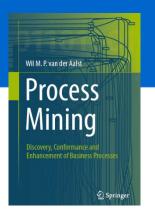
Process Mining: Data Science in Action

Alpha Algorithm: A Process Discovery Algorithm



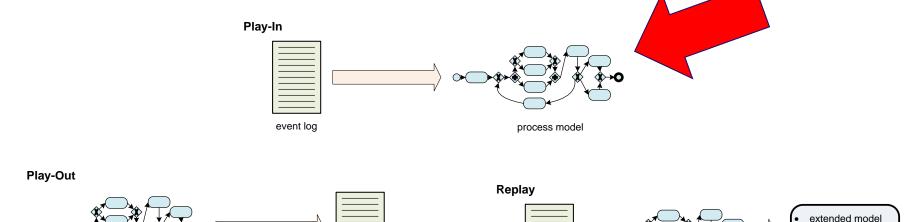
prof.dr.ir. Wil van der Aalst www.processmining.org



Where innovation starts

Process discovery = Play-In

event log



event log

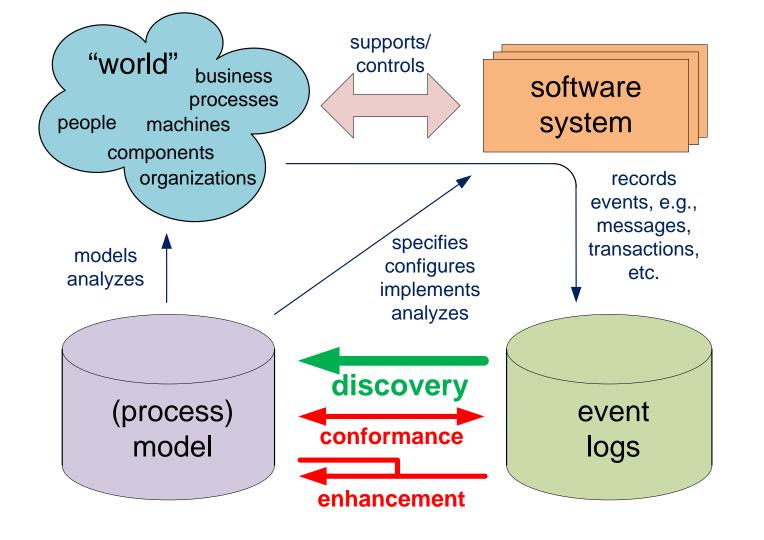
process model



showing times, frequencies, etc. diagnostics predictions

recommendations

process model





Simplifying event logs when focusing on control-flow

order number	activity	timestamp	user	product	quantity
9901	register order	22-1-2014@09.15	Sara Jones	iPhone5S	
9902	register order	22-1-2014@09.18	San Jones	iPhone5S	2
9903	register order	22-1-2014@09.27	Sara Jones	iPhone4S	1
9901	check stock	22-1-2014@09.49	Pete Scott	iPhon oS	1
9901	ship order	22-1-2014@10.11	Sue Fox	iP'ione5S	1
9903	check stock	22-1-2014@10.34	Pete Scott	iPhone4S	1
9901	handle payment	22-1-2014@10.41	Carol Hop	iPhonesS	1
9902	check stock	22-1-2014@10.57	Pete Scott	iPhone5S	2

[\(\text{register_order, check_stock, ship_order, handle_payment \rangle, \) \(\text{register_order, check_stock, cancel_order \rangle, \) \(\text{register_order, check_stock} \) \(\text{...} \)

Simple event log

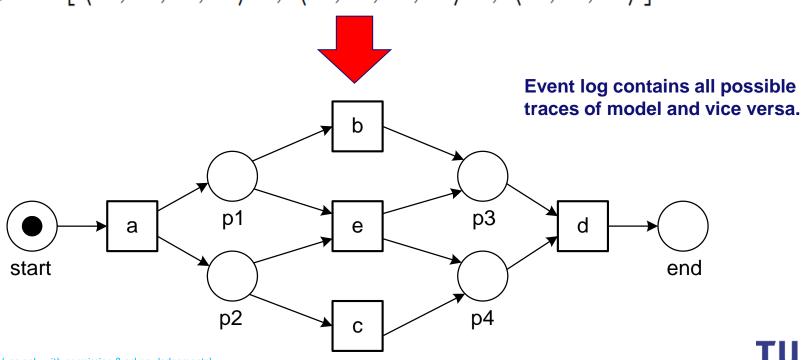
$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$

- An event log is a multiset of traces (same trace may appear multiple times).
- A trace is a sequence of activity names (we abstract from all other attributes, but events are ordered).



