# Lecture Slides


*Essentials of Statistics*
5th Edition

and the Triola Statistics Series

by Mario F. Triola


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# Chapter 6
# Normal Probability Distributions




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# Key Concept

The Central Limit Theorem tells us that for a population with any distribution, the distribution of the sample means approaches a normal distribution as the sample size increases.

The procedure in this section forms the foundation for estimating population parameters and hypothesis testing.


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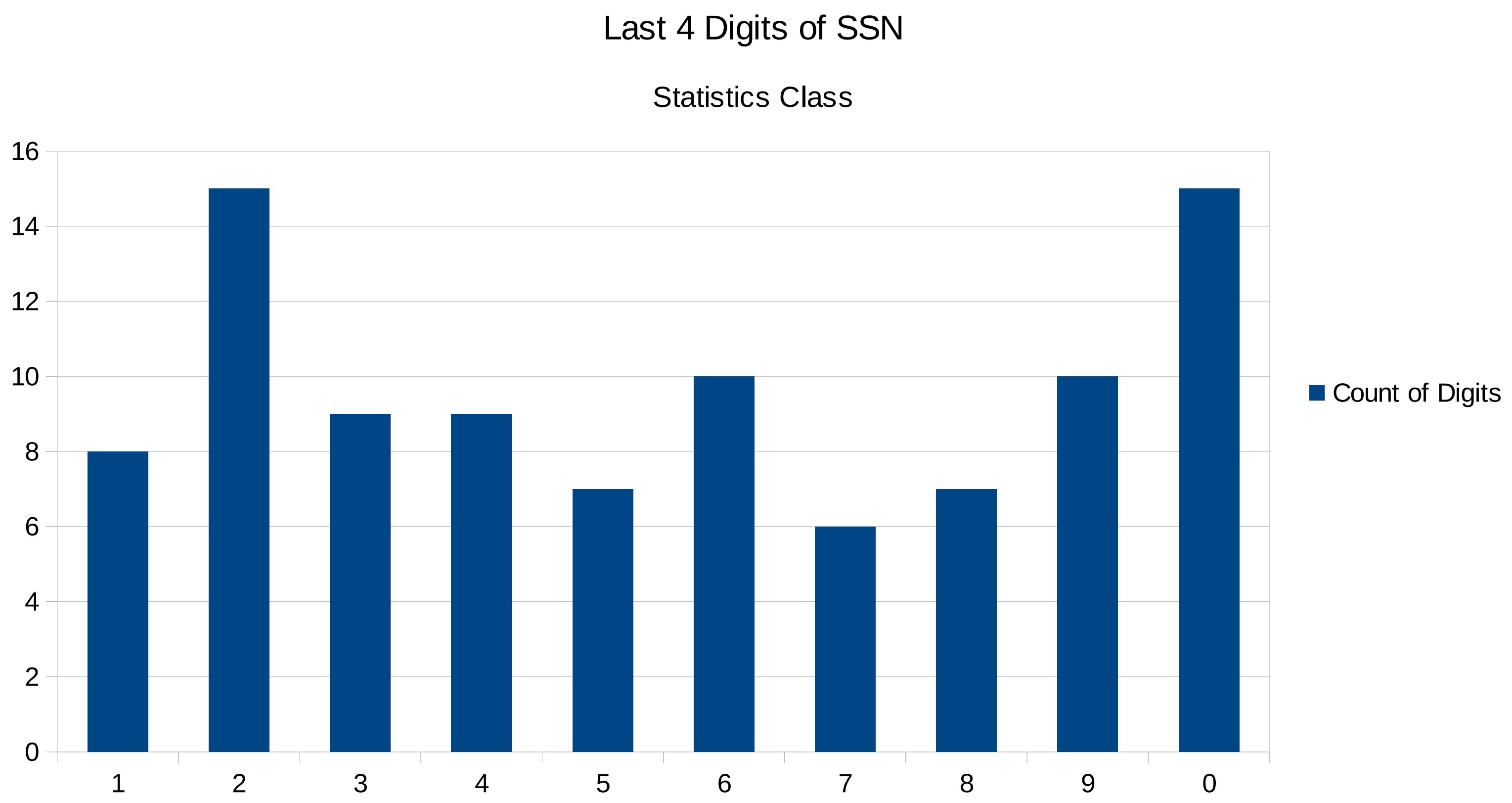



