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A GALACTIC DISK IS NOT A TRUE EXPONENTIAL

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ABSTRACT

Stochastic self-propagating star formation predicts that the distribution of atomic hydrogen in a galactic disk should be relatively independent of radius. The molecular hydrogen density should therefore vary as the total gas density minus a constant. The most likely distribution for the total gas density is approximately $1/r$, the same as the distribution of column density in a halo that produces a flat rotation curve. Thus the molecular column density, the star formation rate per unit area, and the total stellar surface brightness should all vary as r^{-1} – constant. This function resembles an exponential over a wide range of galactic radii. We propose that the approximately exponential surface brightness distribution observed in disk galaxies results from the intrinsic r^{-1} – constant distribution produced by propagating star formation. The exponential fit has no intrinsic physical meaning. In fact, the exponential scale lengths derived for all galaxies are approximately the same when scaled to the galaxy size, and they are equal to the relative scale length predicted from the theory. The observed constancy of the extrapolated central surface brightness of disk galaxies is also obtained from the theory.

Subject headings: galaxies: structure — interstellar: matter — stars: formation

I. INTRODUCTION

Disk galaxies are traditionally analyzed in terms of a bulge and a disk. A number of forms have been used to characterize the bulge, the most commonly used form being the de Vaucouleurs law (1948), in which the log surface brightness varies as $r^{1/4}$, where r is the galactic radius. The form generally used for the disk is an exponential (de Vaucouleurs 1959; Freeman 1970)

$$I(r) = I_D e^{-r/r_D} , \tag{1}$$

where I_D and r_D are constants. In the last 20 years over 100 disk galaxies have been analyzed in terms of an exponential disk (Freeman 1970; Ables 1971; Kormendy 1977; van der Kruit 1979; Burstein 1979; Boroson 1981; van der Kruit and Searle 1981; Elmegreen and Elmegreen 1984). The extrapolated central surface brightnesses obtained from these exponential fits are all approximately equal (Freeman 1970), and the exponential scale lengths are all about the same fraction of the 25th magnitude isophotal radius (see § II). This similarity among galactic disks is surprising because variations in the steepness of the rotation curves (e.g., Rubin, Ford, and Thonnard 1980) indicate that the total mass surface densities can be different in the inner regions of galaxies, where the extrapolated disk intensities are the same.

The total mass distribution in a galaxy, as determined from the rotation curve, is not an exponential. Flat rotation curves (Rubin, Ford, and Thonnard 1980) imply that the mass surface density distributions vary approximately as $1/r$. If galactic disks form by the collapse or gradual accretion of galactic halos with specific angular momentum conserved, and if the halos have power-law mass distributions, then the surface density of the disks should have the same power-law distributions as the surface density of the halos. Thus the form for the mass surface density distribution in a disk with a flat rotation curve should be $1/r$, a power law, not an exponential. Even if most of a galaxy's mass is in the disk, a flat rotation curve still implies a $1/r$ surface density distribution in the disk (Nordsieck 1973). For a constant thickness disk, the volume density will also have a $1/r$ distribution.

The light distribution in a disk should be determined by a combination of the mass distribution and the history of star formation and stellar evolution. Unless the fraction of disk gas turned into stars is the same for all radii, the surface brightness distribution will differ from the mass distribution. Thus a galactic disk could have an exponential-like light distribution with a $1/r$ mass distribution. Any explanation for the observed distribution of surface brightness requires both a theory for disk formation and a theory for star formation in the disk.

