# REPRODUCIBLE ACTIVITIES

The activities in this section are designed to be self-contained units of work. They can be used in class or out of class, as assignments to be completed individually by students or in groups, or as long-term projects that can be completed by students individually or in groups.

The activities are intended to enrich the presentation in the textbook and to guide the students through an investigative or discovery process. It is hoped that, after having completed an activity, a student will have a more in-depth understanding of the underlying mathematical principles and will have developed a richer number sense.

The following activities are available in this supplement. The authors are continually exploring new activities and as they are refined they will be made available in future editions. More information on when and how to use each activity is given in the teaching tips.

**TRANSPOSING DIGITS (p. 18)**

Outcome: Verify that the difference between two numbers that have two digits transposed is divisible by nine.

**FACTORING INTO PAIRS OF FACTORS (pp. 19–22)**

Outcome 1: Find all possible pairs of factors of a given number.
Outcome 2: Distinguish between prime and composite numbers.
Outcome 3: Find numbers that are perfect squares.
Outcome 4: Find the square root of a number using a calculator.

**LOCATING COORDINATES ON A SPHERE (pp. 23–25)**

Outcome: Find locations on a sphere using ordered pairs.

**ARRANGING GLOBAL INTERACTIVE COMMUNICATIONS (p. 26)**

Outcome: Determine an optimum time for a global interactive communication.

**PREPARING A REFERENCE CHART FOR GLOBAL COMMUNICATIONS (p. 27)**

Outcome: Gather and organize data to improve efficiency.

**COMMON FRACTIONS (p. 28)**

Outcome: Distinguish between proper and improper common fractions.

**DECIMAL FRACTIONS (p. 29)**

Outcome: Examine decimal equivalents of common fractions.

**FRACTIONS IN SIMPLEST FORM (p. 30)**

Outcome: Examine equivalent fractions in simplest form.

**FRACTION RELATIONSHIPS (pp. 31–34)**

Outcome: Develop a procedure for comparing fractions to 1, $\frac{1}{2}$, or $\frac{1}{4}$ by inspection.

**SIZE OF FRACTIONS (pp. 35–37)**

Outcome: Categorize fractions based upon their relationship to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.

**ANALYZING NUTRITION LABELS (pp. 38–42)**

Outcome: Use proportions and percents to analyze nutrition labels.

**WHAT PERCENT TAX DO I REALLY PAY? (pp. 43–45)**

Outcome 1: Determine the percent of tax withheld for a given taxable income.
Outcome 2: Determine the percent of income tax for a given annual taxable income.

**CIRCLE, BAR, AND LINE GRAPHS (p. 46)**

Outcome 1: Interpret data from various types of graphs.
Outcome 2: Plan, design, and conduct an investigation in which the findings are reported in writing and graphically.

**CRITIQUING GRAPHS (pp. 47)**

Outcome: Critique graphs found in recent publications.

**ESTIMATING RADICALS (pp. 48–49)**

**Squares and Square Roots**

Outcome 1: Distinguish between squares and square roots and rational and irrational numbers.
Outcome 2: Determine between which two whole numbers a given irrational number will fall.

**RATIONAL EQUATIONS (p. 50)**

**Patient Charting Efficiency**

Outcome: Use rational equations in career applications.

**QUADRATIC EQUATIONS (pp. 51–52)**

**Calculating Vehicular Speed from Skid Marks and Road Conditions**

Outcome: Use quadratic equations in career applications.

**WHAT IS THE NATURAL EXPONENTIAL *e*? (p. 53)**

Outcome: Discover the effect of large values of $n$ in the expression $\left(1+\frac{1}{n}\right)^n$.

**COMPOUNDED AMOUNT AND COMPOUND INTEREST (p. 54)**

Outcome: Compare the two compound amount formulas for compound interest,

$$A = p\left(1+\frac{r}{n}\right)^{nt} \text{ and } A = pe^{nt}.$$

**GRAPHING EQUATIONS PROJECT (pp. 55–56)**

**Instructions for Group Projects for Five-Member Groups**

**GRAPHING EQUATIONS (pp. 57–65)**

**Graphing Activity 1**

Outcome: Examine equations in the form $y = mx$.

**Graphing Activity 2**

Outcome: Examine equations in the form $y = mx + b$.

**Graphing Activity 3**

Outcome: Examine equations in the form $y = k$ and $x = k$.

**Graphing Activity 4**

Outcome: Examine quadratic equations in the form $y = ax^2$.

**Graphing Activity 5**

Outcome: Examine quadratic equations in the form $y = ax^2 + b$.

**Graphing Activity 6**

Outcome: Examine quadratic equations in the form $y = (x+b)^2$.

**GRAPHICAL REPRESENTATION (p. 66)**

**Commodities Market Investing**

Outcome: Use graphing in consumer applications.

**SYSTEMS OF EQUATIONS (p. 67)**

**Making Business Choices**

Outcome: Use systems of equations to make good business choices.

**ESTIMATING MEASURES (pp. 68–69)**

Outcome: Estimate linear and circular measure in inches.

**WHAT IS PI, $\pi$ ? (p. 70)**

Outcome: Discover the relationship between the circumference and diameter of a circle.

## TRANSPOSING DIGITS

**Outcome: Verify that the difference between two numbers that have two digits transposed is divisible by nine.**

We have been told that the difference between two numbers that have two digits transposed is divisible by nine. Is this always true? Does it matter how many digits are in the number or if more than one pair of digits are transposed?

*Find the difference between the following numbers. To avoid using negative numbers, subtract the smaller number from the larger number. Then divide the difference by nine.*

1. 58 and 85
2. 72 and 27
3. 36 and 63
4. 285 and 825
5. 285 and 258
6. 417 and 147
7. 417 and 471
8. 3842 and 8342
9. 3842 and 3482
10. 3842 and 3824
11. 3842 and 8324
12. 13,574 and 31,574
13. 13,574 and 15,374
14. 13,574 and 13,754
15. 13,574 and 13,547
16. 13,574 and 31,754
17. 13,574 and 31,547
18. 13,574 and 15,347

*Summarize your conclusions to the following questions:*
Is it generally true that the difference between two numbers that have two digits transposed is divisible by nine?

Does it matter how many digits are in the number?

Does it matter if more than one pair of digits are transposed?

Verify your conclusions with additional examples.

*Summarize what you learned in this activity:*

## FACTORING INTO PAIRS OF FACTORS

**Outcome 1: Find all possible pairs of factors of a given number.**

Numbers that are multiplied together are called *factors*. The result of the multiplication is called the *product*. In this activity we will limit ourselves to looking at pairs of factors, or two numbers that multiply together to give a particular product. Some numbers may have only one pair of factors while other numbers may have two, three, or more pairs of factors.

We will list pairs of factors of a given number by following a pattern. Start with the number 1. Can it pair with another number to result in a product of the given number? Try 2. Try 3. Continue until you reach the given number.

EXAMPLE 1. List all pairs of factors of 12.

| 12 | |
|---|---|
| 1, | 12 |
| 2, | 6 |
| 3, | 4 |
| 4, | 3 |
| 6, | 2 |
| 12, | 1 |

Since multiplication is commutative, the pairs 3, 4 and 4, 3 are the same. Similarly 2, 6 and 6, 2 are the same, and 1, 12 and 12, 1 are the same.

We can modify our procedure for listing all pairs of factors by replacing the statement, "Continue until you reach the given number" with the following statement: *Continue until the same pair of numbers appears but in the opposite order*. For example, when listing the pairs of factors of 12, if one pair is 3 and 4 and another pair is 4 and 3, all remaining pairs of factors will also be duplicates of other pairs of factors that are already listed. Another way to determine when factors are beginning to repeat is when the first factor is larger than the second. This observation applies only when you begin with 1 and examine every number.

**1.** How many different pairs of factors are there for the number 12? List them.

**2.** State the divisibility rule that shows that 5 cannot pair with another whole number to give a product of 12.

Look at Example 2.

EXAMPLE 2. List all pairs of factors of 210.

| 210 | |
|---|---|
| 1, | 210 |
| 2, | 105 |
| 3, | 70 |
| 5, | 42 |
| 6, | 35 |
| 7, | 30 |
| 10, | 21 |
| 14, | 15 |

**3.** Using individual and com bined divisibility rules, state a reason why each of the following given numbers cannot pair with another whole number to give a product of 210.

(a) 4 (b) 8

(c) 9 (d) 11

(e) 12 (f) 13

(g) 16

**4.** How do you know that ALL the pairs of factors of 210 have been listed?

**5.** For each number listed below, list all pairs of factors of the number.

| | | | | | |
|---|---|---|---|---|---|
| 15 | 24 | 18 | 36 | 13 | 16 |
| 20 | 22 | 17 | 28 | 30 | 23 |
| 25 | 1 | 6 | 8 | 40 | 48 |

*Summarize what you learned about factor pairs.*

**Prime and Composite Numbers**

**Outcome 2: Distinguish between prime and composite numbers.**

A whole number greater than 1 whose only pair of factors is 1 and the number is said to be *prime*. All other whole numbers are said to be *composite*. The smallest prime is 2. Then, all multiples of 2 (4, 6, 8, . . .) are composite. Next, 3 is prime and multiples of 3 (6, 9, 12, . . .) are composite.

**6.** Circle each prime number in the following list of numbers. Then, remove all multiples of the prime by putting a slash through the number.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
| 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 |
| 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | | | | | |

**7.** List the prime numbers that are less than 100.

**8.** List all pairs of factors for the following numbers.

| | | | | | |
|---|---|---|---|---|---|
| 100 ____ | 125 | 132 | 230 | 248 | 143 |
| 144 ____ | 121 | 195 | 350 | 278 | 203 |

*Summarize what you have learned about prime and composite numbers.*

When a number has several different pairs of factors, it is often desirable to find a particular pair that has a certain property. Questions 9 and 10 give some practice in this concept.

**9.** Looking at the lists of factors for the numbers in Exercise 5, find a pair of factors that meet each stated condition.

(a) factors of 24 whose sum is 11
(b) factors of 36 whose difference is 5
(c) factors of 18 whose sum is 9
(d) factors of 28 whose difference is 12
(e) factors of 36 that are the same number

**10.** Looking at the lists of factors for the numbers in Exercise 8, find a pair of factors that meet the stated conditions.

(a) factors of 132 whose difference is 1
(b) factors of 230 whose sum is 33
(c) factors of 350 whose sum is 39
(d) factors of 195 whose difference is 80
(e) factors of 144 that are the same number

**Perfect Squares**

**Outcome 3: Find numbers that are perfect squares.**

When a number has a pair of factors that are the same number, such as $6 \times 6 = 36$, this product is called a *perfect square*.

**11.** Circle each perfect square in the following list of numbers.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
| 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 |
| 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | | | | | |

**12.** For each circled number in Exercise 11, give the pair of like factors. Example: $1 = 1 \times 1$, $4 = 2 \times 2$, etc.

**13.** Extend your list of perfect squares to include all perfect squares between 100 and 1000.

**Square Roots**

**Outcome 4: Find the square root of a number using a calculator.**

A perfect square has a factor pair of identical factors. The number in the identical factors is the principal *square root* of the perfect square. Other numbers also have square roots, but the square roots are not whole numbers. To find the approximate value of a number that is not a perfect square, use your calculator and the square root key $\boxed{\sqrt{\ }}$.

In listing pairs of factors of a given number, how can we be certain we have them all? We have been testing every whole number until we get a pair with the larger factor first or until a pair of factors is repeated but in the opposite order. Let's see if we can find another procedure.