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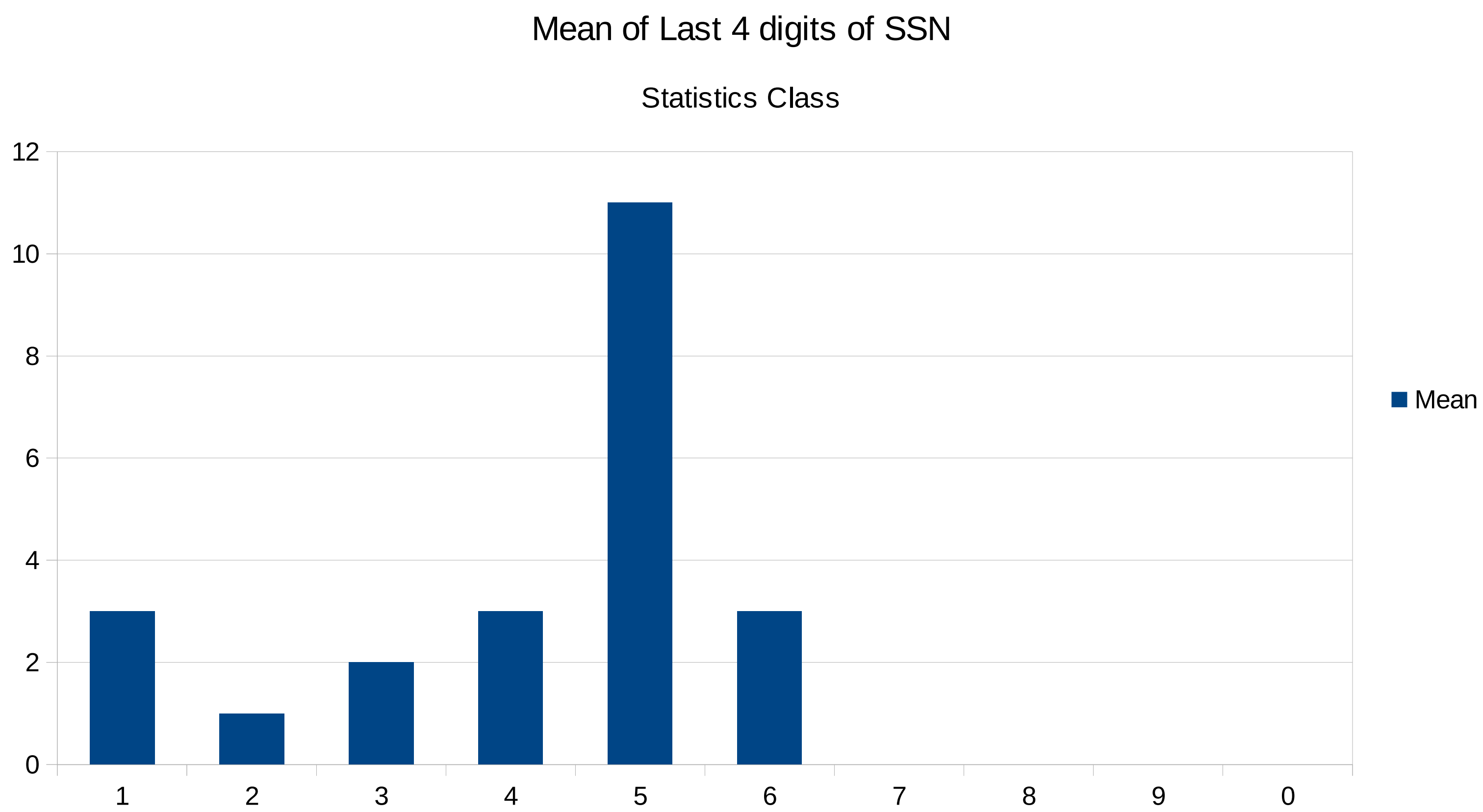



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# Central Limit Theorem

## Given:

1. The random variable $x$ has a distribution (which may or may not be normal) with mean $\mu$ and standard deviation $\sigma$.
2. Simple random samples all of size $n$ are selected from the population. (The samples are selected so that all possible samples of the same size $n$ have the same chance of being selected.)