The theory of stochastic self-propagating star formation (SSPSF) can explain the observed radial distribution of surface brightness in galaxies if the disk mass distribution is similar to the total mass distribution, as obtained from the rotation curve. SSPSF predicts simply that the density of atomic hydrogen should be relatively constant over the optical disk of a galaxy. Thus the molecular hydrogen density should vary as the total gas density minus a constant (other gas phases contribute negligible mass). Since the star formation rate is proportional to the molecular mass (Solomon *et al.* 1981; Young and Scoville 1982), the surface brightness of young stars should

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also vary as the total mass distribution minus a constant. When this function is integrated over time to account for the conversion of gas into stars, the resulting light distribution for the old disk is also $1/r$ minus a constant. Thus the surface brightness in all colors should vary as

$$I(r) = A/r - B\ , \tag{2}$$

where A and B are independent of radius. This function resembles an exponential for a large fraction of the total luminous radius in a galaxy. For a slowly rising circular velocity profile, $v(r) = v_0 r^\alpha$, equation (2) should be replaced by

$$I(r) \approx A'v^2(r)/r - B = Ar^{2\alpha - 1} - B\ . \tag{3}$$

Figure 1 illustrates the predicted distribution of surface brightness from equation (2) for galaxies with different absolute densities (different A) but similar atomic hydrogen densities (normalized to 1). The straight portions of the curves are fitted to exponentials. The exponential fits all have the same extrapolated central value ($e^{2-2\alpha}$, independent of A), and they all have a scale length proportional to $r_{\max}$, the maximum size of the optical disk [i.e., $r_D/r_{\max} = (2 - 2\alpha)^\gamma = 0.25$ for $\alpha = 0$, where $\gamma = -(2 - 2\alpha)/(1 - 2\alpha)$]. These results agree with the observations for galaxies discussed below: the extrapolated central surface brightnesses of galaxies are the same regardless of the true central surface densities, and the scale lengths of the exponential fits are proportional to the optical radii, as determined from the radii of the 25th mag arcsec^{-2} isophote.

The observed distribution of surface brightness in galaxies is reviewed in § II. Section III presents a mean field theory of SSPSF, and the radial profiles obtained from both SSPSF simulations and the mean field theory are illustrated. Section IV shows that the distribution of both molecules and old disk stars should approximate an exponential for all time, even though part of the gas is continuously converted into stars.

II. OBSERVATIONAL DATA ON THE EXPONENTIAL DISK

Simple analytic forms that characterize the apparent correlations between different observable quantities are useful if they

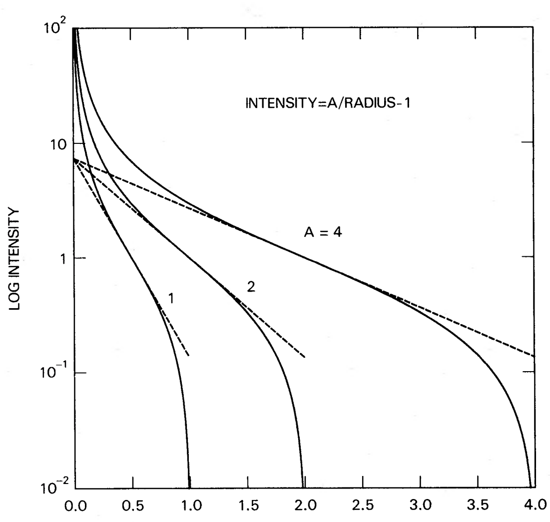

FIG. 1.—A semilog plot of the function $A/r - 1$ for various A, showing the exponential-like falloff at intermediate radii (*dashed lines*), the constant extrapolated central intensity, and the rapid decline in the intensity at large radii.

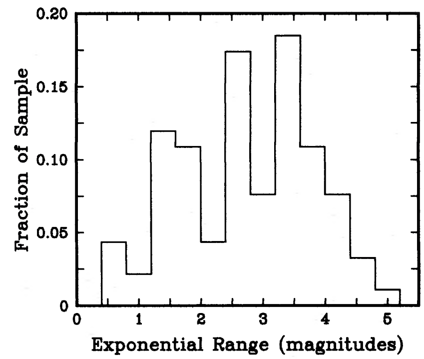

FIG. 2.—Histogram of the range in magnitudes of a good exponential fit of a sample of 114 galaxies from Freeman (1970), Ables (1971), Kormendy (1977), van der Kruit (1979), Burstein (1979), Boroson (1981), van der Kruit and Searle (1981), and Elmegreen and Elmegreen (1984).

point to the physical processes that cause the correlations. One criterion for an empirical fit to be significant is that it must extend over a large range of the function being fitted. Another criterion is that the parameters of the fit cannot depend on each other in some trivial way. From this point of view, the exponential fit to the radial dependence of the surface brightness of a galaxy is not a significant result: the range for most exponential fits is relatively small, and the fitted scale lengths and central surface brightnesses are related by the trivial constraint that the optical edge of a galaxy is the same as the star formation edge.

