Chapter 12

The Real Numbers and Their Representation

Chapter 12: The Real Numbers and Their Representation

12.1 Rational Numbers and Decimal Representation

12.2 Irrational Numbers and Decimal Representation

12.3 Accuracy, Precision, Error and Measuring Instruments

Section 12-3

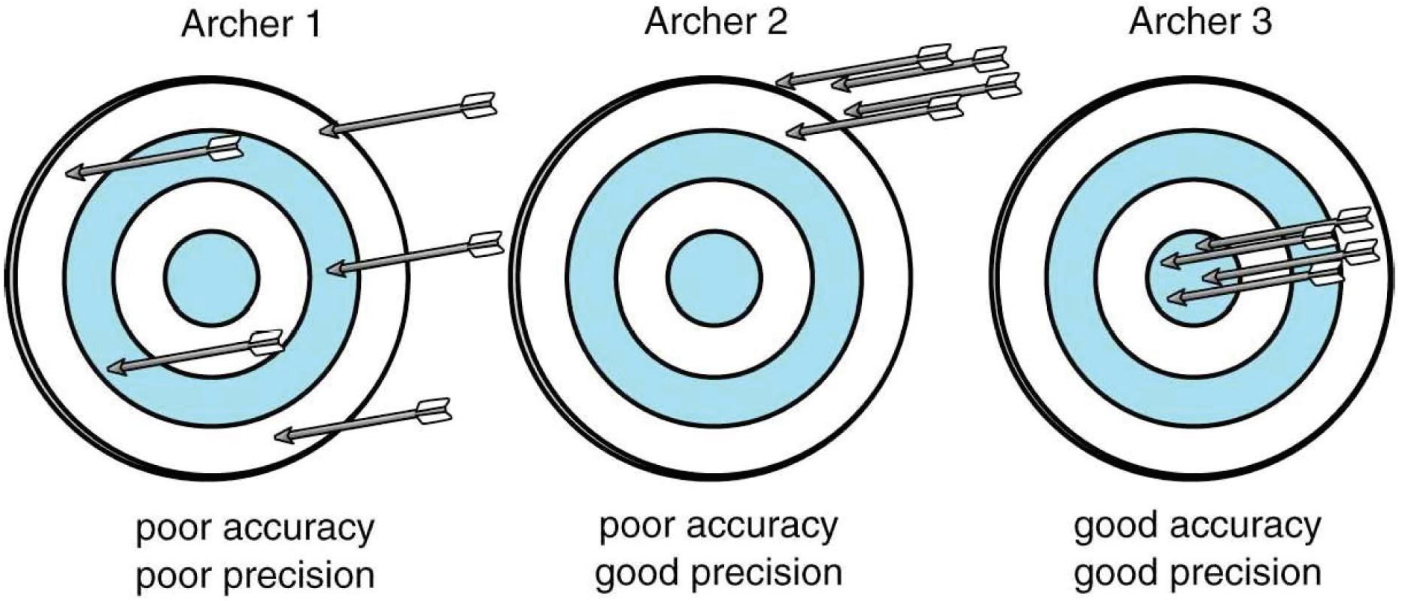
Application of Decimals and Percents

Accuracy, Precision, Error and Measuring Instruments

- Determine the significant digits of a number.
- Find the precision and greatest possible error of a measurement.
- Determine the relative error and percent error of a measurement.
- Determine an appropriate approximation of measurement calculations.
- Read a ruler.
- Find the distance and midpoint of a line segment.

Determine the significant digits of a number

- Accuracy – how close the measured value is to the correct value.
- Precision – the degree to which the process gets consistent results.



Determine the significant digits of a number

- One indicator of precision is the number of **significant digits.**

To determine the significant digits of a number:	
For whole numbers: 1. Start with the leftmost nonzero digit. 2. Count each digit (excluding zeros) through the rightmost nonzero digit.	*Significant digits:* 200 has 1 significant digit 28 has 2 significant digits 1,230 has 3 significant digits 2,005 has 4 significant digits
For decimals and mixed decimals: 1. Start with the leftmost nonzero digit. 2. Count each digit through the last digit.	0.007 has 1 significant digit 2.0 has 2 significant digits 1.20 has 3 significant digits 0.01234 has 4 significant digits

Determine the significant digits of a number

- Indicate the number of significant digits in the following:

a. 5,205 — 4 significant digits

b. 12,000 — 2 significant digits

c. 0.003 — 1 significant digit

d. 0.0105 — 3 significant digits

e. 3.02 — 3 significant digits

Find the precision and greatest possible error of a measurement.

Precision is based on the place value or significant digits of a measure.

The **greatest possible error** or a measurement is half of the precision of the measurement.

