THE ASTROPHYSICAL JOURNAL, 311:1–5, 1986 December 1

HIERARCHICAL STRUCTURE IN THE DISTRIBUTION OF GALAXIES

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Received 1984 February 22; accepted 1986 May 19

ABSTRACT

The distribution of galaxies has a hierarchical structure with power-law correlations. This is usually thought to arise from gravity alone acting on an originally uniform distribution. If, however, the original process of galaxy formation occurs through the stimulated birth of one galaxy due to a nearby recently formed galaxy, and if this process occurs near its percolation threshold, then a hierarchical structure with power-law correlations arises at the time of galaxy formation. If subsequent gravitational evolution within an expanding cosmology is such as to retain power-law correlations, the initial r^{-1} dropoff can steepen to the observed $r^{-1.8}$. The distribution of galaxies obtained by this process produces clustering and voids, as observed.
Subject heading: galaxies: clustering

I. INTRODUCTION

In studying the transition from a uniform universe to the present-day hierarchical distribution of galaxies (and the associated power-law reduced correlation function), one generally looks to gravity. Specifically, one assumes that somehow galaxy-sized mass points have appeared and allows them to evolve under Newtonian gravity appropriately modified to incorporate cosmic expansion (Aarseth, Gott, and Turner 1979; Turner *et al.* 1979; Gott, Turner, and Aarseth 1979; Efstathiou and Eastwood 1981; Miller 1983). In this note we suggest another mechanism, related not to gravity but to the relatively brief process of galaxy formation, by which many of the observed features of the universe automatically emerge, namely hierarchical clustering, large voids, and power-law correlation functions (Peebles 1980; Kirschner *et al.* 1981; Bahcall and Soneira 1983; de Lapparent, Geller, and Huchra 1986). However, despite this qualitative agreement, we do not predict that the observed $r^{-1.8}$ dropoff in the reduced correlation function is established at the time of galaxy formation. Rather, the initial power is close to unity (i.e., an r^{-1} dropoff). Subsequent cosmological and dynamical evolution does not preserve this power, since galaxies near one another are dominated by local gravitational forces and do not diverge as rapidly as relatively distant pairs, thus steepening the distribution. By assuming that the differential divergence yields a distribution that remains a power law, we are able to predict what that power ought to be in the present era. With reasonable values for the age of the galaxies, we find agreement between the predicted and observed powers.

II. PROPAGATING GALAXY FORMATION

Suppose that galaxies form more or less as described by Ostriker and Cowie (1981; see also Bertschinger 1985; Carr and Ikeuchi 1985). The birth of one galaxy causes one or several others to form nearby, which in turn may cause more galaxies to form farther away. We think of the universe as starting as a uniform gas within which there is some small probability of spontaneous galaxy formation. However, if a region of this gas is perturbed by nearby galaxy formation, the probability that this region will itself be a site of galaxy formation is greatly enhanced. This process is modeled as follows: The universe is partitioned into cells whose size is on the order of the intergalactic spacing (at the epoch in question), and each cell has or does not have a galaxy in it. Initially a small number of cells spontaneously produce galaxies. These galaxies are allowed to induce further galaxy formation in neighboring cells with some probability p. Using the same p, the newly formed galaxies may form yet more galaxies, and so forth. The process terminates on a time step in which no new galaxies form. (Cells containing a galaxy cannot be excited a second time.) We are not concerned with a precise connection between this stochastic process and the mechanism of Ostriker and Cowie. For our purposes it is only necessary that such a stimulated process exists with a probability close to the critical value (as discussed in § III). The requirements are similar to those for propagating star formation within a galaxy (Seiden and Gerola 1982; Schulman and Seiden 1982, 1983).

III. PERCOLATION

The process just described is a variant of ordinary bond percolation (Nakanishi and Reynolds 1979). It is well known that this process exhibits a phase transition, and when p is in the neighborhood of the critical value for the phase transition (p_c), the reduced pair correlation functions have a power-law behavior. Specifically, for $p = p_c$, the two-particle correlation function is

$$\xi(r) \propto \frac{1}{r^{d-2+\eta}} , \quad p = p_c , \tag{1}$$

where d is the dimension of the space, in this case 3. For three-dimensional bond percolation, η is known to be less than 0.05 (Wilson and Kogut 1974; Gaunt and Sykes 1983). (The use of ξ for this correlation *function* is conventional in astronomy. However, note that in the percolation and phase transition literature ξ is used to represent a correlation *length*, not a function). Away from criticality (i.e., $p \neq p_c$), ξ has exponential dropoff with correlation length l which is a function of p. However, close to p_c as $l \to \infty$ a more accurate representation, reflecting power-law behavior in ξ, must be used. Specifically (Fisher 1964),

