# Milk and Money Case Study

*Chaman Preet Kaur and Chris Atterbury*

*October 5, 2015*

## Executive Summary

This section is to be written last and contains a short (approx 250 words) summary of the case study as a whole.

## Background

The financial success of dairy farms depends on the price of their milk. The case study helps in understanding how milk prices are decided by the federal and state agencies, how payments are made and focuses on different options that farm owners can explore to hedge the price risk against market fluctuations. The simplest approach is to purchase an option or a futures contract from the Chicago Mercantile Exchange on commodity futures that can ensure minimum floor value for their produce. An option on a futures contract is the right, but not the obligation, to buy or sell a futures contract at a specific price on or before an expiration date.

The price a farm receives for its milk depends on various factors. What the dairy producers actually receive per hundredweight is called the mailbox price. The farmer (Gerard in this case study) needs to determine which of the options available for trade offer the best hedge for his own milk price or his own mailbox price. The federal order groups milk into four classes: Class I (defined as California Class 1), Class II (California Class 2 and Class 3), Class III (California Class 4b), and Class IV (California Class 4a). Options contracts allow hedgers to establish a price floor without giving up the opportunity to benefit from favorable price changes in the future. Buyers pay a price for the option called the premium up front. There are two different types of options:

- Put Options. The right to sell a futures contract at a certain price.
- Call Options. The right to buy a futures contract at a certain price.

The price at which the buyer has the right to buy or sell a futures contract is known as the strike price or exercise price of the option. Payoff options are described in the following ways:

- At the money. When the price of the underlying security (roughly) equals the strike price.
- In the money. When holders do gain a profit if the option is exercised. For a put option, the strike price is more than the market price of the underlying security. For a call option, it is when the strike price is less than the market price of the underlying security.
- Out of the money. When holders do not gain a profit if the option is exercised, so they allow the option to expire unused. The only loss is that of the initial premium and the trading costs of the initial transaction.

Here we use about three and a half years of data from a dairy farmer named Gerard to determine how he can purchase futures options that will be in the money if the mailbox price falls below $12.50.

## Methods

### Initial Considerations

To help Gerard with his put option deliberations, we must analyze the data that he has collected. We have feature columns *Month*, *Mailbox*, *Class.IV*, *Class.III*, *Butter*, *NFDM*. Please refer to **Codeblock 1** in the

*Appendix* to see the first six rows of this data set to get a feel for it. We will not use the *Month* column since we are not doing time series analysis, but simply viewing the data as a collection of measurements of the other feature columns. Also, we must remove the dollar signs from each of the other columns so that we can treat them as numeric columns. Please see **Codeblock 2** in the *Appendix* for details on this approach, including the code itself. With the preprocessing done, we can evaluate the data.

### Determine the Predictor

Gerard plans to purchase put options in just one commodity; the one that is most closely related to his mailbox price. To determine this, we will look at plots of each variable against each other (see **Figure 1** in the *Appendix*), the correlation matrix (see **Codeblock 3** in the *Appendix*), and simple linear regression (SLR) models of each of the potential predictors. We will evaluate the $R^2$ value, mean squared error ($MSE$), and distribution of the error terms (see **Codeblock 4** in the *Appendix*). The results obtained through SLR models were also in conjugation with the p-values of individual variables in the full multiple linear regression model (see **Codeblock 5** in the *Appendix*). Please see the *Results* section for an explanation of what we found.

### Choose put options

We want to choose a put option that leaves Gerard 95% certain that he will be in the money if the mailbox price falls below $12.50. To obtain this certainty, we construct a 95% Confidence Interval and choose the upper bound as our target mailbox price (see **Codeblock 6** in the *Appendix*). Using our SLR model for the commodity most closely related to the mailbox price, we determine the strike price needed for the commodity(see **Codeblock 7** in the *Appendix*). See the *Results* section for an explanation of what we found.

### Final Adjustments for Fees

Premium and trading fees are a part of purchasing dairy futures options. Thus, we finish our work with Gerard by accounting for those costs when determining the desired strike price. Since those fees are subtracted out right away, we simply increase our strike price so that, after the fees are subtracted, we still have the price of our commodity guaranteed at a high enough level to be in the money if the mailbox price falls below $12.50.

With our methods explained, we look at the results.

## Results

(Details the actual model and how it worked. This needs to do all of the leg work so that the Conclusions section can roll through the answers to the questions.)