To find the precision and greatest possible error of a measurement:	
1. Determine the precision of the measurement. 2. The greatest possible error is one-half the precision.	Find the greatest possible error of a measurement of 1¾ ft. Precision: ¼ ft. ½(¼) = 1/8 ft.

Find the precision and greatest possible error of a measurement.

Find the greatest possible error of the measurements.

2 5/8 inches	3.5 cm
• 2 5/8 inches - 1/8 inch precision • ½ (1/8) = 1/16 inch maximum error	• 3.5 cm - Precision = 0.1 cm • 0.5 (0.1) = 0.05cm maximum error

Determine the relative error and percent error of a measurement.

$$absolute\ error = |observed\ value - true\ value|$$

$$relative\ error = \frac{absolute\ error}{true\ value}$$

$$percent\ error = relative\ error * 100\%$$

Determine the relative error and percent error of a measurement.

Example: The blueprint calls for a part to 32.112mm. A measurement of the part is recorded as 32.155mm.

$$absolute\ error = |observed\ value - true\ value|$$

$$Absolute\ error = |32.155 - 32.112| = 0.043\text{mm}$$

Determine the relative error and percent error of a measurement.

Example: The blueprint calls for a part to 32.112mm. A measurement of the part is recorded as 32.155mm.

$$relative\ error = \frac{absolute\ error}{true\ value}$$

$$relative\ error = \frac{0.043mm}{32.112mm} = 0.0013390633$$

Determine the relative error and percent error of a measurement.

Example: The blueprint calls for a part to 32.112mm. A measurement of the part is recorded as 32.155mm.

$$percent\ error = relative\ error\ * 100\%$$

Percent error = 0.134% (rounded)

Determine an appropriate approximation of measurement calculations.

To round the sum or difference of measurements of different precisions:

1. If necessary, change all the measurements to a common unit of measure.
2. Add or subtract common units of measure.
3. Round the result to have the same precision as the *least precise* measurement.

Determine an appropriate approximation of measurement calculations.

Add the measurements: 12.5 m, 38 cm, 2.9m, 43.25cm.

If necessary, change all the measurements to a common unit of measure.

12,500cm, 38cm, 2,900cm, 43.25 cm

Determine an appropriate approximation of measurement calculations.

Add the measurements: 12.5 cm, 38 cm, 2.9m, 43.25cm.

If necessary, change all the measurements to a common unit of measure.

12,500cm, 38cm, 2,900cm, 43.25 cm

Add or subtract common units of measure.

Round the result to have the same precision as the *least precise* measurement.

Determine an appropriate approximation of measurement calculations.

Add the measurements: 12.5 cm, 38 cm, 2.9m, 43.25cm.

Add or subtract common units of measure.

12,500
38.0
2,900
+ 43.25
16,212.25cm

Determine an appropriate approximation of measurement calculations.

Add the measurements: 12.5 cm, 38 cm, 2.9m, 43.25cm.

Round the result to have the same precision as the *least precise* measurement.

16,212.25cm rounds 16.21225m which rounds to 16.2m

Determine an appropriate approximation of measurement calculations.

To round the quotient or product of measurements of different accuracy:

1. Multiply or divide the measurements (same units)
2. Round the product or quotient to have the same number of significant digits as the measurement with the fewest significant digits.

Determine an appropriate approximation of measurement calculations.

Multiply the measurements: (150m)(105m)(50m)

- Multiply or divide the measurements (same units)

  $(150m)(105m)(50m)=787\ 500m^3$

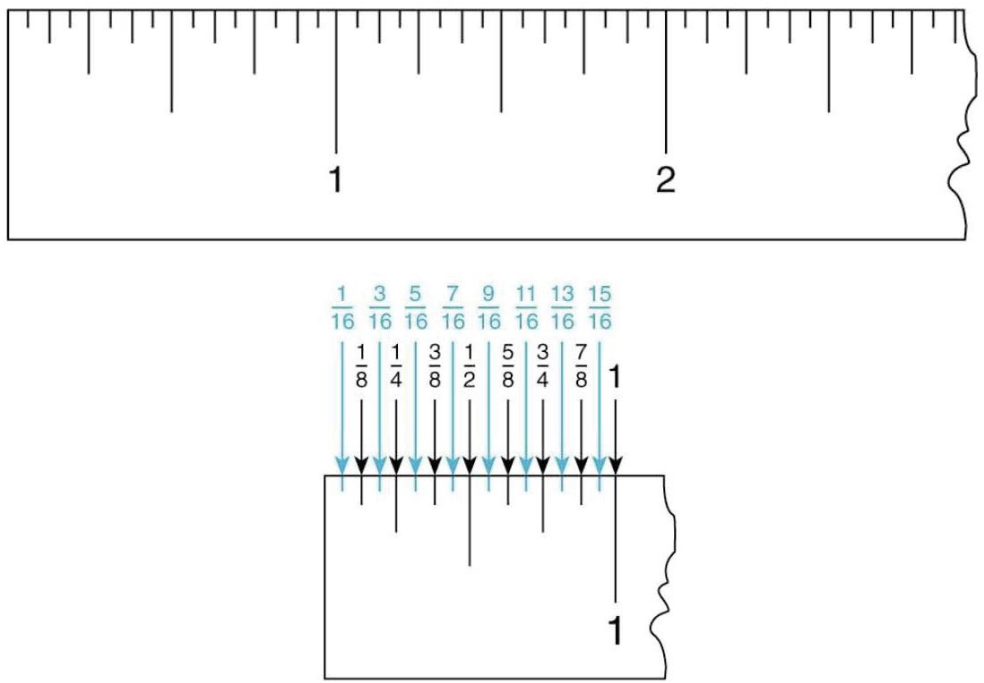
- Round the product or quotient to have the same number of significant digits as the measurement with the fewest significant digits.

