# Using Laplace Transforms for Circuit Analysis

# This Week

This week's sessions are based on Chapter 4 **Circuit Analysis with Laplace Transforms** from Steven T. Karris Signals and Systems: with MATLAB Computing and Simulink Modelling (5th Edition) [You need University Login to access]

# Today's Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

# Circuit Transformation from Time to Complex Frequency

# Resistive Network - Time Domain

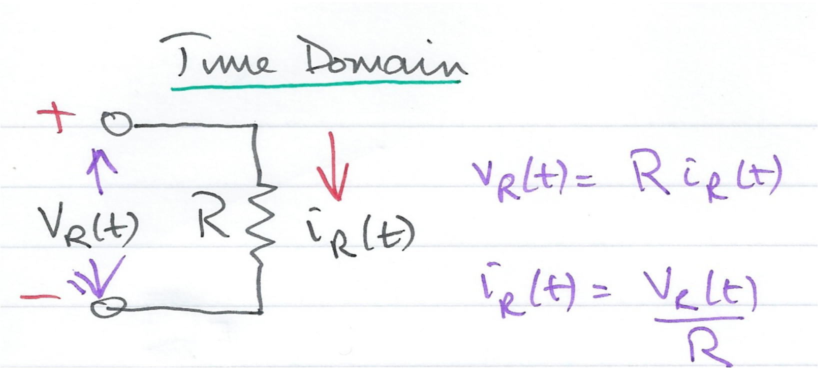


Figure 1: Resistive Network - Time Domain

# Resistive Network - Complex Frequency Domain

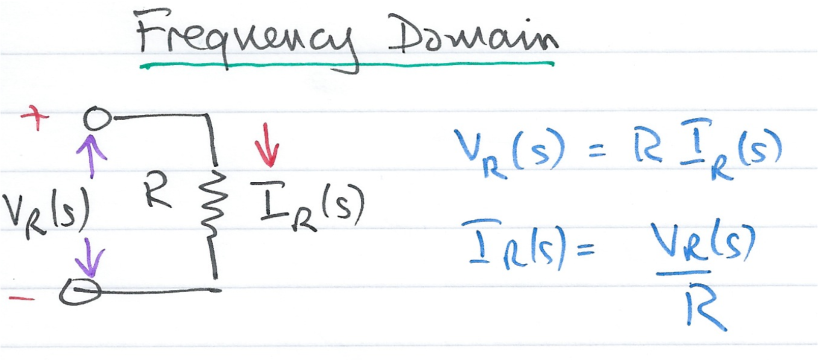


Figure 2: Resistive Network - Complex Frequency Domain

# Inductive Network - Time Domain

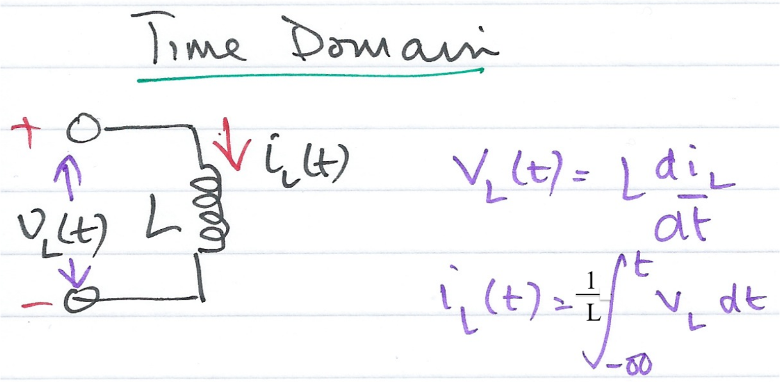


Figure 3: Inductive Network - Time Domain

