Process Mining: Data Science in Action

Transition Systems and Petri Net Properties

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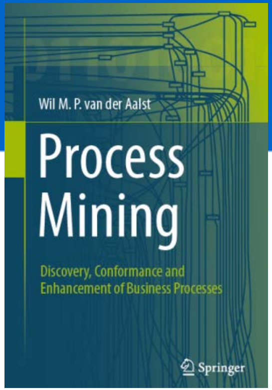

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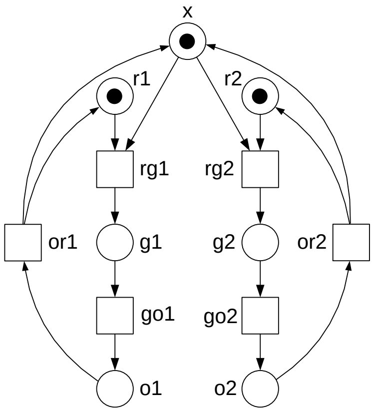
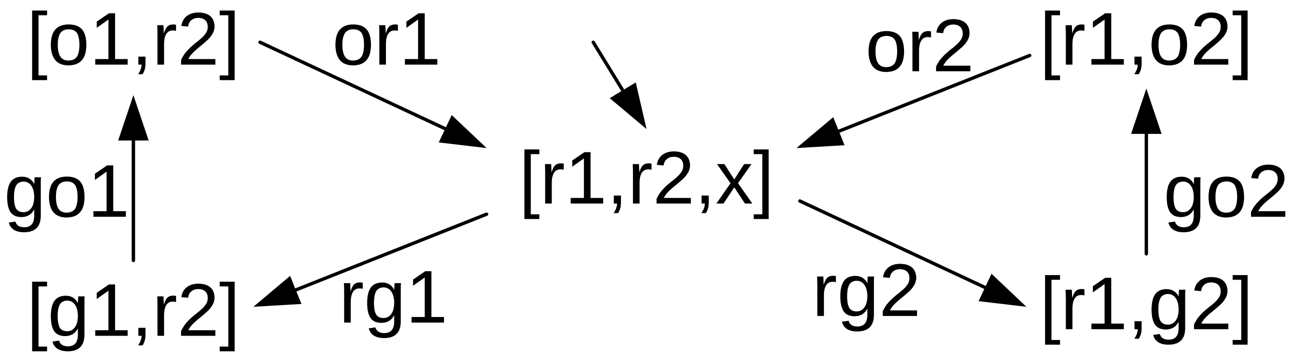
Where innovation starts

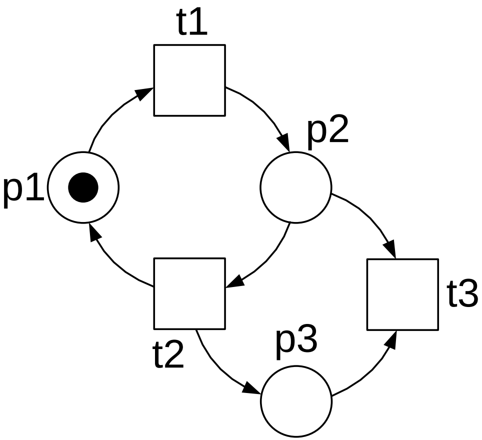
Reachability graph of a Petri net



The reachability graph is a transition system with one initial state (initial marking) and no explicit final marking.



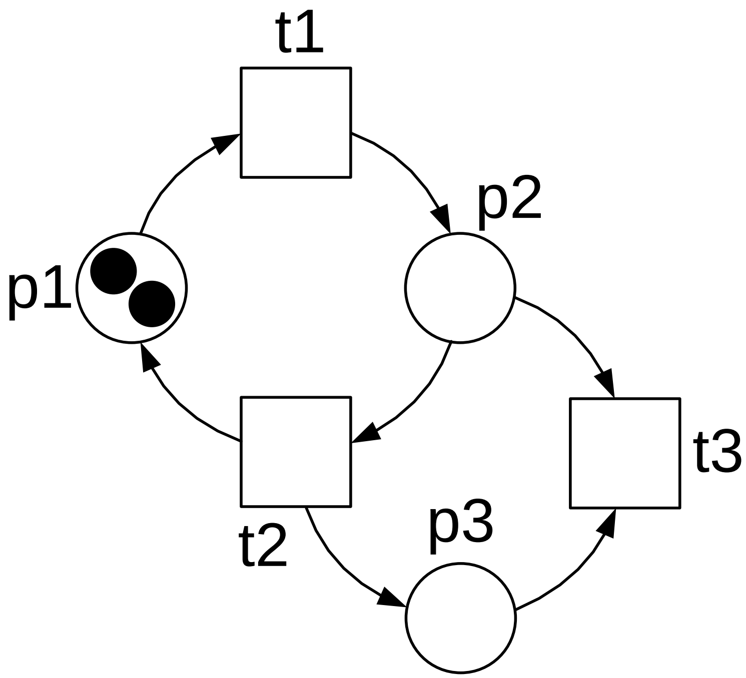
Reachability graph may be infinite



$$\begin{tikzcd}
{[p1]} \arrow[r, "t1"] & {[p2]} \arrow[r, "t2"] & {[p1,p3]} \arrow[r, "t1"] & {[p2,p3]} \arrow[r, "t2"] \arrow[d, "t3"'] & {[p1,p3^2]} \arrow[r, "t1"] & {[p2,p3^2]} \arrow[r, "t2"] \arrow[d, "t3"'] & \ldots \\
 & & & {[\ ]} & & {[p3]} &
\end{tikzcd}$$

Boundedness

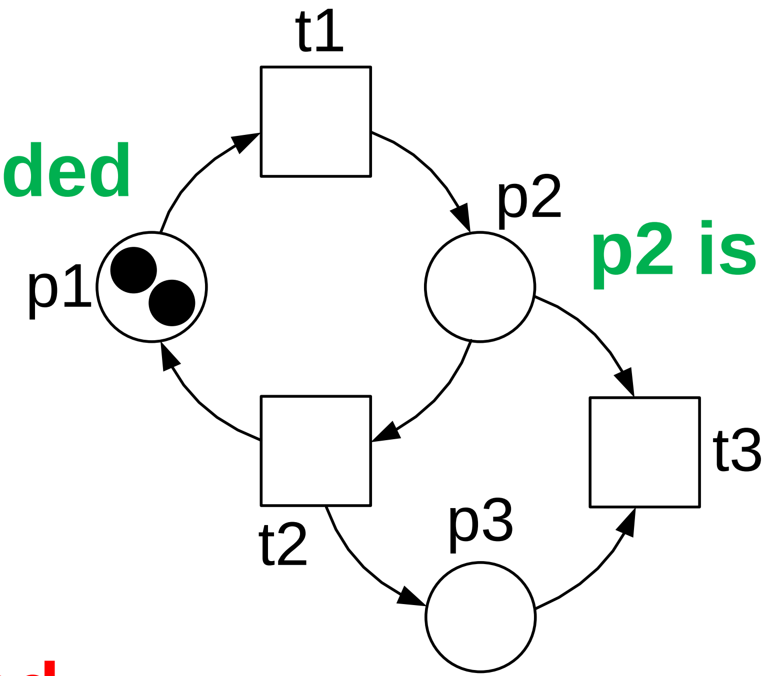
- A place *p* is *k*-bounded if there is no reachable marking with more than *k* tokens in *p*.
- A Petri net is *k*-bounded if all places are *k*-bounded.
- A place/Petri net is bounded if there exists such a *k*.