Figure 2 shows a histogram of the total magnitude range of the exponential fit to the surface brightness of 114 disk galaxies of all Hubble types, from S0's to irregulars. The average range of a good exponential fit is about 2.8 mag (corresponding to 2.5 scale lengths), and 72% of the sample is within $\pm 40\%$ of this value. Thus the range of a typical exponential fit is not large enough to exclude other functional forms. Gunn (1982) suggested, for example, that a flattened uniform ellipsoid fits the light distributions of galactic disks. The exponential is, at best, a convenient representation of a complicated light distribution, and it may be a good approximation for part of that distribution, but it rarely describes the surface brightness distribution over an entire disk. Indeed, many spiral galaxies have light distributions that differ radically from an exponential (Burstein 1979; Boroson 1981; van der Kruit and Searle 1981); some have slower than exponential drop-offs, and others suggest no simple functional forms at all.

The disk scale lengths obtained from an exponential fit also appear to have no intrinsic physical significance. Figure 3 shows a histogram of the normalized exponential scale lengths, r_D/r_{25} for the same sample of 114 galaxies (r_{25} is the galactic radius at a surface brightness of 25 mag arcsec^{-2}, from de Vaucouleurs, de Vaucouleurs, and Corwin 1976). The scale length is even more narrowly defined than the exponential range: 76% of the sample has $r_D/r_{25} = 0.29 \pm 0.1$. Furthermore, as shown in Figure 4, there is no correlation between r_D/r_{25} and galactic type. Freeman's (1970) correlation between the maximum value of r_D and galactic type (r_D decreases from early- to late-type galaxies) arises because the late-type galaxies in Freeman's sample are smaller than the early types. If Freeman's data are rescaled using r_D/r_{25} as in Figure 3, no correlations appear. Thus the exponential scale length in a galaxy does not give information that is independent of the total size of the luminous disk.

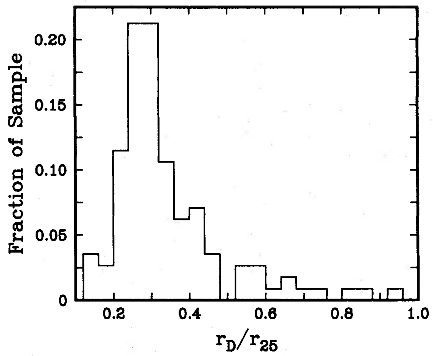

FIG. 3.—Histogram of r_D/r_{25} for the sample of Fig. 1

Freeman (1970) also noted that the extrapolated central surface brightness (at $r = 0$) of the galactic disks of his survey were all around 21.65 mag arcsec^{-2}. This result is related to our observation that r_D/r_{25} is about the same for all galaxies (Fig. 3). In mag arcsec^{-2}, equation (1) can be written as

$$\mu_D = 25 - 1.086 \frac{r_{25}}{r_D} . \tag{4}$$

From the peak in Figure 3, where $r_D/r_{25} = 0.29 \pm 0.05$, we find $\mu_D = 21.3 \pm 0.7$, in agreement with Freeman. Thus the constancy of the central surface brightness follows from the constancy of r_D/r_{25}. Both results follow from equation (2) if the maximum optical radius of a galaxy is where star formation stops, not where the mass distribution ends.

The maximum radius for star formation in a galaxy ($= A/B$ from eq. [2]) is the radius where the gas density becomes so low that the propagation of star formation is impossible or unlikely. Beyond this radius, the surface brightness of a galactic disk should decrease rapidly, as shown in Figure 1. This drop-off is usually not seen in face-on galaxies because it occurs where the disk is already very faint. It has been observed for seven edge-on galaxies by van der Kruit and Searle (1981, 1982), however. In all cases the rapid drop-off occurs at some r_{max}, which is related to r_D by the ratio $r_D/r_{max} = 0.24 \pm 0.04$. This ratio agrees with that obtained from equation (2).

