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PROPAGATING STAR FORMATION AND IRREGULAR STRUCTURE IN SPIRAL GALAXIES

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ABSTRACT

A simple model is proposed which describes the irregular optical appearance often seen in late-type spiral galaxies. If high-mass stars produce spherical shock waves which induce star formation, new high-mass stars will be born which, in turn, produce new shock waves. When this process operates in a differentially rotating disk, our numerical model shows that large-scale spiral-shaped regions of star formation are built up. The structure is seen to be most sensitive to a parameter which governs how often a region of the interstellar medium can undergo star formation. For a proper choice of this parameter, large-scale features disappear before differential rotation winds them up. New spiral features continuously form, so some spiral structure is seen indefinitely. The structure is not the classical two-armed symmetric spiral pattern which the density-wave theory attempts to explain, but it is asymmetric and disorderly.

The mechanism of propagating star formation used in our model is consistent with observations which connect young OB associations with expanding shells of gas. We discuss the possible interaction of this mechanism with density waves.

Subject headings: galaxies: structure — stars: formation

I. INTRODUCTION

The phenomenon of spiral structure in galaxies is intimately connected with star formation. We now know that the bright patches in galactic disks are regions of recently formed, very luminous, high-mass stars. These stars have lifetimes much shorter than typical galactic rotation periods, so they are capable of creating detailed structure despite the effects of differential rotation. What is needed is a mechanism to organize the star formation into a large-scale spiral pattern. The spiral wave picture has thus developed. Lin and his collaborators have suggested that there exists an underlying spiral density wave in galaxies (see Wielen 1974 and references therein). Interstellar clouds falling into the spiral gravitational potential are compressed to the point where they are gravitationally unstable and stars form. This picture seems attractive for symmetric, two-armed spirals.

To obtain a more complete understanding we examine the problem from a different viewpoint. In this paper a model is presented for galactic spiral structure in which star formation is treated as a pseudo-wave phenomenon. It is wavelike in the sense that it is self-propagating. The motivation for this approach is observational evidence reported by several authors including Sancisi (1973) and Berkhuijsen (1974), suggesting that star formation occurs in spherical shells of gas or parts of spherical shells of gas presumably created by supernovae, although other mechanisms, such as H II regions or interstellar bubbles (Castor, McCray, and Weaver 1975), may be responsible. It is assumed that high-mass stars ($\geqslant 10\ M_\odot$) are the progenitors of expanding spherical shells of gas which form stars. We will not concern ourselves with the actual star-formation process, but only with its implications. If high-mass stars are created in these spherical shells, new shells will be formed and the process thus propagates spherically outward. The buildup of large wave fronts is analogous to Huygens's principle for the propagation of light waves. This process will be referred to as self-propagating star formation (for conciseness SPSF). To control the propagation of star formation, what will be called the "regeneration time" is introduced. This is the minimum time between successive star formations for a given region corotating with the galaxy. Thus, after a region undergoes star formation, it must wait one regeneration time before it is allowed to be "triggered" by nearby star formation.

When this process occurs in a differentially rotating disk, regions of star formation are naturally drawn out into spiral features. The wind-up dilemma is avoided because individual features last on the order of one regeneration time. Thus spiral features are continually replaced by new ones as they wind up and disappear. No underlying spiral pattern is assumed in this model, so we expect the spiral structure to be asymmetric and broken. This kind of structure is seen in many galaxies including M101 and many of the objects listed in Arp's (1966) catalog of peculiar galaxies, where outside influences or internal violence are not obvious. Many Sd and Sc galaxies exhibit a patchy appearance and locally asymmetric structure, although their overall appearance is that of the traditional two-arm symmetric spiral. Even within simple versions of the Lin and Shu density-wave model one needs a mechanism to create and maintain the density waves. An SPSF model of spiral structure may be useful in understanding some of these problems. It introduces a "random" spiral structure which may well complement the "ordered" structure induced by density waves.

This model is primarily concerned with the optical structure of spiral galaxies produced by recent star formation. High-resolution radio data have shown that the gas density also presents a spiral structure coinciding with the optical structure (e.g., Allen, Goss, and van Woerden 1973). We have not modeled the SPSF process to specifically say anything about the large-scale structure of the gas density. We expect the small-scale structure to be composed of expanding shells in regions of star formation. The radio resolution is not sufficient to see this structure in external galaxies. One might speculate that a superposition of small shells could produce a large density enhancement. The SPSF process produces enough ordering so that a large "snowplow" might be formed.