# Inductive Network - Complex Frequency Domain

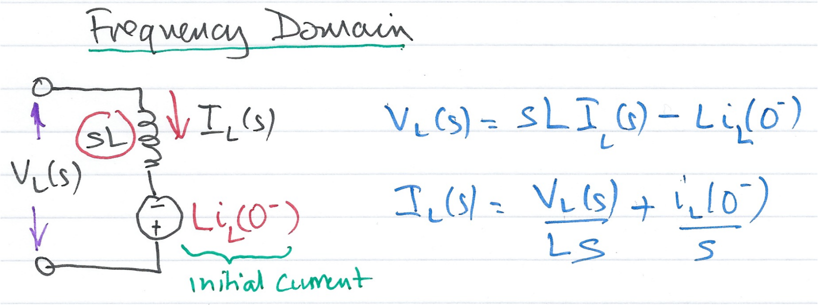


Figure 4: Inductive Network - Complex Frequency Domain

# Capacitive Network - Time Domain

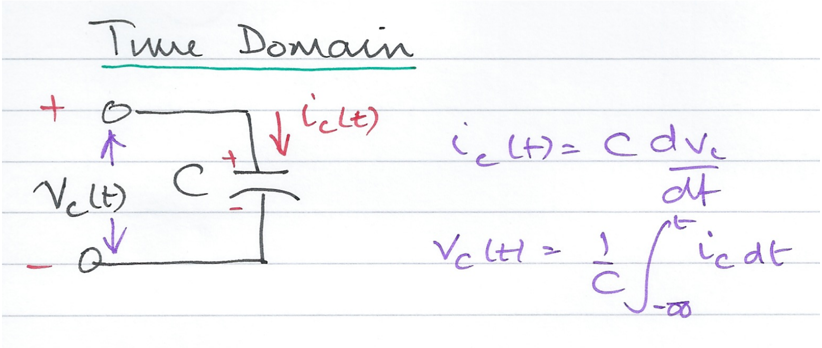


Figure 5: Capacitive Network - Time Domain

# Capacitive Network - Complex Frequency Domain

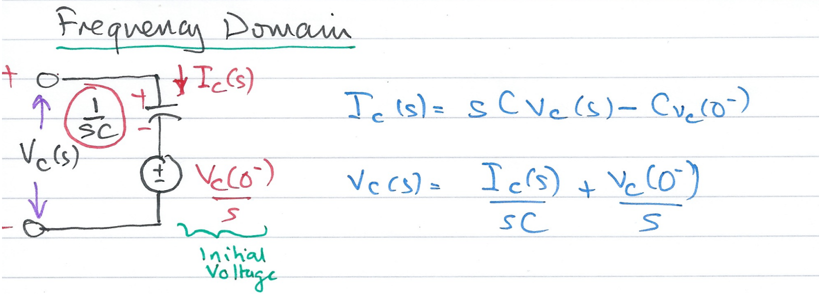


Figure 6: Capacitive Network - Complex Frequency Domain

# Examples

# Example 1

Use the Laplace transform method and apply Kichoff's Current Law (KCL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.

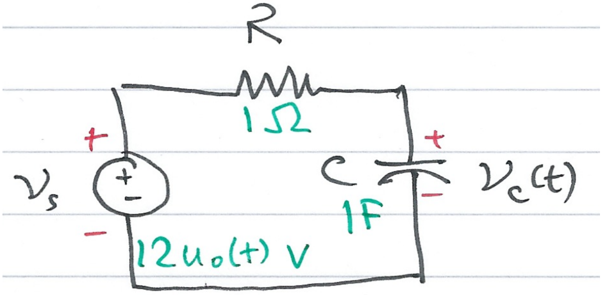


Figure 7: Circuit for Example 1

# Example 2

Use the Laplace transform method and apply Kichoff's Voltage Law (KVL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.

