

# Using Laplace Transforms for Circuit Analysis

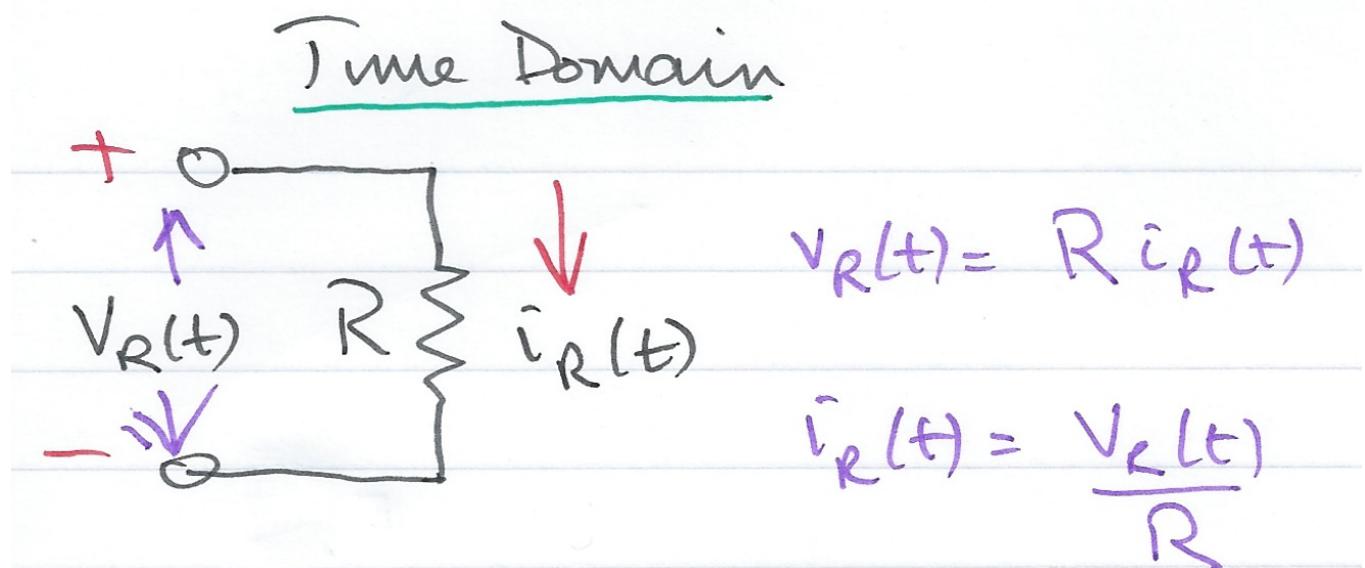
## Today's Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

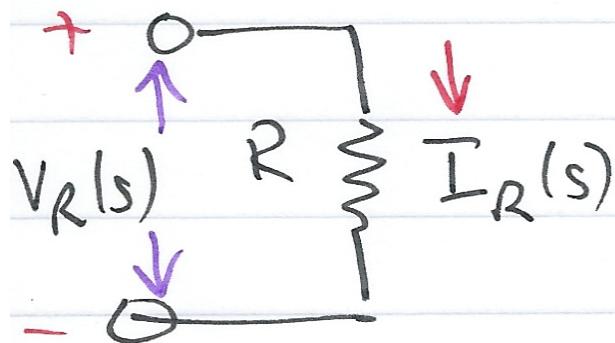
## Circuit Transformation from Time to Complex Frequency