Section II describes the numerical model which was used, and § III discusses the results for this model. In § IV the interaction between the SPSF and density-wave models is discussed and some results are presented. Section V discusses the star-formation process in more detail.

II. NUMERICAL MODEL

We use a very simple model of a disk galaxy to examine the SPSF process. The thickness of the disk is ignored, and only the circular motions around the galactic center are considered. We are not attempting to create a complete galactic model. We wish only to see what types of structures can be produced by SPSF and what parameters are relevant.

The disk is divided into 30 concentric rings and each ring is divided further into approximately square boxes. About 2500 boxes are used. Each ring is rotated according to a chosen rotation curve. A variety of rotation curves were tried; however, the exact form of the curve turned out not to be of critical importance. In practice a rotation curve based on Roberts and Rots's (1974) result for M101 was used most often.

The boxes are labeled as to whether star formation is occurring there, and if not, how long ago star formation did occur. In this way a particular box may be restricted from having star formation until one regeneration time after its last occurrence. At this time, star formation will occur in that box when it propagates from a neighboring box. By "propagate," we mean star formation in one box induces further star formation in all neighboring boxes. Then, from that box, star formation is propagated to all neighboring boxes in which star formation is allowed to occur.

The model operates in discrete time steps. One step consists of rotating the labels in the boxes of each ring according to the rotation curve. Boxes undergoing star formation are then found and relabeled as not undergoing star formation and the four nearest neighboring boxes are examined to determine if one regeneration time has elapsed since star formation last occurred there. If so, they are labeled as undergoing star formation. If not, they are relabeled to indicate that the time since their last period of star formation has increased one time step.

A "picture" is then outputted. A sequence of output is shown in Figure 1. Regions undergoing star formation are printed as crosses (×) in the figures. It is found that more continuous spiral patterns are shown if those regions which underwent star formation one or two time steps ago are also printed out. These boxes correspond to less luminous regions, since the most luminous high-mass stars will have died out within the one time step for typical models. It is found that the spiral structure of an individual picture can often be enhanced by including "older" or "younger" boxes in the printed picture. This seems analogous to varying the exposure time of a photograph since the "older" regions contain only less luminous, low-mass stars. As a matter of convenience and consistency, pictures were printed out using boxes which underwent star formation one and two time steps ago. These regions are designated as asterisks and dots, respectively, in the figures. The galaxy rotates counterclockwise in all the figures. This scheme seems to give the most consistent spiral structures, although we have noticed that some pictures could often be "improved" by including "older" regions or deleting some regions. The problem is that the resolution and contrast available on a line printer do not well simulate those of a photograph.

Originally, no special allowances were made for the nuclear region. Star formation was permitted to propagate through the center of the galaxy. However, the physical conditions in galactic nuclei are so uncertain that we also tried models where SPSF did not operate in the nuclear region. No significant change in the results was seen. In those cases shown in the figures the nucleus is designated by asterisks and star formation is not allowed to propagate in this region.

Of course this very simple model does not include all the features seen in disk galaxies; however, it does not appear that this will significantly affect our conclusions. Noncircular motions and thickness effects can be neglected for our purposes. The discrete nature of this model in time and space introduces some distortion into the model. Because this distortion builds up on a time scale much longer than the typical lifetime of a large-scale feature, we can ignore distortion effects.

The code distorts the position of features on the order of one box per galactic revolution. Large features retain their individuality for the order of one galactic revolution, so we expect them to be distorted by about one box, which is of the order of a few percent of their length. This is also the approximate size of distortions one could expect from noncircular motions, thickness effects, etc.; so, these distortions are not seen as a serious handicap.

We have also considered the effect of "spontaneous" star formation. That is, in some runs star formation was allowed to occur also in random boxes without being induced by neighboring boxes. This sometimes helped to stabilize the star formation rate when the

