

# Using Laplace Transforms for Circuit Analysis

# This Week

This week's sessions are based on Chapter 4 **Circuit Analysis with Laplace Transforms** from Steven T. Karris Signals and Systems: with MATLAB Computing and Simulink Modelling (5th Edition)  
[You need University Login to access]

# Today's Agenda

We look at applications of the Laplace Transform for

- ▶ Circuit transformation from Time to Complex Frequency
- ▶ Complex impedance
- ▶ Complex admittance

# Circuit Transformation from Time to Complex Frequency

## Resistive Network - Time Domain

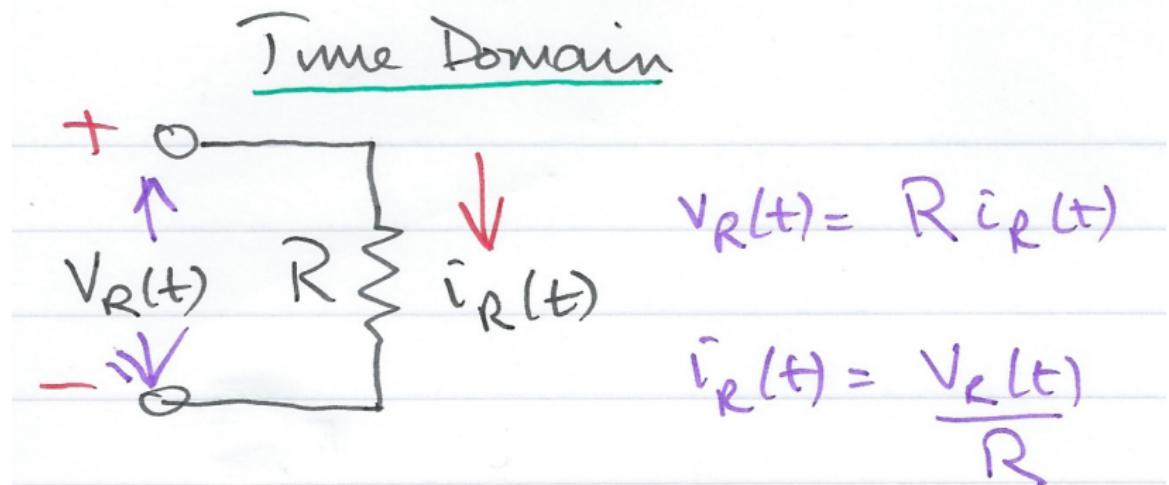
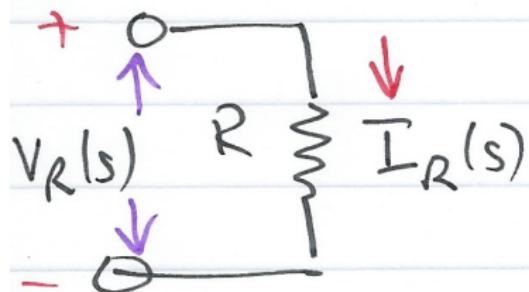


Figure 1: Resistive Network - Time Domain

## Resistive Network - Complex Frequency Domain

Frequency Domain



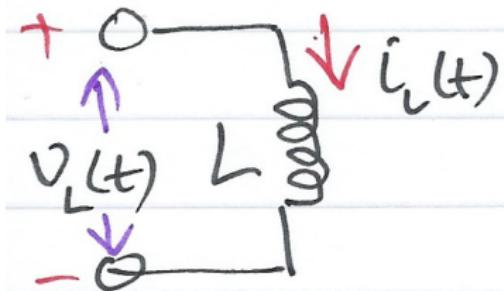
$$V_R(s) = R I_R(s)$$

$$I_R(s) = \frac{V_R(s)}{R}$$

Figure 2: Resistive Network - Complex Frequency Domain

## Inductive Network - Time Domain

### Time Domain



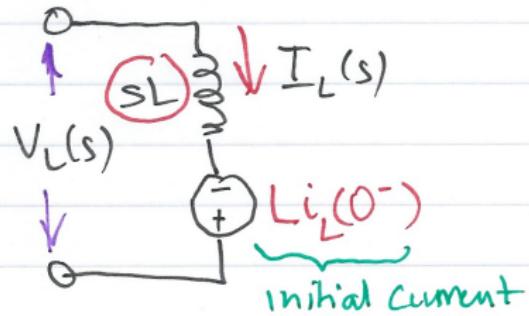
$$V_L(t) = L \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L} \int_{-\infty}^t V_L dt$$

