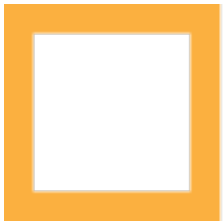
Searching Lists
Algorithms and Datastructures

Marjahan Begum and Anders Kalhauge



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Spring 2017

Outline

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Arrays

Symbol Tables

- List-based
- Array-based
- Hashed

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Expandable arrays

Arrays are by nature of fixed length. How can we make them expandanble and still have direct memory access?

When adding a new element to a full array we could:

1. Copy the array to an array one bigger
2. Copy the array to an array m elements bigger
3. Copy the array to an array of double size

Expandable arrays

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Average time for adding an element will be:

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2.
3.

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2. $O(n)$ - all elements are copied each m^{th} time $O(\frac{n}{m}) = O(n)$
3. $O(1)$ - how can that be?

Expandable arrays

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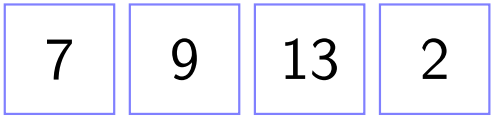
Money in the bank

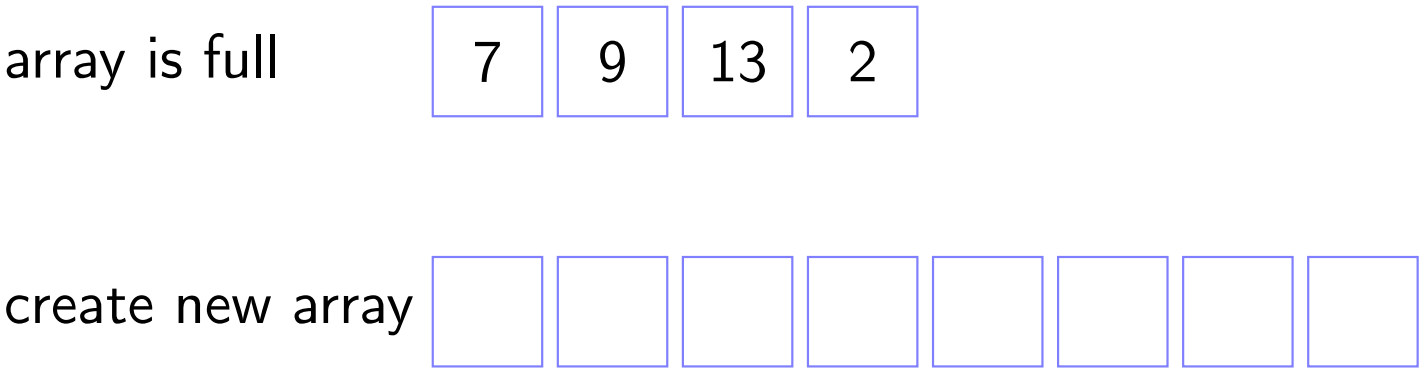
Balance: 4 we hope that is enough to pay for future expansions

array is full



Money in the bank

Balance: $4 - 0 = 4$ (creating new array considered free here)



Expandable arrays

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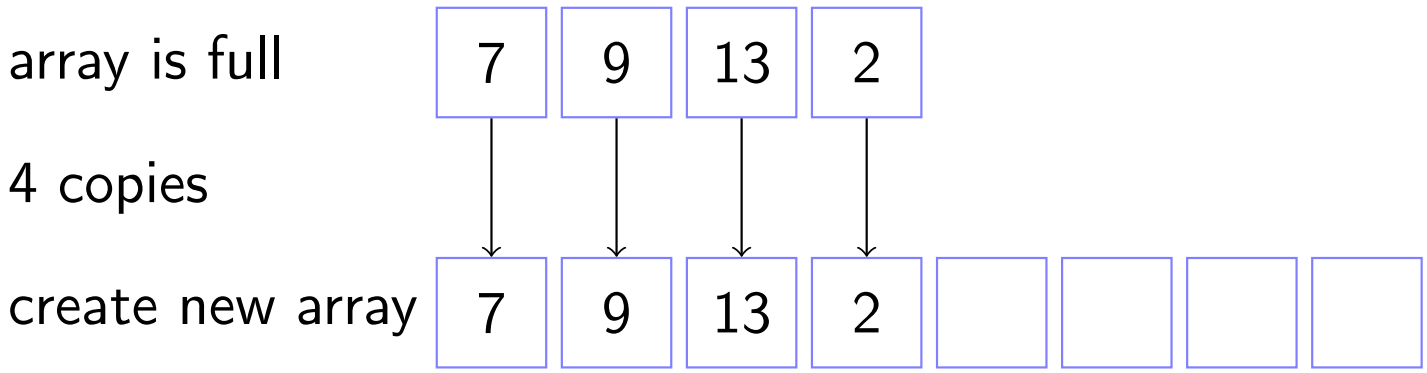
Money in the bank

Balance: $4 - 4 = 0$ (using 1 per copy)



