Using Laplace Transforms for Circuit Analysis

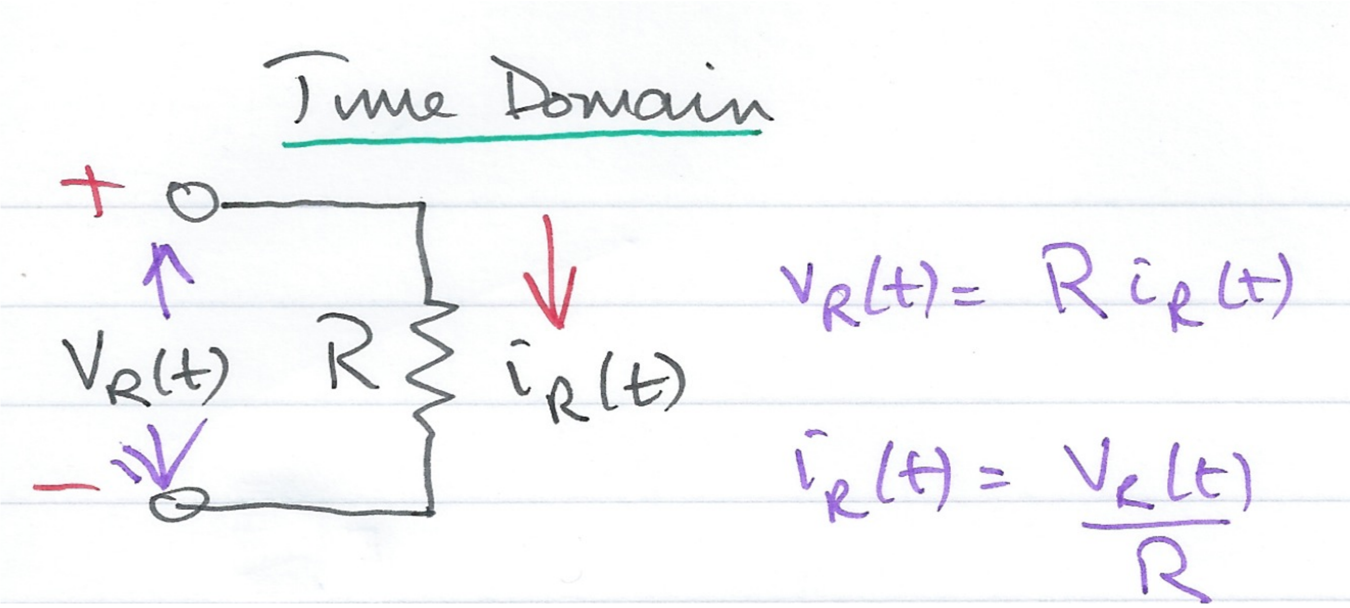
Today's Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

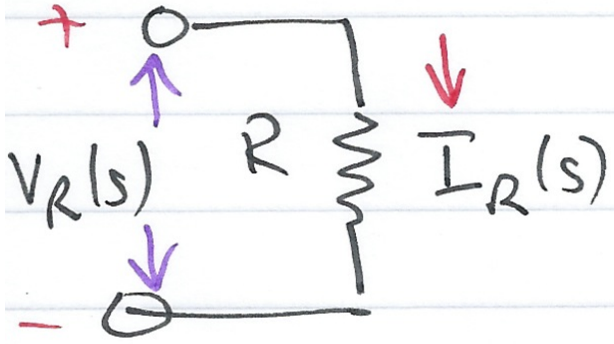
Circuit Transformation from Time to Complex Frequency