$$\xi(r) \propto \frac{e^{-r/l}}{r^{(d-1)/2}}\,, \quad p \neq p_c\,. \tag{2}$$

Thus if the process of galaxy formation is modeled as we described above and if it is occurring for p near criticality, then the distribution of galaxies will be born with an $r^{-\gamma}$ dependence with $\gamma \approx 1$. Moreover, not only will it have this power-law behavior, but on rather large scales the overall structure of galaxies will be hierarchical and characterized by voids and clumps. These are all properties of percolation clusters near criticality (Deutscher, Zallen, and Adler 1983).

To relate the features described above to the structure of the universe, two issues must be treated. First, why should p have been near p_c? Second, the observed value of γ is 1.8 (Peebles 1980); how does the percolation value of $\gamma \approx 1$ become the observed $\gamma \approx 1.8$?

To address the first question we imagine that effective values for spontaneous (p_1) and stimulated (p_2) galaxy formation can be defined, related to quantities like the Jeans length and sound velocity, etc. Because of the initial high temperature early in the history of the universe, the probabilities were too small for percolation to occur, and the few galaxies that may have formed did not cause any burst of galaxy formation. As p rose to and perhaps exceeded p_c, great extended bursts took place, and a substantial fraction of the material was exhausted. Moreover, even where galaxies did not form, gas was heated, lowering the effective value of p in the region. Throughout, the universe continued to expand, and the brief approach to p_c reversed as the natural expansion thinned the gas, lowering the effective value of p. Thus in this optimistic scenario, p rises to the general neighborhood of p_c and then drops. To get power-law dependence for $\xi(r)$ in a finite region it is not necessary to have p equal p_c exactly. Rather, it is sufficient for the region not to be large compared to the correlation length at that p. More specifically, we assume that p_1 is small enough that each spontaneous event is on the average separated by a distance at least as large as the correlation lengths actually observed. That is, the correlations are due to the pure bond percolation process of equation (2). The observed size of the correlated region was found to be of the order of 10 h^{-1} Mpc for galaxy-galaxy correlations (Groth and Peebles 1977; Soneira and Peebles 1978; Davis and Peebles 1983). For cluster-cluster correlations a size of 150 h^{-1} was found (Bahcall and Soneira 1983). This means that spontaneous galaxy formation is rare and that substantially all galaxies that we see arose from a small number of triggering events. Such a scenario would also allow buildup of p to values exceeding p_c.

Based on present observations, we cannot say whether p effectively exceeded p_c. We do not know whether the entire universe, or substantial fractions thereof, is part of a single cluster, or whether several separate triggering events took place and p was close enough to p_c for the correlation length to bridge the distances between them. There is in principle an observational difference between these alternatives. If the triggering events were Poisson-distributed, there will still be gaps; however, on a sufficiently large scale, contemporary correlations would be different.

Nonzero p_1 and p_2 does not correspond to the commonly studied site or site-bond percolation. It would be a new model without a threshold for the occurrence of nonzero asymptotic density.

In order to demonstrate the correlations arising in the percolating system described above, we have carried out a computer simulation of the process. We chose a cube of $51 \times 51 \times 51$ cells. We start by placing a galaxy (spontaneously formed) in the center and then let it induce galaxy formation in the neighboring cells with a probability p. The galaxies thus created go on to create more galaxies in their neighboring cells with the same probability. The process continues until a time step for which no more galaxies are created. No cell is allowed to have more than one galaxy. Figure 1 shows the reduced correlation function as a function of r (the distance between galaxies). As can be seen, the correlation function is $\sim r^{-1}$ up to $r \approx 15$, falling off more rapidly after that. The falloff comes from two sources; the first is the exponential factor in equation (2), and the second is the finite size of the space used in the simulation. In the actual universe, a third source may occur, the overlap of percolating structures arising from separate spontaneous seeds. The importance of the exponential falloff depends on the proximity to p_c. At p_c the exponential vanishes since l is infinite, and we are left with equation (1).