While looking at individual SLR models **Class.III** milk displays the highest **Multiple** $R^2$ value of **0.9268**, followed by **Class.IV** milk of **0.7121**, **Butter** of **0.6178** and **NFDM** of **0.2961** (see **Codeblock 4** in the *Appendix*). The results are in agreement with the correlation matrix and the pairs plot (see **Figure 1** in the *Appendix*) which show highest **correlation** between Mailbox and Class.III milk which is **0.962720** (see **Codeblock 3** in the *Appendix*). These results are again confirmed by the full multiple linear regression model (MLR) output (see **Codeblock 5** in the *Appendix*). The **p-value** for Class.III milk in the MLR model was the **lowest** **(**$2.73e^{-}11$**)** and the most significant. All these approaches confirmed heirarchical order that Class.III milk was the most related to the mailbox price followed by Class.IV, Butter and NFDM.

In an endevour to choose the correct strike price, the construction of 95% confidence interval using $\hat{Y}h$ = 12.50 yielded **(12.48889, 12.51111)** as the lower bound and upper bound values. We use the **upper**

**bound** as our **target mailbox price** making Gerard 95% sure that he is in the money if mailbox price falls below \$12.50. Using mailbox price as 12.51111, we find the Class.III milk price from our previous SLR model developed while calculating the most closely related commodity to mailbox. This yields a **Class.III** price of **\$13.17**.

The **premium fee** and **transactional cost** are **constants** and can be added to the derieved strike price in order to accomodate for extra cost while keeping the model in the money. Hence, we derieve at final strike price by adding the extra premium and transactional costs.

# Conclusions

Integrate the answers to the questions as seam lessly as possible. The goal is that everything is built up to this point so that little justification is needed and other general conclusions can be included.

# Appendix

## Codeblocks

**Codeblock 1**

```
milk <- read.csv("41330727.csv", header = TRUE, stringsAsFactors = FALSE)
head(milk)
```

```
##          Month Mailbox Class.IV Class.III  Butter    NFDM
## 1  January-04  $11.64   $10.97    $11.61 $1.4320 $0.8366
## 2 February-04  $12.25   $12.21    $11.89 $1.7132 $0.8413
## 3    March-04  $14.31   $14.10    $14.49 $2.1350 $0.8518
## 4    April-04  $17.04   $14.57    $19.66 $2.2204 $0.8808
## 5      May-04  $17.29   $14.50    $20.58 $2.0363 $0.9050
## 6     June-04  $16.14   $13.72    $17.68 $1.9300 $0.9188
```

**Codeblock 2**

In **Codeblock 1** we anticipated the need to convert the numeric feature columns. When being read into the data.frame `milk`, those columns are normally interpretated as factors because of the dollar signs. By reading them in as strings, we can use a regular expression to remove all of the dollar signs, then convert the columns from character to numeric types. See below for the code doing this.

```
for (i in 2:length(milk)) {
  milk[[i]] <- gsub("\\$", "", milk[[i]])
  milk[[i]] <- as.numeric(milk[[i]])
}
```

**Codeblock 3**

```
cor(milk[2:length(milk)])
```

```
##              Mailbox  Class.IV Class.III     Butter       NFDM
## Mailbox   1.0000000 0.8438798 0.9627200  0.7859903  0.1720781
## Class.IV  0.8438798 1.0000000 0.8155221  0.5512816  0.5608375
## Class.III 0.9627200 0.8155221 1.0000000  0.7231414  0.2232973
## Butter    0.7859903 0.5512816 0.7231414  1.0000000 -0.2901637
## NFDM      0.1720781 0.5608375 0.2232973 -0.2901637  1.0000000
```

