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STOCHASTIC STAR FORMATION AND THE EVOLUTION OF GALAXIES

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ABSTRACT

The mechanism of stochastic self-propagating star formation has previously been invoked to explain the origin of spiral arms in galaxies. In this paper we extend the application of this mechanism to account for the diversity of morphological types and the evolution of galaxies. The new property that arises from consideration of this mechanism is that the rate of star formation exhibits the critical behavior of a phase transition. This is a general property of the system and is not strongly dependent on the details of the star–interstellar gas interaction. Examination of the properties of this phase transition provides a general scenario for the evolution of galaxies and the origin of the various morphological types.

Subject headings: galaxies: evolution — galaxies: structure — stars: formation

I. INTRODUCTION

The most basic fact in the morphological differentiation of galaxies is the existence of two major types: ellipticals and spirals. The first are rather spheroidal aggregates of stars with little, if any, detectable gas. The latter possess both a spheroidal component and a disk component with a disk-to-bulge ratio and a fractional gas content that varies with the Hubble type (e.g., see de Vaucouleurs 1977). The work of Larson (1976, and references therein) on the dynamical evolution of a protogalactic cloud toward a spiral galaxy indicates that it should proceed in two distinct stages characterized by two different rates of star formation: in the initial phase there is a high rate of star formation that forms the spheroidal component; and in the later phase there is a much lower rate after most of the remaining viscous gas has settled into a disk. In order to reproduce the observed bulge-to-disk ratio, Larson (1976) found that a single functional dependence of the rate of star formation on the density of available gas, such as a power law, is not enough, and that a very sharp cutoff of the initial rate of star formation is required. This general result has been confirmed by recent more detailed three-dimensional calculations by Miller (1978*a*). Several authors (see, e.g., Audouze and Tinsley 1976) have proposed, independent of dynamical arguments, the existence in spiral galaxies of an initial burst of star formation that rapidly enriches the remaining gas, in order to explain the observed low abundance of metal-poor stars. This initial burst should have turned off rather abruptly. This idea, that all galaxies initially undergo a large burst of star formation so that all of them have an early high-luminosity phase, has long been recognized as a crucial point for cosmological tests that rely on galaxies as standard candles.

In a previous paper, Gerola and Seiden (1978, hereafter Paper I), it was shown how stochastic self-propagating star formation (SSPSF) occurring in a differentially rotating disk can produce stable spiral structure. In that paper it was pointed out that unless the probability for stimulated production of stars, P_{st}, lies within a narrow range of values, the model galaxy will either undergo an explosion of star formation or completely cease all activity. The important point is that in this range stable spiral structure is a natural consequence of the SSPSF mechanism.

In this paper we propose that this critical behavior is an intrinsic property of a galaxy undergoing SSPSF, and that such behavior provides a qualitative scenario for the evolution of galaxies into elliptical and spiral forms.

II. PROPERTIES OF SELF-PROPAGATING STOCHASTIC STAR FORMATION

The key parameter that enters the SSPSF model (Gerola and Seiden 1978) is the probability P_{st} that a region of present star formation will induce star formation in a neighboring region. The relation between this probability for stimulated star formation and the physical parameters describing the local interaction between stars and the interstellar medium is not well understood at the present time. It is clear, however, that P_{st} must be a monotonic function of the gas density, since the greater the amount of available gas, the greater the chance of forming stars. At the other extreme, if there is no gas, no stars can be formed. Other physical parameters of the gas, such as temperature and composition, are certain to influence the interaction and therefore P_{st}. But for the purposes of defining the average evolutionary behavior of P_{st} it is enough to accept that it varies monotonically with the density. Therefore one expects that, as the gas in the galaxy is consumed by star formation, the average P_{st} will decrease.