  50m has the fewest signinficant digits – one

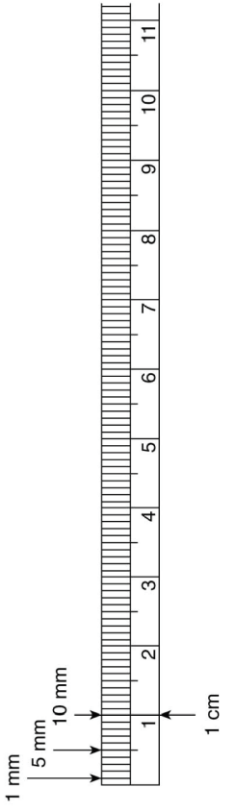
  $787\ 500m^3 \approx 800\ 000m^3$

Read a ruler

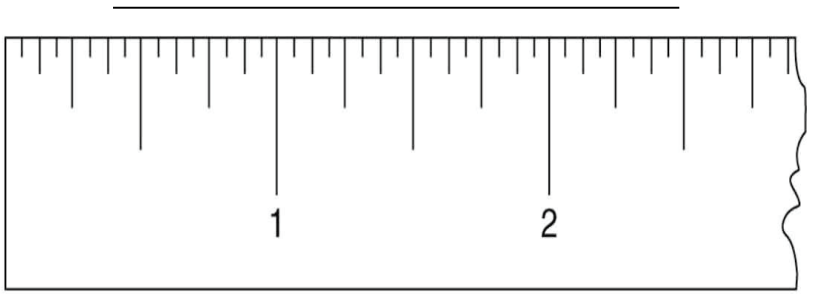
FIGURE 2–8 1 in.



Read a ruler



Find the distance and midpoint of a line segment.

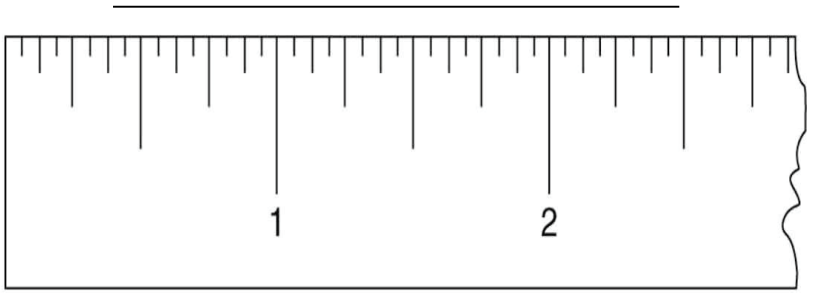


To find the distance between two points on a line:

1. Determine the coordinate for each point.

$$\frac{3}{8}, 2\frac{1}{2}$$

Find the distance and midpoint of a line segment.



To find the distance between two points on a line:

1. Subtract the coordinate of the leftmost (lesser value) point from the rightmost.

$$2\frac{1}{2}-\frac{3}{8}=\frac{5}{2}-\frac{3}{8}=\frac{20}{8}-\frac{3}{8}=\frac{17}{8}=2\frac{1}{8}$$

Find the distance and midpoint of a line segment.

To find the midpoint between two points on a line:

1. Determine the coordinate for each point.
2. Average the coordinates of the endpoints.

$$Midpoint = \frac{P_1 + P_2}{2}$$

Find the distance and midpoint of a line segment.

To find the midpoint between 2.8 cm and 5.6 cm.

1. **Determine the coordinate for each point.**

   2.8 cm 5.6 cm

2. **Average the coordinates of the endpoints.**

$$Midpoint = \frac{P_1 + P_2}{2}$$

$$Midpoint = \frac{2.8cm + 5.6cm}{2}$$

$$Midpoint = \frac{8.4cm}{2} = 4.2cm$$