Resistive Network - Time Domain



Resistive Network - Complex Frequency Domain

Frequency Domain

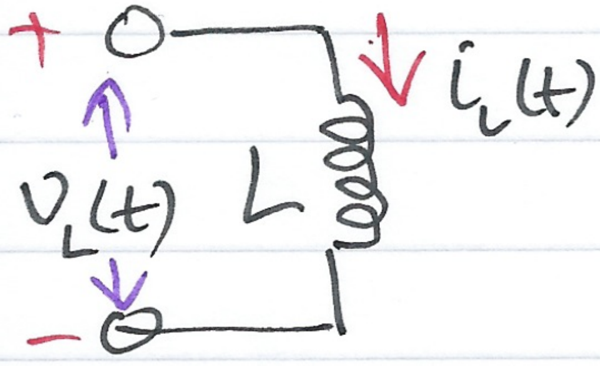


$$V_R(s) = R\,I_R(s)$$

$$I_R(s) = \frac{V_R(s)}{R}$$

Inductive Network - Time Domain

Time Domain

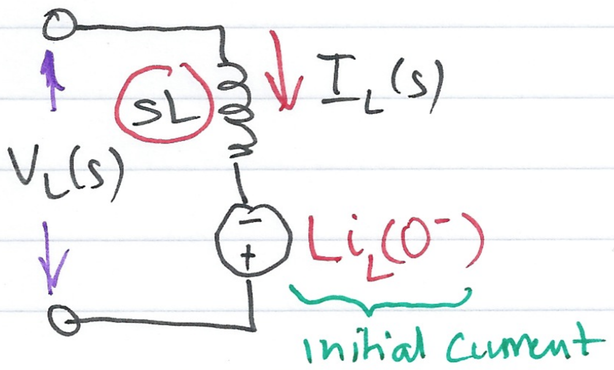


$$V_L(t) = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L\,dt$$

Inductive Network - Complex Frequency Domain

Frequency Domain

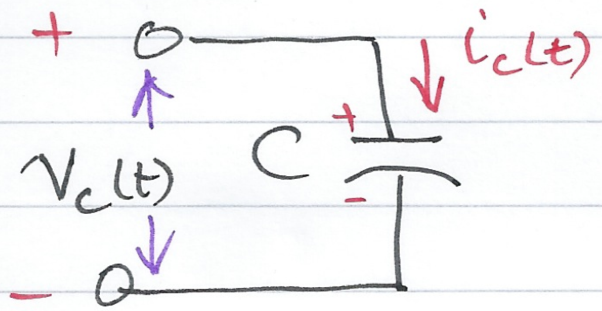


$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

Capacitive Network - Time Domain

Time Domain

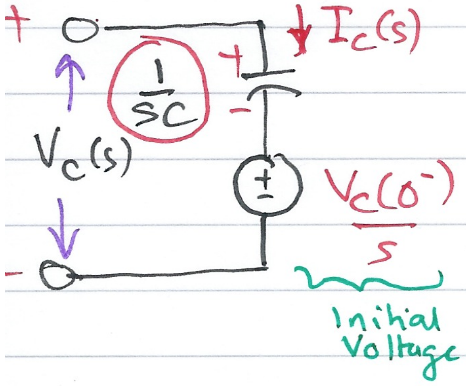


$$i_c(t) = C\frac{dv_c}{dt}$$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c\,dt$$