Boundedness

p1 is 2-bounded

p2 is 2-bounded

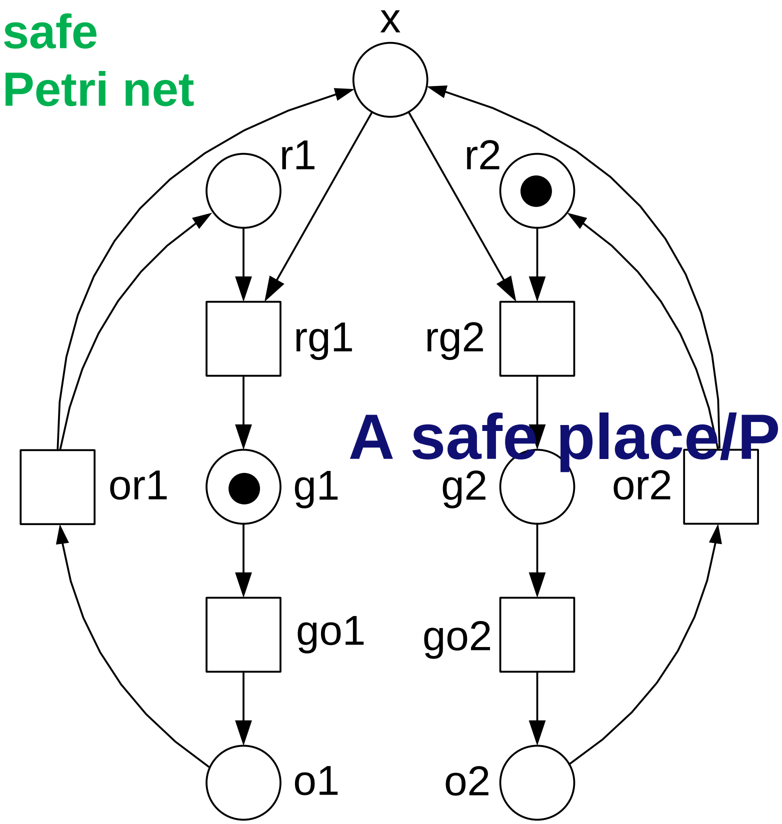


The whole Petri net is unbounded

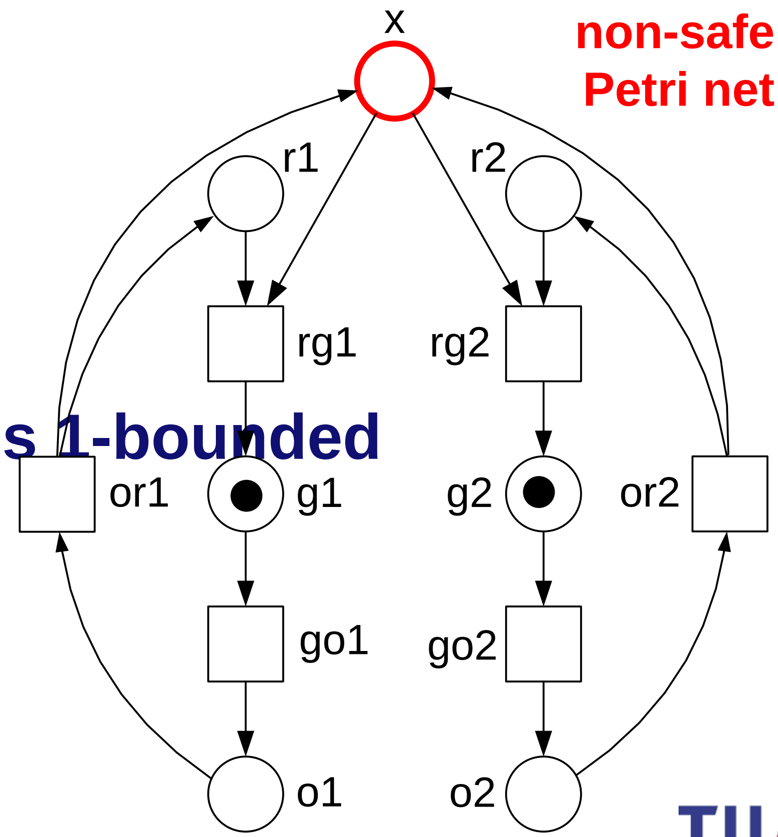
p3 is unbounded

Safeness (= 1-boundedness)

safe Petri net



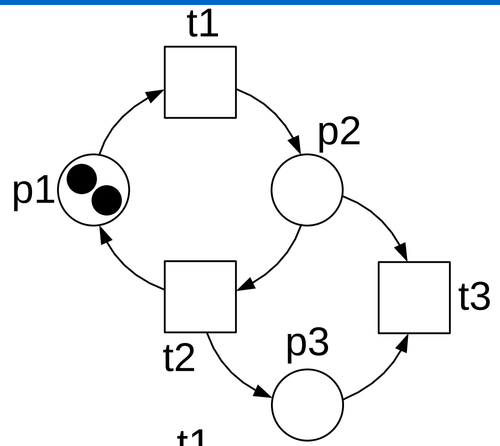
non-safe Petri net

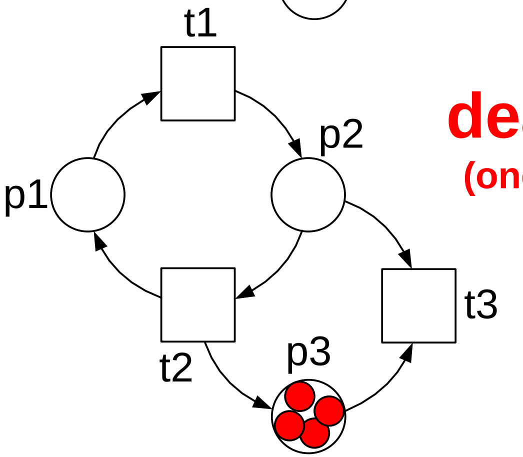


A safe place/Petri net is 1-bounded

Deadlock

- A marking is **dead** if no transition is enabled in it.
- A Petri net has a potential **deadlock** if there is a reachable dead marking.
- A Petri net is **deadlock-free** if each reachable marking enables at least one transition.



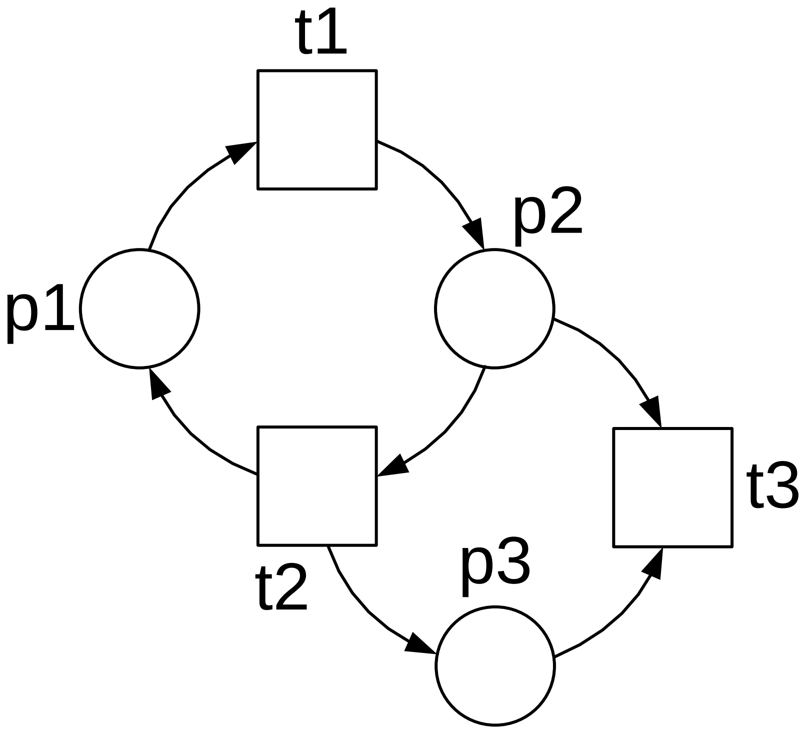


deadlock
(one of many)