**14.** Use your calculator to find the square root of each number in Exercise 8.

**15.** For each number in Exercise 8, compare the factor pair that has the largest first factor with the square root of the original number.

**16.** Make a generalization about when to be sure you have found all factor pairs of a number.

Test your generalization from Exercise 16 with the following numbers. Find all pairs of factors of each number.

**17.** 85 **18.** 120 **19.** 136 **20.** 225

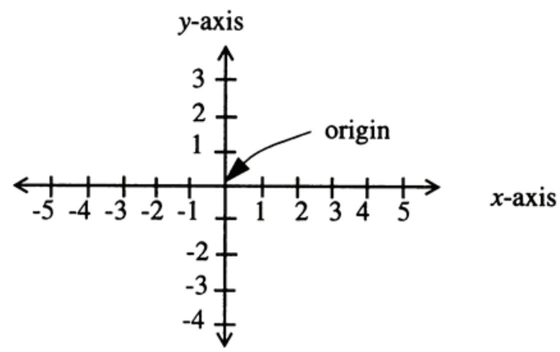
*Summarize what you learned in this activity.*

## LOCATING COORDINATES ON A SPHERE

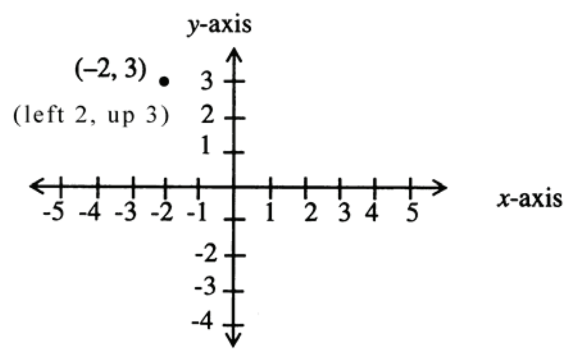
**Outcome: Find locations on a sphere using ordered pairs.**

Materials: World globe

Two number lines can be placed together by aligning zero on each line and making the lines meet to form a square corner. These two number lines are often referred to as the *rectangular coordinate system*. The zero point where the two lines meet is called the *origin*. The horizontal line is called the *x-axis* and the vertical line is the *y-axis*.

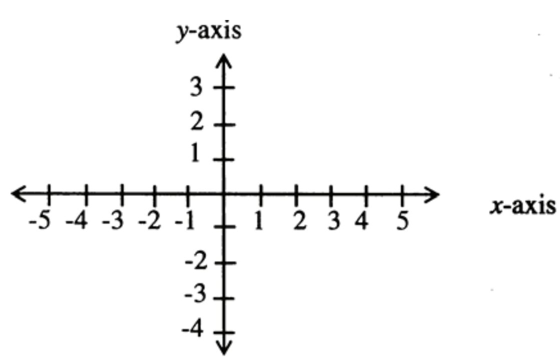



A point in a rectangular coordinate system is located with two directional numbers. The first shows the amount of *horizontal movement* and the second shows the vertical movement. The point indicated by (–2, 3) means that you move two units from the origin to the left. Then, from that point, move three units up.




**1.** Locate the following points on a rectangular coordinate system.

A = (3, –1) B = (–2, –3) C = (–1, 5) D = (4, 2)



On a world globe we have a spherical coordinate system for locating points. Using resources such as an encyclopedia, a world atlas, or the Internet, develop a strategy for locating points on a world globe.

**2.** Locate the following reference points on a world globe.

Equator    Prime Meridian    International Date Line

**3.** How are these points similar to the *x*- and *y*-axes on a rectangular coordinate system?

Equator:

Prime Meridian:

International Date Line:

**4.** What point on a world globe is similar to the origin on a rectangular coordinate system?

**5.** Locate the following reference points on a world globe.

North Pole    South Pole

**6.** How are the directions North, South, East, and West used to identify locations on a world globe?

North:

South:

East:

West:

**7.** How do the four directions compare to the positive and negative directions on the *x*- and *y*-axes on a coordinate plane?

**8.** How is locating points on a sphere different from locating points on a coordinate plane?

**9.** Locate the following cities using the given coordinates.

Juneau, Alaska: 58N/135W    Rio de Janeiro, Brazil: 23S/43W

London, United Kingdom: 51N/0    Phnom Penh, Cambodia: 12N/105E

**10.** Write your strategy for locating a point on a globe.

**11.** Why is this skill useful in the world of business?

Next, find familiar locations on a globe and write these coordinates.

**12.** Give the coordinates of the following cities.

| | Latitude | Longitude |
|---|---|---|
| Memphis | ______ | ______ |
| Seattle | ______ | ______ |
| Miami | ______ | ______ |
| Tokyo | ______ | ______ |
| Nairobi | ______ | ______ |

**13.** What are some resources available for finding the coordinates of global locations?

**14.** Look up the coordinates of five cities and locate the cities on a world globe.

| | City, Country | Coordinates |
|---|---|---|
| City 1: | ____________ | ______ |
| City 2: | ____________ | ______ |
| City 3: | ____________ | ______ |
| City 4: | ____________ | ______ |
| City 5: | ____________ | ______ |

*Summarize what you learned in this activity:*

## ARRANGING GLOBAL INTERACTIVE COMMUNICATIONS

**Outcome: Determine an optimum time for a global interactive communication.**

Materials: World globe, world time zone chart

An interactive teleconference must be arranged for five directors at FedEx. Each director works a normal 8:00 AM to 5:00 PM day. Select a time for the teleconference that fits within everyone's normal working hours, if possible. If not, select a time that will be most convenient for the largest number of directors.

Location of Directors:

| | | |
|---|---|---|
| Memphis, TN | Toronto, Canada | Frankfurt, Germany |
| Tokyo, Japan | San Juan, Puerto Rico | |

Describe your plan for accomplishing this task.

Time for Teleconference for Each Director:

Memphis, TN: ____________

Toronto, Canada: ____________

Frankfurt, Germany: ____________

Tokyo, Japan: ____________

San Juan, Puerto Rico: ____________

Comment on any inconveniences this teleconference might cause.

*Summarize what you learned in this activity:*

## PREPARING A REFERENCE CHART FOR GLOBAL COMMUNICATIONS

**Outcome: Gather and organize data to improve efficiency.**

Materials: World globe, world time zone chart

Businesses that operate throughout the world have to be constantly aware of time differences when communicating with personnel at other locations. An employee at FedEx has been asked to devise a reference guide that other employees may use to quickly determine the time in another part of the world. The guide will give the number of hours to add to or subtract from the local time to determine the time at each listed city. The cities to be referenced are given below. Design a matrix that organizes this information.

Memphis, TN    Spokane, WA    Anchorage, AK    Brussels, Belgium
Lima, Peru    Maui, HI    Sumatra, Indonesia    Manila, The Philippines
Da Nang, Vietnam    Kuala Lumpur, Malaysia

| TO<br>FROM | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |
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Can the strategy used in developing this ten-city guide be used to develop a guide that includes more cities? Explain.

*Summarize what you learned in this activity:*

## COMMON FRACTIONS

**Outcome: Distinguish between proper and improper common fractions.**

Complete the following chart by writing a fraction in each block with the numerator given at the top of the colum n and with the denominator given at the left of each row. Do not simplify any of the fracti ons. This chart will be used as a guide for completing charts that follow.

| Denominator \ Numerator | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $\frac{0}{0}$ | $\frac{1}{0}$ | $\frac{2}{0}$ | | | | | | | |
| 1 | $\frac{0}{1}$ | $\frac{1}{1}$ | | | | | | | | |
| 2 | $\frac{0}{2}$ | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |

Each fraction on this chart except those with a denominator of zero is called a *common fraction*. When the numerator is smaller than the denominator, the common fraction is called a *proper fraction*. When the numerator is the same as or larger than the denominator, the common fraction is called an *improper fraction*.

**1.** Lightly shade or highlight the portion of the chart containing improper fractions.

**2.** Describe in words the portion of the chart that was shaded.

*Summarize what you learned in this activity:*

## DECIMAL FRACTIONS

**Outcome: Examine decimal equivalents of common fractions.**

Complete the following chart by using the common fractions chart and writing each fraction in the equivalent decimal form. Use your calculator to change each fraction to the equivalent decimal form.

*To change a common fraction to an equivalent decimal fraction, divide the numerator by the denominator.* Write exactly what is displayed in your calculator window in each appropriate block.

| | Numerator | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Denominator | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0 | | | | | | | | | | |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |

**1.** Describe your entries in the first row on the chart. What does this mean? Why?

**2.** Describe your entries in the first column on the chart. What does this mean? Why?

**3.** Describe the location of common fractions whose decimal equivalent is 1. When is a common fraction equivalent to 1?

**4.** Describe the location of common fractions whose decimal equivalent is less than 1. When is a common fraction less than 1?

**5.** Describe the location of common fractions whose decimal equivalent is greater than 1. When is a common fraction greater than 1?

**6.** List all groups of common fractions that have the same decimal equivalent. For example, $\frac{1}{2} = 0.5$, $\frac{2}{4} = 0.5$, $\frac{3}{6} = 0.5$, $\frac{4}{8} = 0.5$.

## FRACTIONS IN SIMPLEST FORM

**Outcome: Examine equivalent fractions in simplest form.**

Complete the following chart by using the common fractions chart and writing each common fraction in the simplest form. Proper fractions should be reduced to lowest terms by dividing the numerator and denominator by the same amount. Improper fractions should be changed to an equivalent whole or mixed number by dividing the numerator by the denominator.

Numerator

| Denominator | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | | |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |

**1.** Describe your entries in the first row on the chart. What does this mean? Why?

**2.** Describe your entries in the first column on the chart. What does this mean? Why?

**3.** Describe the location of common fractions equivalent to 1. When is a fraction equivalent to 1?

**4.** Describe the location of all proper fractions in simplest form.

**5.** Describe the location of all improper fractions in simplest form.

**6.** How can you tell if a common fraction is a proper fraction or an improper fraction?

**7.** How can you tell if an improper fraction in simplest form will be a whole number or a mixed number?

**8.** List all groups of common fractions that are equivalent to the same fraction in simplest form. For example,
$\frac{2}{4}=\frac{1}{2}, \frac{3}{6}=\frac{1}{2}, \frac{4}{8}=\frac{1}{2}$.

## FRACTION RELATIONSHIPS

**Outcome: Develop a procedure for comparing fractions to 1, $\frac{1}{2}$, or $\frac{1}{4}$ by inspection.**

Complete this activity by using a calculator to determine the decimal equivalent of a fraction. To find the decimal equivalent, divide the numerator by the denominator.