Goal of Alpha algorithm

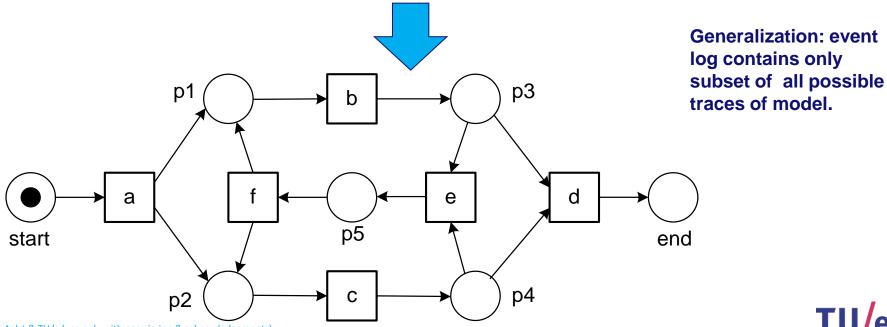
$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$



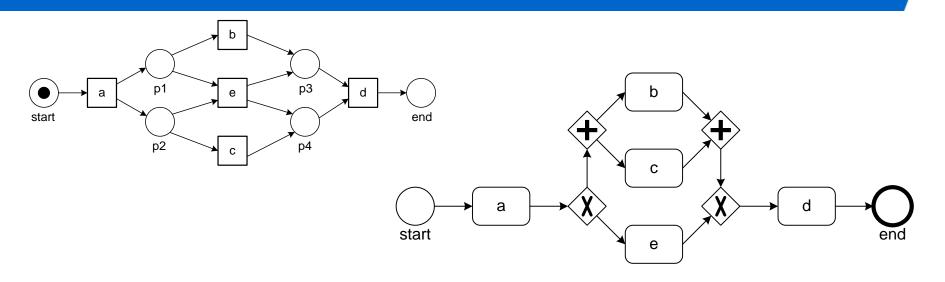


Another example

$$L_2 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^4, \langle a, b, c, e, f, b, c, d \rangle^2, \langle a, b, c, e, f, c, b, d \rangle, \langle a, c, b, e, f, b, c, d \rangle^2, \langle a, c, b, e, f, b, c, e, f, c, b, d \rangle]$$



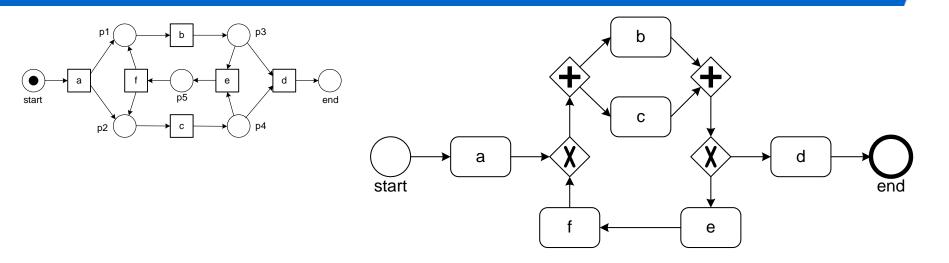
Notation is less relevant (e.g. BPMN)



$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$



Another BPMN example



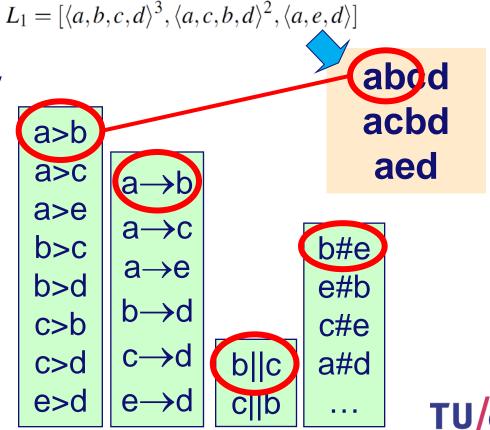
$$L_2 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^4, \langle a, b, c, e, f, b, c, d \rangle^2, \langle a, b, c, e, f, c, b, d \rangle, \langle a, c, b, e, f, b, c, d \rangle^2, \langle a, c, b, e, f, b, c, e, f, c, b, d \rangle]$$



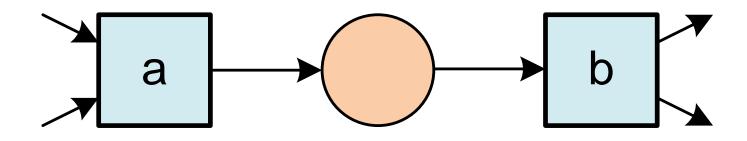


>,→,||,# relations

- Direct succession: x>y iff for some case x is directly followed by y.
- Causality: x→y iff x>y and not y>x.
- Parallel: x||y iff x>y and y>x
- Choice: x#y iff not x>y and not y>x.



