

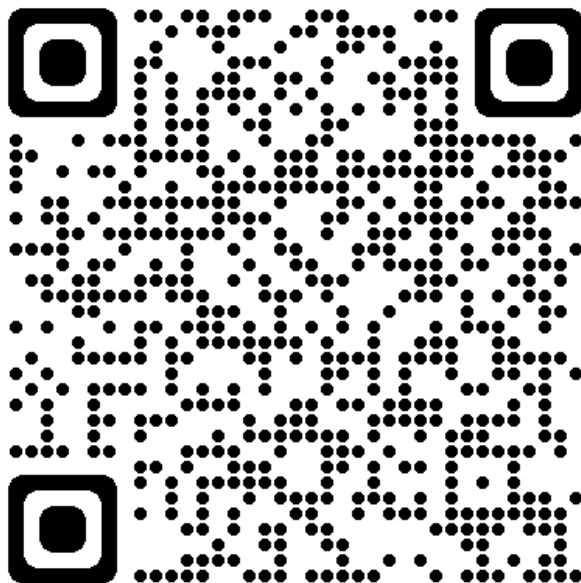
Unit 3.2: Properties and Eigenfunctions of Continuous-Time LTI Systems

Contents

- [Subjects to be covered](#)
- [Properties of Continuous-Time LTI Systems](#)
- [Eigenfunctions of Continuous-Time LTI Systems](#)
- [Examples 6: Properties of Continuous-Time LTI Systems](#)
- [Examples 7: Eigenfunctions of Continuous-Time LTI systems](#)
- [Summary](#)
- [Next Time](#)
- [References](#)

This section is based on Sections 2.3 and 2.4 of [[Hsu, 2020](#)]

Follow along at cpjobling.github.io/eg-150-textbook/lti_systems/lti2



Subjects to be covered

We continue our introduction to continuous-time LTI systems by considering:

- [Properties of Continuous-Time LTI Systems](#)

- [Eigenfunctions of Continuous-Time LTI Systems](#)
- [Examples 6: Properties of Continuous-Time LTI Systems](#)
- [Examples 7: Eigenfunctions of Continuous-Time LTI systems](#)

Properties of Continuous-Time LTI Systems

- [A. Systems with or without memory](#)
- [B. Causality](#)
- [C. Stability](#)

A. Systems with or without memory

Since the output $y(t)$ of a memoryless system depends only on the current input $x(t)$, then, if the system is also linear and time-invariant, this relationship can only be of the form

$$y(t) = Kx(t)$$

where K is a (gain) constant.

Thus the corresponding impulse response $h(t)$ is simply

$$h(t) = K\delta(t)$$

Therefore, if $h(t_0) \neq 0$ for $t_0 \neq 0$, then the continuous-time LTI system has memory.

B. Causality

Causal continuous-time LTI systems

As discussed in Section [Causal and Non-Causal Systems](#), a causal system does not respond to an input event until that event actually occurs.

Therefore, for a causal LTI system, we have

$$h(t) = 0 \quad t < 0$$

Applying the causality condition to the convolution integral, the output of a causal continuous-time LTI system is expressed as

$$y(t) = \int_0^{\infty} h(\tau)x(t - \tau) d\tau$$

Alternatively, applying the causality to the convolution integral (defined in Section [C. Convolution Integral](#))

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau$$

we have

$$y(t) = \int_{-\infty}^t x(\tau)h(t - \tau) d\tau$$

This equation shows that the only values of the input $x(t)$ used to evaluate the output $y(t)$ are those for $\tau \leq t$.

Causal signals

Based on the causality condition, any signal is called *causal* if

$$x(t) = 0 \quad t < 0$$

and is called *anticausal* if

$$x(t) = 0 \quad t > 0$$

Combining the definition of a causal signal with a causal signal, when the input $x(t)$ is causal, the output of a causal continuous-time LTI system is given by

$$y(t) = \int_0^t h(t - \tau)x(\tau) d\tau = \int_0^t x(\tau)h(t - \tau) d\tau$$

C. Stability

The BIBO (bounded-input/bounded-output) stability of an LTI system (Section [Stable Systems](#)) is readily ascertained by the impulse response. It can be shown (Example 6.x) that a continuous-time LTI system is BIBO stable if its impulse response is absolutely integrable; that is,

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

Eigenfunctions of Continuous-Time LTI Systems

In Chapter Systems and Classification of Systems (Example [Example 4.7](#)) we saw that the eigenfunctions of continuous-time LTI system represented by the complex exponentials e^{st} , with s a complex variable.

That is

$$\mathbf{T}\{e^{st}\} = \lambda e^{st}$$

where λ is the eigenvalue of \mathbf{T} associated with e^{st} .

Setting $x(t) = e^{st}$ in the convolution integral, we have

$$y(t) = \mathbf{T}\{e^{st}\}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$y(t) = \left[\int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau \right] e^{st}$$

$$y(t) = H(s) e^{st} = \lambda e^{st}$$

where

$$\lambda = H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$$

Thus, the eigenvalue of a continuous-time LTI system associated with the eigenfunction e^{st} is given by $H(s)$ which is a complex constant whose value is determined by the value of s via the equation

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau.$$

Note from the equation

$$y(t) = H(s) e^{st}$$

that $y(0) = H(s)$ (see ex:4_7).

Looking Ahead

The above results underline the definition of the Laplace transform and Fourier transform. The [Unit 4: Laplace Transforms and their Applications](#) will be discussed later in this course. The Fourier Transform will be introduced in **EG-247 Digital Signal Processing** next year.