Question

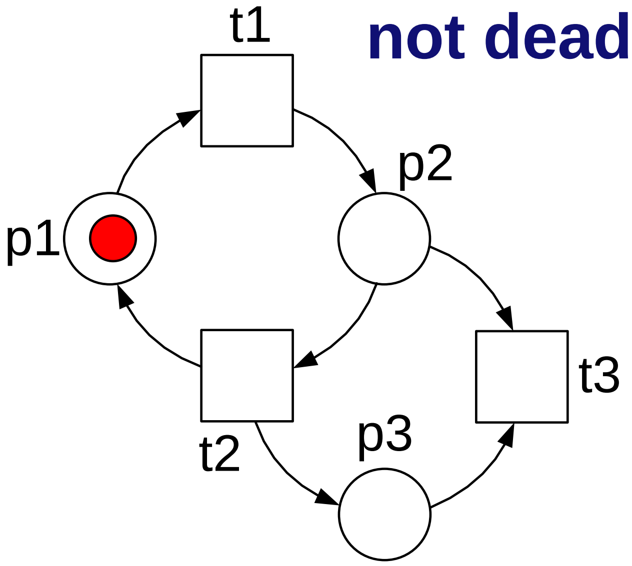
Provide (if possible) an initial making such that

- **the Petri net is unbounded and not deadlock-free,**
- **the Petri net is bounded and not deadlock-free,**
- **the Petri net is unbounded and deadlock-free, and**
- **the Petri net is bounded and deadlock-free.**

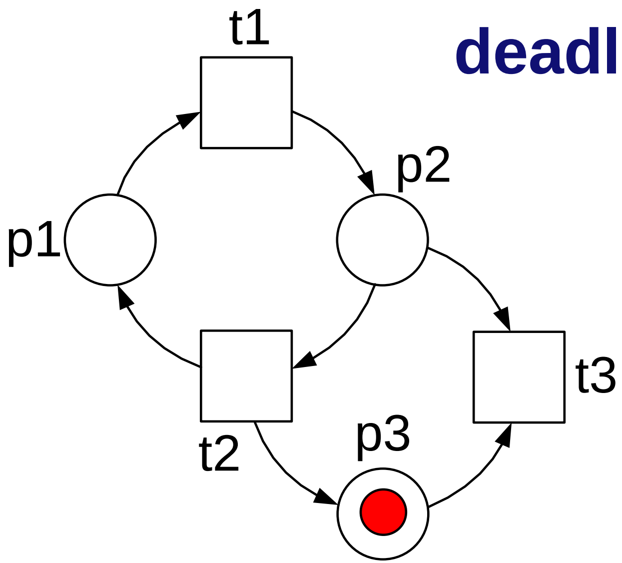


Answer

unbounded and not deadlock-free

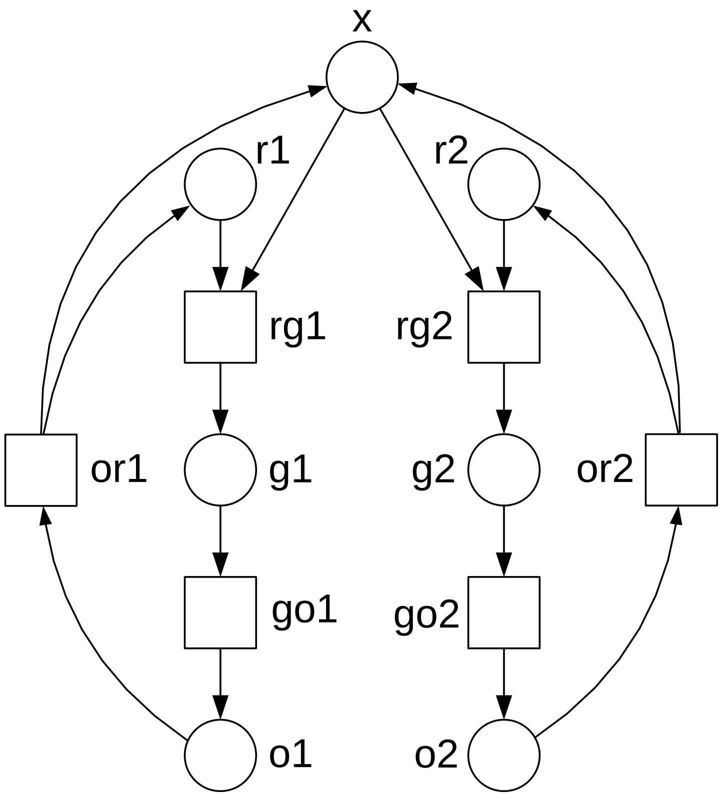


bounded but not deadlock-free



The net will always have a deadlock independent of the initial marking!

Question



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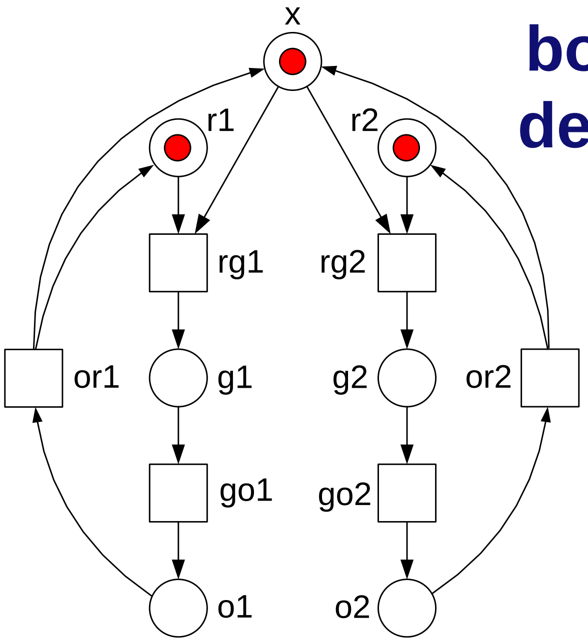
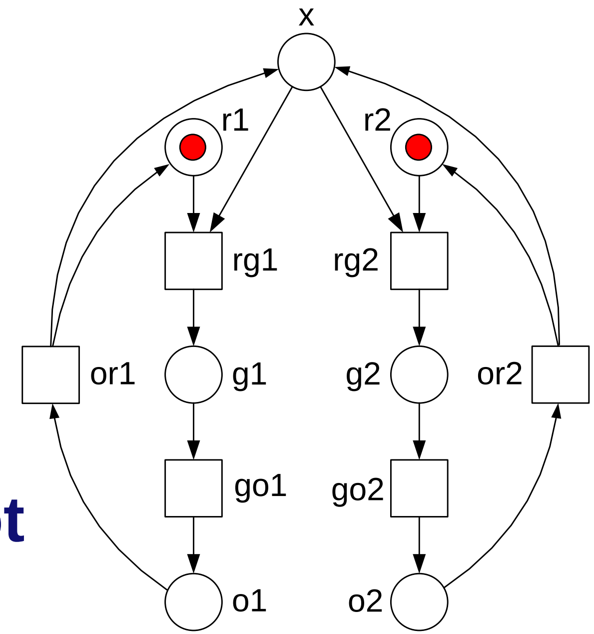
Answer