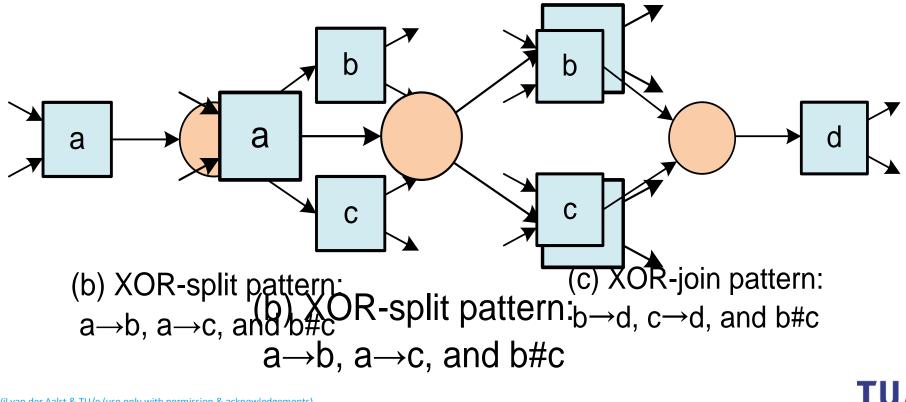
Basic Idea Used by Alpha Algorithm (1)



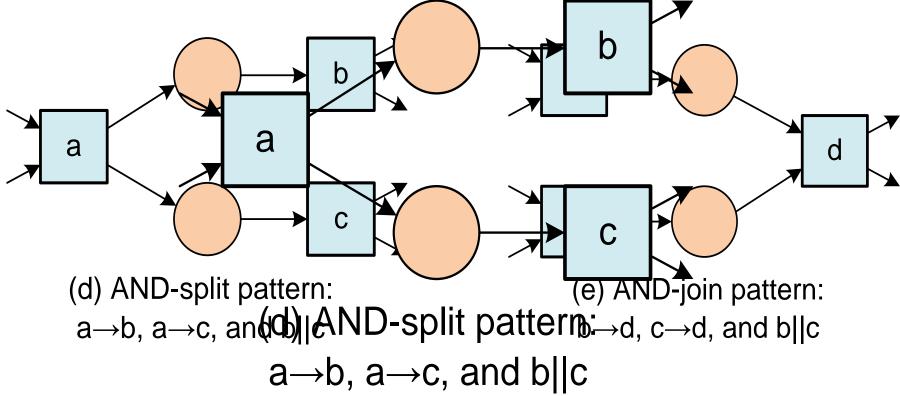
(a) sequence pattern: a→b



Basic Idea Used by Alpha Algorithm (2)



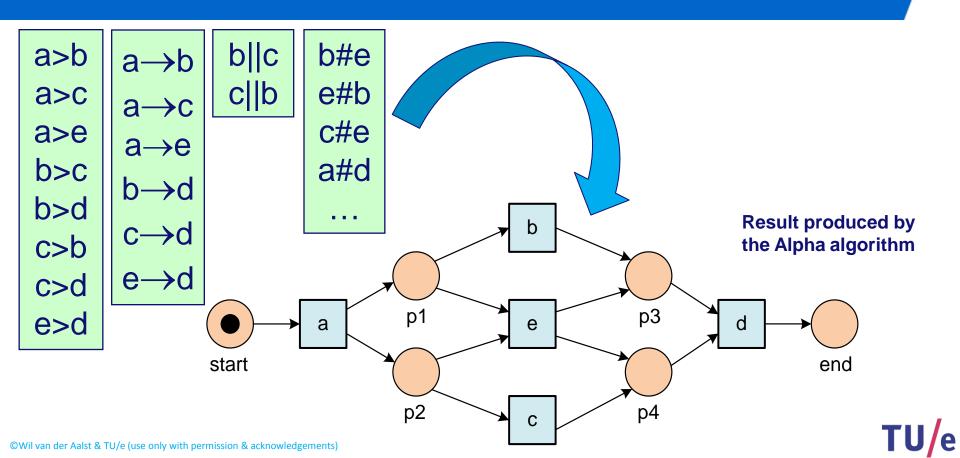
Basic Idea Used by Alpha Algorithm (3)





Example Revisited

 $L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$



Footprint of L₁

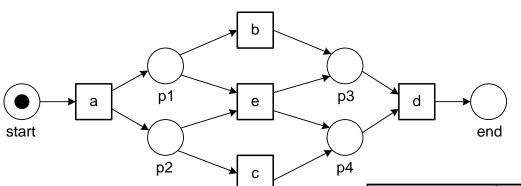
$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$

One of the following: \rightarrow , \leftarrow #, ||

	a	b	<i>c</i> /	d	e
a	$\#_{L_1}$	\rightarrow_{L_1}	\rightarrow_{L_1}	$\#_{L_1}$	\rightarrow_{L_1}
b	\leftarrow_{L_1}	$\#_{L_1}$	$\ _{L_1}$	\rightarrow_{L_1}	$\#_{L_1}$
c	\leftarrow_{L_1}	\parallel_{L_1}	$\#_{L_1}$	\rightarrow_{L_1}	$\#_{L_1}$
d	$\#_{L_1}$	\leftarrow_{L_1}	\leftarrow_{L_1}	$\#_{L_1}$	\leftarrow_{L_1}
e	\leftarrow_{L_1}	$\#_{L_1}$	$\#_{L_1}$	\rightarrow_{L_1}	$\#_{L_1}$