**1.** Circle the fractions from the list below that have a value *less than 1*.

$$\frac{2}{3} \quad \frac{7}{8} \quad \frac{3}{2} \quad \frac{4}{5} \quad \frac{8}{5} \quad \frac{3}{3} \quad \frac{9}{10} \quad \frac{12}{6} \quad \frac{9}{8} \quad \frac{4}{9} \quad \frac{6}{6} \quad \frac{8}{2}$$

$$\frac{0}{8} \quad \frac{5}{1} \quad \frac{6}{7} \quad \frac{15}{16} \quad \frac{9}{6} \quad \frac{4}{9} \quad \frac{2}{1} \quad \frac{15}{4} \quad \frac{7}{3} \quad \frac{2}{5} \quad \frac{7}{10} \quad \frac{6}{5}$$

**2.** Circle the fractions from the list below that have a value *greater than 1*.

$$\frac{1}{4} \quad \frac{5}{6} \quad \frac{4}{5} \quad \frac{5}{4} \quad \frac{8}{8} \quad \frac{7}{3} \quad \frac{7}{15} \quad \frac{11}{12} \quad \frac{3}{8} \quad \frac{9}{4} \quad \frac{1}{6} \quad \frac{1}{2}$$

$$\frac{9}{3} \quad \frac{11}{12} \quad \frac{4}{9} \quad \frac{3}{2} \quad \frac{4}{2} \quad \frac{8}{11} \quad \frac{14}{6} \quad \frac{5}{5} \quad \frac{8}{2} \quad \frac{5}{7} \quad \frac{7}{5} \quad \frac{9}{8}$$

**3.** Write a rule for deciding when a fraction is less than 1.

**4.** Write a rule for deciding when a fraction is greater than 1.

**5.** Write a rule for deciding when a fraction is equal to 1.

**6.** Classify the following fractions as less than 1, great er than 1, or equal to 1, according to your rules in Exercises 3, 4, and 5. Check answers using a calculator.

$$\frac{34}{28} \qquad \frac{23}{25} \qquad \frac{85}{90} \qquad \frac{48}{12}$$

$$\frac{63}{72} \qquad \frac{47}{47} \qquad \frac{45}{50} \qquad \frac{25}{23}$$

The decimal equivalent of $\frac{1}{2} = 0.5$. Complete the following using a calculator to examine the decimal equivalent of each fraction.

**7.** Circle the fractions from the list below that have a value *less than* $\frac{1}{2}$.

$\frac{3}{4}$ $\frac{1}{3}$ $\frac{5}{8}$ $\frac{7}{15}$ $\frac{25}{40}$ $\frac{18}{23}$ $\frac{9}{16}$ $\frac{4}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{7}{12}$

$\frac{5}{12}$ $\frac{4}{9}$ $\frac{6}{10}$ $\frac{6}{7}$ $\frac{5}{7}$ $\frac{3}{12}$ $\frac{9}{20}$ $\frac{8}{16}$ $\frac{5}{11}$ $\frac{19}{40}$ $\frac{23}{42}$

**8.** Circle the fractions from the list below that have a value *greater than* $\frac{1}{2}$.

$\frac{3}{5}$ $\frac{1}{4}$ $\frac{5}{6}$ $\frac{7}{10}$ $\frac{25}{45}$ $\frac{15}{33}$ $\frac{11}{20}$ $\frac{2}{5}$ $\frac{8}{15}$

$\frac{5}{13}$ $\frac{11}{20}$ $\frac{7}{9}$ $\frac{5}{16}$ $\frac{2}{7}$ $\frac{2}{9}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{18}{35}$

**9.** Circle the fractions from the list below that have a value *equal to* $\frac{1}{2}$.

$\frac{4}{8}$ $\frac{3}{5}$ $\frac{12}{24}$ $\frac{15}{25}$ $\frac{10}{15}$ $\frac{14}{28}$ $\frac{13}{25}$

**10.** Write a rule for deciding when a fraction is less than $\frac{1}{2}$.

**11.** Write a rule for deciding when a fraction is greater than $\frac{1}{2}$.

**12.** Write a rule for deciding when a fraction is equal to $\frac{1}{2}$.

**13.** Classify the following fractions as less than $\frac{1}{2}$, greater than $\frac{1}{2}$, or equal to $\frac{1}{2}$ according to the rules written in Exercises 10, 11, and 12.

$\frac{17}{30}$ $\frac{25}{60}$ $\frac{31}{55}$ $\frac{23}{46}$

$\frac{24}{50}$ $\frac{17}{35}$ $\frac{19}{38}$ $\frac{33}{64}$

**14.** Check your answers in Exercise 13 with a calculator.

**15.** Did the rules written in Exercises 10, 11, and 12 work? If not, modify your rules and try Exercise 13 again.

The decimal equivalent of $\frac{1}{4} = 0.25$. Complete the following using a calculator to examine the decimal equivalent of each fraction.

**16.** Circle the fractions from the list below that have a value *less than* $\frac{1}{4}$.

$\frac{4}{15}$ $\frac{5}{20}$ $\frac{3}{10}$ $\frac{6}{24}$ $\frac{7}{25}$ $\frac{6}{30}$ $\frac{11}{40}$ $\frac{12}{50}$

**17.** Circle the fractions from the list below that have a value *greater than* $\frac{1}{4}$.

$\frac{4}{12}$ $\frac{5}{18}$ $\frac{3}{10}$ $\frac{6}{25}$ $\frac{7}{28}$ $\frac{8}{3}$ $\frac{11}{50}$ $\frac{12}{40}$

**18.** Circle the fractions from the list below that have a value *equal to* $\frac{1}{4}$.

$\frac{3}{15}$ $\frac{6}{24}$ $\frac{2}{9}$ $\frac{9}{36}$ $\frac{6}{20}$ $\frac{9}{40}$ $\frac{11}{44}$ $\frac{12}{48}$

**19.** Write a rule for deciding when a fraction is less than $\frac{1}{4}$.

**20.** Write a rule for deciding when a fraction is greater than $\frac{1}{4}$.

**21.** Write a rule for deciding when a fraction is exactly $\frac{1}{4}$.

**22.** Classify the following fractions as less than $\frac{1}{4}$, greater than $\frac{1}{4}$, or equal to $\frac{1}{4}$ according to the rules written in Exercises 19, 20, and 21.

$\frac{17}{30}$ $\frac{25}{60}$ $\frac{31}{55}$ $\frac{23}{46}$

$\frac{24}{50}$ $\frac{17}{35}$ $\frac{19}{38}$ $\frac{33}{64}$

**23.** Check your answer in Exercise 22 by using a calculator.

**24.** Did the rules written in Exercises 19, 20, and 21 work? If not, modify your rules and try Exercise 22 again.

**25.** Write five different fractions for each of the following categories.

(a) less than 1

(b) greater than 1

(c) equal to 1

(d) less than $\frac{1}{2}$

(e) greater than $\frac{1}{2}$

(f) equal to $\frac{1}{2}$

(g) less than $\frac{1}{4}$

(h) greater than $\frac{1}{4}$, but less than $\frac{1}{2}$

(i) equal to $\frac{1}{4}$

*Summarize what you learned in this activity:*

## SIZE OF FRACTIONS

**Outcome: Categorize fractions based upon their relationship to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.**

**1.** Put each fraction from your common fractions chart on page 28 in the proper box. Do not simplify any of the fractions. If you need help deciding in which box a fraction belongs, use your calculator or your decimal fractions chart. When you finish, there should be a total of 100 fractions on the chart.

| Undefined | Exactly 0 | Greater than 0, but less than $\frac{1}{4}$ (0.25) | Exactly $\frac{1}{4}$ (0.25) | Greater than $\frac{1}{4}$ (0.25) but less than $\frac{1}{2}$ (0.5) |
|---|---|---|---|---|
| | | | | |

| Exactly $\frac{1}{2}$ (0.5) | Greater than $\frac{1}{2}$ (0.5), but less than $\frac{3}{4}$ (0.75) | Exactly $\frac{3}{4}$ (0.75) | Greater than $\frac{3}{4}$ (0.75), but less than 1 |
|---|---|---|---|
| | | | |

| Exactly 1 | Greater than 1 |
|---|---|
| | |

List the similarities of the fractions contained in each box. Make a rule for determining when any fraction belongs in each box.

**2.** Undefined

**3.** Exactly 0

**4.** Greater than 0, but less than $\frac{1}{4}$

**5.** Exactly $\frac{1}{4}$

**6.** Greater than $\frac{1}{4}$, but less than $\frac{1}{2}$

**7.** Exactly $\frac{1}{2}$

**8.** Greater than $\frac{1}{2}$, but less than $\frac{3}{4}$

**9.** Exactly $\frac{3}{4}$

**10.** Greater than $\frac{3}{4}$, but less than 1

**11.** Exactly 1

**12.** Greater than 1

**13.** Put each of the following fractions below in the proper box by using your own rules from Exercise 2 through Exercise 12. DO NOT use a calculator.

$\frac{10}{15}$ $\frac{4}{12}$ $\frac{7}{12}$ $\frac{15}{16}$ $\frac{18}{23}$ $\frac{4}{15}$ $\frac{31}{50}$ $\frac{35}{70}$ $\frac{3}{10}$ $\frac{12}{16}$

$\frac{14}{10}$ $\frac{9}{15}$ $\frac{13}{26}$ $\frac{9}{10}$ $\frac{15}{12}$ $\frac{12}{12}$ $\frac{8}{16}$ $\frac{20}{30}$ $\frac{2}{12}$ $\frac{0}{14}$

$\frac{13}{26}$ $\frac{2}{8}$ $\frac{5}{12}$ $\frac{1}{14}$ $\frac{20}{25}$ $\frac{18}{24}$ $\frac{9}{8}$ $\frac{7}{14}$ $\frac{7}{28}$ $\frac{14}{28}$

$\frac{21}{28}$ $\frac{9}{11}$ $\frac{11}{9}$ $\frac{0}{21}$ $\frac{21}{18}$ $\frac{21}{24}$ $\frac{27}{27}$ $\frac{25}{100}$ $\frac{17}{34}$ $\frac{23}{87}$

| Undefined | Exactly 0 | Greater than 0, but less than $\frac{1}{4}$ (0.25) | Exactly $\frac{1}{4}$ (0.25) | Greater than $\frac{1}{4}$ (0.25), but less than $\frac{1}{2}$ (0.5) |
|---|---|---|---|---|
| | | | | |

| Exactly $\frac{1}{2}$ (0.5) | Greater than $\frac{1}{2}$ (0.5), but less than $\frac{3}{4}$ (0.75) | Exactly $\frac{3}{4}$ (0.75) | Greater than $\frac{3}{4}$ (0.75), but less than 1 |
|---|---|---|---|
| | | | |

| Exactly 1 | Greater than 1 |
|---|---|
| | |

**14.** Check the fractions in each box using a calculator and examining the decimal equivalents of each fraction.

**15.** If your rules did not work in all cases, modify your rules. Check with your classmates to see the similarities and differences in your rules.

*Summarize what you learned in this activity:*

## ANALYZING NUTRITION LABELS

**Outcome: Use proportions and percents to analyze nutrition labels.**

In 1990 Congress passed the Nutrition La beling and Education Act. This act requires that nutritional labels following the new standards should app ear on all processed food products. Nutrition information for fresh produce, meat, poultry, and fish will appear in the areas of the store where they are displayed.

To interpret the information given on the label it is important to determine the *base* for the percents given on the label. R egulations also require that *standard* daily nutritional value be used as th e basis for all labels. The column heading "% Daily Value" has an as terisk that guides us to a footnote on the label that reads, "Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs." The label also includes the recommended daily intakes that are used as a basis for a 2,000 and 2,500 calorie diet.

| | | | |
|---|---|---|---|
| **Nutrition Facts** | | | |
| **Serving Size 1/2 cup (114 g)** | | | |
| **Servings per Container 4** | | | |
| **Amount Per Serving** | | | |
| **Calories** 260 Calories from Fat 120 | | | |
| | | | **% Daily Value*** |
| **Total Fat** 13 g | | | **20%** |
| Saturated Fat 5 g | | | **25%** |
| **Cholesterol** 30 mg | | | **10%** |
| **Sodium** 660 mg | | | **28%** |
| **Total Carbohydrate** 31 g | | | **11%** |
| Dietary Fiber 0 g | | | **0%** |
| Sugar 5 g | | | |
| **Protein** 5 g | | | |
| Vitamin A 4% | Vitamin C 2% | | |
| Calcium 15% | Iron 4% | | |
| *Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs: | | | |
| | Calories | 2,000 | 2,500 |
| Total Fat | Less than | 65 g | 80 g |
| Sat Fat | Less than | 20 g | 25 g |
| Cholesterol | Less than | 300 mg | 300 mg |
| Sodium | Less than | 2,400 mg | 2,400 mg |
| Total Carbohydrate | | 300 g | 375 g |
| Dietary Fiber | | 25 g | 30 g |

**Are the recommended daily intakes proportional to the daily calories?**

First examine the recommended daily intake values for Total Fat. If the values are proportional,

$$\frac{65\text{ g}}{80\text{ g}} = \frac{2{,}000\text{ calories}}{2{,}500\text{ calories}}$$

Before we continue, we need to realize that the recommended values have been rounded and if cross products are *reasonably* close, we will say the relationships are proportional. To verify that the Total Fat recommended is or is not proportional to the number of calories per day, check the cross products.

$$65 \times 2{,}500 = 2{,}000 \times 80$$

$$162{,}500 = 160{,}000$$

If we round the values to the nearest ten-thousand, we can say that the recommended daily intake of Total Fat is proportional.

