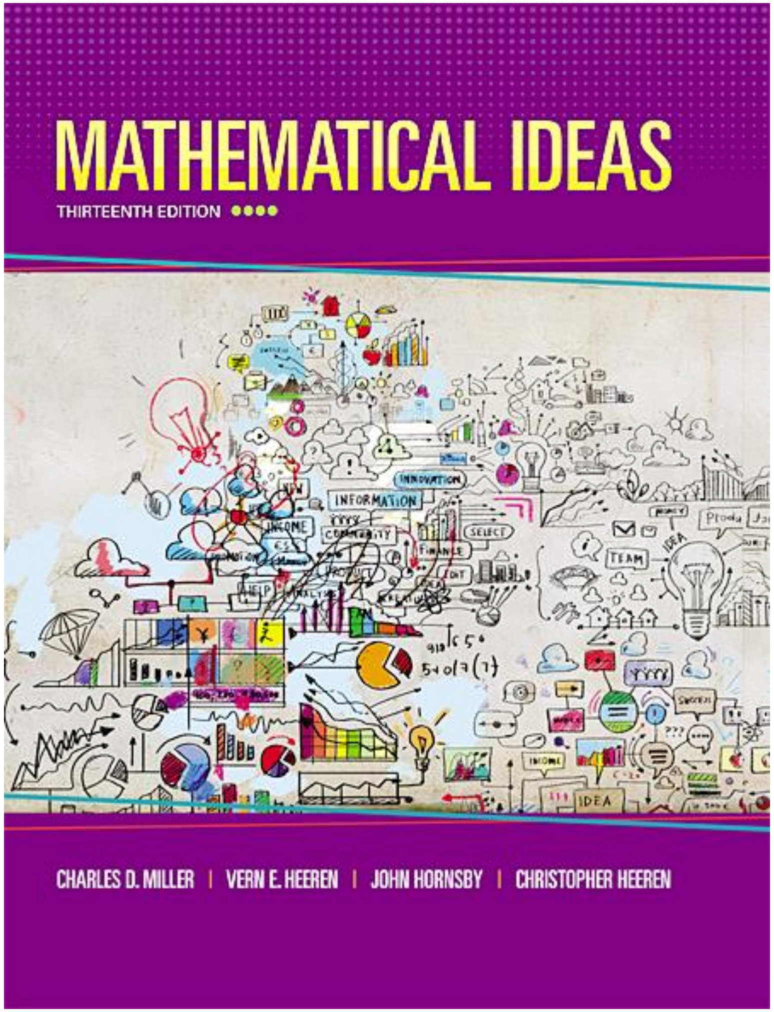
Chapter 7

Personal Financial Management



Chapter 7: Personal Financial Management

Section 13-1

The Time Value of Money

The Time Value of Money

- Know the meaning of interest.
- Calculate simple interest.
- Determine future value and present value.
- Calculate compound interest.
- Find the effective annual yield.
- Use the rate of inflation to compare values over time.

Interest

If we borrow an amount of money today, we will repay a larger amount later. The increase in value is known as **interest**. The money *gains value over time*.

The amount of a loan or a deposit is called the **principal**. The interest is usually computed as a percent of the principal. This percent is called the **rate of interest** (or the **interest rate,** or simply the rate). The rate of interest is assumed to be an annual rate unless otherwise stated.

Interest

Interest calculated only on principal is called **simple interest**. Interest calculated on principal plus any previously earned interest is called **compound interest**.

Simple Interest

If P = principal, r = annual interest rate, and t = time (in years), then the **simple interest** I is calculated as follows.

$$\boldsymbol{I = Prt}$$

Example: Finding Simple Interest

Find the simple interest paid to borrow $4800 for 6 months at 7%.

Solution

$I = Prt = \$4800(0.07)(6/12) = \$168.$

6 months is 6/12 of a year.

Future and Present Value

In the last example, the borrower would have to repay \$4800 + \$168 = \$4968.

The total amount repaid is called the **maturity value** (or the **value**) of the loan. We will refer to it as the **future value**, or **future amount**. The original principal, denoted P, can also be thought of as **present value**.

Future Value for Simple Interest

If a principal P is borrowed at simple interest for t years at an annual interest rate of r, then the **future value** of the loan, denoted $\boldsymbol{A}$, is given by

$$\boldsymbol{A = P(1 + rt)}.$$

Example: Future Value for Simple Interest

Find the future value of $460 in 8 months, if the annual interest rate is 12%.

Solution

$$A = P(1+rt) = \$460\left(1+0.12\left(\frac{8}{12}\right)\right) = \$496.80.$$

Example: Present Value for Simple Interest

If you can earn 6% interest, what lump sum must be deposited now so that its value will be \$3500 after 9 months?

Solution

$$A = P(1 + rt)$$

$$3500 = P\left(1 + 0.06\left(\frac{9}{12}\right)\right)$$

$$P = \frac{\$3500}{1.045} \approx \$3349.28$$

Compound Interest

Interest paid on principal plus interest is called *compound interest*. After a certain period, the interest earned so far is *credited* (added) to the account, and the sum (principal plus interest) then earns interest during the next period.

Compounding Period

Interest can be credited to an account at time intervals other than 1 year. For example, it can be done semiannually, quarterly, monthly, or daily. This time interval is called the **compounding period** (or the **period**).