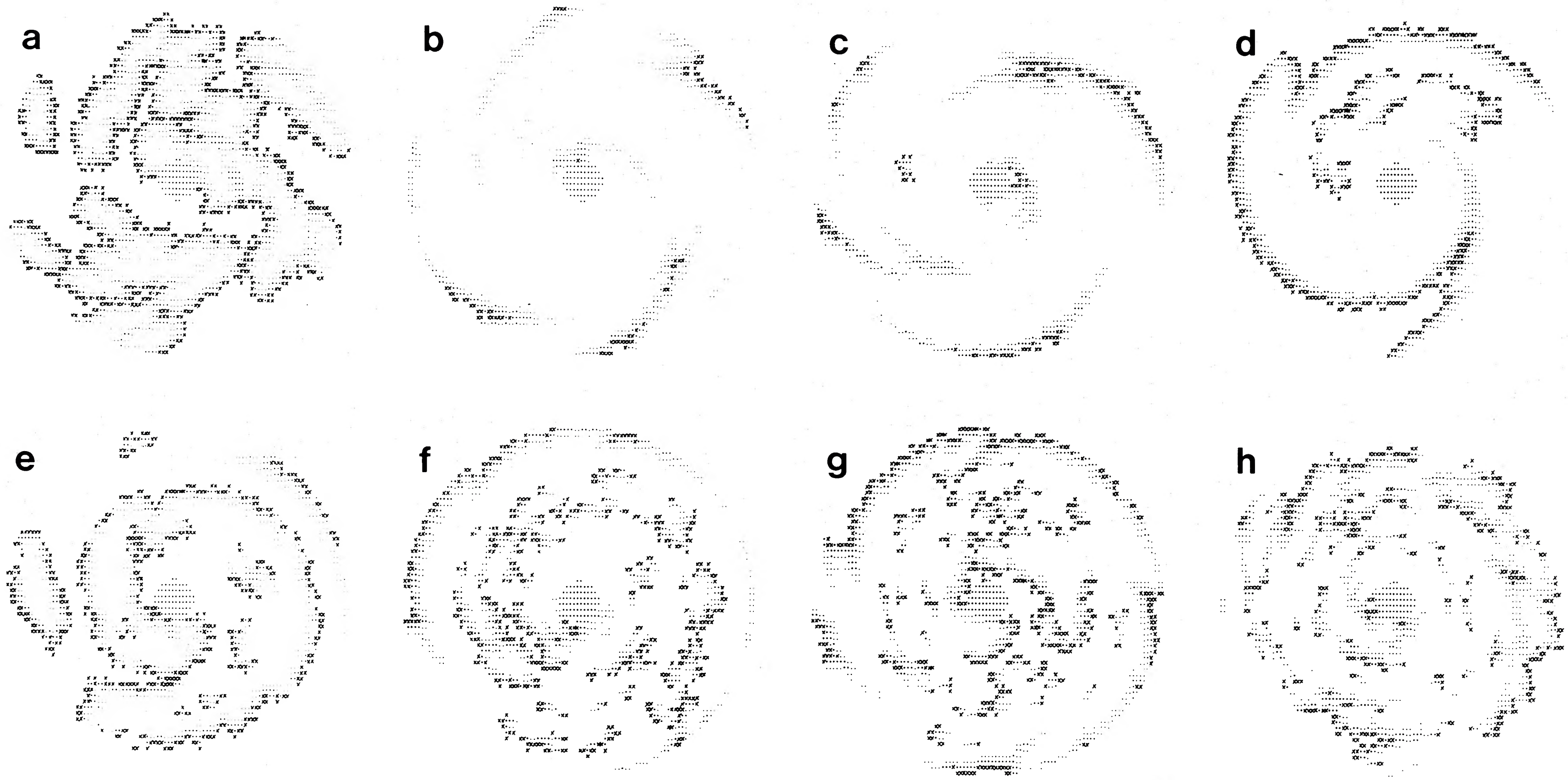

FIG. 1.—This is a representative sample of the output from a run which had some of the better spiral structure. For this run, $\tau_r = 7$, and $s = 0.5$. The ages of the figures, in time steps (one time step = 25 million years for the standard galaxy), are: (*a*) 5, (*b*) 10, (*c*) 30, (*d*) 50, (*e*) 70, (*f*) 100, (*g*) 130, (*h*) 180. Note that the "best" spiral structure is seen between 10 and 100 time steps. Then the structure settles down and consists of shorter features.

spontaneous star formation rate was of the order of one percent of the total star formation rate. In some runs star formation tended to die out without this feature. When the spontaneous star formation rate was more than about one percent of the total star formation rate it tended to disrupt regular features.

III. RESULTS

This simple model has the advantage that only a few parameters govern the operation of the code. One is the ratio of the maximum rotation velocity to the velocity with which star formation propagates:

$$\alpha = v_{\rm rot}{}^{\rm max}/v_{\rm prop} .$$

Another parameter is the regeneration time, τ_r. We will express time units in terms of time steps. One time step, Δt, is

$$\Delta t = \frac{R}{29.5 v_{\rm prop}} ,$$

where R is the radius of the galaxy which in our model is divided into 29 rings concentric around a central circle. Another parameter is the number of boxes which spontaneously undergo star formation per time step, which we will call s.

