Unit 4.7: Transfer Functions for Circuit Analysis

Contents

The preparatory reading for this section is Chapter 4.4 [Karris, 2012] which discusses transfer function models of electrical circuits. We have also adapted content from 3.6 The System Function from [Hsu, 2020].

Follow along at cpjobling.github.io/eg-150-textbook/laplace_transform/7/tf_for_circuits

Agenda

In this unit, we will explore how transfer functions introduced in Unit 4.6: Transfer Functions can be applied to the analysis of circuits.

- Transfer Functions for Circuits
- Examples 14

```
% Initialize MATLAB
clearvars
cd ../matlab
pwd
format compact
```

```
ans =

    '/Users/eechris/code/src/github.com/cpjobling/eg-150-
textbook/laplace_transform/matlab'
```

Transfer Functions for Circuits

When doing circuit analysis with components defined in the complex frequency domain, the ratio of the output voltage $V_{\mathrm{out}}(s)$ to the input voltage $V_{\mathrm{in}}(s)$ *under zero initial conditions* is of great interest.

This ratio is known as the *voltage transfer function* denoted $G_v(s)$:

$$G_v(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)}$$

Similarly, the ratio of the output current $I_{\mathrm{out}}(s)$ to the input current $I_{\mathrm{in}}(s)$ *under zero initial conditions*, is called the *current transfer function* denoted $G_i(s)$:

$$G_i(s) = \frac{I_{\mathrm{out}}(s)}{I_{\mathrm{in}}(s)}$$

In practice, the current transfer function is rarely used, so we will use the voltage transfer function denoted:

$$G(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)}$$

Examples 14

We will work through these and demonstrate the MATLAB solutions in class.

Example 14.1

Derive an expression for the transfer function $G(s)$ for the circuit shown in Fig. 67.

In this circuit R_g represents the internal resistance of the applied (voltage) source v_s, and R_L represents the resistance of the load that consists of R_L, L and C.

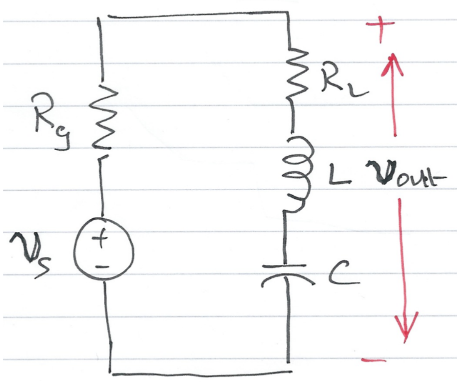


Fig. 67 Circuit for Example 14.1

Sketch of Solution for Example 14.1

- Replace $v_s(t)$, R_g, R_L, L and C by their transformed (*complex frequency*) equivalents: $V_s(s)$, R_g, R_L, sL and $1/(sC)$
- Use the *Voltage Divider Rule* to determine $V_{\text{out}}(s)$ as a function of $V_s(s)$
- Form $G(s)$ by writing down the ratio $V_{\text{out}}(s)/V_s(s)$

Switch to virtual whiteboard in OneNote.

```
Solve in OneNote
```

Worked solution for Example 14.1

Pencast: [ex6.pdf](ex6.pdf) - open in Adobe Acrobat Reader.

Answer for Example 14.1

$$G(s) = \frac{V_{\text{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

Example 14.2

Compute the transfer function for the op-amp circuit shown in Fig. 68 in terms of the circuit constants R_1, R_2, R_3, C_1 and C_2.

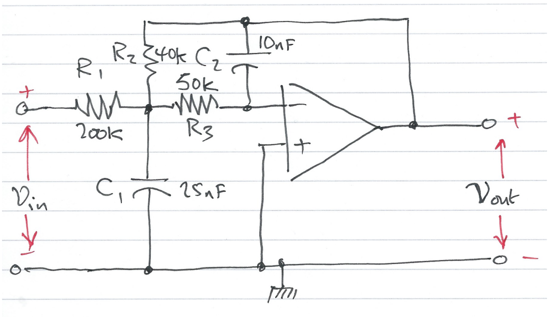


Fig. 68 OpAmp circuit for Example 14.2

Then replace the complex variable s with $j\omega$, and the circuit constants with their numerical values and plot the magnitude

$$|G(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|}$$

versus radian frequency ω rad/s.

Sketch of Solution for Example 14.2

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor R_3

Sketch of Solution for Example 14.2 (continued)

- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for $V_{\text{out}}(s)$ as a function of $V_{\text{in}}(s)$
- Form the reciprocal $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$

Switch to virtual whiteboard in OneNote.

```
Solve in OneNote
```

Answer for Example 14.2

$$G(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)} = \frac{-1}{R_1\left(\left(1/R_1 + 1/R_2 + 1/R_3 + sC_1\right)\left(sC_2R_3\right) + 1/R_2\right)} \quad (41)$$

Worked solution for Example 14.2

Pencast: [ex7.pdf](ex7.pdf) - open in Adobe Acrobat Reader.

Sketch of Solution for Example 14.2 (continued)

- Use MATLAB to calculate the component values, then replace s by $j\omega$.
- Compute $|G(j\omega)|$ and plot on log-linear "paper".