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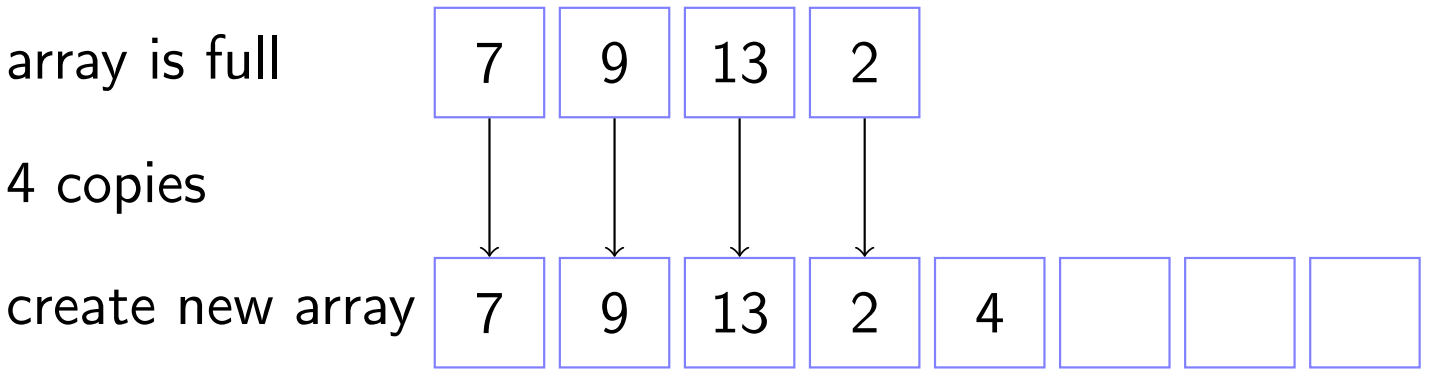
Money in the bank

Balance: $0 + 3 - 1 = 2$ (charging 3 for an insert, using 1)



add element 4

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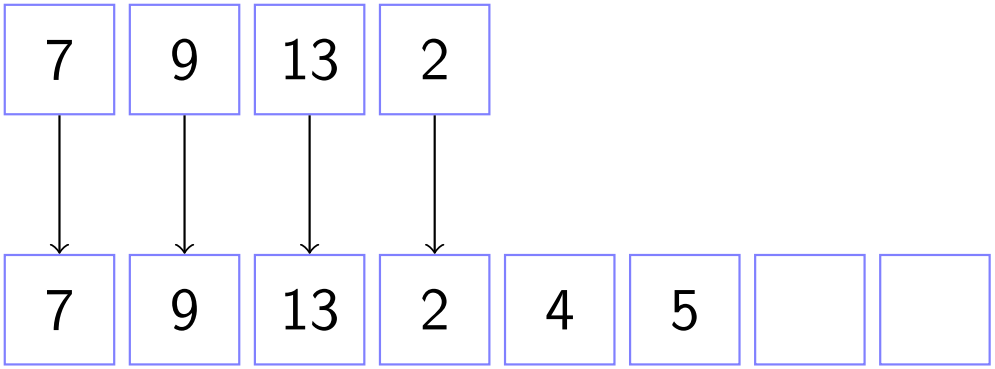
Money in the bank

Balance: $2 + 3 - 1 = 4$ (charging 3 for an insert, using 1)

array is full

4 copies

create new array



add element 4

add element 5

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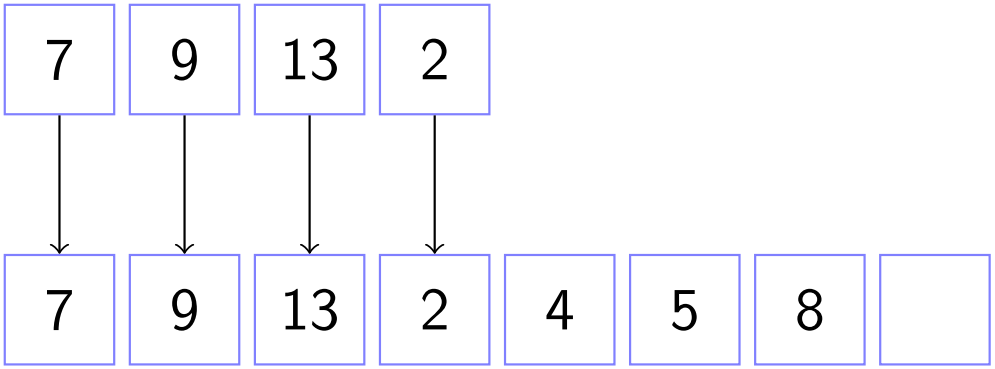
Money in the bank

Balance: $4 + 3 - 1 = 6$ (charging 3 for an insert, using 1)

array is full

4 copies

create new array



add element 4

add element 5

add element 8

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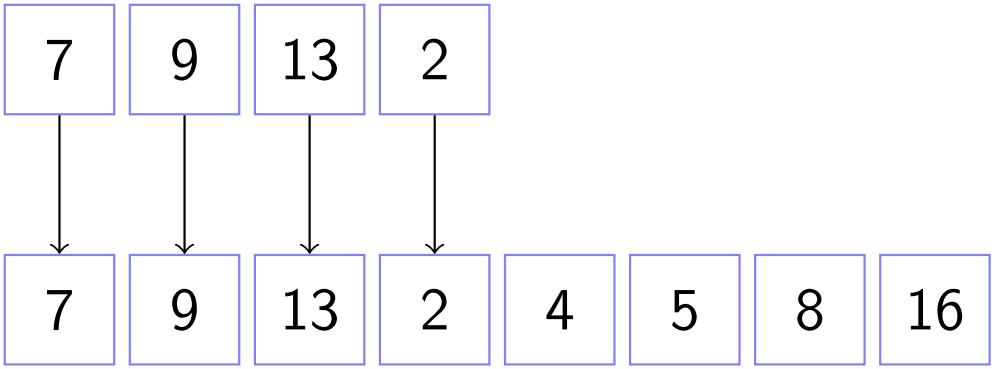
Money in the bank

