Examples 3 - Elementary Signals

Contents

Lecturer: Set up MATLAB

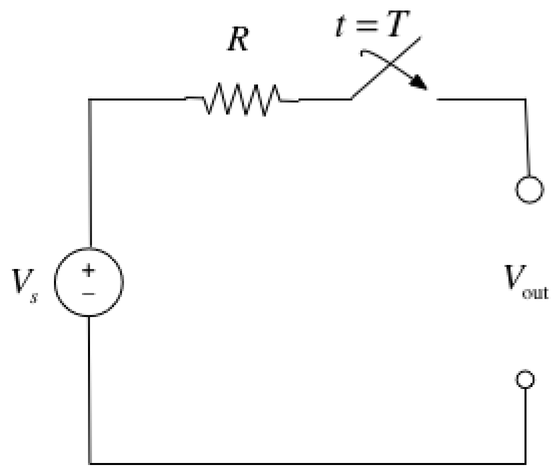
```
clear all
format compact
```

To accompany Unit 2.3: Elementary Signals.

Follow along at cpjobling.github.io/eg-150-textbook/signals_and_systems/elementary_signals/examples3

3.1 Elementary Signals

Consider the network shown in below where the switch is closed at time $t = T$ and all components are ideal.



Express the output voltage V_{out} as a function of the unit step function, and sketch the appropriate waveform.

a) What happens **before** $t = T$?

1. $v_{\text{out}} = \text{undefined}$
2. $v_{\text{out}} = 0$
3. $v_{\text{out}} = V_s$
4. $v_{\text{out}} = V_s/2$
5. $v_{\text{out}} = \infty$

-> Open Poll: 1.2.1

b) What happens **after** $t = T$?

1. $v_{\text{out}} = \text{undefined}$
2. $v_{\text{out}} = 0$
3. $v_{\text{out}} = V_s$
4. $v_{\text{out}} = V_s/2$
5. $v_{\text{out}} = \infty$

-> Open Poll: 1.2.2

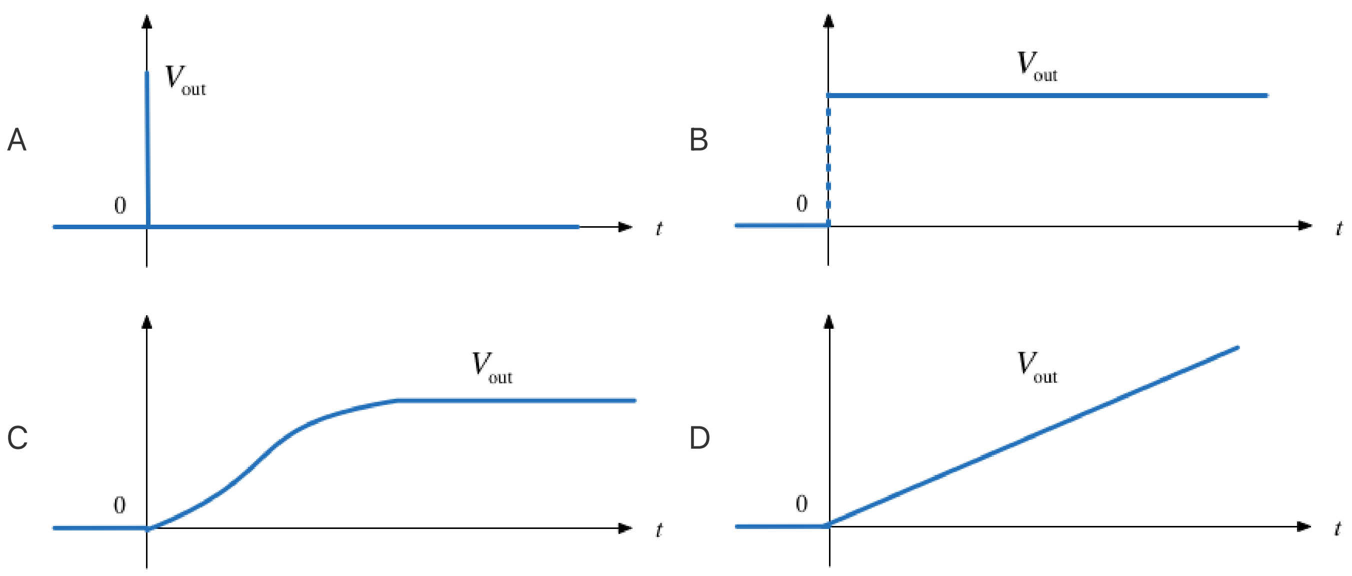
c) What happens **at** $t = T$?

