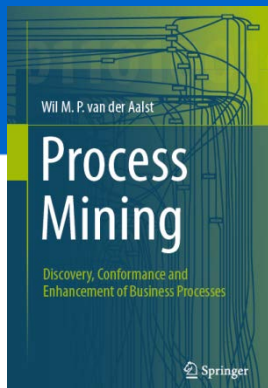


*Process Mining: Data Science in Action*

# Using Regions to Discover Concurrency

prof.dr.ir. Wil van der Aalst  
[www.processmining.org](http://www.processmining.org)

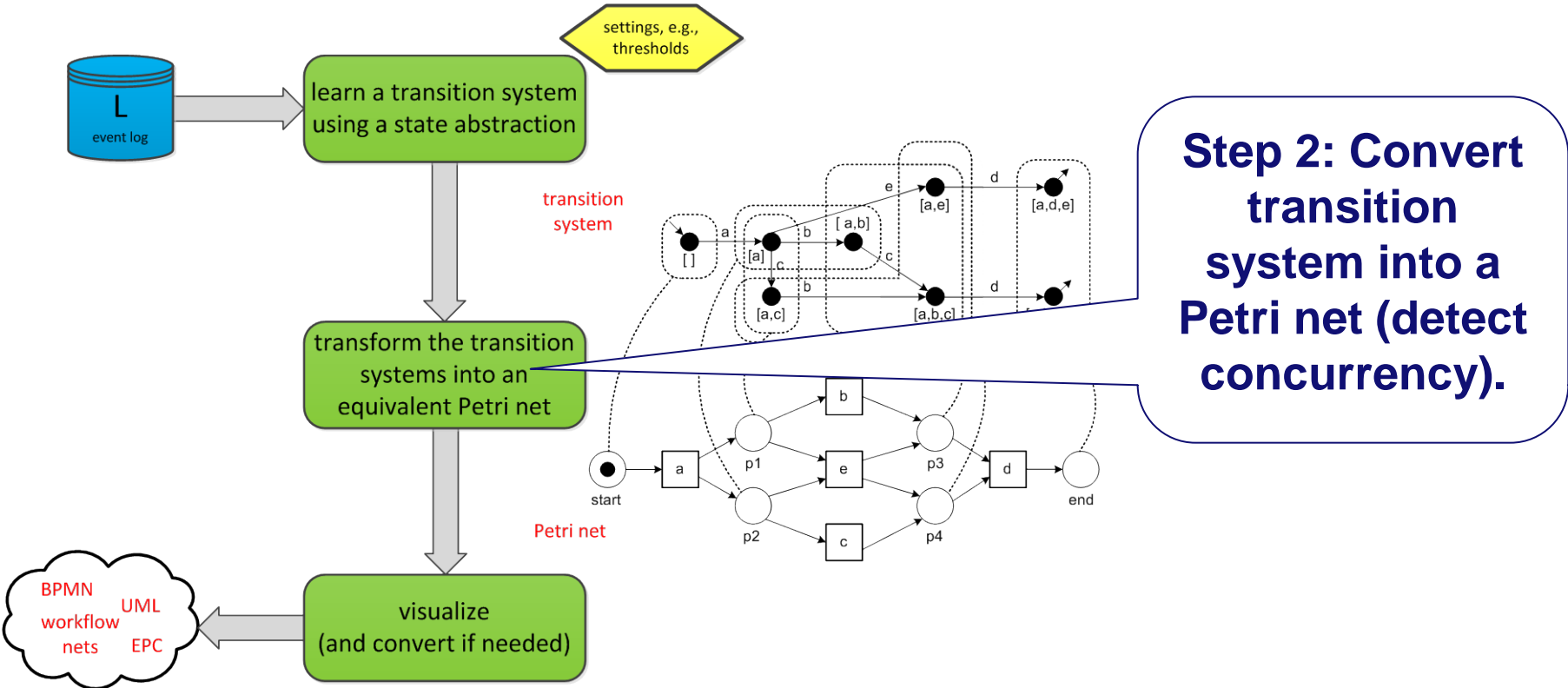


**TU/e**

Technische Universiteit  
**Eindhoven**  
University of Technology

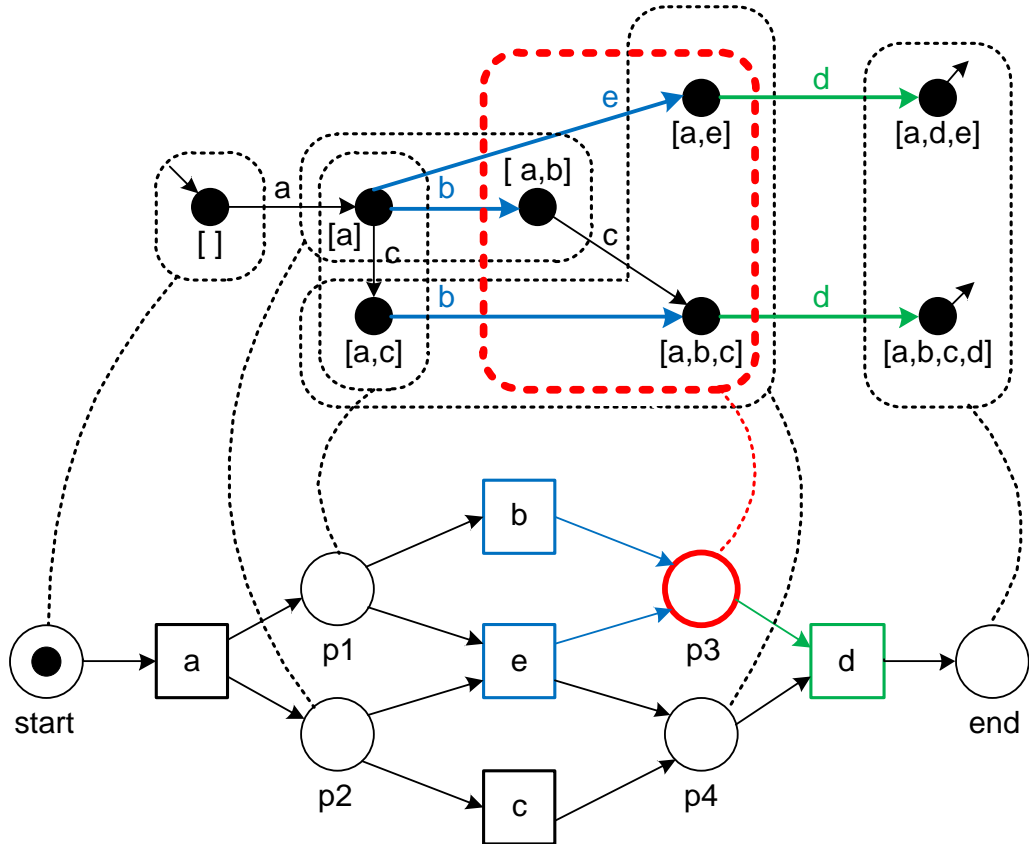
**Where innovation starts**

# State-based regions

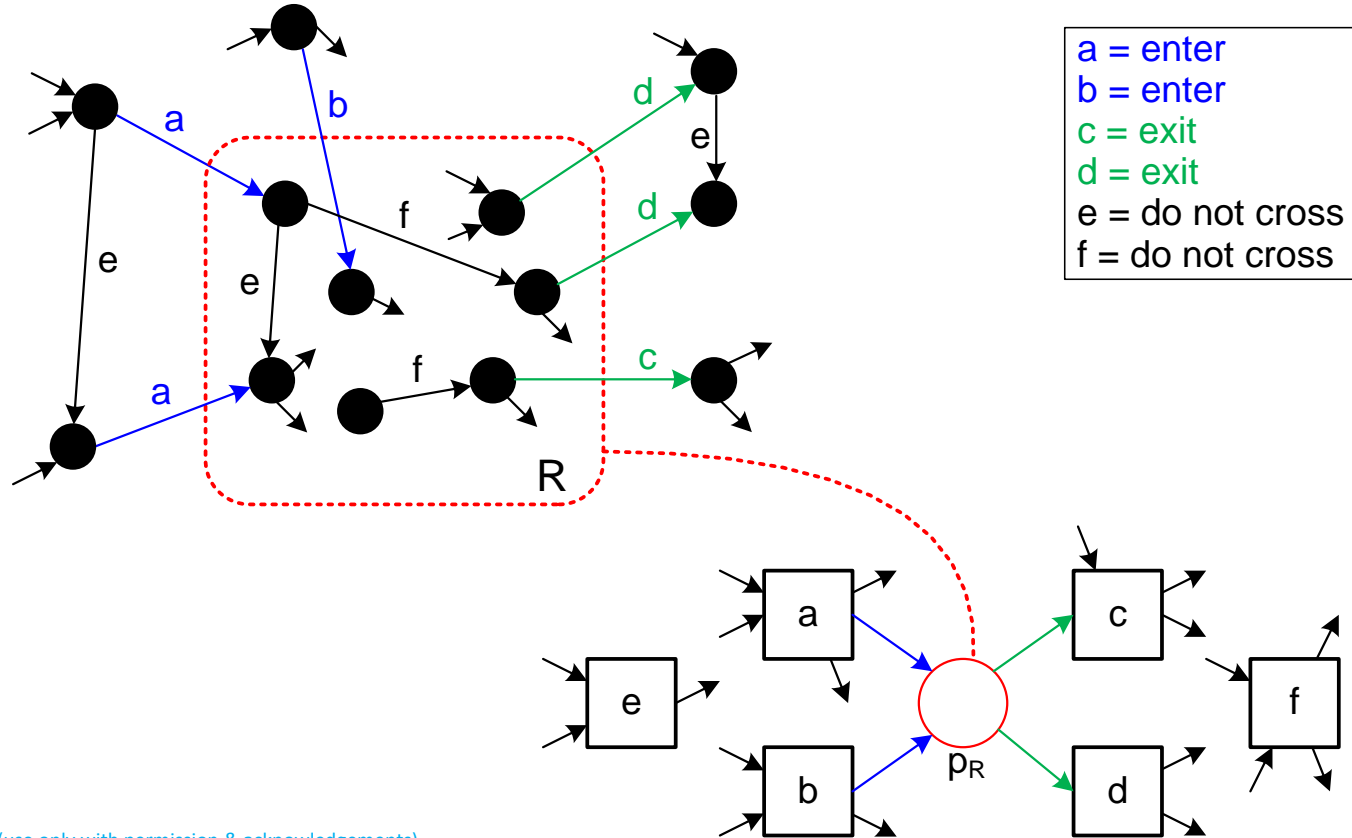


# State-based regions correspond to places

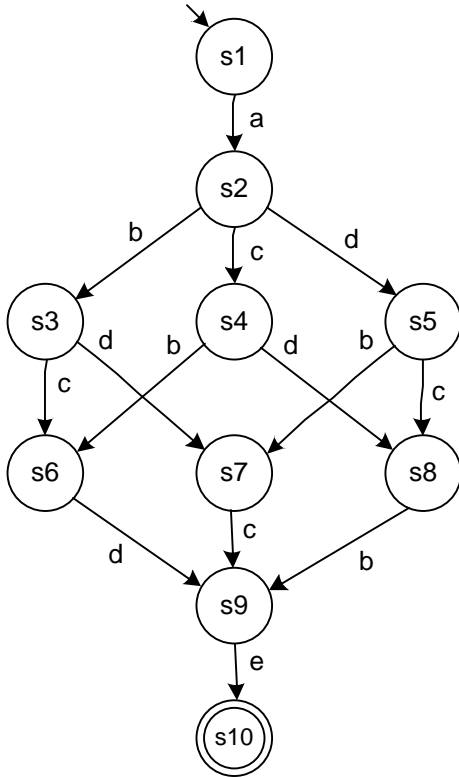
It is all about  
discovering  
concurrency ...



# What is a (state-based) region?



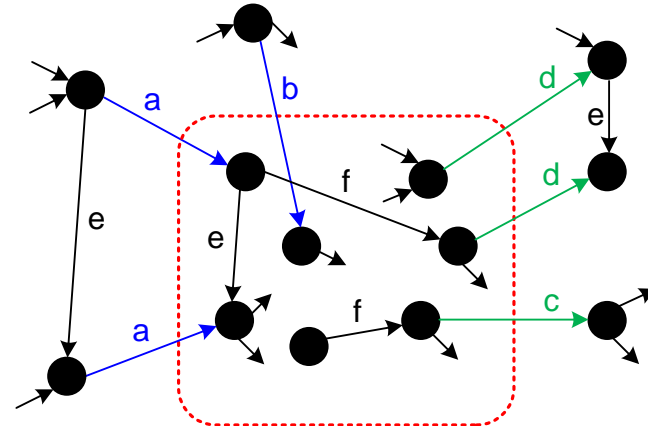
# Starting point: A transition system



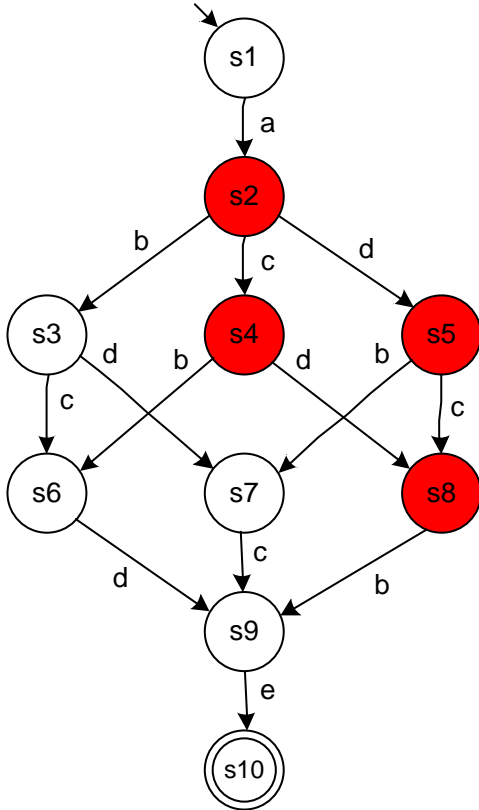
- We assume that there is only **one initial state** (otherwise preprocessing needed).
- It is convenient to also have just one final state that can always be reached (not strictly necessary)
- **All states need to be reachable!**

# Definition

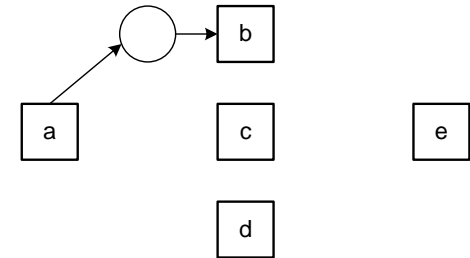