If Total Fat is proportional to Calories, then we can determine the recommended Total Fat intake for diets based on any number of calories.

EXAMPLE 1 Find the recommended intake value of Total Fat for a 1,600 calorie diet. Round the value to the nearest multiple of five.

$$\frac{x}{1{,}600} = \frac{65\text{ g}}{2{,}000} \quad \text{Find cross products.}$$
$$2{,}000x = 1{,}600 \times 65 \quad \text{Multiply } 1{,}600 \times 65.$$
$$2{,}000x = 104{,}000$$
$$x = \frac{104{,}000}{2{,}000} \quad \text{Divide } 104{,}000 \text{ by } 2{,}000.$$
$$x = 52\text{ g}$$

Rounded to the nearest 5 grams, the recommended Total Fat for a 1,600 calorie diet is 50 grams.

**1.** Determine if the following recommended daily intake values are proportional (based on rounded amounts). Show calculations to justify your answers.

Saturated Fat ____________ Cholesterol ____________

Sodium ____________ Total Carbohydrate ____________

Dietary Fiber ____________

**2.** Find the recommended daily intake values for a 1,600 calorie diet for the other recommended values that are proportional.

**3.** Find the recommended daily intake values for a 1,200 calorie diet for the recommended values that are proportional.

**4.** Find the recommended daily intake values for a 1,000 calorie diet for the recommended values that are proportional.

**5.** Is the footnote giving the daily recommended intake value necessary? Could those values have been determined by other information given on the label? Justify your response with a specific example.

**Calories from Fat**

Another important nutritional fact is the percent of calories that are derived from fat. The label gives both the total calories for a defined serving and the calories from fat. The percent can be calculated from these values. It is recommended that less than 30% of the calories that we consume in a day come from fat.

In examining our sample label, we see that 120 calories of the 260 calories are from fat. What percent is this?

$$\begin{aligned} \frac{R}{100} &= \frac{120}{260} \\ 260R &= 12{,}000 \\ R &= \frac{12{,}000}{260} \\ R &= 46.15\% \end{aligned}$$

This product has over 45% of its calories from fat. The product label also gives the number of calories per gram for fat, carbohydrates, and protein. Check the value given for the fat calories per gram to see if it is consistent with the other information given.

9 calories per gram × 13 g = 117 calories from fat

Again, label values are generally rounded.

**6.** Find the number of calories from carbohydrates.

**7.** Find the number of calories from protein.

**8.** Discuss why label values use liberal rounding conventions. In what situations would a more conservative rounding approach be advisable?

Analyze two actual nutrition labels.

**9.** Label 1: Product description ___________________________

Serving size: __________

Percent of calories from fat __________

Total calories from carbohydrates __________

Total calories from protein __________

**10.** Label 2: Product description ___________________________

Serving size: __________

Percent of calories from fat __________

Total calories from carbohydrates __________

Total calories from protein __________

*Summarize what you learned in this activity:*

**Analyzing Nutrition Labels** **Out-of-Class Assignment** **Name**_________________________

**1.** Plan a 500 calorie meal and give the following details. Turn in labels or copies of labels that were used.

| Menu | Serving Size | Total Calories | Fat Calories | Carbohydrate Calories | Protein Calories |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| Menu Totals | | | | | |

**2.** Select a favorite recipe and from nutrition labels complete the following table. Turn in labels or copies of labels that were used.

Recipe for: _______________________

| Ingredient | Amount | Total Calories | Fat Calories | Carbohydrate Calories | Protein Calories |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| Recipe Totals | | | | | |

**3.** In the recipe you selected, define a serving and calculate the total milligrams and the percent of the recommended daily intake of sodium and cholesterol per serving.

Serving:

Sodium: _____mg _____% of recommended daily intake

Cholesterol: _____mg _____% of recommended daily intake

## WHAT PERCENT TAX DO I REALLY PAY?

**Outcome 1: Determine the percent of tax withheld for a given taxable income.**

True or false? As your income increases and you move to a higher tax bracket, your total taxable income is taxed at the higher rate. Before you answer that question, examine the withholding tax for the income in the table below based on a recent IRS Table for Percentage Method of Withholding for a single taxpayer paid monthly (IRS Publication 15, Circular E).

**TABLE 4—MONTHLY Payroll Period**

**(a) SINGLE person** (including head of household)—

If the amount of wages (after subtracting withholding allowances) is: / The amount of income tax to withhold is:

Not over $179 . . . . . . . . . . . . . . . $0

| Over— | But not over— | | of excess over— |
|---|---|---|---|
| $179 | —$904 | $0.00 plus 10% | —$179 |
| $904 | —$3,125 | $72.50 plus 15% | —$904 |
| $3,125 | —$7,317 | $405.65 plus 25% | —$3,125 |
| $7,317 | —$15,067 | $1,453.65 plus 28% | —$7,317 |
| $15,067 | —$32,542 | $3,623.65 plus 33% | —$15,067 |
| $32,542 | | $9,390.40 plus 35% | —$32,542 |

**(b) MARRIED person—**

If the amount of wages (after subtracting withholding allowances) is: / The amount of income tax to withhold is:

Not over $675 . . . . . . . . . . . . . . . $0

| Over— | But not over— | | of excess over— |
|---|---|---|---|
| $675 | —$2,125 | $0.00 plus 10% | —$675 |
| $2,125 | —$6,567 | $145.00 plus 15% | —$2,125 |
| $6,567 | —$12,567 | $811.30 plus 25% | —$6,567 |
| $12,567 | —$18,796 | $2,311.30 plus 28% | —$12,567 |
| $18,796 | —$33,038 | $4,055.42 plus 33% | —$18,796 |
| $33,038 | | $8,755.28 plus 35% | —$33,038 |

Calculate the withholding tax for each amount of monthly income subject to withholding. Then, find the percent that the withholding amount is of the income subject to withholding for one month. Finally, assuming that the monthly income subject to withholding is the same for each month of a year, find the annual income and the annual withholding.

| Monthly Income Subject to Withholding | Withholding | Percent of Income | Annual Income | Annual Withholding |
|---|---|---|---|---|
| 1. $179 | | | | |
| 2. $180 | | | | |
| 3. $904 | | | | |
| 4. $905 | | | | |
| 5. $1,000 | | | | |
| 6. $3,125 | | | | |
| 7. $3,126 | | | | |
| 8. $5,000 | | | | |
| 9. $7,317 | | | | |
| 10. $7,318 | | | | |
| 11. $10,000 | | | | |
| 12. $15,067 | | | | |
| 13. $15,068 | | | | |
| 14. $20,000 | | | | |
| 15. $32,542 | | | | |

| | | | | |
|---|---|---|---|---|
| 16. $32,543 | | | | |
| 17. $35,000 | | | | |
| 18. $40,000 | | | | |
| 19. $50,000 | | | | |
| 20. $100,000 | | | | |

Now, true or false? As your income increases and you move to a higher tax bracket, your total taxable income is taxed at a higher rate.

Explain why you think your answer is correct.

**Outcome 2: Determine the percent of income tax for a given annual taxable income.**

At the end of a year taxpayers are to pay income tax owed for the year based on the total taxable income for the year. The Tax Computation Worksheet for single taxpayers for a recent year is provided from an IRS publication.

**Section A**—Use if your filing status is **Single.** Complete the row below that applies to you.

| **Taxable income.** If line 43 is— | **(a)** Enter the amount from line 43 | **(b)** Multiplication amount | **(c)** Multiply (a) by (b) | **(d)** Subtraction amount | **Tax.** Subtract (d) from (c). Enter the result here and on Form 1040, line 44 |
|---|---|---|---|---|---|
| At least $100,000 but not over $174,400 | $ | × 28% (.28) | $ | $ 6,383.00 | $ |
| Over $174,400 but not over $379,150 | $ | × 33% (.33) | $ | $ 15,103.00 | $ |
| Over $379,150 | $ | × 35% (.35) | $ | $22,686.00 | $ |

Calculate the income tax owed for the year for the following monthly incomes subject to be taxed. Then, find the percent the income tax is of the taxable income.

| Monthly Income Subject to Withholding | Annual Taxable Income | Amount of Income Tax | Percent of Tax |
|---|---|---|---|
| 1. $10,000 | | | |
| 2. $15,067 | | | |
| 3. $15,068 | | | |
| 4. $20,000 | | | |
| 5. $32,542 | | | |
| 6. $32,543 | | | |
| 7. $35,000 | | | |
| 8. $40,500 | | | |
| 9. $50,000 | | | |
| 10. $100,000 | | | |

Compare the tax withheld for a year (outcome 1) with the tax owed for the year. Summarize your findings.

Suppose two persons both had an annual taxable income of $18,000. One person had a seasonal variation in the wages earned and earned $750 in taxable income a month for three months and $1,750 in taxable income a month for nine months. Calculate the amount of tax withheld during the year.

The second person earned $1,500 in taxable income each month and the annual taxable income, the annual withholding, and the annual taxes owed have been found in row 4 on each table. Compare the withholding and tax owed for both persons.

In your own words, explain the difference between the tax withheld from taxable income on a regular basis with the annual income tax owed for the year.

*Summarize what you learned in this activity:*

## CIRCLE, BAR, AND LINE GRAPHS

**Outcome 1: Interpret data from various types of graphs.**

**1.** Circle Graphs

(a) Find three examples of circle graphs in the newspaper or periodical articles. Tape, glue, or staple newspaper articles or a copy to a standard sheet of paper (one article per sheet). Give the name of the newspaper or periodical and the date.

(b) Briefly explain what the graph shows.

(c) Make up three exercises that use the information on the graph to find other (not given) information.

(d) Solve the exercises from part c.

**2.** Bar Graphs

(a) Find three examples of bar graphs in the newspaper or periodical articles. Tape, glue, or staple newspaper articles or a copy to a standard sheet of paper (one article per sheet). Give the name of the newspaper or periodical and the date.
(b) Briefly explain what the graph shows.
(c) Make up three exercises that use the information on the graph to find other (not given) information.
(d) Solve the exercises from part c.

**3.** Line Graphs

(a) Find three examples of line graphs in the newspaper or periodical articles. Tape, glue, or staple newspaper articles or a copy to a standard sheet of paper (one article per sheet). Give the name of the newspaper or periodical and the date.
(b) Briefly explain what the graph shows.
(c) Make up three exercises that use the information on the graph to find other (not given) information.
(d) Solve the exercises from part c.