Capacitive Network - Complex Frequency Domain

Frequency Domain



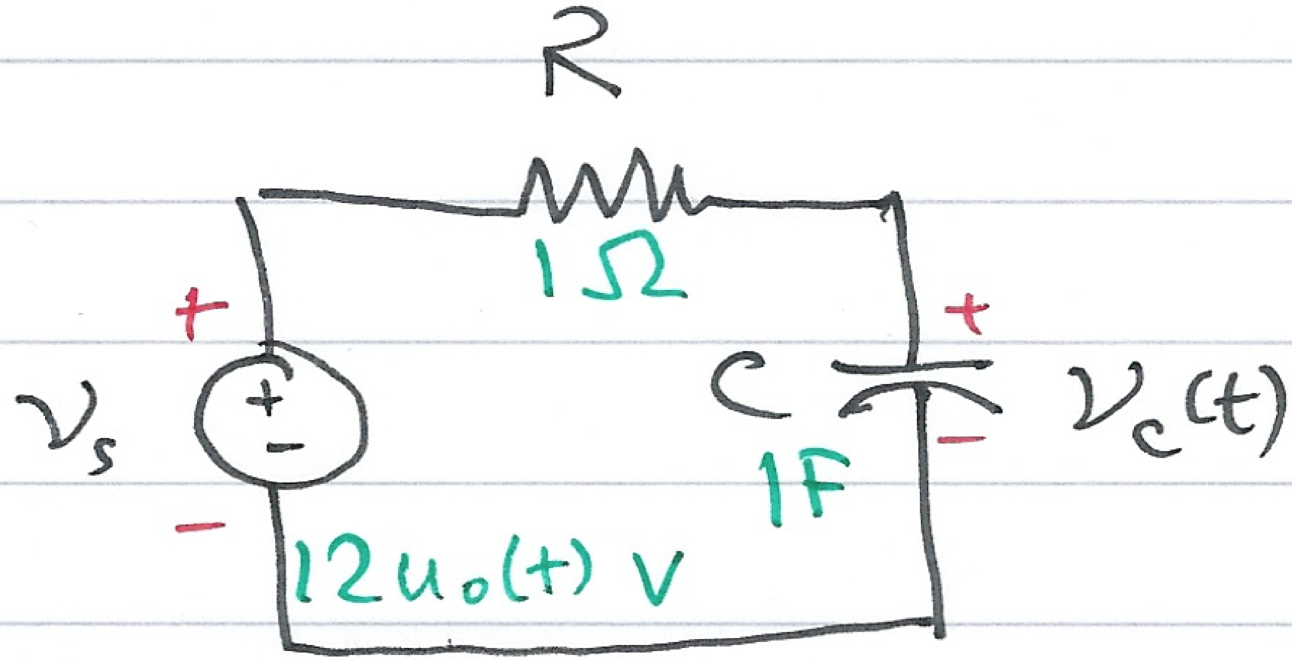
$$I_c(s) = sCV_c(s) - Cv_c(0^-)$$

$$V_c(s) = \frac{I_c(s)}{sC} + \frac{V_c(0^-)}{s}$$

Examples

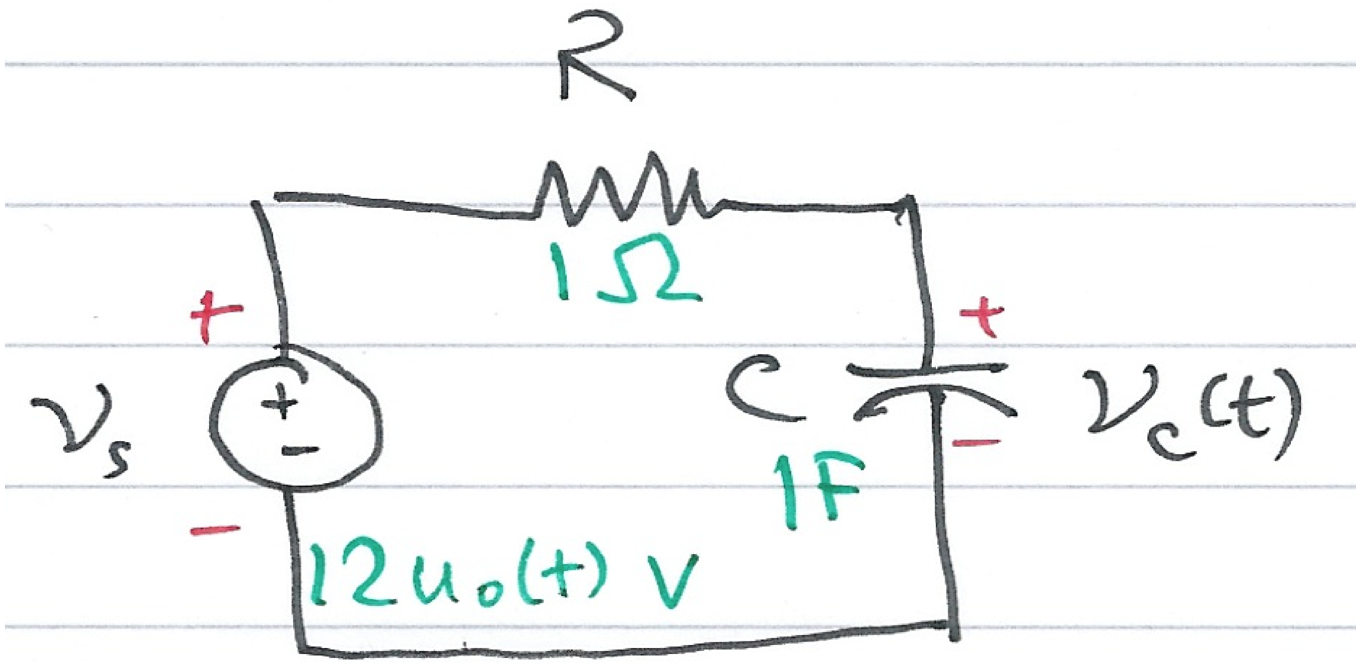
Example 1

Use the Laplace transform method and apply Kirchoff's Current Law (KCL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.