A region is a set of states, such that, if a transition **exits** the region, then all equally labeled transitions **exit** the region, and if a transition **enters** the region, then all equally labeled transitions **enter** the region. All events not entering or exiting the region **do not cross** the region.



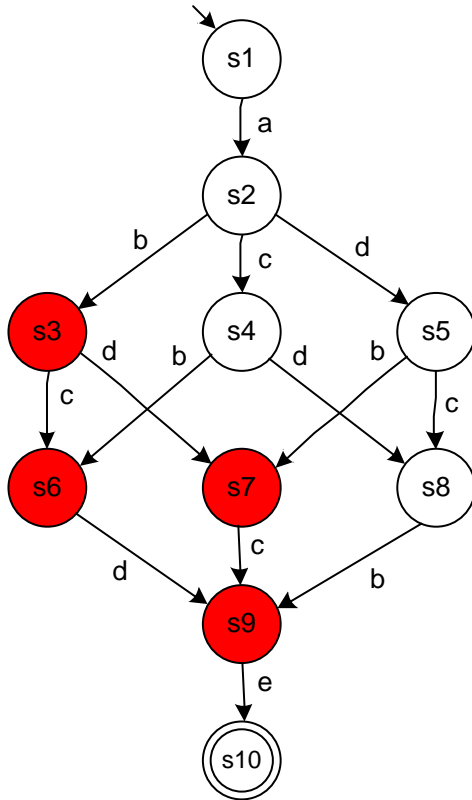
# Example of a region



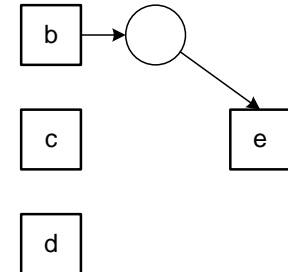
- **a enters**
- **b exits**
- **c does not cross**
- **d does not cross**
- **e does not cross**



# Example of a region

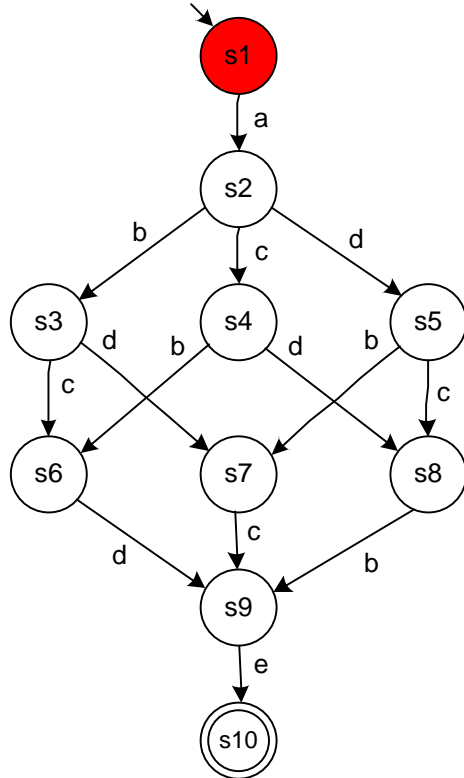


- a does not cross
- b enters
- c does not cross
- d does not cross
- e exits



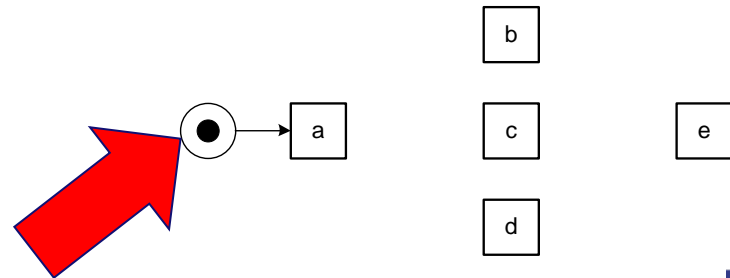


# Example of a region

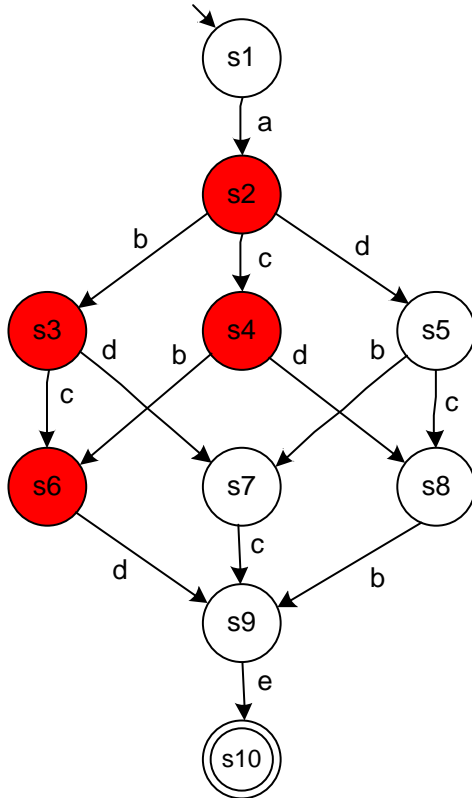


- a exits
- b does not cross
- c does not cross
- d does not cross
- e does not cross

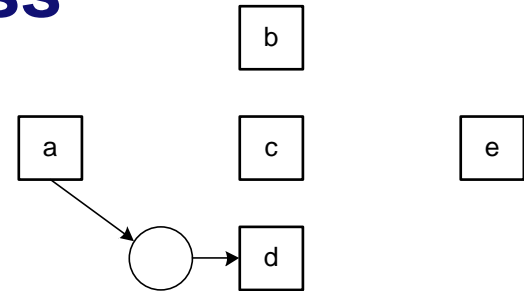
As will be shown later:  
places corresponding  
to regions containing  
the initial state are  
initially marked.