Discovered model has the same footprint



$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$

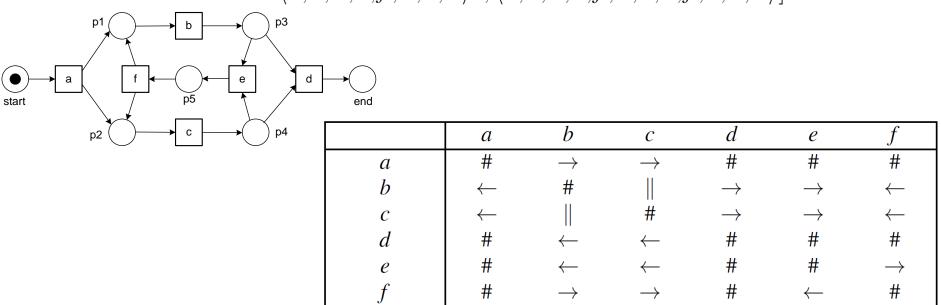
	а	b	\boldsymbol{c}	d	e
a	$\#_{L_1}$	\rightarrow_{L_1}	\rightarrow_{L_1}	$\#_{L_1}$	$ ightarrow_{L_1}$
b	\leftarrow_{L_1}	$\#_{L_1}$	\parallel_{L_1}	\rightarrow_{L_1}	$\#_{L_1}$
\boldsymbol{c}	\leftarrow_{L_1}	\parallel_{L_1}	$\#_{L_1}$	\rightarrow_{L_1}	$\#_{L_1}$
d	$\#_{L_1}$	\leftarrow_{L_1}	\leftarrow_{L_1}	$\#_{L_1}$	\leftarrow_{L_1}
e	\leftarrow_{L_1}	$\#_{L_1}$	$\#_{L_1}$	\rightarrow_{L_1}	$\#_{L_1}$

Log and model agree on footprint



Footprint of L₂

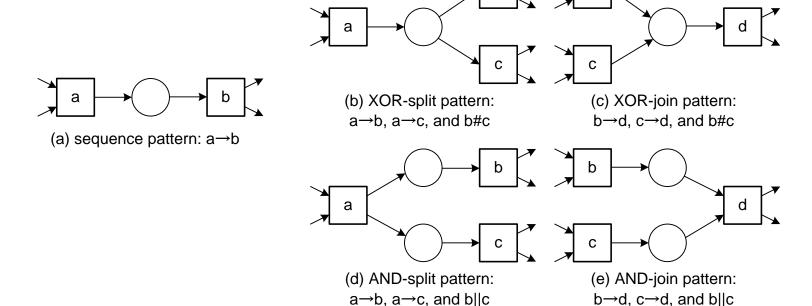
$$L_{2} = [\langle a,b,c,d \rangle^{3}, \langle a,c,b,d \rangle^{4}, \langle a,b,c,e,f,b,c,d \rangle^{2}, \langle a,b,c,e,f,c,b,d \rangle, \langle a,c,b,e,f,b,c,d \rangle^{2}, \langle a,c,b,e,f,b,c,e,f,c,b,d \rangle]$$



Log and model agree on footprint



Summary: Simple process patterns can be discovered from event logs







Let L be an event log over T. $\alpha(L)$ is defined as follows.

$$1.T_{L} = \{ t \in T \mid \exists_{\sigma \in L} t \in \sigma \},\$$

$$2.T_{l} = \{ t \in T \mid \exists_{\sigma \in L} t = \mathit{first}(\sigma) \},\$$

$$3.T_O = \{ t \in T \mid \exists_{\sigma \in L} t = last(\sigma) \},$$

$$4. \, X_L = \{ \, (A,B) \mid A \subseteq T_L \, \land A \neq \emptyset \land B \subseteq T_L \land B \neq \emptyset \land \\ \forall_{a \in A} \forall_{b \in B} \, a \rightarrow_L b \, \land \, \forall_{a1,a2 \in A} \, a_1 \#_L \, a_2 \, \land \, \forall_{b1,b2 \in B} \, b_1 \#_L \, b_2 \, \},$$

5.
$$Y_L = \{ (A,B) \in X_L \mid \forall_{(A',B') \in X_L} A \subseteq A' \land B \subseteq B' \Rightarrow (A,B) = (A',B') \},$$

6.
$$P_L = \{ p_{(A,B)} \mid (A,B) \in Y_L \} \cup \{ i_L, o_L \},$$

$$7.\,F_L = \{\,(a,p_{(A,B)}) \mid \, (A,B) \in Y_L \, \land a \in A \,\} \, \cup \{\,(p_{(A,B)},b) \mid \, (A,B) \in Y_L \, \land b \in B \,\} \, \cup \{\,(i_L,t) \mid \, t \in T_I \} \, \cup \{\,(t,o_L) \mid t \in T_O \}, \, \text{and}$$

8.
$$\alpha(L) = (P_L, T_L, F_L)$$
.