Figure 3: Inductive Network - Time Domain

# Inductive Network - Complex Frequency Domain

## Frequency Domain



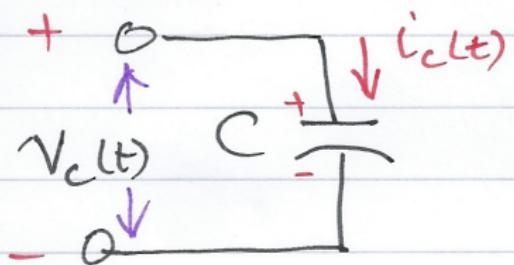
$$V_L(s) = sL I_L(s) - L i_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$

Figure 4: Inductive Network - Complex Frequency Domain

## Capacitive Network - Time Domain

### Time Domain



$$i_c(t) = C \frac{dV_c}{dt}$$

$$V_c(t) = \frac{1}{C} \int_{-\infty}^t i_c dt$$

Figure 5: Capacitive Network - Time Domain

# Capacitive Network - Complex Frequency Domain

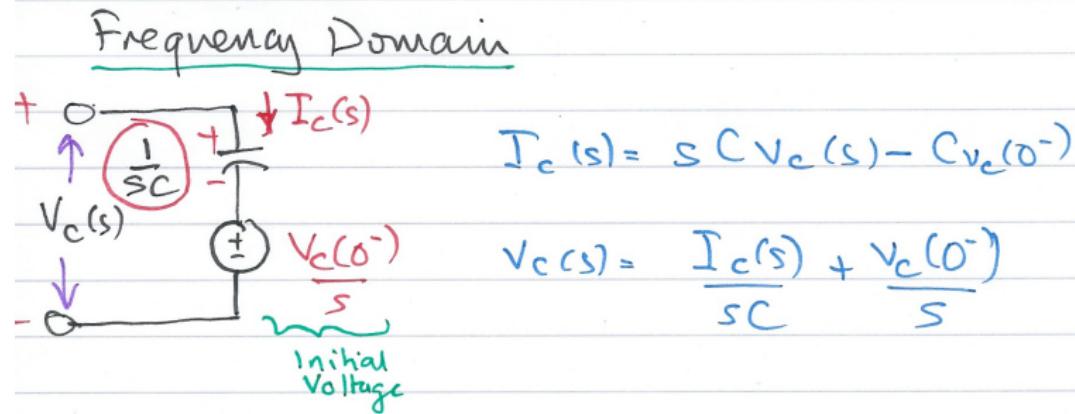


Figure 6: Capacitive Network - Complex Frequency Domain

# Examples

## Example 1

Use the Laplace transform method and apply Kichhoff's Current Law (KCL) to find the voltage  $v_c(t)$  across the capacitor for the circuit below given that  $v_c(0^-) = 6 \text{ V}$ .

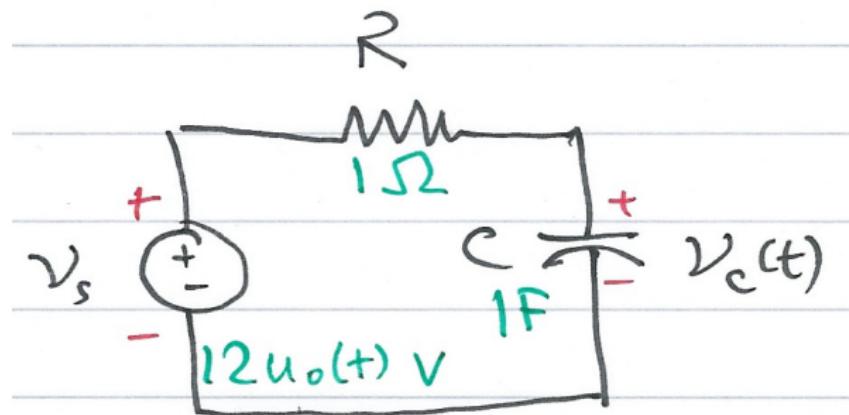


Figure 7: Circuit for Example 1

## Example 2

Use the Laplace transform method and apply Kichhoff's Voltage Law (KVL) to find the voltage  $v_c(t)$  across the capacitor for the circuit below given that  $v_c(0^-) = 6 \text{ V}$ .

