Unit 2.4: Systems and Classification of Systems

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This section is based on Section 1.5 of [Hsu, 2020].

Follow along at cpjobling.github.io/eg-150-textbook/signals_and_systems/systems

Subjects to be covered

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System Representation

A *system* is a mathematical model of a physical process that relates the *input* (or *excitation*) signal to the *output* (or *response*) signal.

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a *transformation* (or *mapping*) of x into y. The transformation is represented by the mathematical notation

$$y = \mathbf{T}x$$

where $\mathbf{T}$ is the *operator* representing some well defined rule by which x is transformed into y.

The relationship is depicted graphically as shown in Fig. 25(a).

Multiple input and/or output systems are possible as shown in Fig. 25(b). In this module we will restrict our attention to the single-input, single-output case.

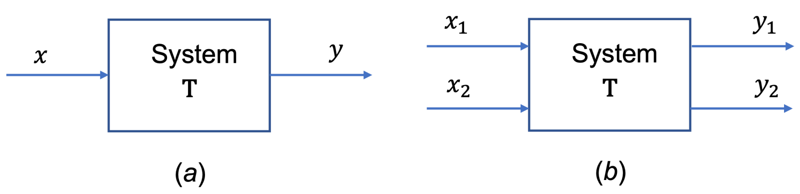

Fig. 25 System with single or multiple inputs and outputs

Deterministic and Stochastic Systems

If the input and output signals x and y are deterministic signals, then the system is called a *deterministic* system.

If the input and output signals x and y are random signals, then the system is called a *stochastic* system.

Continuous-Time and Discrete-Time Systems

If the input and output signals x and y are continuous-time signals, then the system is called a *continuous-time system* (Fig. 26(a)).

If the input and output signals x and y are discrete-time signals or sequences, then the system is called a *discrete-time system* (Fig. 26(b)).

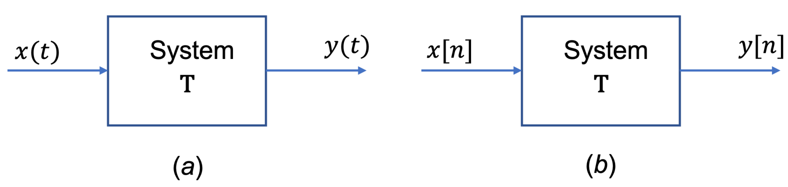

Fig. 26 (a) Continuous-time system; (b) discrete time system.

Note that in a continuous-time system the input $x(t)$ and $y(t)$ are often expressed as a *differential equation* (see Examples 4) and in a discrete-time system $x[n]$ and $y[n]$ are often expressed by a *difference equation*.

Systems with Memory and without Memory

A system is said to be *memoryless* if the output at any time only depends on the input at the same time.

Otherwise the system is said to have *memory*.

A memoryless system

An example of a memoryless system is a resistor R with and the input $x(t)$ taken as the current and the voltage taken as the output $y(t)$.

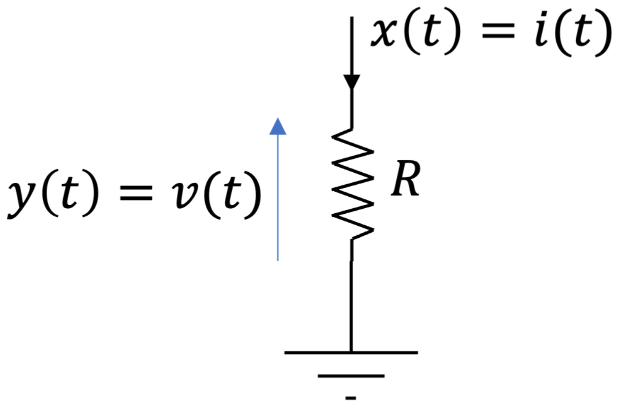

Fig. 27 A memoryless system: a resistor

The input-output relationship (Ohm's law) of a resistor is

$$y(t) = Rx(t)$$

A system with memory

An example of a system with memory is a capacitor C with and the current as the input $x(t)$ taken as the current and the voltage as the output $y(t)$.