### Resistive Network - Time Domain



## Resistive Network - Complex Frequency Domain

### Frequency Domain

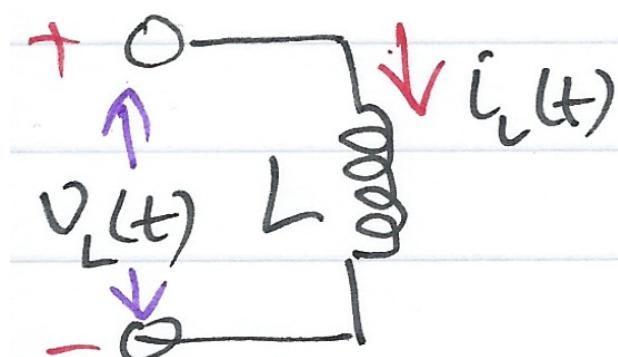


$$V_R(s) = R \bar{I}_R(s)$$

$$\bar{I}_R(s) = \frac{V_R(s)}{R}$$

## Inductive Network - Time Domain

### Time Domain

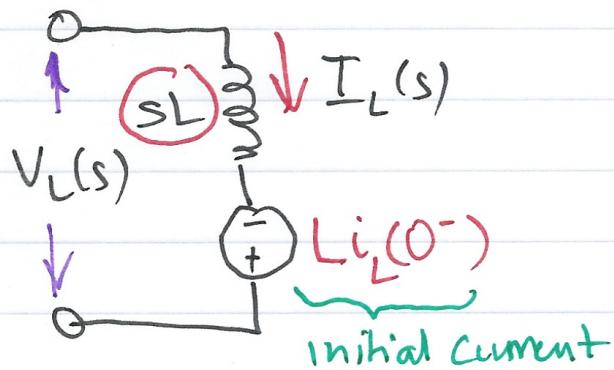


$$V_L(t) = L \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L} \int_{-\infty}^t V_L dt$$

## Inductive Network - Complex Frequency Domain

### Frequency Domain

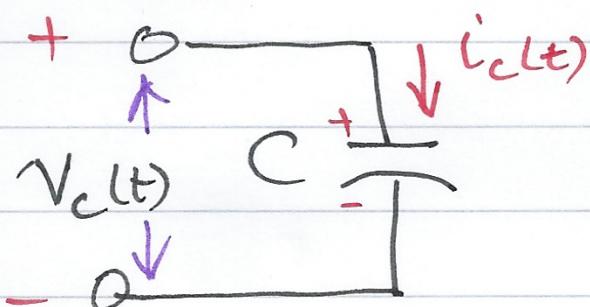


$$V_L(s) = sL I_L(s) - L i_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$