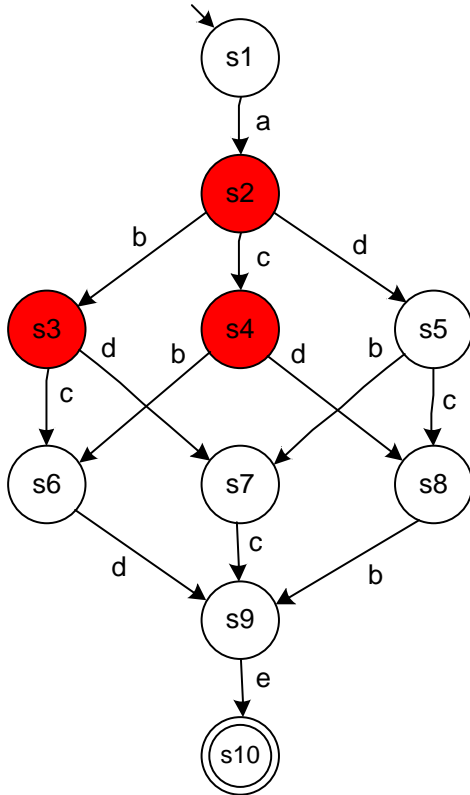
# Example of a region



- **a enters**
- **b does not cross**
- **c does not cross**
- **d exits**
- **e does not cross**

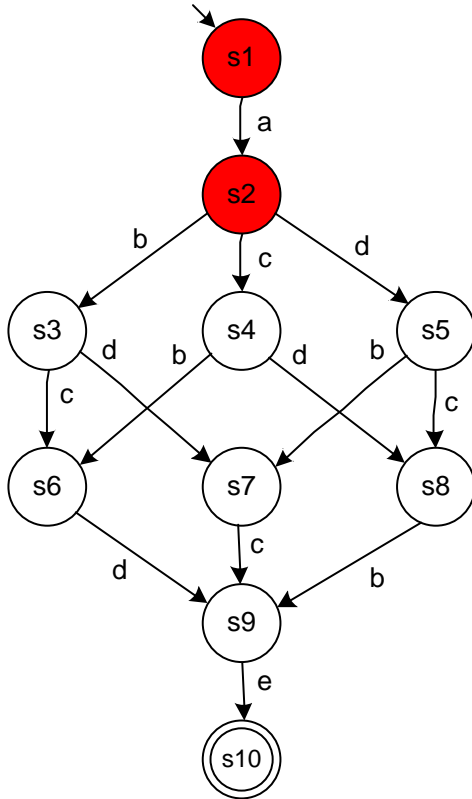


# Not a region



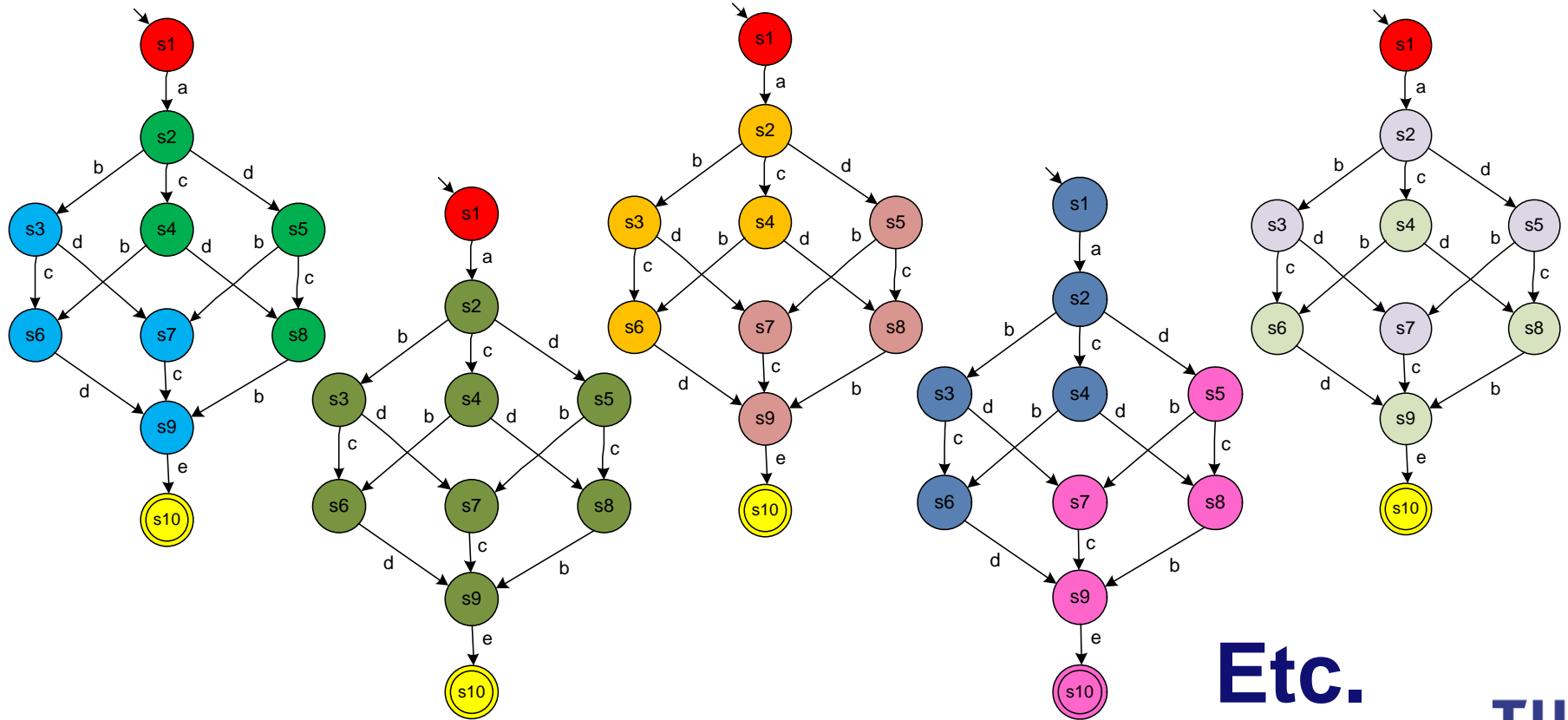
- **a enters**
- **b does not cross and exits**
- **c does not cross and exits**
- **d does not cross and exits**
- **e does not cross**

# Not a region



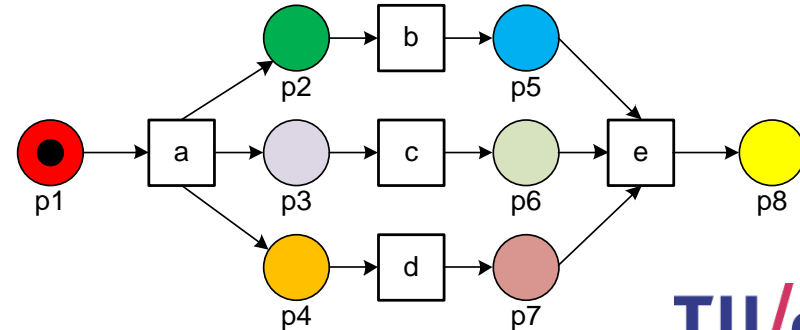
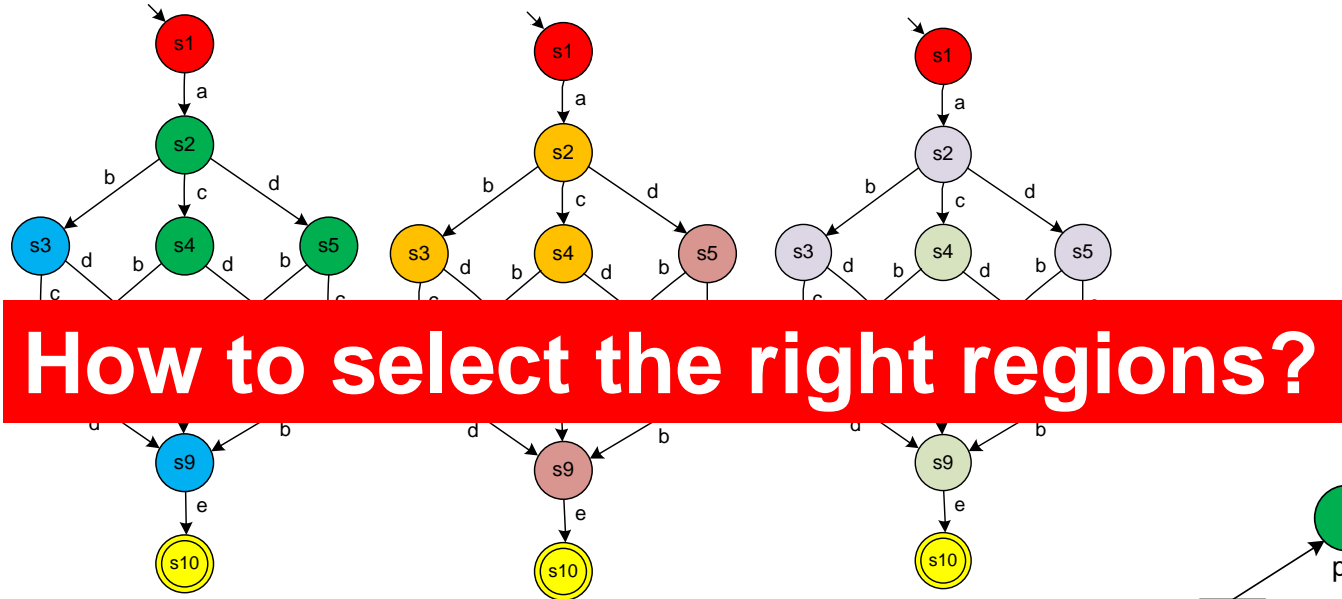
- **a does not cross**
- **b does not cross and exits**
- **c does not cross and exits**
- **d does not cross and exits**
- **e does not cross**

# Multiple regions

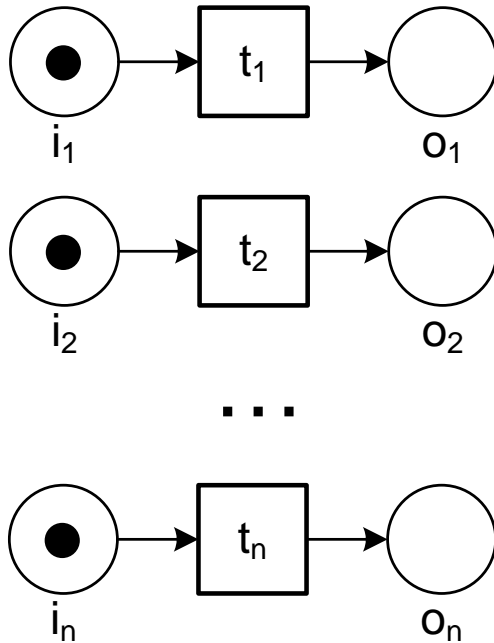


Etc.