bounded and deadlock-free

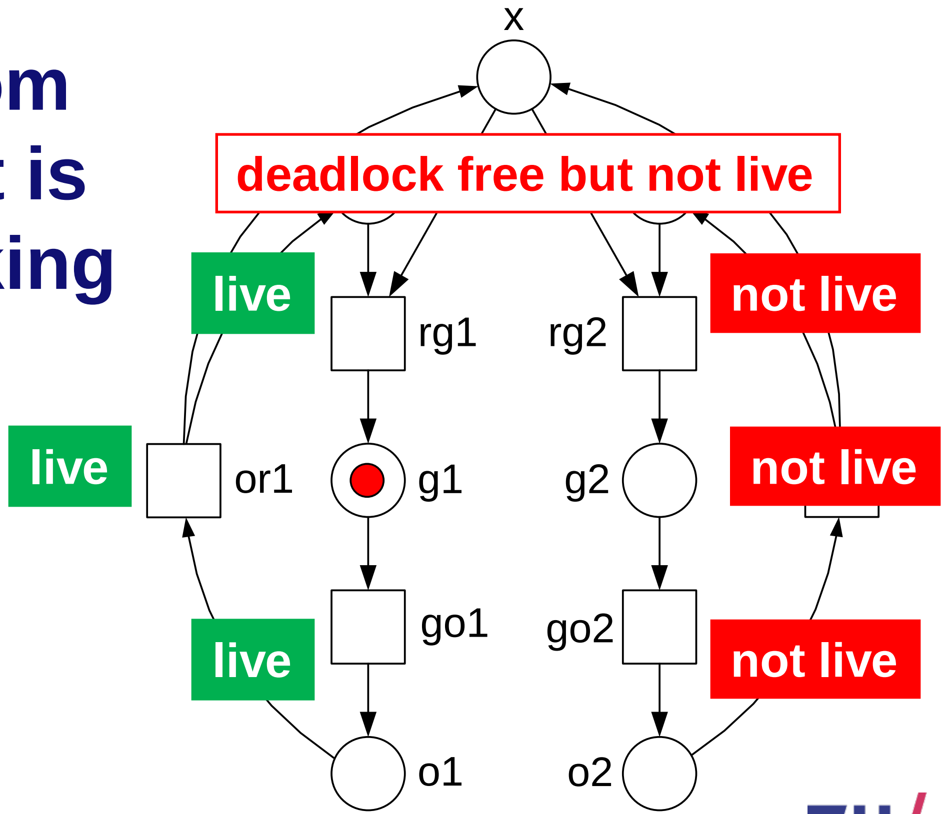


bounded, but not deadlock-free

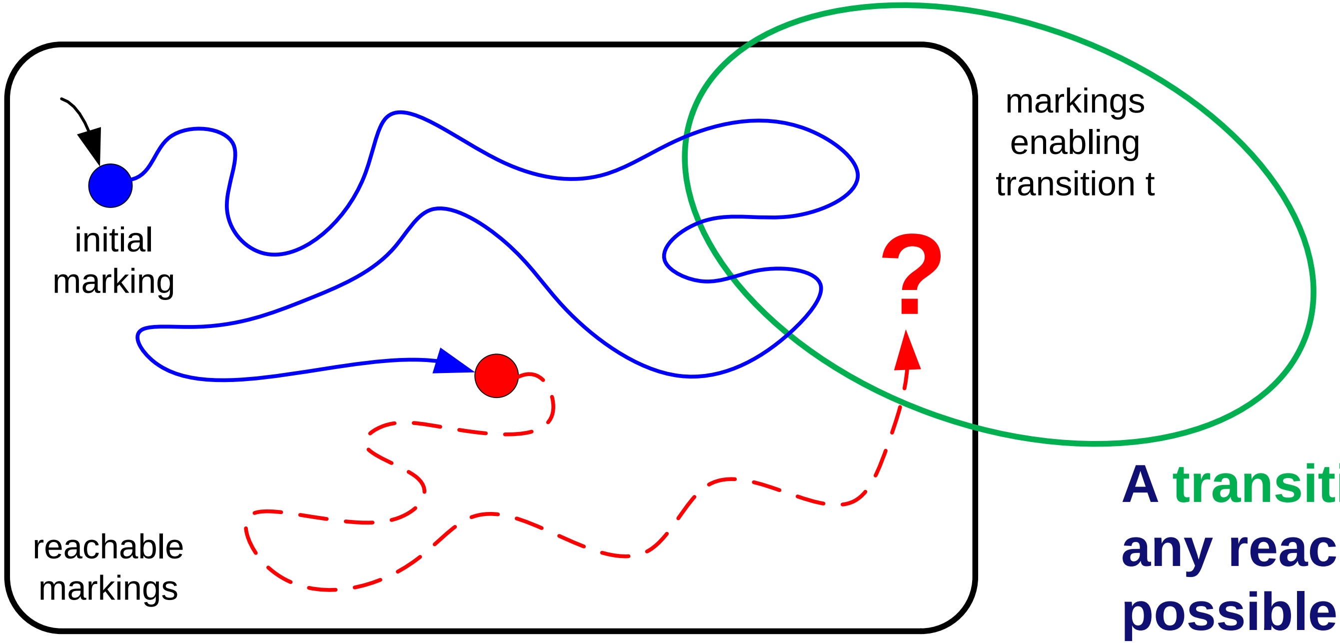
The net will always be bounded independent of the initial marking!

Liveness

- A transition *t* is live if from any reachable marking it is possible to reach a marking that enables *t*.
- A Petri net is live if all transitions are live.
- A Petri net that is live is deadlock-free.



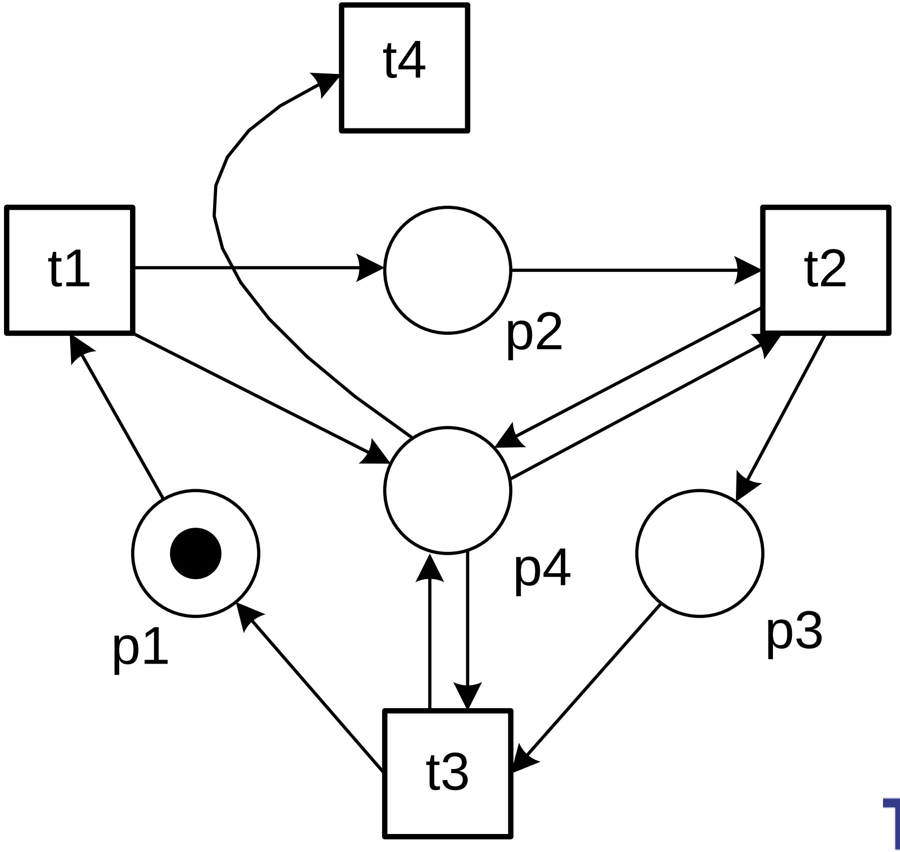
Understanding liveness



A transition *t* is live if from any reachable marking it is possible to reach a marking that enables *t*.
A Petri net is live if all transitions are live.