1. $v_{\text{out}} = \text{undefined}$
2. $v_{\text{out}} = 0$
3. $v_{\text{out}} = V_s$
4. $v_{\text{out}} = V_s/2$
5. $v_{\text{out}} = \infty$

-> Open Poll: 1.2.3

d) What does the response of V_{out} look like?

Circle the picture you think is correct on your handout.

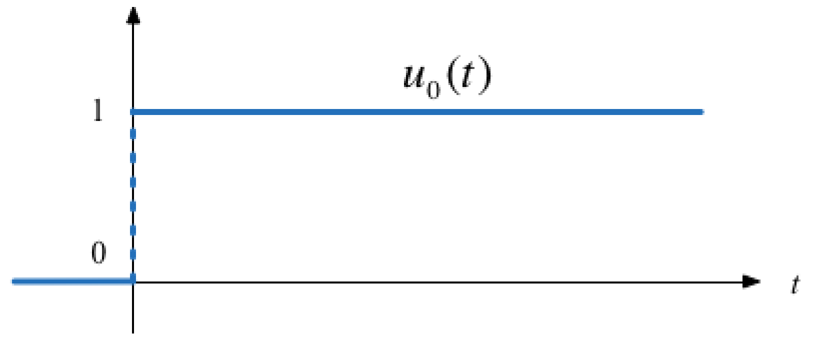


-> Open Poll: 1.2.4

Example 3.2

The Unit Step Function

$$u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



In Matlab

In Matlab, we use the `heaviside` function (Named after Oliver Heaviside).

```
syms t
ezplot(heaviside(t),[-1,1])
heaviside(0)
```

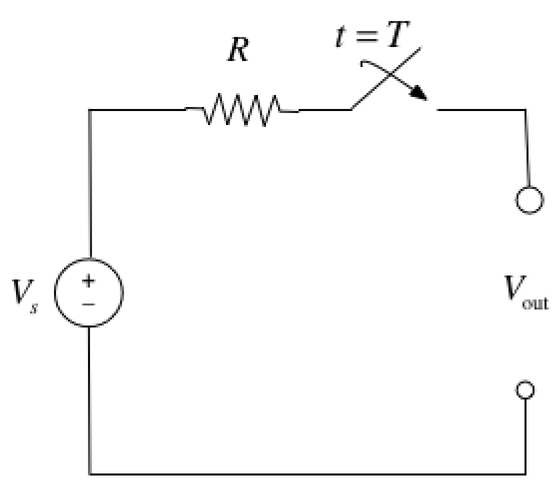
See: show_heaviside.m.

Note that, so it can be plotted, Matlab defines the *heaviside function* slightly differently from the mathematically ideal unit step:

$$\text{heaviside}(t) = \begin{cases} 0 & t < 0 \\ 1/2 & t = 0 \\ 1 & t > 0 \end{cases}$$

Example 3.3: Circuit Revisited

Consider the network shown below, where the switch is closed at time $t = T$.



Express the output voltage v_{out} as a function of the unit step function, and sketch the appropriate waveform.

