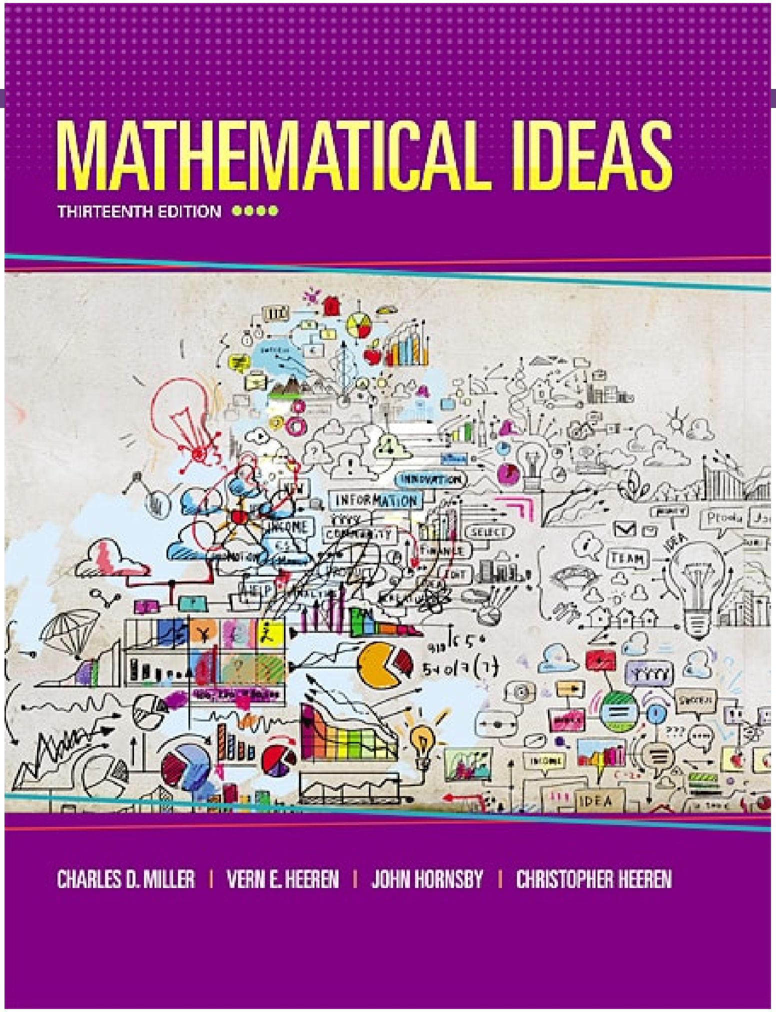
Chapter 2

Introduction to Logic



Chapter 2: Introduction to Logic

Section 2-3

The Conditional and Circuits

The Conditional and Circuits

- Understand the structure of the conditional statement.
- Determine the truth values of conditional statements.
- Express a conditional statement as a disjunction.
- Express the negation of a conditional statement.
- Use circuits to model conditional statements.

Conditionals

A **conditional** statement is a compound statement that uses the connective *if…then*.

The conditional is written with an arrow, so "if p then q" is symbolized:

$$p \to q.$$

We read the above as "p implies q" or "if p then q." The statement p is the **antecedent**, while q is the **consequent**.

Truth Table for The Conditional: If *p*, then *q*

If *p*, then *q*

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for The Conditional: $(p \rightarrow \sim q) \rightarrow (\sim r \rightarrow q)$

Evaluate the conditional:
$(p \rightarrow \sim q) \rightarrow (\sim r \rightarrow q)$.
Given that p=F, q=F and r=F

$$(p \rightarrow \sim q) \rightarrow (\sim r \rightarrow q)$$

$$(F \rightarrow \sim F) \rightarrow (\sim F \rightarrow F)$$

$$(F \rightarrow T) \rightarrow (T \rightarrow F) \; T \rightarrow F$$

$$\boldsymbol{F}$$

Special Characteristics of Conditional Statements

1. $p \to q$ is false only when the antecedent is *true* and the consequent is *false*.
2. If the antecedent is *false*, then $p \to q$ is automatically *true*.
3. If the consequent is *true*, then $p \to q$ is automatically *true*.

