

Using Laplace Transforms for Circuit Analysis

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Digital Technium 123

Office Hours: 12:00-13:00 Mondays

You can view the notes for this presentation in [HTML](#) and [PDF](#).

The source code of this presentation is available in Markdown format from GitHub: [circuit_analysis.md](#).

The GitHub repository [EG-247 Resources](#) also contains the source code for all the Matlab/Simulink examples and the Laboratory Exercises.

This Week

This week's sessions are based on Chapter 4 **Circuit Analysis with Laplace Transforms** from Steven T. Karris [Signals and Systems: with MATLAB Computing and Simulink Modelling \(5th Edition\)](#) [You need University Login to access]

Today's Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

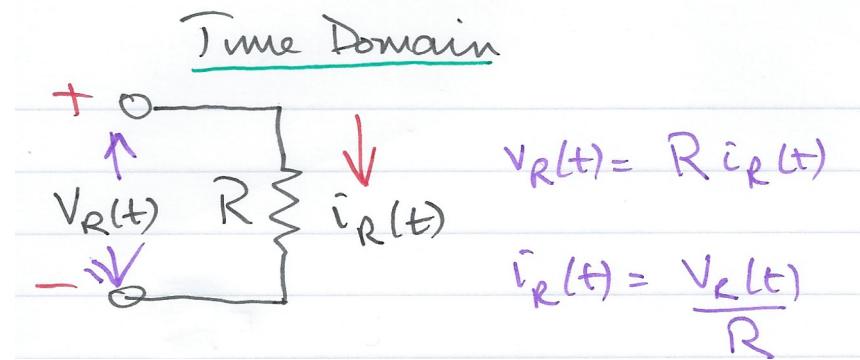


Figure 1: Resistive Network - Time Domain

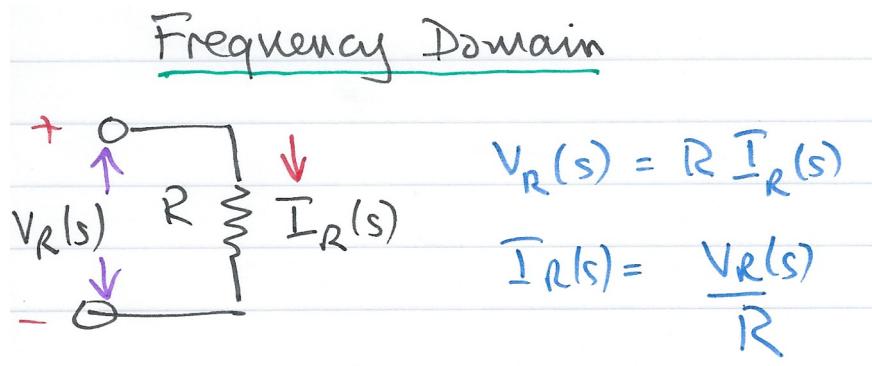


Figure 2: Resistive Network - Complex Frequency Domain

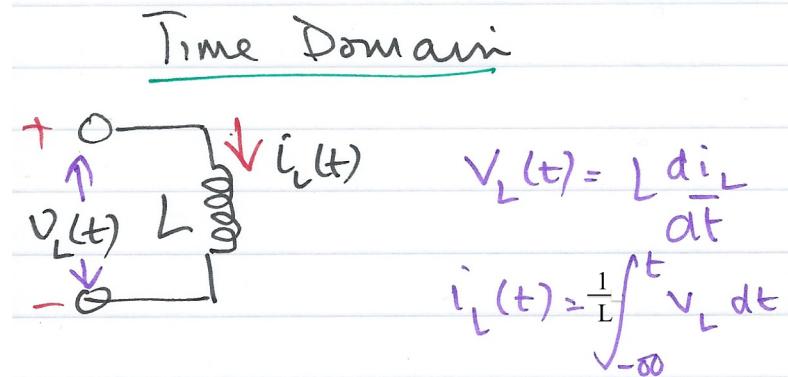


Figure 3: Inductive Network - Time Domain

Frequency Domain

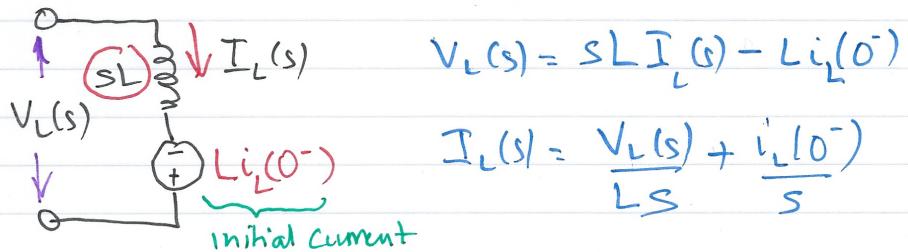


Figure 4: Inductive Network - Complex Frequency Domain