## Capacitive Network - Time Domain

### Time Domain

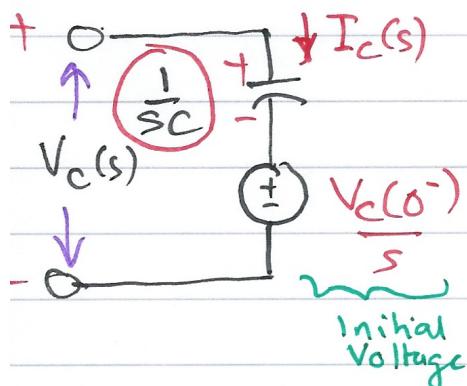


$$i_c(t) = C \frac{dV_c}{dt}$$

$$V_c(t) = \frac{1}{C} \int_{-\infty}^t i_c dt$$

## Capacitive Network - Complex Frequency Domain

### Frequency Domain



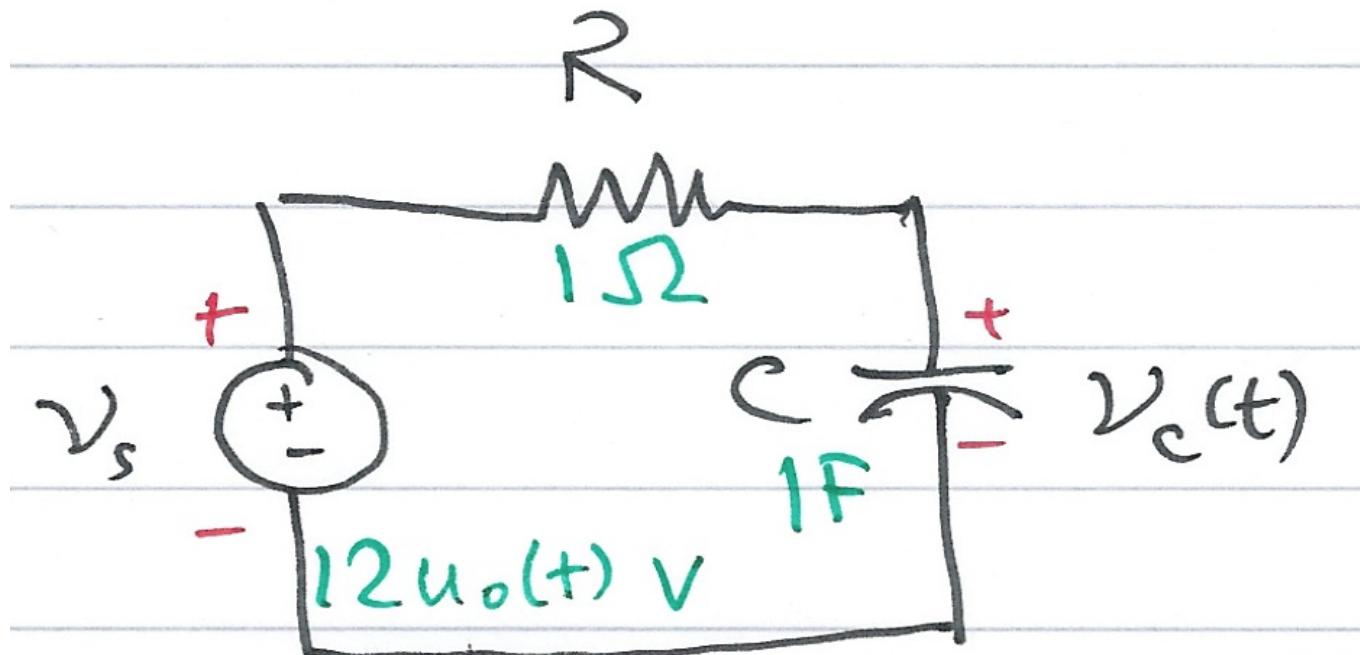
$$I_c(s) = sC V_c(s) - C V_c(0^-)$$

$$V_c(s) = \frac{I_c(s)}{sC} + \frac{V_c(0^-)}{s}$$

### Examples

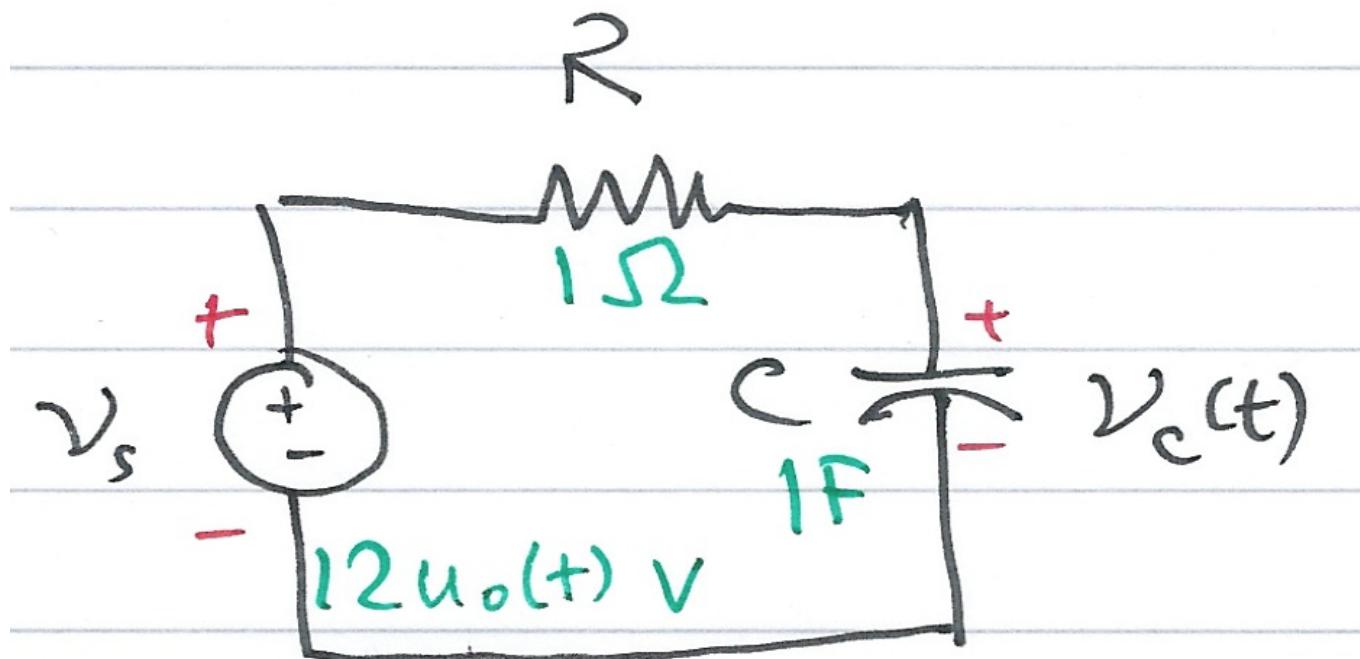
#### Example 1

Use the Laplace transform method and apply Kirchoff's Current Law (KCL) to find the