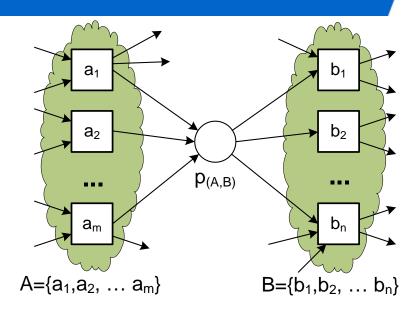
The α-algorithm

Let L be an event log over T. Then, $\alpha(L)$ is defined as follows:

- 1. $T_L = \{ t \in T \mid \exists_{\sigma \in L} t \in \sigma \},$ Each activity in L corresponds to a transition in $\alpha(L)$.
- 2. $T_1 = \{ t \in T \mid \exists_{\sigma \in L} t = \textit{first}(\sigma) \}$ Fix the set of start activities – that is, the first elements of each trace: $\langle t_1, ..., t_n \rangle, ..., \langle t'_1, ..., t'_m \rangle$
- 3. $T_O = \{ t \in T \mid \exists_{\sigma \in L} t = last(\sigma) \}$ Fix the set of end activities – that is, elements that appear last in a trace : $\langle t_1, ..., t_n \rangle, ..., \langle t'_1, ..., t'_m \rangle$



Next steps aim at finding places



Step 4: Calculate pairs (A, B)

Step 5: Delete non-maximal pairs (A, B)

Step 6: Determine places p_(A, B) from pairs (A, B)

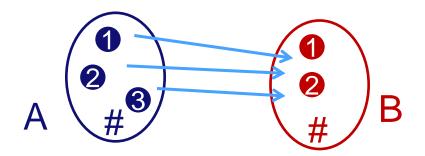


4.
$$X_{L} = \{ (A,B) \mid A \subseteq T_{L} \land A \neq \emptyset \land B \subseteq T_{L} \land B \neq \emptyset \}$$

$$\land \forall_{a \in A} \forall_{b \in B} a \rightarrow_{L} b$$

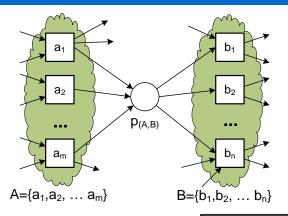
$$\land \forall_{a1,a2 \in A} a_{1} \#_{L} a_{2}$$

$$\land \forall_{b1,b2 \in B} b_{1} \#_{L} b_{2} \},$$



Find pairs (A, B) of sets of activities such that every element $a \in A$ and every element $b \in B$ are causally related (i.e., $a \rightarrow_L b$), all elements in A are independent $(a_1\#_L a_2)$, and all elements in B are independent $(b_1\#_L b_2)$.

Places as footprints

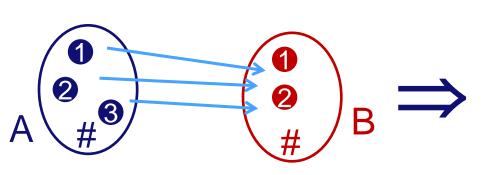


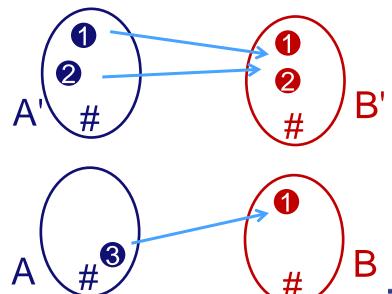
	a_1	a_2	 a_m	b_1	b_2	 b_n
a_1	#	#	 #	\rightarrow	\rightarrow	 \rightarrow
a_2	#	#	 #	\rightarrow	\rightarrow	 \rightarrow
a_m	#	#	 #	\rightarrow	\rightarrow	 \rightarrow
b_1	\leftarrow	\leftarrow	 \leftarrow	#	#	 #
b_2	\leftarrow	\leftarrow	 \leftarrow	#	#	 #
b_n	\leftarrow	\leftarrow	 \leftarrow	#	#	 #

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5.
$$Y_L = \{ (A,B) \in X_L \mid \forall_{(A',B') \in X_L} A \subseteq A' \land B \subseteq B' \Rightarrow (A,B) = (A',B') \}$$

Delete from set X_L all pairs (A, B) that are not maximal!







6.
$$P_L = \{ p_{(A,B)} \mid (A,B) \in Y_L \} \cup \{i_L,o_L\},\$$

 a_2 b_2 $p_{(A,B)}$ b_n a_{m} $B=\{b_1,b_2, ... b_n\}$ $A = \{a_1, a_2, \dots a_m\}$

Determine the place set: Each element (A, B) of Y_L is a place. To ensure the workflow structure, add a source place i_L and a target place o_L





7.
$$F_L = \{ (a,p_{(A,B)}) \mid (A,B) \in Y_L \land a \in A \}$$

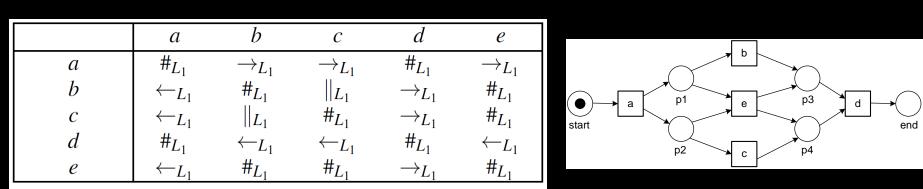
 $\cup \{ (p_{(A,B)},b) \mid (A,B) \in Y_L \land b \in B \}$
 $\cup \{ (i_L,t) \mid t \in T_I \} \cup \{ (t,o_L) \mid t \in T_O \}$

Determine the flow relation: Connect each place $p_{(A,B)}$ with each element a of its set A of source transitions and with each element of its set B of target transitions. In addition, draw an arc from the source place i_L to each start transition $t \in T_I$ and an arc from each end transition $t \in T_O$ to the sink place o_L .