# Selectively chosen regions

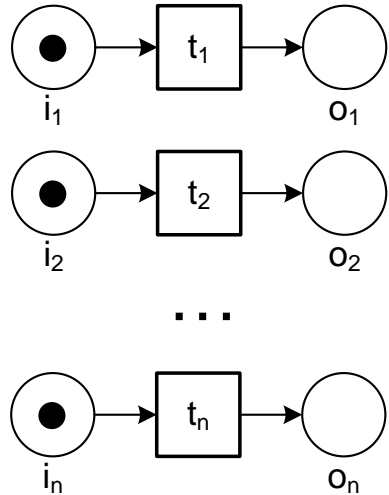


# Question

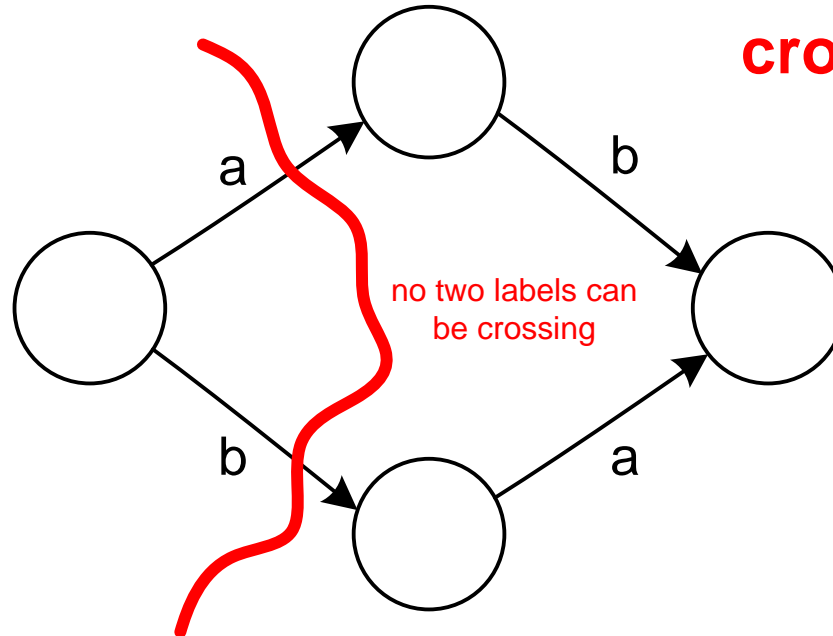


**How many regions  
does the reachability  
graph of this Petri  
net have?**

# Answer: $2(n+1)$ regions



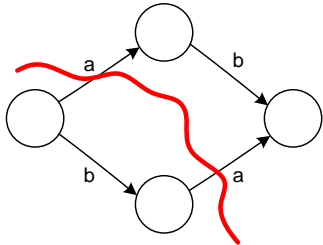
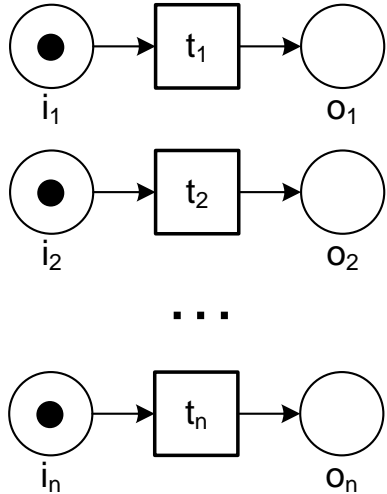
$2^n$  states



At most 1 transition label can be crossing (or none).



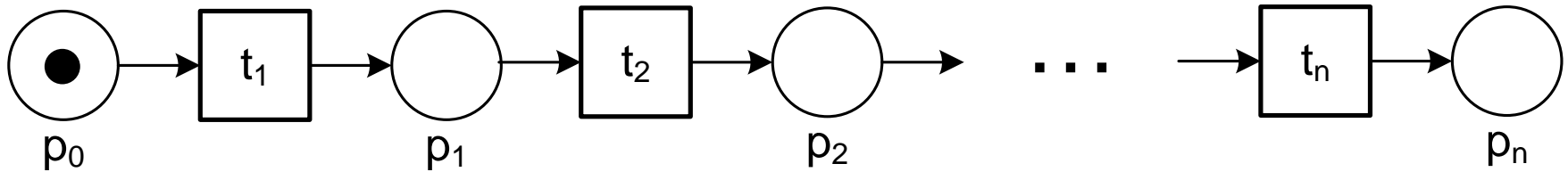
# Answer: $2(n+1)$ regions



- There are  $n$  ways to split the states in two groups based on the label selected to cross.
- Each situation splits the set of states in two regions.
- Also the empty set and all the set of all states are regions.

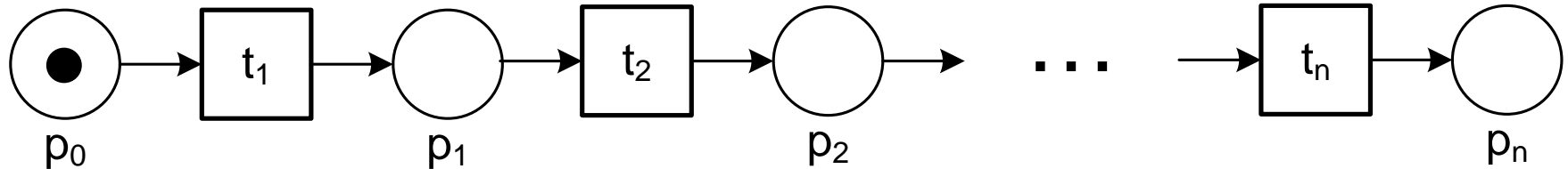
Answer:  **$2(n+1)$  regions.**

# Question



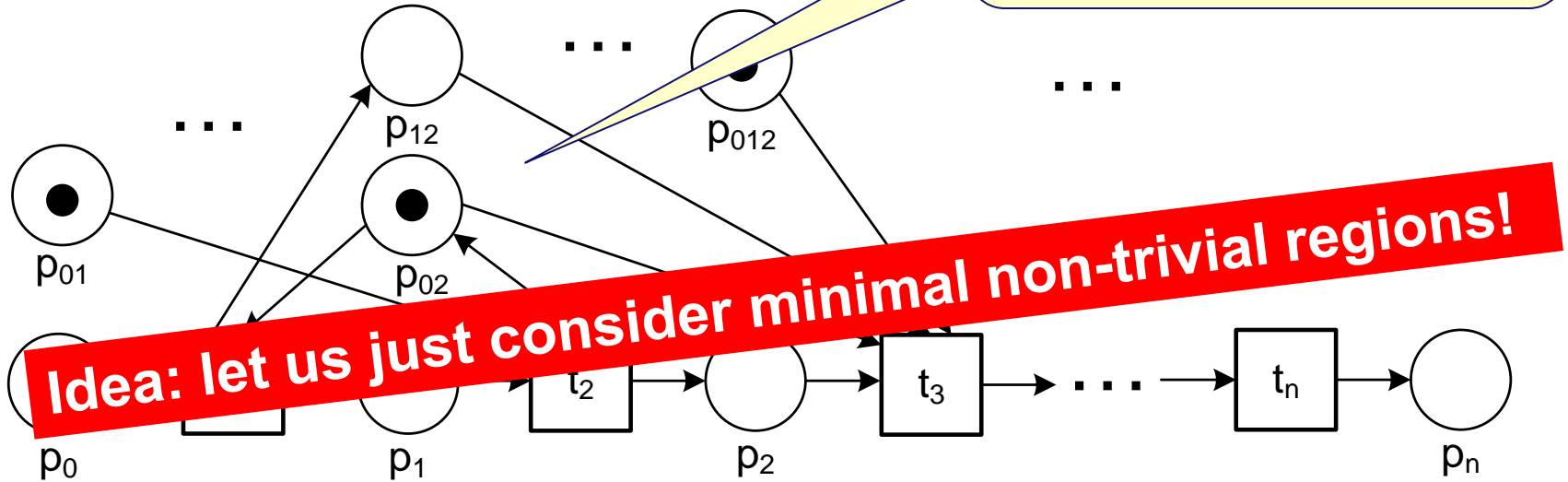
**How many regions does the reachability graph of this Petri net have?**