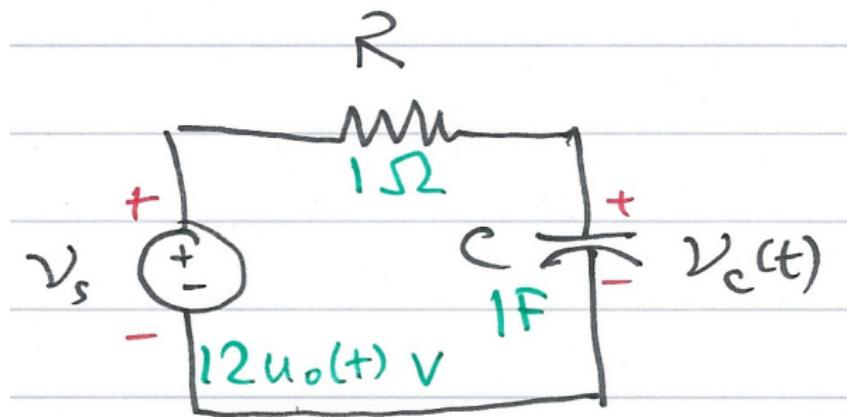


Figure 8: Circuit for Example 2

## Example 3

In the circuit below, switch  $S_1$  closes at  $t = 0$ , while at the same time, switch  $S_2$  opens. Use the Laplace transform method to find  $v_{\text{out}}(t)$  for  $t > 0$ .

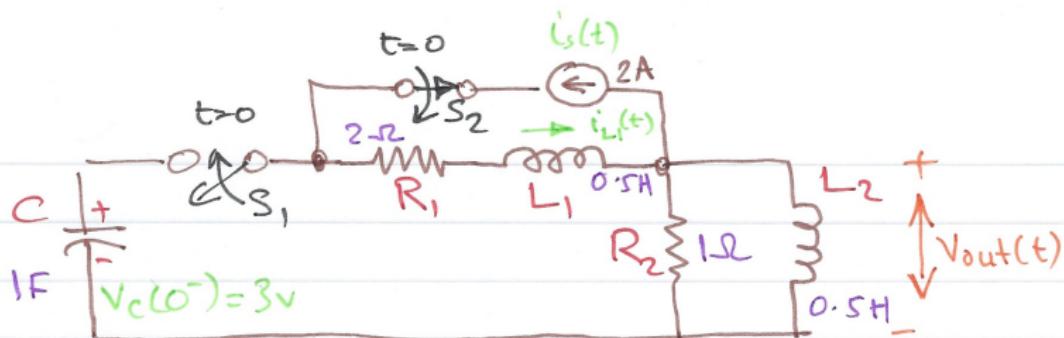


Figure 9: Circuit for Example 3

Show with the assistance of MATLAB (See solution3.m) that the solution is

$$V_{\text{out}} = \left( 1.36e^{-6.57t} + 0.64e^{-0.715t} \cos 0.316t - 1.84e^{-0.715t} \sin 0.316t \right) u_0$$

and plot the result.