Time Domain

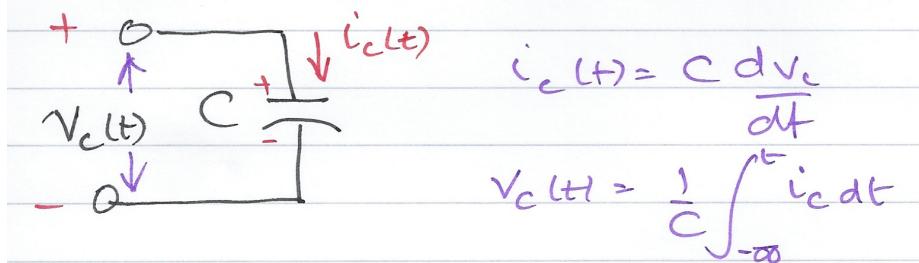


Figure 5: Capacitive Network - Time Domain

Circuit Transformation from Time to Complex Frequency

Resistive Network - Time Domain

Resistive Network - Complex Frequency Domain

Inductive Network - Time Domain

Inductive Network - Complex Frequency Domain

Capacitive Network - Time Domain

Capacitive Network - Complex Frequency Domain

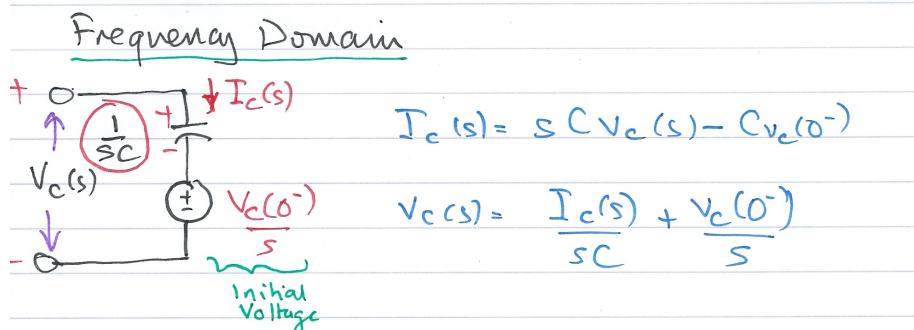


Figure 6: Capacitive Network - Complex Frequency Domain

Examples

Example 1

Use the Laplace transform method and apply Kichhoff's Current Law (KCL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.

Example 2

Use the Laplace transform method and apply Kichhoff's Voltage Law (KVL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that

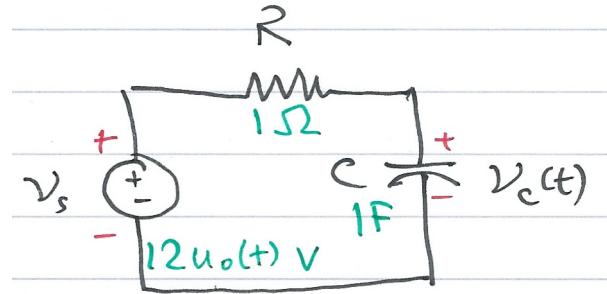


Figure 7: Circuit for Example 1

$$v_c(0^-) = 6 \text{ V.}$$

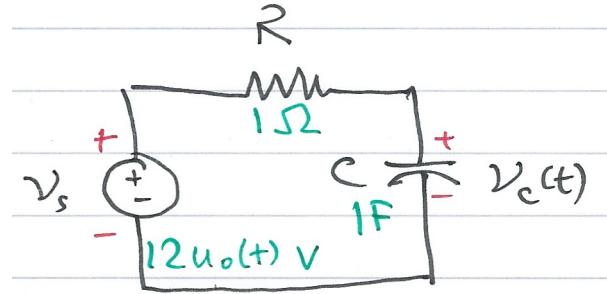


Figure 8: Circuit for Example 2

Example 3

In the circuit below, switch S_1 closes at $t = 0$, while at the same time, switch S_2 opens. Use the Laplace transform method to find $v_{\text{out}}(t)$ for $t > 0$.