The rate at which cells are excited in the SSPSF model, ρ_s, is primarily determined by P_{st}. We will identify this rate with the rate of star formation in the

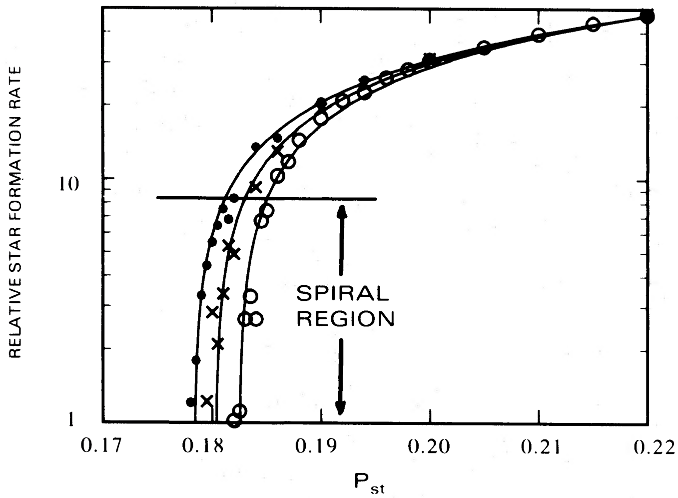

FIG. 1.—Star-formation rate as a function of stimulated probability. The data are for models having flat rotation curves of 0 km s^{-1} (○), 200 km s^{-1} (×), and 400 km s^{-1} (●). The solid lines are fits to the data using formula (1).

galaxy per unit gas density. Figure 1 shows ρ_s as a function of the stimulated probability for three models differing only in rotation curve.[1] The rotation curves are taken as flat with velocities of 0, 200, and 400 km s^{-1} (i.e., the velocity is constant over a major portion of the disk; see Rubin, Ford, and Thonnard 1978). A rotational velocity of zero is not physical in a real galaxy; however, the actual parameter of importance in the model is the differential rotation, $d\omega/dR$, where ω is the angular velocity and R is the radius vector. For the case of flat rotation curves the differential rotation is directly proportional to the velocity itself, so that in the context of our model, zero rotational velocity is merely a convenient way to represent rigid rotation. The differential rotation is not constant with radius, but varies as R^{-2}. However, since we are not varying the form of the rotation curve, the differential rotation at every point simply scales with velocity. Therefore the series in increasing velocity represents increasing differential rotation or shear.

A striking characteristic of Figure 1 is that the transition from a fairly large star-formation rate to zero is very sharp. Furthermore, the area of good spiral formation in the computed models is in the critical region where $d\rho_s/dP_{st}$ is large; the whole region corresponds to only a few percent variation of P_{st}. The question that can be asked is, why is nature such that the stimulated probability finds itself in just that region for all spiral galaxies? We believe that the answer to this question lies in a feedback control mechanism. If such a mechanism exists, the sharp cutoff characteristic of the curves in Figure 1 is what is required to sensitively control the star-formation process.

The feedback may be provided by the star-formation process itself, as suggested by Talbot and Arnett (1975), since the star-formation rate is a function of gas density. If the gas density is high, P_{st} will be large and many stars will be created. The energy generated by the evolution and supernova explosions of all these stars will both heat up the interstellar gas and eject some of it from the plane of the disk; both effects will tend to lower the gas density, thereby reducing the star-formation rate. In contrast, if the gas density goes too low, the star-formation rate will fall until the heated gas cools and the ejected gas falls back into the disk. Therefore we would expect that the behavior shown in Figure 1 combined with the mechanism of heating and ejecting gas could provide the regulatory mechanism which keeps the value of the stimulated probability in the critical range. Although on the average we expect the value of P_{st} to be in the critical range, we also expect the fluctuations to be large, since the process would probably not be critically damped. The important point is that the sharp cutoff characteristic of the curves in Figure 1 is just what is needed to provide sensitive regulation of the star-formation process.

The star-formation rate can also depend on other properties in addition to P_{st}. For example, the three curves in Figure 1 show the effect of differential rotation on the star-formation rate. As can be seen, the curve shifts toward lower values of P_{st} for the differentially rotating galaxies, so that for a given P_{st} the rate ρ_s increases with rotation velocity. In Figure 2 we show explicitly ρ_s as a function of rotational velocity. The exact shape of this curve will depend strongly on the value chosen for P_{st}, as can be easily discerned from Figure 1. The important

[1] These calculations were performed for a model galaxy of 49 rings (7351 cells) with a spontaneous probability of zero, a refractory period of one, a time step of 10 million years, and a galactic radius of 10 kpc. The general form of the curves in Fig. 1 is not dependent on the values of these parameters. Changes in the parameters will shift the curves to another area of the (ρ_s versus P_{st})-plane and can change the shape of the curves to a small degree. In particular, in Paper I the area of good spiral arm formation was at $P_{st} = 0.29 \pm 0.04$, whereas in Fig. 1 it is 0.1816 ± 0.0013 (for $V = 200$ km s^{-1}). This difference is primarily due to the fact that in Paper I the refractory period was 11 and the spontaneous probability was 2×10^{-4}.

