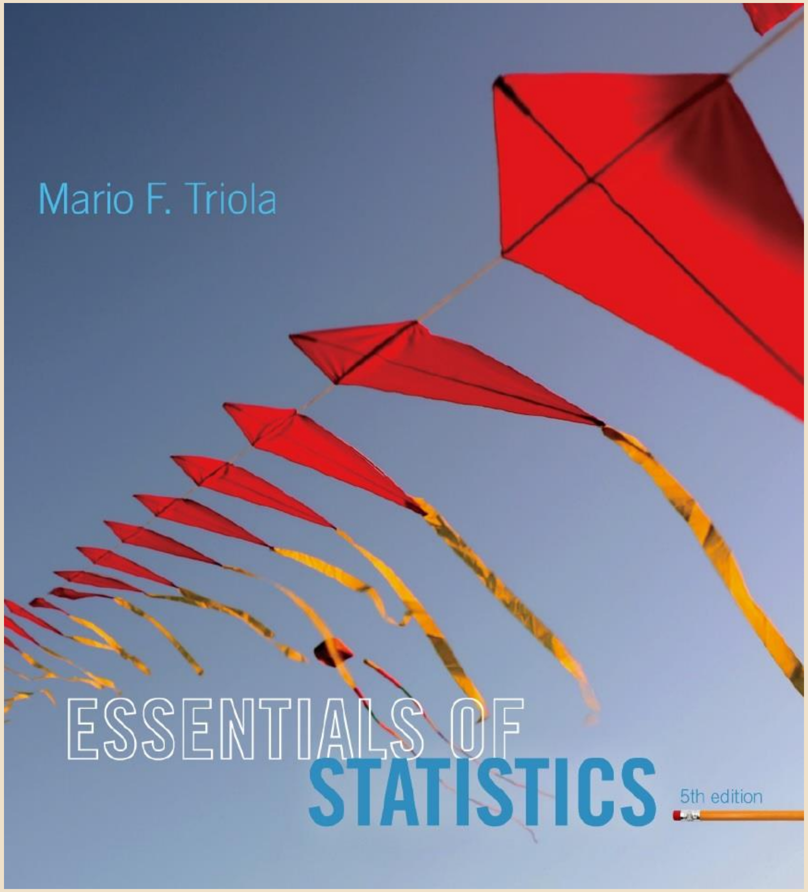
Lecture Slides



Essentials of Statistics
5th Edition

and the Triola Statistics Series

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Chapter 4
Probability

4-1 Review and Preview

4-2 Basic Concepts of Probability

4-3 Addition Rule

4-4 Multiplication Rule: Basics

4-5 Multiplication Rule: Complements and Conditional Probability

4-6 Counting

4-7 Probabilities Through Simulations

4-8 Bayes' Theorem

Definitions

A prior probability is an initial probability value originally obtained before any additional information is obtained.

A posterior probability is a probability value that has been revised using additional information that is later obtained.

Bayes' Theorem

The probability of event *A*, given that event *B* has subsequently occurred, is

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{[P(A) \cdot P(B|A)] + [P(\bar{A}) \cdot P(B|\bar{A})]}$$

Example

In Orange County, 51% are males and 49% are females.

One adult is selected at random for a survey involving credit card usage.

a. Find the prior probability that the selected person is male.

b. It is later learned the survey subject was smoking a cigar, and 9.5% of males smoke cigars (only 1.7% of females do). Now find the probability the selected subject is male.

Example – continued

Notation:

$$M = \text{ male} \qquad \bar{M} = \text{female}$$

$$C = \text{ cigar smoker} \qquad \bar{C} = \text{ not a cigar smoker}$$

a. Before the extra information obtained in part (b), we know 51% of the adults are male, so *P*(*M*) = 0.51.

Example – continued

b. Based on the additional information:

$$P(M) = 0.51$$

$$P\left(\bar{M}\right) = 0.49$$

$$P\left(C \mid M\right) = 0.095$$

$$P\left(C \mid \bar{M}\right) = 0.017$$

We can now apply Bayes' Theorem:

Example – continued

$$P(M \mid C) = \frac{P(M) \cdot P(C \mid M)}{\left[P(M) \cdot P(C \mid M)\right] + \left[P(\bar{M}) \cdot P(C \mid \bar{M})\right]}$$

$$= \frac{0.51 \cdot 0.095}{[0.51 \cdot 0.095] + [0.49 \cdot 0.017]}$$

$$= 0.853 \ (\text{rounded})$$

Bayes' Theorem Generalized

The preceding formula used exactly two categories for event *A*, but the formula can be extended to include more than two categories.

We must be sure the multiple events satisfy two important conditions:

1. The events must be disjoint (with no overlapping).
2. The events must be exhaustive, which means they combine to include all possibilities.