In the outer, nonluminous part of a galaxy, the surface density of atomic hydrogen should vary in direct proportion to the total mass surface density obtained from the rotation curve.

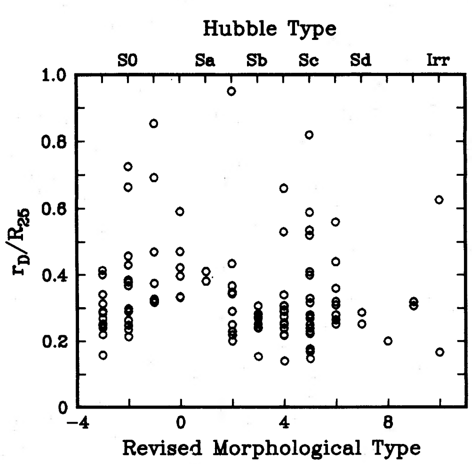

FIG. 4.—Scatter plot of r_D/r_{25} vs. Hubble type for the sample of Fig. 1

This follows from our assumption that the mass distribution in the disk is everywhere proportional to the total mass distribution, and that in the outer part, the disk should be mostly atomic gas. This predicted H I distribution has been observed by Bosma (1981). Figure 7 in Bosma (1981) shows that the ratio of the surface density of the total galactic mass (obtained from the rotation curve) to the surface density of atomic hydrogen is constant at large radii. The total H I density beyond the star formation region is not exponential—it varies as $1/r$.

III. STOCHASTIC SELF-PROPAGATING STAR FORMATION (SSPSF)

a) Feedback Control Resulting from an Interchange between Atomic and Molecular Gas Phases

The SSPSF theory has been extended recently to include explicitly two phases of gas in the interstellar medium (Seiden 1983). The main result for present purposes is that the interaction between interstellar gas and the star-forming processes results in a flat distribution of atomic hydrogen, where the atomic density is relatively independent of radius over the part of the galaxy where star formation occurs. This result is independent of the original distribution of total gas. The atomic hydrogen does not fall below this critical value until far out in the galaxy, where the total gas density is approximately equal to the constant atomic value. Beyond that point, of course, the atomic gas density must fall.

The reason for this result is that SSPSF controls the division of interstellar gas into two states, "active" and "inactive." Active gas is defined as the component through which star formation can propagate. We view star formation as a two-step process, where molecular clouds are formed first, and then stars are formed in the molecular clouds. Since most of the observed giant molecular clouds contain young stars, the second step must be relatively fast. The rate-determining process is the creation of the molecular clouds. For this reason, the atomic hydrogen may be viewed as the active gas: this is the phase of interstellar gas that can be stimulated by current star formation sites into forming new molecular clouds and new stars.

The inactive gas is the component of interstellar matter whose star formation rate cannot be changed. The most abundant form for inactive gas is in molecular clouds. Star formation proceeds in a molecular cloud at a rate determined by internal processes, which will not be affected substantially by the presence of star formation in neighboring, detached clouds. Molecular clouds simply form stars until they erode, when the remaining molecular gas converts back into atomic gas.

This theory of two phases implies that the star formation rate will be very sensitive to the density of atomic hydrogen; this is the gas that fuels the propagation process. The interchange between phases establishes a feedback mechanism that keeps the density of atomic hydrogen at whatever value is necessary to sustain a steady state cloud (star) formation rate. If the atomic density is too low, then the cloud formation rate will be too low also, and molecular clouds will erode faster than they form. This will result in an increase in the atomic hydrogen density. If the atomic density is temporarily too high, SSPSF will form molecular clouds at a faster than average rate, and this will decrease the atomic density back to its preferred value. Thus the atomic hydrogen density should be pinned to a constant value for all galactic radii out to the position where the total gas density falls below this constant. Observations of H I emission in the disks of spiral galaxies are consistent with this behavior (for a more complete discussion see Seiden 1983).