# Answer



**Every subset of states forms a region.  
Hence, there are  $2^{(n+1)}$  regions.**

# Illustration



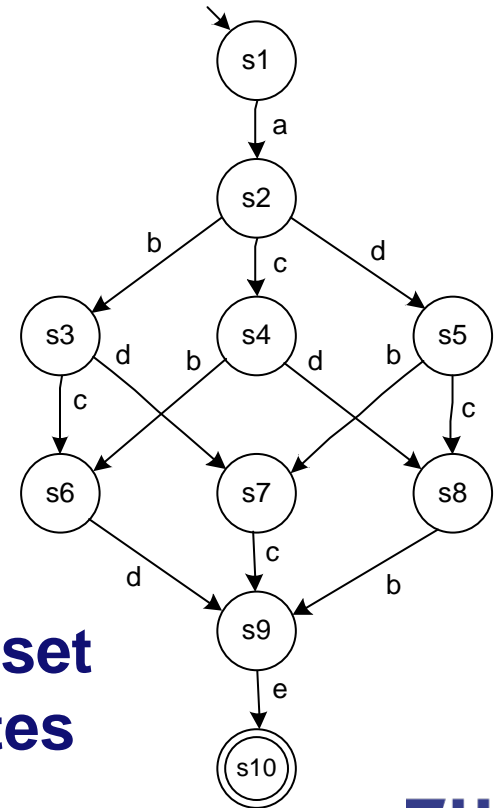
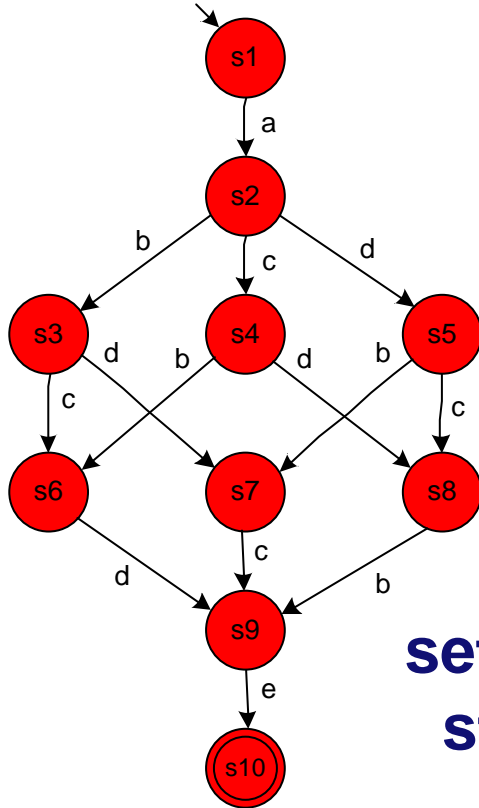
For a sequence of 10 activities we have  $2^{(10+1)} = 2048$  regions.

# Selecting the "interesting regions" only ...

- We only include the **non-trivial minimal** regions!
- **Non-trivial**
  - The empty set and the set of all states are regions by definition.
  - These trivial regions carry no information and should not be included.
- **Minimal**
  - A region is minimal if it cannot be decomposed into smaller (non-trivial) ones.
  - Non-minimal regions are implied by smaller ones and should not be included.

# Trivial regions: All or nothing

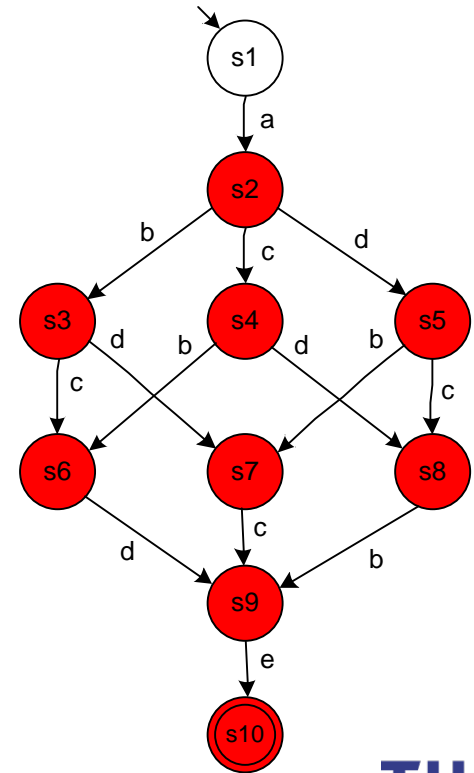
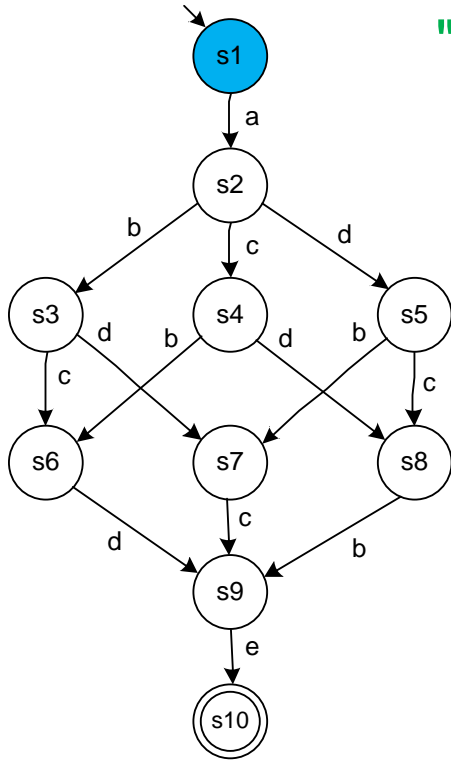
a,b,c,d,e do not cross



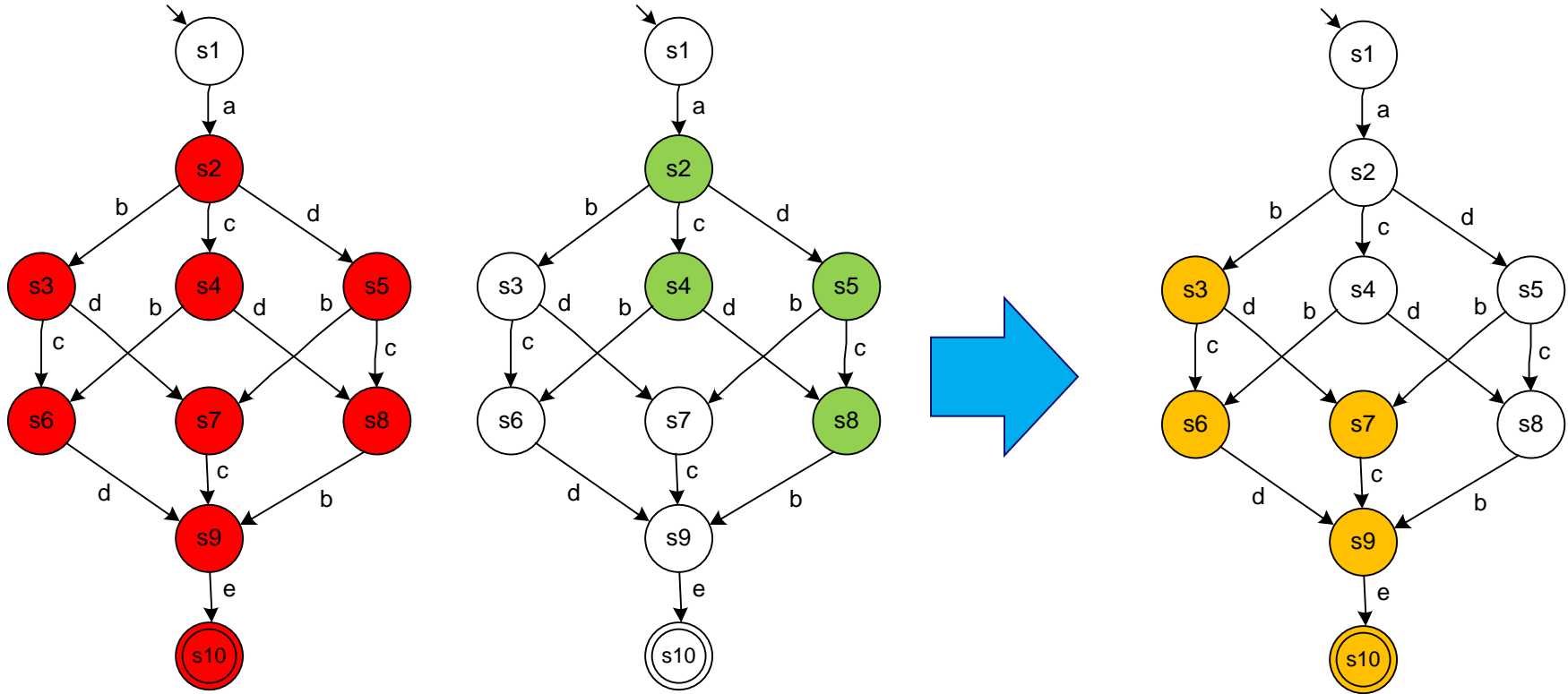
# Complement:

If  $r$  is a region, then  $S \setminus r$  is a region

"exits" and "enters" are swapped

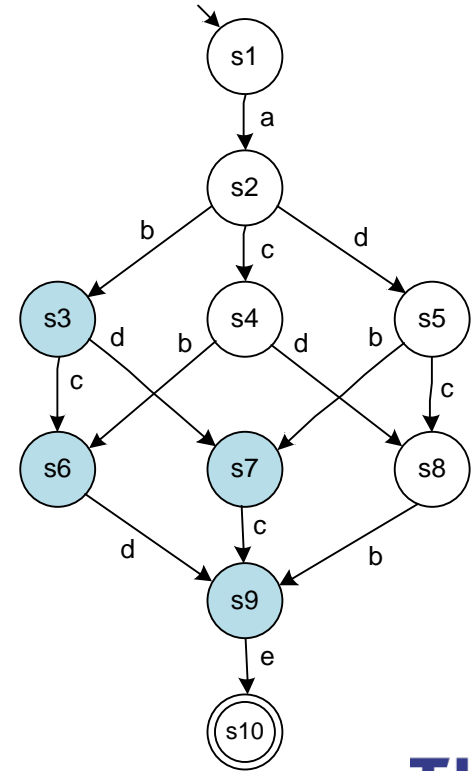
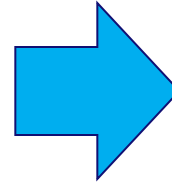
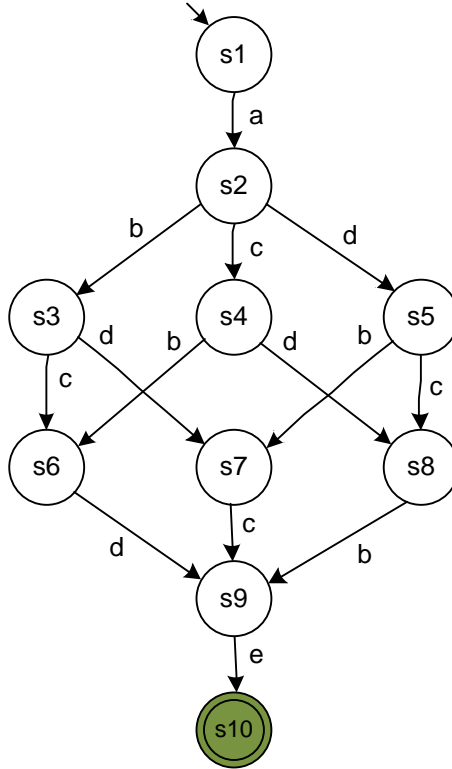
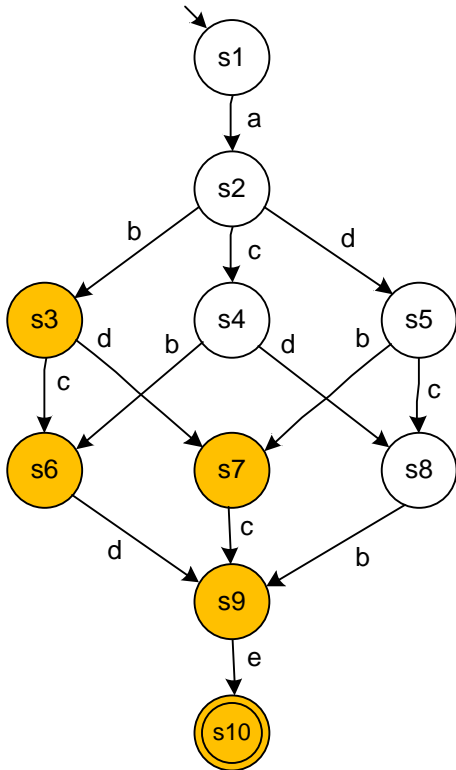


