Generating SU(2) matrix

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Generating SU(2) matrix-Pendelton's method

We use "Heat bath algorithm" to generate $SU(2)$ matrix.Following are the Pendelton's method:

Step A. Generate two uniformly distributed pseudo-random numbers R and R' in the unit interval.

Step B. Set $X \leftarrow -\frac{(lnR)}{\alpha}, X^{'} \leftarrow -\frac{(lnR^{'})}{\alpha}$;

Step C. Set $C \leftarrow \cos^2(2\pi R^{''})$ with $R^{''}$ another random number in [0,1];

Step D. Let $A \leftarrow XC$;

Step E. Let $\delta \leftarrow X^{'} + A$;

Step F. If $R^{'''2} > 1 - \frac{1}{2}\delta$ for $R^{'''}$ pseudo random and uniform in (0,1], go back to step A;

Step G. Set $a_0 \leftarrow 1 - \delta$.

Once we generate the value of a_0 we generate the randum point on the surface of 4 dimensional sphere with the value of a_0 already known.

$$a_0^2 + a_1^2 + a_2^2 + a_3^2 = 1 \tag{1}$$

Such that SU(2) matrix is :

$$SU(2) = \begin{bmatrix} a_0 + ia_3 & a_2 + ia_1 \\ -a_2 + ia_1 & a_0 - ia_3 \end{bmatrix} \tag{2}$$

Now form equation (1),

$$a_1^2 + a_2^2 + a_3^2 = 1 - a_0^2 = a^2 \tag{3}$$

This is equivalent to the 3 dimensional sphere. One genereate the random point in the three dimentional spher by following steps:

Step I - Generate a random no t_1 in between [-1,1] and get random angle $\theta = \cos^{-1}(t)$.

Step II - Generate a random angle ϕ by generating randum no $t_2 \in [0,1]$ and taking $\phi = 2\pi t_2$.

Step III - Define;

$$\begin{aligned} a_1 &= a\cos\theta\sin\phi \\ a_2 &= a\cos\theta\cos\phi \\ a_3 &= a\sin\theta \end{aligned} \tag{4}$$

Then the structure of matrix looks like:

$$SU(2) = \begin{bmatrix} a_0 + ia\sin\theta & a\cos\theta\cos\phi + i\cos\theta\sin\phi \\ -a\cos\theta\cos\phi + i\cos\theta\sin\phi & a_0 - ia\sin\theta \end{bmatrix}$$
$$or, SU(2) = \begin{bmatrix} a_0 + ia\sin\theta & a\cos\theta e^{i\phi} \\ -a\cos\theta e^{-i\phi} & a_0 - ia\sin\theta \end{bmatrix} \tag{5}$$

Distribution of eigen values of SU(2)- matrix

For toally randum SU(2) matrix,the angular parameterη in the eigen value $e^{\pm i\eta}$ follows the distribution function

$$\rho(\eta) = \frac{2}{\pi}\sin^2\eta \tag{6}$$

In Heat bath algorithm, the distribution function shows interesting property. Here we consider the wilson loop W containing N number of plaquettes. For a given value of α, the ratio $\frac{N}{\alpha}$ playes a significant role on analysing the nature of distribution function for eigen value.We can summarize in following points:

1- For a fixed value of $x = \frac{N}{\alpha}$ when we keep on increasing value of N and α , after a certain increment we get the distribution function coinciding for furthur increase.

2- For different value of x we can see the distribution being peaked and flatten.

