Test 4 Review

Let $\log x = 2$ $\log y = 3$ $\log 2 \approx 0.3$

1) $\log 4 = \log(2 \cdot 2)$
$= \log 2 + \log 2$
$\approx 0.3 + 0.3$
≈ 0.6

2) $\log(2x^2y)$
$\log 2 + \log(x^2) + \log y$
$\log 2 + 2\log x + \log y$
$\approx 0.3 + 2 \cdot 2 + 3$
$\approx 0.3 + 4 + 3$
≈ 7.3

3) $\log_b\left(\frac{x^3y}{z^2}\right) = \log_b(x^3y) - \log z^2$

$= \log_b x^3 + \log_b y - \log z^2$

$= 3\log_b x + \log_b y - 2\log z$

4) $\log_5 \sqrt[3]{\frac{x^2y}{25}}$

$\log_5 \left(\frac{x^2y}{25}\right)^{\frac{1}{3}}$

$\frac{1}{3}\left[\log_5\left(\frac{x^2y}{25}\right)\right]$

$\frac{1}{3}\left[\log_5(x^2y) - \log_5 25\right]$

$\frac{1}{3}\left[\log_5 x^2 + \log_5 y - \log_5 25\right]$

$\frac{1}{3}\left[2\log_5 x + \log_5 y - 2\right]$

$\frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \frac{2}{3}$

5) $5\log_b x + 6\log_b y$

$\log_b x^5 + \log_b y^6$

$\log_b(x^5y^6)$

6) $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

$\frac{1}{2}(\log_5(xy)) - \log_5(x+1)^2$

$\log_5\sqrt{xy} - \log_5(x+1)^2$

$\log_5\left[\frac{\sqrt{xy}}{(x+1)^2}\right]$

7) $\log x + 6\log(x-4) - \log 15 - \log(x+2)$

$\log x + 6\log(x-4) - [\log 15 + \log(x+2)]$

$\log x + \log(x-4)^6 - [\log[15(x+2)]]$

$\log(x(x-4)^6) - \log(15x+30)$*

$\log\left[\frac{x(x-4)^6}{15x+30}\right]$

* Not needed, but saves space

9) $\log_\pi 63 = \frac{\log 63}{\log \pi} \approx 3.6193$

11) $2^{\log_2 2} = 2$

13) $\log (2\times10^{-4})$

$\log 2 + \log 10^{-4}$

$\log 2 - 4$

≈ -3.6990

15) $\log_x 25 = 2$

$x^2 = 25$

$x^2 = 5^2$

$x = 5$

17) $\ln x - 4\ln 3 = \ln \frac{5}{x}$

$\ln x - \ln 3^4 = \ln 5/x$

$\ln \frac{x}{81} = \ln 5/x$

$\frac{x}{81} = \frac{5}{x}$

$x^2 = 405$

$x = \sqrt{405}$

19) $4^{x+3} = 5^{2-x}$

$\ln 4^{x+3} = \ln 5^{2-x}$

$(x+3)\ln 4 = (2-x)\ln 5$

$x\ln 4 + 3\ln 4 = 2\ln 5 - x\ln 5$

$x\ln 4 + x\ln 5 = 2\ln 5 - 3\ln 4$

$x(\ln 4 + \ln 5) = \ln 5^2 - \ln 4^3$

$x = \frac{\ln(\frac{25}{81})}{\ln 20} \approx -0.3924$

8) $\log_5 13 = \frac{\ln 13}{\ln 5} \approx 1.5937$

10) $\log_3(\log_2 8)$

$\log_3(3)$