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# Central Limit Theorem – cont.

## Conclusions:

1. The distribution of sample $\overline{x}$ will, as the sample size increases, approach a normal distribution.

2. The mean of the sample means is the population mean $\mu$.

3. The standard deviation of all sample means is $\sigma / \sqrt{n}$ .


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# Practical Rules Commonly Used

1. For samples of size *n* larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation becomes closer to a normal distribution as the sample size *n* becomes larger.
2. If the original population is normally distributed, then for any sample size *n*, the sample means will be normally distributed (not just the values of *n* larger than 30).


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# Notation

The mean of the sample means

$$\mu_{\bar{x}} = \mu$$

The standard deviation of sample mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(often called the standard error of the mean)


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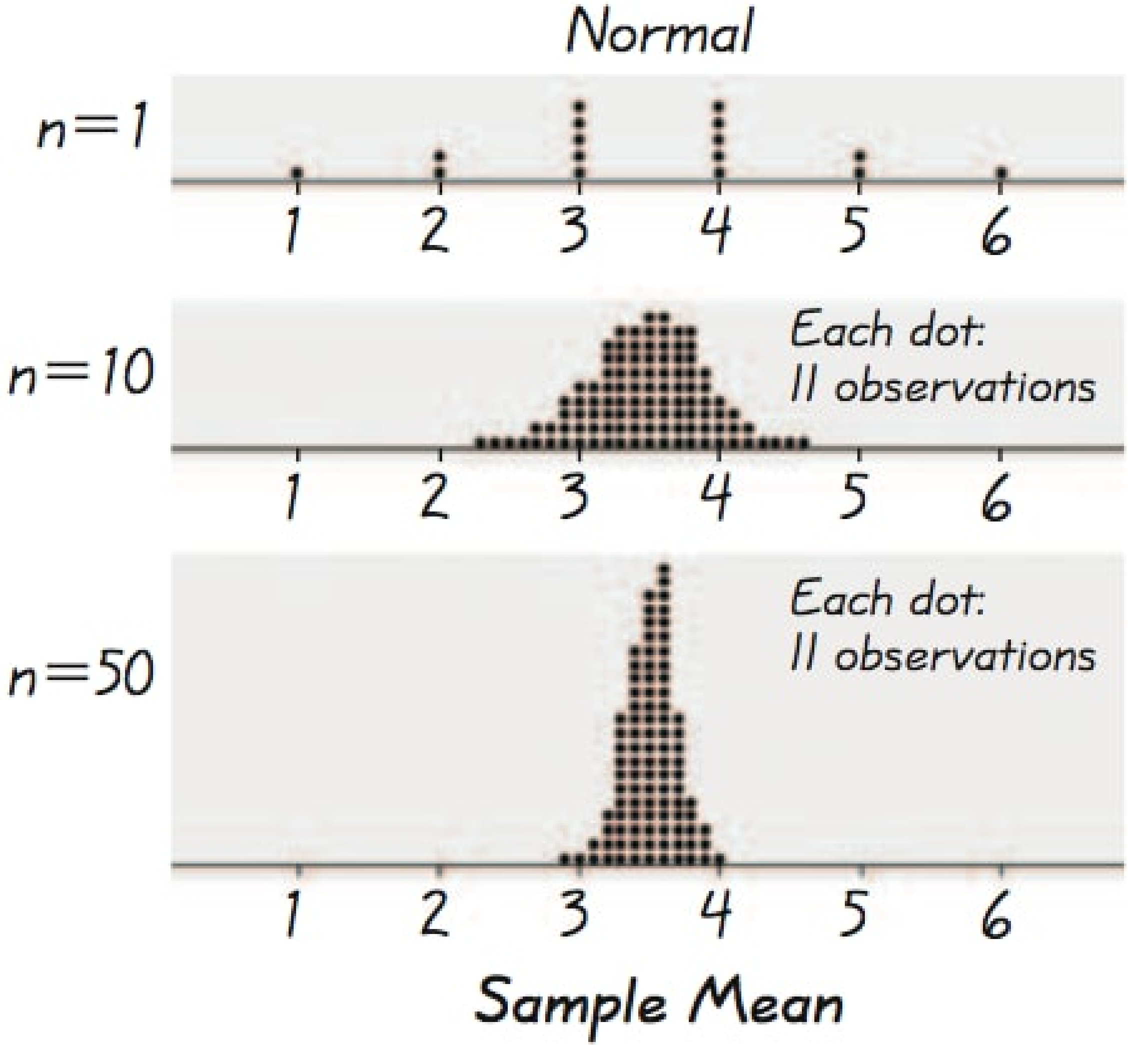
# Example - Normal Distribution

