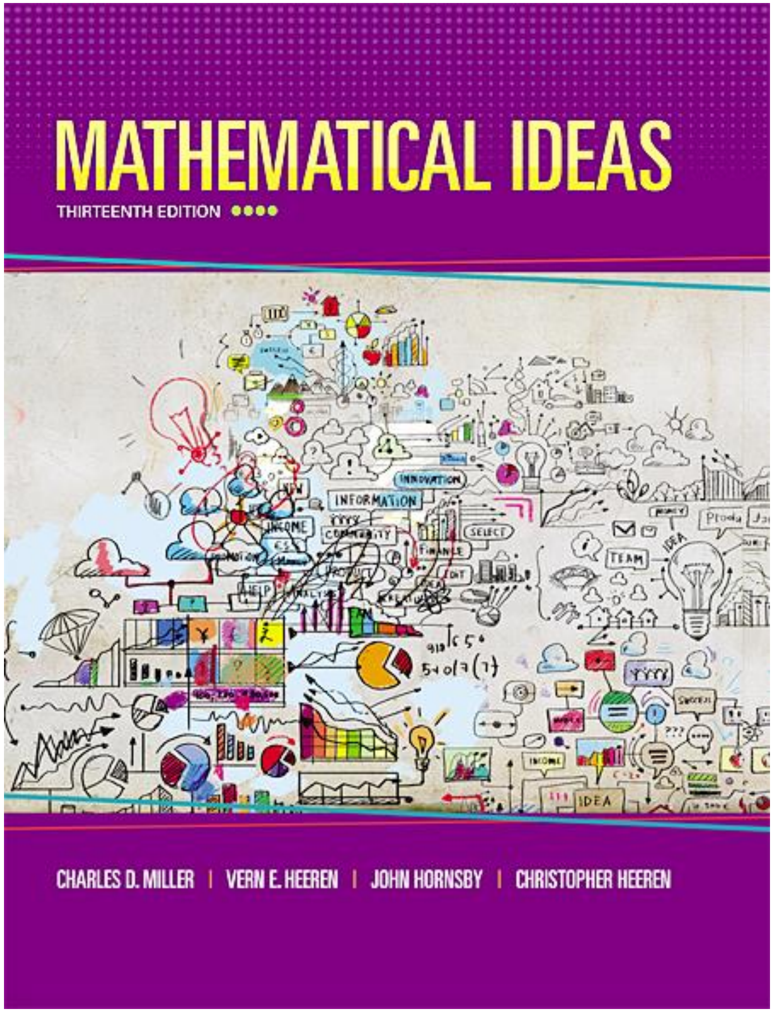
# Chapter 10

## Number Theory



# Chapter 10: Number Theory

# Section 10-2

Selected Topics from Number Theory

# Selected Topics from Number Theory

- Understand and identify perfect numbers.
- Understand and identify deficient and abundant numbers.
- Understand amicable (friendly) numbers.
- State and evaluate Goldbach's conjecture.
- Understand and identify twin primes.
- State and evaluate Fermat's Last Theorem.

# Perfect Numbers

A natural number is said to be **perfect** if it is equal to the sum of its proper divisors.

6 is perfect because $6 = 1 + 2 + 3$.

8 is not because $8 \neq 1 + 2 + 4$.

# Deficient and Abundant Numbers

A natural number is **deficient** if it is greater than the sum of its proper divisors. It is **abundant** if it is less than the sum of its proper divisors.

# Example: Identifying Deficient and Abundant Numbers

Decide whether 12 is deficient or abundant.

## Solution

The proper divisors of 12 are 1, 2, 3, 4, and 6. Their sum is 16. Because 16 > 12, the number 12 is abundant.

# Amicable (Friendly) Numbers

The natural numbers $a$ and $b$ are **amicable**, or **friendly**, if the sum of the proper divisors of $a$ is $b$, and the sum of the proper divisors of $b$ is $a$.

The smallest pair of amicable numbers is 220 and 284.

# Goldbach's Conjecture (Not Proved)

Every even number greater than 2 can be written as the sum of two prime numbers.

# Example: Expressing Numbers as Sums of Primes

Write each even number as the sum of two primes.

a) 12 b) 40

## Solution

a) $12 = 5 + 7$

b) $40 = 17 + 23$

# Ramujan & Hardy: Taxicab numbers

1729 can be written as the sum of two cubes in two different ways:

$$1^3 + 12^3 = 1729$$
$$9^3 + 10^3 = 1729$$

Show that 85 can be written as the sum of two squares in two different ways:

