Lecture Slides

Essentials of Statistics
5th Edition

and the Triola Statistics Series

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Chapter 4
Probability

Key Concept

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

If the outcome of the first event A somehow affects the probability of the second event B, it is important to adjust the probability of B to reflect the occurrence of event A.

Notation

$P(A$ and $B) =$

$P($event A occurs in a first trial and

   event B occurs in a second trial)

$P(B \mid A)$ represents the probability of event B occurring after event A has already occurred.

Formal Multiplication Rule

❖ $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Intuitive Multiplication Rule

When finding the probability that event *A* occurs in one trial and event *B* occurs in the next trial, multiply the probability of event *A* by the probability of event *B*, but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

Caution

When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly.

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Multiplication Rule for Several Events

In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.

Dependent and Independent

Two events *A* and *B* are independent if the occurrence of one does not affect the *probability* of the occurrence of the other.

(Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.)

If *A* and *B* are not independent, they are said to be dependent.

Dependent Events

Two events are dependent if the occurrence of one of them affects the *probability* of the occurrence of the other, but this does not necessarily mean that one of the events is a *cause* of the other.

Treating Dependent Events as Independent

Some calculations are cumbersome, but they can be made manageable by using the common practice of treating events as independent when small samples are drawn from large populations. In such cases, it is rare to select the same item twice.

The 5% Guideline for Cumbersome Calculations

If a sample size is no more than 5% of the size of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).

Example

Suppose 50 drug test results are given from people who use drugs:

Positive Test Results:	44
Negative Test Results:	6
Total Results:	50

If 2 of the 50 subjects are randomly selected *without replacement*, find the probability that the first person tested positive and the second person tested negative.

Example – continued

If 2 of the 50 subjects are randomly selected *without replacement*, find the probability that the first person tested positive and the second person tested negative.

Positive Test Results:	44
Negative Test Results:	6
Total Results:	50

$$P\left(\text{positive test result for first person}\right) = \frac{44}{50}$$

$$P\left(\text{negative test result for second person}\right) = \frac{6}{49}$$

$$P\left(\text{1st selection is positive and 2nd is negative}\right) = \frac{44}{50} \cdot \frac{6}{49} = 0.108$$

Example

When two different people are randomly selected from those in your class, find the indicated probability by assuming birthdays occur on the same day of the week with equal frequencies.

a. Probability that two people are born on the same day of the week.

b. Probability that two people are both born on Monday.

Example – continued

a. Probability that two people are born on the same day of the week.

Because no particular day is specified, the first person can be born on any day. The probability that the second person is born on the same day is 1/7, so the probability both are born on the same day is 1/7.

b. Probability that two people are both born on Monday.

The probability the first person is born on Monday is 1/7, and the same goes for the second person. The probability they are both born on Monday is: $\dfrac{1}{7} \cdot \dfrac{1}{7} = \dfrac{1}{49}$.

Tree Diagrams

A tree diagram is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are sometimes helpful in determining the number of possible outcomes in a sample space, if the number of possibilities is not too large.

Tree Diagrams

This figure summarizes the possible outcomes for a true/false question followed by a multiple choice question.

Note that there are 10 possible combinations.
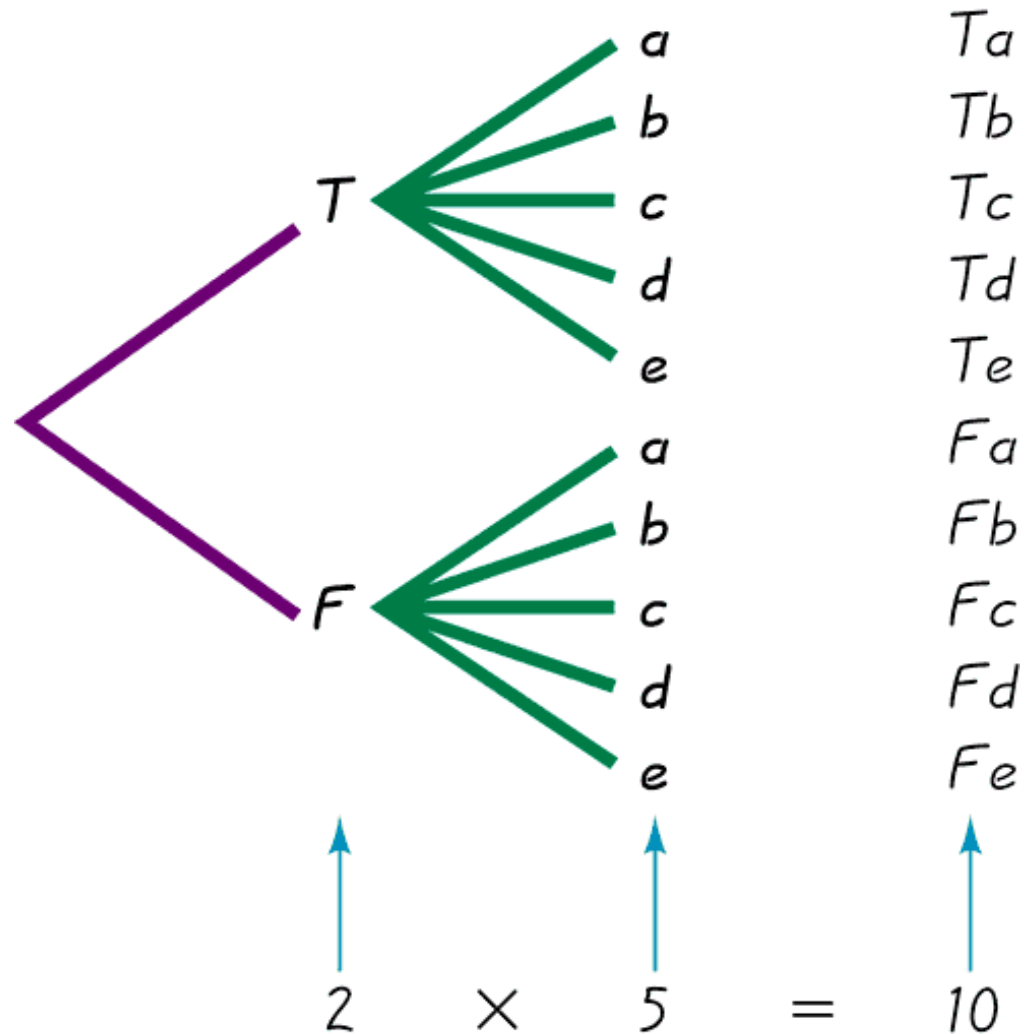
Summary of Fundamentals

❖ In the addition rule, the word "or" in $P(A$ or $B)$ suggests addition. Add $P(A)$ and $P(B)$, being careful to add in such a way that every outcome is counted only once.

❖ In the multiplication rule, the word "and" in $P(A$ and $B)$ suggests multiplication. Multiply $P(A)$ and $P(B)$, but be sure that the probability of event B takes into account the previous occurrence of event A.

Applying the Multiplication Rule

Start

P(A and B)
Multiplication rule

Are
A and B
independent
?

Yes
P(A and B) = P(A) · P(B)

No
P(A and B) = P(A) · P(B | A)

Be sure to find the probability of
event B by assuming that
event A has already occurred.