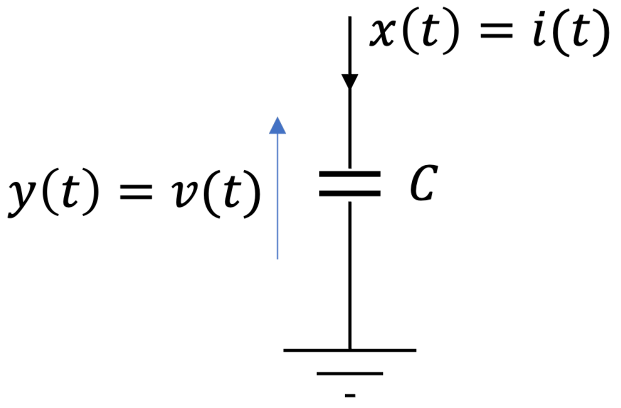

Fig. 28 A system with memory: a capacitor

Then:

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau$$

Causal and Non-Causal Systems

A system is called *causal* if its output at the present time depends only on the present and/or past values of the input.

Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system.

A system is called *noncausal* (or *anticipative*) if its output at the present time depends on future values of the input.

An example of a noncausal system is

$$y(t) = x(t+1)$$

Note that all memoryless systems are causal but not all *vice versa*.

Linear Systems and Nonlinear Systems

If an operator $\mathbf{T}$ satisfies the following two conditions, then $\mathbf{T}$ is called a *linear operator* and the system represented by the linear operator $\mathbf{T}$ is called a *linear system*:

Properties of Linear Systems

1. Additivity

Given that $\mathbf{T}\left\{x_1\right\} = y_1$ and $\mathbf{T}\left\{x_2\right\} = y_2$,

then

$$\mathbf{T}\left\{x_1 + x_2\right\} = y_1 + y_2$$

for any signals x_1 and x_2.

2. Homogeneity (or *Scaling*)

$$\mathbf{T}\left\{\alpha x\right\} = \alpha y$$

for any signals x and any scalar α.

Nonlinear systems

Any system that does not satisfy the additivity and homogeneity conditions is classified as a *nonlinear system*.

Superposition property

The additivity and homogeneity conditions can combined in a single condition (known as the *superposition property*) as

$$\mathbf{T}\left\{\alpha_1 x_1 + \alpha_2 x_2\right\} = \alpha_1 y_1 + \alpha_2 y_2$$

where α_1 and α_2 are arbitrary scalars.

Example linear systems

Examples of linear systems are the resistor and capacitor discussed earlier.

Example nonlinear systems

Examples of nonlinear systems are

$$y = x^2$$

$$y = \cos x$$

Zero input property

Note that a consequence of the homegenity (or scaling) property of linear systems is that a *zero input yields a zero output*. This follows readilty by setting $\alpha = 0$ in the equation $\mathbf{T}\left\{\alpha x\right\} = \alpha y$. This is another important property of linear systems.

Time-Invariant and Time-Varying Systems

A system is called *time-invariant* if a time-shift (delay or advance) in the input signal causes the same time-shift in the output signal.

Thus for a continuous-time system, the system is time-invariant if

$$\mathbf{T}\left\{x(t-\tau)\right\} = y(t-\tau)$$

for any real value of τ.

Time-varying system

A system that does not satisfy the equation $\mathbf{T}\left\{x(t-\tau)\right\} = y(t-\tau)$ is called a *time-varying system*.

Testing for time-invariance

To check for time invariance, we can compare the time-shifted output with the output produced by the time-shifted input (See Example 4.2: Capacitor circuit and ex4_3).

Linear Time-Invariant Systems

If a system is linear and also time-invariant it is called a *linear time-invariant* (LTI) system.

All ths systems analysed in in the rest of the module and in **EG-247 Digital Signal Processing** and **EG-243 Control Systems** next year will be LTI systems.

