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This Matlab Notebook presents an analytical procedure for phase-lead design. It is based on Section 7.8 of Phillips and Harbor “*Feedback Control Systems,*” Prentice Hall, 1988[[1]](#footnote-1). For the procedure it is convenient to write the compensator transfer function as

 (1)

In this procedure we choose , , and  such that given , the equation

 (2)

is satisfied; that is we are designing a compensator that places a root of the closed-loop characteristic equation at .

In equation () we have four unknowns, including *K*, and only two relationships (magnitude and phase) that must be satisfied. Hence, we can arbitrarily assign values to two of the unknowns. *K* is easily eliminated since



so if we assume that  for the design procedure we eliminate one of the unknowns. The other unknown that can be eliminated is  which can be seen to be the DC gain of the compensator. Its value can therefore be chosen to satisfy the steady-state error requirements of the design and we need only to determine values for  and .

The design proceeds as follows. First, we express the desired closed loop pole position

 (3)

and

 (4)

Then the design equations (derived in Appendix B of Phillips and Harbor, 1988) are

 (5)

Given , , and the desired closed-loop pole location , () gives the remaining compensator coefficients. This procedure places a closed-loop pole at ; however, the locations of the remaining poles are unknown and may be unsatisfactory. In fact, some may be unstable!

For the case that is either  or , equations () must be modified to give the single equation

 (6)

where the plus sign applies to the case  and the minus sign applies to . For this case, the value of either  or  can also be assigned. An example is now given to illustrate the procedure.

## Example

[This example is presented using actual Matlab commands. The example is the satellite attitude control problem given in lectures, but this document can be used to solve similar problems if use is made of the Matlab Notebook interface. All that is necessary is to change the definitions for the plant transfer function , the feedback transfer function , the desired closed-loop pole position  and the DC gain . The command *Evaluate M-book* from the *Notebook* menu should then be executed to recalculate the values]

### Definitions (change these to change design)

The plant transfer function is :

G = tf([1],[1 0 0]);

The feedback transfer function is  :

H= tf(1,1);

So *G*(*s*)*H*(*s*) is:

GH=series(G,H)

Transfer function:

1

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s^2

The desired closed-loop poles are:

s1 = -2 + 2j;

s1 =

-2.0000 + 2.0000i

Now the DC gain of this type 2 system will be:



For the purpose of illustration let us arbitrarily take a value of :

a0 = 8/3

a0 =

2.6667

### Calculations (shouldn’t need to change these commands)

Polar form of 

m\_s1=abs(s1), p\_s1 = (angle(s1)\*180/pi + 90) % degrees

m\_s1 =

2.8284

p\_s1 =

225

Transfer function evaluated at in polar form:

#### **[numGH,denGH] = tfdata(GH,'v');**

#### **GHs1=polyval(numGH,s1)/polyval(denGH,s1)**

GHs1 =

0 + 0.1250i

Magnitude:

mGHs1=abs(GHs1)

mGHs1 =

0.1250

Phase:

pGHs1=angle(GHs1)\*180/pi - 180% degrees

pGHs1 =

-90

Hence angles are:

beta = p\_s1\*pi/180

psi = pGHs1\*pi/180 % radians

beta =

3.9270

psi =

-1.5708

From ()

a1 = (sin(beta) + a0\*mGHs1\*sin(beta - psi))/(m\_s1\*mGHs1\*sin(psi))

a1 =

2.6667

b1 = (sin(beta + psi) + a0\*mGHs1\*sin(beta))/(-(m\_s1)\*sin(psi))

b1 =

0.1667

Compensator is therefore given by

numD = [a1, a0], denD = [b1, 1]

numD =

2.6667 2.6667

denD =

0.1667 1.0000

which in normal form:



has

Kc = a1/b1, z0 = a0/a1, p0 = 1/b1

Kc =

16.0000

z0 =

1

p0 =

6.0000

Now make a transfer function

D = tf(Kc\*[1, z0],[1, p0])

Transfer function:

16 s + 16

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s + 6

## Evaluation of Design

### Open loop transfer function:

Go = series(D,GH)

Transfer function:

16 s + 16

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s^3 + 6 s^2

### Root locus:

rlocus(Go)



### Closed-loop transfer function:

DG=series(D,G)

Transfer function:

16 s + 16

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s^3 + 6 s^2

Gc=feedback(DG,H)

Transfer function:

16 s + 16

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s^3 + 6 s^2 + 16 s + 16

### Step response:

step(Gc)



As an exercise, you should examine the effect of designing for a range of DC gains in the range .

1. The proofs of the formulae given are derived in Appendix B of that text. [↑](#footnote-ref-1)