1

12) $\log(2\times10^4) = \log(20000)$

or $\log 2 + \log 10^4$

$\log 2 + 4$

≈ 4.3010

14) $\ln(2\times e^4)$

$\ln 2 + \ln e^4$

$\ln 2 + 4$

≈ 4.6931

16) $\log x + \log(x-3) = 1$

$\log(x^2-3x) = 1$

$x^2-3x = 10^1$

$x^2-3x-10 = 0$

$(x+2)(x-5) = 0$

$x = -2, 5$

-2 doesn't work

Check $\log 5 + \log(5-3) \stackrel{?}{=} 1$

$\log 5 + \log(2) \stackrel{?}{=} 1$

$\log 10 = 1$ ✓

18) $5^{x+1} = 7^x$

$\ln(5^{x+1}) = \ln(7^x)$

$(x+1)\ln 5 = x\ln 7$

$x\ln 5 + \ln 5 = x\ln 7$

$\ln 5 = x\ln 7 - x\ln 5$

$\ln 5 = x(\ln 7 - \ln 5)$

$\ln 5 = x\ln 7/5$

$\frac{\ln 5}{\ln 7/5} = x$

$x \approx 4.7833$

20) $A = P(1+\frac{r}{n})^{nt}$ $A = \$60000$

$r = 7\%$ quarterly quarterly 4 times 5 years

$A = \$60000\left(1+\frac{0.07}{4}\right)^{4\cdot 5}$

$= 60000(1+0.0175)^{20}$

$= \$84886.69$

Compounded is better. $800.31 more

$A = Pe^{rt}$

$t = 6.75$

$A = 60000\, e^{(0.0675)(5)}$

84086.38

21) $2A = Pe^{0.025t}$

$2 = e^{0.025t}$

$\ln 2 = 0.025t \ln e$

$\frac{\ln 2}{0.025} = t$

$27.7259 \approx t$

23) a) $2^5 = 32$

$\log_2 32 = 5$

b) $100^{\frac{1}{2}} = 10$

$\log_{100} 10 = \frac{1}{2}$

c) $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

$\log_{\frac{3}{4}} \frac{4}{3} = -1$

22) $y = y_0 e^{kt}$

first find k, 5% per day means that at $t = 1$ $y = 0.95y_0$

$570 = 600\, e^{k\cdot 1}$

$\frac{570}{600} = e^k$

$0.95 = e^k$

$\ln 0.95 = \ln e^k$

$\ln 0.95 = k$

$-0.0513 = k$

a) $y = 600\, e^{-0.0513k}$

b) $t = 12$

$y = 600\, e^{-0.0513(12)}$

$y = 324.2$

c) $y = 0.5y_0$

$y = 0.5(900)$

$y = 450$

NOW find t

$450 = 900\, e^{-0.0513t}$

$0.5 = e^{-0.0513t}$

$\ln(0.5) = \ln(e^{-0.0513t})$

$\ln(0.5) = -0.0513t \ln e$

$\frac{\ln 0.5}{-0.0513} = t$

$13.5 = t$

24) $\log_9 27 = \frac{3}{2}$

a) $9^{\frac{3}{2}} = 27$

b) $\log 3.45 \approx .5378$

$10^{0.5378} \approx 3.45$

c) $\ln 45 \approx 3.8067$

$e^{3.8067} \approx 45$

25) $\log_2 x + \log_2 (x+2) = 3$

$\log_2 [x^2 + 2x] = 3$

$x^2 + 2x = 2^3$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \quad x = 2$