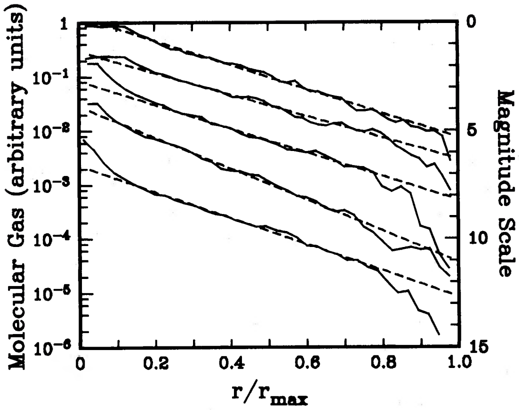

FIG. 5.—Results of SSPSF simulations for the radial dependence of the molecular hydrogen.

b) SSPSF Simulations

SSPSF simulations can be used to illustrate the evolution of stars and gas in a galaxy. Figure 5 shows the results of five such simulations. The parameters are the same as in Seiden (1983). The initial total gas distribution was taken to be proportional to $1/r$ for all runs. The figure illustrates the exponential radial dependence of the molecular component and the star formation rate; exponential ranges up to 5.5 mag are shown. These exponential regions are more extensive than those in Figure 1 because the simulation uses a realistic rotation curve with $v(r)$ decreasing in the inner region. The simulation also follows the conversion of gas into stars, so it accounts for galactic evolution. The resulting surface brightness distributions are in excellent agreement with the observations of galaxies.

c) An Approximate Mean Field Theory for SSPSF

In the SSPSF simulations, stochastic processes are used to model cloud formation and cloud-to-intercloud conversions in a galaxy. We now describe an analytical method that approximates these stochastic processes (see Seiden and Gerola 1982).

Consider the coupled evolution of stars and gas in a galactic system approximated by discrete spatial cells. Each cell α has three variables: $\sigma_\alpha(t)$ represents the presence ($\sigma = 1$) or absence ($\sigma = 0$) of a star cluster, $D_\alpha(t)$ represents the density of atomic hydrogen, and $\tilde{D}_\alpha(t)$ represents the density of molecular hydrogen. The evolution equations for star clusters and gas are, respectively (Schulman and Seiden 1982),

$$\sigma_\alpha(t+1) = 1 - \prod_\beta [1 - A_{\alpha\beta t}\, \sigma_\beta(t)] \tag{5}$$

and

$$D_\alpha(t) = D_\alpha(t-1)[1 - \sigma_\alpha(t)] + \frac{\tilde{D}_\alpha(t-1)}{\tau} . \tag{6}$$

The product index β is for cells that are neighbors of cell α; τ is the decay time of molecular hydrogen into atomic hydrogen (assumed to be constant for all cells); and $A_{\alpha\beta t}$ is a zero-one valued random variable whose average value is the effective probability for star formation to propagate from cell β to cell α. We assume, as in Seiden (1983), that this effective stimulated probability is proportional to the atomic gas density; then

$$P_{\mathrm{eff},\alpha} \equiv \langle A_{\alpha\beta t} \rangle = \min\left[1, \frac{D_\alpha(t)}{D_C}\right], \tag{7}$$

where D_C is a critical gas density. The density of molecular gas is given by

$$\tilde{D}_\alpha(t) = \bar{D}_\alpha - D_\alpha(t) , \tag{8}$$

with $\bar{D}_\alpha$ equal to the total gas density, which, for this discussion, is considered to be constant in time for each cell (but different for cells at different radii). The fraction of gas in the atomic state is

$$d_\alpha(t) = D_\alpha(t)/\bar{D}_\alpha , \tag{9}$$

so from equation (6) we have

$$d_\alpha(t) = d_\alpha(t-1)[1 - \sigma_\alpha(t)] + \frac{1}{\tau}[1 - d_\alpha(t-1)] . \tag{10}$$