**Outcome 2: Plan, design, and conduct an investigation in which the findings are reported in writing and graphically.**

**4.** Design a project and gather appropriate data. Make a report of y our findings and display data graphically. Use as many different types of graphs as is practical.

*Summarize what you learned in this activity:*

## CRITIQUING GRAPHS

**Outcome: Critique graphs found in recent publications.**

Collect a variety of graphs from several sources (newspapers, magazines, the Internet, etc.). Select two graphs to critique: one you evaluate to be a good graphical representation of the information presented and the other to be a poor graphical representation of the information presented. Answer the following questions about your selected graphs.

***Sample of good graphical representation.***
***(Attach graph or copy of graph.)***

1. What type of graph is used to display the information?
2. Briefly describe the purpose of the graph.
3. Do the titles and notes explain the data satisfactorily? Explain why or why not.
4. What additional information would have been helpful?
5. What suggestions would you make for a better presentation of these data?
6. What questions would you ask the researcher about how the data were collected?

***Sample of poor graphical representation.***
***(Attach graph or copy of graph.)***

1. What type of graph is used to display the information?
2. Briefly describe the purpose of the graph.
3. Do the titles and notes explain the data satisfactorily? Explain why or why not.
4. What additional information would have been helpful?
5. What suggestions would you make for a better presentation of these data?
6. What questions would you ask the researcher about how the data were collected?

*Summarize what you learned in this activity:*

# ESTIMATING RADICALS

### Squares and Square Roots

**Outcome 1: Distinguish between squares and square roots and rational and irrational numbers.**

**1.** Complete the following table of numbers and their squares.

| ***n*** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $n^2$ | | | | | | | | | | |

| ***n*** | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $n^2$ | | | | | | | | | | |

Table 1

The numbers represented by $n^2$ are called *perfect squares*. The principal *square root* of each $n^2$ is $n$.

**2.** Using your calculator, complete the following table, rounding answers to the nearest thousandth.
some calculators: $n$ $\boxed{\sqrt{\ }}$ other calculators: $\boxed{\sqrt{\ }}$ $n$ $\boxed{=}$

| ***n*** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\sqrt{n}$ | | | | | | | | | | |

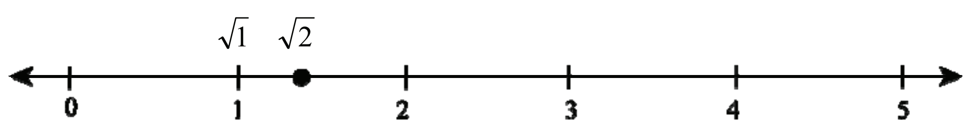
| ***n*** | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\sqrt{n}$ | | | | | | | | | | |

| ***n*** | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|
| $\sqrt{n}$ | | | | | |

Table 2

The square root of a perfect square is a *rational number*. The square root of a number that is not a perfect square is an *irrational number*. The decimal equivalents of irrational numbers will never terminate or repeat. We normally use approximations that are found by rounding.

**3.** Using the answers in Table 2, position the principal square roots of each value of ***n*** on the following number line.



**Outcome 2: Determine between which two whole numbers an irrational number will fall.**

| | |
|---|---|
| $\sqrt{1}=1,\ \sqrt{4}=2$ | 2 and 3 are between 1 and 4.<br>$\sqrt{2}$ and $\sqrt{3}$ are between $\sqrt{1}$ and $\sqrt{4}$ or between 1 and 2. |
| $\sqrt{4}=2,\ \sqrt{9}=3$ | 5, 6, and 7 are between 4 and 9.<br>$\sqrt{5}$, $\sqrt{6}$, and $\sqrt{7}$ are between $\sqrt{4}$ and $\sqrt{9}$ or between 2 and 3. |
| $\sqrt{9}=3,\ \sqrt{16}=4$ | 10, 11, 12, 13, 14, and 15 are between 9 and 16.<br>$\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$, and $\sqrt{15}$ are between $\sqrt{9}$ and $\sqrt{16}$ or between 3 and 4. |

**4.** Using the table and the fact that $\sqrt{n^2}=n$, decide between which two whole numbers the principal square roots of the following numbers will fall.

EXAMPLE: Since 30 is between 25 and 36, $\sqrt{30}$ is between $\sqrt{25}$ and $\sqrt{36}$ or between 5 and 6.

(a) $\sqrt{40}$ (b) $\sqrt{46}$ (c) $\sqrt{105}$ (d) $\sqrt{380}$

(e) $\sqrt{250}$ (f) $\sqrt{300}$ (g) $\sqrt{150}$ (h) $\sqrt{214}$

(i) $\sqrt{375}$ (j) $\sqrt{180}$

*Summarize what you learned in this activity:*

# RATIONAL EQUATIONS

**Patient Charting Efficiency**

**Outcome: Use rational equations in career applications.**

Accurate, up-to-date, and complete patient data has always been an important part of medical treatment, whether in a doctor's office or in a hospital setting. It is vitally important in the Intensive Care Unit (ICU) of a hospital where patients' lives may hang by a thread.

Teaching hospitals accept and train a certain number of student nursing interns from affiliated medical schools. Suppose you and your team of personnel consultants have been hired by a teaching hospital to help them decide which of two nursing interns should be assigned to the ICU. After observing the two interns, Pat and Rosa, your team found them to be about equal in terms of procedure expertise, bedside manner, nursing knowledge/nomenclature, work ethic, attitude, academic background, experience, and hygiene. Your team decides to test them on one of the most important and time-consuming tasks: charting.

This hospital does not yet have computerized charting, and currently uses one night-shift nurse named Janet to visit and chart its ICU patients. **Janet spends 60–90 minutes (70 minutes average) to chart two ICU patients, and 90–120 minutes (100 minutes average) to chart her maximum of three ICU patients.**

**Pat and Janet work side-by-side one night to care for two patients. Your team found that it took them 48 minutes, working together, to chart these patients.**

**The next night Janet and Rosa worked side-by-side to care for three patients. (An additional patient was admitted who required about the same amount of care as each of the other two patients.) Your team found it took Janet and Rosa together 66 minutes to chart these patients.**

**1.** Inspect the data given and state your opinion on which nursing intern seems to be the faster charter.

**2.** Estimate how long you think it would have taken Pat to chart the two patients alone. Write and solve an algebraic equation to find Pat's time to chart these two patients when working alone.

**3.** Estimate how long you think it would have taken Rosa to chart the three patients when working alone. Write and solve an algebraic equation to find Rosa's time to chart the three patients alone.

**4.** Estimate which intern is the faster charter based on your findings in exercises 2 and 3.

**5.** Verify your estimate by writing and solving an equation to decide who should be hired in order to get the faster charter.

*Summarize what you learned from this activity:*

# QUADRATIC EQUATIONS

**Calculating Vehicular Speed from Skid Marks and Road Conditions**

**Outcome: Use quadratic equations in career applications.**

Law enforcement officers, insurance investigators, and private investigators use two formulas to determine the speed of a motor vehicle based on skid marks and road conditions. The first formula is the formula for calculating the *drag factor* based on road conditions. The drag factor, or coefficient of friction, is a number between 0.03 (poor stopping conditions) and 1.3 (ideal stopping conditions).

The drag factor varies according to road surface type (asphalt, concrete, gravel, dirt, or grass), age of road, weather conditions (precipitation, wind, air temperature, air pressure, relative humidity), road surface conditions (wet, dry, ice, hail, oil, sand, dirt/gravel or other debris on road), grade of road, etc. Officers determine the drag factor of specific road conditions by conducting drag factor tests under similar conditions.

Each test consists of braking to a stop from a specified speed (usually 25 –35 mph), then measuring the length of the skid marks. At least two tests are conducted and the longest skid distance is used in the formula if the two distances are within 5%. The formula for calculating the drag factor ($f$) is $\boldsymbol{f = s^2/30D}$, where $s$ is the test speed and $D$ is the length of the skid marks. Usually the weight of the vehicle is negligible in determining speed, but weight can be a factor under extremely poor stopping conditions such as black ice, oil slick, or oil mixed with water (which happens at the beginning of a rain occurring after a long dry period).

Once a drag factor is determined, the estimated speed that the vehicle was traveling is calculated using a second formula. Under most stopping conditions the distance of the skid marks $D$ is related to the speed of the car $s$, the drag factor $\boldsymbol{f}$, and the percentage of braking before stop $n$ (written as a decimal), by the formula: $\boldsymbol{D = s^2/30fn}$.

EXAMPLE Suppose tests indicate the drag factor on a wet, 15-year-old asphalt highway is 0.5. Estimate the speed of a car, traveling in a 45 mph speed limit zone, which left 388 ft skid marks showing continuous braking before coming to a stop.

$\boldsymbol{D = s^2/30fn}$

$388 = s^2/30(0.5)(1.00)$ Continuous braking: $n = 100\% = 1.00$

$388 = s^2/15$

$5{,}820 = s^2$

$\pm 76 = s$ The car was traveling approximately 76 mph.

**1.** Your car was run off an icy road by an out-of-control car, which left continuous skid marks 572 feet long before it stopped. The police report found the drag factor to be 0.13. Approximate the speed of the car that ran you off the road.

**2.** Solve the formula $D = s^2/30\,fn$ for $s$.

**3.** Use the rearranged version of the formula to estimate your speed if you started braking when you entered the deep sand shoulder of the road (which has a drag factor of 0.35 under these conditions) and continuously braked to a stop leaving 129 feet of skid marks.

**4.** Research the drag factors of dry concrete and deep sand. Which of these two surfaces takes longer to stop on, with other factors being equal? Explain why runaway truck ramps along mountain highways are often made of deep sand. What other surface would be appropriate? Why?

**5.** Research the difference between glare ice and black ice. Which of these road surface conditions takes longer to stop on, with other factors being equal?

Contact your State Highway Patrol Office or local police department if you want to verify your findings.

*Summarize what you learned in this activity:*

## WHAT IS THE NATURAL EXPONENTIAL *e*?

**Outcome: Discover the effect of large values of *n* in the expression $(1 + 1/n)^n$.**

When a quantity grows or decays exponentially, that means that the rate of increase or decrease is affected by an exponent. Several formulas dealing with exponential growth or decay use the natural exponential $e$. This irrational number, like $\pi$, is a specific value. The value is the limit of the expression $\left(1+\frac{1}{n}\right)^n$ as $n$ gets larger and larger. To see this relationship, we will evaluate this expression for several values of $n$.

**1.** Evaluate the expression $\left(1+\frac{1}{n}\right)^n$ for the following values of $n$.

| $n$ | $\left(1+\frac{1}{n}\right)^n$ |
|---|---|
| 1 | $\left(1+\frac{1}{1}\right)^1 = (1+1)^1 = 2^1 = 2$ |
| 10 | $\left(1+\frac{1}{10}\right)^{10} = (1+0.1)^{10} = (1.1)^{10} = 2.59374246$ |
| 100 | |
| 1,000 | |
| 10,000 | |
| 100,000 | |
| 1,000,000 | |
| 10,000,000 | |
| 100,000,000 | |

**2.** Find the calculator value of $e$.
To determine the calculator value of $e$, find the value of $e^1$ using your calculator.
most calculators: $\boxed{e^x}$ 1 $\boxed{=}$

some calculators: 1 $\boxed{e^x}$

(If the label is above the key, as $\overset{e^x}{\boxed{\phantom{x}}}$, access the $e^x$ function by first pressing a shift, inverse, or second function key.)