These three parameters govern the operation of the code. Runs were made with several different values for these parameters. The regeneration time, τ_r, is by far the most important parameter, while the results were less sensitive to changes in α and s.

To be specific, a "standard galaxy" will be taken to have a radius of 15 kpc and a maximum rotational velocity of 200 km s^{-1}. One time step for this model is about 25 million years, for $\alpha = 10$.

A trial-and-error procedure was adopted to determine the set of parameters which gave the best spiral structure. We will discuss the physical meaning of these values in § V.

Throughout the investigation α was usually chosen to be 10, although a few runs were also tried between $\alpha = 1$ and $\alpha = 21$. The best spiral patterns seemed to result from α between 7 and 13.

Lower values of α (slow rotation or fast propagation) make the pattern very random, and higher values tended to make a ring structure concentric with the center. The degree of spiral structure is a subjective judgment, so it is difficult to say more than this. These values for α give a star formation propagation velocity in the range of 15–29 km sec^{-1} for our "standard galaxy." (Old supernova remnants expand at a velocity of about 20 km s^{-1}, which is encouraging.) In all the following discussion we will hold α to the value of 10.

Spontaneous star formation was originally introduced to ensure that star formation did not die out. Typically s was given a value of about 1. For $s \geqslant 2$ the pattern became obviously more random. There were usually about 300 boxes undergoing star formation at any time, so $s = 2$ corresponds to about 1 percent of the total star formation rate. Spontaneous star formation did not really make the structure more spiral, but it does show that the structures produced by SPSF are stable to small disturbances.

The general statements that follow are based on the authors' experience with several hundred pages of output and are meant to indicate trends seen in the structure rather than absolute rules.

The pattern is most sensitive to changes in τ_r. Values used ranged from 3–10, with the best spiral patterns seen between $\tau_r = 6$ and $\tau_r = 8$. For $\tau_r < 6$ the pattern tended to consist of very short features, although they did show a spiral structure (Fig. 2*a*). As τ_r is decreased, the pattern becomes completely random. For $\tau_r > 8$ the features tend to get longer; however, they start to wrap around themselves and show a more circular structure (Fig. 2*b*). As τ_r is increased a system of concentric rings is often formed. In the range $\tau_r = 6$ to $\tau_r = 8$ a wide variety of shapes and forms are seen. In the best runs, half or two-thirds of the pictures contain spiral patterns. The remaining are usually circular or random patterns containing primarily short features. The model as it stands is apparently not able to produce a constant spiral pattern. It should be noted, however, that (1) the contrast is not adjusted for each picture as can be done in photographs, and (2) there are disk galaxies which show structure which is not strictly "spiral." The spiral patterns we generate are, with rare exception, trailing spirals, and usually they are asymmetric. Features comparable in length to a galactic radius are often seen. A frequently seen pattern is a ring with a radius of one-third to half a galactic radius with arms coming off it. Another is a multitude of short spiral arms near the outer edges of the disk.

Figure 2*c* shows the locations of only the crosses for a model in which $\tau_r = 7$ and $s = 1$. The boxes labeled with crosses contain all the stars in the galaxy with lifetimes less than one time step, which is about 25 million years for the standard galaxy. This corresponds to stars of spectral type up to about B3, so that all stars capable of creating H II regions are contained in these boxes. Figure 2*c* is then essentially a map of H II regions for this model. This can be compared with Hodge's (1969) maps of H II regions in external galaxies. Note that H II regions are not good tracers of spiral structure.

A typical run starts out with about 40 randomly chosen boxes designated as undergoing star formation. The process grows from there. After about 10–20 time steps (cycles) a spiral pattern begins to take shape. The best and most consistent spiral patterns are seen after about 15 and before 100 time steps. For the standard model, this corresponds to between 0.4 and 2.5 billion years. After about 100 time steps the system is relaxed and the structure does not evolve, although individual pictures are different. At this point about 300 boxes (or one-eighth of the area of the galaxy) is undergoing star formation. A typical run lasts about 200 steps, although as many as twice that have been run on occasion. After the system is relaxed, spiral patterns are seen at least half the time in the better runs. These occur randomly amidst the nonspiral