Now suppose cell α forms a cluster at $t = 0$ and then forms another cluster again at some later time $t = S$; in general, S will be a random variable. For this case,

$$\sigma_\alpha(0) = \sigma_\alpha(S) = 1 , \quad \sigma_\alpha(t) = 0 , \quad \text{for} \quad 0 < t < S . \tag{11}$$

From equation (10) it follows that

$$d_\alpha(0) = [1 - d_\alpha(-1)]/\tau , \tag{12}$$

$$d_\alpha(1) = d_\alpha(0) + [1 - d_\alpha(0)]/\tau , \tag{13}$$

and so on, and, in general,

$$d_\alpha(t) = 1 - \gamma^t[1 - d_\alpha(0)] , \quad 0 < t < S , \tag{14}$$

where $\gamma = 1 - 1/\tau$.

Since, from equation (6), $D_\alpha(t)/D_C$ varies with time, p_{eff} for cell α will also vary with time. Thus we define $p(t)$ to be the value of p_{eff} during the time between the first ($t = 0$) and second ($t = S$) cluster formations. By equations (7) and (14),

$$p_\alpha(t) = \min\left[\!\left[1, \frac{\bar{D}_\alpha}{D_C}\{1 - \gamma^t[1 - d_\alpha(0)]\}\right]\!\right] . \tag{15}$$

The probability $P_\alpha(S)$ of cluster formation in cell α at time step $S = 1$ is, therefore, by equation (5),

$$P_\alpha(S=1) = \langle \sigma_\alpha(1) \rangle = 1 - \left\langle \prod_\beta [1 - p_\alpha(0)\sigma_\beta(0)] \right\rangle , \tag{16}$$

with β running over the neighbors of α, and the average, $\langle\rangle$, representing an average over the random variable $A_{\alpha\beta t}$.

For the spatial dependence we make a mean field approximation, replacing $\sigma_\beta(t)$ by a mean density of clusters $\rho(t)$, which is not necessarily an integer. We also assume that the neighbors β have their time dependence uncorrelated with the time dependence of cell α so that $\rho(t)$ is in turn replaced by the constant ρ, which is the equilibrium value of the mean fraction of cells that form stars per time step (or the mean star formation rate). Therefore,

$$P_\alpha(S=1) = 1 - [1 - p_\alpha(0)\rho]^6 . \tag{17}$$

The power 6 enters because in a two-dimensional grid, each cell usually has six (partially contacting) neighbor cells, each of which has probability $p_\alpha(0)\rho$ of stimulating cell α.

Now we introduce the following notation (α subscripts deleted):

$$B_t = 1 - p(t)\rho , \quad A_{-1} = 1 , \quad A_t = [B_t \cdots B_0]^6 , \quad t \geq 0 , \tag{18}$$

and

$$a_1 \equiv P(S=1) = 1 - A_0 , \tag{19}$$

and, in general,

$$a_k \equiv P(S = k) = A_{k-2} - A_{k-1} \,. \tag{20}$$

The value of ρ will now be determined self-consistently; it is the time average of $\sigma_\alpha(t)$, with the ρ-dependent probability for S given by equations (18)–(20). Thus

$$\rho = \lim_{T\to\infty} \left\langle \frac{1}{T} \sum_{t=1}^{T} \sigma_\alpha(t) \right\rangle , \tag{21}$$

for some long time period T, during which many clusters form in cell α. To evaluate ρ we suppose $\sigma_\alpha(t) = 1$ on exactly n separate occasions during this period T. On each of these occasions, a cluster will have been created after some period which we call k. Thus there is a sequence of numbers $k_1, \ldots, k_n$ that are intervals between the creation of clusters. The probability that some k_j takes a particular numerical value k is the quantity calculated in equation (20). Therefore, over a long period, the number of times a number k appears in the sequence $k_1, \ldots, k_n$ is na_k. Since by their definition the numbers $k_1, \ldots, k_n$ sum to T, that sum can be regrouped to give