Balance: $6 + 3 - 1 = 8$ (charging 3 for an insert, using 1)

array is full

4 copies

create new array



add element 4

add element 5

add element 8

add element 16

Expandable arrays

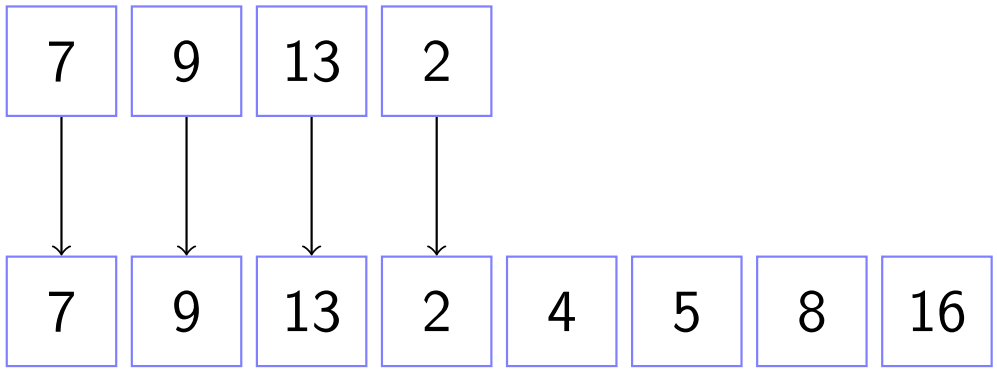
Money in the bank

Balance: 8 enough to pay for 8 copies

array is full

4 copies

create new array



add element 4

add element 5

add element 8

add element 16

$$O(3) = O(1)$$

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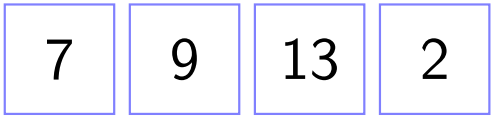
Constant payload

Payload: 0

array is full



Expandable arrays

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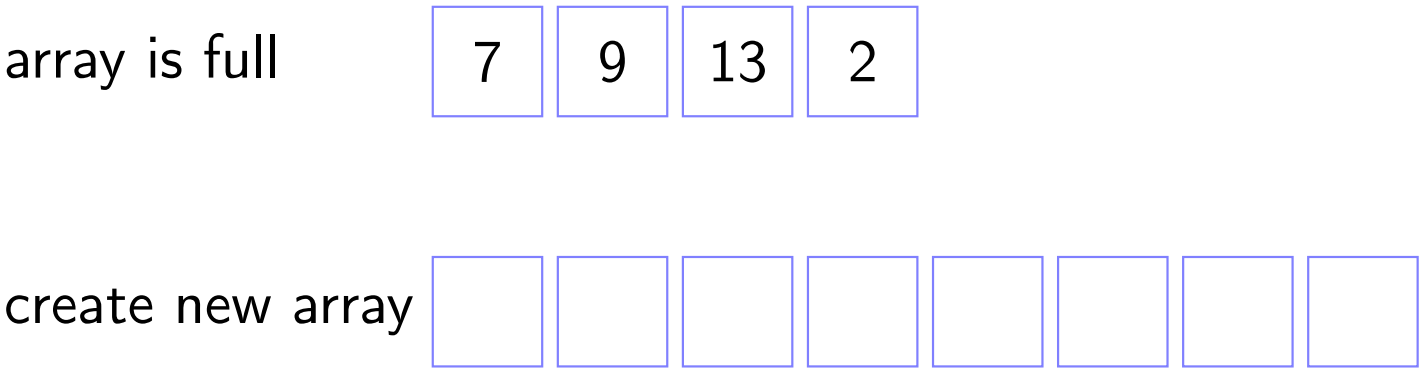
Constant payload

Payload: 0 (creating new array considered free here)



Expandable arrays

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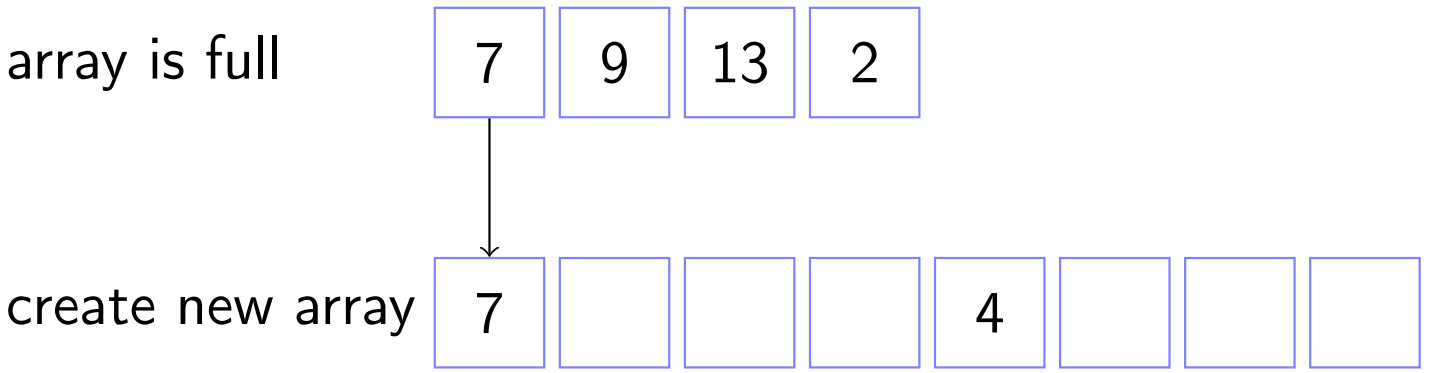
Constant payload

