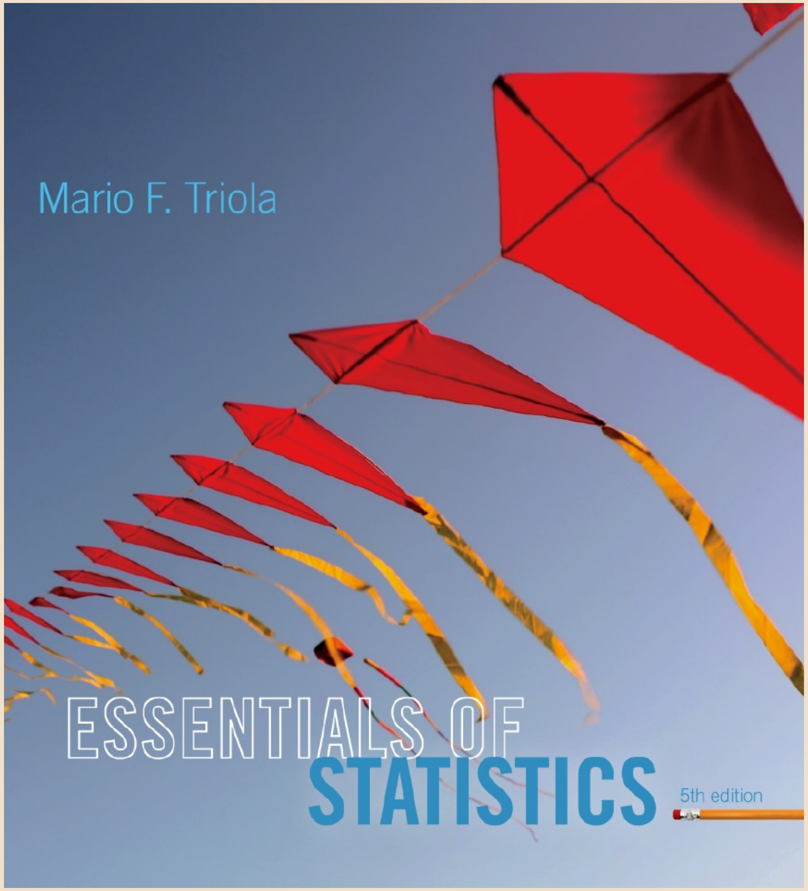
Lecture Slides



Essentials of Statistics
5th Edition

and the Triola Statistics Series

by Mario F. Triola

Chapter 4
Probability

Key Concept

In many probability problems, the big obstacle is finding the total number of outcomes, and this section presents several methods for finding such numbers without directly listing and counting the possibilities.

Counterfeit Coin Problem

- A king has 12 bags of gold coins.
- Inside one bag of gold is a counterfeit coin.
- Using only a balance which can tell which of two items is heavier or that they have the same weight how can you find the bag containing the counterfeit coin, without opening the bags?
- Once you find the bag can you find the coin?

Notation

The factorial symbol ! denotes the product of decreasing positive whole numbers.

For example,

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

By special definition, 0! = 1.

Fundamental Counting Rule

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.

Example

A byte is a sequence of eight numbers, all either 0 or 1.

The number of possible bytes is $2^8 = 256$.

Permutations versus Combinations

- When different orderings of the same items are to be counted separately, we have a permutation problem.

- When different orderings are not to be counted separately, we have a combination problem.

Factorial Rule

Number of different permutations (order counts) of *n* different items can be arranged when all *n* of them are selected. (This factorial rule reflects the fact that the first item may be selected in *n* different ways, the second item may be selected in $n - 1$ ways, and so on.)

Example

A history pop quiz asks students to arrange the following presidents in chronological order: Hayes, Taft, Polk, Taylor, Grant, Pierce.

If an unprepared student totally guesses, what is the probability of guessing correctly?

$$\text{Possible arrangements:}\quad 6! = 720$$

$$P(\text{guessing correctly}) = \frac{1}{720} = 0.00139$$

Permutations Rule
(when items are all different)

Requirements:

1. There are *n* different items available. (This rule does not apply if some of the items are identical to others.)
2. We select *r* of the *n* items (without replacement).
3. We consider rearrangements of the same items to be different sequences. (The permutation of ABC is different from CBA and is counted separately.)

If the preceding requirements are satisfied, the number of permutations (or sequences) of *r* items selected from *n* available items (without replacement) is

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Permutations Rule
(when some items are identical to others)

Requirements:

1. There are n items available, and some items are identical to others.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are n_1 alike, n_2 alike, . . . n_k alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1!\,n_2!\cdots n_k!}$$

Combinations Rule

Requirements:

1. There are *n* different items available.
2. We select *r* of the *n* items (without replacement).
3. We consider rearrangements of the same items to be the same. (The combination of ABC is the same as CBA.)

If the preceding requirements are satisfied, the number of combinations of *r* items selected from *n* different items is

$$_nC_r = \frac{n!}{(n-r)!r!}$$

Example

In the Pennsylvania Match 6 Lotto, winning the jackpot requires you select six different numbers from 1 to 49. The winning numbers may be drawn in any order. Find the probability of winning if one ticket is purchased.

Number of combinations: $_{n}C_{r} = \frac{n!}{(n-r)!r!} = \frac{49!}{43!6!} = 13{,}983{,}816$

$$P(\text{winning}) = \frac{1}{13{,}983{,}816}$$