Examples 3 - Elementary Signals

Contents

Lecturer: Set up MATLAB

```
clear all
format compact
```

To accompany Unit 2.3: Elementary Signals.

Follow along at cpjobling.github.io/eg-150-textbook/signals_and_systems/elementary_signals/examples3

3.1: Other forms of unit step

MATLAB Example

We will solve this example by hand and then give the solution in the MATLAB lab.

Use the MATLAB functions `subplot`, `heaviside` and `fplot` to reproduce Fig. 22. We've done the first row for you.

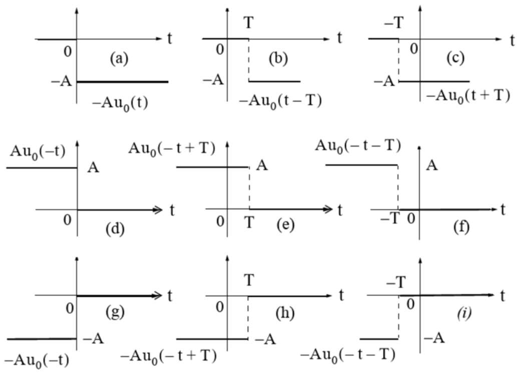


Figure 1.8. Other forms of the unit step function

Fig. 22 Other forms of unit step function (Figure 1.8 [Karris, 2012])

```
syms t
u0(t) = heaviside(t); % allows us to type u0(t) in our formulae
A = 2; T = 2; % we need numerical values to get a successful plot
```

a). $-Au_0(t)$

```
subplot(331)
fplot(-A*u0(t)),title('a)')
```

b). $-A(t-T)$

```
subplot(332)
fplot(-A*u0(t - T)),title('b)')
```

c). $-A(t+T)$

```
subplot(333)
fplot(-A*u0(t + T)),title('c)')
```

d). $A(-t)$

e). $A(-t+T)$

f). $A(-t-T)$

g). $-A(-t)$

h). $-A(-t+T)$

i). $-A(-t-T)$

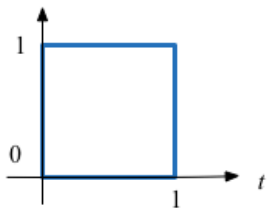
3.2: Synthesis of Signals from Unit Step

MATLAB Example

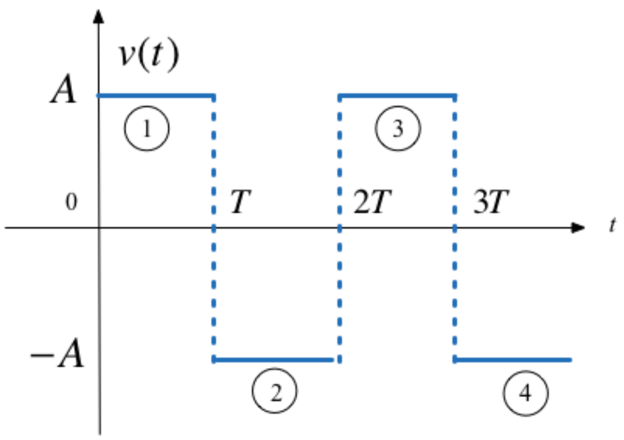
We will solve this example by hand and then give the solution in the MATLAB lab.

Unit step functions can be used to represent other time-varying functions such as rectangular pulses, square waves and triangular pulses.

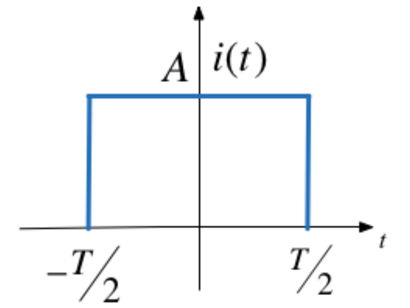
a) Synthesize Rectangular Pulse



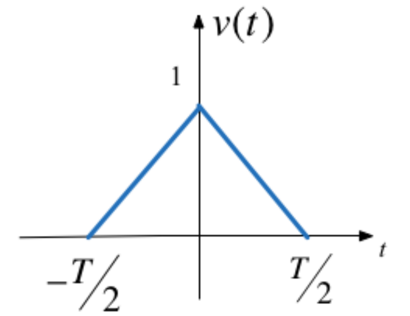
b) Synthesize Square Wave



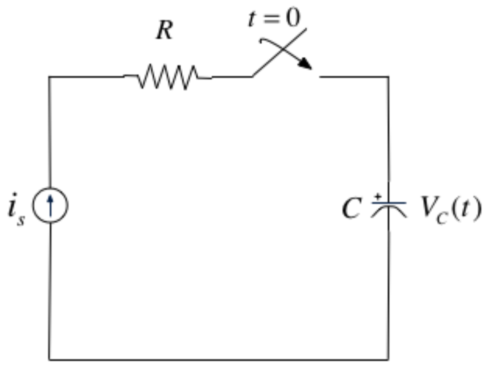
c) Synthesize Symmetric Rectangular Pulse