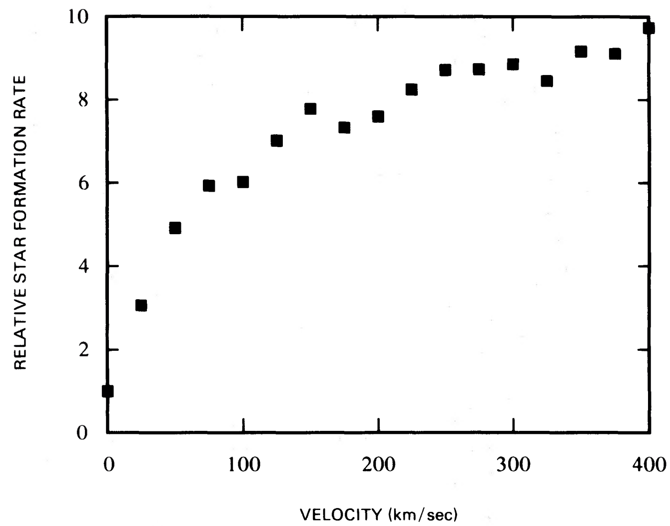

FIG. 2.—Star-formation rate as a function of velocity for flat rotation curves ($P_{st} = 0.1825$)

point that the figure shows, however, is that the greater the differential rotation, the greater the star-formation rate will be. That this should be so can be easily understood. The larger the differential rotation the faster the new material will be brought into contact with active star-forming regions, thereby propagating the "infection." That is, in a differentially rotating disk the neighbors of an active region continually change, allowing the active region to affect other previously unaffected areas.

We can get a better feeling for the data of Figure 1 by replotting them as shown in Figure 3. The behavior of all three sets of data fall on a universal curve given by

$$\rho_s \sim (P_{st} - P_{st}^{(C)})^k , \qquad (1)$$

where $P_{st}^{(C)}$ is the critical value of P_{st} where $\rho_s \rightarrow 0$. The constant k is the same for all the data and equals 0.66. The value of $P_{st}^{(C)}$ is different for each data set, as is clear in Figure 1.

This type of behavior is typical for the phenomenon of phase transitions and, in particular, our system is an example of a "percolation phase transition" (Shante and Kirkpatrick 1971), since the SSPSF process can be thought of as "percolating" over the galactic disk through the course of time.

The concept of percolation is not totally foreign to the astrophysical literature; Cox and Smith (1974) applied it in their analysis of the existence of interconnected hot interstellar tunnels produced by the superposition of supernova shells. However, in order to gain a better understanding of the process, it will be easier to discuss a simpler and more familiar model, the forest fire. A fire is maintained in a forest by hopping from tree to tree. We cannot definitely state whether fire will hop between any two particular

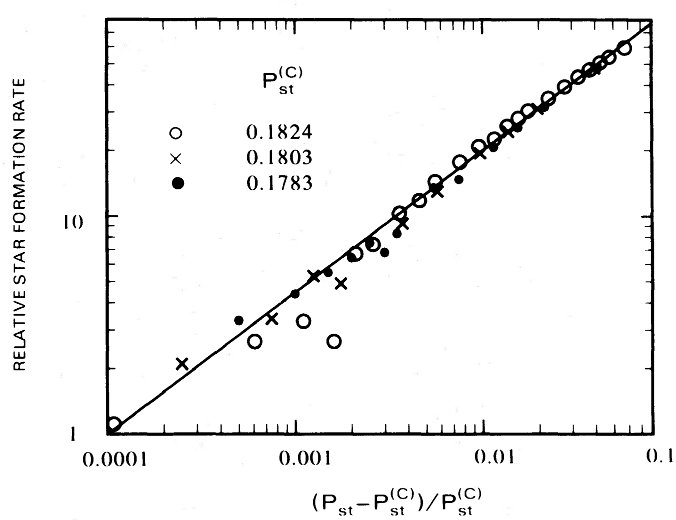

FIG. 3.—Star-formation rate as a function of $P_{st} - P_{st}^{(C)}$. The data are for models having flat rotation curves of 0 km s^{-1} (○), 200 km s^{-1} (×), and 400 km s^{-1} (●).