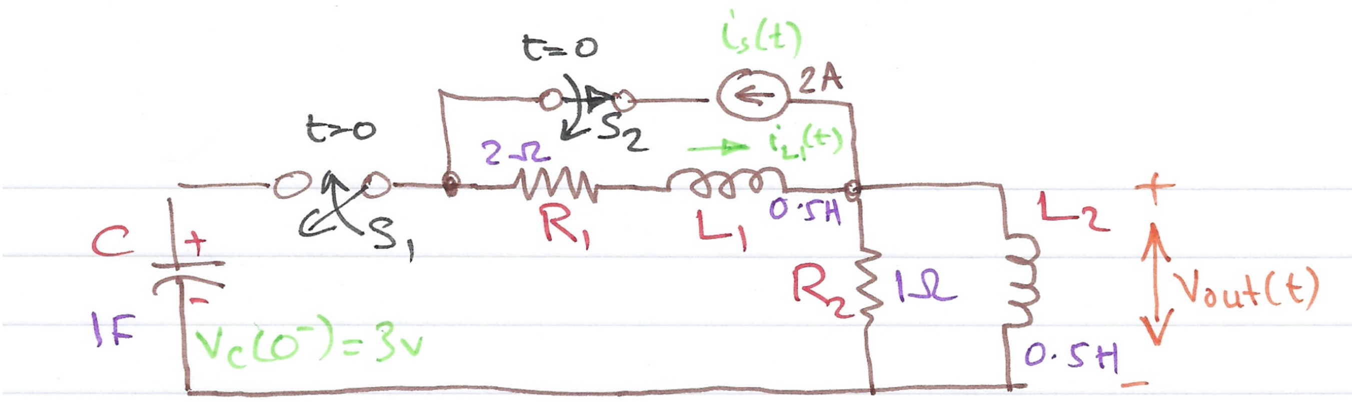
Example 2

Use the Laplace transform method and apply Kirchoff's Voltage Law (KVL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.



Example 3

In the circuit below, switch S_1 closes at $t = 0$, while at the same time, switch S_2 opens. Use the Laplace transform method to find $v_{\text{out}}(t)$ for $t > 0$.