**3.** How does the calculator value of $e$ compare to your calculated value when $n$ is 100,000,000?

*Summarize what you learned in this activity:*

# COMPOUNDED AMOUNT AND COMPOUND INTEREST

**Outcome: Compare the two compound amount formulas for compound interest, $A = p(1 + r/n)^{nt}$, and $A = pe^{nt}$**

The formula for finding the compound amount is: $A = p\left(1+\frac{r}{n}\right)^{nt}$.

In the formula, $A$ is the accumulated amount, $p$ is the principal, $t$ is time in years, $r$ is the decimal equivalent of the rate per year, and $n$ is the number of compounding periods per year.

We will examine the effect of the number of compounding periods per year by evaluating the expression for several different periods for a principal of $1 at a rate of 10% per year for 1 year. Complete the chart.

| **Compounding Period** | ***n*** | $A = 1\left(1+\frac{0.1}{n}\right)^n$ |
|---|---|---|
| Annually | 1 | $A = 1\left(1+\frac{0.1}{n}\right)^n$ |
| Semi-annually | 2 | $A = 1\left(1+\frac{0.1}{2}\right)^2 = 1.1025$ |
| Quarterly | 4 | |
| Monthly | | |
| Weekly | | |
| Daily | | |
| Hourly | | |
| Every minute | | |
| Every second | | |

Calculator pattern: $\boxed{(}\ 1\ \boxed{+}\ .1\ \boxed{\div}\ n\ \boxed{)}\ \boxed{x^y}\ n\ \boxed{=}$

The formula for finding the accumulated amount by compounding continuously (every instant) is $A = pe^{nt}$, where $p$ is the principal, $r$ is the decimal equivalent of the rate per year, and $t$ is the number of years.

**1.** Find the accumulated amount, compounded continuously, of $1 at 10% per year for 1 year.
Calculator: $\boxed{e^x}\ .1\ \boxed{=}$ (Multiply by 1 mentally.)

**2.** Compare the accumulated amounts for a $1 investment at 10% per year for 1 year using the two formulas.

**3.** If you were earning interest on an investment, under what conditions would you prefer using the formula, $A = p\left(1+\frac{r}{n}\right)^{nt}$? If you were making the calculations for or estimating interest on an investment earning daily interest, which formula would you choose and why?

*Summarize what you learned in this activity:*

## GRAPHING EQUATIONS PROJECT (See Graphing Activities 1–6.)

**Instructions for Group Project**
**For Five-Member Groups**

One group member will serve as group recorder for each activity. Rotate recorders with each activity. Each of the other four members will graph a set of equations using a computer or calculator.

**Activity 1: Linear Equations in the form $y = mx$**

Member 1: Graph equations 1–6 on page 57. All six graphs should appear on the axes at the same time.

Be able to identify each equation to your group members. Leave the graphs on the screen until your group has answered questions 7 and 8.

Member 2: Graph equations 9–12 on page 57. All four graphs should appear on the axes at the same time.

Be able to identify each equation to your group members. Leave the graphs on the screen until your group has answered question 13.

Member 3: Graph equations 14–19 on page 57, using the computer. All six graphs should appear on the axes at the same time.

Be able to identify each equation to your group members. Leave the graphs on the screen until your group has answered questions 20 and 21.

Member 4: Graph equations 22–25 on page 58. All four graphs should appear on the axes at the same time.

Be able to identify each equation to your group members. Leave the graphs on the screen until your group has answered question 26.

Recorder: Record the results of each of the four sets of graphs.

Entire Group: Discuss 8, 13, 20, 21, 26, and 27–33; reach a consensus, and record your responses.

**Activity 2: Linear Equations in the form $y = mx + b$**

Member 1: Graph 1–3 on page 59. Discuss 4.
Member 2: Graph 5–7 on page 59. Discuss 8.
Member 3: Graph 9–11 on page 59. Discuss 12.
Member 4: Graph 13–15 on page 60. Discuss 16–18.
Recorder: Record the results of each of the four sets of graphs.

Entire Group: Without the use of a computer or calculator, draw a freehand sketch of the graphs of equations 19–30. Reach a consensus on questions 4, 8, 12, and 16–18; record your responses.

**Activity 3: Linear Equations in the form $y = k$ and $x = k$**

Member 1: Graph 1–3 on page 61. Discuss 4.
Member 2: Graph 5–7 on page 61. Discuss 8-9.
Member 3: Graph 10–12 on page 61. Discuss 13.
Member 4: Graph 14–16 on page 62. Discuss 17–18.
Recorder: Record the results of each of the four sets of graphs.

Entire Group: Reach a consensus on questions 4, 8, 9, 13, 17, and 18; record your responses.

Without the use of a com puter or cal culator, draw a freehand sketch of the graphs for equations 19–30.

**Activity 4: Quadratic Equations in the form $y = ax^2$**

Member 1: Graph 1–3 on page 63. Discuss 7 and 8.
Member 2: Graph 4–6 on page 63. Discuss 9.
Member 3: Graph 10–12 on page 63. Discuss 16 and 17.
Member 4: Graph 13–15 on page 63. Discuss 18–20.

Recorder: Record the results of each of the four sets of graphs.

Entire Group: Reach a consensus on questions 7–9 and 16–20 and record your responses.

**Activities 5 and 6: Quadratic Equations in the form $y = ax^2 + b$ and $y = (x + b)^2$**

Member 1: Graph 1–3 for each activity on pages 64 and 65. Discuss 4.
Member 2: Graph 5–7 for each activity on pages 64 and 65. Discuss 8.
Member 3: Graph 9–14 for each activity on pages 64 and 65. Discuss 13.
Member 4: Graph 15–20 for each activity on pages 64 and 65. Discuss 17.
Recorder: Record the results of each of the four sets of graphs.

Entire Group: Reach a consensus on questions 7–9 and 16–20 and record your responses.

# GRAPHING EQUATIONS

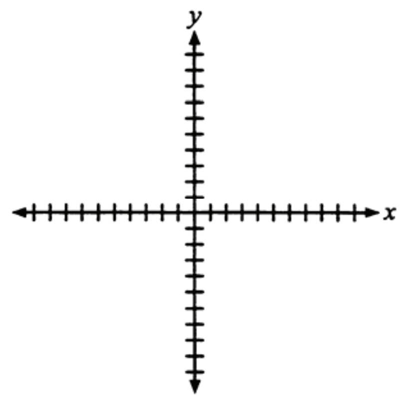
## Graphing Activity 1

**Outcome: Examine equations in the form $y = mx$.**

Use a graphing calculator or computer to graph the equations. Record a sketch of graphs 1–6 on the same axes, labeling each graph.

**1.** $y = x$ **2.** $y = 2x$ **3.** $y = 3x$

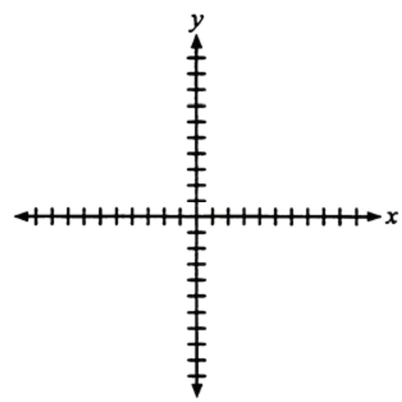
**4.** $y = 5x$ **5.** $y = 8x$ **6.** $y = 10x$




**7.** In your own words describe the graph of $y = x$.

**8.** In your own words compare graphs 2–6 to the graph of $y = x$.

Record a sketch of graphs 9–12 on the same axes, labeling each graph.

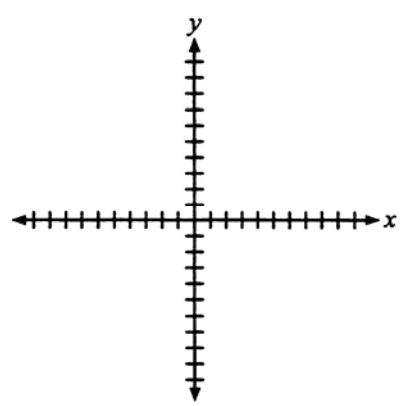



**9.** $y = x$ **10.** $y = \frac{1}{2}x$

**11.** $y = \frac{1}{3}x$ **12.** $y = \frac{1}{4}x$

**13.** In your own words compare graphs 10–12 to the graph of $y = x$.

Record a sketch of graphs 14–19 on the same axes, labeling each graph.



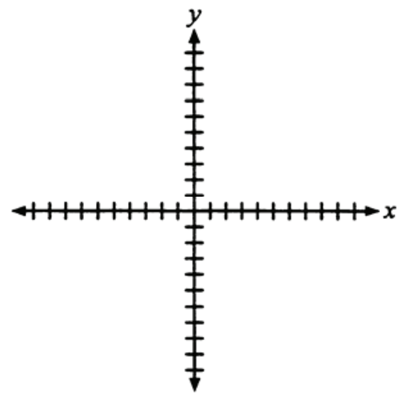

**14.** $y = -x$ **15.** $y = -2x$ **16.** $y = -3x$

**17.** $y = -5x$ **18.** $y = -8x$ **19.** $y = -10x$

**20.** In your own words describe the graph $y = -x$.

**21.** In your own words compare graphs 15–19 to the graph of $y = -x$.

Record a sketch of graphs 22–25 on the same axes, labeling each graph.




**22.** $y = -x$ **23.** $y = -\frac{1}{2}x$

**24.** $y = -\frac{1}{4}x$ **25.** $y = -\frac{1}{3}x$

**26.** In your own words compare graphs 23–25 to the graph of $y = -x$.

In the equation of the line $y = mx$. $m$ represents the slope (slant or steepness) of the line. Complete 27–32 about the graphs of the lines whose equations are in the form $y = mx$.

**27.** Make a general statement about the slopes of lines when $m$ is positive ($m > 0$).

**28.** Make a general statement about the slopes of lines when $m$ is negative ($m < 0$).

**29.** Make a general statement about the slopes of lines when $m$ is greater than 1 ($m > 1$) and gets larger and larger.

**30.** Make a general statement about the slopes of lines when $m$ is a fraction between 0 and 1 ($0 < m < 1$) and gets smaller and smaller (closer to 0).

**31.** Make a general statement about the slopes of lines when $m$ is less than $-1$ ($m < -1$) and the absolute value of $m$ gets larger and larger.

**32.** Make a general statement about the slopes of lines when $m$ is a fraction between 0 and $-1$ ($-1 < m < 0$) and gets closer and closer to 0.

**33.** In an equation in the form $y = mx$, summarize the information given by the value of $m$.

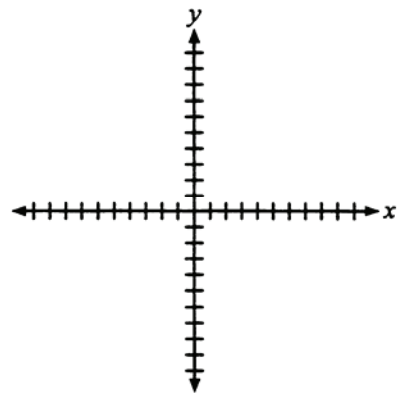
*Summarize what you learned in this activity:*

**Graphing Activity 2**

**Outcome: Examine equations in the form $y = mx + b$.**

Use a graphing calculator or computer to graph the equations. Record a sketch of graphs 1 –3 on the same axes, labeling each graph.

