Statistical Inference Project 1

Exploring Asymptotics of the Exponential Distribution

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In this document I will investigate the exponential distribution in R and compare it with the Central Limit Theorem.

The Central Limit Theorem states that given a distribution with mean mu and variance σ^2, the sampling distribution of the mean approaches a normal distribution with mean mu and variance $\frac{\sigma^2}{N}$. Counter-intuitively, the sampling distribution of the mean approaches a normal distribution regardless of the shape of the original distribution.

```
library(ggplot2) #load the required packages
```

Let's set the parameters:

```
lambda <- 0.2 #the rate of the exponential distribution
mu <- 1/lambda #the mean of the exponential distribution
n <- 40 #numbers of exponential numbers to be drawn
nsim <- 1000 #numbers of simulations to be performed
```

Sample and Theoretical Mean

The theoretical mean of an exponential distribution with rate λ is:

$\mu = \frac{1}{\lambda}$

```
mu <- 1/lambda
mu
```

```
## [1] 5
```

Let $\bar{X}$ be the sample mean of 1000 sets of 40 random exponential distributions with $\lambda = 0.2$

```
set.seed(144) #set the seed for reproducibility

expDist <- matrix(data= NA, nrow = nsim, ncol = n) #empty matrix with 1000 rows and 40 columns
for(i in 1:nsim) {#for each row of the matrix,
    expDist[i,] <- rexp(n = n, rate = lambda) #draw 40 random numbers from the exponential distribution
}
means <- rowMeans(expDist)
xbar <- mean(means)
xbar
```

```
## [1] 4.998169
```

$\bar{X}$ and μ are almost identical, as the Central Limit Theorem predicts.

Sample and Theoretical Variance

The expected variance of our sample of 1000 means is $\frac{\frac{1}{\lambda^2}}{40} = 0.625$.

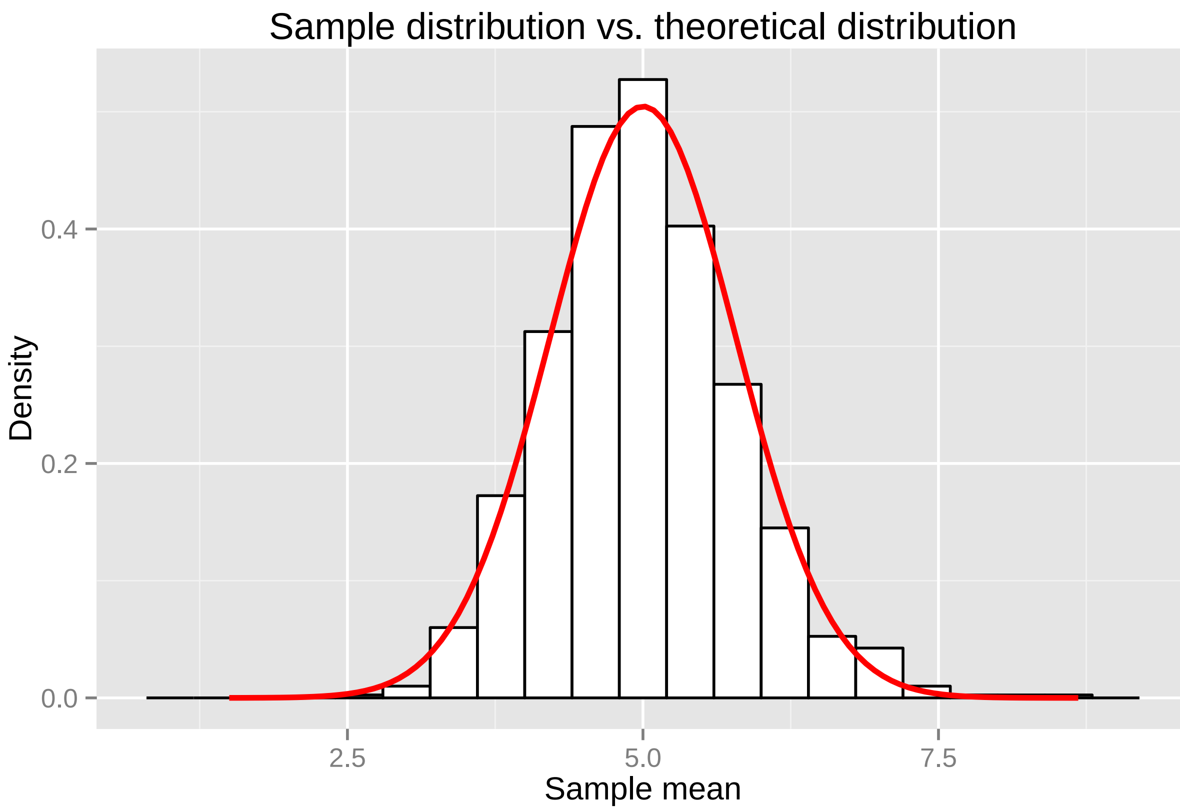
```r
var(means)
```

```
## [1] 0.645807
```

The theoretical and the sample variance are very similar. This matches the expectations of the Central Limit Theorem.

Distribution

The Central Limit Theorem states that the means of our 1000 simulations should approximate a distribution: $N(\frac{1}{0.2}, \frac{\frac{1}{0.2}}{\sqrt{40}})$. The histogram



Again, our sample and theoretical distributions are very close to one another.