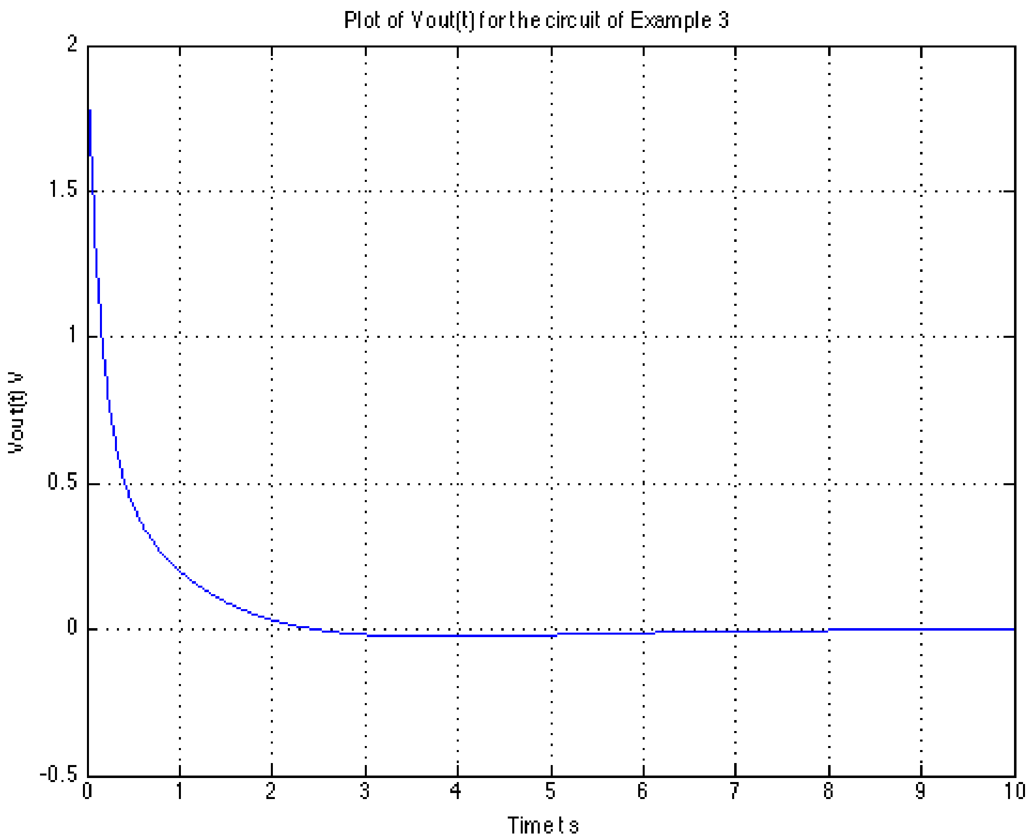
Show with the assistance of MATLAB (See solution3.m (matlab/solution3.m)) that the solution is

$$V_{\text{out}} = \left(1.36e^{-6.57t} + 0.64e^{-0.715t}\cos 0.316t - 1.84e^{-0.715t}\sin 0.316t\right) u_0(t)$$

and plot the result.

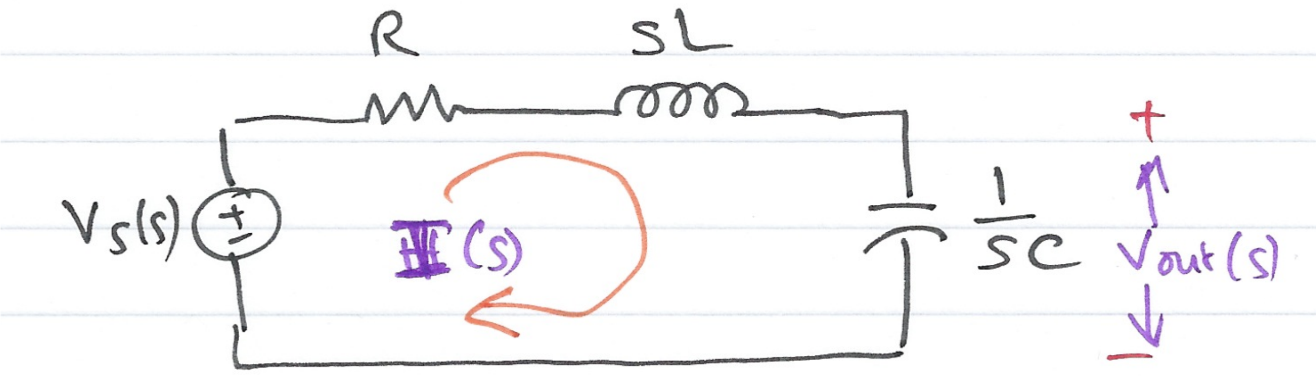
See Blackboard for worked solution.

Plot of time response



Complex Impedance Z(s)

Consider the s-domain RLC series circuit, wehere the initial conditions are assumed to be zero.