**Codeblock 4**

```
lm_classiv <- lm(Mailbox ~ Class.IV, data = milk)
print (summary(lm_classiv))
```

```
##
## Call:
## lm(formula = Mailbox ~ Class.IV, data = milk)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.2378 -0.6067 -0.1703  0.5040  2.4422
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.25861    1.21771   1.034    0.308
## Class.IV     0.93719    0.09541   9.822 4.24e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9673 on 39 degrees of freedom
## Multiple R-squared:  0.7121, Adjusted R-squared:  0.7048
## F-statistic: 96.48 on 1 and 39 DF,  p-value: 4.24e-12
```

```
lm_classiii <- lm(Mailbox ~ Class.III, data = milk)
print (summary(lm_classiii))
```

```
##
## Call:
## lm(formula = Mailbox ~ Class.III, data = milk)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.02598 -0.30017 -0.01168  0.26231  1.16295
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.18135    0.49832   4.377 8.72e-05 ***
## Class.III    0.78400    0.03527  22.226  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4877 on 39 degrees of freedom
## Multiple R-squared:  0.9268, Adjusted R-squared:  0.925
## F-statistic:   494 on 1 and 39 DF,  p-value: < 2.2e-16
```

```r
lm_butter <- lm(Mailbox ~ Butter, data = milk)
print (summary(lm_butter))
```

```
##
## Call:
## lm(formula = Mailbox ~ Butter, data = milk)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9798 -0.6459 -0.1431  0.6679  3.3427
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.5167     0.9742   5.663 1.52e-06 ***
## Butter        5.0459     0.6355   7.940 1.14e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.115 on 39 degrees of freedom
## Multiple R-squared:  0.6178, Adjusted R-squared:  0.608
## F-statistic: 63.04 on 1 and 39 DF,  p-value: 1.14e-09
```

```r
lm_nfdm <- lm(Mailbox ~ NFDM, data = milk)
print (summary(lm_nfdm))
```

```
##
## Call:
## lm(formula = Mailbox ~ NFDM, data = milk)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9163 -1.3683  0.0148  0.8255  4.3243
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   11.822      1.228   9.625 7.48e-12 ***
## NFDM           1.264      1.159   1.091    0.282
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.776 on 39 degrees of freedom
## Multiple R-squared:  0.02961,    Adjusted R-squared:  0.004729
## F-statistic:  1.19 on 1 and 39 DF,  p-value: 0.282
```

**Codeblock 5**

```r
lm_full <- lm(Mailbox ~ ., data = milk[-1])
print (summary(lm_full))
```

```
##
## Call:
```

```
## lm(formula = Mailbox ~ ., data = milk[-1])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.61028 -0.22891  0.04805  0.17654  1.10697
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.13895    0.48970   2.326  0.02578 *
## Class.IV     0.39648    0.11167   3.550  0.00109 **
## Class.III    0.53240    0.05632   9.453 2.73e-11 ***
## Butter       0.46185    0.54053   0.854  0.39851
## NFDM        -1.12574    0.58859  -1.913  0.06378 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3713 on 36 degrees of freedom
## Multiple R-squared:  0.9608, Adjusted R-squared:  0.9565
## F-statistic: 220.8 on 4 and 36 DF,  p-value: < 2.2e-16
```

**Codeblock 6**

The code below uses the following equation to determine the upper bound of the 95% Confidence Interval: $\hat{Y}_h + t_{(1-\alpha/2;n-2)} s\{\hat{Y}_h\}$

```
s_2 <- anova(lm_classiii)[["Mean Sq"]][[2]]
x_bar <- mean(milk[["Class.III"]])
x_h <- (12.50 - coef(lm_classiii)[[1]]) / coef(lm_classiii)[[2]]
num <- (x_h - x_bar) ^ 2
denom <- sum((milk[["Class.III"]] - x_bar) ^ 2)
val <- 1 / 41 + num / denom
s <- s_2 * val
mbox_upper <- 12.50 + s * qt(0.95, 39)
print (mbox_upper)
```

```
## [1] 12.51111
```

**Codeblock 7**

```
b_0 <- coef(lm_classiii)[[1]]
b_1 <- coef(lm_classiii)[[2]]
classiii_price <- (mbox_upper - b_0) / b_1
print (classiii_price)
```

```
## [1] 13.1758
```

**Codeblock 7**

```
lm_lclassiii <- lm(Mailbox ~ log(Class.III), data = milk)
print (summary(lm_lclassiii))
```

```
##
## Call:
## lm(formula = Mailbox ~ log(Class.III), data = milk)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.7134 -0.2889 -0.0434  0.1699  1.0264
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -17.1652     1.1587  -14.81   <2e-16 ***
## log(Class.III)  11.5397     0.4407   26.18   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4183 on 39 degrees of freedom
## Multiple R-squared:  0.9462, Adjusted R-squared:  0.9448
## F-statistic: 685.6 on 1 and 39 DF,  p-value: < 2.2e-16
```

```r
lm_classiiil <- lm(log(Mailbox) ~ Class.III, data = milk)
print (summary(lm_classiiil))
```

```
##
## Call:
## lm(formula = log(Mailbox) ~ Class.III, data = milk)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.10438 -0.03004  0.00229  0.02740  0.09311
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.745586   0.045550   38.32   <2e-16 ***
## Class.III   0.058742   0.003224   18.22   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04458 on 39 degrees of freedom
## Multiple R-squared:  0.8949, Adjusted R-squared:  0.8922
## F-statistic: 331.9 on 1 and 39 DF,  p-value: < 2.2e-16
```