## Plot of time response

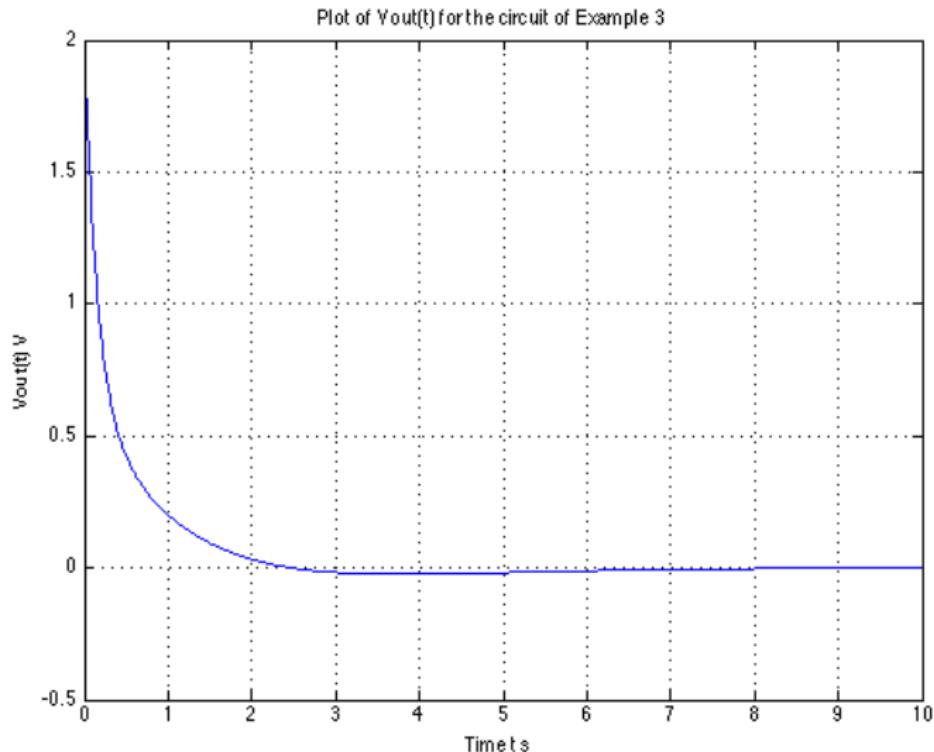


Figure 10: Plot of time response

## Complex Impedance $Z(s)$

Consider the  $s$ -domain RLC series circuit, where the initial conditions are assumed to be zero.

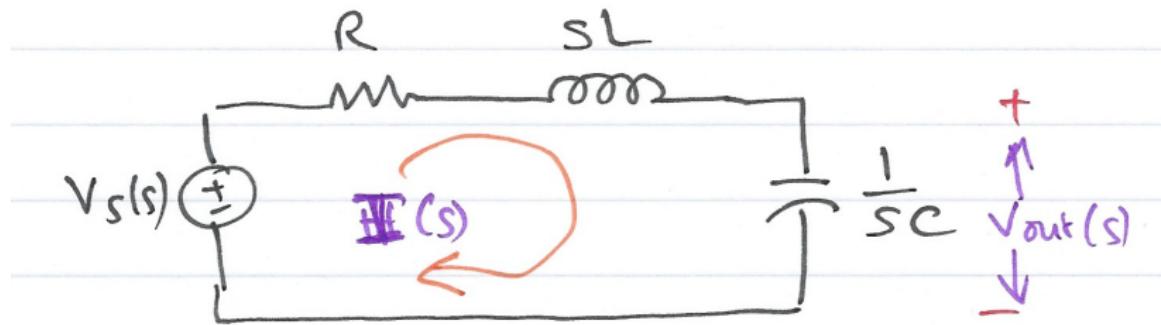


Figure 11: Complex Impedance  $Z(s)$

For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents the total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio  $V_s(s)/I(s)$  as  $Z(s)$ , we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The  $s$ -domain current  $I(s)$  can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since  $s = \sigma + j\omega$  is a complex number,  $Z(s)$  is also complex and is known as the *complex input impedance* of this RLC series circuit.

## Exercise

Use the previous result to give an expression for  $V_c(s)$

## Example 4

For the network shown below, all the complex impedance values are given in  $\Omega$  (ohms).

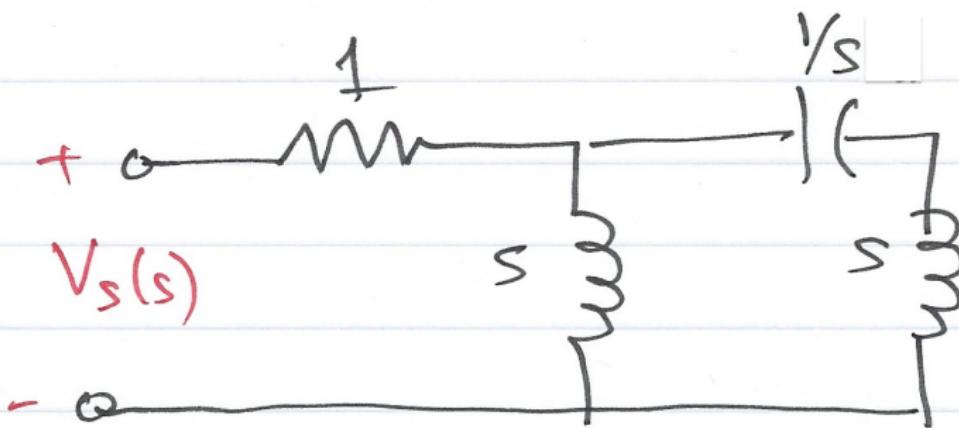


Figure 12: Circuit for example 4

Find  $Z(s)$  using:

1. nodal analysis
2. successive combinations of series and parallel impedances

## Complex Admittance $Y(s)$

Consider the  $s$ -domain GLC parallel circuit shown below where the initial conditions are zero.