See Blackboard for worked solution.

Show with the assistance of MATLAB (See [solution3.m](#)) that the solution is

$$V_{\text{out}} = (1.36e^{-6.57t} + 0.64e^{-0.715t} \cos 0.316t - 1.84e^{-0.715t} \sin 0.316t) u_0(t)$$

and plot the result.

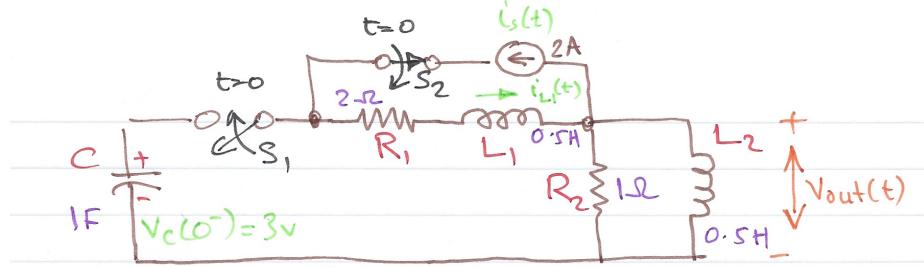


Figure 9: Circuit for Example 3

Plot of time response

Complex Impedance $Z(s)$

Consider the s -domain RLC series circuit, where the initial conditions are assumed to be zero.

For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents the total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s -domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

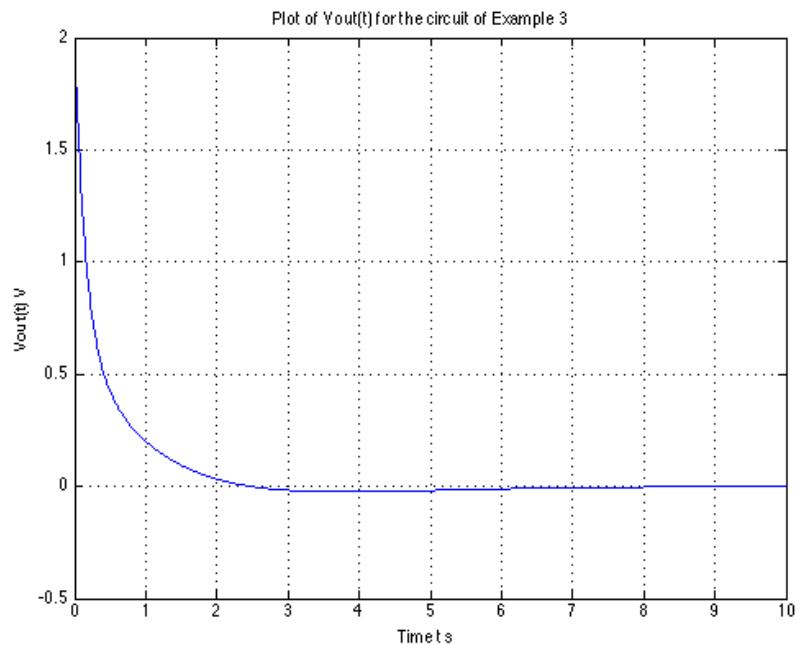


Figure 10: Plot of time response

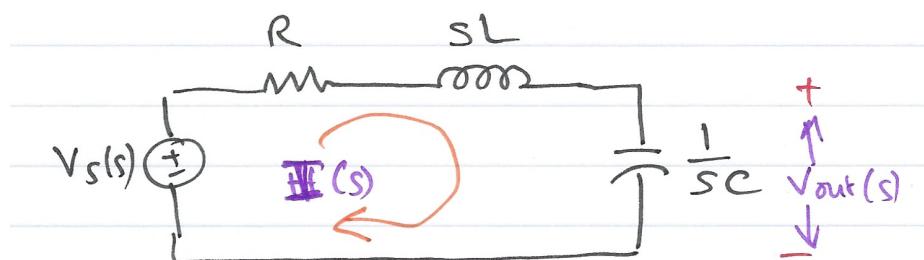


Figure 11: Complex Impedance $Z(s)$

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Exercise

Use the previous result to give an expression for $V_c(s)$

Example 4

For the network shown below, all the complex impedance values are given in Ω (ohms).