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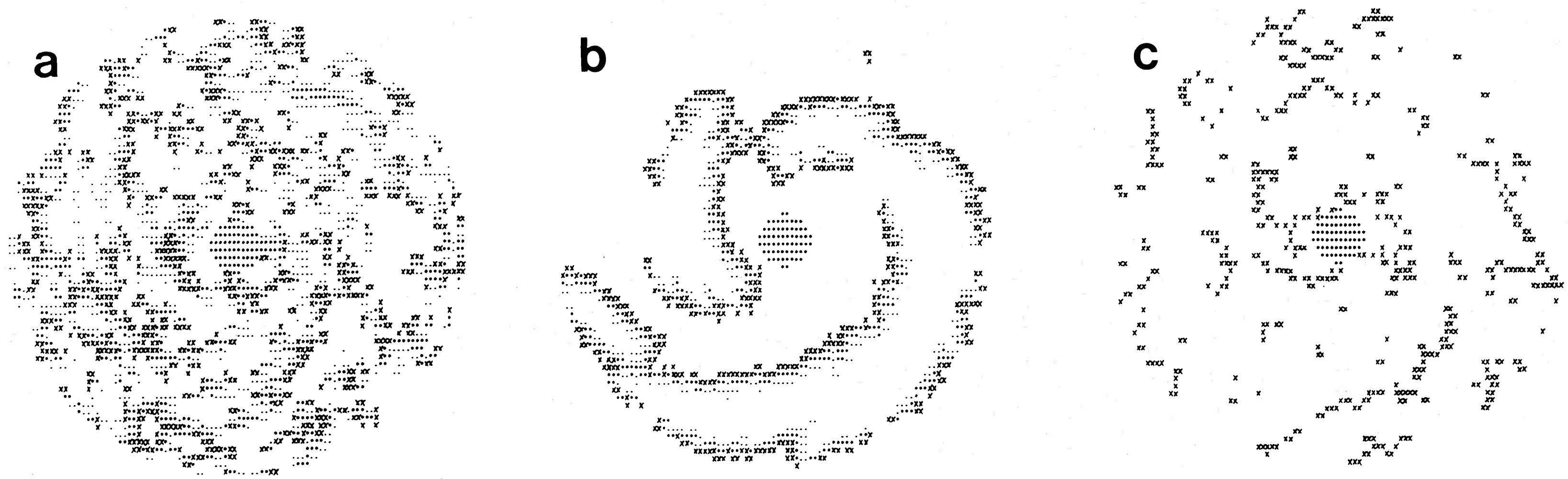

FIG. 2.—These figures from other runs show some of the effects of varying the parameters from those of Fig. 1. A more random structure is seen when τ_r is decreased. In (*a*), $\tau_r = 4$, and $s = 1$. The features are very short. This figure is 100 time steps old. A more circular and less random pattern is seen when τ is increased. In (*b*), $\tau_r = 10$, and $s = 1$. The features are very long. This figure is 60 time steps old. In (*c*), we see the effect of marking only regions which have undergone star formation in the last time step. This figure essentially indicates the positions of stars capable of producing H II regions and might be compared to Hodge's (1969) maps of H II regions in external galaxies. This run is 80 time steps old and had parameters set to $\tau_r = 7$, and $s = 1$.

patterns and do not die out with time. Figure 1 shows the evolution of a model which had a significant degree of spiral structure. An effort was made to show a representative rather than an extraordinary run. Figures 1*g* and 1*h* are representative of the "final" structure; that is, further time evolution would not significantly alter the pattern.

A few runs were made in which τ_r was made to be a function of distance to the center of the disk and a function of time. However, the introduction of these features so greatly increased the number of parameters that the results became difficult to interpret. In any case, no startling results were noticed.

This very simple model seems to make tantalizingly suggestive spiral patterns; however, they do not seem "pretty" enough. That is, they are not the "typical" two-arm symmetric spiral. Also, rather irregular forms occur with a frequency which is apparently not observed in external galaxies. However, we can hardly expect "photographic" pictures to come out of the code. We have not built in any spiral structure, yet the model, based on a small-scale phenomenon, can produce spiral structure on a galactic scale (provided, of course, that a differential rotation occurs). The results do suggest SPSF may be occurring—especially in "atypical" galaxies. An additional ordering process seems necessary to explain symmetric, two-arm spirals.

IV. DENSITY WAVES

In an effort to obtain more symmetric and consistent spiral structures, a density wave was introduced into the model. This was done in the simplest way possible to get an idea of how the SPSF process might interact with the density wave. For this mechanism, star formation is not caused by the density wave as such, but star formation is made more probable in a spiral pattern. This is done by decreasing the regeneration time, τ_r, in that area and so increasing the star formation rate. Physically, then, τ_r is connected to the parameters governing star formation, such as gas density.