$x = -4$ doesn't work

$x = 2$ does

$\{2\}$

26) $\{(-5, 1), (-5, -18), (-1, 17)\}$

27) yes

28) $f^{-1}(x)$ is the inverse of f

so $f(f^{-1}(4)) = 4$

29) See below

30) $f(x) = \sqrt{x+9}$

$y = \sqrt{x+9}$

$x = \sqrt{y+9}$

$x^2 = y + 9$

$x^2 - 9 = y$

$f^{-1}(x) = x^2 - 9$

31) $f(x) = 4x - 32$

$y = 4x - 32$

$x = 4y - 32$

$x + 32 = 4y$

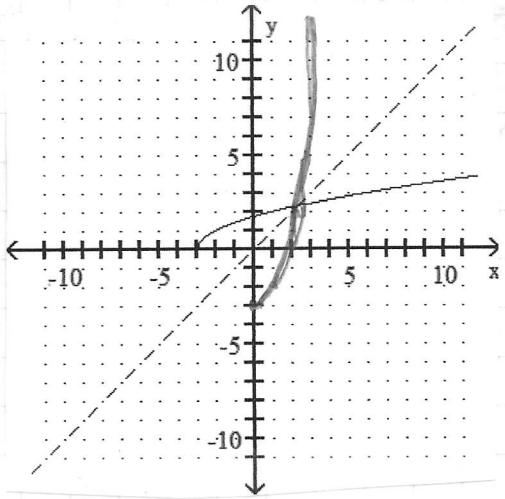
$4y = x + 32$

$y = \frac{1}{4}x + 8$

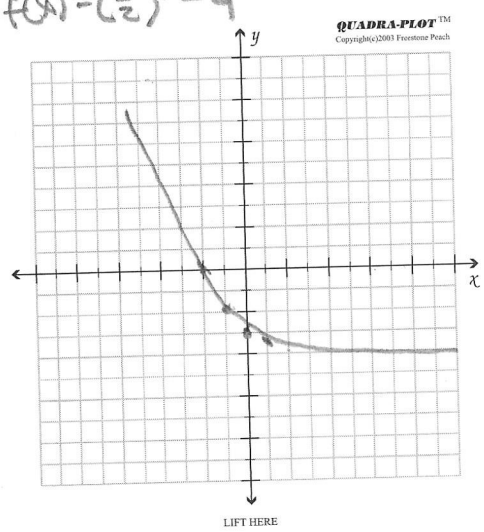
$f^{-1}(x) = \frac{1}{4}x + 8$

32) $f(x) = \frac{1}{x+2}$ find inverse

29)

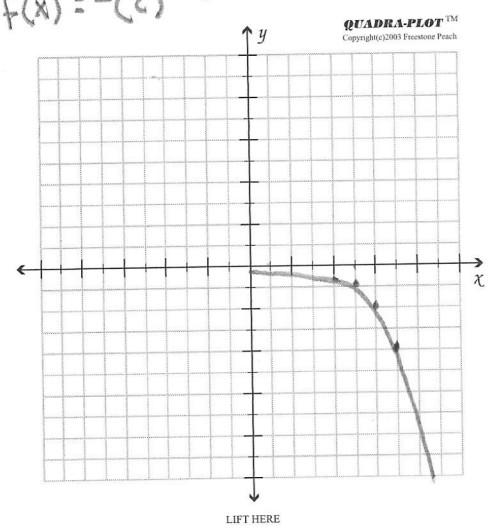


33 $f(x) = (\frac{1}{2})^x - 4$



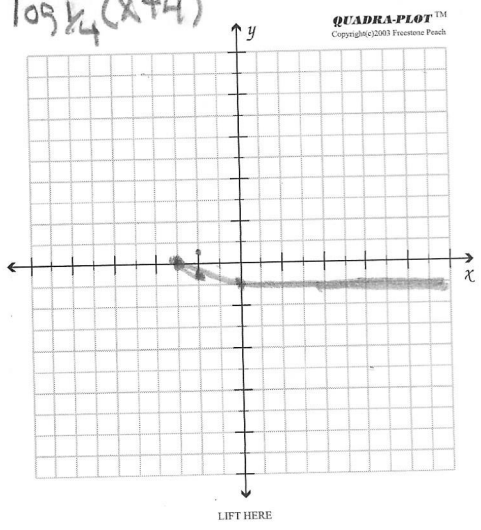
x	f(x)
0	−3
1	$-3\frac{1}{2}$
2	$-3\frac{3}{4}$
−1	−2
−2	0

34) $f(x) = -(2)^{x-5}$



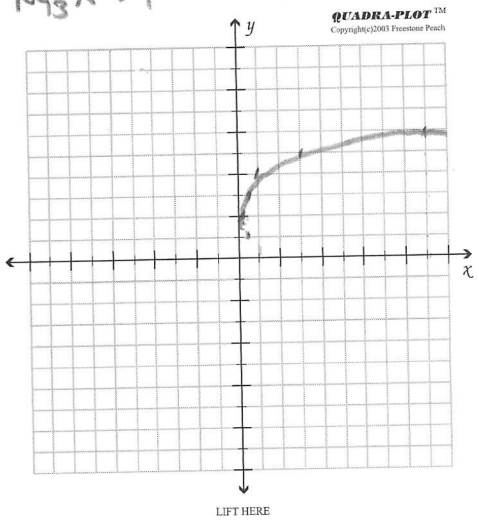
x	f(x)
5	−1
6	−2
7	−4
4	$-\frac{1}{2}$
3	$-\frac{1}{4}$

35) $\log_{\frac{1}{4}}(x+4)$



x	f(x)
−3	0
−2	$-\frac{1}{2}$
−0	−1
12	−2

36) $\log_3 x + 4$



x	$\log_3 x$	$\log_3 x + 4$
$\frac{1}{9}$	−2	2
$\frac{1}{3}$	−1	3
1	0	4
3	1	5
9	2	6