voltage  $v_c(t)$  across the capacitor for the circuit below given that  $v_c(0^-) = 6 \text{ V}$ .



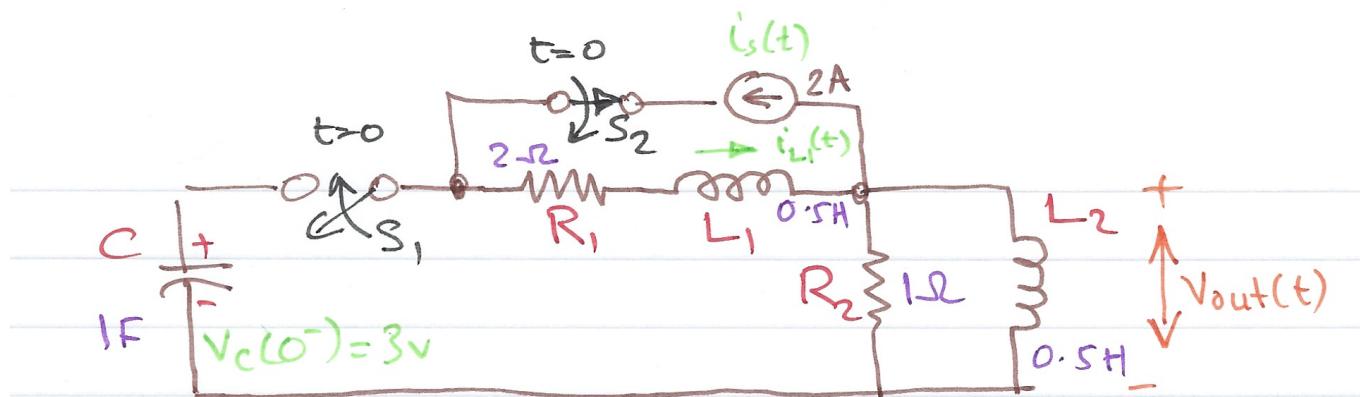
### Example 2

Use the Laplace transform method and apply Kirchoff's Voltage Law (KVL) to find the voltage  $v_c(t)$  across the capacitor for the circuit below given that  $v_c(0^-) = 6 \text{ V}$ .



### Example 3

In the circuit below, switch  $S_1$  closes at  $t = 0$ , while at the same time, switch  $S_2$  opens. Use the Laplace transform method to find  $v_{\text{out}}(t)$  for  $t > 0$ .