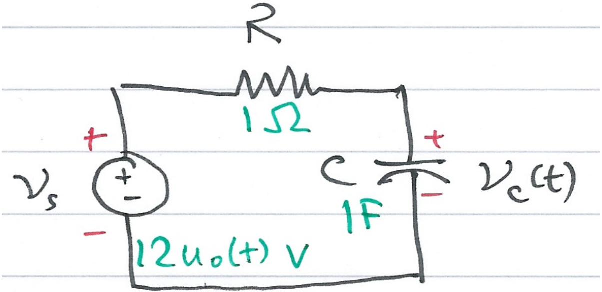


Figure 8: Circuit for Example 2

# Example 3

In the circuit below, switch $S_1$ closes at $t = 0$, while at the same time, switch $S_2$ opens. Use the Laplace transform method to find $v_{\text{out}}(t)$ for $t > 0$.

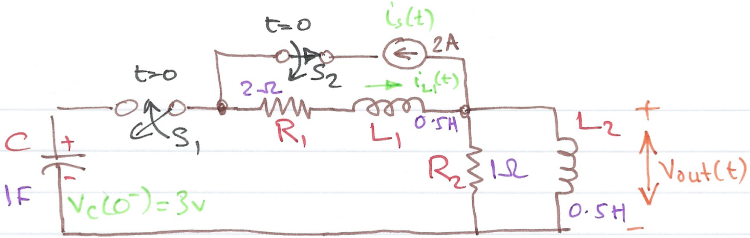


Figure 9: Circuit for Example 3

Show with the assistance of MATLAB (See solution3.m) that the solution is

$$V_{\mathrm{out}} = \left(1.36e^{-6.57t} + 0.64e^{-0.715t}\cos 0.316t - 1.84e^{-0.715t}\sin 0.316t\right) u_0$$

and plot the result.

# Plot of time response

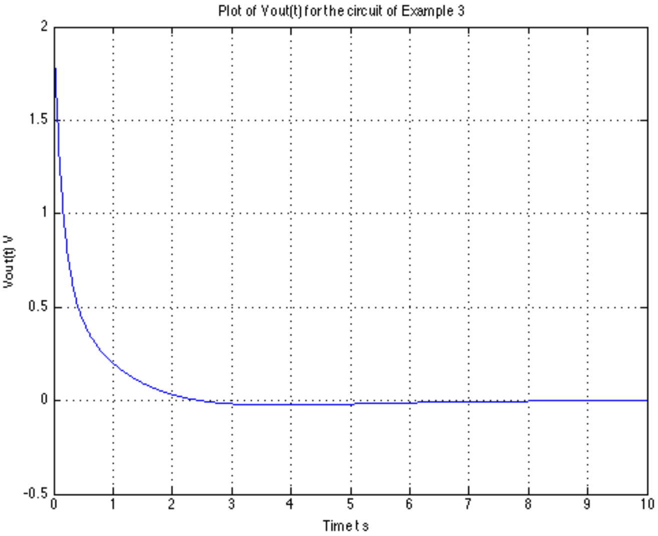


Figure 10: Plot of time response

# Complex Impedance Z(s)

Consider the $s$-domain RLC series circuit, wehere the initial conditions are assumed to be zero.

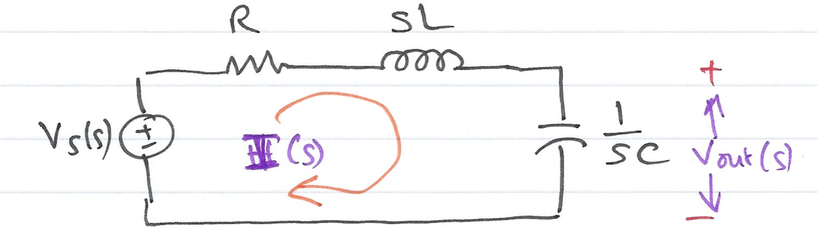


Figure 11: Complex Impedance $Z(s)$

For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

respresents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The $s$-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

# Exercise

Use the previous result to give an expression for $V_c(s)$

# Example 4

For the network shown below, all the complex impedence values are given in Ω (ohms).