IV. COSMIC EXPANSION AND GRAVITY

Next we discuss the evolution of the power in the power law from the value 1 at the time of galaxy formation to its presently observed value of ~ 1.8. Work on the evolution of the distribution of galaxies has involved numerical calculation of the evolution of a large number of mass points under gravity on the background of an expanding universe. The expansion is significant, since the era for the initiation of galaxy formation is estimated to be in the range $5 < z < 100$. Gott, Turner, and Aarseth (1979) found that they were able to get a roughly power-law reduced correlation function with a very approximate r^{-2} fall off. Efstathiou and Eastwood (1981), on the other hand, found a falloff of approximately the right magnitude, but it was not well represented by a power law.

The above-referenced calculations are indicative but do not bear directly on our work, since, first, they use homogeneous initial conditions, whereas the point of our work is that the appropriate initial state already has hierarchical structure; second, our envisioned formation process involves the conversion of inactive matter into galaxies. There will be some sweeping (by shock waves) of matter from intergalactic regions, but the larger "voids" will still have substantial amounts of matter, albeit not in the form of visible, concentrated galaxies. A newly formed galaxy in our scenario will not behave like a mass point but rather, if one works on a background uniform mass cosmology, the galaxy looks like a concentrated relative mass enhancement (the formerly dispersed matter from which this galaxy was made) within a region of depletion. Two such galaxies not perfectly centered within their respective regions will attract by a weak r^{-2} dipole potential (rather than r^{-1}). Therefore, existing evolutionary calculations are again inappropriate.

We have taken a phenomenological approach to the problem of the evolution of the correlation function. We assume that at all times $\xi(r)$ has *some* power-law dropoff and estimate the value of that power today, given that it was close to unity at the time of galaxy formation. Measuring the dis-

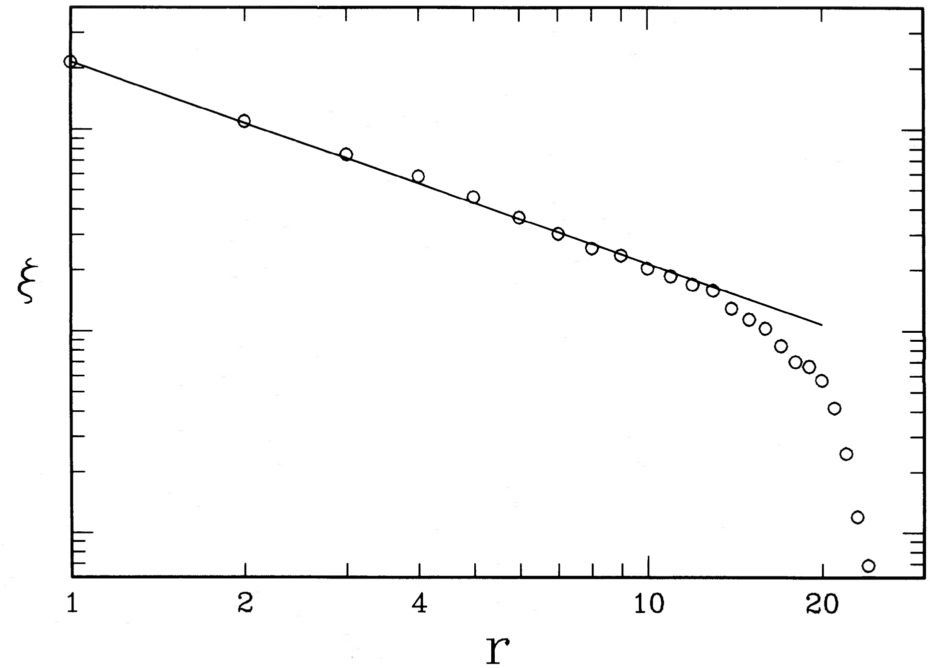

FIG. 1.—log-log plot of the reduced correlation function for a computer simulation of bond percolation in a 51 × 51 × 51 cube for $p = 0.251$ ($p_c = 0.2479$). The solid line is r^{-1}.

tance r from our own galaxy, the number of galaxies (n) in a volume d^3r about r is

$$n(r)d^3r = n_0[1 + \xi(r)]d^3r\ , \tag{3}$$

where n_0 is the asymptotic ($r \to \infty$) density. On a log $n(r)$ versus log r plot, the initial (i.e., time of galaxy formation) curve will be a straight line of slope -1 for quite some distance until it flattens out with the domination of n_0. Subsequent dynamical behavior is complicated. The most distant galaxies diverge at a rate given simply by the expansion rate of the universe. But the mutual gravitational attraction of closer pairs (or that of their clusters) leads to smaller divergence, until at some small scale the overall expansion has no effect. Statistically speaking, at this proximity, relative distances are constant.