The Matlab Bit

Set up the symbols we will be using. In this case just the Laplace complex frequency s.

```
syms s
```

Now define the values of the components

```
R1 = 200*10^3;
R2 = 40*10^3;
R3 = 50*10^3;

C1 = 25*10^(-9);
C2 = 10*10^(-9);
```

Define the transfer function derived from analysis (Eq. `eg:ex14.2`)

```
den = R1*((1/R1+ 1/R2 + 1/R3 + s*C1)*(s*R3*C2) + 1/R2)
```

```
den =
```

```
100*s*((7555786372591433*s)/302231454903657293676544 + 1/20000) + 5
```

Simplify coefficients of s in the denominator. Note `sym2poly` converts a symbolic polynomial with numerical coeficients into a MATLAB polynomial.

```
format long
denG = sym2poly(den)
```

```
denG =
   0.000002500000000   0.005000000000000   5.000000000000000
```

Now define the denominator

```
numG = -1;
```

Plot the frequency response

For convenience, define coefficients a and b:

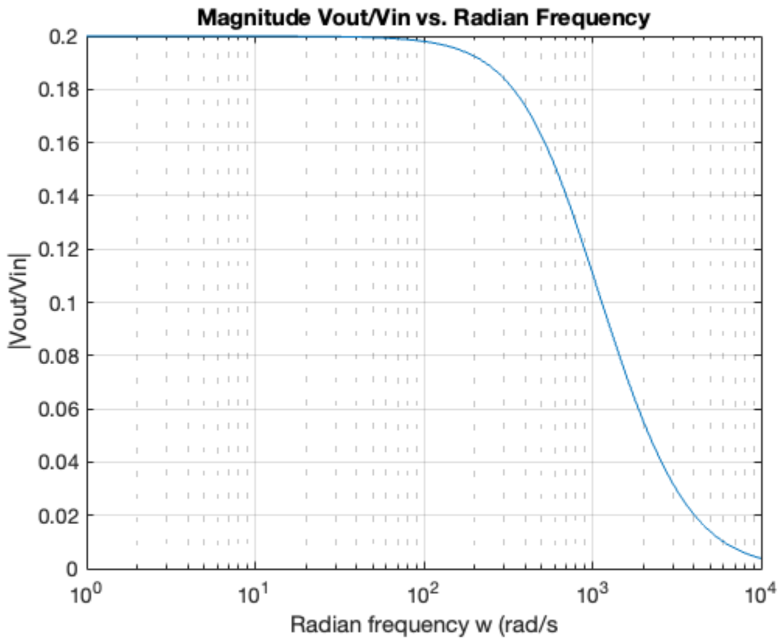
```
a = denG(1);
b = denG(2);
```

$$G(j\omega) = \frac{-1}{a\omega^2 - jb\omega + 5}$$

```
w = 1:10:10000;
Gw = -1./(a*w.^2 - j.*b.*w + denG(3));
```

Plot $|G(j\omega)|$ against ω on log-lin "graph paper".

```
semilogx(w, abs(Gw))
xlabel('Radian frequency w (rad/s')
ylabel('|Vout/Vin|')
title('Magnitude Vout/Vin vs. Radian Frequency')
grid
```



Note that this is a low-pass filter. Sinusoids at low frequencies are passed with a gain of 0.2. For frequencies above around 100 ra/s, the filter starts to reduce the attenuation of the passed signal. At 10,000 rad/s, the attenuation is 1/10 of the attenuation at 1 rad/s.

Summary

In this unit, we will explored how transfer functions introduced in Unit 4.6: Transfer Functions can be applied to the analysis of circuits.

- Transfer Functions for Circuits
- Examples 14

Take Away

The ratio of the output voltage $V_{\mathrm{out}}(s)$ to the input voltage $V_{\mathrm{in}}(s)$ under zero initial conditions is of great interest. We call this ratio the *voltage transfer function*

$$G_v(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)}$$

We can consider other ratios such as the *current transfer function*

$$G_i(s) = \frac{I_{\mathrm{out}}(s)}{I_{\mathrm{in}}(s)}$$

but in practice this is rarely used.

Next time

We explore the facilties provided by other toolboxes in MATLAB, most notably the *Control Systems Toolbox* and the simulation tool Simulink in Unit 4.8: Computer-Aided Systems Analysis and Simulation. We will also look at some of the problems you have studied in **EG-152 Analogue Design** hopefully confirming some of the results you have obbserved in the lab.

- Unit 4.8: Computer-Aided Systems Analysis and Simulation

References

[Hsu20] Hwei P. Hsu. *Schaums outlines signals and systems*. McGraw-Hill, New York, NY, 2020. ISBN 9780071634724. Available as an eBook. URL: https://www.accessengineeringlibrary.com/content/book/9781260454246.

[Kar12] Steven T. Karris. *Signals and systems with MATLAB computing and Simulink modeling*. Orchard Publishing, Fremont, CA., 2012. ISBN 9781934404232. Library call number: TK5102.9 K37 2012. URL: https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197.

Matlab Solutions

For convenience, single script MATLAB solutions to the examples are provided and can be downloaded from the accompanying MATLAB folder.

- ex:14.2 [example_14.2.mlx]