## Figures

**Figure 1**

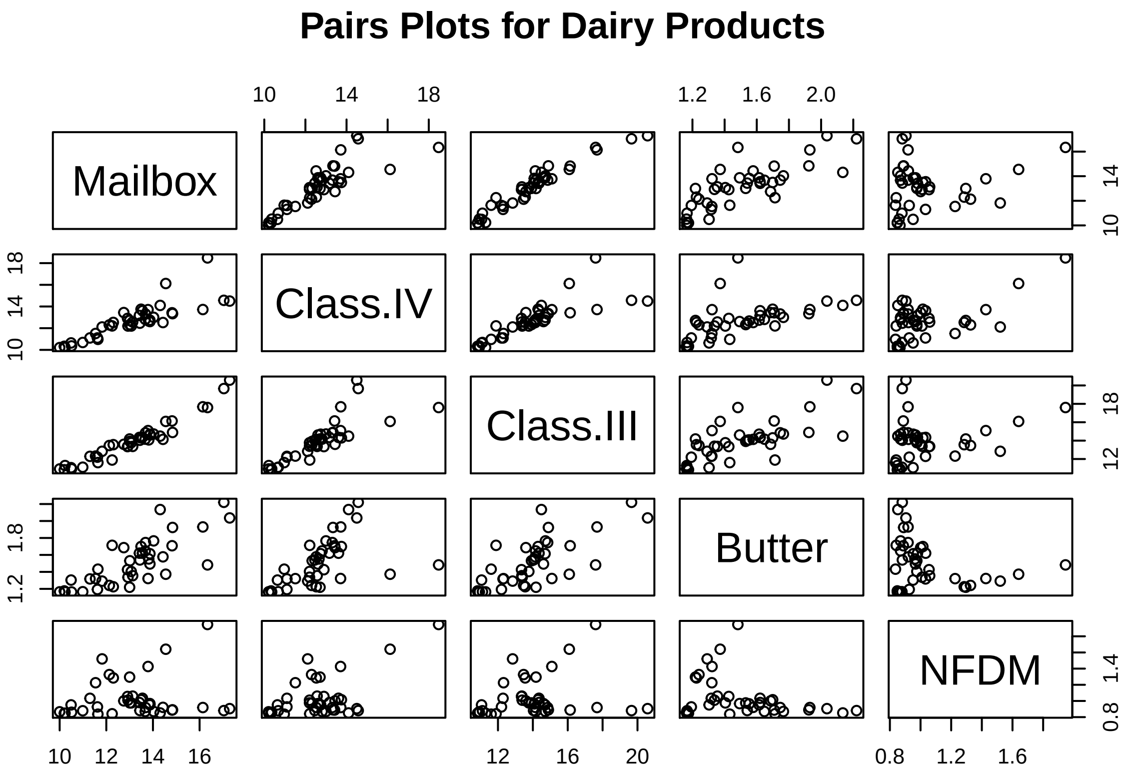



**Figure 2**

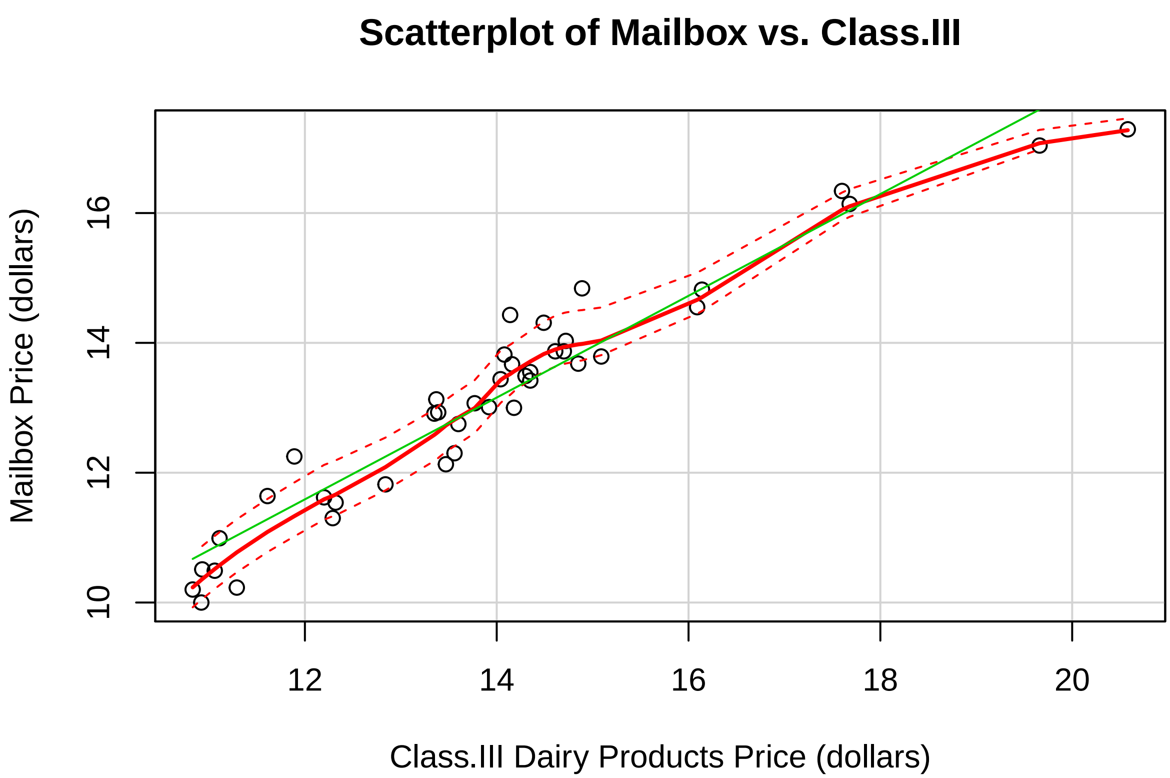
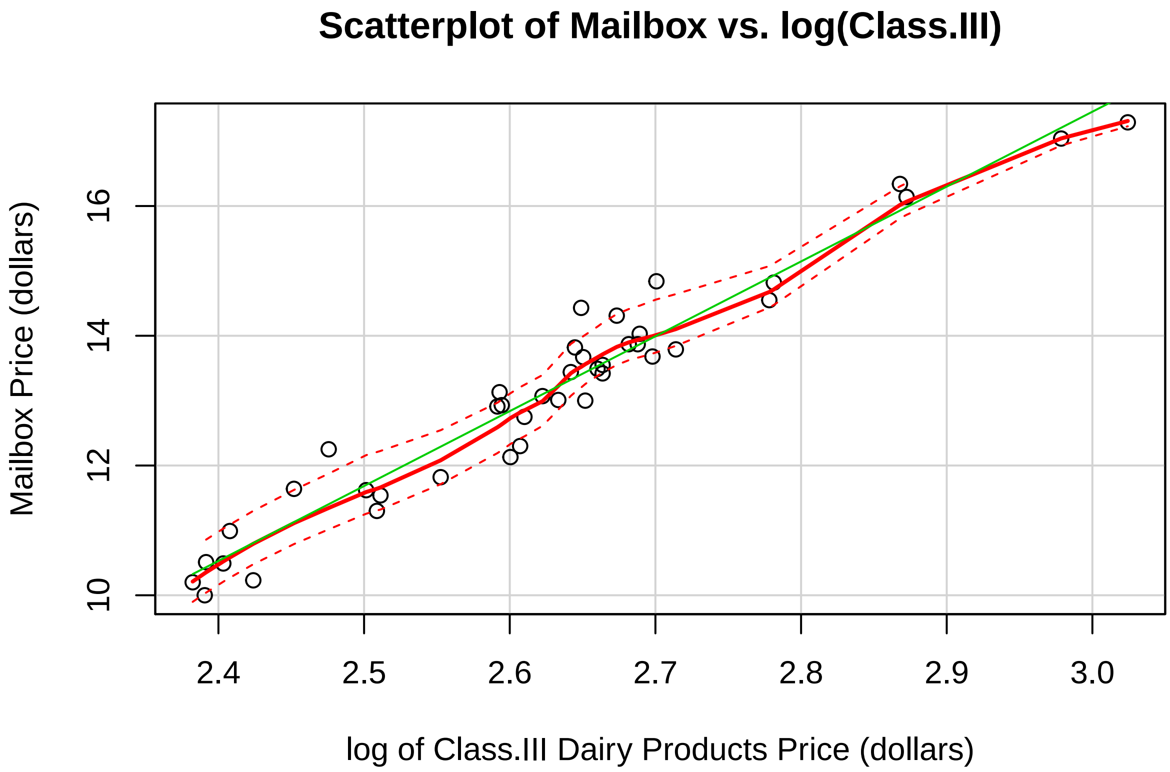



**Figure 3**


Scatterplot of Mailbox vs. log(Class.III)
Mailbox Price (dollars)
10
12
14
16
2.4
2.5
2.6
2.7
2.8
2.9
3.0
log of Class.III Dairy Products Price (dollars)