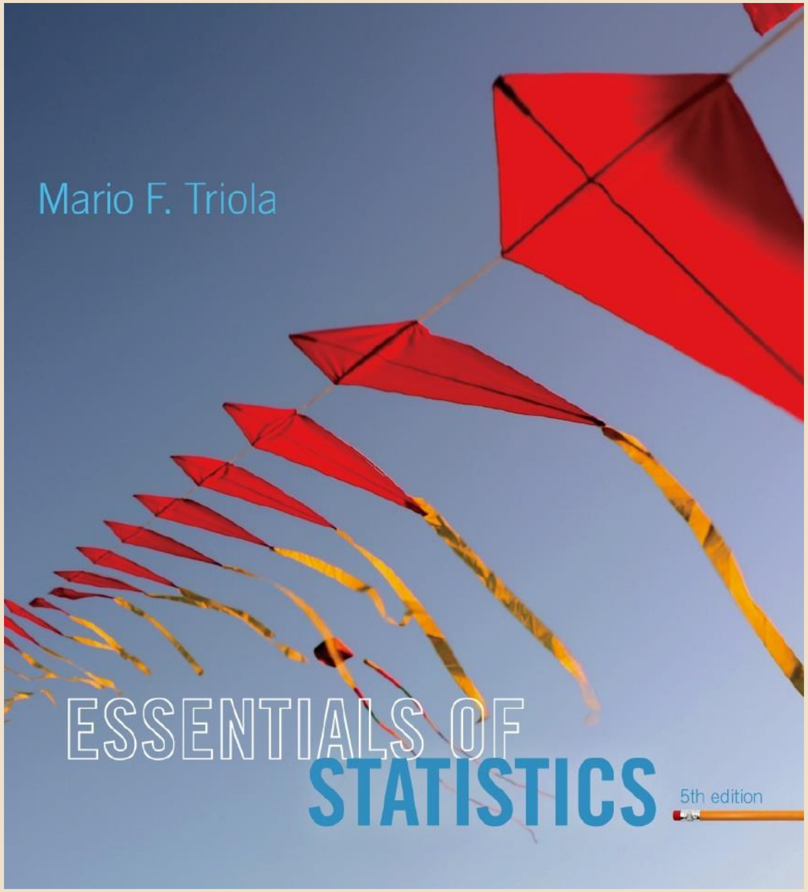
Lecture Slides



Essentials of Statistics
5th Edition

and the Triola Statistics Series

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Chapter 5
Probability Distributions

5-1 Review and Preview

5-2 Probability Distributions

5-3 Binomial Probability Distributions

5-4 Parameters for Binomial Distributions

Key Concept

This section presents a basic definition of a binomial distribution along with notation and methods for finding probability values.

Binomial probability distributions allow us to deal with circumstances in which the outcomes belong to two relevant categories such as acceptable/defective or survived/died.

Binomial Probability Distribution

A binomial probability distribution results from a procedure that meets all the following requirements:

1. The procedure has a fixed number of trials.
2. The trials must be independent. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into two categories (commonly referred to as success and failure).
4. The probability of a success remains the same in all trials.

Notation for Binomial Probability Distributions

S and F (success and failure) denote the two possible categories of all outcomes; *p* and *q* will denote the probabilities of S and F, respectively, so

$$P(\mathrm{S}) = p$$ (p = probability of success)

$$P(\mathrm{F}) = 1 - p = q$$ (q = probability of failure)

Notation (continued)

n denotes the fixed number of trials.

x denotes a specific number of successes in *n* trials, so *x* can be any whole number between 0 and *n*, inclusive.

p denotes the probability of success in one of the *n* trials.

q denotes the probability of failure in one of the *n* trials.

P(*x*) denotes the probability of getting exactly *x* successes among the *n* trials.

Caution

- Be sure that *x* and *p* both refer to the same category being called a success.
- When sampling without replacement, consider events to be independent if $n < 0.05N$.