Matlab - Code

```
%This program generates ths SU(2) matrices using
% pendelton's method for various purposes:
% By Dibakar sigdel          Date: November-5-2013
%  SUB - Lattice gauge theory

tic;
row = 5;
LL = 100;
MM = 100000;
MN = 100;
pi = acos(-1.0);
kk = 1;

nct1 = zeros(MN,1);
nct2 = zeros(MN,1);
nct3 = zeros(MN,1);
nct4 = zeros(MN,1);
nct5 = zeros(MN,1);

for kk = 1:row

if kk == 1;
ratio = 1.0;
beta = ratio*(LL);
end
if kk == 2;
ratio = 5.0;
beta = ratio*(LL);
end
if kk == 3
ratio = 10.0;
beta = ratio*(LL);
end
if kk == 4
ratio = 25.0;
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```
beta = ratio*(LL);
end
if kk == 5
ratio = 50.0;
beta = ratio*(LL);
end

%generating SU(2) matrix and plot of eigen value of W matrix

eta = zeros(MM,1);
m = ones(MM,1);

for k = 1: MM;

    tt = 0;
    PM = eye(2,2);

    while m(k) < LL;
        success = 0;

while success < 1;
r1 = rand;
r2 = rand;
x1 = -log(r1)/beta;
x2 = -log(r2)/beta;
r3 = rand;
C = (cos(2*pi*r3))^2;
A = x1*C;
delta = x2 +A;
r4 = rand;
   if (r4)^2 < (1-(0.5*delta));
   success = success +1;
   tt = tt+1;
   a0 = (1-delta);
   a = sqrt(1-(a0)^2);
   theta = acos(2*rand-1);
   phi = pi*2*(rand);

   a1 = a*sin(theta)*cos(phi);
   a2 = a*sin(theta)*sin(phi);
```

```
   a3 = a*cos(theta);
   end

end

   SU2 = [a0 + 1i*a3 a2 + 1i*a1;-a2 + 1i*a1 a0 - 1i*a3 ];
   PM = PM*SU2;


  m(k) = m(k) +1;
   end



 eta(k) = acos(real(PM(1,1)));

end


%Plot of histogram.......

n = zeros(MN,1);
dx = (1/MN)*pi;

for k = 1:MM;
    for p = 1:MN;
      if eta(k)> dx*p;
         if eta(k) < dx*(p+1);
           n(p) = n(p) +1;
         end
      end
    end
end

nct = zeros(MN,1);

for p = 1:MN ;
nct(p) = (n(p)/(MM*dx));
end
```

```
x = zeros(MN,1);

for p = 1:MN
    x(p) = dx*p;
end

if kk == 1;
for p = 1:MN ;
nct1(p) = nct(p);
end
end
if kk == 2;
for p = 1:MN ;
nct2(p) = nct(p);
end
end
if kk == 3;
for p = 1:MN ;
nct3(p) = nct(p);
end
end
if kk == 4;
for p = 1:MN ;
nct4(p) = nct(p);
end
end
if kk == 5;
for p = 1:MN ;
nct5(p) = nct(p);
end
end

end
toc;

 plot(x,nct1,'r',x,nct2,'b',x,nct3,'g',x,nct4,'k',x,nct5,'m');
```

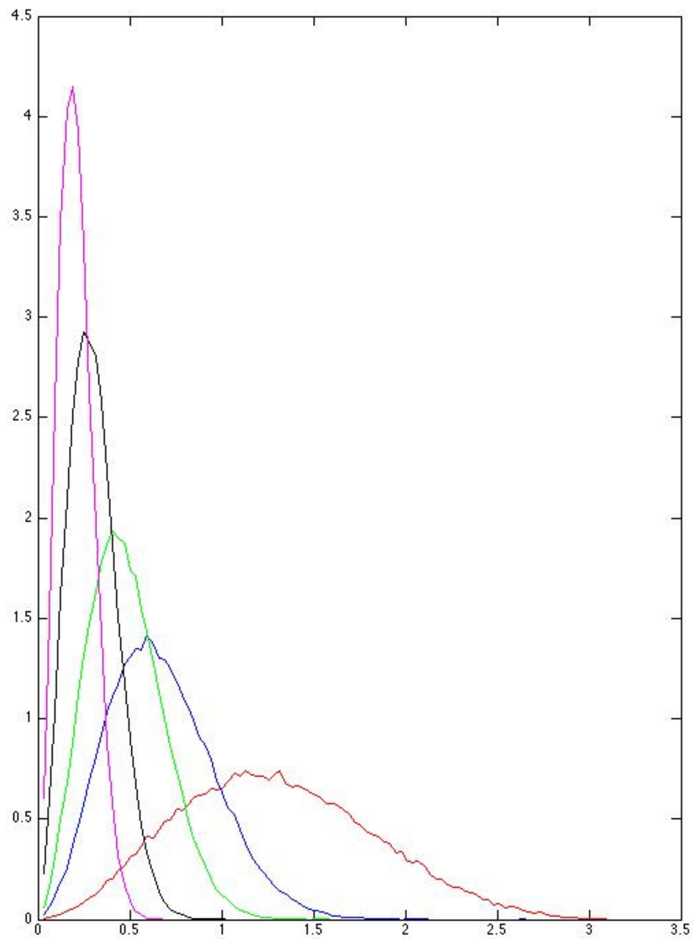

Figure 1: Plot of distribution of η for different value of x