If  $r_1$  and  $r_2$  are regions, and  $r_1$  is a subset of  $r_2$ , then  $r_2 \setminus r_1$  is a region.

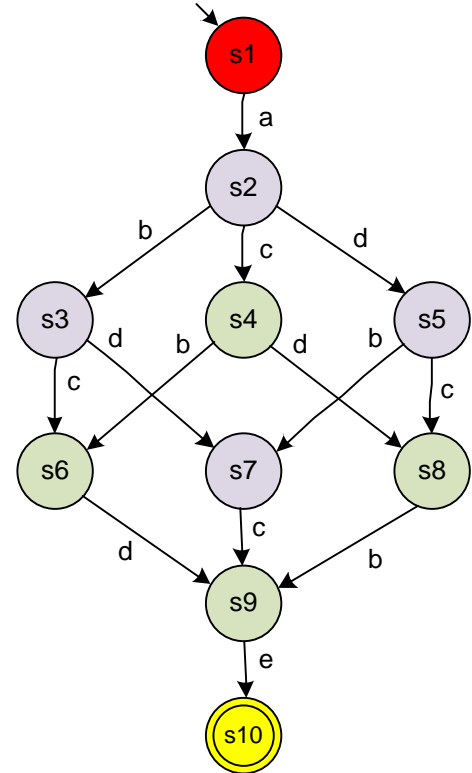
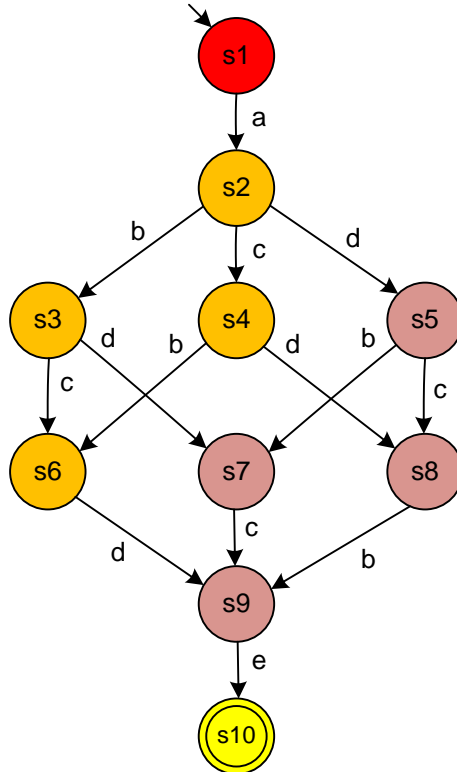
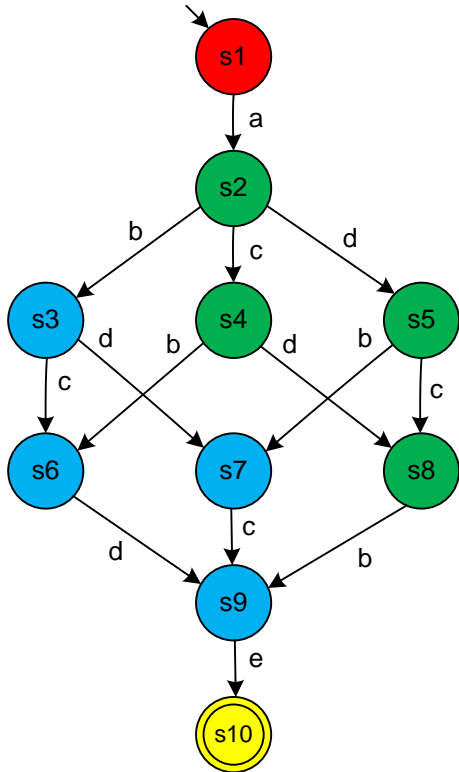




# Not minimal yet ...



# Running example: 8 minimal non-trivial regions

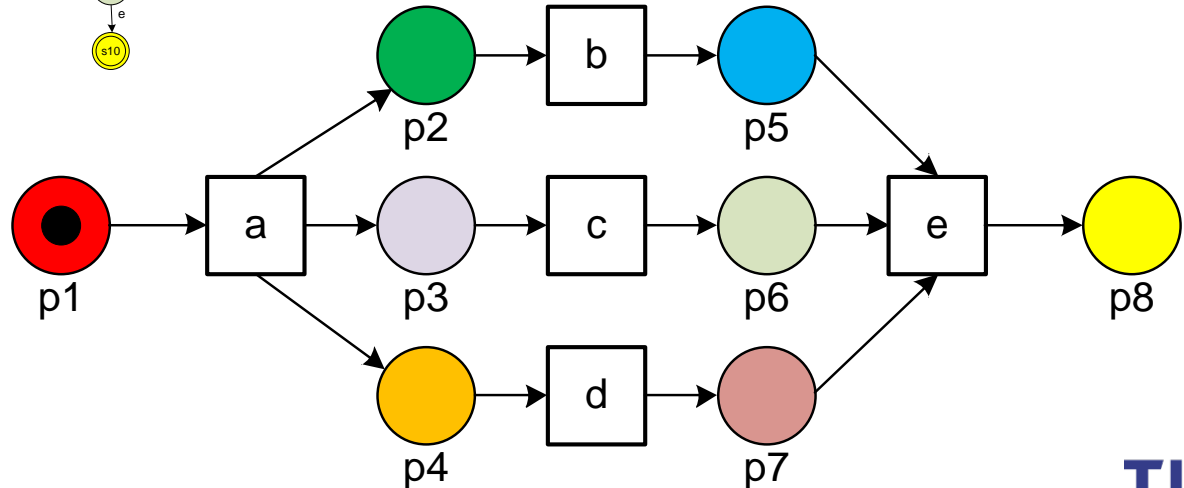
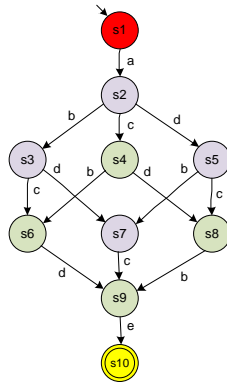
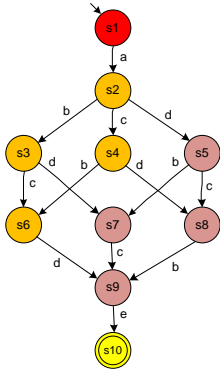
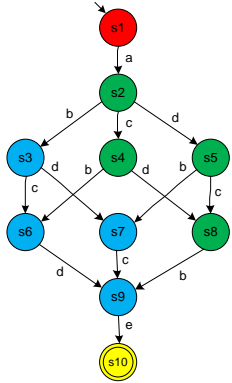