Stable Systems

A system is *bounded-input/bounded-output* (BIBO) *stable* if for any bounded input signal x defined by

$$|x| \le k_1$$

the corresponding output y is also bounded defined by

$$|y| \le k_2$$

where k_1 and k_2 are finite real constants.

Unstable systems

An *unstable* system is one in which not all bounded inputs lead to a bounded output.

For example, consider the system where output

$$y(t) = t x(t)$$

and input $x(t)$ is the unit step $u_0(t)$

In this case $x(t) = 1$ (so is bounded) but the output $y(t)$ increases without bound as t increases.

Feedback Systems

A special class of systems of great importance consists of systems having *feedback*.

In a *feedback system*, a portion of the output signal is fed back and added to the input as shown in Fig. 29.

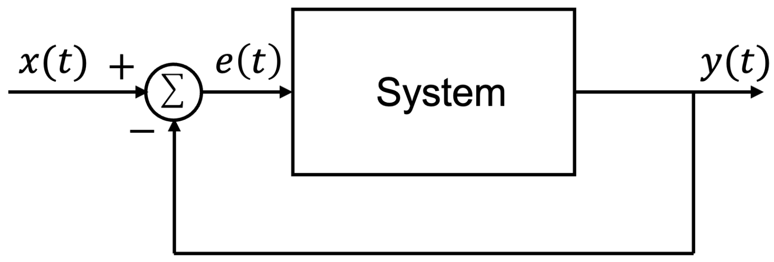

Fig. 29 A feedback system with negative feedback: $e(t) = x(t) - y(t)$.

You will see examples of systems with feedback when you study op-amp circuits in **EG-152 Practical Electronics**, the simple closed-loop systems to be studied in **EG-142 Instrumentation and Control**. Feedback, and its impact on system stability, is also the basis of control theory to be studied next year in **EG-243 Control Systems**.

Examples 4

MATLAB Example

We will solve this example by hand and then give the solution in the MATLAB lab.

Example 4.1: RC Circuit

Consider the RC circuit shown in Fig. 30. Find the relationship between the input $x(t)$ and the output $y(t)$

(a) If $x(t) = v_s(t)$ and $y(t) = v_c(t)$.

(b) If $x(t) = v_s(t)$ and $y(t) = i(t)$.

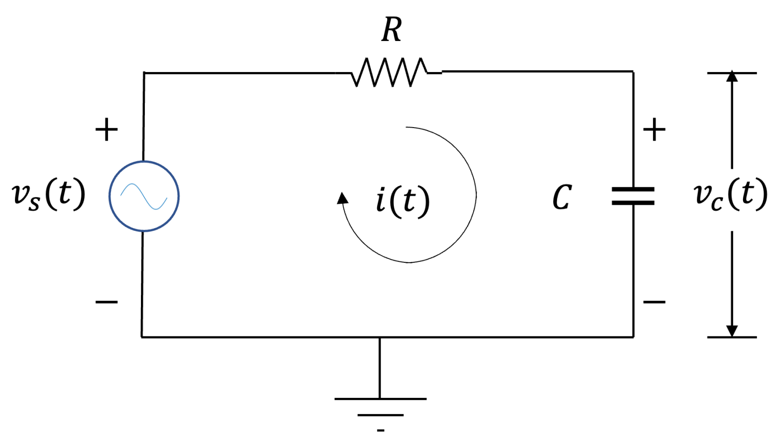

Fig. 30 RC circuit

For the answer, refer to the lecture recording or see solved problem 1.32 in {cite}schaum.

Example 4.2: Capacitor circuit

MATLAB Example

We will solve this example by hand and then give the solution in the MATLAB lab.

Consider the capacitor shown in Fig. 31. Let the input $x(t) = i(t)$ and the output $y(t) = v_c(t)$.

(a) Find the input-output relationship.

(b) Determine whether the system is (i) memoryless, (ii) causal, (iii) linear, (iv) time invariant, or (v) stable.