Examples 6: Properties of Continuous-Time LTI Systems

Example 6.1

The signals in [Fig. 35](#) (a) and (b) are the input $x(t)$ and the output $y(t)$, respectively, of a certain continuous-time LTI system.

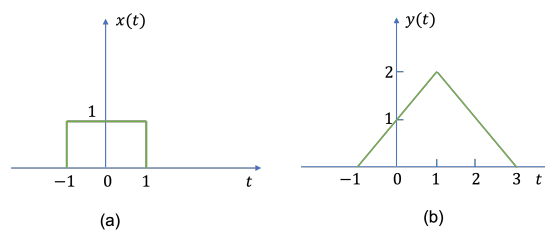


Fig. 35 Input and output signals for a continuous-time LTI system

Sketch the output to the following inputs:

(a) $x(t - 2)$;

(b) $\frac{1}{2}x(t)$.

For the answer, refer to the lecture recording or see solved problem 2.9 in in [\[Hsu, 2020\]](#).

Example 6.2

Consider a continuous-time LTI system whose step response is given by

$$y(t) = e^{-t}u_0(t)$$

Determine and sketch the output of the system to the input $x(t)$ shown in [Fig. 36](#).

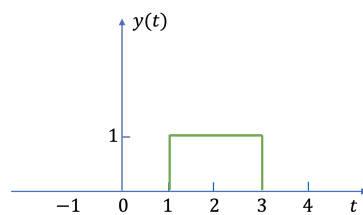


Fig. 36 Signal for Exercise 6.2

For the answer, refer to the lecture recording or see solved problem 2.2 in in [\[Hsu, 2020\]](#).

Example 6.3

Consider a continuous-time LTI system system described by

$$y(t) = \mathbf{T}\{x(t)\} = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau) d\tau$$

(a) Find and sketch the impulse response $h(t)$ of the system.

(b) Is this system causal?

For the answer, refer to the lecture recording or see solved problem 2.11 in in [\[Hsu, 2020\]](#).

Example 6.4

Let $y(t)$ be the output of a continuous-time LTI system with input $x(t)$. Find the output of the system if this input is $x'(t)$, where $x'(t)$ is the first derivative of $x(t)$.

For the answer, refer to the lecture recording or see solved problem 2.12 in in [\[Hsu, 2020\]](#).

Example 6.5

Verify the BIBO stability condition ([C. Stability](#)) for continuous-time LTI systems.

For the answer, refer to the lecture recording or see solved problem 2.13 in [\[Hsu, 2020\]](#).

Example 6.6

The system shown in [Fig. 37\(a\)](#) is formed by connecting two systems in cascade. The impulse responses of the two systems are $h_1(t)$ and $h_2(t)$, respectively, and

$$h_1(t) = e^{-2t}u_0(t) \quad h_2(t) = 2e^{-t}u_0(t)$$

(a) Find the impulse response $h(t)$ of the overall system shown in [Fig. 36\(b\)](#).

(b) Determine if the overall system is BIBO stable.

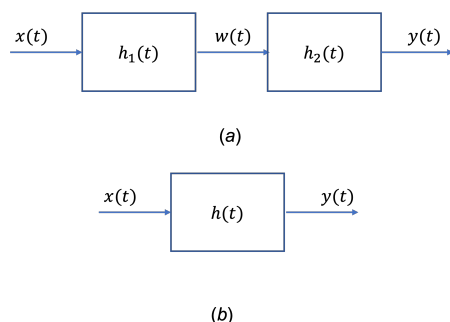


Fig. 37 A cascade connection of two continuous-time LTI systems

For the answer, refer to the lecture recording or see solved problem 2.14 in [\[Hsu, 2020\]](#).

Examples 7: Eigenfunctions of Continuous-Time LTI systems

Example 7.1

Consider a continuous-time LTI system with the input-output relation given by

$$y(t) = \int_{-\infty}^t e^{-(t-\tau)} x(\tau) d\tau$$

(a) Find the impulse response $h(t)$ of this system.

(b) Show that the complex exponential e^{st} is an eigenfunction of the system.

(c) Find the eigenvalue of the system corresponding to e^{st} using the impulse response $h(t)$ obtained in part (a).

For the answer, refer to the lecture recording or see solved problem 2.15 in [\[Hsu, 2020\]](#).

Example 7.2

Consider the continuous-time LTI system described by

$$y(t) = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau) d\tau$$

- (a) Find the eigenvalue of the system corresponding to the eigenfunction e^{st} .
- (b) Repeat part (a) by using the impulse response $h(t)$ of the system.

For the answer, refer to the lecture recording or see solved problem 2.16 in [[Hsu, 2020](#)].

Example 7.3

Consider a stable continuous-time LTI system with impulse response $h(t)$ that is real and even. Show that $\cos \omega t$ and $\sin \omega t$ are eigenfunctions of this system with the same eigenvalue.

For the answer, refer to the lecture recording or see solved problem 2.17 in [[Hsu, 2020](#)].

Summary

We have continued our introduction the continuous-time LTI systems by considering

- [Properties of Continuous-Time LTI Systems](#)
- [Eigenfunctions of Continuous-Time LTI Systems](#)

Take aways

Next Time

We will conclude our look at continuous-time LTI systems by considering

- [Unit 3.3: Systems Described by Differential Equations](#)

References

[Hsu20]([1,2,3,4,5,6,7,8,9](#)) Hwei P. Hsu. *Schaums outlines signals and systems*. McGraw-Hill, New York, NY, 2020. ISBN 9780071634724. Available as an eBook. URL: <https://www.accessengineeringlibrary.com/content/book/9781260454246>.

By Dr Chris P. Jobling

© Copyright Swansea University (2023).

This page was created by [Dr Chris P. Jobling](#) for [Swansea University](#).