# Basic algorithm to construct a Petri net

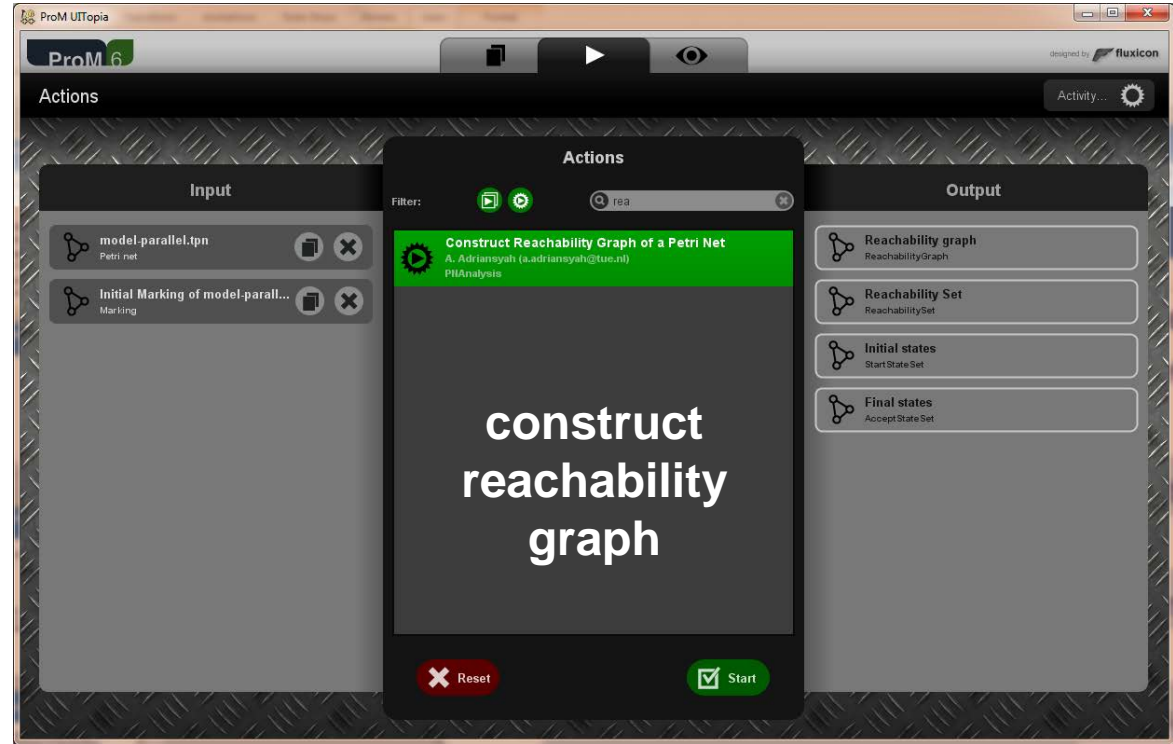
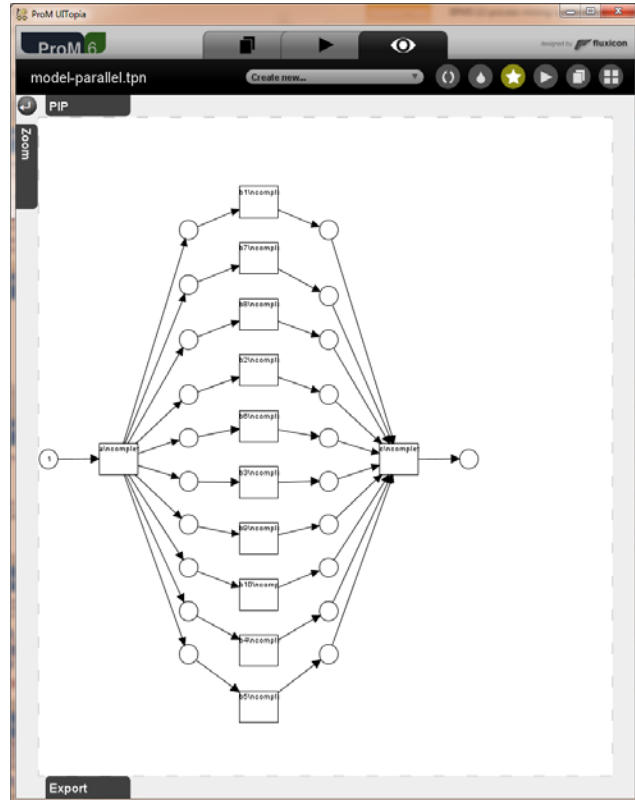
1. For each transition label in the transition system, a **transition** is added to the Petri net.
2. The **minimal non-trivial regions** are computed.
3. For each minimal non-trivial region in the transition system, a **place** is added to the Petri net.
4. The corresponding **arcs** are generated.
5. A **token** is added to each place that corresponds to a region containing the initial state.

The resulting Petri net is called the **minimal saturated net**.

# Applying the algorithm yields the expected result

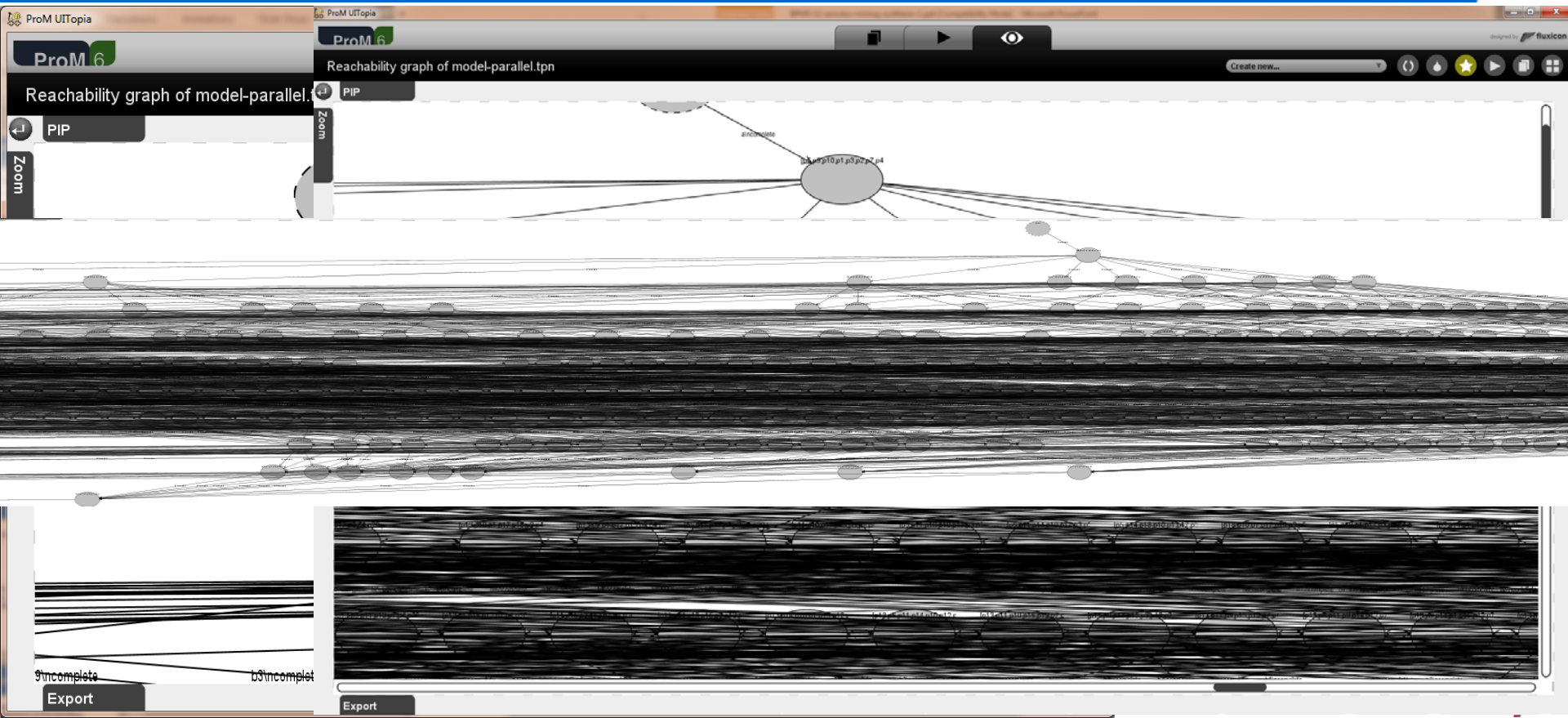


# Rediscovery experiment with 10 parallel activities



past with multiset abstraction

# Reachability graph ( $1+2^{10}+1 = 1026$ states)



# Apply "Covert to Petri net using regions"

The image displays two screenshots of the ProM 6.10.0 software interface, demonstrating the process of converting a model to a Petri net using regions.

**Left Screenshot: Actions Menu**

The interface shows the 'Actions' menu with the following options:

- Convert to Petri Net using Regions** (B.F. van Dongen (b.f.v.dongen@tue.nl))
- Remove edgepoints** (B.F. van Dongen (b.f.v.dongen@tue.nl))
- Unpack Aggregate Type** (M. Westergaard (m.westergaard@tue.nl))

The 'Convert to Petri Net using Regions' option is highlighted in green. Below the menu, there are 'Reset' and 'Start' buttons.

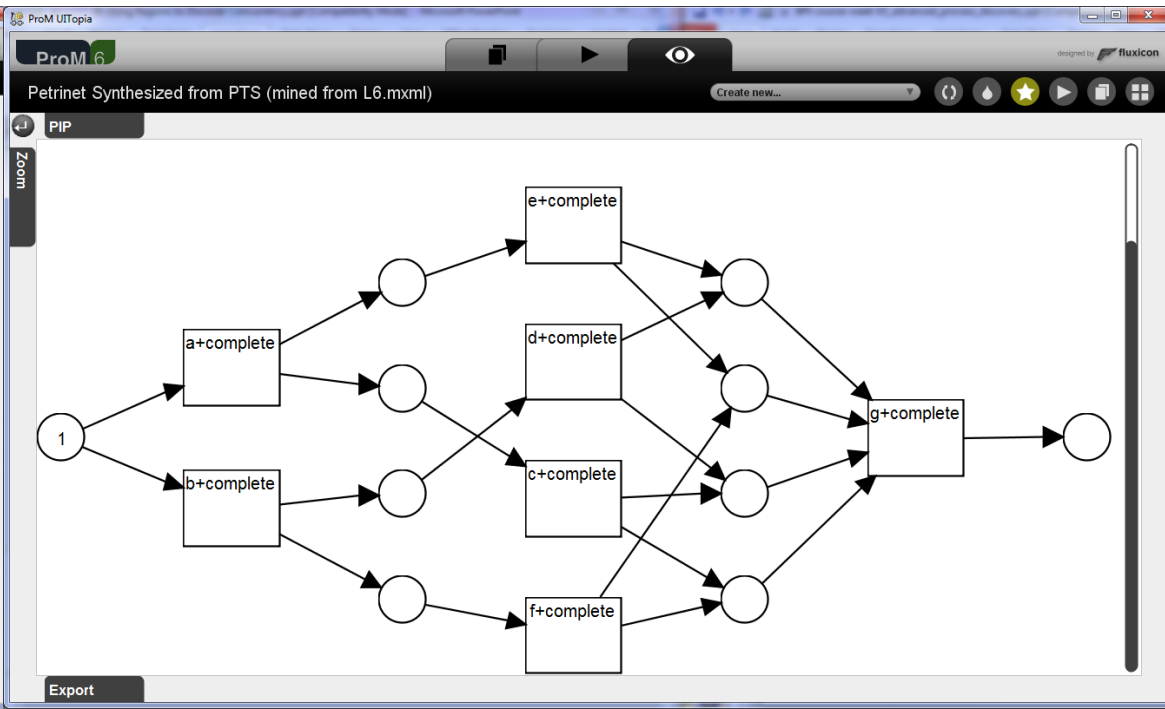
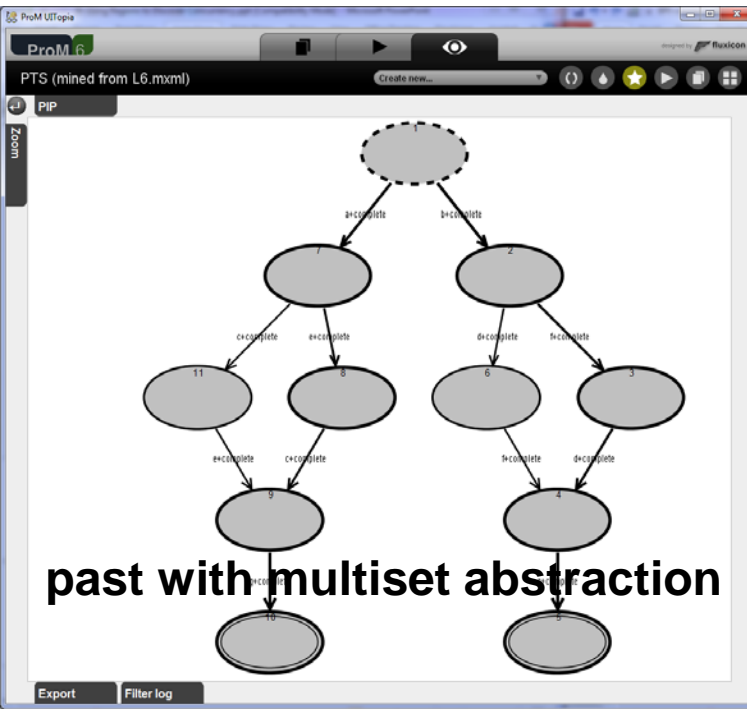
**Right Screenshot: Petri net Synthesized from StateSpace of n**