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.





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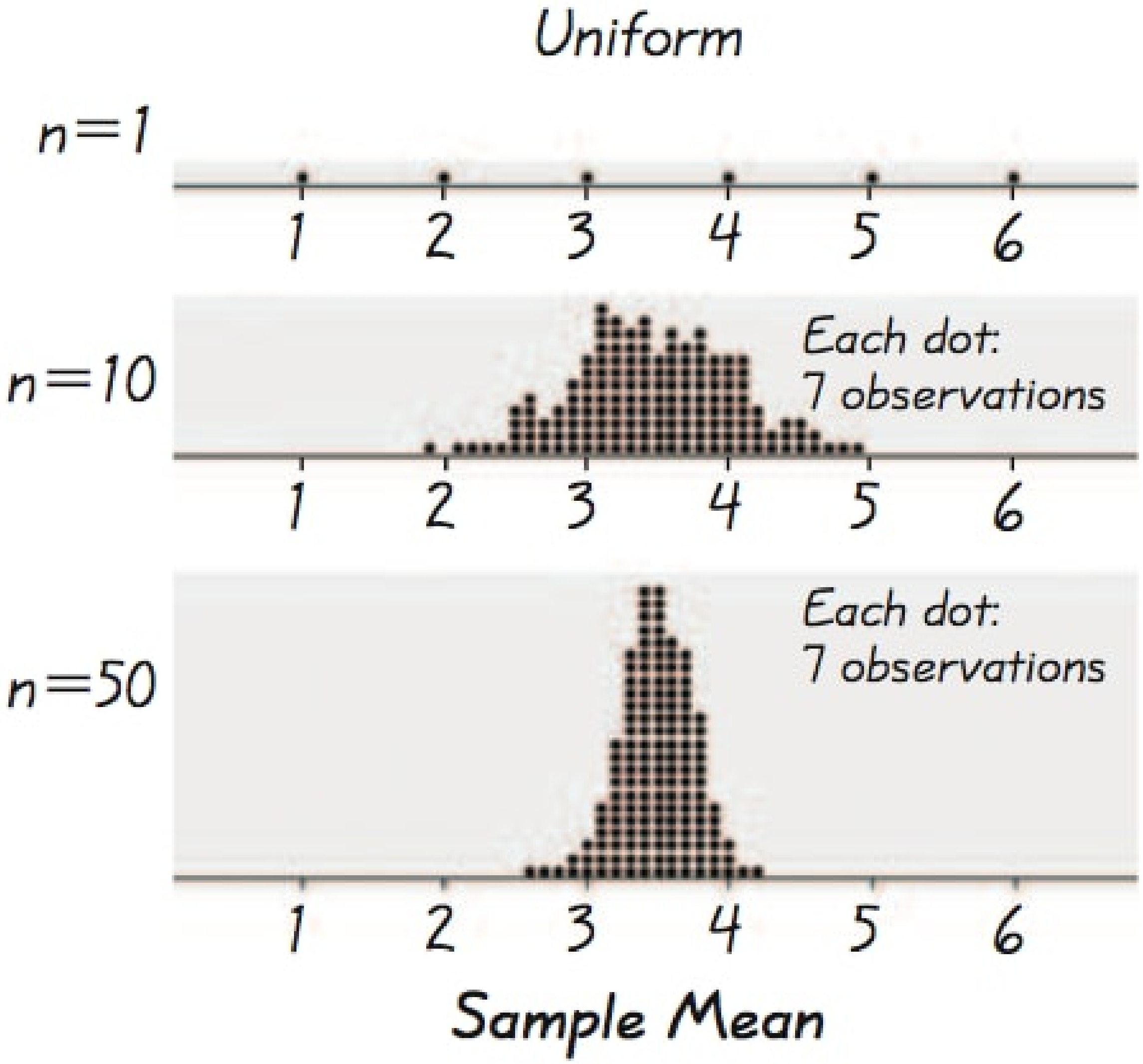
# Example - Uniform Distribution