Payload: $1 + 1 = 2$ (1 for copying and 1 for inserting)



Expandable arrays

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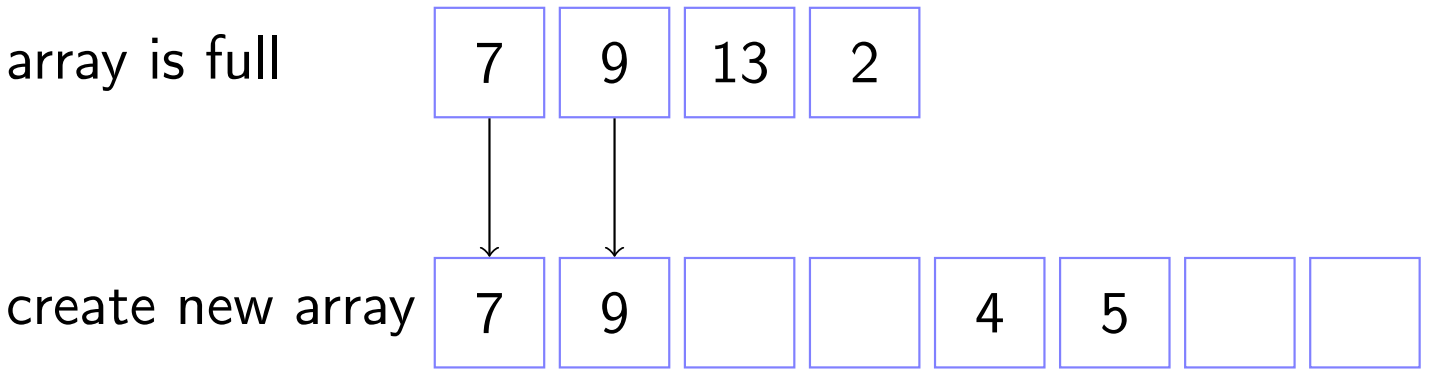


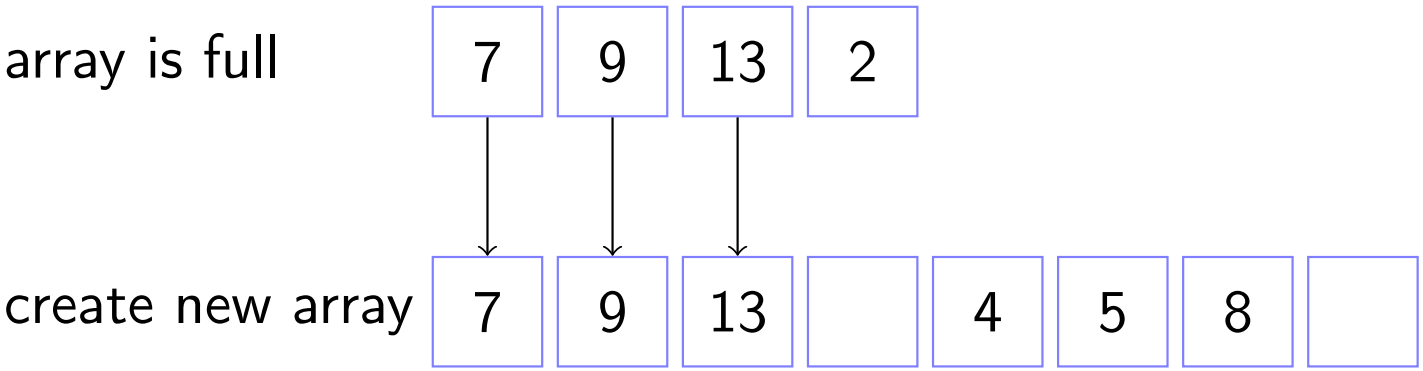
copy 7 insert 4

copy 9 insert 5

Expandable arrays

Constant payload

Payload: $1 + 1 = 2$ (1 for copying and 1 for inserting)



copy 7 insert 4

copy 9 insert 5

copy 13 insert 8

Expandable arrays

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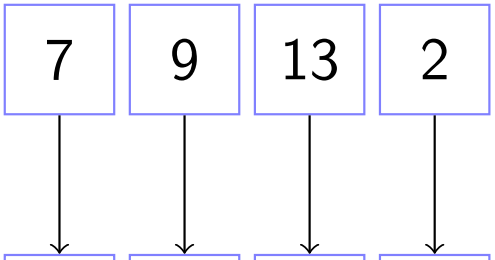
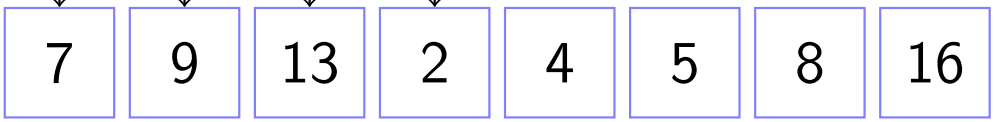
Constant payload