$$\sum_{k\geq 1} k(na_k) = T \,. \tag{22}$$

But n/T has the limit ρ for large T and n, so equation (22) becomes

$$\sum_{k=1}^{\infty} ka_k = 1/\rho \,. \tag{23}$$

Equation (23) is the self-consistency condition, and, in effect, is the equation of state for the star formation rate density, ρ. Using equation (20), equation (23) can be simplified to

$$\frac{1}{\rho} = \sum_{k=1}^{\infty} A_{k-2} = 1 + \sum_{k=0}^{\infty} \prod_{j=0}^{k} [1 - p(j)\rho]^6$$
$$= 1 + [1 - p(0)\rho]^6 + [1 - p(0)\rho]^6[1 - p(1)\rho]^6 + \cdots \,. \tag{24}$$

The foregoing development provides a dependence of ρ on D_C, $\bar{D}$, and τ which is a mean field theory insofar as spatial relations are concerned, but takes into account detailed time dependence. The quantity $d_\alpha(0)$ also enters equation (24), via its appearance in $p_\alpha(t)$ from equation (15). This quantity is the fraction of the total gas in cell α that has been converted from old molecular clouds into an atomic intercloud medium in the same time step when a new molecular cloud was induced to form. This quantity is a random variable, dependent on the past history of star formation, and so it presents a significant complication to the solution of equation (24). Fortunately it is numerically small ($\langle 1/\tau$, cf. eq. [12]), and, for most applications, it can be ignored in comparison to 1. Thus ρ may be determined as a function of $\bar{D}/D_C$ and τ alone.

Using the same methods, a simple expression can be obtained for the time-averaged density of the two components of the gas. It can be shown that

$$\langle \tilde{D} \rangle = \bar{D}[1 - d(0)]\rho \sum_{k=0}^{\infty} A_{k-1} \gamma^k \tag{25}$$

and

$$\langle D \rangle = \bar{D} - \langle \tilde{D} \rangle \,. \tag{26}$$

Figure 6 shows the molecular distribution from equation (25) for five different initial gas distributions, $\bar{D}(r)$, of the form $r^{-\nu}$. The lines have been drawn so that their thickness indicates a 10% variation (corresponding to 0.1 mag, very good for galactic photometry). The dashed lines are exponential fits to the curves. The exponential fits cover a range of 2.9 mag for $\nu = 0.1$ to 4.6 mag for $\nu = 3$. Acceptable exponential-like regions are obtained with $0.2 \leq \nu \leq 2$. Outside this region the scale length is outside the range of the observations. The exponential-like variation of $\tilde{D}$ with r again results from the shape of the fraction $r^{-\nu}$ − constant, since equations (24) and (25) produce a nearly constant value of $\langle D \rangle$.

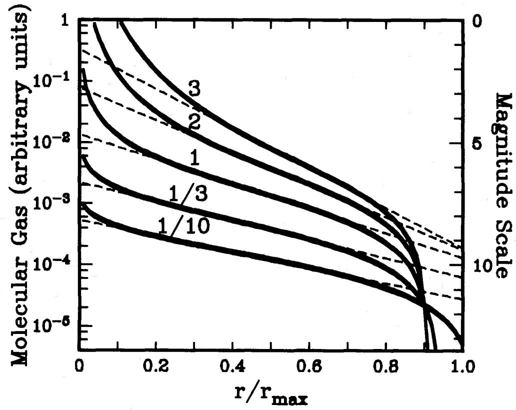

FIG. 6.—Radial dependence of molecular hydrogen from the mean field theory of SSPSF for initial gas distributions of the form $r^{-\nu}$. The value of ν for each curve is given in the figure. The thickness of the curves corresponds to a 10% (0.1 mag) variation in the ordinate.