Consider what would happen to $n(r)$ if there were uniform expansion at all distances. Let the new, expanded coordinate

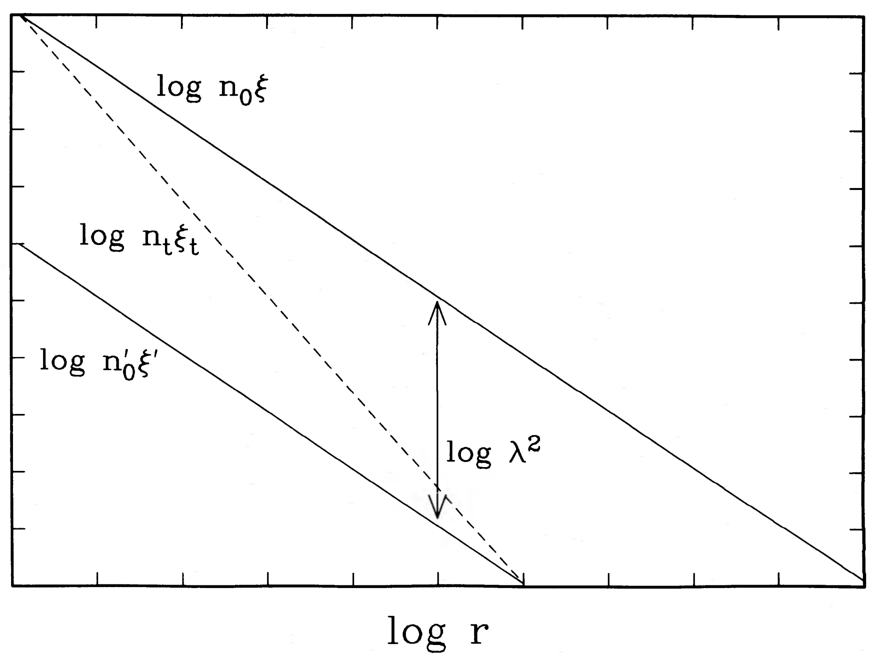

FIG. 2.—log-log plots of various reduced correlation functions. *Upper solid curve:* $n_0\,\xi$, the initial r^{-1} distribution. *Lower solid curve:* $n_0'\xi'$, the image of $n_0\,\xi$ under uniform expansion. *Dashed line:* $n_t\,\xi_t$, representing the form seen today.

be r', so that $r' = \lambda r$ with $\lambda = 1 + z$. Since the number of galaxies in any particular comoving volume remains constant, we have

$$n_0[1 + \xi(r)]d^3r = n_0'[1 + \xi'(r')]d^3r' . \qquad (4)$$

Clearly, $d^3r' = \lambda^3 d^3r$. Equation (4) holds for all r and in particular for r large enough that ξ and ξ' are negligible. Thus $n_0' = \lambda^{-3}n_0$. Consider, however, the region where $\xi \gg 1$. Here we have $n_0'\xi'(r') = n_0\,\xi(r)\lambda^{-3}$ or, if $n_0\,\xi \approx K/r$, it follows that

$$n_0'\xi'(r') = \lambda^{-3}n_0\,\xi\left(\frac{r'}{\lambda}\right) \approx \frac{K}{\lambda^2 r'} . \qquad (5)$$

Not surprisingly, $n_0'\xi'$ is still a straight line on a log-log plot (in the appropriate range of r) and is simply displaced downward by an amount log λ^2. Both $n_0\,\xi$ and $n_0'\xi'$ are drawn in Figure 2 for r in the range such that $\xi \gg 1$ and $\xi' \gg 1$.

Now in fact there is not uniform expansion at all distances, and for the true correlation function the upper curve in Figure 2 is displaced downward by a full log λ^2 only at the largest distances, which we designate R_L. At small distances, where the cosmic expansion is unimportant (R_S), the upper curve is still appropriate. The true curve, called $n_t\,\xi_t$ will lie somewhere between $n_0\,\xi$ and $n_0'\xi'$.