Payload: $1 + 1 = 2$ (1 for copying and 1 for inserting)

array is full



create new array



copy 7 insert 4

copy 9 insert 5

copy 13 insert 8

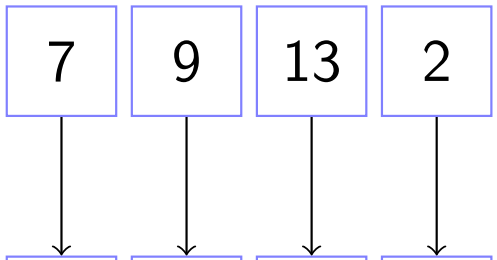
copy 2 insert 16

Expandable arrays

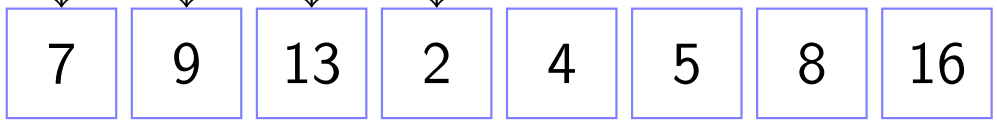
Constant payload

Payload: 8 in total for 4 insertions

array is full



create new array



copy 7 insert 4

copy 9 insert 5

copy 13 insert 8

copy 2 insert 16

$O(2) = O(1)$

Exercise 1 - Complexity

- What would the complexity (big-O) be if we:
  - Triple the array size instead of doubling it?
  - Only made the new array 50% bigger?
- Bearing in mind that most modern memory is paged[1], consider why doubling the array size is not such a bad idea?

[1] typically in 2^n sized pages

Exercise 2 - Implementation

1. Create a Java class `FlexibleArray` that uses the "Constant payload" algorithm.

```
public class FlexibleArray<T> {
   ...
  public T get(int index) { ... }
  public void set(int index, T element) { ... }
  public void add(T element) { ... }
  public int size() { ...}
  }
```

Note that to create a new array of type `T` you must:

```
private T[] arrayOfT = (T[])new Object[1000];
```

2. Measure the time it takes to add 10.000, 100.000, and 1.000.000 elements.
3. Measure Javas build-in `ArrayList` with the same data.

Arrays

Symbol Tables

- List-based
- Array-based
- Hashed

Symbol tables

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`Map` in Java

- Lookup a value based on a key
- No `null` keys
- No `null` values
- No duplicate keys

```
public interface SymbolTable<K,V> {
  void put(K key, V value);
  V get(K key);
  int size();
  Iterable<K> keys();
  default void delete(K key) { put(key, null); }
  default boolean contains(K key) {
    return get(key) != null;
    }
  default boolean isEmpty() { return size() == 0; }
  }
```

Ordered Symbol Tables

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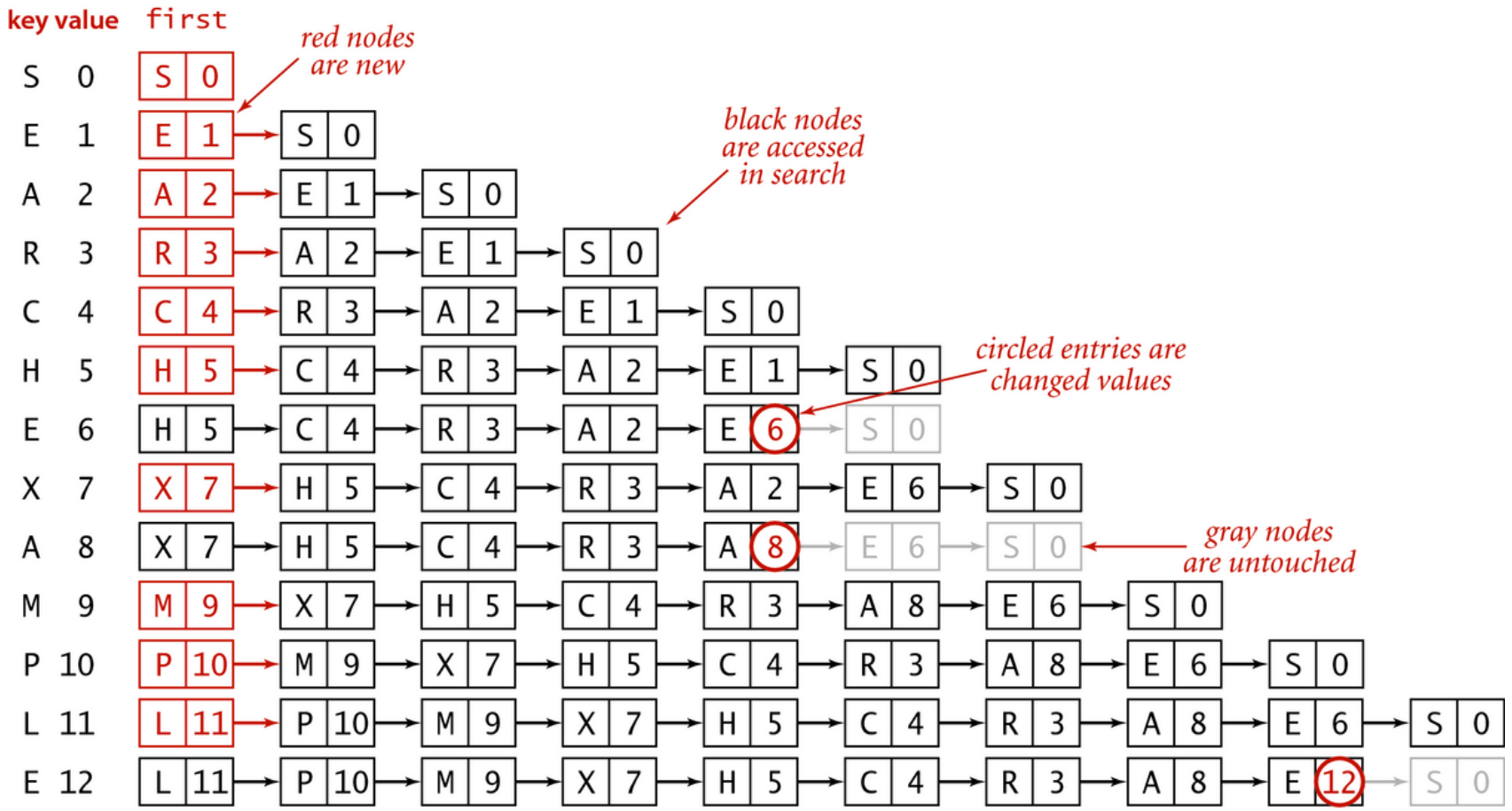
`OrderedMap` in Java

```
public interface
    OrderedSymbolTable<K extends Comparable<K>, V>
    extends SymbolTable<K,V> {
  K min();
  K max();
  K floor(K key);
  K ceiling(K key);
  int rank(K key);
  K select(int rank);
  void deleteMin();
  void deleteMax();
  int size(K low, K high);
  Iterable<K> keys(K low, K high);
  }
```