Future Value for Compound Interest

If P dollars are deposited at an annual interest rate of r, compounded n times per year, and the money is left on deposit for a total of m periods (or $t = m/n$ years), then the **future value, *A*** (the final amount on deposit), is given by:

$$A = P\left(1+\frac{r}{n}\right)^{m} \text{ or } \quad A = P\left(1+\frac{r}{n}\right)^{nt}$$

Example: Finding Future Value for Compound Interest

Find the future value of \$8560 at 4% compounded quarterly for 8 years.

Solution

$P = \$8560,\ r = 4\% = 0.04,\ n = 4.$

Over 8 years $m = 8n = 8(4) = 32.$

$$A = P\left(1+\frac{r}{n}\right)^{m} = \$8560\left(1+\frac{0.04}{4}\right)^{32} \approx \$11,769.49.$$

Example: Finding Present Value for Compound Interest

What amount must be deposited today, at 5% compounded monthly, so that it will be $18,000 in 20 years?

Solution

$$\$18000 = P\left(1+\frac{0.05}{12}\right)^{240}$$

$$P = \frac{\$18000}{\left(1+\frac{0.05}{12}\right)^{240}} \approx \$6635.60$$

Example: Video Daily Double??
Futurama: “A Fishful of Dollars”

Watch the video and answer the following question:

How much money does Phillip J Fry have in his account?

Solve: $A = P(1 + \frac{r}{n})^{nt}$

Principal: 0.93

rate: 2¼ %

Time: 1000 years

Example: Video Daily Double?? Futurama: "A Fishful of Dollars"

Watch the video and answer the following question:

How much money does Phillip J Fry have in his account?

Solve: $A = P(1 + \frac{r}{n})^{nt}$

$$A = 0.93(1 + 0.0225)^{1000}$$
$$A = 0.93(1.0225)^{1000}$$
$$A = 0.93 * 4605923064$$

A=$4 283 508 450

Principal: 0.93

rate: 2¼ %

Time: 1000 years

Effective Annual Yield

Savings institutions often give two quantities when advertising the rates. The first, the actual annualized interest rate, is the **nominal rate** (the "stated" rate). The second quantity is the equivalent rate that would produce the same final amount, or future value, at the end of 1 year if the interest being paid were simple rather than compound. This is called the "effective rate," or the **effective annual yield**.

Effective Annual Yield

A nominal interest rate of *r*, compounded *n* times per year, is equivalent to the following **effective annual yield.**

$$Y = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Example: Effective Annual Yield

What is the effective annual yield of an account paying a nominal rate of 4.2%, compounded monthly?

Solution

$$Y = \left(1 + \frac{0.042}{12}\right)^{12} - 1 \approx .0428 = 4.28\%$$

Inflation

In terms of the equivalent number of goods or services that a given amount of money will buy, it is normally more today than it will be later. In this sense, the money *loses value over time*. This periodic increase in the cost of living is called **price inflation**.

Unlike account values under interest compounding, which make sudden jumps at just certain points in time, price levels tend to fluctuate gradually over time. It is appropriate, for inflationary estimates, to use a formula for continuous compounding.

Inflation

Inflation in an economy usually is expressed as a monthly or annual rate of price increases, estimated by government agencies in a systematic way. In the United States, the Bureau of Labor Statistics publishes **consumer price index (CPI)** figures, which reflect the prices of certain items purchased by large numbers of people (see table on page 714 of the text).

Future Value for Continuous Compounding

If an initial deposit of *P* dollars earns continuously compounded interest at an annual rate *r* for a period of *t* years, then the **future value**, ***A***, is calculated as follows.

$$A = Pe^{rt}$$

Example: Future Value for Continuous Compounding

Suppose that a cup of your favorite coffee is $1.25. If the inflation rate persists at 2% over time, find the approximate cost of the coffee in 25 years.

Solution

$$A = Pe^{rt} = \$2.25e^{(0.02)(25)} \approx \$3.71$$

The coffee will cost about $3.71.

Inflation Proportion

For a consumer product or service subject to average inflation,

$$\frac{\text{Price in year A}}{\text{Price in year B}} = \frac{\text{CPI in year A}}{\text{CPI in year B}}$$

Example: Inflation

If your mother paid \$3,000 in tuition in 1980 at the same college that you will be attending and paying \$9,300 in 2005, compare the school's tuition increase to inflation over that same period of time.

Solution

Let x represent what we expect the tuition to be in 2005 if it had increased at the average rate since 1980.

$$\frac{\text{Price in year 2005}}{\text{Price in year 1980}} = \frac{\text{CPI in year 2005}}{\text{CPI in year 1980}}$$

Example: Inflation

Solution (continued)

$$\frac{x}{\$3000} = \frac{195.3}{82.4} \qquad x \approx \$7110.44.$$

Now compare with the actual 2005 tuition.

$$\frac{\$9300}{\$7110.44} \approx 1.31$$

Tuition at the school increased approximately 31% more than the average CPI-U rate.

Example: Inflation

An estimation of the **years to double**, which is the number of years it takes for the general level of prices to double for a given annual rate of inflation, is given by

$$\text{years to double} \approx \frac{70}{\text{annual inflation rate}}.$$

Example: Estimating Years to Double by the Rule of 70

Estimate the number of years to double for an annual inflation rate of 2.1%

Solution

$$\text{Years to double} \approx \frac{70}{2.1} = 33.33$$

With an inflation rate of 2.1%, prices would double in about 34 years.