Solution

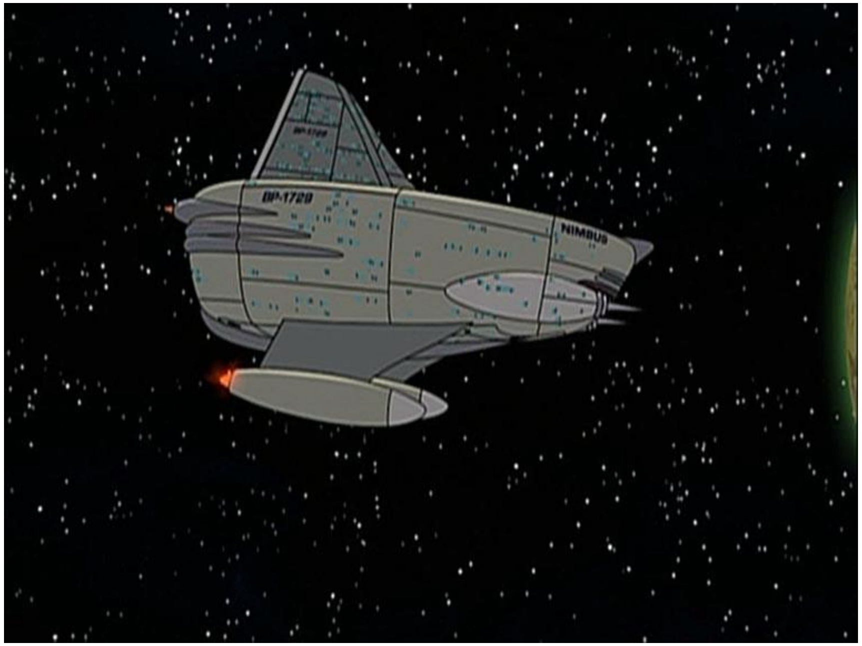
$$2^2 + 9^2 = 85$$
$$2^2 + 3^4 = 85$$

# Ramujan & Hardy: Taxicab numbers

Futurama:

The serial number of the nimbus is seen as 1729.

Number of box containing universe populated by bobbleheads.



# Twin Primes

**Twin primes** are prime numbers that differ by 2.

Examples: 3 and 5, 11 and 13

# Twin Primes Conjecture (Not Proved)

There are infinitely many pairs of twin primes.

# Fermat's Last Theorem

For *any* natural number $n \geq 3$, there are no triples $(a, b, c)$ that satisfy the equation:

$$a^n + b^n = c^n.$$

# Example: Using a Theorem Proved by Fermat

*Every odd prime can be expressed as the difference of two squares in one and only one way.*

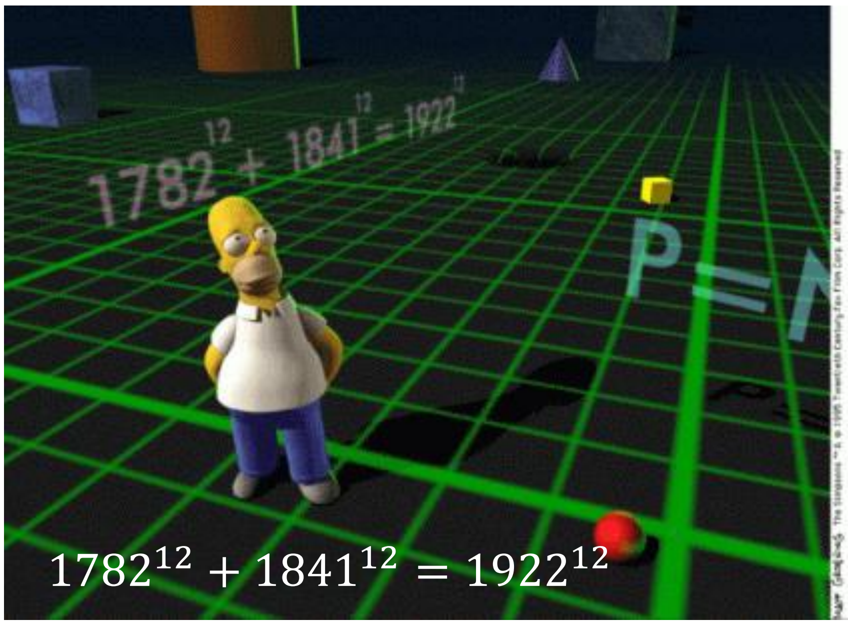
Express 7 as the difference of two squares.

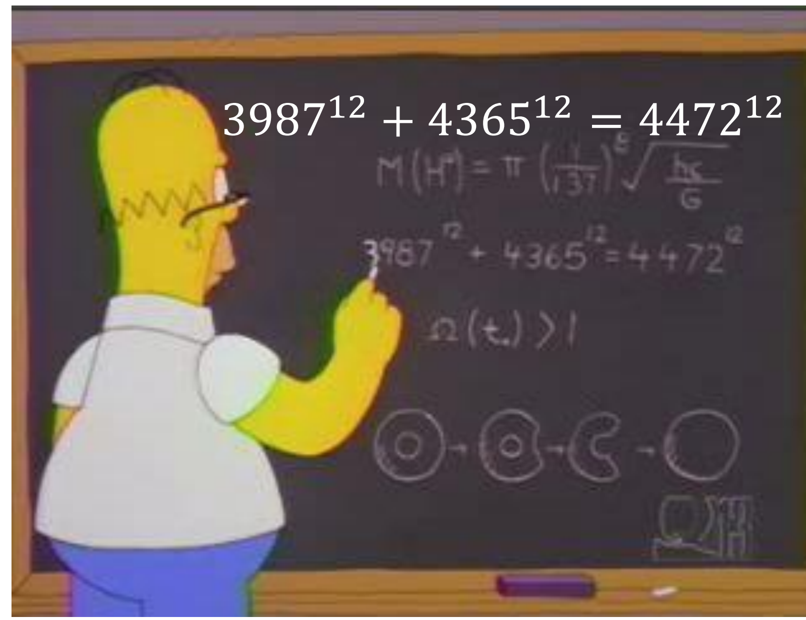
## Solution

$$7 = 16 - 9 = 4^2 - 3^2$$

# Fermat Near Misses

## Turn to the Simpsons




How can we prove that these equations are untrue?