8.
$$\alpha(L) = (P_L, T_L, F_L)$$



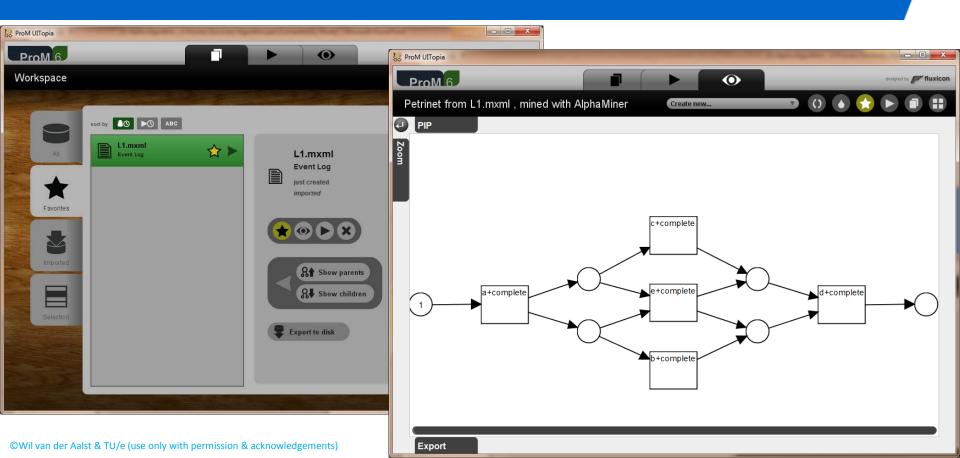
$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$



$$X_{L_1} = \{(\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, e\}), (\{a\}, \{c, e\}), (\{b\}, \{d\}), (\{c\}, \{d\}), (\{c\}, \{d\}), (\{b, e\}, \{d\}), (\{c, e\}, \{d\})\}\}$$

$$Y_{L_1} = \{(\{a\}, \{b, e\}), (\{a\}, \{c, e\}), (\{b, e\}, \{d\}), (\{c, e\}, \{d\})\}$$

ProM's output for event log L₁



Question: Give footprint matrix for event log L₃

$$L_{3} = [\langle a, b, c, d, e, f, b, d, c, e, g \rangle,$$

$$\langle a, b, d, c, e, g \rangle^{2},$$

$$\langle a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g \rangle]$$



Answer: Footprint matrix for event log L₃

	a	b	$\boldsymbol{\mathcal{C}}$	d	e	f	g
а	#	\rightarrow	#	#	#	#	#
b	\leftarrow	#	\rightarrow	\rightarrow	#	\leftarrow	#
c	#	\leftarrow	#		\rightarrow	#	#
d	#	\leftarrow		#	\rightarrow	#	#
e	#	#	\leftarrow	\leftarrow	#	\rightarrow	\rightarrow
f	#	\rightarrow	#	#	\leftarrow	#	#
g	#	#	#	#	\leftarrow	#	#

$$\langle a,b,d,c,e,g \rangle^2,$$

 $\langle a,b,c,d,e,f,b,c,d,e,f,b,d,c,e,g \rangle]$

 $L_3 = [\langle a, b, c, d, e, f, b, d, c, e, g \rangle,$



Question:

Apply the 8 steps of the Alpha algorithm.

$$L_{3} = [\langle a, b, c, d, e, f, b, d, c, e, g \rangle,$$

$$\langle a, b, d, c, e, g \rangle^{2},$$

$$\langle a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g \rangle]$$

Let L be an event log over T. $\alpha(L)$ is defined as follows.
$1. T_{L} = \{ t \in T \mid \exists_{\sigma \in L} t \in \sigma \},$
2. $T_1 = \{ t \in T \mid \exists_{\sigma \in L} t = first(\sigma) \},$
$3. T_{O} = \{ t \in T \mid \exists_{\sigma \in L} t = last(\sigma) \},$
$4. X_{L} = \{ (A,B) \mid A \subseteq T_{L} \land A \neq \emptyset \land B \subseteq T_{L} \land B \neq \emptyset \land$
$\forall_{a \in A} \forall_{b \in B} a \to_{L} b \land \forall_{a1,a2 \in A} a_1\#_L a_2 \land \forall_{b1,b2 \in B} b_1\#_L b_2 \},$
$5.\ Y_L = \{\ (A,B) \in X_L \ \ \forall_{(A',B') \in X_L} A \subseteq A' \land B \subseteq B' \Rightarrow (A,B) = (A',B') \},$
6. $P_L = \{ p_{(A,B)} \mid (A,B) \in Y_L \} \cup \{i_L,o_L\},$
7. $F_L = \{ (a, p_{(A,B)}) \mid (A,B) \in Y_L \land a \in A \} \cup \{ (p_{(A,B)},b) \mid (A,B) \in A \} $
$Y_{L} \wedge b \in B$ $\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$
$8. \alpha(L) = (P_1, T_1, F_1).$