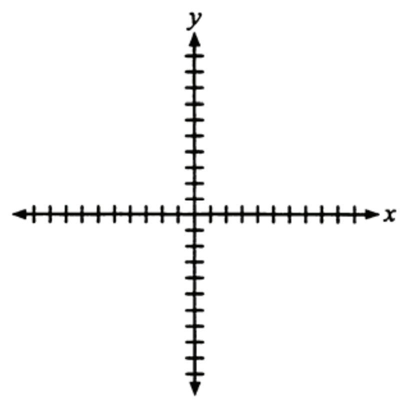
**1.** $y = x$ **2.** $y = x + 1$ **3.** $y = x - 2$




**4.** Describe the similarities and differences among graphs 1–3.

Record a sketch of graphs 5–7 on the same axes, labeling each graph.

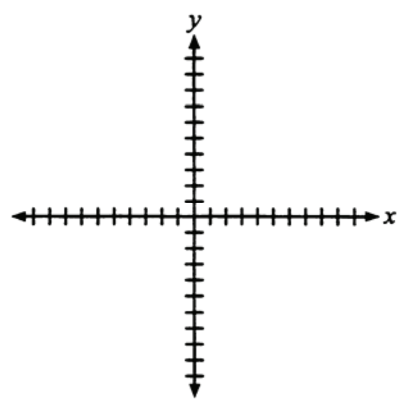
**5.** $y = 4x$ **6.** $y = 4x + 3$ **7.** $y = 4x - 1$




**8.** Describe the similarities and differences among graphs 5–7.

Record a sketch of graphs 9–11 on the same axes, labeling each graph.

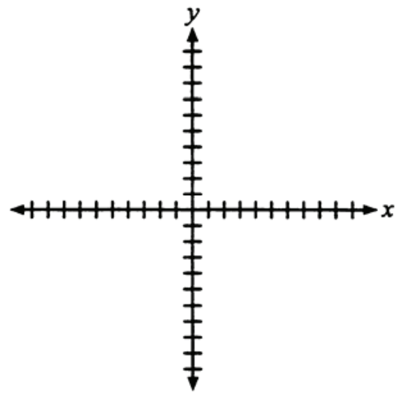
**9.** $y = -3x$ **10.** $y = -3x + 2$ **11.** $y = -3x + 1$




**12.** Describe the similarities and differences among graphs 9–11.

Record a sketch of graphs 13–15 on the same axes, labeling each graph.

**13.** $y = -\frac{2}{3}x$ **14.** $y = -\frac{2}{3}x + 2$ **15.** $y = -\frac{2}{3}x - 2$




**16.** Describe the similarities and differences among graphs 13–15.

**17.** In an equation of the form $y = mx + b$, summarize the information given by the value of $b$.

**18.** In an equation of the form $y = mx + b$, summarize the information given by the value of $m$.

Without the use of a computer or calculator, draw a freehand sketch of the graphs of the following equations.

**19.** $y = x$ **20.** $y = x + 4$ **21.** $y = \frac{1}{2}x$

**22.** $y = -x$ **23.** $y = -x + 3$ **24.** $y = -2x$

**25.** $y = 4x - 2$ **26.** $y = \frac{3}{4}x + 5$ **27.** $y = \frac{1}{3}x - 4$

**28.** $y = -5x - 1$ **29.** $y = \frac{7}{8}x - 2$ **30.** $y = -\frac{3}{2}x + \frac{1}{2}$

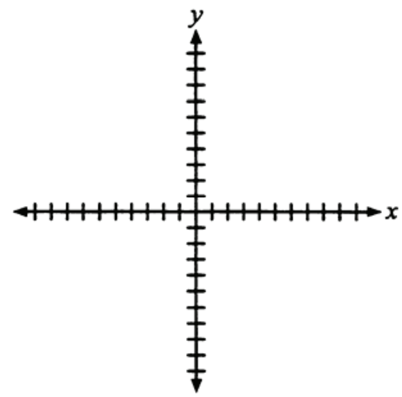
*Summarize what you learned in this activity:*

**Graphing Activity 3**

**Outcome: Examine equations in the form *y* = *k* and *x* = *k*.**

Use a graphing calculator or com puter to graph the equations. Record a sketch of graphs 1 −3 on the sam e axes, labeling each graph.

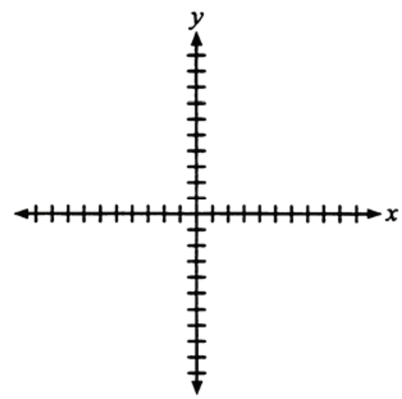
**1.** $y = 1$ **2.** $y = 2$ **3.** $y = \frac{1}{2}$




**4.** In your own words describe the graphs in 1−3.

Record a sketch of graphs 5−7 on the same axes, labeling each graph.

**5.** $y = -3$ **6.** $y = -2$ **7.** $y = -\frac{1}{4}$

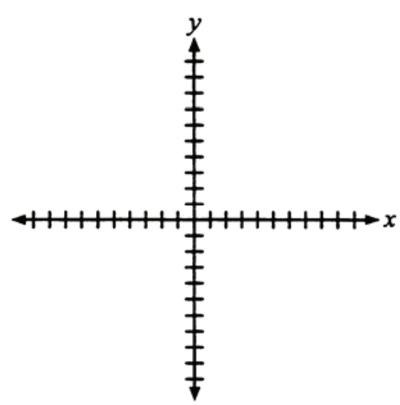



**8.** In your own words describe the graphs in 5−7.

**9.** Make a general statement about graphs of lines whose equations are in the form $y = k$.

Record a sketch of graphs 10 −12 on the sam e axes, labeling each graph. Graphs 10 −12 **cannot** be done using m ost computer programs or graphing calculators. Use other methods.

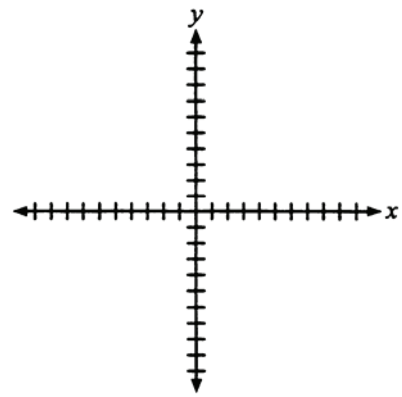
**10.** $x = 1$ **11.** $x = 2$ **12.** $x = \frac{3}{2}$




**13.** In your own words describe the graphs in 10−12.

Record a sketch of graphs 14–16 on the same axes, labeling each graph. Graphs 14–16 **cannot** be done using most computer programs or graphing calculators. Use other methods.

**14.** $x = -2$ **15.** $x = -3$ **16.** $x = -\frac{1}{4}$




**17.** In your own words describe the graphs in 14–16.

**18.** Make a general statement about the graphs of the lines whose equations are in the form $x = k$.

Without the use of a computer or calculator, draw a freehand sketch of the graphs of the following equations.

**19.** $y = -4$ **20.** $y = \frac{3}{4}$ **21.** $x = 6$

**22.** $x = -\frac{4}{3}$ **23.** $y = 3$ **24.** $x = -5$

**25.** $x = 0$ **26.** $y = 0$ **27.** $y = \frac{5}{2}$

**28.** $x = -\frac{9}{4}$ **29.** $y = 8$ **30.** $x = \frac{8}{3}$

*Summarize what you learned in this activity:*

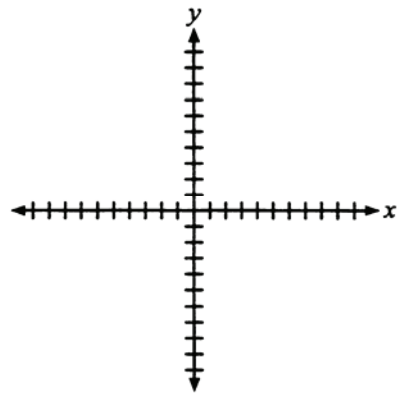
**Graphing Activity 4**

**Outcome: Determine patterns formed by quadratic equations in the form $y = ax^2$.**

Use a graphing calculator or computer to graph the equations. Record a sketch of graphs 1–6 on the same axes, labeling each graph.

**1.** $y = x^2$ **2.** $y = 2x^2$ **3.** $y = 3x^2$

**4.** $y = \frac{1}{2}x^2$ **5.** $y = \frac{1}{4}x^2$ **6.** $y = \frac{1}{3}x^2$




**7.** In your own words describe the graph of $y = x^2$.

**8.** In your own words compare graphs 2–3 to the graph of $y = x^2$.

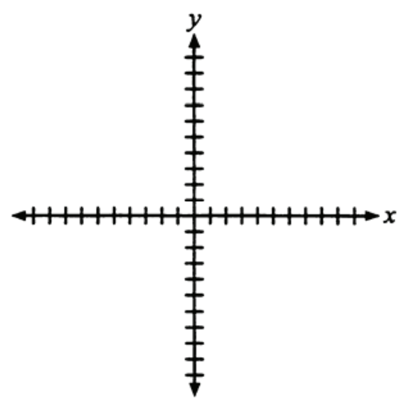
**9.** In your own words compare graphs 4–6 to the graph of $y = x^2$.

Record a sketch of graphs 10–15 on the same axes, labeling each graph.

**10.** $y = -x^2$ **11.** $y = -2x^2$

**12.** $y = -3x^2$ **13.** $y = -\frac{1}{2}x^2$

**14.** $y = -\frac{1}{3}x^2$ **15.** $y = -\frac{1}{4}x^2$




**16.** In your own words describe the graph of $y = -x^2$.

**17.** In your own words compare the graphs 11–12 to the graph of $y = -x^2$.

**18.** In your own words compare the graphs 13–15 to the graph of $y = -x^2$.

**19.** Make a general statement about the graphs of $y = x^2$ and $y = -x^2$.

**20.** What information does *m* give in an equation in the form $y = mx^2$?

*Summarize what you learned in this activity:*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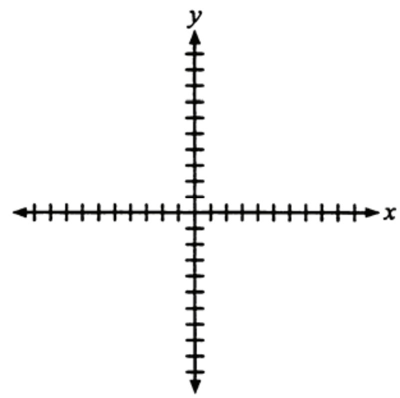
**Graphing Activity 5**

**Outcome: Determine patterns formed by equations in the form $y = ax^2 + b$.**

Use a graphing calculator or computer to graph the equations. Record a sketch of graphs 1 –3 on the same axes, labeling each graph.

**1.** $y = x^2 + 1$ **2.** $y = x^2 + 3$

**3.** $y = x^2 + 4$

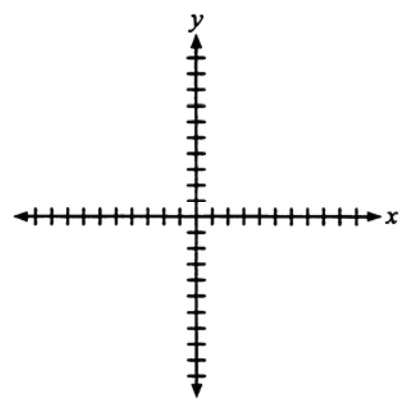



**4.** In your own words describe the similarities and differences among graphs 1–3.

Record a sketch of graphs 5–7 on the same grid, labeling each graph.