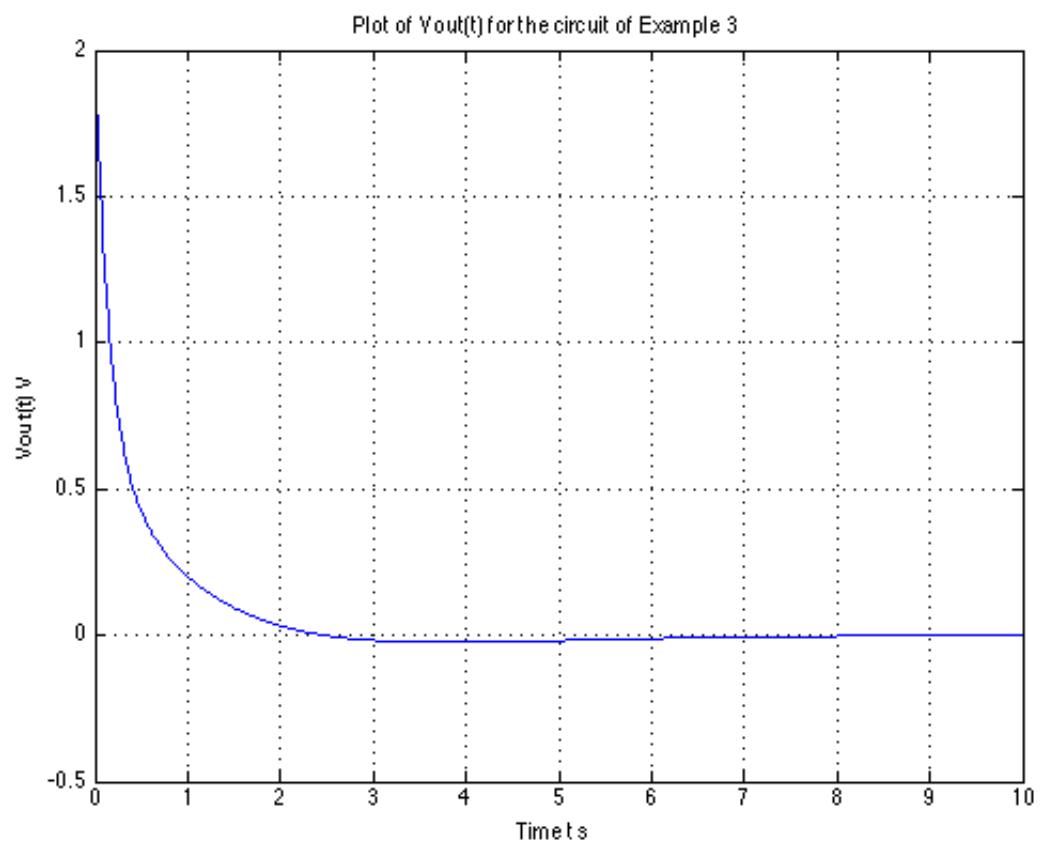
Show with the assistance of MATLAB (See [solution3.m \(matlab/solution3.m\)](#)) that the solution is

$$V_{\text{out}} = (1.36e^{-6.57t} + 0.64e^{-0.715t} \cos 0.316t - 1.84e^{-0.715t} \sin 0.316t) u_0(t)$$

and plot the result.

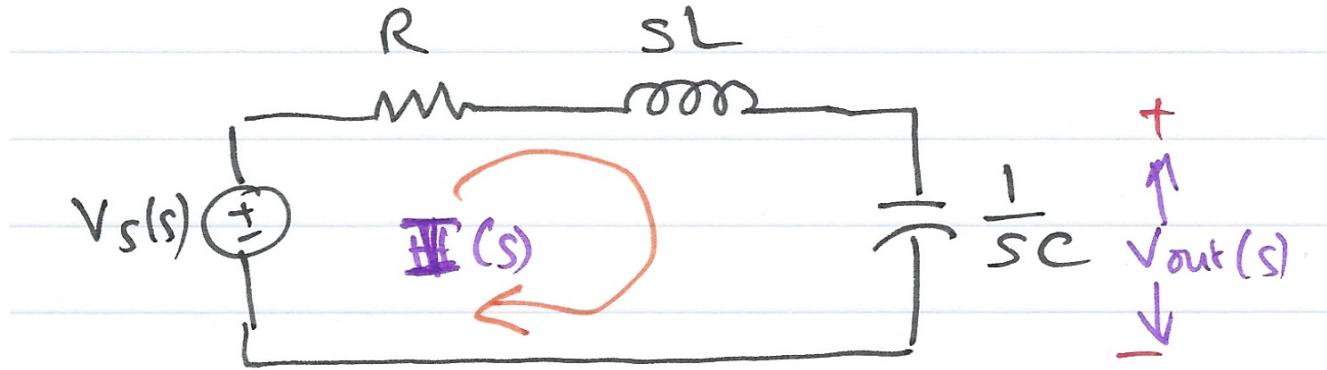
See Blackboard for worked solution.