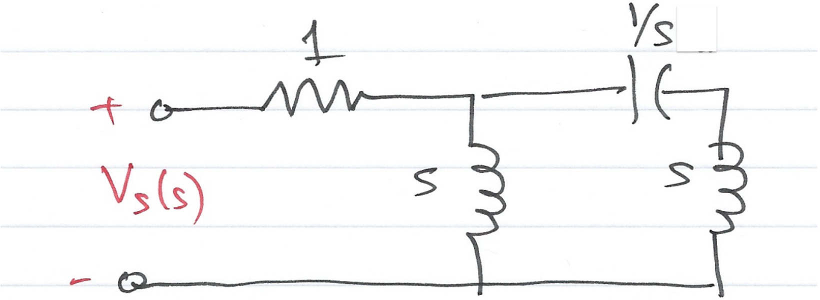


Figure 12: Circuit for example 4

Find $Z(s)$ using:

1. nodal analysis
2. successive combinations of series and parallel impendences

# Complex Admittance Y(s)

Consider the $s$-domain GLC parallel circuit shown below where the initial conditions are zero.

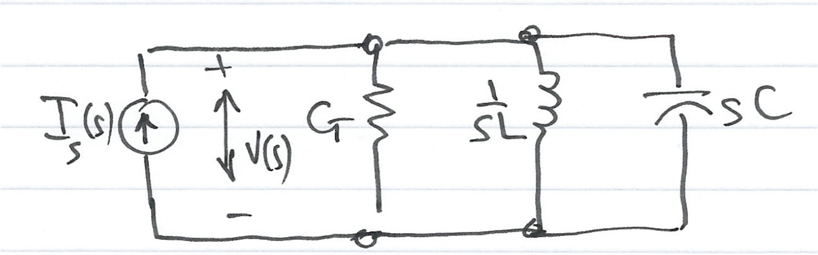


Figure 13: Complex admittance $Y(s)$

For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right) V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The $s$-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

# Example 5 - Do It Yourself

Compute $Z(s)$ and $Y(s)$ for the circuit shown below. All impedence values are in Ω (ohms). Verify your answers with MATLAB.

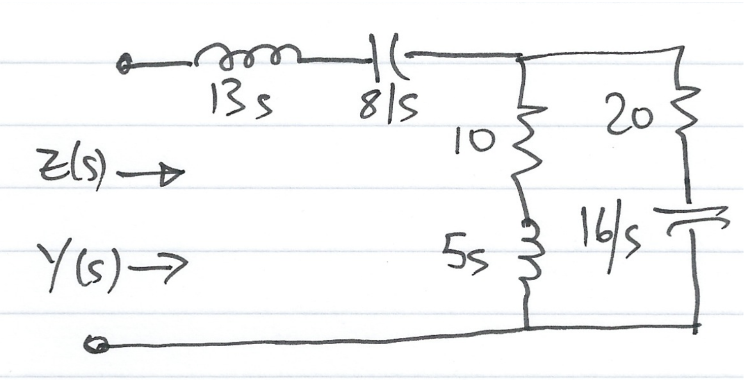


Figure 14: Circuit for Example 5

# Answer 5

$$Z(s) = \frac{65s^4 + 490s^3 + 528s^2 + 400s + 128}{s(5s^2 + 30s + 16)}$$

$$Y(s) = \frac{1}{s} = \frac{s(5s^2 + 30s + 16)}{65s^4 + 490s^3 + 528s^2 + 400s + 128}$$

Matlab verification: solution5.m

# Next Lesson

- Transfer Functions of Circuits (Notes PDF, Slides PDF)

# Homework

Do the end of the chapter exercises (Section 4.7 - questions 1 to 4) from the textbook. Don't look at the answers until you have attempted the problems.

# Lab Work

In the lab, week on Friday, we will see the use of Matlab and Simulink in the solution of circuit problems.