	а	b	С	d	e	f	g
а	#	\rightarrow	#	#	#	#	#
b	(#	\rightarrow	\rightarrow	#	\leftarrow	#
c	#	\leftarrow	#		\rightarrow	#	#
d	#	\leftarrow		#	\rightarrow	#	#
e	#	#	\leftarrow	\leftarrow	#	\rightarrow	\rightarrow
f	#	\rightarrow	#	#	\leftarrow	#	#
g	#	#	#	#	\leftarrow	#	#



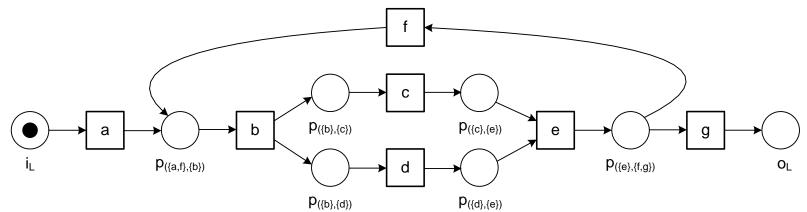
Model for L₃ discovered by the Alpha algorithm

$$L_{3} = [\langle a, b, c, d, e, f, b, d, c, e, g \rangle,$$

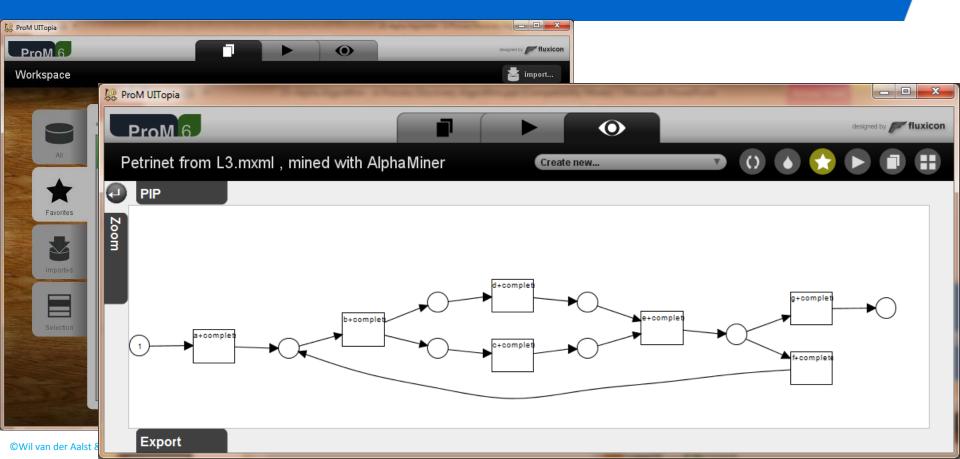
$$\langle a, b, d, c, e, g \rangle^{2},$$

$$\langle a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g \rangle]$$

	а	b	\mathcal{C}	d	e	f	g
а	#	\rightarrow	#	#	#	#	#
b	\leftarrow	#	\rightarrow	\rightarrow	#	\leftarrow	#
c	#	\leftarrow	#		\rightarrow	#	#
d	#	\leftarrow		#	\rightarrow	#	#
e	#	#	\leftarrow	\leftarrow	#	\rightarrow	\rightarrow
f	#	\rightarrow	#	#	\leftarrow	#	#
g	#	#	#	#	\leftarrow	#	#

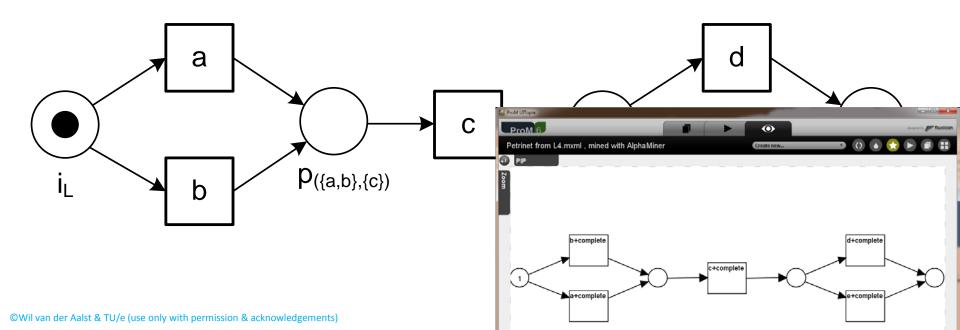


ProM's output for event log L₃



Another event log L₄

$$L_4 = [\langle a, c, d \rangle^{45}, \langle b, c, d \rangle^{42}, \langle a, c, e \rangle^{38}, \langle b, c, e \rangle^{22}]$$



