Using Laplace Transforms for Circuit Analysis

This Week

This week's sessions are based on Chapter 4 **Circuit Analysis with Laplace Transforms** from Steven T. Karris Signals and Systems: with MATLAB Computing and Simulink Modelling (5th Edition) [You need University Login to access]

Today's Agenda

We look at applications of the Laplace Transform for

- Circuit transformation from Time to Complex Frequency
- Complex impedance
- Complex admittance

Circuit Transformation from Time to Complex Frequency

Resistive Network - Time Domain

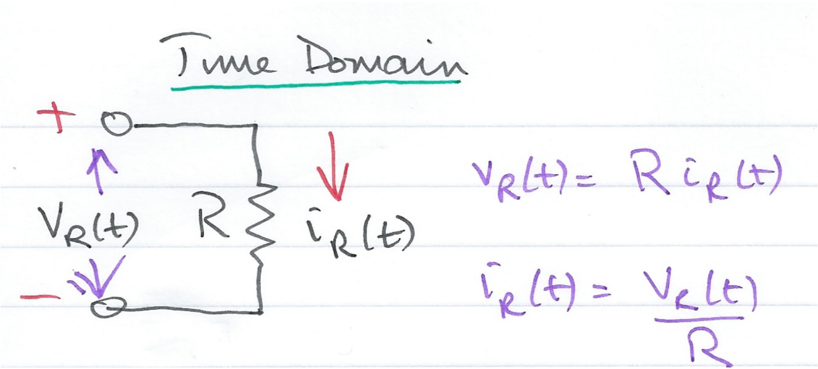


Figure 1: Resistive Network - Time Domain

Resistive Network - Complex Frequency Domain

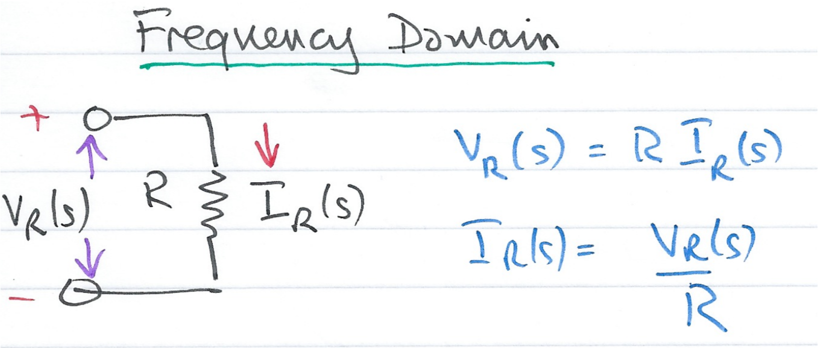


Figure 2: Resistive Network - Complex Frequency Domain

Inductive Network - Time Domain

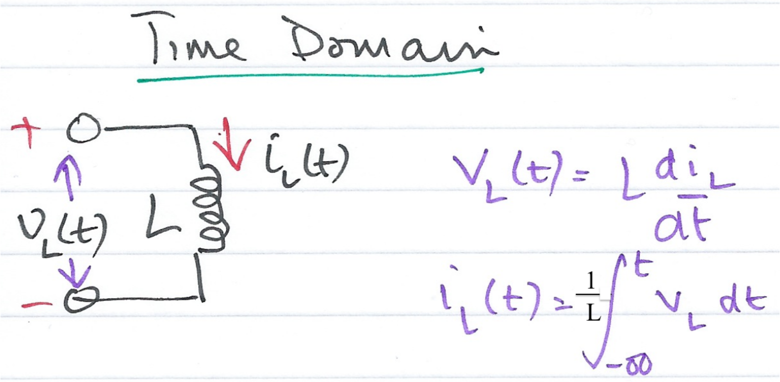
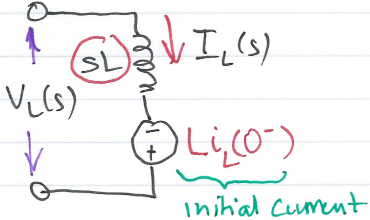