trees, but can only determine the probability for such a hop taking place. A major factor determining this probability is the distance between trees. If the distance between trees is large, it is difficult for the flames from one tree to reach another and the fire will die out in a short time. If, on the other hand, the distance is small enough, the fire propagates very readily and can consume the whole forest. The transition between a fire that dies out and one that propagates occurs very suddenly. That is, there is a critical distance between trees above which the fire dies, but below which the fire can propagate quite easily. The rate at which the fire propagates depends on how much the distance differs from the critical value, and the rate increases very rapidly as this difference increases.

A property of phase transitions, important for the present discussion, is the dependence of formula (1) on the dimension of the system. The greater the dimensionality of the system, the smaller the critical probability; or, for the forest fire example, the larger the critical distance. The reason for this is easy to understand; if the probability for igniting a neighboring tree is P and there are N neighboring trees, the number of trees ignited will be NP. If the system is one-dimensional, then we have neighbors in one direction only. In two dimensions, on the other hand, we can also propagate to the "right" or "left"; in three dimensions the directions "above" and "below" are also available. The result is that N will increase with the number of dimensions. In order to propagate we must have $NP \geq 1$; therefore as the dimensionality increases P can decrease and the critical distance will increase.

We should note that a forest is effectively two-dimensional, since it may spread to great distances horizontally but is limited vertically to the height of the trees. The same is true for a disk galaxy, since its height is much smaller than its diameter.

We can now return to the behavior of the galaxy itself. Although our disk galaxies are two-dimensional, the percolation problem is three-dimensional because the system percolates in time as well as in the two space dimensions. The critical exponent k in formula (1) is not an arbitrary parameter but is directly given by percolation theory (Shante and Kirkpatrick 1971) as $d - 1/d$ where d is the dimensionality of the percolation problem. In our case this yields a value of two-thirds, in excellent agreement with the slope of 0.66 found for Figure 3. For a three-dimensional galaxy, like an elliptical, we would have a four-dimensional percolation problem with $k = \frac{3}{4}$ and a lower value for $P_{\mathrm{st}}^{(C)}$.

III. EVOLUTION OF GALAXIES

On the basis of the properties of the percolation phase transition described in § II, we will now outline the course of evolution of a galaxy. We assume that the galaxy begins as a protogalactic cloud undergoing gravitational self-contraction. At some point the protogalaxy will have a high enough gas density to start forming stars. Since it is in the three-dimensional phase, the star-formation rate can be large for low gas densities because $P_{\mathrm{st}}^{(C)}$ is small. If the contraction rate is small relative to the star-formation rate, the gas is converted into stars quite rapidly and the star-formation rate remains large down to quite low gas densities. Therefore the galaxy uses up its gas before it can contract into a disk, and what is left is a spheroidal stellar system that becomes an elliptical galaxy, as suggested by the calculations of Larson (1976) and Miller (1978*b*).

If the protogalaxy should contract faster it still will form stars at a high rate during much of the contraction phase. However, when it becomes disklike, the effective dimensionality is reduced and $P_{\mathrm{st}}^{(C)}$ is raised to a much higher value. This has two important effects; first the raising of $P_{\mathrm{st}}^{(C)}$ will cause the star-formation rate to be greatly reduced, sharply cutting off the initial burst of star formation. Second, the amount of gas remaining after this cutoff will be much higher than for elliptical galaxies, because the cutoff occurs at a relatively earlier stage in the evolution. Therefore fuel will be available for further star formation. In the flattened, roughly two-dimensional, regime, the star formation will proceed at a much slower rate, being kept there by the feedback mechanism discussed earlier.

This scenario reproduces the major points discussed in § I. That is, we find two types of galaxies, ellipticals with very little gas and star formation, and spirals with considerably more gas and active star formation. Second, all galaxies undergo an initial burst of star formation which, in the case of disk galaxies, is turned off quite sharply.

