Homework Chapter 1

21. **MATLAB.** At the beginning of Section 1.3, we developed a mathematical model for the toss of a single die. The probability of any one of the six faces landing up is $1/6 \approx 0.167$. If we toss a die 100 times, we expect that each face should land up between 16 and 17 times. Save the following MATLAB script in an M-file and run it to simulate the toss of a fair die. For now, do not worry about how the script works. You will learn more about histograms in Chapter 6.

```
% Simulation of Tossing a Fair Die
%
n = 100;                    % Number of tosses.
X = ceil(6*rand(1,n));
minX = min(X);              % Save to avoid re-
maxX = max(X);              % computing min & max.
e = [minX:maxX+1]-0.5;
H = histc(X,e);
nbins = length(e) - 1;
bin_centers = [minX:maxX];
bar(bin_centers,H(1:nbins),'w')
```

Did each face land up between 16 and 17 times? Modify your *M*-file to try again with $n = 1000$ and $n = 10000$.

23. A letter of the alphabet (a–z) is generated at random. Specify a sample space Ω and a probability measure P. Compute the probability that a vowel (a, e, i, o, u) is generated.

29. Let A and B be events for which $\mathsf{P}(A)$, $\mathsf{P}(B)$, and $\mathsf{P}(A \cup B)$ are known. Express the following in terms of these probabilities:

    (a) $\mathsf{P}(A \cap B)$.

    (b) $\mathsf{P}(A \cap B^c)$.

    (c) $\mathsf{P}(B \cup (A \cap B^c))$.

    (d) $\mathsf{P}(A^c \cap B^c)$.

30. Let Ω be a sample space equipped with two probability measures, P_1 and P_2. Given any $0 \le \lambda \le 1$, show that if $\mathsf{P}(A) := \lambda \mathsf{P}_1(A) + (1-\lambda)\mathsf{P}_2(A)$, then P satisfies the four axioms of a probability measure.

53. If $\mathsf{P}(C)$ and $\mathsf{P}(B \cap C)$ are positive, derive the **chain rule of conditional probability**,

$$\mathsf{P}(A \cap B|C) = \mathsf{P}(A|B \cap C)\,\mathsf{P}(B|C).$$

Also show that

$$\mathsf{P}(A \cap B \cap C) = \mathsf{P}(A|B \cap C)\,\mathsf{P}(B|C)\,\mathsf{P}(C).$$

54. The university buys workstations from two different suppliers, Mini Micros (MM) and Highest Technology (HT). On delivery, 10% of MM's workstations are defective, while 20% of HT's workstations are defective. The university buys 130 MM workstations and 70 HT workstations for its computer lab. Suppose you walk into the computer lab and randomly sit down at a workstation.

(a) What is the probability that your workstation is from MM? From HT?

(b) What is the probability that your workstation is defective?

(c) Given that your workstation is defective, what is the probability that it came from Mini Micros?

61. (a) If two sets A and B are disjoint, what equation must they satisfy?

(b) If two events A and B are independent, what equation must they satisfy?

(c) Suppose two events A and B are disjoint. Give conditions under which they are also independent. Give conditions under which they are not independent.

62. A certain binary communication system has a bit-error rate of 0.2; i.e., in transmitting a single bit, the probability of receiving the bit in error is 0.2. To transmit messages, a three-bit repetition code is used. In other words, to send the message **1**, 111 is transmitted, and to send the message **0**, 000 is transmitted. At the receiver, if two or more 1s are received, the decoder decides that message **1** was sent; otherwise, i.e., if two or more zeros are received, it decides that message **0** was sent. Assuming bit errors occur independently, find the probability that the decoder puts out the wrong message.

77. How many 8-bit words are there with four ones (and four zeros)?

83. ***m*-ary pick-*n* lottery.** In this game, a player chooses n m-ary digits. In the lottery drawing, n m-ary digits are chosen at random. If the n digits selected by the player match the random n digits of the lottery drawing in any order, the player wins. If the player has selected n digits with k_0 zeros, k_1 ones, $\dots$, and k_{m-1} copies of digit $m-1$, where $k_0+\cdots+k_{m-1}=n$, what is the probability of winning? In the case of $n=4$, $m=10$, and a player's choice of the form $xxyz$, what is the probability of winning; for $xxyy$; for $xxxy$?