IV. TIME-DEPENDENT EVOLUTION OF THE DISK SURFACE BRIGHTNESS

Star formation changes the relative proportions of gas mass and star mass in a galaxy, so it changes the distribution of the gas if the star formation rate per unit gas is nonuniform. The star formation mechanism described here, however, preserves its initial $A/r - B$ radial dependence for all time. Thus, the molecular distribution, the star formation rate, and the total surface brightness all approximate an exponential throughout the life of the galaxy. This important result follows from our assumption that the star formation rate is proportional to the total gas density minus a constant. Since this star formation rate is also proportional to the rate of change of the total gas density, we have

$$d\bar{D}(r)/dt = -C[\bar{D}(r) - B] \tag{27}$$

for star formation rate C per unit molecular gas mass. The solution to this equation is

$$\bar{D}(r, t) = B + [\bar{D}(r, t = 0) - B]e^{-Ct} , \tag{28}$$

so the molecular hydrogen distribution at time t is

$$\bar{D}(r, t) - B = [\bar{D}(r, t = 0) - B]e^{-Ct} \,. \tag{29}$$

The star formation rate at time t is this quantity multiplied by C, so the total mass surface density of stars, $D_{\rm stars}(r, t)$, is the integral of the star formation rate over time, or

$$D_{\rm stars}(r, t) = C[\bar{D}(r, t = 0) - B](1 - e^{-Ct}) \,. \tag{30}$$

The surface brightness of new stars is proportional to the star formation rate, and the surface brightness of old stars is proportional to the integrated star formation rate. Both of these brightness distributions have a radial dependence proportional to $\bar{D}(r, t = 0) - B$, which is the $A/r - B$ form we have been discussing (i.e., where the initial disk surface density distribution is proportional to the total mass surface density distribution). Thus the conversion of gas into stars preserves the approximately exponential distribution of the initial star formation rate for all time.

The same would be true if the galactic disk accreted matter with a rate whose radial dependence is proportional to $\bar{D}(r, t = 0)$. Suppose the accretion rate is $\alpha\bar{D}(r, t = 0)$, so

$$d\bar{D}(r)/dt = -C[\bar{D}(r) - B] + \alpha\bar{D}(r, t = 0) . \quad (31)$$

Then the molecular gas distribution and star formation rates are proportional to

$$\bar{D}(r, t) - B = \bar{D}(r, t = 0)\left[\frac{\alpha}{C} + e^{-Ct}\left(1 - \frac{\alpha}{C}\right)\right] - Be^{-Ct} \quad (32)$$

and

$$D_{\text{stars}}(r, t)$$
$$= \bar{D}(r, t = 0)\left[\alpha t + (1 - e^{-Ct})\left(1 - \frac{\alpha}{C}\right)\right] - B(1 - e^{-Ct}) . \quad (33)$$

In these cases, equation (2) still applies, so the star formation rate and disk brightness always approximate an exponential, but the effective values of A and B are time dependent. The radial extent of the luminous disk (and r_D) would therefore increase with time.

V. CONCLUSIONS

When self-propagating star formation regulates the conversion of gas into stars, the atomic hydrogen density in a galaxy should be pinned at a constant value, independent radius. This implies that the radial dependences of the molecular column density, the star formation rate, and the surface brightness of old disk stars should all be proportional to the total surface density distribution minus a constant. For a flat or slowly rising rotation curve, this radial dependence is given by equations (2) or (3). Since these functions resemble an exponential for a large range of radii, the observed exponential distributions of galactic disks are the result of propagating star formation.

At large radii the drop-off in the surface brightness and star formation rate should be much faster than exponential, in agreement with the observations of van der Kruit and Searle (1981, 1982). However, the surface density of atomic hydrogen should continue beyond the optical disk, varying in direct proportion to the total mass surface density obtained from the rotation curve. This behavior also agrees with observations. At small radii, the surface brightness of a galactic disk should increase above the extrapolated exponential. This rise may be masked by a bulge, however, and it will be diminished if the rotation curve is slowly rising in the inner part. We predict that galaxies with slowly rising rotation curves in the inner parts should have exponential-like brightness variations that extend to very small radii (as in M33, for example).

Galactic disk properties that vary with radius as $A/r - B$ also account for both the observed constant value of the ratio r_D/r_{25} and the observed constant value of the central surface brightness. These properties are observed even though the true extents of the galactic mass distributions, and the true central mass surface densities, vary from galaxy to galaxy.

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