It is difficult to know exactly how large the difference in star-formation rate will be between the two regimes. This is due to the fact that we observe that the two-dimensional regime is a well-controlled fairly stable phase, but we do not know whether this is also true of the initial three-dimensional regime. It is possible that this early phase is dominated by the dynamic effects of the collapse which could let the gas density and star-formation rate get quite high. We can, however, assume that the initial phase is regulated by the same feedback control as the disk phase in order to estimate the star-formation rate in the context of the present model. This rate will be a lower limit to the rates possible in the three-dimensional phase.

The star formation rate is determined by the solution of formula (1) with the following two equations:

$$P_{\mathrm{st}} = f(\rho_g)\,, \tag{2}$$

and

$$\rho_g = f(1/P_{\mathrm{st}}) = f(1/\rho_s) \tag{3}$$

(the f simply denotes a monotonic functional dependence on its argument). Equation (2) is just the dependence of P_{st} on the gas density, ρ_g, discussed before, while equation (3) represents the feedback mechanism. Now by formula (1) the star-formation rate depends on the difference between P_{st} and $P_{\mathrm{st}}^{(C)}$. However, since $P_{\mathrm{st}}^{(C)}$ is lower for the three-dimensional galaxy, $P_{\rm st}$ itself must be lower. By equation (2) this implies a smaller value of ρ_g and, therefore, by equation (3) a larger value of ρ_s. The point is that the steady-state star-formation rate in the three-dimensional phase will be greater than in the disk phase, the difference being proportional to a function of the difference of the critical probabilities. Since the condition for stable propagation is that on the average $NP = 1$, we expect the difference in $P_{\rm st}^{(C)}$ to vary as $1/N$. If we partition the three-dimensional space in a manner analogous to our partitioning of the two-dimensional model (Paper I), we find a factor of about 3 for the ratio of the critical probabilities.

In the critical region, the rate of star formation is sensitive to the velocity of rotation (Fig. 2); the higher the maximum velocity of rotation, the higher the rate of star formation. This property of the SSPSF mechanism predicts a well-defined set of correlations among properties of spiral galaxies. If all spiral galaxies are coeval, those that rotate faster (on the average, early-type galaxies; Rubin *et al.*) will have a smaller fractional gas content than spiral galaxies with a lower velocity of rotation. By the same token, due to their higher rate of star formation, spiral galaxies with higher velocities of rotation would appear redder than those with lower velocities because of the integrated effect of the larger number of old red stars (see, e.g., Searle, Sargent, and Bagnuolo 1973; Tinsley and Larsen (1978)). Finally, as discussed by Gerola and Seiden (1978), the differential rotation directly determines the morphological type. It is gratifying that observations of the gas mass (Roberts 1975), color (de Vaucouleurs 1977), and morphological type (Rubin *et al.*) are in accord with these predictions. A more detailed treatment of these correlations in the context of the SSPSF model is given in a separate paper (Seiden and Gerola 1979).

IV. CONCLUSIONS

The SSPSF model is able to give a quite good qualitative picture of the evolution and differentiation of galactic forms. It explicitly shows how the difference between elliptical and spiral galaxies arises from the competition between the large initial-star-formation rate in the three-dimensional regime and the collapse rate of the gas cloud to a disk form. If the star-formation rate is large enough in comparison to the collapse rate, an elliptical galaxy will result. If, on the other hand, the collapse is faster, a disk galaxy will result.

Furthermore, in the disk phase the model shows that the star-formation rate is a function of the differential rotation of the galaxy. We expect that this can account for the observed correlations of rotation velocity, color, and gas fraction with morphological type.

The consideration of the phase transition nature of the galactic system, in addition to providing a natural explanation for the elliptic and spiral forms, also provides the characteristics necessary for a feedback mechanism which can stabilize disk galaxies in the region of the formation of distinct spiral arms.

The scenario described in this paper derives from general properties of the SSPSF mechanism. The only assumption is that on a time average, $P_{\rm st}$ is a monotonic function of the density. The descriptions given here are by necessity qualitative; however, we believe the total picture is suggestive enough to be considered seriously. Whether this model does indeed provide a correct description of the evolution of galaxies will, of course, depend on obtaining a good quantitative description of these processes.

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