Figure 3: Inductive Network - Time Domain

Inductive Network - Complex Frequency Domain

Frequency Domain



$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

Figure 4: Inductive Network - Complex Frequency Domain

Capacitive Network - Time Domain

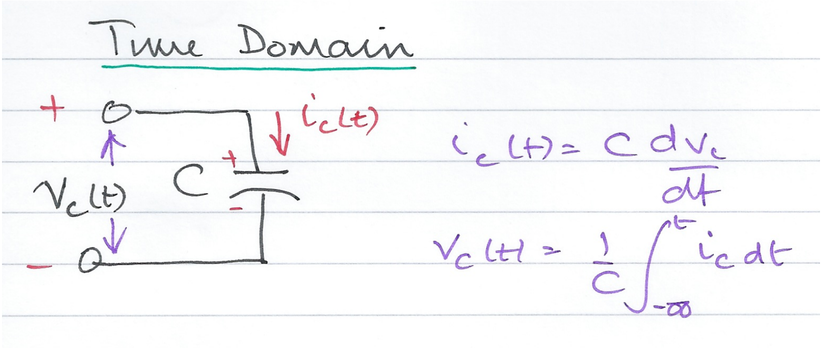


Figure 5: Capacitive Network - Time Domain

Capacitive Network - Complex Frequency Domain

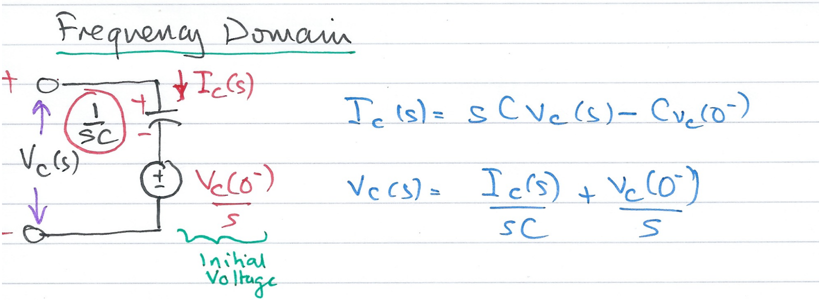


Figure 6: Capacitive Network - Complex Frequency Domain

Examples

Example 1

Use the Laplace transform method and apply Kichoff's Current Law (KCL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.

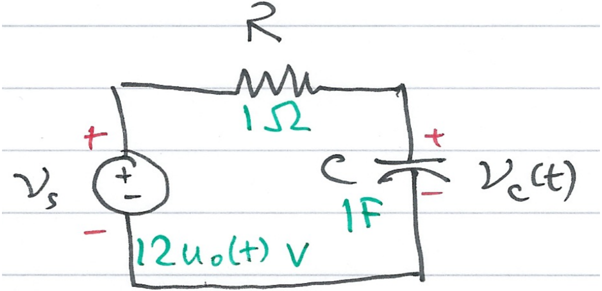


Figure 7: Circuit for Example 1

Example 2

Use the Laplace transform method and apply Kichoff's Voltage Law (KVL) to find the voltage $v_c(t)$ across the capacitor for the circuit below given that $v_c(0^-) = 6$ V.

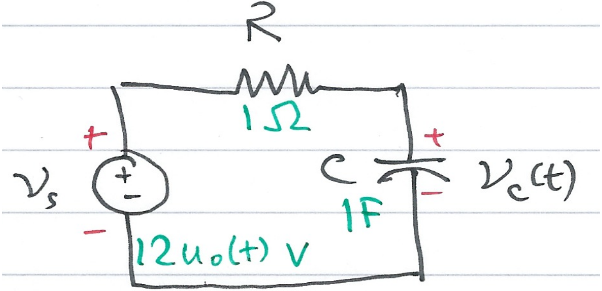


Figure 8: Circuit for Example 2

Example 3

In the circuit below, switch S_1 closes at $t = 0$, while at the same time, switch S_2 opens. Use the Laplace transform method to find $v_{\text{out}}(t)$ for $t > 0$.

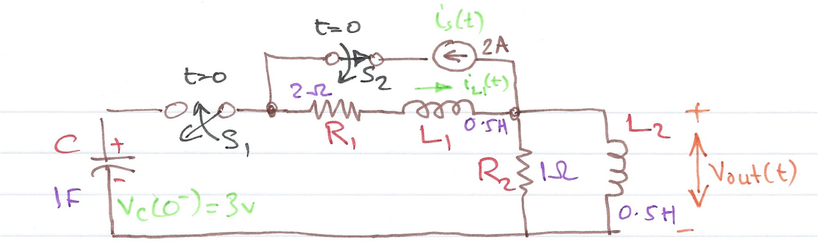


Figure 9: Circuit for Example 3

See Blackboard for worked solution.

Show with the assistance of MATLAB (See solution3.m) that the solution is

$$V_{\mathrm{out}} = \left(1.36e^{-6.57t} + 0.64e^{-0.715t}\cos 0.316t - 1.84e^{-0.715t}\sin 0.316t\right) u_0$$

and plot the result.

Plot of time response

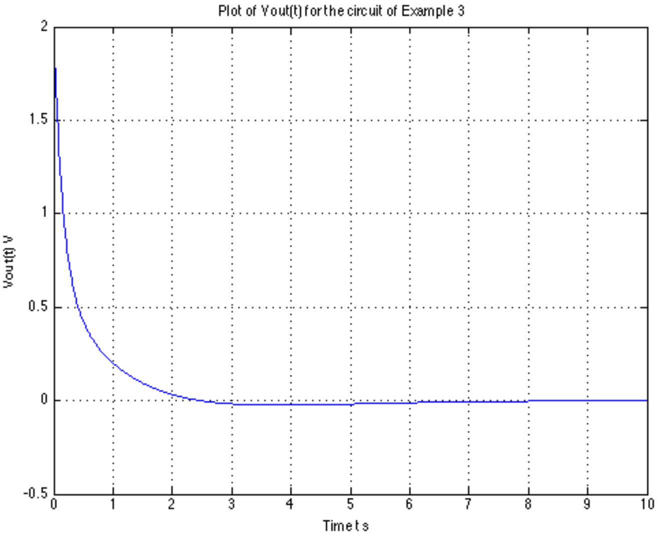


Figure 10: Plot of time response

Complex Impedance Z(s)

Consider the s-domain RLC series circuit, wehere the initial conditions are assumed to be zero.