List-based Symbol Tables



Trace of linked-list ST implementation for standard indexing client

List-based Symbol Tables

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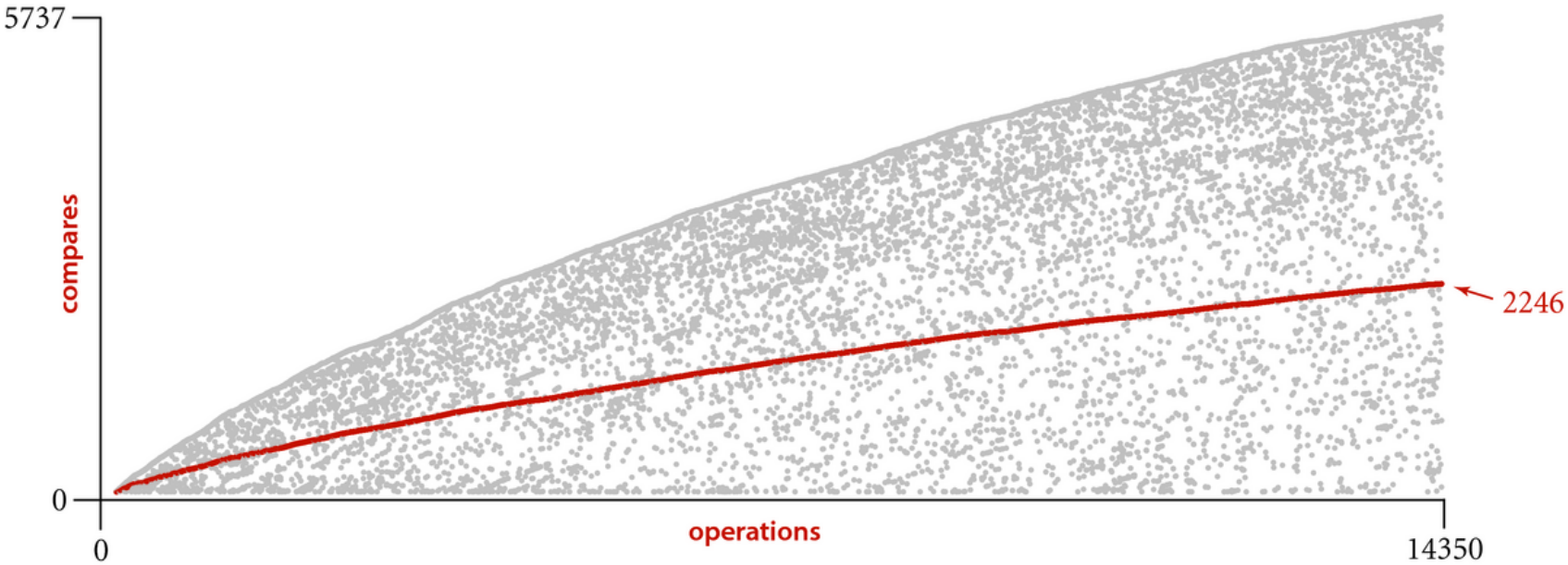
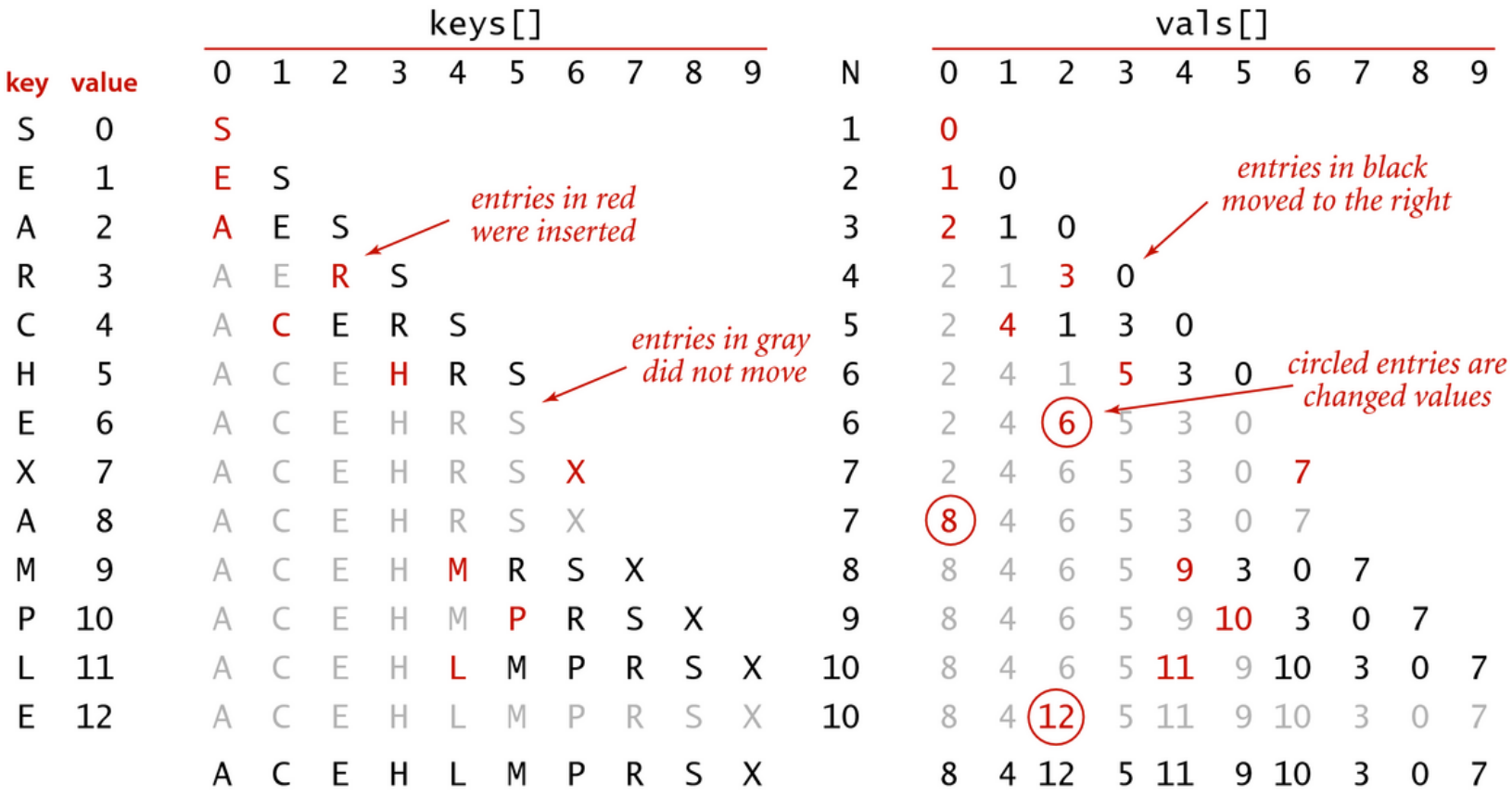
Costs