For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

$$Z(s) = R + sL + \frac{1}{sC}.$$

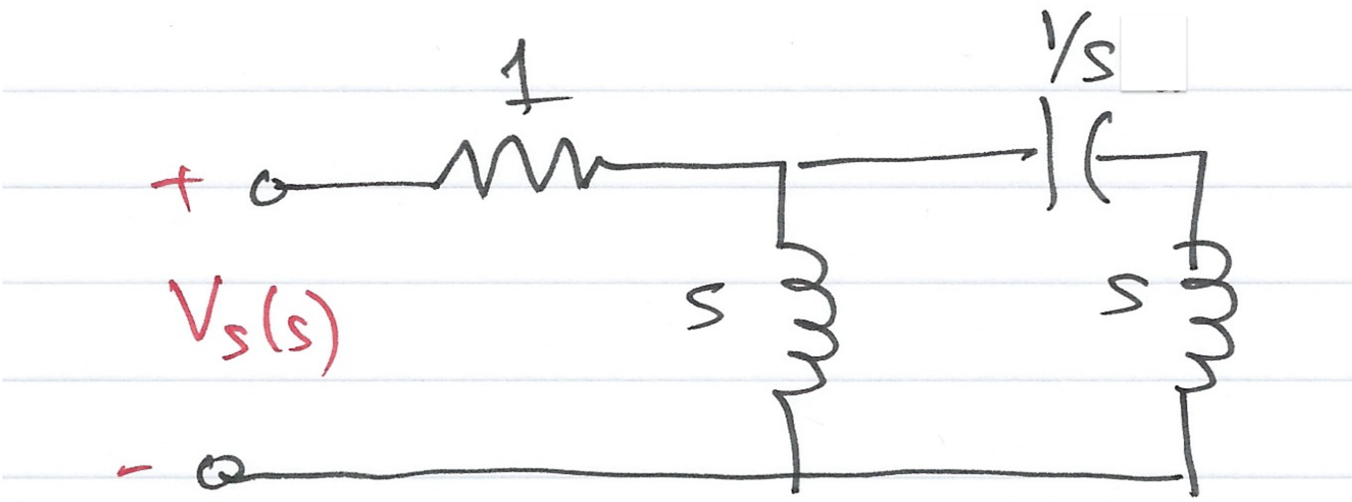
Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Exercise

Use the previous result to give an expression for $V_c(s)$

Example 4

For the network shown below, all the complex impedence values are given in Ω (ohms).

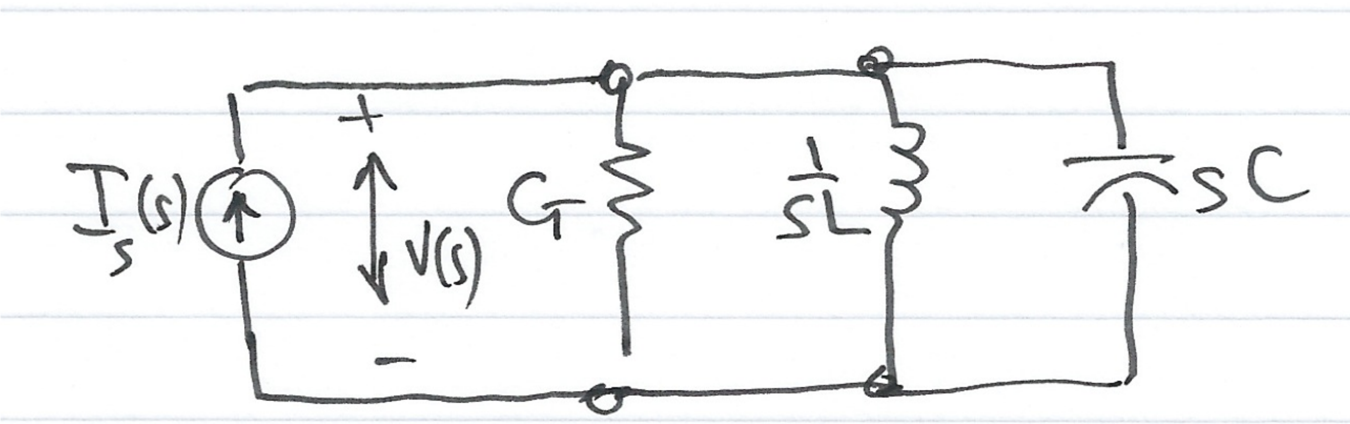


Find $Z(s)$ using:

1. nodal analysis
2. successive combinations of series and parallel impedances

Complex Admittance Y(s)

Consider the s-domain GLC parallel circuit shown below where the initial conditions are zero.