If we now assume that the present distribution $n_t\,\xi_t$ is some power law $\tilde{K}/r^\gamma$, then $n_t\,\xi_t$ will be a straight line on our log-log plot (shown as a dashed line in Fig. 2). By our previous considerations, we have located two points on this line (viz., at $r = R_S$, $n_t\,\xi_t(R_S) = n_0\,\xi(R_S) = K/R_S$ and at $r = R_L$, $n_t\,\xi_t(R_L) = n_0'\xi'(R_L) = K/\lambda^2 R_L$). The negative slope of $n_t\,\xi_t$ is γ and is then easily calculated to be

$$\gamma + 1 + \frac{2 \log \lambda}{\log (R_L/R_S)} . \qquad (6)$$

We estimate the values of R_L and R_S as follows. Choose R_L to be the distance where the recession velocity is the velocity of light. At this distance there has not been enough time for any gravitational interaction. Therefore, $R_L = 3000\ h^{-1}$ Mpc. The typical peculiar velocity of galaxies is ~ 300 km s^{-1}. This means that gravity and cosmic expansion should be roughly equal at a recessional velocity of this value. We then choose 1/10 of this distance as the point where cosmic expansion can be neglected, yielding $R_S = 0.3\ h^{-1}$ Mpc or $R_L/R_S = 10^4$. Sub-

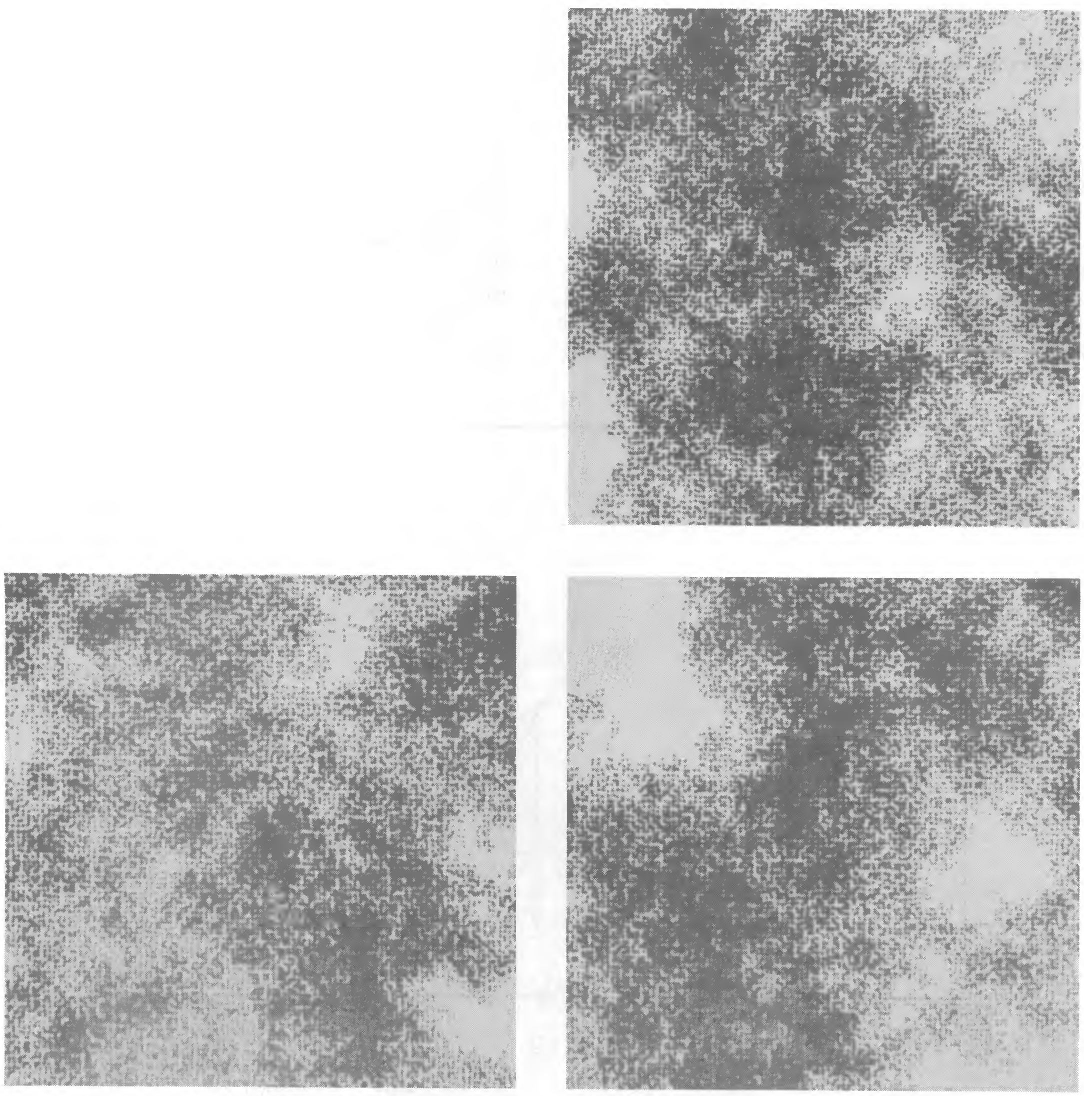

FIG. 3.—Galaxy distribution arising from a simulation in a $101 \times 101 \times 101$ cube with $p = 0.251$. The three views are projections along the three axes of the cube. The symbol size is proportional to the number of galaxies in the line of sight.

stituting this into equation (6) yields a value of $\lambda = 40$ for $\gamma = 1.8$.