Costs for `java FrequencyCounter 8 < tale.txt` **using** `SequentialSearchST`

Array-based Symbol Tables



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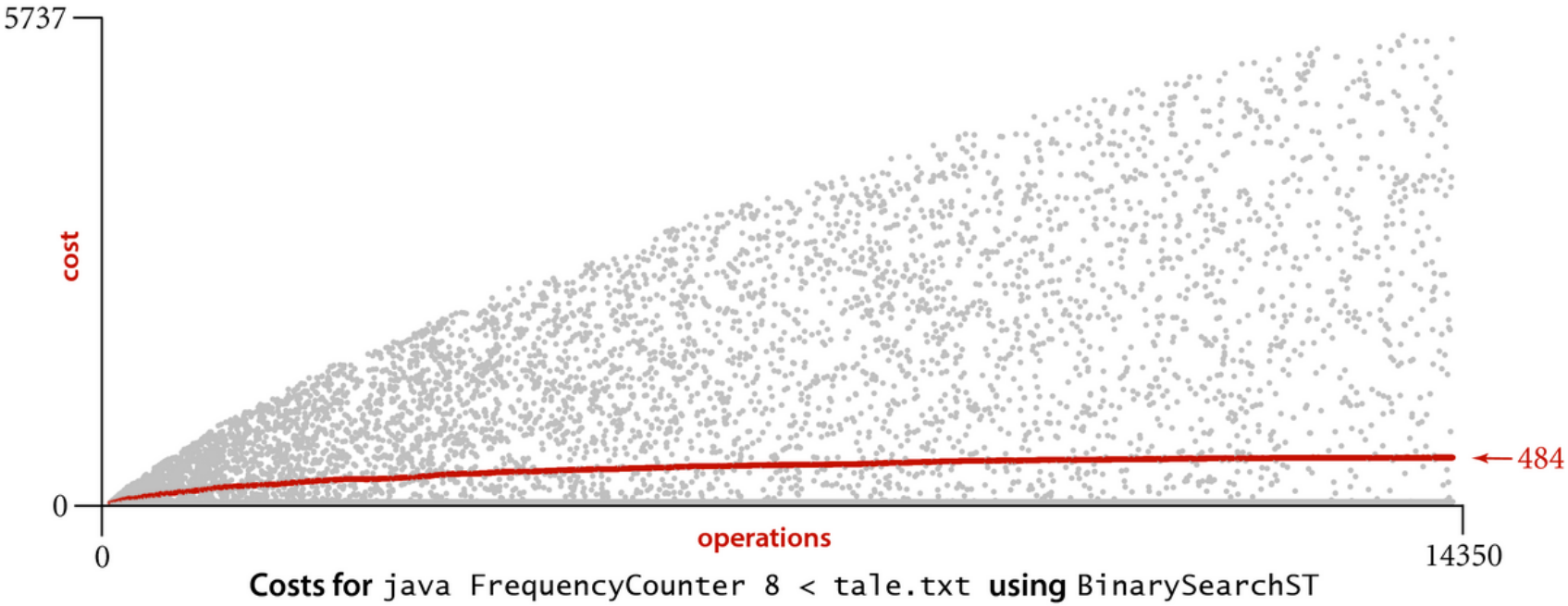
Costs