Event log L₅

$$L_5 = [\langle a, b, e, f \rangle^2, \langle a, b, e, c, d, b, f \rangle^3, \langle a, b, c, e, d, b, f \rangle^2, \\ \langle a, b, c, d, e, b, f \rangle^4, \langle a, e, b, c, d, b, f \rangle^3]$$

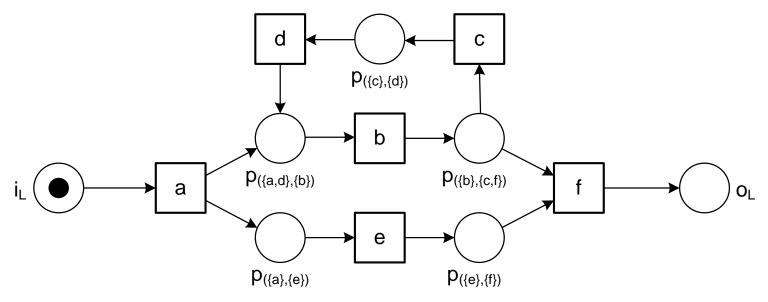
	а	b	С	d	e	f
а	#	\rightarrow	#	#	\rightarrow	#
b	\leftarrow	#	\rightarrow	\leftarrow		\rightarrow
c	#	\leftarrow	#	\rightarrow		#
d	#	\rightarrow	\leftarrow	#	jj –	#
e	\leftarrow				#	\rightarrow
f	#	\leftarrow	#	#	\leftarrow	#



```
T_L = \{a, b, c, d, e, f\}
     T_I = \{a\}
     T_I = \{f\}
  (\{d\},\{b\}),(\{e\},\{f\}),(\{a,d\},\{b\}),(\{b\},\{c,f\})\}
    Y_L = \{(\{a\}, \{e\}), (\{c\}, \{d\}), (\{e\}, \{f\}), (\{a,d\}, \{b\}), (\{b\}, \{c,f\})\}\}
  P_L = \{p_{(\{a\},\{e\})}, p_{(\{c\},\{d\})}, p_{(\{e\},\{f\})}, p_{(\{a,d\},\{b\})}, p_{(\{b\},\{c,f\})}, i_L, o_L\}\}
  F_L = \{(a, p_{(\{a\}, \{e\})}), (p_{(\{a\}, \{e\})}, e), (c, p_{(\{c\}, \{d\})}), (p_{(\{c\}, \{d\})}, d), (c, p_{(\{c\}, \{d\})}), 
                                            (p_{(\{a,d\},\{b\})},b),(b,p_{(\{b\},\{c,f\})}),(p_{(\{b\},\{c,f\})},c),(p_{(\{b\},\{c,f\})},f),
                                            (i_L,a),(f,o_L)
\alpha(L) = (P_L, T_L, F_L)
```

Discovered model

 $L_5 = [\langle a, b, e, f \rangle^2, \langle a, b, e, c, d, b, f \rangle^3, \langle a, b, c, e, d, b, f \rangle^2,$ $\langle a, b, c, d, e, b, f \rangle^4, \langle a, e, b, c, d, b, f \rangle^3]$



$$X_L = \{(\{a\}, \{b\}), (\{a\}, \{e\}), (\{b\}, \{c\}), (\{b\}, \{f\}), (\{c\}, \{d\}), (\{d\}, \{b\}), (\{e\}, \{f\}), (\{a, d\}, \{b\}), (\{b\}, \{c, f\})\}\}$$



 $Y_L = \{(\{a\}, \{e\}), (\{c\}, \{d\}), (\{e\}, \{f\}), (\{a,d\}, \{b\}), (\{b\}, \{c,f\})\}\}$

Summary

- The Alpha algorithm provides a basic process discovery approach.
- It has many limitations. These will be discussed later.
- However, it nicely illustrates the key ingredients of process discovery.
- Hence, it is important to understand the algorithm and practice using concrete examples.

Part I: Preliminaries Chapter 1 Introduction Chapter 2 Process Modeling and Analysis Chapter 4 Getting the Data Chapter 5 Process Discovery: An Introduction Chapter 6 Advanced Process Models Chapter 5 Process Discovery: An Introduction Chapter 6 Advanced Process Discovery Tech



Part III: Beyond Process Discovery

Chapter 7
Conformance
Checking

Chapter 8
Mining Additional
Perspectives

Chapter 9
Operational Support

Part IV: Putting Process Mining to Work

Chapter 10
Tool Support

Chapter 11
Analyzing "Lasagna Processes"

Chapter 12 Analyzing "Spaghetti Processes"

Part V: Reflection

Chapter 13
Cartography and
Navigation

Chapter 14
Epilogue



nhancement of Business Proc

Wil M. P. van der Aalst

Process

Mining