For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right) V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

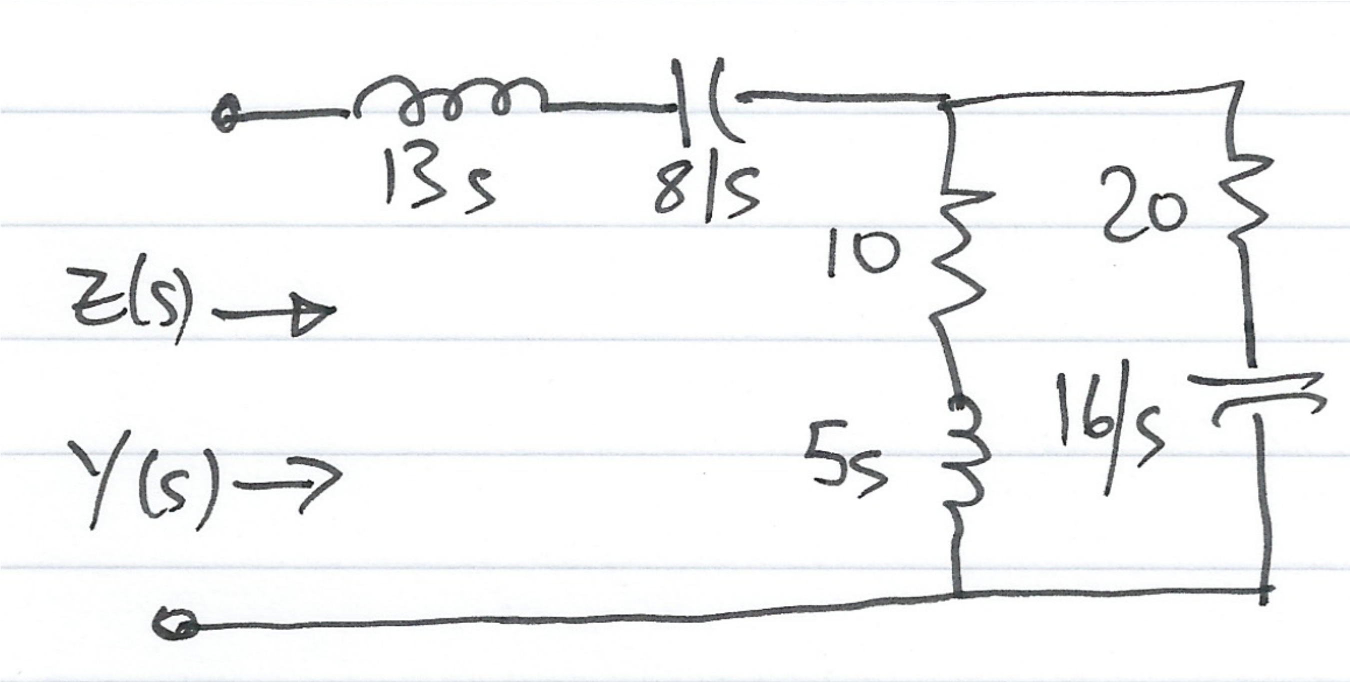
where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Example 5 - Do It Yourself

Compute $Z(s)$ and $Y(s)$ for the circuit shown below. All impedence values are in Ω (ohms). Verify your answers with MATLAB.



Answer 5

$$Z(s) = \frac{65s^4 + 490s^3 + 528s^2 + 400s + 128}{s(5s^2 + 30s + 16)}$$

$$Y(s) = \frac{1}{s} = \frac{s(5s^2 + 30s + 16)}{65s^4 + 490s^3 + 528s^2 + 400s + 128}$$

Matlab verification: solution5.m (matlab/solution5.m)