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Problem

We want to access a symbol table with n keys, using a key $k \in K$ as was it an index to an array with n elements.

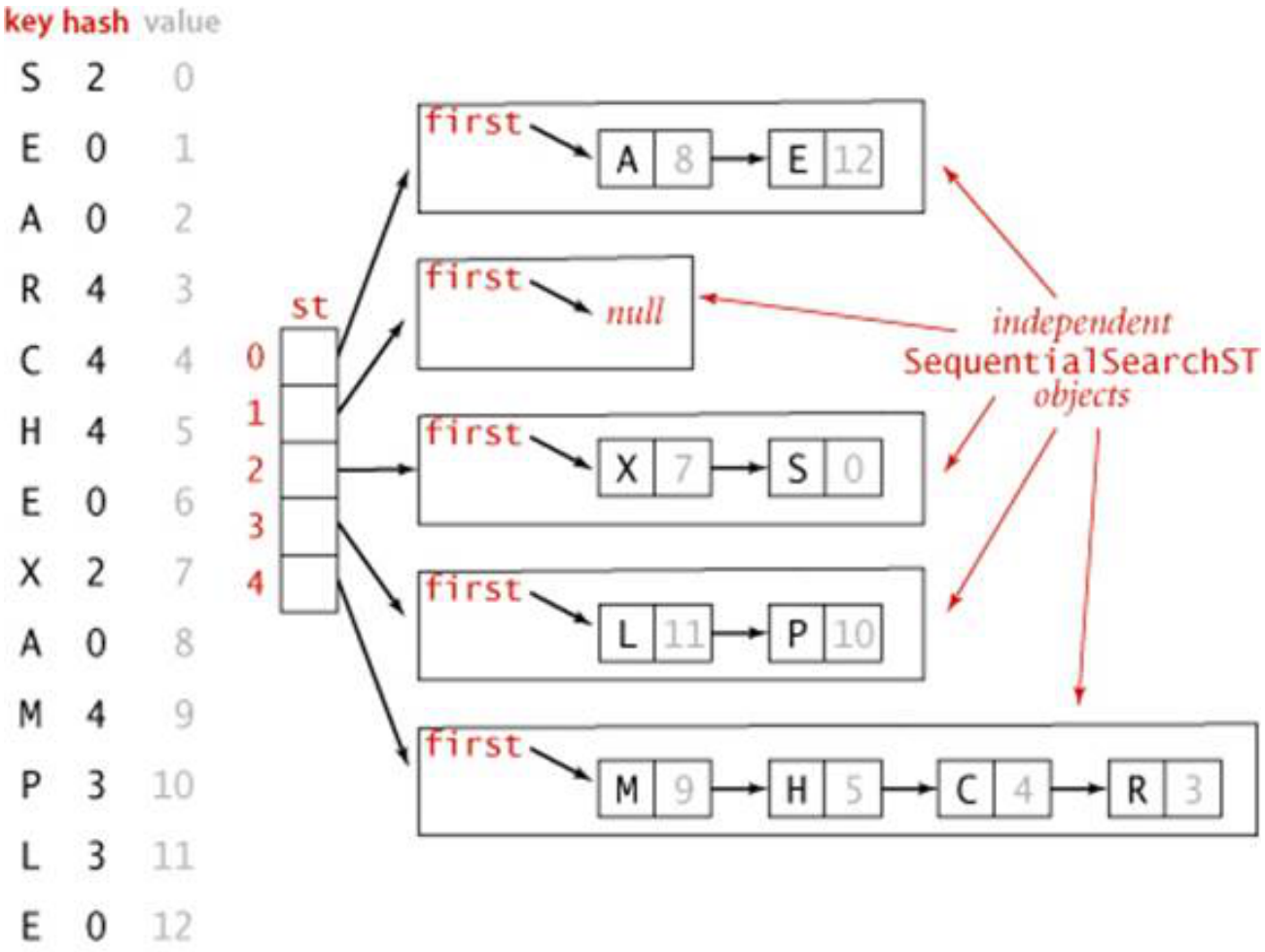
Perfect solution

By **magic** we find a mapping from the keys K to $\{0, \ldots, n-1\}$.
We call it the hash function: $h : K \to \{0, \ldots, n-1\}$.

Realistic Solution

Find a hash function that maps the keys K uniformly over $\{0, \ldots, m-1\}$ where $m > n$.

Chained hashing



Hashing with separate chaining for standard indexing client

Other hashing aspects

- Universal hashing
- Static tables
- Perfect hashing