A two-arm logarithmic spiral was used with an inclination angle of about 15° which corotated with the outermost ring in the disk (Roberts, Roberts, and Shu 1975). The value of α was set to 10 and τ_r was 7 except in a band centered on the spiral pattern which had a thickness perpendicular to the spiral of about 1/15 the galactic radius, or about 1 kpc for the standard model. In this region τ_r was set to different values. For one run we set τ_r to 4 in the spiral pattern and found no significant effects of the imposed pattern (Fig. 3*a*). When τ_r was set to 2 in the density-wave region, the effect was very strong (Fig. 3*b*). The imposed spiral pattern came through very clearly on every picture. This run produced the picture most like "typical" spiral galaxies, although the spiral pattern was induced. As seen in Figure 3*b*, the arms are split and the outer regions of the disk are rather patchy. The effect is not unlike that seen in M101.

We do not claim to have made a model of a spiral galaxy here since the pattern was essentially forced to be a spiral. We can say that for this model the density wave must act to decrease the regeneration time by a factor of 2 or 3.

V. SELF-PROPAGATING STAR FORMATION

Let us take a closer look at the SPSF process as it must be operating to create the spiral patterns seen in the previous sections. Sancisi (1973) and Berkhuijsen (1974), among others, have reported observing groups of young stars which are apparently physically connected to expanding shells of hydrogen. Let us take Sancisi's data on the OB association, Per OB2. A shell of gas is seen expanding asymmetrically at a velocity between 5 and 50 km s^{-1}. It is expanding most slowly in a region of high density where presumably the snowplow effect slowed the shock wave as it ran into an interstellar cloud. Ahead of this region lies the OB association which is expanding at 12 km s^{-1}. This is consistent with the association forming at an earlier time when the gas shell was expanding at a faster velocity, i.e., 12 km s^{-1}. The radius of the shell is about 20 pc, it is about 5 pc thick, and it contains about $10^4\ M_\odot$ in gas. The association contains about $1.5 \times 10^3\ M_\odot$ in stars and the system is between 1.5 and 4 million years old. The stars are spread around only one side of the shell.

If we assume that high-mass stars become supernovae and form new shells of hydrogen, the star formation process will propagate. In order for the system to grow, more than one high-mass star must be formed in the shell. This seems to be the case in the Per OB2 association since it is an OB association and it contains about $1.5 \times 10^3\ M_\odot$.

In the code we use a disk divided into boxes which are about 500 pcs on a side for the standard model, so the resolution is too poor to represent one of the expanding gas shells. We assume that star formation propagates from one box to all neighboring boxes. This, of course, implicitly assumes a minimum efficiency in the SPSF process. By the time star formation has propagated a few hundred parsecs from the progenitor, the newly born stars need to be distributed in a roughly spherical shell. This can occur if each supernova creates several high-mass stars in one direction, as seems to be the case in Per OB2. Each of the newly born high-mass stars may then supernova and create new stars in different directions. After several cycles a roughly spherical shell of star formation could be built up. Given the size of the shell near Per OB2, it appears we would need of the order of 10 cycles for star formation to fill up one of our 500 pc boxes. If each high-mass star creates n new high-mass stars, then one star could induce the formation of about n^{10} stars by the time SPSF filled a box. This is approximately 6×10^4 for $n = 3$, so even with such a low efficiency a spherical shell could be built up after several cycles.

We can get a rough value for the supernova rate by insisting that the standard galaxy be in a steady state, its mass in stars, $M_{\text{gal}}*$, be $10^{11}\ M_\odot$, and its age be 10^{10} years. Let us assume that only stars above $10\ M_\odot$