We can get a lower bound on λ from another consideration. The universe must be old enough for the stimulated propagation to have taken place. As mentioned earlier, Bahcall and Soniera (1983) see $r^{-1.8}$ cluster-cluster correlations out to 150 h^{-1}. Bertschinger (1985) has estimated that the detonation wave associated with this process is ~ 3000 km s^{-1}. Therefore, at $z = 40$ the propagation time would be of the order of 10^9 yr.

It is worth remarking on where specifically percolation theory has made a prediction in the foregoing calculation. It is in the slope (-1) of the two curves that bracket $n_t \xi_t$. For example, had we assumed homogeneity at the time of galaxy formation ($n_0 \xi =$ const, or $\gamma_{\text{initial}} = 0$), then under uniform expansion a log λ^3 lowering of the curve would take place, and if one again assumes that dynamical evolution maintains *some* power law, then the value one gets for the contemporary γ is $(3 \log \lambda)/\log (R_L/R_S)$. For the values of R_L, R_S, and γ used earlier, we obtain $\lambda = 250$. This is a large but perhaps not unreasonable value (Bertschinger 1985; Carr and Ikeuchi 1985).

V. GALAXY MAPS

The clustering and voids produced by this percolation process can be seen in Figure 3. This figure is the result of a simulation in a 101 × 101 × 101 cube. Two-dimensional projections along all three cube axes are shown. Although clusters and voids are clearly displayed, a close comparison to observed galaxy maps is complicated by the fact that the observations are magnitude-limited and we have not generated a galaxy luminosity function for our simulation. Therefore, to have a figure more easily comparable with published galaxy maps, we have reduced the thickness of the cube in one direction so that the projected galaxy density is that observed. This is shown in Figure 4. It resembles Figure 3 of Soneira and Peebles (1978) and Figure 1*c* of de Lapparent, Geller, and Huchra (1986). One should not expect the comparison to be too good, since Figure 4 has r^{-1} correlations, not $r^{-1.8}$; however, the clustering and voids are clearly shown.

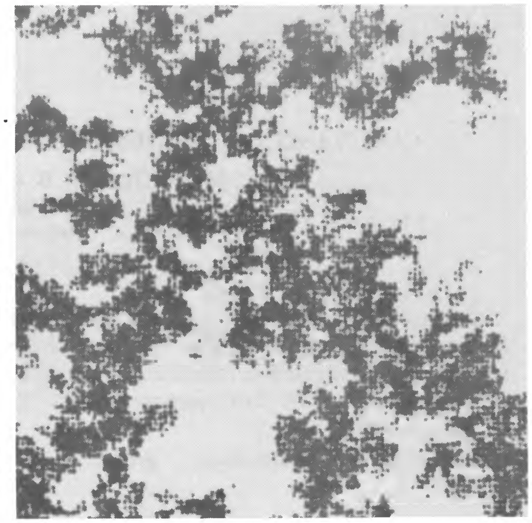

FIG. 4.—A projection of galaxies seen in a 101 × 101 × 6 slice of the model of Fig. 3. This slice gives the same average galaxy density as, and has an area ~3% of, Fig. 3 of Soneira and Peebles (1978).

VI. CONCLUSIONS

We have suggested a mechanism by which a hierarchical galaxy distribution arises naturally. Our mechanism uses a percolation model for the spread of galaxy formation, and if this spread takes place just above the percolation threshold, it will yield an r^{-1} dropoff in the reduced correlation function. Assuming that subsequent dynamical evolution preserves the power-law dropoff, we predict that power currently to be in the neighborhood of 1.8. The galaxy distribution arising from this process produces clustering and voids similar to those observed.

We wish to thank H. van Beijeren, B. G. Elmegreen, Y. Gefen, G. Grinstein, C. M. Newman, P. J. E. Peebles, V. Privman, and P. Shapiro for useful discussions.

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