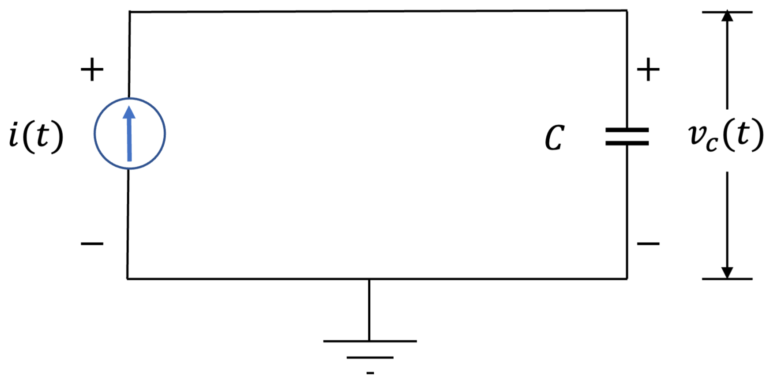

Fig. 31 A capacitor citcuit.

For the answer, refer to the lecture recording or see solved problem 1.33 in {cite}schaum.

> **MATLAB Example**
>
> We will solve this example by hand and then give the solution in the MATLAB lab.

Example 4.3: Signal modulator

Consider the system shown in Fig. 32. Determine whether it is (a) memoryless, (b) causal, (c) linear, (d) time invariant, or (e) stable.

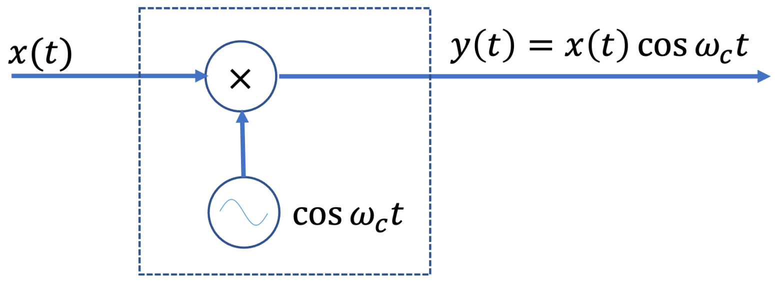

Fig. 32 A signal modulator

For the answer, refer to the lecture recording or see solved problem 1.34 in {cite}schaum.

Example 4.4

A system has the input-output relationship given by

$$y = \mathbf{T}\left\{x\right\} = x^2$$

Show that this system is nonlinear.

For the answer, refer to the lecture recording or see solved problem 1.35 in {cite}schaum.

Example 4.5

Consider the system whose input-output relationship is given by the linear equation

$$y = ax + b$$

where x and y are the inout and output respectively and s and b are constant. Is this system linear?

For the answer, refer to the lecture recording or see solved problem 1.40 in {cite}schaum.

Example 4.6

(a) Show that the causality for a continuous-time linear system is equivalent to the following statement: For any time t_0 and any input $x(t)$ with $x(t) = 0$ for $t \leq t_0$, the output $y(t)$ is zero for $t \leq t_0$.

(b) Find a nonlinear system that is causal but does not satisfy this condition.

(c) Find a nonlinear system that satisfies this condition but is not causal.

For the answer, refer to the lecture recording or see solved problem 1.43 in {cite}schaum.

> **MATLAB Example**
>
> We will solve this example by hand and then give the solution in the MATLAB lab.

Example 4.7

Let $\mathbf{T}$ represent a continuous-time LTI system. Then show that

$$\mathbf{T}\left\{e^{st}\right\} = \lambda e^{st}$$

where s is a complex variable and λ is a complex constant.

For the answer, refer to the lecture recording or see solved problem 1.44 in {cite}schaum.

Summary

In this lecture we have started our look at systems and the classification of systems.

In particular we have looked at

- System Representation
- Deterministic and Stochastic Systems
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Next time

- Unit 3.2: Properties and Eigenfunctions of Continuous-Time LTI Systems

References

[Hsu20] Hwei P. Hsu. *Schaums outlines signals and systems*. McGraw-Hill, New York, NY, 2020. ISBN 9780071634724. Available as an eBook. URL: https://www.accessengineeringlibrary.com/content/book/9781260454246.

By Dr Chris P. Jobling