Chapter 1 - Newton's Laws of Motion

$$\mathbf{F} = m\ddot{r} \qquad x = r\cos\phi \qquad y = r\sin\phi$$

$$r = \sqrt{x^2+y^2} \quad \phi = \arctan(y/x) \quad \dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\frac{d\hat{\mathbf{r}}}{dt}$$

$$\mathbf{v} \equiv \dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\phi}\hat{\phi} \quad v_r = \dot{r}$$

$$v_\phi = r\dot{\phi} = r\omega \quad \mathbf{a} \equiv \ddot{\mathbf{r}} = \frac{d}{dt}\dot{\mathbf{r}} = \frac{d}{dt}(\dot{r}\hat{\mathbf{r}} + r\dot{\phi}\hat{\phi})$$

$$\frac{d\hat{\phi}}{dt} = -\dot{\phi}\hat{\mathbf{r}} \qquad \mathbf{a} = -r\dot{\phi}^2\hat{\mathbf{r}} + r\ddot{\phi}\hat{\phi} = -r\omega^2\hat{\mathbf{r}} + r\alpha\hat{\phi}$$

Chapter 6 - Calculus of Variations

$$S = \int_{x1}^{x2} f[y(x), y'(x), x]dx$$

Stationary wrt variations of path iff

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0 \qquad \frac{\partial f}{\partial y} = 0 \implies \frac{\partial f}{\partial \dot{y}} = c$$

$$S = \int_{x1}^{x2} f[x(t), y(t), x'(t), y'(t), t]dx \implies$$

$$\frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial f}{\partial x'} \qquad \frac{\partial f}{\partial y} = \frac{d}{dt}\frac{\partial f}{\partial y'}$$

Chapter 7 - Lagrange's Equations

$$\mathcal{L} = T - U \qquad \frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

Generalized Momentum

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

Chapter 8 - Two Body Central Force Problems

$$\vec{r} = \vec{r_1} - \vec{r_2} \qquad \mu = \frac{m_1 m_2}{M}$$

$$M = m_1 + m_2 \qquad F_g = \frac{GM\mu}{r^2}$$

$$U_{eff}(r) = U(r) + U_{ef}(r) = U(r) + \frac{\ell^2}{2\mu r^2}$$

$$\mu\ddot{\vec{r}} = -\frac{d}{dr}U_{eff}(r) \qquad K = K_{cm} + K_{rel} = \frac{1}{2}(M\dot{R}^2 + \mu\dot{r}^2)$$

Transformed Radial Equation

$$u = \frac{1}{r} \qquad u''(\phi) = -u(\phi) - \frac{\mu}{\ell^2 u(\phi)^2}F$$

Kepler Orbits

$$r(\phi) = \frac{c}{1+\epsilon\cos(\phi)} \qquad \gamma = Gm_1m_2 \qquad c = \frac{\ell}{\mu\gamma}$$

$$E = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2 - 1) \qquad r_{max}\frac{c}{(1-\epsilon)} \qquad r_{min} = \frac{c}{(1+\epsilon)}$$

$$t = 2\pi\sqrt{\frac{r^3}{GM}}$$

Chapter 9 - Rotating Reference Frames

$$m\ddot{\vec{r}} = mg_0 + 2m\dot{\vec{r}} \times \vec{\Omega} + m\left(\vec{\Omega} \times \vec{r}\right)\vec{\Omega}$$

Math Stuff - Coordinate Systems

Cylindrical

$$x = s\cos\phi \qquad y = s\sin\phi \qquad z = z$$

$$s = \sqrt{x^2+y^2} \qquad \phi = \arctan\frac{y}{x} \qquad z = z$$

$$\hat{x} = \cos\phi\hat{s} - \sin\phi\hat{\phi} \qquad \hat{y} = \sin\phi\hat{s} - \cos\phi\hat{\phi} \qquad \hat{z} = \hat{z}$$

$$\hat{s} = \cos\phi\hat{x} + \sin\phi\hat{y} \qquad \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y} \qquad \hat{z} = \hat{z}$$

Polar

$$x = sr\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2} \quad \phi = \arctan\frac{y}{x} \quad \theta = \arctan\frac{\sqrt{x^2+y^2}}{z}$$

$$\hat{x} = \sin\theta\cos\phi\hat{r} - \sin\phi\hat{\phi} + \cos\theta\cos\phi\hat{\theta}$$

$$\hat{y} = \sin\theta\sin\phi\hat{r} + \cos\phi\hat{\phi} + \cos\theta\sin\phi\hat{\theta}$$

$$\hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$$

$$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$$

$$\hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y} \quad \hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z}$$