# Day 15 – Relations and Functions

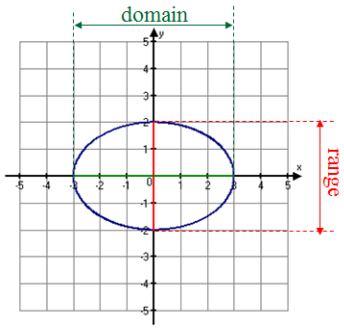
## Definitions

- Relation—A relation is a set of ordered pairs.
- Function—A function is a relation in which, for each value of the first component of the ordered pairs, there is *exactly one* value of the second component.
- If the value of the variable $y$ depends on the value of the $x$ variable, then $y$ is the **dependent variable** and $x$ is the **independent variable**. ($x$, $y$)

## Domain and Range

- In a relation, the set of all values of the independent variable ($x$) is the **domain**; the set of all values of the dependent variable ($y$) is the **range**.

### Finding Domains and Ranges from Graphs




## Definitions

- Agreement on Domain
  Unless specified otherwise, the domain of a relation is assumed to be all real numbers that produce real numbers when substituted for the independent variable.

- Vertical Line Test
  If each vertical line intersects a graph in at most one point, then the graph is that of a function.

## Function Notation

- $y = f(x)$ is called **function notation**
- We read $f(x)$ as "$f$ of $x$" (The parentheses do not indicate multiplication.) The letter $f$ stand for function.
- $f(x)$ is just another name for the dependent variable $y$.
- **Example**: We can write $y = 10x + 3$
  as $f(x) = 10x + 3$

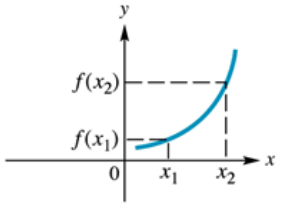
## Variations of the Definition of Function

- A **function** is a relation in which, for each value of the first component of the ordered pairs, there is exactly one value of the second component.
- A **function** is a set of ordered pairs in which no first component is repeated.
- A **function** is a rule or correspondence that assigns exactly one range value to each domain value.

## Increasing, Decreasing, and Constant Functions

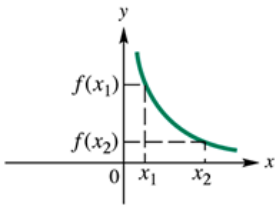
- Suppose that a function $f$ is defined over an interval $I$. If $x_1$ and $x_2$ are in $I$,

(a) $f$ **increases** on $I$ if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$;
(b) $f$ **decreases** on $I$ if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$;
(c) $f$ is **constant** on $I$ if, for every $x_1$ and $x_2$, $f(x_1) = f(x_2)$.



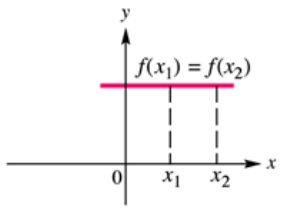

Whenever $x_1 < x_2$, and $f(x_1) < f(x_2)$, $f$ is increasing.

**(a)**




Whenever $x_1 < x_2$, and $f(x_1) > f(x_2)$, $f$ is decreasing.

**(b)**




For every $x_1$ and $x_2$, if $f(x_1) = f(x_2)$, then $f$ is constant.

**(c)**

## Symmetry with Respect to an Axis

- The graph of an equation is **symmetric with respect to the *y*-axis** if the replacement of $x$ with $-x$ results in an equivalent equation.
- The graph of an equation is **symmetric with respect to the *x*-axis** if the replacement of $y$ with $-y$ is an equivalent equation.

## Symmetry with Respect to the Origin

- The graph of an equation is **symmetric with respect to the origin** if the replacement of both $x$ with $-x$ and $y$ with $-y$ results in an equivalent equation.

## Even and Odd Functions

- A function $f$ is called an **even function** if $f(-x) = f(x)$ for all $x$ in the domain of $f$. (Its graph is symmetric with respect to the $y$-axis.)

- A function $f$ is called an **odd function** if $f(-x) = -f(x)$ for all $x$ in the domain of $f$. (Its graph is symmetric with respect to the origin.)