The interface shows the resulting Petri net diagram, titled 'Petri net Synthesized from StateSpace of n'. The diagram is a Petri net with places (rectangles) and transitions (circles). The places are labeled with names like 'a\_incomple', 'b\_incomple', 'c\_incomple', etc. The transitions are labeled with names like 'a', 'b', 'c', etc. The diagram is a complex Petri net structure.

# Example

(same result as Alpha algorithm)

$$L_6 = [\langle a, c, e, g \rangle^2, \langle a, e, c, g \rangle^3, \langle b, d, f, g \rangle^2, \langle b, f, d, g \rangle^4]$$

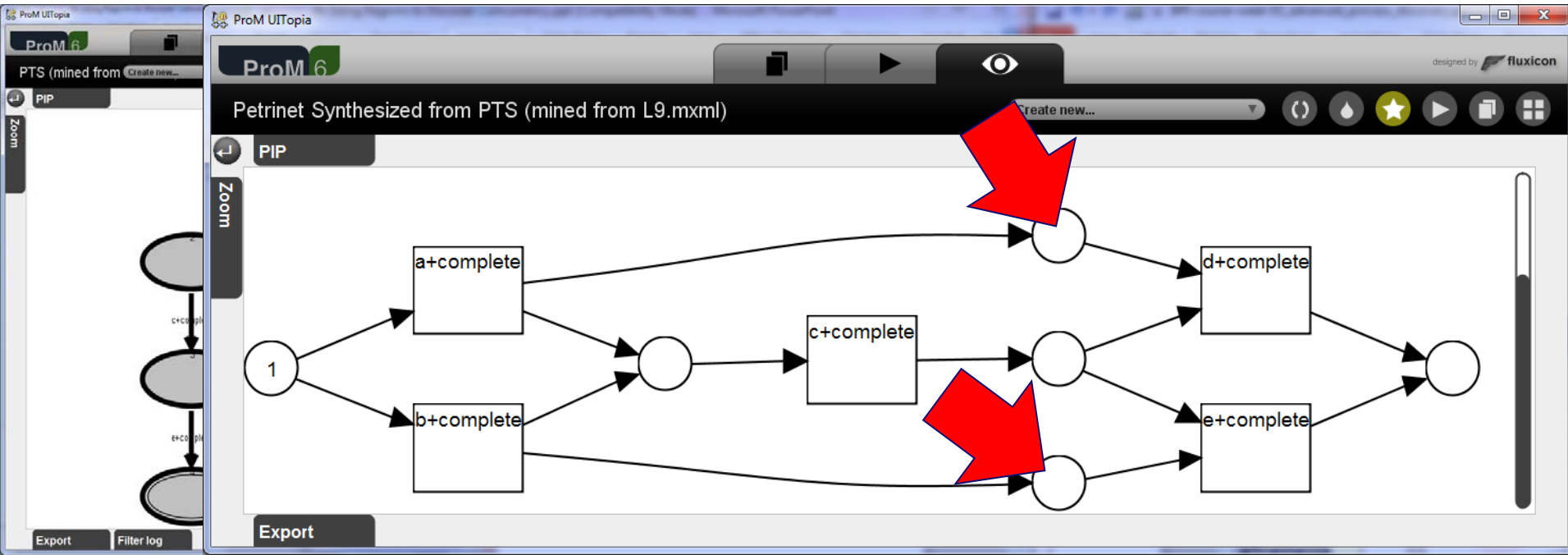




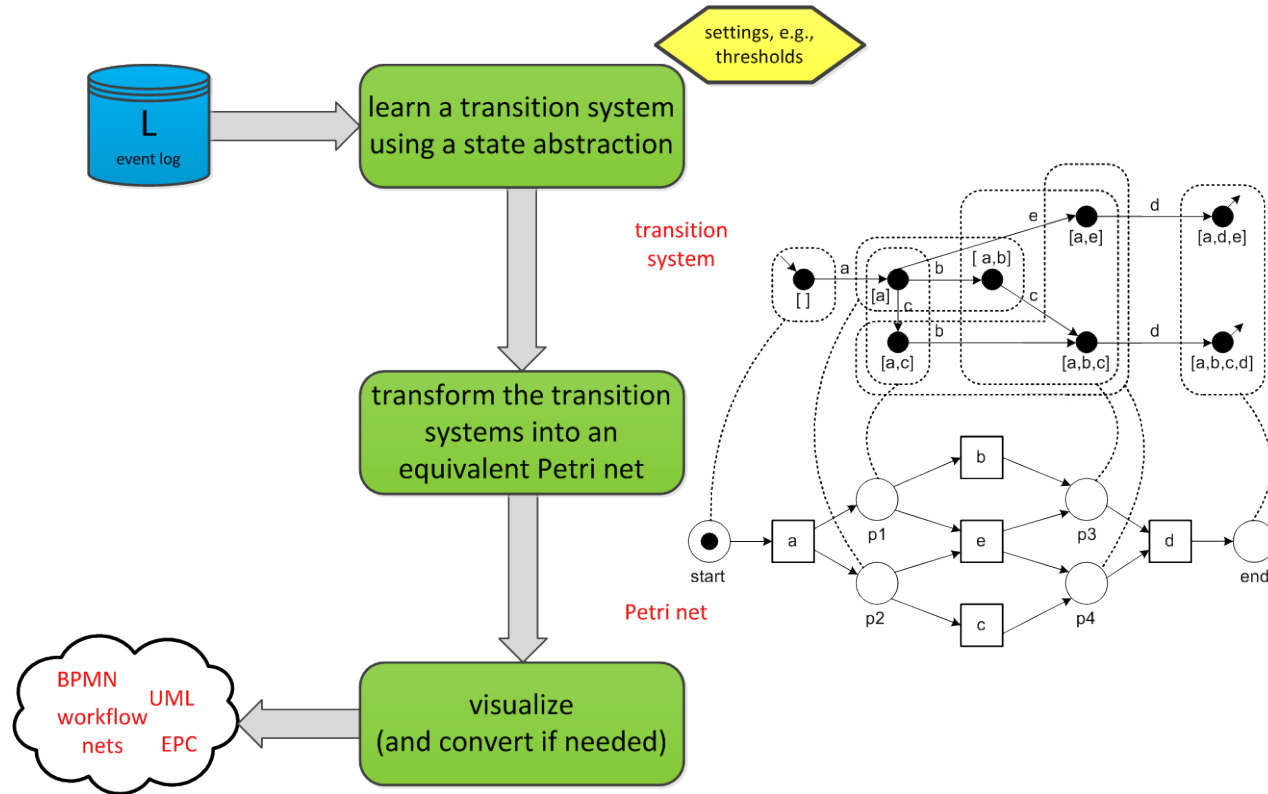
# Example

(improved result compared to Alpha algorithm)

$$L_9 = [\langle a, c, d \rangle^{45}, \langle b, c, e \rangle^{42}]$$



# Next: limitations of the approach



### *Part I: Preliminaries*

**Chapter 1**  
Introduction

**Chapter 2**  
Process Modeling and  
Analysis

**Chapter 3**  
Data Mining

### *Part III: Beyond Process Discovery*

**Chapter 7**  
Conformance  
Checking

**Chapter 8**  
Mining Additional  
Perspectives

**Chapter 9**  
Operational Support

### *Part II: From Event Logs to Process Models*

**Chapter 4**  
Getting the Data

**Chapter 5**  
Process Discovery: An  
Introduction

**Chapter 6**  
Advanced Process  
Discovery Issues

### *Part IV: Putting Process Mining to Work*

**Chapter 10**  
Tool Support

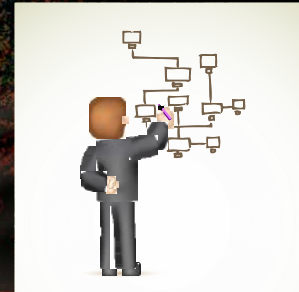
**Chapter 11**  
Analyzing “Lasagna  
Processes”

**Chapter 12**  
Analyzing “Spaghetti  
Processes”

### *Part V: Reflection*

**Chapter 13**  
Cartography and  
Navigation

**Chapter 14**  
Epilogue



Wil M. P. van der Aalst

# Process Mining

Discovery, Conformance and  
Enhancement of Business Processes

 Springer