Solve Radical Equations

1. Isolate RADICAL
2. Raise both SIDES TO INDEX of radical.
3. Solve Equation
4. Check solution with original Equation.

$\sqrt{p-4} + 7 = 10$

$(\sqrt{p-4})^2 = (3)^2$ ①

$p - 4 = 9$

$p = 13$ ③

$\sqrt{(13)-4} + 7 = 10$ ④

$\sqrt{9} + 7 = 10$

$3 + 7 = 10$

$10 = 10$ ✓

①

$\sqrt{2x+6} + 3 = x + 2$

$(\sqrt{2x+6})^2 = (x-1)^2$ ①

$2x + 6 = (x-1)(x-1)$ ②

$2x + 6 = x^2 - 2x + 1$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5$ $\quad x = -1$

$\sqrt{2(5)+6} + 3 = 5 + 2$

$\sqrt{10+6} + 3 = 5 + 2$

$\sqrt{16} + 3 = 5 + 2$

$4 + 3 = 5 + 2$

$7 = 7$ ✓

Check $x = -1$

$\sqrt{2(-1)+6} + 3 = (-1) + 2$

$\sqrt{-2+6} + 3 = -1 + 2$

$\sqrt{4} + 3 = -1 + 2$

$2 + 3 = -1 + 2$

$5 = 1$

X

$x = \{5\}$

2 RADICALS

$$\sqrt{3c+1} - \sqrt{c-1} = 2$$

$$\left(\sqrt{3c+1}\right)^2 = \left(2 + \sqrt{c-1}\right)^2 \rightarrow \left(2+\sqrt{c-1}\right)\left(2+\sqrt{c-1}\right)$$

$$4 + 2\sqrt{c-1} + 2\sqrt{c-1} + c - 1$$

$$3c+1 = 2^2 + 4\sqrt{c-1} + \left(\sqrt{c-1}\right)^2$$

$$\left(2+\sqrt{c-1}\right)\left(2+\sqrt{c-1}\right)$$

$$= 4 + 2\sqrt{c-1} + 2\sqrt{c-1} + (c-1)$$

$$3c+1 = 3 + 4\sqrt{c-1} + c$$

$$2c - 2 = 4\sqrt{c-1}$$

$$(c-1)^2 = \left(2\sqrt{c-1}\right)^2 \quad 2^2\left(\sqrt{c-1}\right)^2$$

$$c^2 - 2c + 1 = 4(c-1) = 4c - 4$$

$$c^2 - 6c + 5 = 0$$

$$(c-5)(c-1) = 0$$

$$c = \{1, 5\}$$

$$\sqrt{3(1)+1} - \sqrt{(1)-1} = 2$$

$$\sqrt{3+1} - \sqrt{0} = 2$$

$$\sqrt{4} - 0 = 2$$

$$2 = 2$$

✓

$$\sqrt{3(5)+1} - \sqrt{5-1} = 2$$

$$\sqrt{15+1} - \sqrt{4} = 2$$

$$\sqrt{16} - \sqrt{4} = 2$$

$$4 - 2 = 2$$

✓

$$c = \{1, 5\}$$

Quadratic in Form

$$\left(2+\frac{3}{x}\right)^2 + 10\left(2+\frac{3}{x}\right) + 9 = 0$$

Let $a = 2+\frac{3}{x}$

$$a^2 + 10a + 9 = 0$$

$$(a+1)(a+9) = 0$$

$a+1=0$

$2+\frac{3}{x}+1=0$

$x\left(3+\frac{3}{x}\right)=(0)x$

$3x+3=0$

$x=-1$

$a+9=0$

$2+\frac{3}{x}+9=0$

$11+\frac{3}{x}=0$

$11x+3=0$

$x=-\frac{3}{11}$

$$\left(x^2-23\right)^2 + 6\left(x^2-23\right) - 7 = 0$$

Let $a = x^2-23$

$$a^2 + 6a - 7 = 0$$

$$(a+7)(a-1) = 0$$

$a+7=0$

$x^2-23+7=0$

$x^2-16=0$

$(x-4)(x+4)=0$

$x=\pm 4$

$a-1=0$

$x^2-23-1=0$

$x^2-24=0$

$x^2=24$

$x=\pm\sqrt{24}=\pm\sqrt{4}\sqrt{6}=\pm 2\sqrt{6}$

$w^{2/3} - 6w^{1/3} = 27$

$w^{2/3} - 6w^{1/3} - 27 = 0$

$(w^{1/3})^2 - 6(w^{1/3}) - 27 = 0$

Let $w^{1/3} = a$

$a^2 - 6a - 27 = 0$

$(a-9)(a+3) = 0$

$a - 9 = 0$

$w^{1/3} - 9 = 0$

$w^{1/3} = 9$

$(\sqrt[3]{w})^3 = (9)^3$

$w = 729$

$a + 3 = 0$

$w^{1/3} + 3 = 0$

$w^{1/3} = -3$

$(\sqrt[3]{w})^3 = (-3)^3$

$w = -27$

$3c^4 + 14c^2 - 5 = 0$

Let $a = c^2$

$3(c^2)^2 + 14(c^2) - 5 = 0$

$3a^2 + 14a - 5 = 0$

$3a^2 + 15a - a - 5 = 0$

$3a(a+5) - 1(a+5) = 0$

$(3a-1)(a+5) = 0$

$3a - 1 = 0$ $\qquad$ $a + 5 = 0$

$3a = 1$ $\qquad$ $a = -5$

$a = \frac{1}{3}$

$c^2 = \frac{1}{3}$ $\qquad$ $c^2 = -5$

$c = \pm\sqrt{\frac{1}{3}}$ $\qquad$ $c = \pm\sqrt{-5} = \sqrt{-1}\sqrt{5}$

$= \pm\frac{\sqrt{1}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$ $\qquad$ $= \pm i\sqrt{5}$

$c = \pm\frac{\sqrt{3}}{3}$ $\qquad$ $c = \pm i\sqrt{5}$