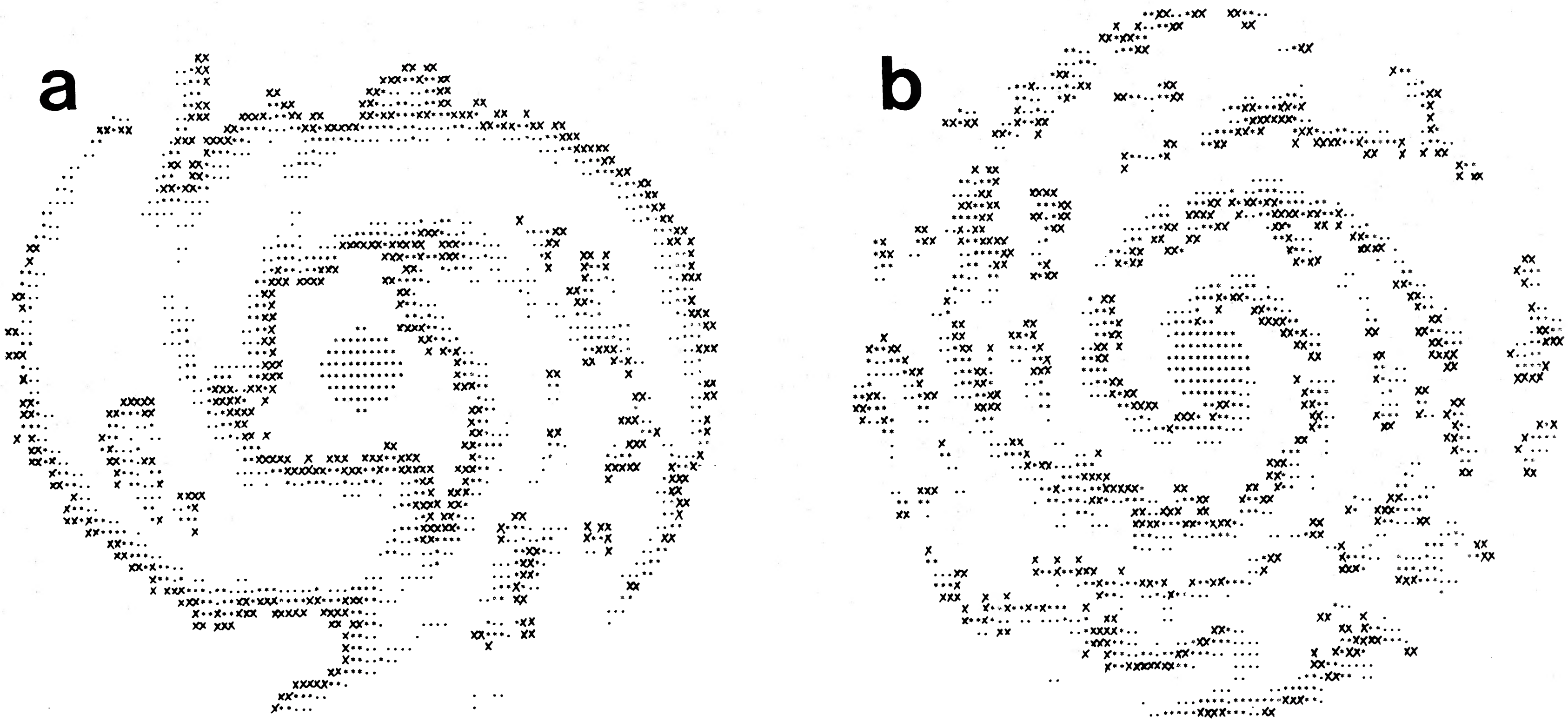

FIG. 3.—An underlying spiral pattern was introduced to produce these figures. In (*a*), $s = 0.5$ and $\tau_r = 7$, except in the spiral pattern where $\tau_r = 4$. The underlying pattern had little effect. This figure is 120 time steps old. All parameters were kept the same in (*b*) except $\tau_r = 2$ in the spiral pattern. The resulting structure closely follows the underlying pattern. The age here is 60 time steps.

become supernovae and all stars are formed by SPSF. Using Tinsley's (1972) formalism, we will choose an initial mass function of the form,

$$\phi(m) = 1.35 m_L^{1.35} m^{-(2.35)} ,$$

where

$$\int_0^\infty \phi(m) dm = 1 ,$$

and

$$m_L = 0.1 \, M_\odot .$$

The total mass of stars with mass greater than 10 $M_\odot$, M_{10}, is then

$$M_{10} = \int_{10}^{50} dm m \phi(m) M_{\rm gal}^* ,$$

so

$$M_{10} \approx 0.033 M_{\rm gal}^* ,$$

where for our analysis we have taken 50 $M_\odot$ to be the upper limit of stellar masses. This is 3×10^8 stars of 10 $M_\odot$, or a supernova rate of about three per hundred years. This is also close to the value observed. In the steady state each supernova would have to produce on the average a mass in stars of 300 $M_\odot$. This is lower than the value seen in Per OB2, but there is probably a selection effect for recognizing only objects which have produced copious groups of stars.

Observational evidence has been presented above for the SPSF process operating on a small scale involving individual stars. There is also evidence that it does operate on a very large scale. Rickard (1968) has found that a large section of the interstellar medium in the Cas-Per spiral arm in our Galaxy exhibits a unified peculiar motion. He describes a model of this phenomenon which consists of a ring in the galactic plane about 1600 pc in diameter expanding at 20–30 km s^{-1}. The ring contains about 10^7 $M_\odot$ and according to Rickard may have been caused by the collective effect of several hundred supernovae. Similar objects have been found in the Large Magellanic Cloud by Westerlund and Mathewson (1966). They report a ring of H I with a diameter of 1 kpc in which are embedded two supernova remnants and a series of stellar associations. We could conceivably be seeing the SPSF mechanism in action here.