Example 3.4: Simple Signal Operations

a) Amplitude Scaling

Sketch $Au_0(t)$ and $-Au_0(t)$

b) Time Reversal

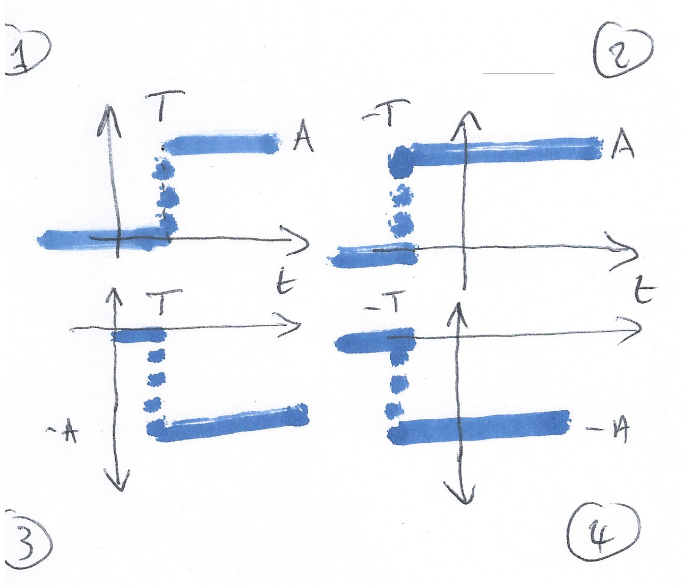
Sketch $u_0(-t)$

c) Time Delay and Advance

Sketch $u_0(t-T)$ and $u_0(t+T)$

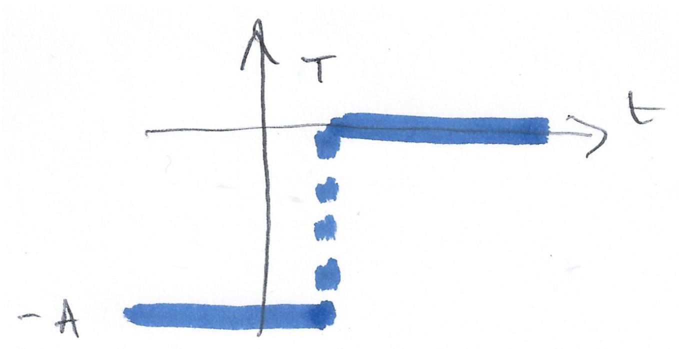
Example 3.5

a) Which of these signals represents $-Au_0(t+T)$?



-> Open Poll: 1.2.5

b) What is represented by



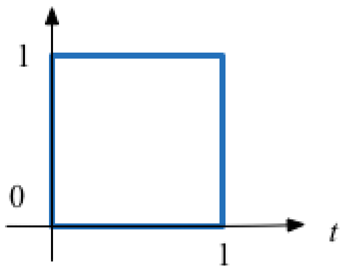
1. $-Au_0(t+T)$
2. $-Au_0(-t+T)$
3. $-Au_0(-t-T)$
4. $-Au_0(t-T)$

-> Open Poll: 1.2.6

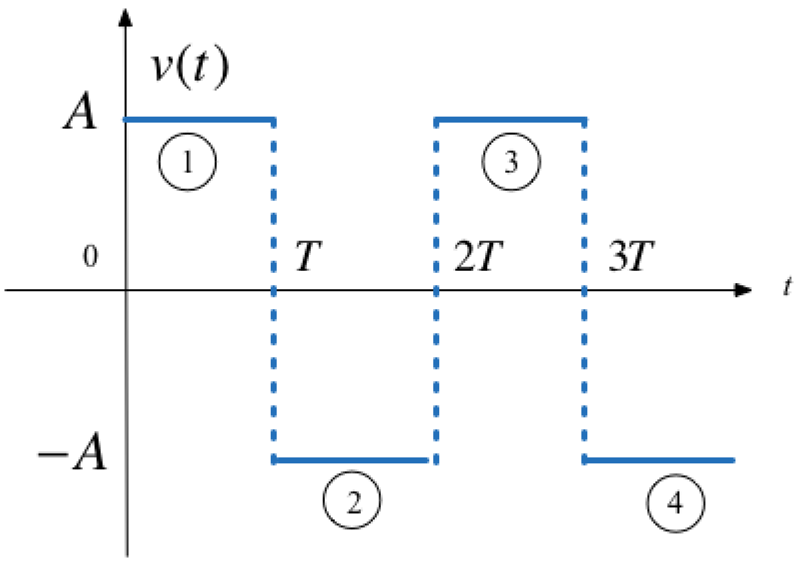
3.6: Synthesis of Signals from Unit Step

Unit step functions can be used to represent other time-varying functions such as rectangular pulses, square waves and triangular pulses.