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.





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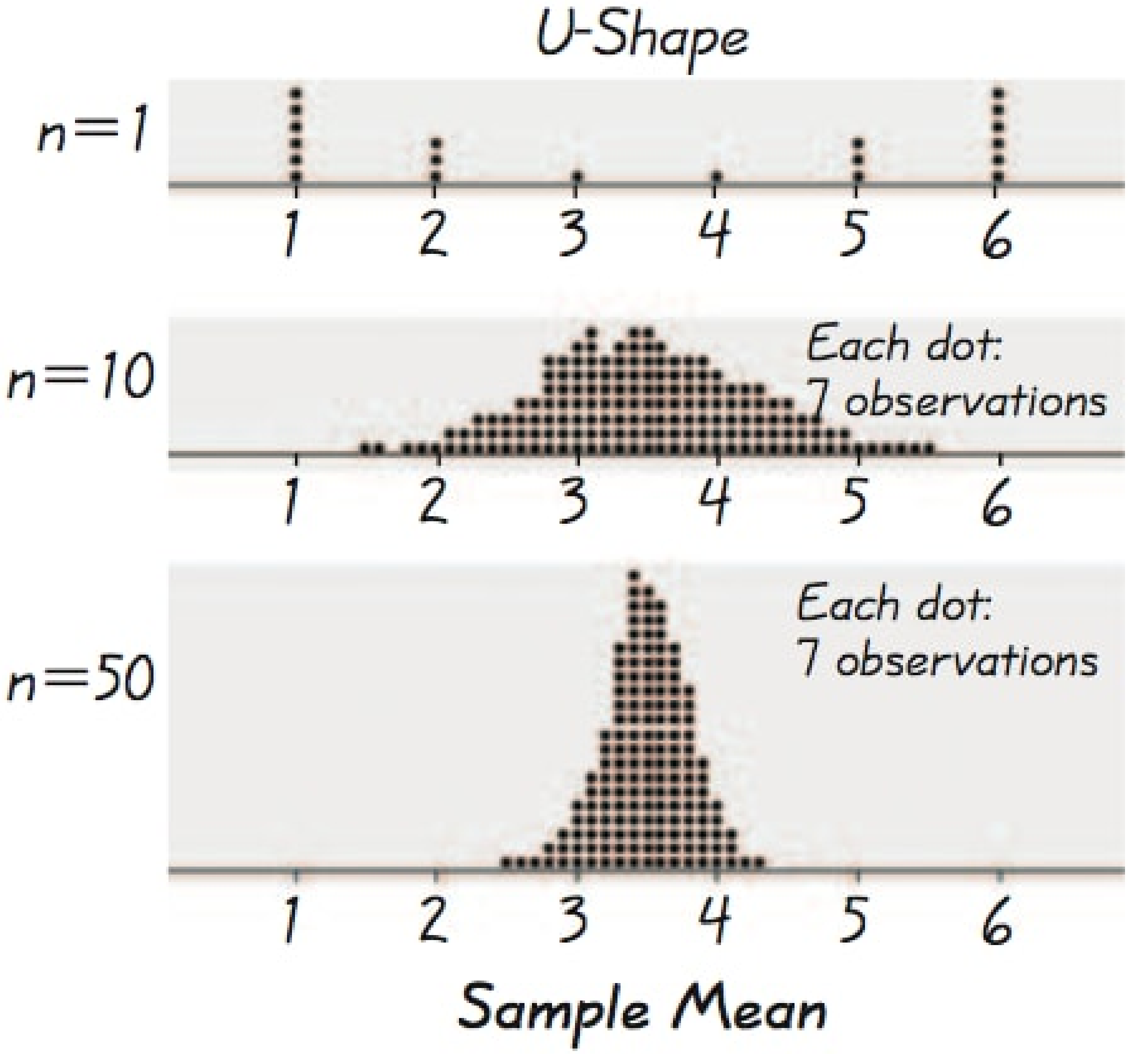
# Example - U-Shaped Distribution