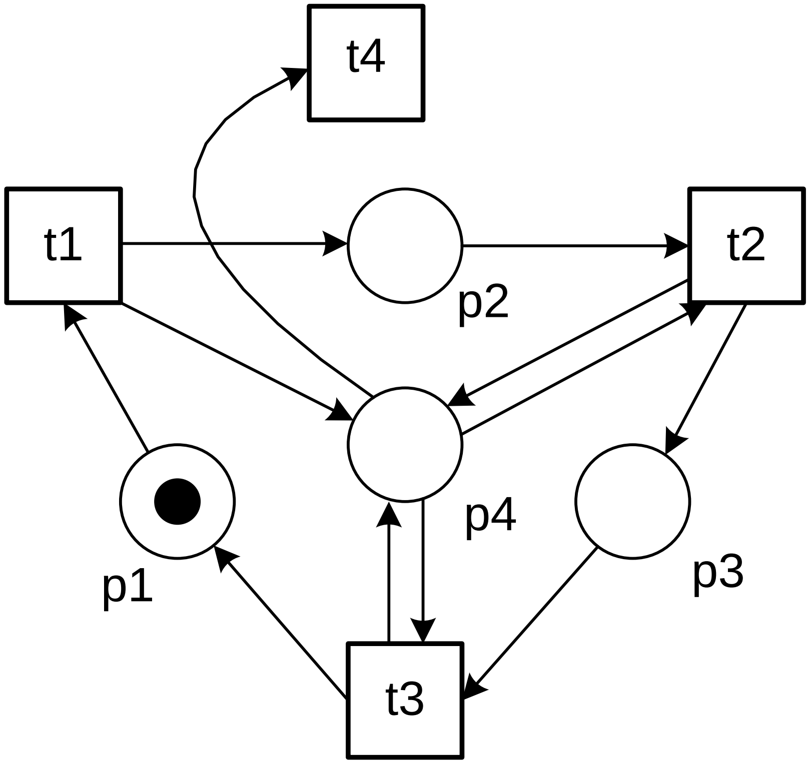
Question

- **Is the Petri net bounded?**
- **Is the Petri net safe?**
- **Is the Petri net deadlock free?**
- **Is the Petri net live?**



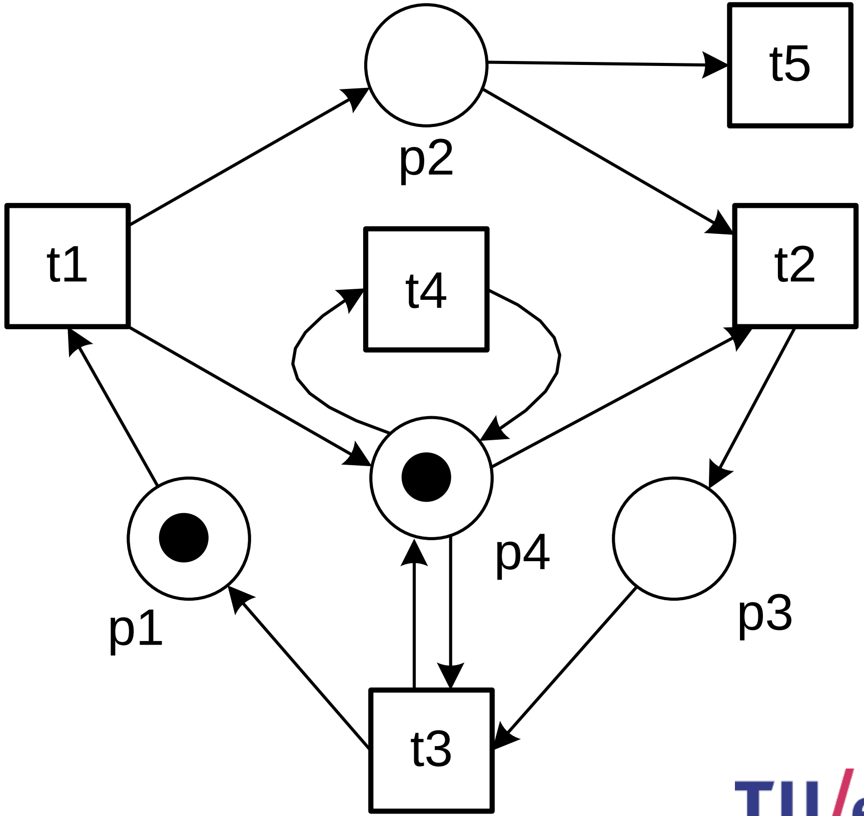
Answer

- **The Petri net is not bounded because any number of tokens can be put in *p4*.**
- **Hence, also not safe (= 1-bounded).**
- **The Petri net is not deadlock free, e.g., [p2] is reachable.**
- **Hence, also not live.**



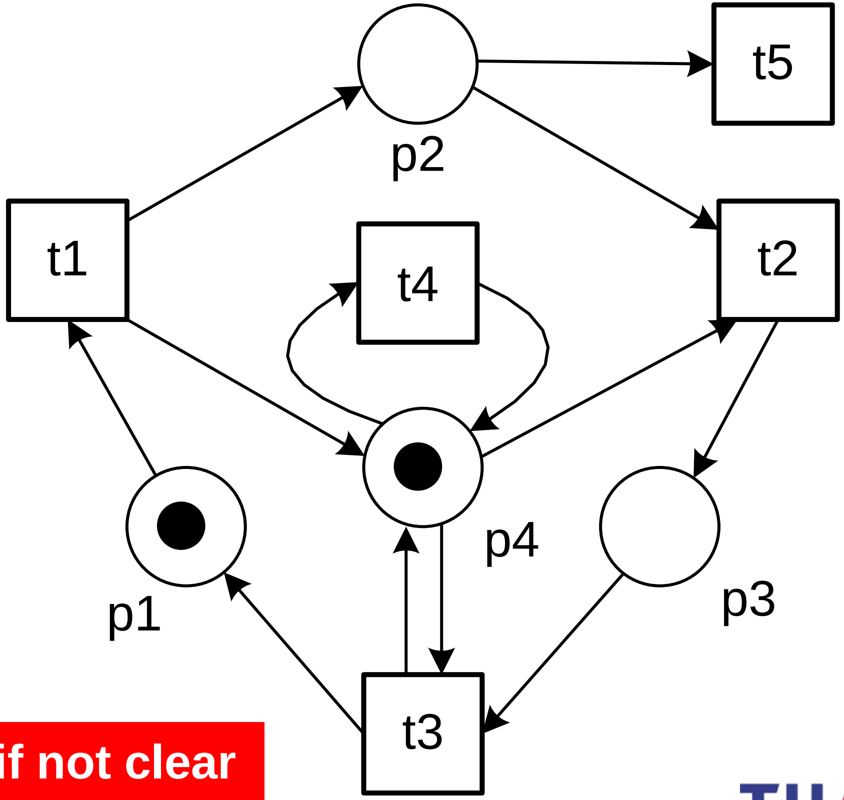
Question

- **Is the Petri net bounded?**
- **Is the Petri net safe?**
- **Is the Petri net deadlock free?**
- **Is the Petri net live?**



Answer

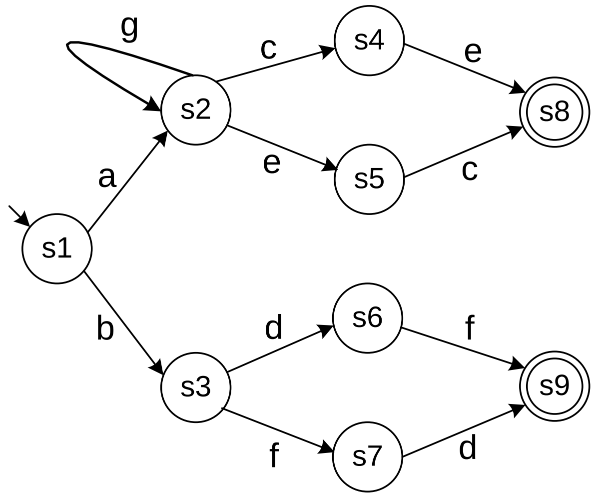
- The Petri net is **bounded**, but **not safe** (*p4* is 2-bounded).
- The Petri net is **deadlock-free**, but **not live** (after firing *t5* only *t4* can fire).



draw reachability graph if not clear

Transition systems

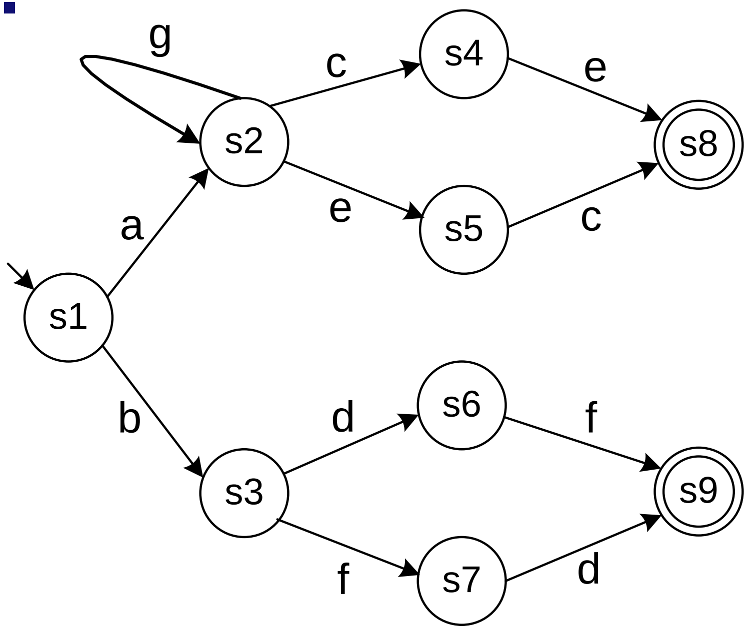
- A reachability graph is a special kind of transition system.
- Firing sequences correspond to paths in the transition system.
- A transition system is composed of states and transitions.
- There may be 1 or more initial states and 0 or more final states (more general than Petri nets).