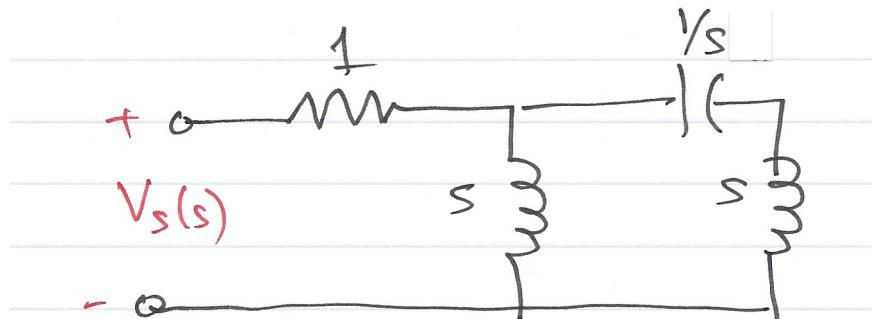


Figure 12: Circuit for example 4

Find $Z(s)$ using:

1. nodal analysis
2. successive combinations of series and parallel impedances

Complex Admittance $Y(s)$

Consider the s -domain GLC parallel circuit shown below where the initial conditions are zero.

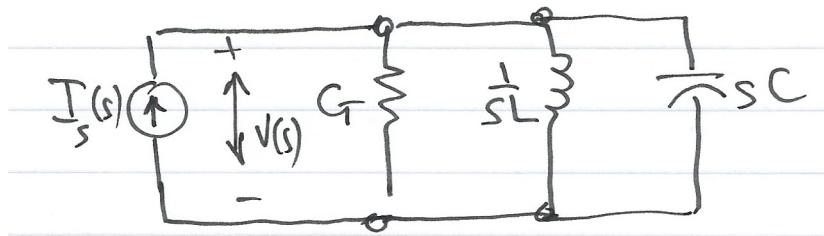


Figure 13: Complex admittance $Y(s)$

For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC \right) V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s -domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Example 5 - Do It Yourself

Compute $Z(s)$ and $Y(s)$ for the circuit shown below. All impedance values are in Ω (ohms). Verify your answers with MATLAB.

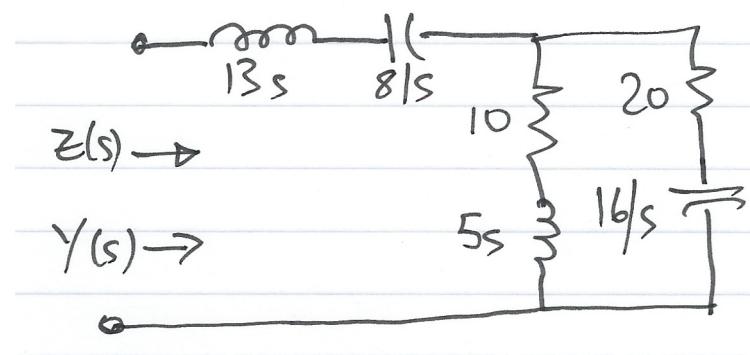


Figure 14: Circuit for Example 5

Answer 5

$$Z(s) = \frac{65s^4 + 490s^3 + 528s^2 + 400s + 128}{s(5s^2 + 30s + 16)}$$

$$Y(s) = \frac{1}{s} = \frac{s(5s^2 + 30s + 16)}{65s^4 + 490s^3 + 528s^2 + 400s + 128}$$

Matlab verification: [solution5.m](#)

Next Lesson

- Transfer Functions of Circuits ([Notes PDF](#), [Slides PDF](#))

Homework

Do the end of the chapter exercises (Section 4.7 - questions 1 to 4) from the textbook. Don't look at the answers until you have attempted the problems.

Lab Work

In the lab, week on Friday, we will see the use of Matlab and Simulink in the solution of circuit problems.