## Plot of time response



# Complex Impedance $Z(s)$

Consider the  $s$ -domain RLC series circuit, where the initial conditions are assumed to be zero.



For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents the total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio  $V_s(s)/I(s)$  as  $Z(s)$ , we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The  $s$ -domain current  $I(s)$  can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

$$Z(s) = R + sL + \frac{1}{sC}.$$

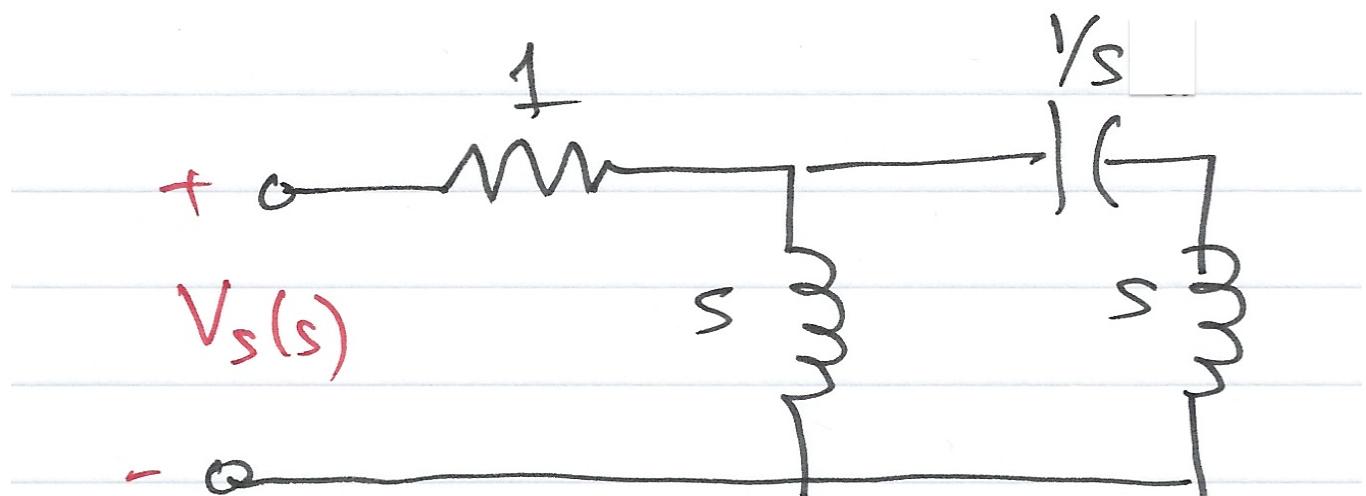
Since  $s = \sigma + j\omega$  is a complex number,  $Z(s)$  is also complex and is known as the *complex input impedance* of this RLC series circuit.

## Exercise

Use the previous result to give an expression for  $V_c(s)$

#### Example 4

For the network shown below, all the complex impedance values are given in  $\Omega$  (ohms).

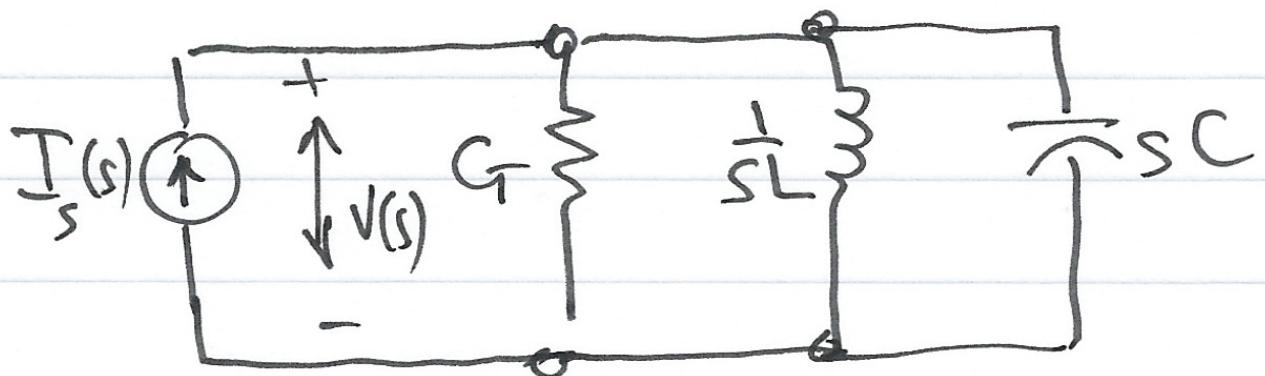


Find  $Z(s)$  using:

1. nodal analysis
2. successive combinations of series and parallel impedances

## Complex Admittance $Y(s)$

Consider the  $s$ -domain GLC parallel circuit shown below where the initial conditions are zero.