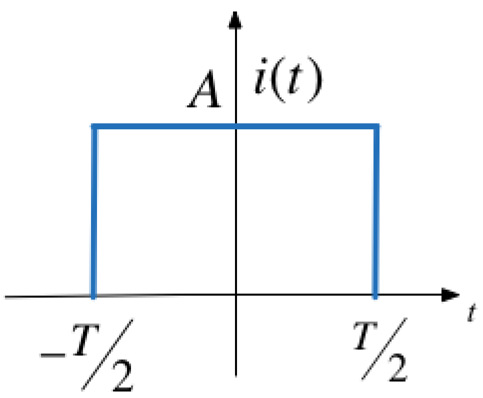
a) Synthesize Rectangular Pulse



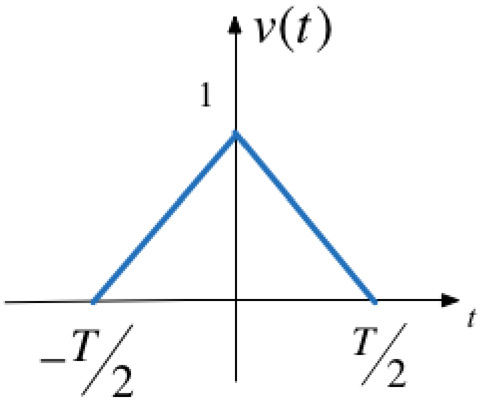
b) Synthesize Square Wave



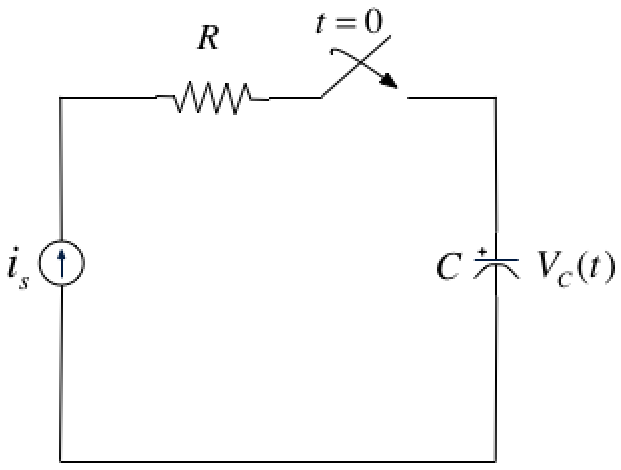
c) Synthesize Symmetric Rectangular Pulse



d) Synthesize Symmetric Triangular Pulse



Example 3.7: The Ramp Function



In the circuit shown above i_s is a constant current source and the switch is closed at time $t = 0$.

Show that the voltage across the capacitor can be represented as

$$v_C(t) = \frac{i_s}{C} t u_0(t)$$

and sketch the wave form.

The unit ramp function is defined as

$$u_1(t) = \int_{-\infty}^{t} u_0(\tau)d\tau$$

so

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

and

$$u_0(t) = \frac{d}{dt}u_1(t)$$

Note

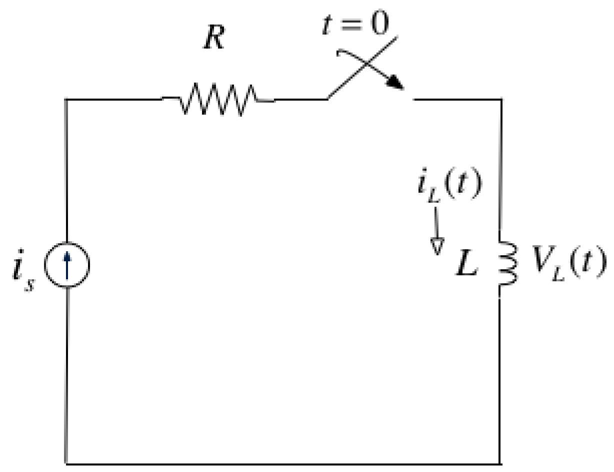
Higher order functions of t can be generated by the repeated integration of the unit step function.

For future reference, you should determine $u_2(t)$, $u_3(t)$ and $u_n(t)$ for yourself and make a note of the general rule:

$$u_{n-1} = \frac{1}{n}\frac{d}{dt}u_n(t)$$

Details are given in equations 1.26—1.29 in the textbook.

Example 3.8: The Dirac Delta Function



In the circuit shown above, the switch is closed at time $t = 0$ and $i_L(t) = 0$ for $t < 0$. Express the inductor current $i_L(t)$ in terms of the unit step function and hence derive an expression for $v_L(t)$.

Notes

To solve this problem we need to invent a function that represents the derivative of the unit step function. This function is called $\delta(t)$ or the *dirac delta* function (named after Paul Dirac).

The delta function

The unit impulse or the delta function, denoted as $\delta(t)$, is the derivative of the unit step.

This function is tricky because $u_0(t)$ is discontinuous at $t = 0$ but it must have the properties

$$\int_{-\infty}^{t} \delta(\tau) d\tau = u_0(t)$$

and

$\delta(t) = 0$ for all $t \neq 0$.

Sketch of the delta function

Example 3.9: Important properties of the delta function

See the accompanying notes.