The most important parameter has been seen to be the regeneration time, τ_r. For the standard galaxy $\tau_r = 7$ corresponds to 1.7×10^8 years. We have considered the possibility that this was the time needed for stars to expel a significant amount of mass back to the interstellar medium. This seems unlikely because, for reasonable initial mass functions, it was found that on this time scale stars probably contribute only a few percent to the gas density in the interstellar medium.

Another possibility is that τ_r represents the time needed for the interstellar medium to relax after star formation. The shock wave which causes some stars to form can also make the interstellar medium hot and turbulent—conditions which are probably not conducive to gravitational collapse to form stars. Also the gas density behind the shock is low. If supernovae blow holes with radii of about 20 pc in the interstellar medium which is at a temperature of 100 K, then the time it takes gas to diffuse back in will be of the order 10^8 years.

Supernova shocks may also disrupt interstellar clouds while at the same time forming stars from them. A cold cloud has a gravitational binding energy of

$$E_B \approx \frac{3}{5} \frac{GM^2}{R} ,$$

$$E_B \approx 3 \times 10^{45} \text{ ergs} ,$$

where typical values of the mass have been taken to be 1000 $M_\odot$ and the radius to be 15 pc (Allen 1973). Supernovae liberate about 10^{52} ergs and the expanding shells of gas seen by Sancisi (1973) contain about 10^{49} ergs in kinetic energy. There seems to be no problem in disrupting clouds. Once a region is swept clean of clouds it will have to be repopulated before star formation can occur there again.

Repopulation may occur from diffusion of clouds from other regions of the galaxy. The velocity dispersion of clouds is about 14 km s^{-1} (Spitzer 1968), so they can travel significant distances (several kpc) in the galaxy on a time scale of 10^8 years.

We note that differential rotation may also play a role in mixing the interstellar medium. This process would operate on a time scale of about one galactic revolution or on the order of 10^8 years.

It seems reasonable that the regeneration time should be dependent on the physical state of the interstellar medium. In particular, a relatively higher gas density should be associated with a shorter regeneration time since star formation is presumably dependent on gas density. The results tend to agree with observations in this respect. Small values for τ_r yielded very irregular structures possibly resembling irregular galaxies which have relatively high gas density, and large values for τ_r yielded ring-type structures not unlike that seen in S0/Sa galaxies which have relatively low gas density.

VI. CONCLUSION

This very simple investigation of self-propagating star formation suggests that it is capable of creating spiral-like optical structures in a galactic environment. It does not seem that this process alone can reproduce spiral structure like that observed, at least not for more than half or two-thirds of the time. It is quite successful in describing some of the "disorderly" aspects not evident in density-wave theory and thus supplements that theory. SPSF produces the best spiral structure on a time scale of 10^9 years when one starts from a random distribution and typical galactic parameters. This indicates that the process may be important in the early stages of a spiral galaxy before a density wave can develop. We expect that this process may induce the formation of a density wave.

Observational evidence connecting expanding shells of hydrogen to star formation has been seen in our Galaxy and also in external spiral galaxies by Monnet (M33) and Baade (NGC 6822 and K 342) (comments after Sancisi 1973). These objects are always seen in the outermost portion of the galaxy. This may simply be due to the difficulty in detecting large diffuse objects (Downes 1971; Ilovaisky and Lequeux 1972), or it may be real. Lin (1970, 1971) has suggested that density waves may be produced by star formation in the outer regions of a spiral galaxy. Here spiral arms tend to break up and become quite patchy, not unlike the patterns seen in the figures. Toomre (1969) has shown that density waves will die out within a few galactic revolutions. SPSF may be a replenishing mechanism for density waves. The large, 1 kpc rings may also be evidence of SPSF operating on a large scale.

The nature of the SPSF mechanism as formulated here unfortunately does not seem to lend itself to a simple mathematical description. Caution should be used in drawing conclusions from numerical experiments; nevertheless, the results are encouraging.

We have examined an aspect of the problem not treated by the density-wave theory or by galactic evolution calculations (e.g., Talbot and Arnett 1975). Real galaxies do not have perfect geometrical structure. Perhaps the most encouraging aspect of the SPSF model is that it relates the irregular visual characteristics of spiral galaxies to some parameters subject to theoretical interpretation.

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