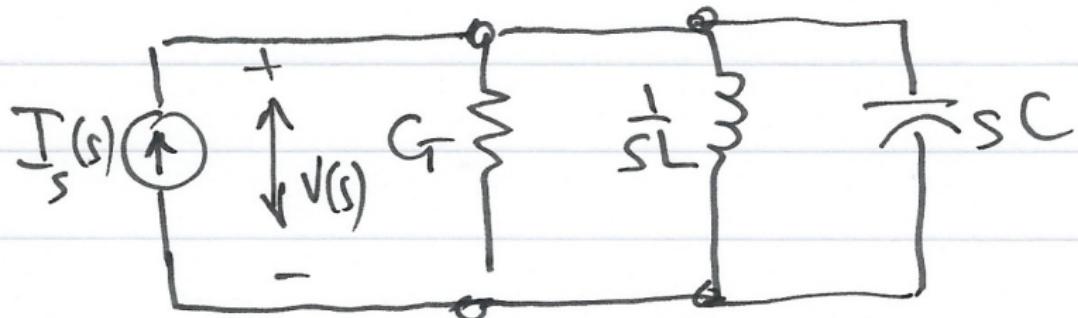


Figure 13: Complex admittance  $Y(s)$

For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left( G + \frac{1}{sL} + sC \right) V(s) = I_s(s)$$

Defining the ratio  $I_s(s)/V(s)$  as  $Y(s)$  we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The  $s$ -domain voltage  $V(s)$  can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$  is complex and is known as the *complex input admittance* of this GLC parallel circuit.

## Example 5 - Do It Yourself

Compute  $Z(s)$  and  $Y(s)$  for the circuit shown below. All impedance values are in  $\Omega$  (ohms). Verify your answers with MATLAB.

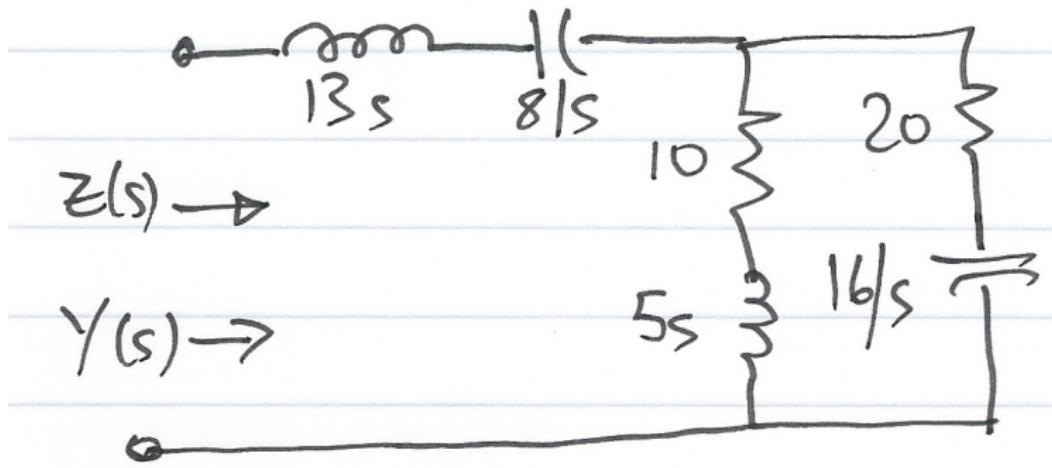


Figure 14: Circuit for Example 5

## Answer 5

$$Z(s) = \frac{65s^4 + 490s^3 + 528s^2 + 400s + 128}{s(5s^2 + 30s + 16)}$$

$$Y(s) = \frac{1}{s} = \frac{s(5s^2 + 30s + 16)}{65s^4 + 490s^3 + 528s^2 + 400s + 128}$$

Matlab verification: solution5.m

## Next Lesson

- ▶ Transfer Functions of Circuits (Notes PDF, Slides PDF)

# Homework

Do the end of the chapter exercises (Section 4.7 - questions 1 to 4) from the textbook. Don't look at the answers until you have attempted the problems.

# Lab Work

In the lab, week on Friday, we will see the use of Matlab and Simulink in the solution of circuit problems.