Evaluate the following expressions

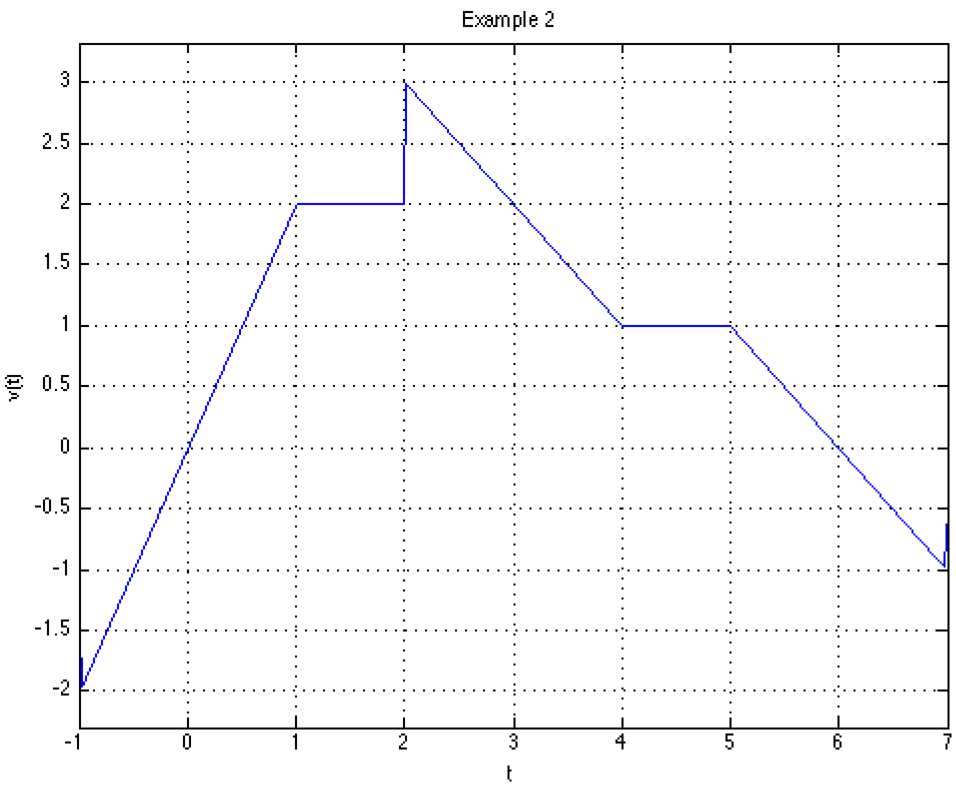
a) $3t^4\delta(t-1)$

b)

$$\int_{-\infty}^{\infty} t\delta(t-2)dt$$

$$t^2\delta'(t-3)$$

Example 3.10



a) Express the voltage waveform $v(t)$ shown above as a sum of unit step functions for the time interval $-1 < t < 7$ s

b) Using the result of 3.10(a), compute the derivative of $v(t)$ and sketch its waveform.

Lab Work

In the first lab, next Tuesday, we will solve further elemetary signals problems using MATLAB and Simulink following the procedure given between pages 1-17 and 1-22 of the Karris. We will also explore the `heaviside` and `dirac` functions.

Answers to in-class questions

Mathematically

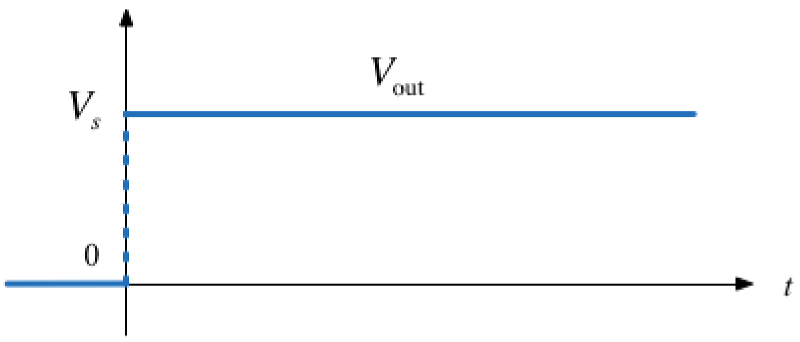
3.1(a). $v_{\text{out}} = 0$ when $-\infty < t < 0$ (answer 2)

3.1(b). $v_{\text{out}} = V_s$ when $0 < t < \infty$ (answer 3)

3.1(c). $v_{\text{out}} = \text{undefined}$ when $t = 0$ (answer 1)

V_{out} jumps from 0 to V_s instantanously when the switch is closed. We call this a discontinuous signal!

3.1(d): The correct image is:



Example 3.5(a): Answer 3.

Example 3.5(b): Answer 2.

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