Table A2: Model and Data Moments Robustness

Panel A: 1979.Q4-2001.Q1 Calibration

	Baseline	$\phi = 1$	$\gamma = 1$	$\gamma = 1, \phi = 1$	$\zeta = 0$	$\kappa = 0.019$
Stocks						
Equity Premium	7.33	6.91	5.87	5.69	7.14	7.03
Equity Vol	14.95	13.95	15.16	14.43	14.40	14.19
Equity SR	0.49	0.50	0.39	0.39	0.50	0.50
AR(1) pd	0.96	0.95	0.95	0.95	0.95	0.95
1 YR Excess Returns on pd	-0.38	-0.43	-0.41	-0.43	-0.41	-0.42
1 YR Excess Returns on pd (R2)	0.06	0.07	0.05	0.05	0.06	0.06
Bonds						
Return Vol.	15.82	15.39	12.89	12.59	10.12	12.66
Nominal Bond-Stock Beta	0.86	0.85	0.42	0.37	0.65	0.71
Real Bond-Stock Beta	0.05	0.06	0.05	0.05	0.09	0.06
1 YR Excess Returns on slope	1.26	1.24	0.25	0.24	-0.36	0.15
1 YR Excess Returns on slope (R2)	0.01	0.01	0.00	0.00	0.00	0.00
Macroeconomic Volatilities						
Std. Annual Cons. Growth	0.76	0.73	0.89	0.87	0.98	0.75
Std. Annual Change Fed Funds Rate	1.64	1.63	1.62	1.62	2.15	1.65
Std. Annual Change 10-Year Subj. Infl. Forecast	0.73	0.71	0.64	0.62	0.87	0.58

This table shows robustness results for the calibration moments shown in Table 2 in the main paper. Panel A sets all parameters to their 1979.Q4-2001.Q1 calibration values and changes the indicated parameters one at a time. Panel B sets all parameters to their 2001.Q2-2019.Q4 calibration values and changes the indicated parameters one at a time.

Model and Data Moments Robustness (continued)

	Panel B: 2001.Q2-2019.Q4 Calibration					
	Baseline	$\phi = 1$	$\gamma = 1$	$\gamma = 1, \phi = 1$	$\zeta = 0$	$\kappa = 0.019$
Stocks						
Equity Premium	9.15	9.15	5.69	5.68	9.15	9.17
Equity Vol	19.29	19.29	15.69	15.68	19.29	19.32
Equity SR	0.47	0.47	0.36	0.36	0.47	0.47
AR(1) pd	0.93	0.93	0.95	0.95	0.93	0.93
1 YR Excess Returns on pd	-0.38	-0.38	-0.32	-0.32	-0.38	-0.38
1 YR Excess Returns on pd (R2)	0.14	0.14	0.10	0.10	0.14	0.14
Bonds						
Return Vol.	2.12	2.16	1.42	1.44	2.62	1.89
Nominal Bond-Stock Beta	-0.09	-0.09	-0.07	-0.07	-0.10	-0.09
Real Bond-Stock Beta	-0.08	-0.08	-0.07	-0.07	-0.08	-0.08
1 YR Excess Returns on slope	-0.31	-0.31	-0.16	-0.15	-0.39	-0.27
1 YR Excess Returns on slope (R2)	0.01	0.01	0.00	0.00	0.00	0.01
Macroeconomic Volatilities						
Std. Annual Cons. Growth	1.59	1.62	1.44	1.47	1.58	1.62
Std. Annual Change Fed Funds Rate	0.65	0.68	0.56	0.58	0.61	0.71
Std. Annual Change 10-Year Subj. Infl. Forecast	0.12	0.12	0.10	0.10	0.09	0.06

This table shows robustness results for the calibration moments shown in Table 2 in the main paper. Panel A sets all parameters to their 1979.Q4-2001.Q1 calibration values and changes the indicated parameters one at a time. Panel B sets all parameters to their 2001.Q2-2019.Q4 calibration values and changes the indicated parameters one at a time.