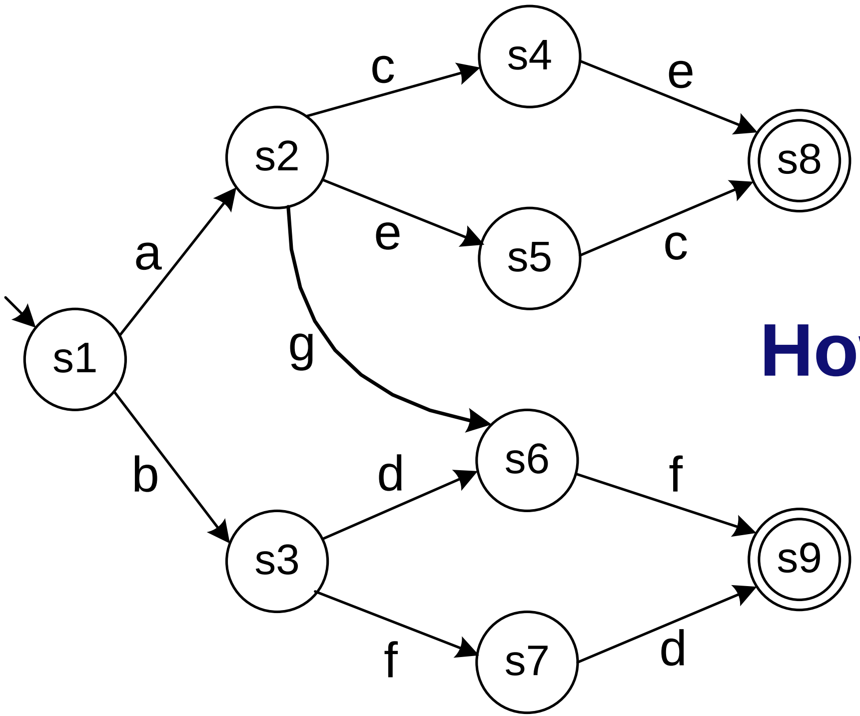
One initial state: *s1*
Two final states: *s8* and *s9*

Complete traces

- ⟨a,g,g,c,e⟩ is a **complete** trace.
- ⟨b,d,f⟩ is a **complete** trace.
- ⟨a,g,g,g,g⟩ is an **incomplete** trace.
- ⟨b,f⟩ is an **incomplete** trace.
- The transition system has **infinitely** many (in)complete traces.



Question

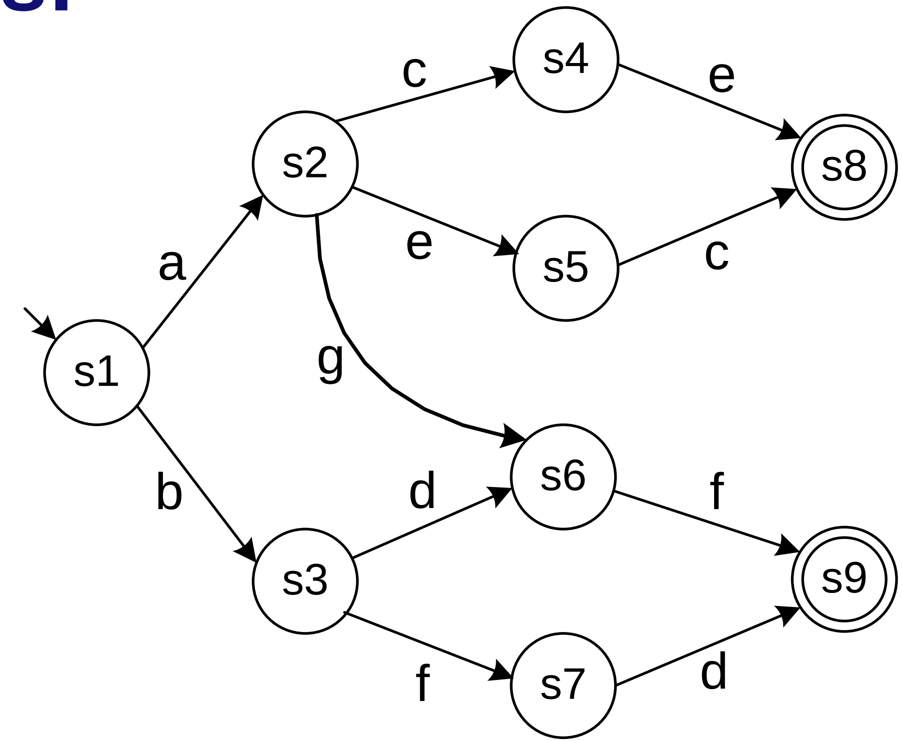


How many complete traces?

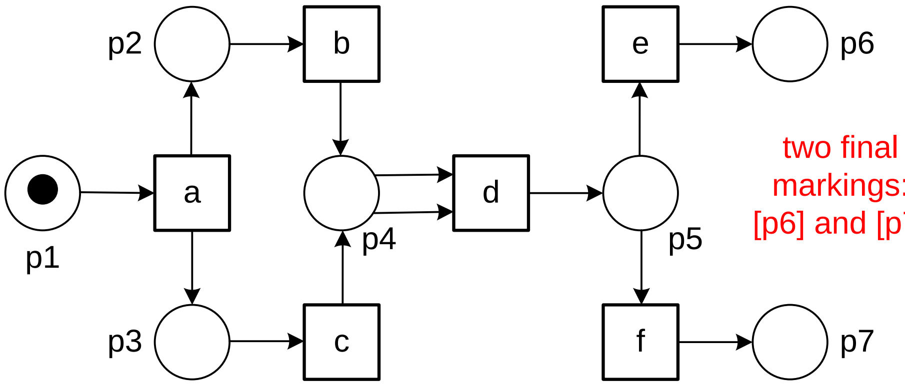
Answer

Five complete traces:

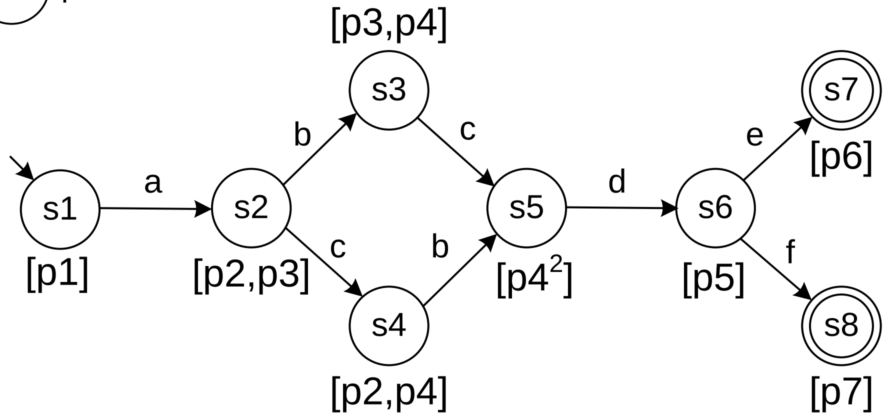
- ⟨a,c,e⟩
- ⟨a,e,c⟩
- ⟨a,g,f⟩
- ⟨b,d,f⟩
- ⟨b,f,d⟩