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.





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# Important Point

As the sample size increases, the sampling distribution of sample means approaches a normal distribution.


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The Columbus Dispatch

# Ohio State to pay $1 million settlement in student's dorm-elevator death

Wednesday
Posted Apr 13, 2011 at 12:01 AM
Updated Apr 13, 2011 at 9:49 AM

## Ohio State University has agreed to pay $1 million to the family of a freshman student crushed by a residence-hall elevator more than four years ago.

Ohio State University has agreed to pay $1 million to the family of a freshman student crushed by a residence-hall elevator more than four years ago.

Andy "Polo" Polakowski died on Oct. 20, 2006, when he was crushed as he was trying to step off an elevator in Stradley Hall. His upper body was wedged between the ceiling of the elevator and the floor of the third-floor lobby as the elevator suddenly descended from the third floor with its doors open.

His parents, Janina and Kazimierz, sued the university in the Ohio Court of Claims three years after their only son's death. They said the school was negligent in maintaining, inspecting, repairing and operating the elevator.

A relative at the Polakowskis' home in Millcreek Township near Erie, Pa., said the family would not comment on the settlement.

In a statement yesterday, Ohio State called Andy Polakowski's death "a tragic loss" and said the university hoped the conclusion of the lawsuit would bring his family a measure of comfort.

"His passing serves as a continual reminder of the need for constant vigilance and education regarding student safety," the statement concluded.



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| Weights of Adults | | |
|---|---|---|
| | **Males** | Females |
| **μ** | 182.9lb | 165.0lb |
| **σ** | 40.8 | 45.6 |
| **Distrbution** | Normal | Normal |


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# Example – Elevators

Suppose an elevator has a maximum capacity of 16 passengers with a total weight of 2500 lb.

Assuming a worst case scenario in which the passengers are all male, what are the chances the elevator is overloaded?

Assume male weights follow a normal distribution with a mean of 182.9 lb and a standard deviation of 40.8 lb.

a. Find the probability that 1 randomly selected male has a weight greater than 156.25 lb.

b. Find the probability that a sample of 16 males have a mean weight greater than 156.25 lb (which puts the total weight at 2500 lb, exceeding the maximum capacity).


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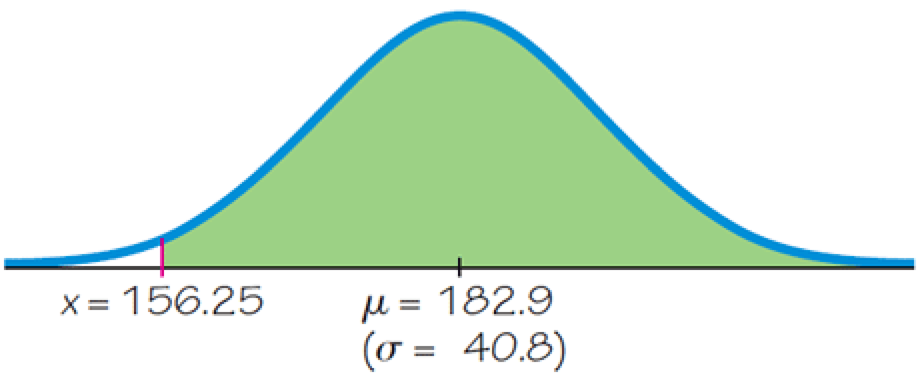
# Example – Elevators

a. Find the probability that 1 randomly selected male has a weight greater than 156.25 lb.