For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$
$$\left( G + \frac{1}{sL} + sC \right) V(s) = I_s(s)$$

Defining the ratio  $I_s(s)/V(s)$  as  $Y(s)$  we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The  $s$ -domain voltage  $V(s)$  can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

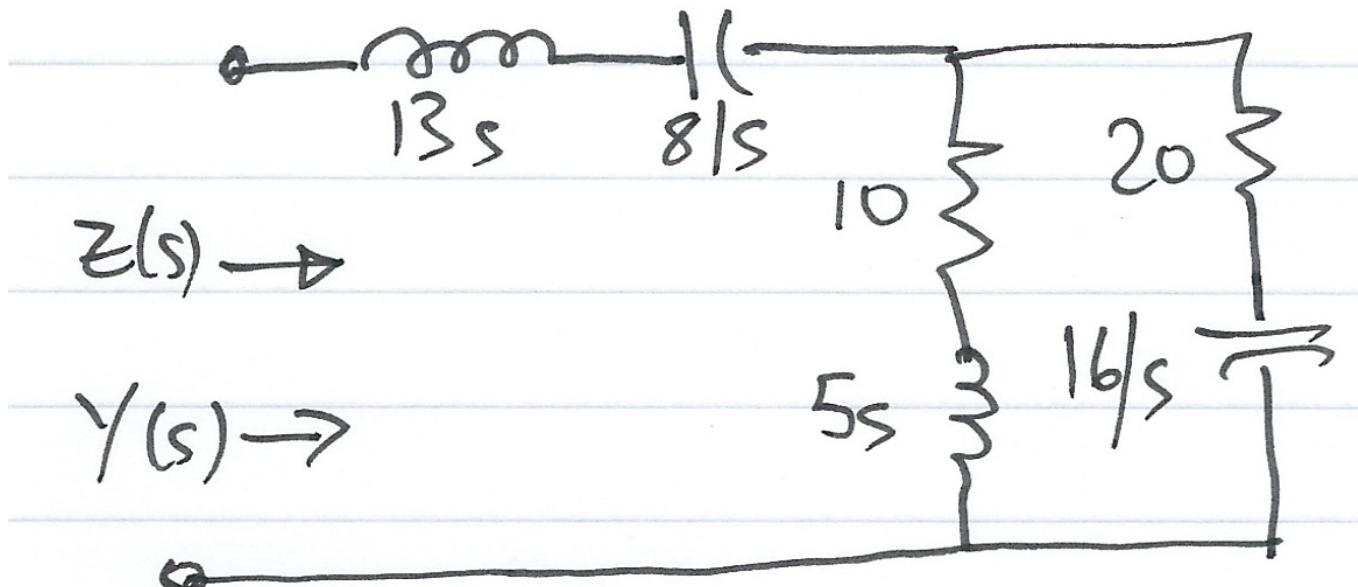
where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$  is complex and is known as the *complex input admittance* of this GLC parallel circuit.

### Example 5 - Do It Yourself

Compute  $Z(s)$  and  $Y(s)$  for the circuit shown below. All impedance values are in  $\Omega$  (ohms). Verify your answers with MATLAB.



## Answer 5

$$Z(s) = \frac{65s^4 + 490s^3 + 528s^2 + 400s + 128}{s(5s^2 + 30s + 16)}$$
$$Y(s) = \frac{1}{s} = \frac{s(5s^2 + 30s + 16)}{65s^4 + 490s^3 + 528s^2 + 400s + 128}$$

Matlab verification: [solution5.m \(matlab/solution5.m\)](#)