Example: Determining Whether Conditionals Are True or False

Decide whether each statement is True or False (T represents a true statement, F a false statement).

a) $T \rightarrow (4 < 2)$ b) $(8 = 1) \rightarrow F$

Solution

a) False

b) True

Tautology

A statement that is always true, no matter what the truth values of the components, is called a **tautology**. They may be checked by forming truth tables.

Writing a Conditional as a Disjunction: "Or" Statement

$p \rightarrow q$ is equivalent to $\sim p \vee q$.

Negation of a Conditional $p \to q$

The negation of $p \to q$ is $p \vee \sim q$

Example: Determining Negations

Determine the negation of each statement.

a) If I’m hungry, I will eat.

b) All dogs have fleas.

Solution

a) I’m hungry and I will not eat.

b) Restate as an *if…then*:

If it is a dog, then it has fleas.

Negation: It is a dog and it does not have fleas.

Example: Determining Statements Equivalent to Conditionals

Write the conditional as an equivalent statement without using *if* . . . *then*.

If the Indians win the pennant, then Johnny will go to the World Series.

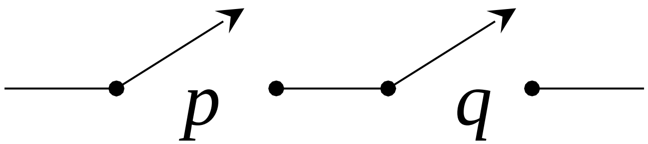
Solution

Let p represent "The Indians win the pennant" and q represent "Johnny will go to the World Series.

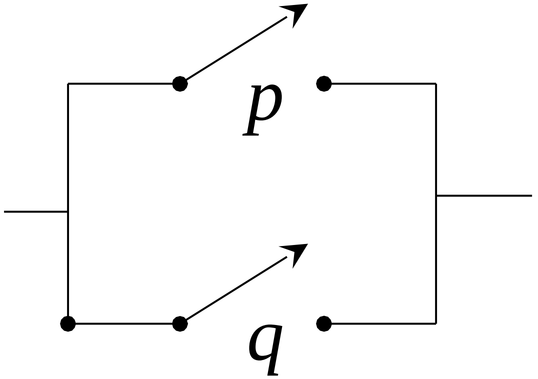
Restate: The Indians do not win the pennant or Johnny will go to the World Series.

Circuits

Logic can be used to design electrical circuits.



Series circuit



Parallel circuit

Equivalent Statements Used to Simplify Circuits

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \to q \equiv \sim q \to \sim p$$

$$p \to q \equiv \sim p \vee q$$

$$p \vee p \equiv p \qquad p \wedge p \equiv p$$

$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

Equivalent Statements Used to Simplify Circuits

If T represents any true statement and F represents any false statement, then:

$$p \vee \mathrm{T} \equiv \mathrm{T}$$

$$p \wedge \mathrm{F} \equiv \mathrm{F}$$

$$p \vee \sim p \equiv \mathrm{T}$$

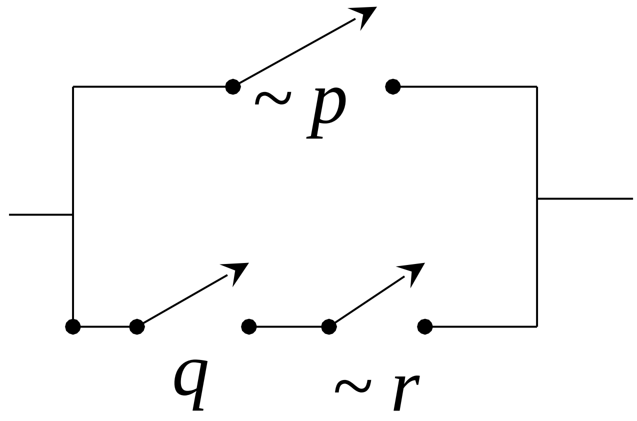
$$p \wedge \sim p \equiv \mathrm{F}.$$

Example: Drawing a Circuit for a Conditional Statement

Draw a circuit for $p \to (q \wedge \sim r)$.

Solution

$$p \to (q \wedge \sim r) \equiv \sim p \vee (q \wedge \sim r)$$



Bellamy Brothers (1979)