Example

- When an adult is randomly selected, there is a 0.85 probability that this person knows what Twitter is.
- Suppose we want to find the probability that exactly three of five randomly selected adults know of Twitter.
- Does this procedure result in a binomial distribution?

  Yes. There are five trials which are independent. Each trial has two outcomes and there is a constant probability of 0.85 that an adult knows of Twitter.

Methods for Finding Probabilities

We will now discuss three methods for finding the probabilities corresponding to the random variable x in a binomial distribution.

Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

$$\text{for } x = 0,\ 1, 2, \ldots, n$$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

Method 2: Using Technology

STATDISK, Minitab, Excel, SPSS, SAS and the TI-83/84 Plus calculator can be used to find binomial probabilities.

STATDISK

Binomial Probability

Num Trials, n: 5

Success Prob, p: 0.75

Evaluate

Mean:	3.7500
St Dev:	0.9682
Variance:	0.9375

x	P(x)	P(x or fewer)	P(x or greater)
0	0.0009766	0.0009766	1.0000000
1	0.0146484	0.0156250	0.9990234
2	0.0878906	0.1035156	0.9843750
3	0.2636719	0.3671875	0.8964844
4	0.3955078	0.7626953	0.6328125
5	0.2373047	1.0000000	0.2373047

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MINITAB

x	P(x)
0	0.000977
1	0.014648
2	0.087891
3	0.263672
4	0.395508
5	0.237305

Method 2: Using Technology

STATDISK, Minitab, Excel and the TI-83 Plus calculator can all be used to find binomial probabilities.

EXCEL

	A	B
1	0	0.000977
2	1	0.014648
3	2	0.087891
4	3	0.263672
5	4	0.395508
6	5	0.237305

TI-83 PLUS Calculator

L1	L2	L3 2
0	9.8E-4	------
1	.01465	
2	.08789	
3	.26367	
4	.39551	
5	.2373	

L2(7) =

Method 3: Using Table A-1 in Appendix A

Part of Table A-1 is shown below. With n = 12 and p = 0.80 in the binomial distribution, the probabilities of 4, 5, 6, and 7 successes are 0.001, 0.003, 0.016, and 0.053 respectively.

n	x	p 0.80
	4	0.001
	5	0.003
	6	0.016
	7	0.053

→

x	p
4	0.001
5	0.003
6	0.016
7	0.053

Strategy for Finding Binomial Probabilities

- Use computer software or a TI-83/84 Plus calculator, if available.
- If neither software nor the TI-83/84 Plus calculator is available, use Table A-1, if possible.
- If neither software nor the TI-83/84 Plus calculator is available and the probabilities can't be found using Table A-1, use the binomial probability formula.

Example

Given there is a 0.85 probability that any given adult knows of Twitter, use the binomial probability formula to find the probability of getting exactly three adults who know of Twitter when five adults are randomly selected.

We have:

$$n = 5, \quad x = 3, \quad p = 0.85, \quad q = 0.15$$

We want:

$$P(3)$$

Example

We have:

$$n = 5, \quad x = 3, \quad p = 0.85, \quad q = 0.15$$

$$P(3) = \frac{5!}{(5-3)!3!} \cdot 0.85^{3} \cdot 0.15^{5-3}$$

$$= \frac{5!}{2!3!} \cdot 0.614125 \cdot 0.0225$$

$$= (10)(0.614125)(0.0225)$$

$$= 0.138$$

Rationale for the Binomial Probability Formula

$$P(x) = \underbrace{\frac{n!}{(n-x)!x!}}_{\substack{\text{The number of}\\ \text{outcomes with}\\ \text{exactly } x \text{ successes}\\ \text{among } n \text{ trials}}} \cdot p^x \cdot q^{n-x}$$

Rationale for the Binomial Probability Formula

$$P(x) = \underbrace{\frac{n!}{(n-x)!x!}}_{\text{Number of outcomes with exactly } x \text{ successes among } n \text{ trials}} \cdot \underbrace{p^x \cdot q^{n-x}}_{\text{The probability of } x \text{ successes among } n \text{ trials for any one particular order}}$$