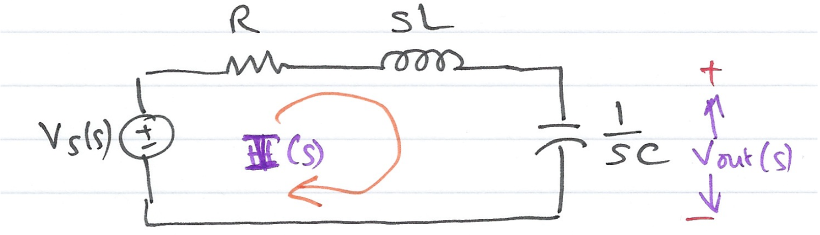


Figure 11: Complex Impedance $Z(s)$

For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

respresents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Exercise

Use the previous result to give an expression for $V_c(s)$

Example 4

For the network shown below, all the complex impedence values are given in Ω (ohms).

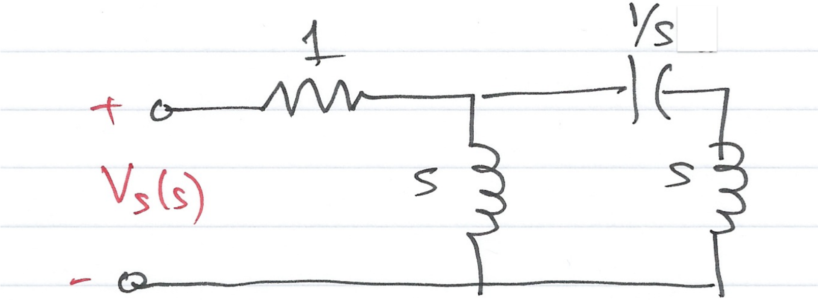


Figure 12: Circuit for example 4

Find $Z(s)$ using:

1. nodal analysis
2. successive combinations of series and parallel impendences

Complex Admittance Y(s)

Consider the *s*-domain GLC parallel circuit shown below where the initial conditions are zero.

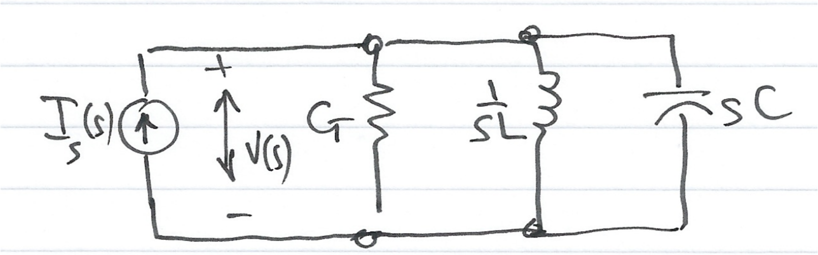


Figure 13: Complex admittance $Y(s)$

For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right)V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.

Example 5 - Do It Yourself

Compute $Z(s)$ and $Y(s)$ for the circuit shown below. All impedence values are in Ω (ohms). Verify your answers with MATLAB.

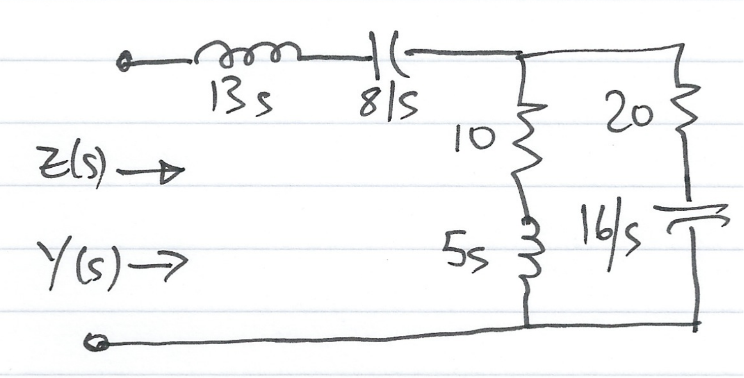


Figure 14: Circuit for Example 5

Answer 5

$$Z(s) = \frac{65s^4 + 490s^3 + 528s^2 + 400s + 128}{s(5s^2 + 30s + 16)}$$

$$Y(s) = \frac{1}{s} = \frac{s(5s^2 + 30s + 16)}{65s^4 + 490s^3 + 528s^2 + 400s + 128}$$

Matlab verification: solution5.m

Next Lesson

- Transfer Functions of Circuits (Notes PDF, Slides PDF)

Homework

Do the end of the chapter exercises (Section 4.7 - questions 1 to 4) from the textbook. Don't look at the answers until you have attempted the problems.

Lab Work

In the lab, week on Friday, we will see the use of Matlab and Simulink in the solution of circuit problems.