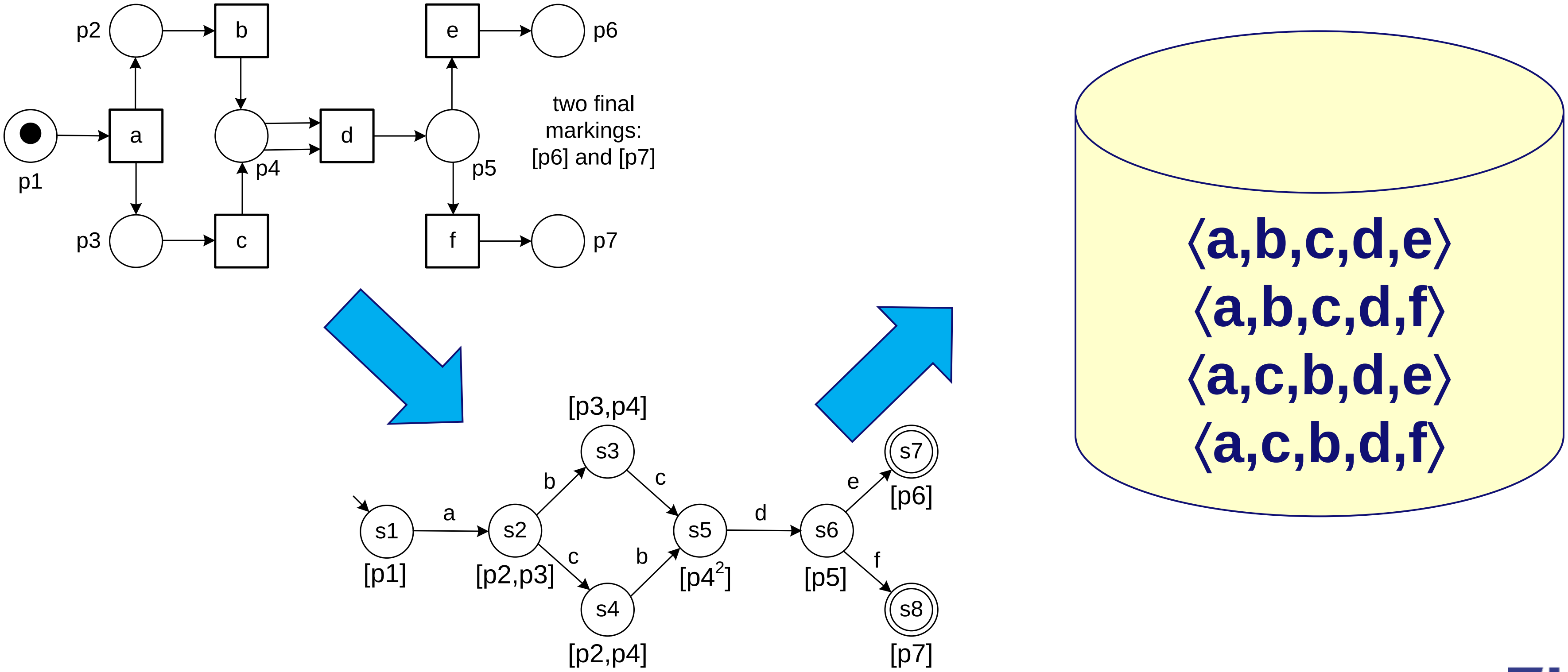
A Petri net may also have a designated set of final markings



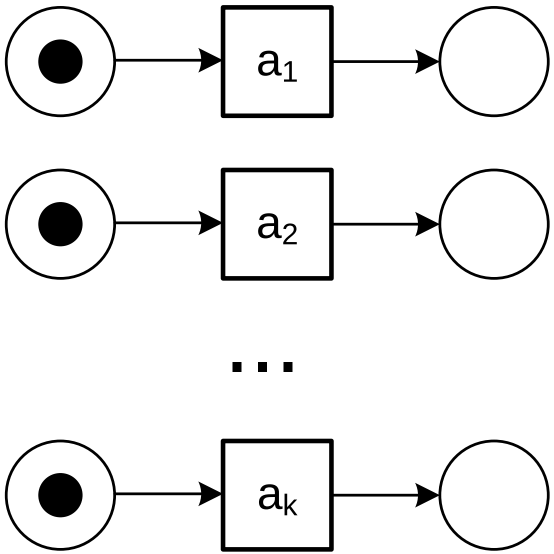
two final markings: [p6] and [p7]



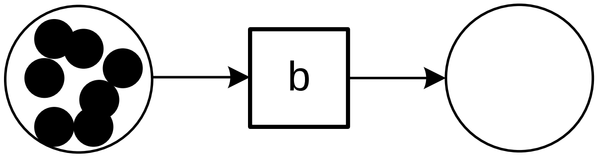
Play-out


p1
p2
p3
p4
p5
p6
p7
a
b
c
d
e
f
two final markings: [p6] and [p7]
s1
s2
s3
s4
s5
s6
s7
s8
[p1]
[p2,p3]
[p3,p4]
[p2,p4]
[p4^2]
[p5]
[p6]
[p7]
⟨a,b,c,d,e⟩
⟨a,b,c,d,f⟩
⟨a,c,b,d,e⟩
⟨a,c,b,d,f⟩

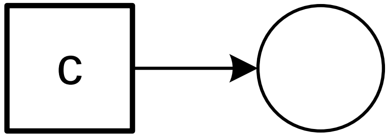
Transition system may be exponential in size of Petri net (or even infinite)



2^k states



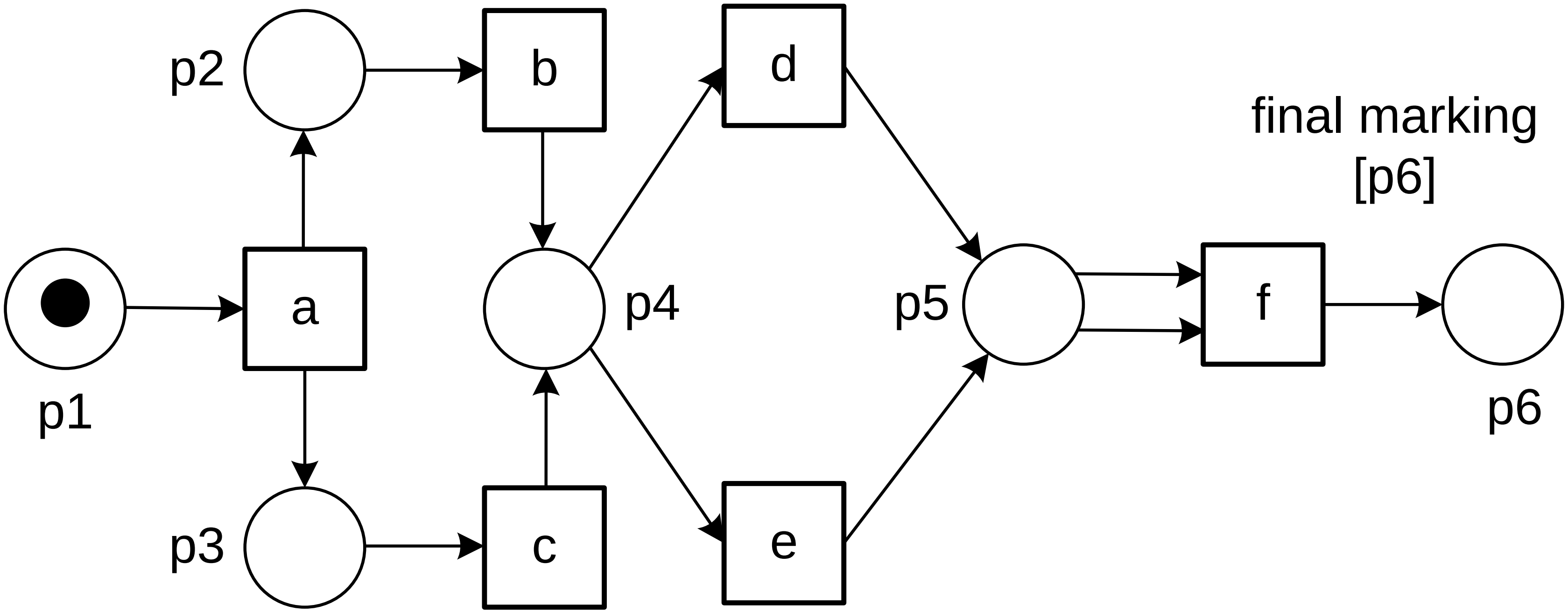
(k+1) states



infinitely many states

Play-out model:

Give the transition system and all complete traces that are possible



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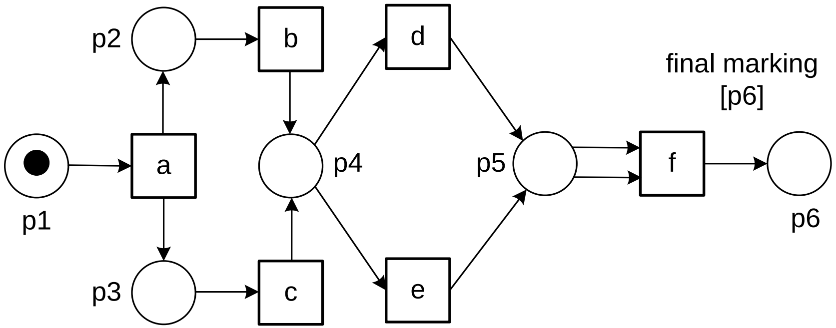
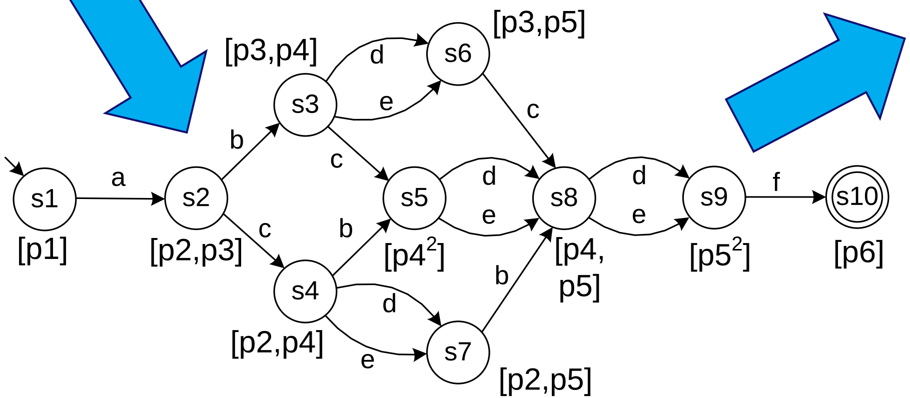
Resulting transition system and set of complete traces





16 possible traces

⟨a,b,c,d,d,f⟩
⟨a,b,c,d,e,f⟩
⟨a,b,c,e,d,f⟩
⟨a,b,c,e,e,f⟩
⟨a,c,b,d,d,f⟩
⟨a,c,b,d,e,f⟩
⟨a,c,b,e,d,f⟩
⟨a,c,b,e,e,f⟩
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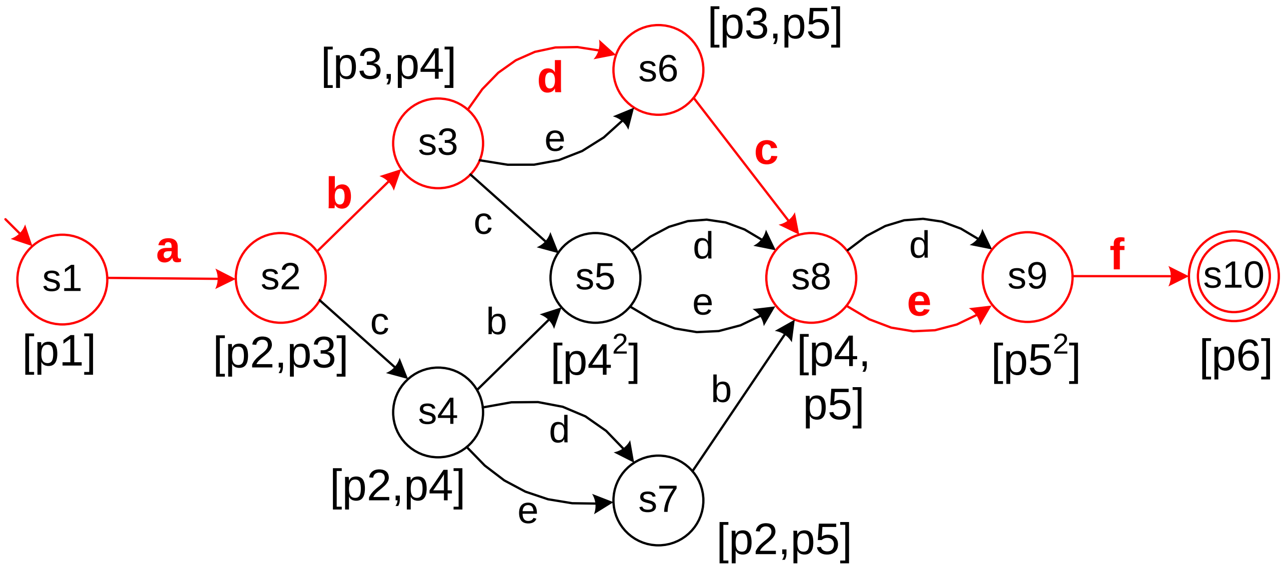
⟨a,c,e,b,e,f⟩

Example path



⟨a,b,c,d,d,f⟩
⟨a,b,c,d,e,f⟩
⟨a,b,c,e,d,f⟩
⟨a,b,c,e,e,f⟩
⟨a,c,b,d,d,f⟩
⟨a,c,b,d,e,f⟩
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Part I: Preliminaries

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Process Modeling and Analysis

Chapter 3
Data Mining

Part II: From Event Logs to Pr Models

Chapter 4
Getting the Data

Chapter 5
Process Discovery: An Introduction

Chapter 6
Advanced Process Discovery Techniques

Part III: Beyond Process Discovery

Chapter 7
Conformance Checking

Chapter 8
Mining Additional Perspectives

Chapter 9
Operational Support

Part IV: Putting Process Mining to Work

Chapter 10
Tool Support

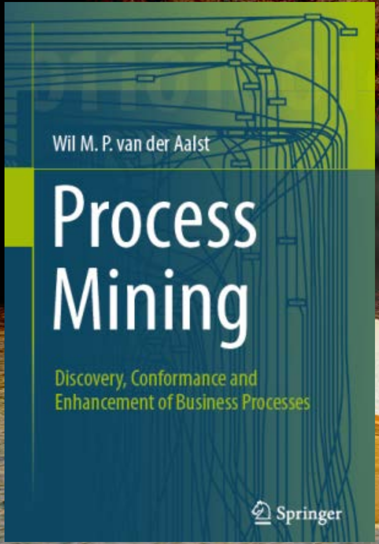
Chapter 11
Analyzing “Lasagna Processes”

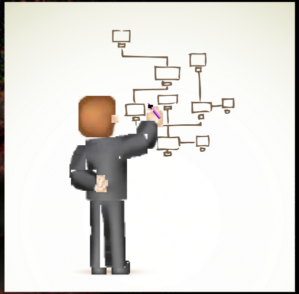
Chapter 12
Analyzing “Spaghetti Processes”

Part V: Reflection

Chapter 13
Cartography and Navigation

Chapter 14
Epilogue





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