d) Synthesize Symmetric Triangular Pulse



Example 3.3: The Ramp Function



In the circuit shown above i_s is a constant current source and the switch is closed at time $t = 0$.

Show that the voltage across the capacitor can be represented as

$$v_C(t) = \frac{i_s}{C} t u_0(t)$$

and sketch the wave form.

The unit ramp function is defined as

$$u_1(t) = \int_{-\infty}^{t} u_0(\tau)d\tau$$

so

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

and

$$u_0(t) = \frac{d}{dt}u_1(t)$$

Note

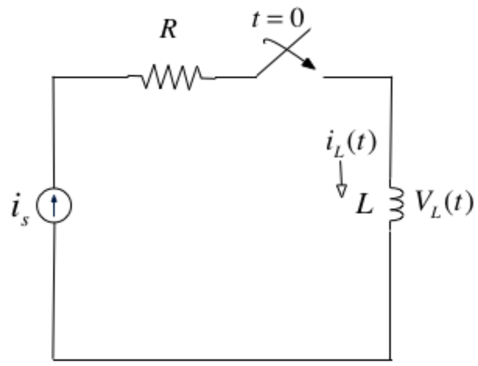
Higher order functions of t can be generated by the repeated integration of the unit step function.

For future reference, you should determine $u_2(t)$, $u_3(t)$ and $u_n(t)$ for yourself and make a note of the general rule:

$$u_{n-1} = \frac{1}{n}\frac{d}{dt}u_n(t)$$

Details are given in equations 1.26—1.29 in the textbook.

Example 3.4: The Dirac Delta Function



In the circuit shown above, the switch is closed at time $t = 0$ and $i_L(t) = 0$ for $t < 0$. Express the inductor current $i_L(t)$ in terms of the unit step function and hence derive an expression for $v_L(t)$.

Notes

To solve this problem we need to invent a function that represents the derivative of the unit step function. This function is called $\delta(t)$ or the *dirac delta* function (named after Paul Dirac).

The delta function

The unit impulse or the delta function, denoted as $\delta(t)$, is the derivative of the unit step.

This function is tricky because $u_0(t)$ is discontinuous at $t = 0$ but it must have the properties

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u_0(t)$$

and

$\delta(t) = 0$ for all $t \neq 0$.

Sketch of the delta function

Example 3.5: Important properties of the delta function

> **MATLAB Example**
>
> We will solve this example by hand and then give the solution in the MATLAB lab.

See the accompanying notes.

Evaluate the following expressions

a) $3t^4\delta(t-1)$

b)

$$\int_{-\infty}^{\infty} t\delta(t-2)dt$$

$$t^2\delta'(t-3)$$

Example 3.6

MATLAB Example

We will solve this example by hand and then give the solution in the MATLAB lab.

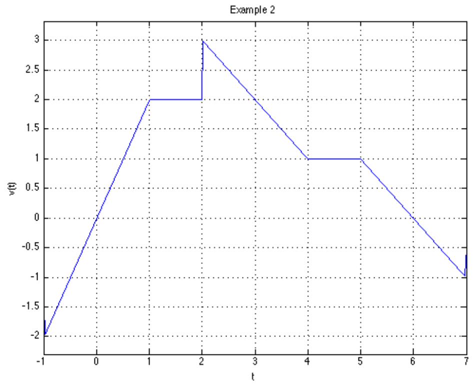


Fig. 23 Signal to be symthesized for Example 3.6

a) Express the voltage waveform $v(t)$ shown in Fig. 23 as a sum of unit step functions for the time interval $-1 < t < 7$ s

b) Using the result of 3.6(a), compute the derivative of $v(t)$ and sketch its waveform.

Lab Work

In the second lab we will solve the examples indicated in these examples.

By Dr Chris P. Jobling