Use the methods presented in Section 6.3. We can convert to a *z* score and use Table A-2.

$$z = \frac{x - \mu}{\sigma} = \frac{156.25 - 182.9}{40.8} = -0.65$$

Using Table A-2, the area to the right is 0.7422.



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# Example – Elevators

b. Find the probability that a sample of 16 males have a mean weight greater than 156.25 lb.

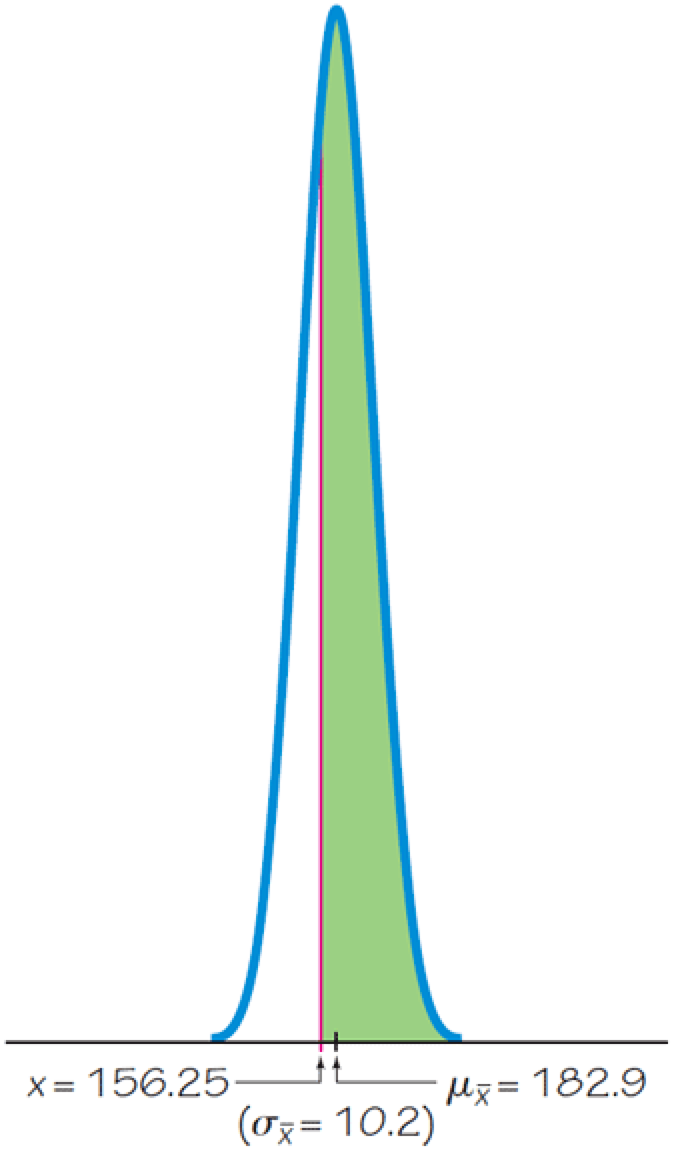
Since the distribution of male weights is assumed to be normal, the sample mean will also be normal.

$$\mu_{\bar{x}} = \mu_x = 182.9$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{40.8}{\sqrt{16}} = 10.2$$

Converting to *z*:

$$z = \frac{156.25 - 182.9}{10.2} = -2.61$$





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# Example – Elevators

b. Find the probability that a sample of 16 males have a mean weight greater than 156.25 lb.

While there is 0.7432 probability that any given male will weigh more than 156.25 lb, there is a 0.9955 probability that the sample of 16 males will have a mean weight of 156.25 lb or greater.

If the elevator is filled to capacity with all males, there is a very good chance the safe weight capacity of 2500 lb. will be exceeded.


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# Correction for a Finite Population

When sampling without replacement and the sample size $n$ is greater than 5% of the finite population of size $N$ (that is, $n > 0.05N$), adjust the standard deviation of sample means by multiplying it by the finite population correction factor:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \underbrace{\sqrt{\frac{N-n}{N-1}}}_{\text{finite population correction factor}}$$


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