**5.** $y = x^2 - 1$ **6.** $y = x^2 - 3$

**7.** $y = x^2 - 4$




**8.** In your own words describe the similarities and differences among graphs 5–7.

Without the use of a calculator or computer, draw a freehand sketch of the following.

**9.** $y = x^2$ **10.** $y = -x^2$ **11.** $y = 5x^2$ **12.** $y = -5x^2$

**13.** $y = \frac{1}{5}x^2$ **14.** $y = -\frac{1}{5}x^2$ **15.** $y = x^2 + 5$ **16.** $y = x^2 - 2$

**17.** $y = 2x^2 + 1$ **18.** $y = -x^2 + 2$ **19.** $y = -x^2 - 3$ **20.** $y = -3x^2 + 1$

*Summarize what you learned in the activity:*

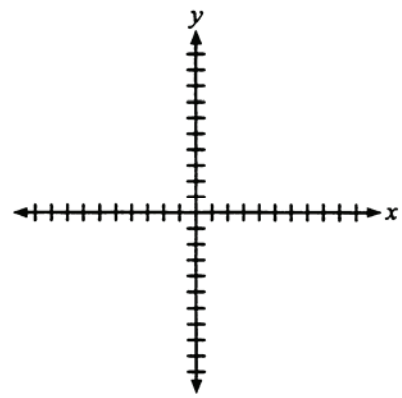
**Graphing Activity 6**

**Outcome: Determine patterns formed by equations in the form $y = (x + b)^2$.**

Use a graphing calculator or computer to graph the equations. Record a sketch of graphs 1–3 on the same axes, labeling each graph.

**1.** $y = (x + 1)^2$ **2.** $y = (x + 3)^2$

**3.** $y = (x + 4)^2$

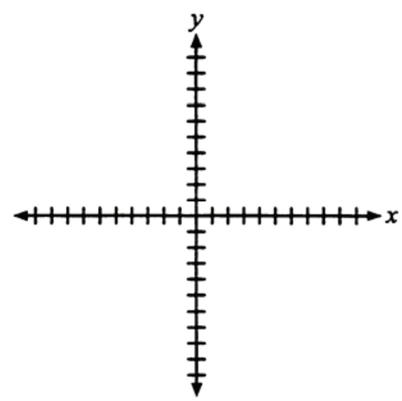



**4.** In your own words describe the similarities and differences among graphs 1–3.

Record a sketch of graphs 5–7 on the same axes, labeling each graph.

**5.** $y = (x - 1)^2$ **6.** $y = (x - 3)^2$

**7.** $y = (x - 4)^2$




**8.** In your own words describe the similarities and differences among graphs 5–7.

Without the use of a calculator or computer, draw a freehand sketch of the following.

**9.** $y = (x + 6)^2$ **10.** $y = (x + 2)^2$ **11.** $y = (x + 5)^2$ **12.** $y = -5x^2$

**13.** $y = (5x + 2)^2$ **14.** $y = (3x + 4)^2$ **15.** $y = (x - 2)^2$ **16.** $y = (x - 5)^2$

**17.** $y = (x + 6)^2$ **18.** $y = (2x - 1)^2$ **19.** $y = (3x - 2)^2$ **20.** $y = (4x - 3)^2$

*Summarize what you learned in this activity:*

# GRAPHICAL REPRESENTATION

**Commodities Market Investing**

**Outcome: Use graphing in consumer applications.**

Suppose you and your team of investors (who live in the Mississippi River Delta region around Memphis, TN) invested a total of $10,000 in locally grown cotton last year, and held the investment for 10 months. According to your records, your team's total profit $P$ after $t$ months of investing was approximated by the equation: $\boldsymbol{P = 100t^2 - 400t}$

EXAMPLE: *Find your team's profit 1 month after investing.*

$P = 100(1^2) - 400(1) \qquad (t = 1)$
$P = 100(1) - 400$
$P = 100 - 400$
$P = -300$
*Your team's $10,000 investment lost $300 after 1 month.*

Use $\boldsymbol{P = 100t^2 - 400t}$ to calculate the following, and state your interpretation of your answers in complete sentences.

**1.** Graph the profit equation, counting by 1's on the horizontal $t$ axis and by 500's on the vertical $P$ axis. Why is the graph valid only between $t = 0$ and $t = 10$?

**2.** Use the graph to estimate the total profit after the following months: a) 3 b) 5 c) 7.5 d) 10

**3.** Use the profit equation to find the exact total profit after the following months: a) 3 b) 5 c) 7.5 d) 10

**4.** Explain any differences between your answers in exercises 2 and 3.

**5.** Find when this investment "broke even" (*Hint:* when profit is equal to zero) by using a) the graph and b) the equation.

**6.** Shade the region(s) in which the investment lost money and label it "loss." Shade the region(s) in which the investment made money and label it "profit."

**7.** The lowest this investment went was $400 below the purchase price. Find the month when this occurred.

**8.** If you could have held this investment for a maximum of a year at the same rate of growth, would you have done better? Show why or why not. State the maximum possible profit for the year.

**9.** Calculate the annual interest rate this investment earned during the 10 months of investment.

**10.** Suppose each member of your team invested a different amount to total $10,000. Assign an amount that each team member invested and prorate the amount of profit for each team member based on those investment amounts.

*Summarize what you learned in this activity:*

# SYSTEMS OF EQUATIONS

**Making Business Choices**

**<u>Outcome:</u> Use systems of equations to make good business choices.**

A student is trying to make some choices regarding a new business venture. Your team will advise this student. The student decides that he wants to work 20 hr per week, and he can afford to spend \$135 per week in materials.

In helping your classmate with his economic forecasting, let $x$ = the number of 12″ base centerpieces and $y$ = the number of 16″ base centerpieces he can make per week in his home business.

**1.** Write a linear equation using $x$ and $y$ to show his 20 hr/wk time allotment.
**2.** Write a linear equation using $x$ and $y$ to show his \$135/wk investment allotment.
**3.** Solve the system of linear equations from problems 1 and 2 by the graphing method and estimate how many of each type of centerpiece he should make each week in order to meet both his time and money allotments.
**4.** Solve the system to find an exact solution. This "ideal" solution represents how many of each type of centerpiece should be made weekly in order to take exactly 20 hours and cost exactly \$135. If this solution isn't the same as in problem 3, explain why.
**5.** If we assume that local florists will buy all he can make, how much revenue will be received from the solution found in problem 4?
**6.** How much did it cost him to make these centerpieces?
**7.** Since profit = revenue − cost, find the profit made on these centerpieces. Use this figure to calculate the student's hourly wage. Would he make at least as much in this 20 hr/wk part-time venture as he currently makes in his full-time minimum wage job?
**8.** Why is the part-time wage found in problem 7 misleading? What other material costs or labor time have not been considered?

If the 20 hr/wk and \$135/wk factors are disregarded, derive the following economic forecasting formulas appropriate for different production levels.

**9.** Let $R$ = revenue from florists, and write a linear equation in $x$ and $y$ to show the revenue received from any number of 12″ and 16″ centerpieces. This equation is called the Revenue Formula.
**10.** Let $C$ = cost of centerpiece materials, and write a linear equation in $x$ and $y$ to show the materials cost of making any number of 12″ and 16″ centerpieces. This equation is called the Cost Formula.
**11.** Let $P$ = profit, and write a profit formula in $x$ and $y$ to show the profit made on any number of 12″ and 16″ centerpieces. This equation is called the Profit Formula.
**12.** Using your profit formula from problem 11, which centerpiece is more profitable to make?
**13.** Use the profit formula to find the profit made on the sale of eight 12″ and ten 16″ centerpieces.
**14.** Use the cost formula to find the cost of making five 12″ and twelve 16″ centerpieces.
**15.** Use the revenue formula to find the revenue on four 12″ and nine 16″ centerpieces.

*Summarize what you learned in this activity:*

## ESTIMATING MEASURES

**Outcome: Estimate linear and circular measure in inches.**

Materials: Empty tennis ball can, tennis ball, tape measure.

**1.** Examine the tennis ball can and estimate in inches the following measures.

Height: ________ Circumference: ________

**2.** Use a tape measure to find a close approximation in inches for the measure of the tennis can.

Height: ________ Circumference: ________

**3.** Compare your estimates with the measured results. Were the results as you expected? Explain.

**4.** Measure the diameter of a tennis ball.

Diameter of tennis ball: ________

**5.** Explain the procedure you used to find the diameter.

**6.** Relate the height of the tennis can to the diameter of the tennis ball.

**7.** Relate the circumference of the tennis can to the diameter of the tennis ball.

**8.** Write a rule or procedure for estimating the circumference of a circle when the diameter is known.

**9.** Write a rule or procedure for estimating the diameter of a circle when the circumference is known.

**Estimating Measures** **Out-of-Class Assignment** **Name**________________________

Find five circular objects whose circumference can be easily measured with a tape measure. Trace the circumference of each object on paper. Make the traced circle dark enough to be seen from the back side of the paper or cut out the circle. Fold each circle in half. Measure and record the diameter of each object. Estimate the circumference of each object, measure each circumference, and compare the measured results with your estimates.

| | Description of Object | Measured Diameter | Estimated Circumference | Measured Circumference |
|---|---|---|---|---|
| Object 1: | | | | |
| Object 2: | | | | |
| Object 3: | | | | |
| Object 4: | | | | |
| Object 5: | | | | |

Find five different circular objects: Measure and record the circumference of each object and estimate each diameter. Measure each diameter and compare the results with your estimates.

| | Description of Object | Measured Circumference | Estimated Diameter | Measured Diameter |
|---|---|---|---|---|
| Object 1: | | | | |
| Object 2: | | | | |
| Object 3: | | | | |
| Object 4: | | | | |
| Object 5: | | | | |

*Summarize what you learned in this activity:*

## WHAT IS PI, π?

**Outcome: Discover the relationship between the circumference and diameter of a circle.**

**1.** Find the calculator value of $\pi$.
some calculators: $\boxed{\pi}$ other calculators: $\boxed{\pi}$ $\boxed{\text{EXE}}$

(If the label is above a key, as $\overset{\pi}{\boxed{\phantom{xx}}}$, access the $\pi$ function by first pressing a shift, inverse, or 2nd function key.)

**2.** What is π?

(a) Find 10 circular objects of different sizes and trace the circles onto a sheet of paper.

(b) Use a flexible rule, like a tape measure, to measure the circumference ($C$) of each circular object. If no flexible rule is available, use string and a rigid rule.

(c) Cut out the 10 circles of different sizes.

(d) Fold the cut-out circles in half and measure the diameter ($d$).

(e) Calculate $\frac{C}{d}$. Use decimal equivalents rounded to the nearest thousandth and record your results on the chart in Exercise 3.

(f) Find the difference between $\pi$ and the calculated value, $\frac{C}{d}$. (Use the calculator value of $\pi$.)

error $= \pi - \frac{C}{d}$

(g) Find the percent of error (rounded to the nearest tenth of a percent).

$$\frac{\text{percent of error}}{100} = \frac{\text{amount of error}}{\pi}$$

**3.** Complete the chart with the results from Exercise 2.

Circular Object Number

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| b. $C$ | | | | | | | | | | |
| d. $d$ | | | | | | | | | | |
| e. $\frac{C}{d}$ | | | | | | | | | | |
| f. error | | | | | | | | | | |
| g. % error | | | | | | | | | | |

**4.** Use your findings to argue that the following statement is true or false:
The quotient of the circumference and diameter of any size circle is a fixed value or $\pi$.

**5.** What is a reasonable percent error for this investigation? Check the measurements and calculations for measurements with an unreasonable percent error.

*Summarize what you learned in this activity:*