E Details for Delta Method Standard Errors

This Section provides details for the Delta method used to construct standard errors in Table 2. Standard errors are computed separately for each of the calibration steps, taking all other parameters as given. Let $\hat{\Psi}$ denote the vector of twelve (13 for the second subperiod) empirical target moments, and $\Psi(\sigma_x, \sigma_\pi, \sigma_i, \gamma^x, \gamma^\pi, \rho^i; \zeta)$ the vector of model moments computed analogously on model-simulated data. I choose subperiod-specific monetary policy parameters γ^x, γ^π, and ρ^i and shock volatilities σ_x, σ_π, and σ_i while holding the inflation expectations parameter constant at $\zeta = 0$ to minimize the objective function:

$$\left\| \frac{\hat{\Psi} - \Psi(\sigma_x, \sigma_\pi, \sigma_i, \gamma^x, \gamma^\pi, \rho^i; \zeta = 0)}{SE(\hat{\Psi})} \right\|^2 . \tag{A164}$$

Let $\hat{H}_{SE}$ is the numerical Jacobian

$$\hat{H}_{SE} \quad = \quad \Delta\left(\Psi/SE\left(\hat{\Psi}\right)\right), \tag{A165}$$

i.e. the vector of derivatives of the normalized objective function with respect to each parameter. I calculate this derivative numerically by using a derivative step of 0.05 for most parameters (0.005 for σ_x to avoid setting it to negative values) and a simulation length of 1000, just as in the SMM estimation. I then average the simulated value for $\hat{H}$ across 20000 independent simulations to reduce simulation noise. Because the weighting function in the GMM estimation is the identity (after normalizing all moments into z-scores using the standard errors of the empirical moments computed in the data), standard GMM results imply that the variance-covariance matrix of the parameters is

$$\hat{V}_{params} = \hat{M}_{SE}^{-1} \hat{H}_{SE} \hat{V} \hat{H}_{SE}' \hat{M}_{SE}^{-1\prime}. \tag{A166}$$

Here $\hat{V}$ is the variance-covariance matrix of the target moments, and $\hat{M}_{SE} = \hat{H}_{SE}'\hat{H}_{SE}$. Under the simplifying assumption that the target moments are uncorrelated, $\hat{V}$ collapses to the identity and the expression for $\hat{V}_{params}$ becomes

$$\hat{V}_{params} \quad = \quad (\hat{H}_{SE}\hat{H}_{SE}')^{-1}. \tag{A167}$$

The standard errors reported in Table 2 do not make this simplifying assumption, instead computing $\hat{V}$ as the correlation matrix across 1000 separate simulations of the model, where again each simulation has sample length 100. The standard errors reported in Table 2 in the main paper are then obtained by substituting into A166 and

$$SE(\sigma_x, \sigma_\pi, \sigma_i, \gamma^x, \gamma^\pi, \rho^i) \;\; = \;\; diag\left(\sqrt{\hat{V}_{params}}\right). \tag{A168}$$

A joint hypothesis test for $\gamma_x = 0.5$ and $\gamma_\pi = 1.1$ is run by comparing $[\gamma_x - 0.5, \gamma_\pi - 1.1]\,\hat{V}_{params}(1:2,1:2)^{-1}\,[\gamma_x - 0.5, \gamma_\pi - 1.1]' \sim \chi^2_2$, where $\hat{V}_{params}(1:2,1:2)$ is the upper $[2 \times 2]$ block of $\hat{V}_{params}$.

The standard errors for ζ is obtained similarly, holding all other parameter values fixed. The relevant moment is the Campbell-Shiller coefficient b^{CS}. By standard GMM algebra, the standard error of ζ is

$$SE(\zeta) \;\; = \;\; \left\{\frac{db^{CS}}{d\zeta}\right\}^{-1} SE\left(\hat{b}^{SE}\right), \tag{A169}$$

i.e. the ratio of the standard error of the Campbell-Shiller coefficient in the data to the slope of the model coefficient with respect to ζ.

For the 1980s period, the empirical Newey-West standard error with 4 lags equals $SE(\hat{b}^{SE}) = 1.38$. The numerical derivative is computed as $\frac{db^{CS}}{d\zeta} = \frac{b^{CS}(\zeta=0.6) - b^{CS}(\zeta=0)}{0.6} = \frac{1.26-(-0.36)}{0.6} = 2.70$, giving the standard error for ζ as $SE(\zeta) = 1.38/2.70 = 0.51$.

For the 2000s period, the empirical Newey-West standard error with 4 lags equals $SE(\hat{b}^{SE}) = 1.18$. The numerical derivative is computed as $\frac{db^{CS}}{d\zeta} = \frac{b^{CS}(\zeta=0.6) - b^{CS}(\zeta=0)}{0.6} = \frac{-0.39-(-0.31)}{0.6} = 0.44$, giving the standard error for ζ as $SE(\zeta) = 1.18/0.44 = 2.67$.

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